CBCS Scheme

USN | | | | | | | | | | | 17MAT11

First Semester B.E. Degree Examination, Dec.2017/Jan.2018

Engineering Mathematics – I

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\cos x \cos 2x$. (06 Marks)

b. Find the angle between the curves $r = a\log\theta$, $r = \frac{a}{\log\theta}$. (07 Marks)

c. Find the radius of curvature of the curve $r = a(1+\cos\theta)$. (07 Marks)

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)

b. With usual notations prove that the pedal equation in the form $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. (07 Marks)

c. Find the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ at the point (a, 0). (07 Marks)

Module-2

3 a. Find the Taylor's series of $\log x$ in powers of $(x-1)$ upto fourth degree terms. (06 Marks)

b. If $U = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = \sin 2U$ by using Euler's theorem. (07 Marks)

c. If $U = x+3y^2$, $V = 4x^2yz$, $W = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point (1, -1, 0). (07 Marks)

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$. (06 Marks)

b. Find the Maclaurin's expansion of $\log(\sec x)$ upto x^4 terms. (07 Marks)

c. If $z = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$, prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. (07 Marks)

Module-3

5 a. A particle moves along the curve $\bar{r} = (t^3-4t)\hat{i} + (t^2+4t)\hat{j} + (8t^2-3t^3)\hat{k}$. Find the velocity and acceleration vectors at time t and their magnitudes at t = 2. (06 Marks)

b. If $\bar{f} = (x+y+1)\bar{i} + \bar{j} - (x+y)\bar{k}$, prove that $\bar{f}.\text{curl}\,\bar{f} = 0$. (07 Marks)

c. Prove that $\text{div}(\text{curl}\,\overline{A}) = 0$. (07 Marks)

OR

6 a. A particle moves along the curve $\bar{r} = 2t^2\bar{i} + (t^2 - 4t)\hat{j} + (3t-5)\bar{k}$. Find the components of velocity and acceleration along $\bar{i} - 3\bar{j} + 2\bar{k}$ at t = 2. **(06 Marks)**

b. If $\bar{f} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find div $\bar{f}$ and curl $\bar{f}$. **(07 Marks)**

c. Prove that $\text{curl}(\text{grad}\,\phi) = 0$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}} dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$. **(07 Marks)**

c. Find the orthogonal trajectories of $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int \cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x\, dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes in the surroundings of temperature 40°C. Find the temperature of the body after 40 minutes from the original instant. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$$

by reducing it to echelon form. **(06 Marks)**

b. Using the power method find the largest eigenvalue and the corresponding eigenvector of matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ taking $(1, 1, 1)^T$ as the initial eigenvector. Perform five iterations. **(07 Marks)**

c. Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$ is regular. Also, find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the following system of equations by using Gauss-Jordan method:
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ **(06 Marks)**

b. Diagnolize the matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$. **(07 Marks)**

c. Obtain the canonical form of $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ using orthogonal transformation. **(07 Marks)**

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CBCS Scheme

USN 2VD16ME029 | **15MAT11**

First Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $y = e^{-x}\sin x\cos 2x$. **(06 Marks)**

b. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cut each other orthogonally. **(05 Marks)**

c. Find the radius of curvature of the curve $x^2y = a(x^2+y^2)$ at the point (-2a, 2a). **(05 Marks)**

OR

2 a. If $y = \sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$ **(06 Marks)**

b. Find the pedal equation of $r = 2(1+\cos\theta)$. **(05 Marks)**

c. Find the radius of curvature of $r^n = a^n \sin n\theta$. **(05 Marks)**

Module-2

3 a. Expand $\tan^{-1}x$ in powers of $(x-1)$ upto the fourth degree term. **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left[\frac{1}{x} - \frac{\log(1+x)}{x^2}\right]$ **(05 Marks)**

c. If $z = f(x+ct) + g(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = C^2.\frac{\partial^2 z}{\partial x^2}$. **(05 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of $e^{\sin x}$ upto the form containing x^4. **(06 Marks)**

b. If $z = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(05 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve whose parametric equations are $x = t^3+1$, $y = t^2$ and $z = 2t+5$. Find the components of its velocity and acceleration at time t = 1 in the direction of $i + j + 3k$. **(06 Marks)**

b. If $\phi = 2x^3y^2z^4$, find Div(Grad ϕ). **(05 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ, such that $\vec{F} = \nabla\phi$. **(05 Marks)**

OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at P(1, −2, −1) in the direction of $2i - j - 2k$. **(06 Marks)**

b. If $\vec{F} = (x+y+1)i + j - (x+y)k$. Show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(05 Marks)**

c. If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at the point (1, −1, 1). **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x dx$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of curves $y^2 = Cx^3$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} - \frac{2}{x}y = \frac{y^2}{x^3}$. **(05 Marks)**

c. A body is heated to 110°C and placed in air at 10°C. After one hour its temperature becomes 60°C. How much additional time is required for it to cool to 30°C? **(05 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$. **(06 Marks)**

b. Solve the following system of equations by Gauss Jordan method:
$x + 2y + z = 3$, $2x + 3y + 3z = 10$, $3x - y + 2z = 13$ **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss-Seidal method:
$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. Perform three iterations. **(06 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(05 Marks)**

c. Reduce the quadratic form,
$3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ into the canonical form. **(05 Marks)**

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CBCS Scheme

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15MAT21

Second Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{4dy}{dx} - 4y = \sinh(2x+3)$ by inverse differential operator method. **(05 Marks)**

b. Solve $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = xe^{3x} + \sin 2x$ by inverse differential operator method. **(05 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters. **(06 Marks)**

OR

2 a. Solve $y'' - 2y' + y = x\cos x$ by inverse differential operator method. **(05 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 4y = x^2 + 2^{-x} + \log 2$ by inverse differential operator method. **(05 Marks)**

c. Solve $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 4y = 2x^2 + 3e^{-x}$ by the method of undetermined coefficients. **(06 Marks)**

Module-2

3 a. Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$. **(05 Marks)**

b. Solve $y - 2px = \tan^{-1}(xp^2)$. **(05 Marks)**

c. Solve $xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\frac{dy}{dx} + xy = 0$. **(06 Marks)**

OR

4 a. Solve $(2x+5)^2y'' - 6(2x+5)y' + 8y = 6x$. **(05 Marks)**

b. Solve $y = 2px + y^2p^3$. **(05 Marks)**

c. Solve the equation : $(px-y)(py+x) = a^2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$. **(06 Marks)**

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function given $z = yf(x) + x\phi(y)$. **(05 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions $\frac{\partial z}{\partial x} = \log(1+y)$ when $x = 1$, and $z = 0$ when $x = 0$. **(05 Marks)**

c. Derive one dimensional heat equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(06 Marks)**

OR

6 a. Obtain the partial differential equation given $f\left(\frac{xy}{z}, z\right) = 0$. **(05 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the conditions that $z = 1$ and $\frac{\partial z}{\partial x} = y$ when $x = 0$. **(05 Marks)**

c. Obtain the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables for the positive constant. **(06 Marks)**

Module-4

7 a. Evaluate $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} xyz \, dz\, dy\, dx$. **(05 Marks)**

b. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. **(05 Marks)**

c. Derive the relation between beta and gamma function as $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

OR

8 a. Evaluate $\int_0^a \int_y^a \frac{x\, dx\, dy}{x^2 + y^2}$ by changing the order of integration. **(05 Marks)**

b. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\, dx\, dy$ by changing into polar co-ordinates. **(05 Marks)**

c. Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ by using Beta-Gamma functions. **(06 Marks)**

Module-5

9 a. Find the Laplace transform of $te^{2t} + \dfrac{\cos 2t - \cos 3t}{t} + t \sin t$. **(05 Marks)**

b. Express the function $f(t) = \begin{cases} \pi - t, & 0 < t \le \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(05 Marks)**

c. Solve $y'' + 6y' + 9y = 12t^2 e^{-3t}$ subject to the conditions, $y(0) = 0 = y'(0)$ by using Laplace transform. **(06 Marks)**

