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10MAT11

First Semester B.E. Degree Examination, June/July 2017

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The n^{th} derivative of $\cos^2 x$ is

A) $2^n \cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^{n-1} \cos\left(2x + \frac{n\pi}{2}\right)$

C) $2^{n-1} \cos(2x + n\pi)$ D) $2^{n-1} \cos\left(\frac{n\pi}{2}\right)$

ii) The Maclaurin's series of $f(x) = K$ (constant) is

A) $f(x) = K$ B) $f(x) = 0$ C) does not exist D) $f(x) = K!$

iii) The value of C of the Cauchy mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[4, 5]$ is

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{1}{2}$

iv) The n^{th} derivative of $y = x^{n-1}.\log x$ is

A) $y_n = \frac{n!}{x}$ B) $y_n = \frac{(n+1)!}{x}$ C) $y_n = \frac{(n-1)!}{x}$ D) $y_n = \frac{n!}{x^2}$

b. If $x = \tan(\log y)$, prove that $(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$. **(06 Marks)**

c. Expand $\log(\sec x)$ by Maclaurin's series expansion upto the term containing x^4. **(05 Marks)**

d. State and prove Lagrange's mean value theorem. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\lim_{x\to\infty} \left[a^{1/x} - 1\right] x$ is of the following form

A) $0 \times \infty$ B) ∞^0 C) 0^∞ D) $\infty - \infty$

ii) If S is the arc length of the curve $x = g(y)$ then $\frac{ds}{dy}$ is

A) $\sqrt{1 + y_1}$ B) $\sqrt{1 + y_1^2}$

C) $\sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2}$ D) $\sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

iii) The angle between radius vector and the tangent for the curve $r = a(1 - \cos\theta)$ is

A) $\frac{\theta}{2}$ B) $-\frac{\theta}{2}$ C) $\frac{\pi}{2} + \theta$ D) $\frac{\pi}{2} - \frac{\theta}{2}$

iv) Two polar curves are said to be orthogonal if

A) $\phi_1 \cdot \phi_2 = 0$ B) $\tan\phi_1 \cdot \tan\phi_2 = -1$

C) $\frac{\phi_1}{\phi_2} = \frac{\pi}{2}$ D) $\phi_1 \cdot \phi_2 = -1$

b. If $y = \frac{ax}{a+x}$, then show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$ where ρ is the radius of curvature at any point (x, y). **(06 Marks)**

c. Evaluate $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]^{\frac{1}{x^2}}$. **(05 Marks)**

d. Derive an expression for the radius of curvature in polar form. **(05 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $z = x^2 + y^2$ then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

A) 0 B) 2 C) 2y D) 2x

ii) The Taylor's series of f(x, y) = xy at (1, 1) is

A) $1 + [(x-1)+(y-1)]$ B) $1 + [(x-1)+(y-1)] + [(x-1)(y-1)]$

C) $(x-1)(y-1)$ D) None of these

iii) If $z = f(x, y)$ then the relative error in z is

A) $\frac{\delta z}{x}$ B) $\delta z - y$ C) $\frac{\delta z}{z}$ D) $z - \delta z$

iv) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

A) r B) $\frac{1}{r}$ C) 1 D) −1

b. Find the extreme values of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(06 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$. **(05 Marks)**

d. The diameter and altitude of a can in the form of a right circular cylinder are found to be 4.5 cms and 8.25 cms respectively. The possible error in each measurement is 0.1 cm. Find the approximate error in the volume and lateral surface area. **(05 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The gradient, divergence, curl are respectively

A) scalar, scalar, vector B) vector, scalar vector

C) scalar, vector, vector D) vector, vector, scalar

ii) $\vec{F} = y^2 z\hat{i} + z^2 x\hat{j} + x^2 y\hat{k}$ is

A) constant vector B) solenoidal C) scalar D) none of these

iii) curl grad ϕ is

A) grad curl ϕ B) curl grad ϕ + grad curl ϕ

C) zero D) does not exist

iv) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then curl $\vec{r}$ is

A) 0 B) 1 C) −1 D) ∞

b. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find div$\vec{F}$ and curl $\vec{F}$. **(06 Marks)**

c. Prove that curl $(\phi\vec{F}) = \phi$ curl $\vec{F}$ + grad $\phi \times \vec{F}$. **(05 Marks)**

d. Prove that the cylindrical coordinate system is orthogonal. **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of the integral $\int_0^{\pi/2} \sin^7 x\,dx$ is