OR

10 a. Find he inverse Laplace form of $\dfrac{7s + 4}{4s^2 + 4s + 9}$. **(05 Marks)**

b. Find the Laplace transform of the full wave rectifier f(t) = E sin ωt, 0 < t < π/ω having period π/ω. **(05 Marks)**

c. Obtain the inverse Laplace transform of the function $\dfrac{1}{(s-1)(s^2+1)}$ by using convolution theorem. **(06 Marks)**

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14MAT21

Second Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting ONE full question from each module.***

Module – 1

1 a. Solve initial value problem $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+29x=0$ given $x(0)=0$, $\frac{dx}{dt}(0)=15$. **(06 Marks)**

b. Solve the differential equation,
$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=e^{2x}+\sin x+x$. **(07 Marks)**

c. Solve $y''-4y'+3y=20\cos x$ using method of undetermined coefficients. **(07 Marks)**

2 a. Solve $(D^2+4)y=x^2+\cos 2x+2^{-x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2}+y=\sec x$ by using the method of variation of parameters. **(07 Marks)**

c. Solve $(D^2-1)y=(1+x^2)e^x$. **(07 Marks)**

Module – 2

3 a. Solve simultaneous differential equations,
$\frac{dx}{dt}+5x-2y=t$; $\frac{dy}{dt}+2x+y=0$. **(06 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. **(07 Marks)**

c. Solve $p^2+p(x+y)+xy=0$. **(07 Marks)**

4 a. Solve $y+px=x^4p^2$. **(06 Marks)**

b. Obtain the solution of differential equation, $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$. **(07 Marks)**

c. Solve $y=xp+\sqrt{4+p^2}$ for general and singular solutions. **(07 Marks)**

Module – 3

5 a. Form the partial differential equation by eliminating arbitrary functions,
$xyz=f(x^2+y^2+z^2)$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2}=a^2z$ given that if $x=0$, $\frac{\partial z}{\partial x}=a\sin y$ and $\frac{\partial z}{\partial y}=0$. **(07 Marks)**

c. Evaluate by changing the order of integration $\int_0^a\int_0^{2\sqrt{ax}}x^2\,dy\,dx$. **(07 Marks)**

6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. (06 Marks)

b. Solve PDE by direct integration method. $\frac{\partial^2 z}{\partial x \partial t} = e^{-t}\cos x$ given $z = 0$ when $t = 0$ and $\frac{\partial z}{\partial t} = 0$ when $x = 0$. (07 Marks)

c. Obtain solution of one dimensional wave equation, $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables. (07 Marks)

Module – 4

7 a. Find the area between parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (06 Marks)

b. Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta = \pi$. (07 Marks)

c. Prove that cylindrical coordinates system is orthogonal. (07 Marks)

8 a. Evaluate $\int_0^1 x^6 (1-x^2)^{\frac{1}{2}} dx$. (06 Marks)

b. Express the vector $zi - 2xj + yk$ in cylindrical co-ordinates. (07 Marks)

c. Find the volume bounded by the surface $z^2 = a^2 - x^2$ and the planes $x = 0$, $y = 0$, $z = 0$ and $y = b$. (07 Marks)

Module – 5

9 a. Find Laplace transform of,

(i) $te^{-t}\sin(4t)$ (ii) $\frac{\cos at - \cos bt}{t}$. (06 Marks)

b. Find inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$. (07 Marks)

c. Express the function, $f(t) = \begin{cases} \pi - t & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. (07 Marks)

10 a. Find inverse Laplace transform of the following using convolution theorem $L^{-1}\left[\frac{s}{(s-1)(s^2+4)}\right]$. (06 Marks)

b. Given $f(t) = \begin{cases} E & 0 < t < \frac{a}{2} \\ -E & \frac{a}{2} < t < a \end{cases}$ where $f(t+a) = f(t)$. Show that $L\{f(t)\} = \frac{E}{S}\tanh\left(\frac{aS}{2}\right)$. (07 Marks)

c. Using Laplace transform method, solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 12t^2 e^{-3t}$. (07 Marks)

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CBCS Scheme

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15MAT31

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. Hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ **(08 Marks)**

b. The turning moment T units of the Crank shaft of a steam engine is a series of values of the crank angle θ in degrees. Find the first four terms in a series of sines to represent T. Also calculate T when θ = 75°. **(08 Marks)**

θ:	0°	30°	60°	90°	120°	150°	180°
T:	0	5224	8097	7850	5499	2626	0

OR

2 a. Find the Fourier Series expansion of the periodic function.

$$f(x) = \begin{cases} l + x, & -l \le x \le 0 \\ l - x, & 0 \le x \le l \end{cases}.$$ **(06 Marks)**

b. Obtain a half-range cosine series for $f(x) = x^2$ in $(0, \pi)$. **(05 Marks)**

c. The following table gives the variations of periodic current over a period:

t sec:	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A amp:	1.98	1.30	1.05	1.30	-0.88	-0.25

Show that there is a direct current part 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(05 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and evaluate $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$ **(06 Marks)**

b. Find the Fourier cosine transform of, $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$. **(05 Marks)**

c. Obtain the inverse Z-transform of the following function, $\frac{z}{(z-2)(z-3)}$. **(05 Marks)**

OR

4 a. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \alpha\right)$. **(06 Marks)**

b. Solve $u_{n+2} - 5u_{n+1} + 6u_n = 36$ with $u_0 = u_1 = 0$, using Z-transforms. **(05 Marks)**

c. If Fourier sine transform of f(x) is $\frac{e^{-a\alpha}}{\alpha}$, $\alpha \ne 0$. Find f(x) and hence obtain the inverse Fourier sine transform of $\frac{1}{\alpha}$. **(05 Marks)**

Module-3

5 a. Calculate the Karl Pearson's co-efficient for the following ages of husbands and wives: **(06 Marks)**

Husband's age x:	23	27	28	28	29	30	31	33	35	36
Wife's age y:	18	20	22	27	21	29	27	29	28	29

b. By the method of least square, find the parabola $y = ax^2 + bx + c$ that best fits the following data: **(05 Marks)**

x:	10	12	15	23	20
y:	14	17	23	25	21

c. Using Newton-Raphson method, find the real root that lies near $x = 4.5$ of the equation $\tan x = x$ correct to four decimal places. (Here x is in radians). **(05 Marks)**

OR

6 a. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and the coefficient of correlation between x and y. **(06 Marks)**

b. Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares: **(05 Marks)**

x:	1	5	7	9	12
y:	10	15	12	15	21

c. Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to three decimal places. **(05 Marks)**

Module-4

7 a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46: **(06 Marks)**

Age:	45	50	55	60	65
Premium (in Rupees):	114.84	96.16	83.32	74.48	68.48

b. Using Newton's divided difference interpolation, find the polynomial of the given data: **(05 Marks)**

x	3	7	9	10
f(x)	168	120	72	63

c. Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(05 Marks)**

OR

8 a. Find the number of men getting wages below ₹ 35 from the following data: **(06 Marks)**

Wages in ₹ :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	9	30	35	42

b. Find the polynomial f(x) by using Lagrange's formula from the following data: **(05 Marks)**

x:	0	1	2	5
f(x):	2	3	12	147

c. Compute the value of $\int_{0.2}^{1.4}(\sin x - \log_e x + e^x)dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. **(05 Marks)**

Module-5

9 a. A vector field is given by $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$. **(06 Marks)**

b. If C is a simple closed curve in the xy-plane not enclosing the origin. Show that $\int_C \vec{F}.d\vec{R} = 0$, where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$. **(05 Marks)**

c. Derive Euler's equation in the standard form viz., $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(05 Marks)**

OR

10 a. Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{R}$ where $\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. **(06 Marks)**

b. Show that the geodesics on a plane are straight lines. **(05 Marks)**

c. Find the curves on which the functional $\int_0^1 \left((y')^2 + 12xy\right)dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremized. **(05 Marks)**