A) $\frac{35}{16}$ B) $\frac{16}{35}$ C) $-\frac{16}{35}$ D) $\frac{18}{35}$

ii) $x^2 + y^2 = x^2y^2$ is symmetric about

A) x - axis B) y - axis C) $y = x$ D) All A, B, C

iii) The value of $\int_0^{\pi} \sin^4 x dx$ is

A) $\frac{3\pi}{8}$ B) $\frac{3}{8}$ C) $\frac{\pi}{16}$ D) $\frac{\pi}{4}$

iv) Asymptote to the curve $y^2(a - x) = x^3$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) none of these

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ using differentiation under integral sign, find $\int_0^1 \frac{x^3 - 1}{\log x} dx$. **(06 Marks)**

c. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x dx$. **(05 Marks)**

d. Find the surface area generated by an arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the x-axis. **(05 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) For the differential equation $\left[\frac{d^3y}{dx^3}\right]^2 + \left[\frac{d^2y}{dx^2}\right]^6 + y = x^4$ the order and degree respectively are

A) 2, 6 B) 3, 2 C) 2, 4 D) none of these

ii) The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A) $e^x + e^{-y} = c$ B) $e^{-x} + e^{-y} = c$ C) $e^x + e^y = c$ D) $e^{x+y} = c$

iii) The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ where P, Q are functions Y is

A) $e^{\int pdy}$ B) $e^{\int pdx}$ C) $e^{\int Qdy}$ D) none of these

iv) If the differential equation of the given family remains unaltered after replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ then given family of curves is said to be

A) not orthogonal B) self orthogonal C) reciprocal D) none of these

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(06 Marks)**

c. Solve $\left[x\tan\left(\frac{y}{x}\right) - y\sec^2\left(\frac{y}{x}\right)\right]dx + x\sec^2\left(\frac{y}{x}\right)dy = 0$. **(05 Marks)**

d. Find the orthogonal trajectory of $r^n = a^n \sin n\theta$. **(05 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) Which of the following is not an elementary transformation

A) adding two columns
B) adding two rows
C) squaring all elements of the matrix
D) multiplying a row by a non-zero number

ii) The exact solution of the system of equations $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$ by inspection is

A) (-1, 1, 1) B) (-1, -1, -1) C) (1, 1, 1) D) (0, 0, 0)

iii) If r is the rank of the matrix [A] of order $m \times n$ then r is

A) $r \le$ minimum of (m, n)
B) $r \le n$
C) $r > n$
D) $r \ge m$

iv) Which of the following is in the normal form

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ D) all of these

b. Find the rank of the following matrix by reducing it to the normal form

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$ **(06 Marks)**

c. Find the value of K such that the following system equations possess a non-trivial solution

$4x+9y+z=0$, $Kx+3y+Kz=0$, $x+4y+2z=0$. **(05 Marks)**

d. Solve the following system of equations by Gauss Jordan method:

$x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$ **(05 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) A square matrix A is called orthogonal if

A) $A = A^2$ B) $A^1 = A$ C) $AA^1 = I$ D) none of these

ii) The eigen values of the matrix A exist if

A) A is a square matrix
B) A is singular matrix
C) A is any matrix
D) A is null matrix

iii) The matrix of the quadratic form $a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$ is

A) $\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{11} \end{bmatrix}$ B) $\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$ C) $\begin{bmatrix} 1 & a_{11} \\ a_{11} & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

iv) If the eigen vector is (1, 1, 1) then its normalized form is

A) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ B) $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ C) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ D) $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

b. Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ **(06 Marks)**

c. Diagonalize the matrix, $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(05 Marks)**

d. Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_2x_3$ into sum of squares. **(05 Marks)**

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14MAT11

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. Find the n^{th} derivative of $y = \sin^2 x \sin h^3 x + \log_{10}(x^2 - 3x + 2)$. **(07 Marks)**

b. Find the pedal equation for the curve $r = a + b\cos\theta$. **(06 Marks)**

c. Obtain radius of curvature for the parametric curve, $x = a(t - \sin t)$ $y = a(1 - \cos t)$. **(07 Marks)**

2 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. Hence obtain $y_n(0)$. **(07 Marks)**

b. Find the angle of intersection between the curves $r = 2\sin\theta$; $r = 2(\sin\theta + \cos\theta)$. **(06 Marks)**

c. Find the radius of curvature for the polar curve $r^2 = a^2 \cos 2\theta$. **(07 Marks)**

Module-2

3 a. Evaluate : $\lim\limits_{x\to 0}(\cos x)^{\cot^2 x}$. **(06 Marks)**

b. Determine Maclarin's series for the function for $f(x) = \log(1 + \cos x)$ upto term containing x^4. **(07 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then obtain the value of $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$. **(07 Marks)**

4 a. Find total derivative of u with respect to t where $u = \tan^{-1}x/y$, $x = e^t - e^{-t}$, $y = e^t + e^{-t}$. **(06 Marks)**

b. If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$, find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. Determine whether u, v and w are functionally dependent. **(07 Marks)**

c. If x y z be the angles of a triangle, show that the maximum value of sin x sin y sin z is $\frac{3\sqrt{3}}{8}$. **(07 Marks)**

Module-3

5 a. A particle moves along $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$, where 't' denotes time. Find the magnitudes of velocity and acceleration at time $t = 2$. **(07 Marks)**

b. Assuming the validity of differentiation under integral sign prove that $\int_0^\infty e^{-x^2}\cos\alpha x\,dx = \frac{\sqrt{\pi}}{2}e^{-\alpha^2/4}$. **(07 Marks)**

c. Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$, using general rules of tracing the curve. **(06 Marks)**

6 a. If $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ find curl $\vec{F}$. Is $\vec{F}$ irrotational? **(07 Marks)**

b. Prove that if $\vec{F}$ is a vector point function div (curl $\vec{F}$) = 0. **(07 Marks)**

c. If $\vec{r}$ is a position vector of a point in space obtain div $\vec{r}$ and curl $\vec{r}$. **(06 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(07 Marks)**

b. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$, where 'n' is a positive integer. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(06 Marks)**

8 a. Show that family $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ with λ as a parameter is self orthogonal. **(07 Marks)**

b. Evaluate : $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx$. **(07 Marks)**

c. Solve : $(y^2e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} + 3y^2)\,dy = 0$. **(06 Marks)**

Module-5

9 a. Solve by gauss elimination method :

$2x - 3y + 4z = 7$
$5x - 2y + 2z = 7$
$6x - 3y + 10z = 23$. **(07 Marks)**

b. Reduce the quadratic form : $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_12x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form by orthogonal transformation. **(07 Marks)**

c. Find the largest eigen value and corresponding eigen vector by Rayeligh's power method performing five iterations, with $x^{(0)} = [1, 1, 1]^T$ for $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **(06 Marks)**

10 a. Solve by LU decomposition method :

$10x + y + z = 12$
$2x + 10y + z = 13$
$2x + 2y + 10z = 14$. **(07 Marks)**

b. Diagonalze the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. Hence find A^4. **(07 Marks)**

c. Solve by Gauss Seidel iteration method :

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$
Perform 3 iterations. **(06 Marks)**

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CBCS Scheme

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15MAT11

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the n^{th} derivative of $\frac{x}{(x-1)^2(x+2)}$. **(06 Marks)**

b. Find the angle of intersection of the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(05 Marks)**

c. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. **(05 Marks)**

OR

2 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that $(x^2-1)\, y_{n+2} + (2n+1)\, xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Obtain the pedal equation of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

c. Find the derivative of arc length of $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right)$ and $y = a\sin t$. **(05 Marks)**

Module-2

3 a. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e(1.1)$, correct to four decimal places. **(06 Marks)**

b. If $z = \sin(ax+y) + \cos(ax-y)$, prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$. **(05 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