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USN

MATDIP301

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Advanced Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Find the modulus and amplitude of $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Express the complex number $2 + 3i + \frac{1}{1-i}$ in the form a + ib. **(07 Marks)**

c. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x^2}{2x^2 + 7x + 6}$. **(07 Marks)**

c. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)\, y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. **(07 Marks)**

3 a. If ϕ is the angle between the tangent and radius vector to the curve $r = f(\theta)$ at any point (r, θ), prove that $\tan\theta = \frac{rd\theta}{dr}$. **(06 Marks)**

b. Find the angle of intersection between the curves $r^n = a^n\cos n\theta$ and $r^n = b^n \sin n\theta$. **(07 Marks)**

c. Using Maclaurin's series, expand tan x up to the term containing x^5. **(07 Marks)**

4 a. If $Z = f(x + ct) + \phi(x - ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = C^2 \frac{\partial^2 z}{\partial x^2}$. **(06 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ tan u. **(07 Marks)**

c. If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

PART – B

5 a. Obtain the reduction formula for $\int \cos^n x\,dx$. **(06 Marks)**

b. Using reduction formula evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}dx$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_0^1 e^{x+y}\,dx\,dy$. **(07 Marks)**

6 a. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\ dxdydz$. (07 Marks)

b. Prove that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

c. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (06 Marks)

7 a. Solve $3e^x \tan y\, dx + (1-e^x)\sec^2 y dy = 0$. (06 Marks)

b. Solve $(2x + 3y + 4)dx - (4x + 6y + 5)\, dy = 0$. (07 Marks)

c. Solve $\dfrac{dy}{dx} + y \tan x = \cos x$. (07 Marks)

8 a. Solve $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = -2 \cos hx$. (06 Marks)

b. Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$. (07 Marks)

c. Solve $\dfrac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$. (07 Marks)

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CBCS SCHEME

USN 2VD17EC410 **15MATDIP31**

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express complex numbers $\dfrac{(5-3i)(2+i)}{4+2i}$ in the form a + ib. (06 Marks)

b. If $x = \cos\theta + i\sin\theta$, then show that $\dfrac{x^{2n}-1}{x^{2n}+1} = i\tan\theta$ (05 Marks)

c. Prove that the area of the triangle whose vertices are A, B, C is $\frac{1}{2}\left[B\times C + C\times A + A\times B\right]$. (05 Marks)

OR

2 a. Find the cube root of $\sqrt{3} + i$. (06 Marks)

b. Find the modulus and amplitude of $\dfrac{3+i}{2+i}$ (05 Marks)

c. Prove that the vectors $i-2j+3k$, $-2i+3j-4k$ and $i-3j+5k$ are coplanar. (05 Marks)

Module-2

3 a. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$. (06 Marks)

b. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ (05 Marks)

c. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$. (05 Marks)

OR

4 a. Find the pedal equation $r = a(1+\cos\theta)$. (06 Marks)

b. Expand $\tan x$ in ascending powers of x. (05 Marks)

c. If $u = x+y+z$, $v = y+z$, $w = z$ then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ (05 Marks)

Module-3

5 a. Evaluate $\int_0^{\pi/2}\sin^n x\,dx$. (06 Marks)

b. Evaluate $\int_0^a \dfrac{x^3}{\sqrt{a^2-x^2}}\,dx$. (05 Marks)

c. Evaluate $\int_1^2\int_1^3 xy^2\,dxdy$ (05 Marks)

OR

6 a. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dx\,dy\,dz$ **(06 Marks)**

b. Evaluate $\int_0^{\pi/6} \cos^4 3x\,dx.$ **(05 Marks)**

c. Evaluate $\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\,dx.$ **(05 Marks)**

Module-4

7 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the velocity and acceleration at t = 1 in the direction $i - 3j + 2k$. **(06 Marks)**

b. Find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point (1, –1, 2). **(05 Marks)**

c. Show that the vector $f = (2x - 5y)i + (x - y)j + (3x - z)k$ is a solenoidal. **(05 Marks)**

OR

8 a. If $f(x, y, z) = 3x^2y - y^3z^2$ then find grad f at the point (1, –2, –1). **(06 Marks)**

b. Evaluate (i) div R, (ii) curl R, if $R = xi + yj + zk$. **(05 Marks)**

c. Find a, if $(axy - z^2)i + (x^2 + 2yz)j + (y^2 - axz)k$ is an irrotational vector. **(05 Marks)**

Module-5

9 a. Solve $(x^2 + y^2)dx + 2xydy = 0$ **(06 Marks)**

b. Solve $(e^x + 1)\cos x\,dx + e^y \sin x\,dy = 0$ **(05 Marks)**

c. Solve $(1 + xy)ydx + (1 - xy)xdy = 0$ **(05 Marks)**

OR

10 a. Solve $(x\log x)\frac{dy}{dx} + y = 2\log x$ **(06 Marks)**

b. Solve $(x + 2y^3)\frac{dy}{dx} = y$ **(05 Marks)**

c. Solve $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$ **(05 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

USN [| | | | | | | | |]

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions selecting atleast TWO questions from each part.***

PART – A

1 a. Find the direction cosines ℓ, m, n of the line :
$x + y + z + 1 = 0$
$4x + y - 2z + 2 = 0.$ **(06 Marks)**

b. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. **(07 Marks)**

c. Find the angle between the line $\frac{x+4}{4} + \frac{y-3}{-3} = \frac{z+2}{1}$ and the plane $2x + 2y - z + 15 = 0$. **(07 Marks)**

2 a. Find the equation of the plane which passes through the points A(0, 1, 1), B(1, 1, 2), C(–1, 2, –2). **(06 Marks)**

b. Find the equation of the plane which passes through the point(3, –3, 1) and normal to the line joining the points (3, 2, –1) and (2, –1, 5). **(07 Marks)**

c. Find the equations to the two planes which bisects the angle between the planes :
$3x - 4y + 5z = 3$
$5x + 3y - 4z = 9.$ **(07 Marks)**

3 a. Find the sides and the angle A of the triangle whose vertices are $\overline{OA} = I - 2J + 2K$, $\overline{OB} = 2I + J - K$, $\overline{OC} = 3I - J + 2K$. **(06 Marks)**

b. Show that the points $-6I + 3J + 2K$, $3I - 2J + 4K$, $5I + 7J + 3K$ and $-13I + 17J - k$ are coplanar. **(07 Marks)**

c. Prove that : $\left[\overline{B}\times\overline{C}, \overline{C}\times\overline{A}, \overline{A}\times\overline{B}\right] = \left[\overline{A}\,\overline{B}\,\overline{C}\right]^2$. **(07 Marks)**

4 a. A particle moves along the curve $x = t^2 + 1$, $y = t^2$, $z = 2t + 3 + \sin(\pi t)$ where t is the time. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. If $\overline{A} = (\cos t)I + (\sin t)J + (4t)K$ and $\overline{B} = (t^3 + 1)I + J + (8t^2 - 3t^3)K$ then find :
i) $\frac{d}{dt}(\overline{A} + \overline{B})$ ii) $\frac{d}{dt}(\overline{A}\cdot\overline{B})$. **(07 Marks)**

c. If $\phi = 3x^2y - y^3z^2$, find grad ϕ at (1, –2, 1). Also find a unit normal vector to the surface $3x^2y - y^3z^2 = 6$ at (1, –2, 1). **(07 Marks)**

PART – B

5 a. If $\overline{A} = xyz\,I + 3x^2y\,J + (xz^2 - y^2z)K$ then find curl $\overline{A}$ at (1, 2, 3). **(06 Marks)**

b. Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point A(1, –1, –1) in the direction of $2\hat{i} + 3\hat{j} + 2\hat{k}$. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$ and $\overline{r} = xI + yJ + zK$ then find div $(u\overline{r})$ in terms of u. if $\vec{f} = \nabla(x^3 + y^3 + 2^3 - 3xyz)$ find $\nabla\cdot\vec{f}$ and $\nabla\times\vec{f}$. **(07 Marks)**

6 a. Find the Laplace transform of f(t) defined as :

$$f(t)=\begin{cases}\frac{t}{6}, & \text{when} \quad 0<t<6 \\ 1, & \text{when} \quad t<6\end{cases}.$$ **(05 Marks)**

b. Find : i) $L(\cos^2 t)$ ii) $L(t \sin h\, at)$ iii) $L\left(\frac{1}{t}\sin 2t\right)$. **(15 Marks)**

7 a. Find : $L(e^{2t}\cos 3t)$. **(06 Marks)**

b. Find : $L^{-1}\left(\frac{2h-5}{9s^2-25}\right)$. **(07 Marks)**

c. Find : $L^{-1}\left(\frac{s^2+4}{x^2+9}\right)$. **(07 Marks)**

8 a. Using Laplace transforms, find the solution of the initial value problem $y''-4y'+4y=64\sin 2t$, $y(0)=0$, $y'(0)=1$. **(10 Marks)**

b. Using Laplace transforms, solve $y''+9y=\cos 2t$, $y(0)=1$, $y'(0)=\frac{12}{5}$. **(10 Marks)**

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CBCS Scheme

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15MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by applying elimentary row transformations. (06 Marks)

b. Solve the following system of equations by Gauss-elimination method: $x + y + z = 9$, $x - 2y + 3z = 8$ and $2x + y - z = 3$. (05 Marks)

c. Find the inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ using Cayley-Hamilton theorem. (05 Marks)

OR

2 a. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to echelon form. (06 Marks)

b. Solve the following system of equations by Gauss-elimination method: $x + y + z = 9$, $2x - 3y + 4z = 13$ and $3x + 4y + 5z = 40$. (05 Marks)

c. Find the eigen values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (05 Marks)

Module-2

3 a. Solve $(D^4 - 2D^3 + 5D^2 - 8D + 4)y = 0$. (05 Marks)

b. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$. (05 Marks)

c. Solve by the method of variation of parameters $y'' + a^2y = \sec ax$. (06 Marks)

OR

4 a. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x$. (05 Marks)

b. Solve $(D^2 + 5D + 6)y = \sin x$. (05 Marks)

c. Solve by the method of undetermined coefficients $y'' + 2y' + y = x^2 + 2x$ (06 Marks)

Module-3

5 a. Find the Laplace transform of $\cos t.\cos 2t.\cos 3t$. (06 Marks)

b. Find the Laplace transform $f(t) = \frac{Kt}{T}$, $0 < t < \pi$, $f(t + T) = f(t)$. (05 Marks)

c. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit step function, and hence find $L[f(t)]$. (05 Marks)

OR

6 a. Find the Laplace transform of (i) $t\cos at$, (ii) $\dfrac{1-e^{-at}}{t}$. (06 Marks)

b. Find the Laplace transform of a periodic function a period 2a, given that
$$f(t) = \begin{cases} t, & 0 \le t < a \\ 2a - t, & a \le t < 2a \end{cases} \quad f(t+2a) = f(t).$$ (05 Marks)

c. Express $f(t) = \begin{cases} 1, & 0 < t < 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transform. (05 Marks)

Module-4

7 a. Find the inverse Laplace transform of (i) $\dfrac{(s+2)^3}{s^6}$, (ii) $\dfrac{s+5}{s^2-6s+13}$. (06 Marks)

b. Find inverse Laplace transform of $\log\left[\dfrac{s^2+4}{s(s+4)(s-4)}\right]$. (05 Marks)

c. Solve by using Laplace transforms $\dfrac{d^2y}{dt^2} + k^2y = 0$, given that y(0) = 2, y′(0) = 0. (05 Marks)

OR

8 a. Find the inverse Laplace transform of $\dfrac{4s+5}{(s+1)^2(s+2)}$. (06 Marks)

b. Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{s+a}{b}\right)$. (05 Marks)

c. Using Laplace transforms solve the differential equation $y'' + 4y' + 3y = e^{-t}$ with y(0) = 1, y′(0) = 1. (05 Marks)

Module-5

9 a. If A and B are any two events of S, which are not mutually exclusive then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (05 Marks)

b. The probability that 3 students A, B, C, solve a problem are ½, ⅓, ¼ respectively. If the problem is simultaneously assigned to all of them, what is the probability that the problem is solved? (05 Marks)

c. In a class 70% are boys and 30% are girls. 5% of boys, 3% of girls are irregular to the classes. What is the probability of a student selected at random is irregular to the classes and what is the probability that the irregular student is a girl? (06 Marks)

OR

10 a. If A and B are independent events then prove that $\overline{A}$ and $\overline{B}$ are also independent events. (05 Marks)

b. State and prove Baye's theorem. (05 Marks)

c. A Shooter can hit a target in 3 out of 4 shots and another shooter can hit the target in 2 out or 3 shoots. Find the probability that the target is being hit:
(i) when both of them try (ii) by only one shooter. (06 Marks)

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CBCS SCHEME

USN | 17MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x}{(x+1)(2x-3)}$. **(06 Marks)**

b. Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersects orthogonally. **(07 Marks)**

c. Find the Pedal equation of the curve $r = a(1+\cos\theta)$. **(07 Marks)**

OR

2 a. If $x = \tan y$ prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. With usual notation, prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(07 Marks)**

c. Find the radius of curvature of the curve $y^2 = \dfrac{a^2(a-x)}{x}$ at the point (a, 0) **(07 Marks)**

Module-2

3 a. Find the Taylor's series of $\log_e x$ about $x = 1$ upto the term containing fourth degree. **(06 Marks)**

b. If $u = \sin^{-1}\left[\dfrac{x^2y^2}{x+y}\right]$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3\tan u$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$. **(06 Marks)**

b. Find the Maclaurin's expansion of $\sqrt{1+\sin 2x}$ upto fourth degree term. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$ **(07 Marks)**

Module-3

5 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ where t denotes time. Find the velocity and acceleration at $t = 2$. **(06 Marks)**

b. If $\vec{f} = (x+y+az)\hat{i} + (bx+2y-z)\hat{j} + (x+cy+2z)\hat{k}$ is irrotational find a, b, c. Hence find the scalar potential ϕ such that $\vec{f} = \nabla\phi$. **(07 Marks)**

c. Prove that curl (grad ϕ) = 0. **(07 Marks)**

OR

6 a. If $\overrightarrow{f} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ show that $\overrightarrow{f} \cdot \text{curl}\,\overrightarrow{f} = 0$. **(06 Marks)**

b. If $\overrightarrow{f} = \text{grad}(x y^3 z^2)$ find $\text{div}\,\overrightarrow{f}$ and $\text{curl}\,\overrightarrow{f}$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl}\,\overrightarrow{A}) = 0$ **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^a x\sqrt{ax - x^2}\,dx$ **(06 Marks)**

b. Solve $r\sin\theta - \cos\theta\,\frac{dr}{d\theta} = r^2$ **(07 Marks)**

c. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$. **(06 Marks)**

b. Solve $(x^2 + y^2 - x)\,dx + xy\,dy = 0$. **(07 Marks)**

c. Water at temperature 10°C takes 5 minutes to warm upto 20°C in a room temperature 40°C. Find the temperature after 20 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ by reducing it to echelon form. **(06 Marks)**

b. Find the largest eigen value and the corresponding eigen vector for

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

taking $(1\ \ 0\ \ 0)^T$ as initial vector by using power method. (Carry out six iterations) **(07 Marks)**

c. Show that the transformation $y_1 = 2x - 2y - z$, $y_2 = -4x + 5y + 32$ and $y_3 = x - y - z$ is regular and find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the equations $20x + y - 2z = 17$; $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by using Gauss-Seidel method. (Carry out 3 iterations) **(06 Marks)**

b. Diagonalise the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ **(07 Marks)**

c. Reduce the quadratic form $3x^2 - 2y^2 - z^2 + 12yz + 8xz - 4xy$ into canonical form, using orthogonal transformation. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | | **15MAT11**

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of the $\sin^3 x \cos^2 x$. **(06 Marks)**

b. Find angle between the pair of curves $r = 6\cos\theta$ and $r = 2(1+\cos\theta)$. **(05 Marks)**

c. Show that for the curve $r(1-\cos\theta) = 2a$ the radius of curvature is $\frac{2}{\sqrt{a}} r^{3/2}$. **(05 Marks)**

OR

2 a. Show that $\left(\frac{2\rho}{a}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ for the curve $y = \frac{ax}{a+x}$. **(06 Marks)**

b. Find the Pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(05 Marks)**

c. If $y = \log(x + \sqrt{1+x^2})$ prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$. **(05 Marks)**

Module-2

3 a. Expand Log(1+cosx) by Maclaurin's series upto the term containing x^4. **(06 Marks)**

b. Evaluate $\lim_{x \to \pi/2} (\sin x)^{\tan x}$. **(05 Marks)**

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = u$ **(05 Marks)**

OR

4 a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that $xu_x + yu_y = \sin 2u$. **(06 Marks)**

b. If $z = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$ **(05 Marks)**

c. Expand tanx in Taylor's series upto three in powers of $\left(x - \frac{\pi}{4}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1-t^3$, $y = 1+t^2$ and $z = 2t-5$, determine velocity and acceleration at t = 1. Also find the components of velocity and acceleration in the direction $2i + j + 2k$. **(06 Marks)**

b. Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at $(2,-1,2)$. **(05 Marks)**

c. Prove that $\text{Div}(\phi \vec{A}) = \phi(\text{div}\, \vec{A}) + \text{grad}\phi \bullet \vec{A}$ **(05 Marks)**

OR

6 a. Find the unit tangent vector and normal vector to the curve $\vec{r} = \cos 2t\, i + \sin 2t\, j + t\, k$ at $x = \frac{1}{\sqrt{2}}$. **(06 Marks)**

b. Find the $\text{curl}\left(\text{curl}\,\vec{A}\right)$, where $\vec{A} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ at the point (1, 0, 2). **(05 Marks)**

c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function of ϕ such that $\vec{F} = \nabla\phi$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\, dx$. **(06 Marks)**

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(05 Marks)**

c. Show that the family of the curves $y^2 = 4a(x+a)$ is self orthogonal. **(05 Marks)**

OR

8 a. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(05 Marks)**

b. Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\, d\theta$. **(05 Marks)**

c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(06 Marks)**

Module-5

9 a. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, by using the power method by taking initial vector as $[1, \ 1, \ 1]^T$ **(06 Marks)**

b. Find the rank of the matrix by reducing into the normal form, $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. **(05 Marks)**

c. Solve the following system of equation by Gauss seidel method: $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. **(05 Marks)**

OR

10 a. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

b. Solve by Gauss elimination method, $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$. **(05 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into the canonical form. **(05 Marks)**

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USN | | | | | | | | | | | 14MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If $y = a\cos(\log_e x) + b\sin(\log_e x)$, show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

b. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cut each other orthogonally. **(06 Marks)**

c. Find the radius of curvature at the point (a, 0) on the curve $xy^2 = a^3 - x^3$. **(07 Marks)**

OR

2 a. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$. **(06 Marks)**

b. Define curvature of a curve and derive an expression for the radius of curvature in the polar form. **(07 Marks)**

c. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's series for $e^x \cos x$ upto the term containing x^4. **(06 Marks)**

b. If $w = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$, show that $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$. **(07 Marks)**

c. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

OR

4 a. Find the constants 'a' and 'b' such that $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3}$ may be equal to unity. **(07 Marks)**

b. If $u = \log_e\left(\frac{x^5+y^5+z^5}{x^2+y^2+z^2}\right)$, then show that by using Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3$. **(06 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **(07 Marks)**

Module-3

5 a. Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point (1, -1, 2). **(07 Marks)**

b. Show that $\text{Div}(\text{curl}\,\vec{A}) = \vec{0}$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

OR

6 a. A vector field is given by $\vec{f} = (x^2 + xy^2)i + (y^2 + x^2)j$. Show that the field is irrotational and find the scalar potential. **(07 Marks)**

b. If $\vec{r} = xi + yj + zk$, show that i) div $\vec{r} = 3$ ii) curl $\vec{r} = \vec{0}$ **(06 Marks)**

c. Evaluate $\int_0^{\infty}\left(\frac{e^{-\alpha x}\sin x}{x}\right)dx$ and hence show that $\int_0^{\infty}\left(\frac{\sin x}{x}\right)dx = \frac{\pi}{2}$, by using differentiation under integral sign. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. **(06 Marks)**

b. Solve: $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(07 Marks)**

c. Find the orthogonal trajectories of a system of confocal and coaxial parabolas $y^2 = 4a(x + a)$. **(07 Marks)**

OR

8 a. Solve: $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. **(07 Marks)**

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}dx$, using reduction formulae. **(06 Marks)**

c. Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes. **(07 Marks)**

Module-5

9 a. Reduce the following matrix to Echelon form and hence find the Rank,

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ **(06 Marks)**

b. Solve by LU decomposition method $x + 2y + 3z = 14$, $2x + 3y + 4z = 20$, $3x + 4y + z = 14$. **(07 Marks)**

c. Determine the largest eigen-value and the corresponding eigen vector of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ with the initial eigen vector be $[1,\ 1,\ 0]^1$ using Rayleigh's power method. Perform six iterations. **(07 Marks)**

OR

10 a. Solve $3x + 8y + 29z = 71$, $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$ by using Gauss-Seidel method. Carryout 3 iterations. **(06 Marks)**

b. Reduce the matrix $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 6xz + 2xy + 2yz$ to the canonical form and specify the matrix of transformation. **(07 Marks)**

15

CBCS Scheme

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17MAT21

Second Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x}$. **(06 Marks)**

b. Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 4y = 3e^{x}$. **(07 Marks)**

c. Solve by the method of variation of parameter $y'' + y = \frac{1}{1+\sin x}$. **(07 Marks)**

OR

2 a. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + \sin x + x$. **(06 Marks)**

b. Solve $y'' + 4y' + 5y = -2\cosh x$: find y when $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$. **(07 Marks)**

c. Solve by the method of undetermined coefficient $(D^2 - 3D + 2)y = x^2 + e^x$. **(07 Marks)**

Module-2

3 a. Solve $x\frac{d^2y}{dx^2} - \frac{2y}{x} = x + \frac{1}{x^2}$ **(06 Marks)**

b. Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ **(07 Marks)**

c. Find the general and singular solution for $xp^2 + xp - yp + 1 - y = 0$. **(07 Marks)**

OR

4 a. Solve $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$. **(06 Marks)**

b. Solve $xy\left\{\left(\frac{dy}{dx}\right)^2 + 1\right\} = (x^2 + y^2)\frac{dy}{dx}$. **(07 Marks)**

c. Find the general solution by reducing to Clairaut's form $(px - y)(x + py) = 2p$ using $U = x^2$ and $V = y^2$. **(07 Marks)**

Module-3

5 a. Find the partial differential equation of all spheres $(x-a)^2 + (y-b)^2 + z^2 = c^2$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional wave equation with usual notations. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function from $z = y\phi(x) + x\psi(y)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$: given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. **(07 Marks)**

c. Find the various possible solution for one dimensional heat equation by the method of separation of variables. **(07 Marks)**

Module-4

7 a. Prove that $\Gamma(1/2) = \sqrt{\pi}$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$ **(07 Marks)**

c. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **(06 Marks)**

b. Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **(07 Marks)**

c. Prove that with usual notations $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

Module-5

9 a. Find the Laplace transform of $\frac{\cos 2t - \cos 3t}{t}$. **(06 Marks)**

b. Express the function in terms of unit step function and hence find its Laplace transform

$$f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$$

(07 Marks)

c. Find $L^{-1}\left\{\frac{s+3}{s^2-4s+13} + \log_e\left(\frac{s+1}{s-1}\right)\right\}$. **(07 Marks)**

OR

10 a. Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases}$$ of period 2π. **(06 Marks)**

b. Using convolution theorem obtain the inverse Laplace transform of $\frac{s}{(s+2)(s^2+9)}$. **(07 Marks)**

c. Solve the equation $y'' - 3y' + 2y = e^{3t}$: $y(0) = 1$ and $y'(0) = 0$ using Laplace transform technique. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | **15MAT21**

Second Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – II

Time: 3 hrs.   Max. Marks: 80

Note: 1. Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Solve : $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$ **(05 Marks)**

b. Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$ **(05 Marks)**

c. Solve $\frac{d^2y}{dx^2} + y = \sec x.\tan x$ by the method of variation of parameters. **(06 Marks)**

OR

2 a. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$, using inverse differential operator method. **(05 Marks)**

b. Solve $\frac{d^2y}{dx^2} - 4y = x.\sin 2x$, using inverse differential operator method. **(05 Marks)**

c. Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x} + \sin x$$ **(06 Marks)**

Module-2

3 a. Solve : $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ **(05 Marks)**

b. Solve : $p^2 + p(x + y) + xy = 0$ **(05 Marks)**

c. Solve : $x - yp = ap^2$ by solving for x **(06 Marks)**

OR

4 a. Solve : $(3x + 2)^2\frac{d^2y}{dx^2} + 5(3x + 2)\frac{dy}{dx} - 3y = x^2 + x + 1$ **(06 Marks)**

b. Solve : $p^2 + 2py\cot x - y^2$ by solving for p. **(05 Marks)**

c. Solve the equation $(px - y)(x - py) = 2p$ by reducing it into Clairaut's form by taking a substitution $x^2 = u$ and $y^2 = v$. **(05 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating arbitrary constants

$$(x - a)^2 + (y - b)^2 = z^2\cot^2\alpha$$, where 'α' is the parameter. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$, when $x = 0$ and $z = 0$ when y is an odd multiple of $\pi/2$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Derive the one-dimensional wave equation in the form $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(05 Marks)**

OR

6 a. Form a partial differential equation by eliminating the arbitrary function from $z = f(x + at) + g(x - at)$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the conditions that $z = 1$ and $\frac{\partial z}{\partial x} = y$ when $x = 0$. **(06 Marks)**

c. Derive the one dimensional heat equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(04 Marks)**

Module-4

7 a. Evaluate $\int_1^2 \int_x^3 (xy + e^y)\, dydx$ **(05 Marks)**

b. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dxdy$ by changing the order of integration. **(05 Marks)**

c. Obtain the relation between beta and gamma function in the form $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$ **(06 Marks)**

OR

8 a. Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} x^2\, dydx$ by changing to polar coordinate. **(05 Marks)**

b. Evaluate $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dydxdz$ **(05 Marks)**

c. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta} \cdot d\theta$ **(06 Marks)**

Module-5

9 a. Evaluate (i) $L\{t^3 + 4t^2 - 3t + 5\}$ (ii) $L\{\cos t \cdot \cos 2t \cdot \cos 3t\}$ **(06 Marks)**

b. Find the Laplace transform of $L\{e^{3t} \cdot \sin 5t \cdot \sin 3t\}$ **(05 Marks)**

c. Solve the equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$ under the conditions $y(0) = 1$, $y'(0) = 0$. **(05 Marks)**

OR

10 a. Evaluate : $L^{-1}\left\{\frac{4s + 5}{(s + 1)^2 (s + 2)}\right\}$ **(06 Marks)**

b. Find $L^{-1}\left\{\frac{1}{(s + 1)(s^2 + 1)}\right\}$ by using convolution theorem. **(05 Marks)**

c. Express the function in terms of unit step function and hence find their Laplace transform

$$f(t) = \begin{cases} 1, & 0 \le t \le 1 \\ t, & 0 < t \le 2 \\ t^2, & t > 2 \end{cases}$$ **(05 Marks)**

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USN | | | | | | | | | | | **14MAT21**

Second Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module – 1

1 a. Solve: $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = e^{3t}\cosh 2t + 2^t$. **(06 Marks)**

b. Solve : $y'' - 4y' + 4y = 8\cos 2x$. **(07 Marks)**

c. Solve : $y'' + 4y = x^2 + e^{-x}$ by the method of undetermined coefficients. **(07 Marks)**

OR

2 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

b. Solve : $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$. **(07 Marks)**

c. Solve by the method of variation of parameters $y'' + 2y' + 2y = e^{-x}\sec^3 x$. **(07 Marks)**

Module – 2

3 a. Solve : $\frac{dx}{dt} + 2y = -\sin t$. $\frac{dy}{dt} - 2x = \cos t$. **(06 Marks)**

b. Solve : $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = \sin(\log x)$. **(07 Marks)**

c. Solve : $xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$. using solvable for P. **(07 Marks)**

OR

4 a. Solve : $\frac{dy}{dx} + y = z + e^x$, $\frac{dz}{dx} + z = y + e^x$. **(06 Marks)**

b. Solve : $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$. **(07 Marks)**

c. Show that the equation, $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation. Hence obtain the general and singular solution. **(07 Marks)**

Module – 3

5 a. Form the partial differential equation by eliminating the arbitrary function in $\phi(x + y + z,\ x^2 + y^2 - z^2) = 0$ **(06 Marks)**

b. Derive one dimensional wave equation in the form. $\frac{\partial^2 u}{\partial t^2} = C^2\frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

c. Evaluate : $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$ given that u = 0 when t = 0 and $\frac{\partial u}{\partial t} = 0$ at x = 0. Also show that $u \to \sin x$ as $t \to \infty$. **(06 Marks)**

b. Derive one dimensional heat equation in the form. $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_x^{2-x} xy\,dy\,dx$ by changing the order of integration. **(07 Marks)**

Module – 4

7 a. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. **(06 Marks)**

b. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Express the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. **(07 Marks)**

OR

8 a. Find the volume generated by the revolution of the cardioide $r = a(1+\cos\theta)$ about the initial line. **(06 Marks)**

b. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$ **(07 Marks)**

c. Show that spherical polar coordinate system is orthogonal. **(07 Marks)**

Module – 5

9 a. Find the Laplace transform of $2^t + \frac{\cos 2t - \cos 3t}{t} + t\sin t$. **(06 Marks)**

b. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a, \end{cases}$ $f(t+2a) = f(t)$

then (i) Sketch the graph of f(t) as a periodic function.

(ii) Show that $L\{f(t)\} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$. **(07 Marks)**

c. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method. **(07 Marks)**

OR

10 a. Find the Laplace transform of $t^2 e^{-3t} \sin 2t$. **(06 Marks)**

b. Express the following function interms of Heaviside unit step function and hence find its Laplace transform:

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$$ **(07 Marks)**

c. Using convolution theorem obtain the inverse Laplace transform of:

$\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. **(07 Marks)**

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CBCS Scheme

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15MAT31

Third Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the Fourier series for the function :

$$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+----$. **(08 Marks)**

b. Obtain the half–range cosine series for the function $f(x) = (x - 1)^2$, $0 \le x \le 1$. Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+-----$. **(08 Marks)**

OR

2 a. Find the Fourier series of the periodic function defined by $f(x) = 2x - x^2$, $0 < x < 3$. **(06 Marks)**

b. Show that the half range sine series for the function $f(x) = \ell x - x^2$ in $0 < x < \ell$ is

$$\frac{8\ell^2}{\pi^3}\sum_{0}^{\infty}\frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{\ell}\right)\pi x.$$ **(05 Marks)**

c. Express y as a Fourier series upto 1st harmonic given :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(05 Marks)

Module-2

3 a. Find the Fourier transform of

$$f(x)=\begin{cases}1-|x|, & |x|\le 1\\ 0, & |x|>1\end{cases}$$

and hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. **(06 Marks)**

b. Find the Fourier Sine and Cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$. **(05 Marks)**

c. Solve by using z – transforms $y_{n+1}+\frac{1}{4}y_n=\left(\frac{1}{4}\right)^n$ $(n \ge 0)$, $y_0 = 0$. **(05 Marks)**

OR

4 a. Find the Fourier transform of $f(x) = e^{-|x|}$. **(06 Marks)**

b. Find the Z – transform of sin(3n + 5). **(05 Marks)**

c. Find the inverse Z – transform of : $\dfrac{z}{(z-1)(z-2)}$. **(05 Marks)**

Module-3

5 a. Find the correlation coefficient and the equation of the line of regression for the following values of x and y. **(06 Marks)**

x	1	2	3	4	5
y	2	5	3	8	7

b. Find the equation of the best fitting straight line for the data : **(05 Marks)**

x	0	1	2	3	4	5
y	9	8	24	28	26	20

c. Use Newton – Raphson method to find a real root of the equation $x \log_{10} x = 1.2$ (carry out 3 iterations). **(05 Marks)**

OR

6 a. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

b. Fit a second degree parabola to the following data : **(05 Marks)**

x	1	2	3	4	5
y	10	12	13	16	19

c. Use the Regula–Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$, correct to 3 decimal places. **(05 Marks)**

Module-4

7 a. Given Sin45° = 0.7071, Sin50° = 0.7660, Sin55° = 0.8192, Sin60° = 0.8660 find Sin57° using an appropriate interpolation formula. **(06 Marks)**

b. Construct the interpolation polynomial for the data given below using Newton's divided difference formula :

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

(05 Marks)

c. Use Simpson's $\frac{1}{3}$rd rule with 7 ordinates to evaluate $\int_2^8 \dfrac{dx}{\log_{10} x}$. **(05 Marks)**

OR

8 a. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304, find f(38) using Newton's forward interpolation formula. **(06 Marks)**

b. Use Lagrange's interpolation formula to fit a polynomial for the data :

x	0	1	3	4
y	−12	0	6	12

Hence estimate y at x = 2. **(05 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$. **(05 Marks)**

Module-5

9 a. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem in a plane. **(06 Marks)**

b. Verify Stoke's theorem for the vector $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(05 Marks)**

c. Find the extremal of the functional : $\int_{x_1}^{x_2} [y' + x^2(y')^2]\,dx$. **(05 Marks)**

OR

10 a. Verify Green's theorem in a plane for $\oint_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(06 Marks)**

b. If $\vec{F} = 2xyi + yz^2j + xzk$ and S is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3 evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$. **(05 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x[1+(y')^2}\,dx$. **(05 Marks)**

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CBCS Scheme

USN   **15MAT41**

Fourth Semester B.E. Degree Examination, June/July 2018

Engineering Mathematics – IV

Time: 3 hrs.   Max. Marks: 80

Note: 1. ***Answer any FIVE full questions, choosing one full question from each module.***
2. ***Use of statistical tables is permitted.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Use Taylor's series method to find y at x = 1.1, considering terms upto third degree given that $\frac{dy}{dx} = x + y$ and y(1) = 0. **(05 Marks)**

b. Using Runge-Kutta method, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$; y(0) = 1, taking h = 0.2. **(05 Marks)**

c. Given $\frac{dy}{dx} = x^2 - y$, y(0) = 1 and the values y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918, evaluate y(0.4), using Adams-Bashforth method. **(06 Marks)**

OR

2 a. Using Euler's modified method, find y(0.1) given $\frac{dy}{dx} = x - y^2$, y(0) = 1, taking h = 0.1. **(05 Marks)**

b. Solve $\frac{dy}{dx} = xy$; y(1) = 2, find the approximate solution at x = 1.2, using Runge-Kutta method. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ with the following data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, compute y at x = 0.8, using Milne's method. **(06 Marks)**

Module-2

3 a. Using Runge-Kutta method of order four, solve $y'' = y + xy'$, y(0) = 1, y′(0) = 0 to find y(0.2). **(05 Marks)**

b. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\, dx = 0$, if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Given $y'' = 1 + y'$; y(0) = 1, y′(0) = 1, compute y(0.4) for the following data, using Milne's predictor-corrector method.

y(0.1) = 1.1103	y(0.2) = 1.2427	y(0.3) = 1.399
y′(0.1) = 1.2103	y′(0.2) = 1.4427	y′(0.3) = 1.699.

(05 Marks)

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(05 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(06 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in polar form. **(05 Marks)**

b. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$, using Cauchy's residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps $z = \infty, i, 0$ on to $w = 0, i, \infty$. **(06 Marks)**

OR

6 a. State and prove Cauchy's integral formula. **(05 Marks)**

b. If $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$, find the corresponding analytic function $f(z) = u + iv$. **(05 Marks)**

c. Discuss the transformation $w = z^2$. **(06 Marks)**

Module-4

7 a. Derive mean and standard deviation of the binomial distribution. **(05 Marks)**

b. If the probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individual (i) exactly 3 (ii) more than 2 individuals will suffer a bad reaction. **(05 Marks)**

c. The joint probability distribution for two random variables X and Y is as follows:

X \ Y	-3	-2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Determine: i) Marginal distribution of X and Y ii) Covariance of X and Y iii) Correlation of X and Y **(06 Marks)**

OR

8 a. Derive mean and standard deviation of exponential distribution. **(05 Marks)**

b. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. Given $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.14757) = 0.43$. **(05 Marks)**

c. The joint probability distribution of two random variables X and Y is as follows:

Y \ X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute: i) E(X) and E(Y) ii) E(XY) iii) COV(X, Y) iv) $\rho(X, Y)$ **(06 Marks)**

Module-5

9 a. Explain the terms: i) Null hypothesis ii) Type I and Type II errors. **(05 Marks)**

b. The nine items of a sample have the values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? **(05 Marks)**

c. Given the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ then show that A is a regular stochastic matrix. **(06 Marks)**

OR

10 a. A die was thrown 9000 times and of these 3220 yielded a 3 or 4, can the die be regarded as unbiased? **(05 Marks)**

b. Explain: i) Transient state ii) Absorbing state iii) Recurrent state **(05 Marks)**

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run, how often does he study? **(06 Marks)**

CBCS Scheme

USN

15MATDIP31

Third Semester B.E. Degree Examination, June/July 2018
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$. **(05 Marks)**

b. Prove that $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n=\cos n\theta+i\sin n\theta$. **(05 Marks)**

c. If $z=\cos\theta+i\sin\theta$, then show that $x^n+\frac{1}{x^n}=2\cos n\theta$, $x^n-\frac{1}{x^n}=2i\sin n\theta$. **(06 Marks)**

OR

2 a. Find the sine of the angle between $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$. **(05 Marks)**

b. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ **(05 Marks)**

c. Show that (3, –2, 4), (6, 3, 1), (5, 7, 3) and (2, 2, 6) are coplanar. **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\sin(3x)\cos x$. **(05 Marks)**

b. Find the angle between radius vector and tangent to the curve $\gamma^m\cos m\theta=a^m$. **(05 Marks)**

c. Find the pedal equation of $\gamma=a(1+\cos\theta)$. **(06 Marks)**

OR

4 a. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin(2u)$. **(05 Marks)**

b. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. **(05 Marks)**

c. If $u=x+y$, $v=y+z$, $w=z+x$, find $J\left(\frac{uvw}{xyz}\right)$. **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\cos^6 x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}}\,dx$ **(05 Marks)**

c. Evaluate $\int_0^1 x^5(1-x^2)^{5/2}\,dx$. **(06 Marks)**

OR

6 a. Evaluate $\int_1^2\int_3^4 (xy+e^y)\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^y xyz\,dx\,dy\,dz$. **(06 Marks)**

Module-4

7 a. Find the angle between the tangents to the curve $x=t^2, y=t^3, z=t^4$ at $t=2$, and $t=3$. **(05 Marks)**

b. Find the unit normal to the curve $\vec{\gamma} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\,\hat{k}$. **(05 Marks)**

c. Find the velocity and acceleration to the curve $\vec{\gamma} = t^2\,\hat{i} - t^3\,\hat{j} + t^4\,\hat{k}$ at $t=1$. **(06 Marks)**

OR

8 a. Find the directional derivative of $\varphi = x^3y^3z^3$ at (1, 2, 1) in the direction of $\hat{i}+2\hat{j}+2\hat{k}$. **(05 Marks)**

b. Find the unit normal to the surface $xy + x + zx = 3$ at $(1,1,1)$. **(05 Marks)**

c. If $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$, find div $\vec{F}$. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$. (05 Marks)

b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. (05 Marks)

c. Solve $y(x + y)dx + (x + 2y - 1)dy = 0$. (06 Marks)

OR

10 a. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. (05 Marks)

b. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. (05 Marks)

c. Solve $(x^2 + y^2)\frac{dy}{dx} = xy$. (06 Marks)

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USN | | | | | | | | | | MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2018

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the ratio in which the point C, (9, 8, -10) divides the line segment joining the points A(5, 4, -6) and B(3, 2, -4). **(06 Marks)**

b. If cos α, cos β, cos γ are the direction cosines of a straight line, prove that (i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ (ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$. **(07 Marks)**

c. Find the constant K such that the angle between the lines with direction ratios (-2, 1, -1) and (1, -K, 1) is 90°. **(07 Marks)**

2 a. Show that the angles between the diagonals of a cube is $\theta = \cos^{-1}(1/3)$. **(06 Marks)**

b. Find the equation of the plane through the points (1, 0, -1) and (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. **(07 Marks)**

c. Show that the points A(-6, 3, 2), B(3, -2, 4), C(5, 7, 3) and D(-13, 17, -1) are coplanar. Also find the equation of the plane containing them. **(07 Marks)**

3 a. Find the angle between the vectors $\bar{a} = 2i + 6j + 3k$, $\bar{b} = 12i - 4j + 3k$. **(06 Marks)**

b. Find the area of a parallelogram whose adjacent sides are $i - 2j + 3k$ and $2i + j - 4k$. **(07 Marks)**

c. Find a unit vector perpendicular to both vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = 7i - 5j + k$. **(07 Marks)**

4 a. Show that the four points whose position vectors are $3i - 2j + 4k$, $6i + 3j + k$, $5i + 7j + 3k$ and $2i + 2j + 6k$ are coplanar. **(06 Marks)**

b. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of i + j + 3k. **(07 Marks)**

c. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of vector i + 2j + 2k. **(07 Marks)**

5 a. Find div F and curl F where $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(06 Marks)**

b. Show that $F = x(y - z)i + y(z - x)j + z(x - y)k$ is solenoidal. **(07 Marks)**

c. Find the constants a and b so that the vector $\bar{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. **(07 Marks)**

6 a. Find the Laplace transforms of $1 + 2t^3 - 4e^{3t} + 5e^{-t}$. **(07 Marks)**

b. Find the Laplace transform of $t^2 \sin^2 t$. **(07 Marks)**

c. Find the Laplace transform of $\frac{\sin at}{t}$. **(06 Marks)**

7 a. Find the inverse Laplace transform of $\frac{3s-4}{16-s^2}$. **(06 Marks)**

b. Find the inverse Laplace transform of $\frac{1}{s^2+4s+9}$. **(07 Marks)**

c. Evaluate $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\}$. **(07 Marks)**

8 a. Obtain the Laplace transforms of $f'(t)$, $f''(t)$. **(08 Marks)**

b. Solve the differential equation using Laplace transforms $y''-3y'+2y=1-e^{2t}$ under the conditions $y(0)=1$, $y'(0)=0$. **(12 Marks)**

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CBCS Scheme

USN | | | | | | | | | | |

15MATDIP41

Fourth Semester B.E. Degree Examination, June/July 2018
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: *Answer any FIVE full questions, choosing one full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ by reducing to echelon form. **(06 Marks)**

b. Use Cayley-Hamilton theorem to find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **(05 Marks)**

c. Apply Gauss elimination method to solve the equations $x + 4y - z = -5$; $x + y - 6z = -12$; $3x - y - z = 4$ **(05 Marks)**

OR

2 a. Find all the eigen values and eigen vector corresponding to the largest eigen value of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. **(06 Marks)**

b. Find the rank of the matrix by elementary row transformations $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$. **(05 Marks)**

c. Solve the system of linear equations $x + y + z = 6$; $2x - 3y + 4z = 8$, $x - y + 2z = 5$ by Gauss elimination method. **(05 Marks)**

Module-2

3 a. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters. **(06 Marks)**

b. Solve $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$, given $x(0) = 0$, $\frac{dx}{dt}(0) = 15$. **(05 Marks)**

c. Solve $(D^2 + 5D + 6)y = e^x$. **(05 Marks)**

OR

4 a. Solve by the method of undetermined coefficients $(D^2 - 2D + 5)y = 25x^2 + 12$. **(06 Marks)**

b. Solve $(D^2 + 3D + 2)y = \sin 2x$. **(05 Marks)**

c. Solve $(D^2 - 2D - 1)y = e^x \cos x$. **(05 Marks)**

Module-3

5 a. Find the Laplace transforms of. (i) $t\cos^2 t$ (ii) $\frac{1 - e^{-t}}{t}$ **(06 Marks)**

b. Find the Laplace transforms of. (i) $e^{-2t}(2\cos 5t - \sin 5t)$ (ii) $3\sqrt{t} + \frac{4}{\sqrt{t}}$. **(05 Marks)**

c. Express the function, $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(05 Marks)**

OR

6 a. Find the Laplace transform of the periodic function defined by $f(t) = E\sin\omega t$, $0 < t < \frac{\pi}{\omega}$ having period $\frac{\pi}{\omega}$. **(06 Marks)**

b. Find the Laplace transform of $2^t + t\sin t$. **(05 Marks)**

c. Find the Laplace transform of $\frac{2\sin t\sin 5t}{t}$. **(05 Marks)**

Module-4

7 a. Using laplace transforms method, solve $y'' - 6y' + 9 = t^2e^{3t}$, $y(0) = 2$, $y'(0) = 6$. **(06 Marks)**

b. Find the inverse Laplace transforms of: (i) $\frac{s^2 - 3s + 4}{s^3}$ (ii) $\frac{s+3}{s^2 - 4s + 13}$ **(05 Marks)**

c. Find the inverse Laplace transforms of: (i) $\log\left(\frac{s+1}{s-1}\right)$ (ii) $\frac{s^2}{(s-2)^3}$ **(05 Marks)**

OR

8 a. Solve the simultaneous equations $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$ being given $x = y = 0$ when $t = 0$. **(06 Marks)**

b. Find the inverse Laplace transforms of $\cot^{-1}\left(\frac{s}{2}\right)$. **(05 Marks)**

c. Find the inverse Laplace transforms of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. **(05 Marks)**

Module-5

9 a. For any three arbitrary events A, B, C prove that,
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$ **(04 Marks)**

b. A class has 10 boys and 5 girls. Three students are selected at random, one after the other. Find probability that, (i) first two are boys and third is girl (ii) first and third boys and second is girl. (iii) first and third of same sex and the second is of opposite sex. **(06 Marks)**

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. (i) what is the probability that mathematics is being studied ? (ii) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? (iii) a boy? **(06 Marks)**

OR

10 a. State and prove Bayes theorem. **(04 Marks)**

b. A problem in mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved? **(06 Marks)**

c. A pair of dice is tossed twice. Find the probability of scoring 7 points. (i) Once, (ii) at least once (iii) twice. **(06 Marks)**

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