OR

4 a. If $u(x+y) = x^2 + y^2$, then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$. **(06 Marks)**

b. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$. **(05 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\, u_x + y\, u_y + z\, u_z = 0$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{f} = (x + y + az)\,\hat{i} + (bx + 2y - z)\,\hat{j} + (x + cy + 2z)\,\hat{k}$, find a, b, c such that $\vec{f}$ is irrotational. **(05 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point P(2, -1, 2). **(05 Marks)**

OR

6 a. Find the directional derivative of $xy^3 + yz^3$ at (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{u} \times \vec{v}$ is a solenoidal vector. **(05 Marks)**

c. For any scalar field ϕ and any vector field $\vec{f}$, prove that curl $(\phi\vec{f}) = \phi$ curl $\vec{f}$ + (grad ϕ) $\times$ $\vec{f}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x \, dx$, where n is a positive integer, hence evaluate $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)\, dx + xy dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of circles $r = 2a\cos\theta$, where 'a' is a parameter. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^{9/2}} dx$. **(06 Marks)**

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(05 Marks)**

c. Water at temperature 10^0C takes 5 minutes to warm upto 20^0C in a room temperature 40^0C. Find the temperature after 20 minutes. **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss Elimination Method. **(06 Marks)**
$x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$.

b. Find the dominant eigen value and the corresponding eigen vector by power method $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, perform 5 iterations, taking initial eigen vector as $[1\ 1\ 1]^1$. **(05 Marks)**

c. Show that the transformation $y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Write down the inverse transformation. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss – Seidel method. **(06 Marks)**
$10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$.

b. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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10MAT21

Second Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions on any single page of the answer booklet.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) Solution of one of the factors of $p^2 - p - 1 = 0$ with $p = \frac{dy}{dx}$ is y = ______.

A) $\left(\frac{1+\sqrt{5}}{2}\right)x^4 + C$, B) $\left(\frac{2+\sqrt{5}}{2}\right)x^3 + C$ C) $\left(\frac{2-\sqrt{5}}{2}\right)x^2 + C$ D) $\left(\frac{1-\sqrt{5}}{2}\right)x + C$

ii) On solving for x in $P = \tan\left(x - \frac{P}{1+P^2}\right)$ the solution for y = ______.

A) $C + \frac{1}{1+P^2}$ B) $C - \frac{2}{1+P^2}$ C) $C - \frac{3}{1+P^2}$ D) $C - \frac{1}{1+P^2}$

iii) The solution for Clairut's form of the differential equation, $(y - Px)(P - 1) = P$ is y = ______.

A) $Cx - \frac{C}{C-1}$ B) $Cx + \frac{C}{C-1}$ C) $C^2 - \frac{Cx}{C-1}$ D) $C^2 + \frac{Cx}{C-1}$

iv) If the given equation is solvable for y then it is of the form,

A) $y = f(x, p)$ B) $x = f(y, p)$ C) $x = f(y/p)$ D) $x = f(p/y)$

b. Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$ with solvable for P. **(05 Marks)**

c. Solve $(px - y)(py + x) = \alpha^2 p$ with $x^2 = u$ and $y^2 = v$ using Clairut's form. **(05 Marks)**

d. Solve $y = x + a\tan^{-1} p$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) Solution of $(D^3 - 2D + 4)y=0$ is y = ______.

A) $C_1e^{2x} + C_2e^{-x}\cos x + C_3e^{-x}\sin x$ B) $C_1e^{2x} + C_2e^{-x}\cos x$
C) $C_1e^{-2x} + C_2e^{x}\cos x + C_3e^{x}\sin x$ D) $C_1\cos x + C_2\sin x$

ii) Particular integral of $(D^2 + 1)y = \sin 2x$ is y_p = ______.

A) $\frac{1}{3}\sin 2x$ B) $\sin 2x$ C) $\cos 2x$ D) $-\frac{1}{3}\sin 2x$

iii) Particular integral of $(D - 1)y = \sinh x$ is y = ______.

A) $\frac{1}{2}(xe^{x} + e^{-x})$ B) $\frac{1}{2}xe^{-x}$ C) $\frac{1}{2}(e^{-x} + e^{x})$ D) $\frac{1}{2}$

iv) The displacement in the simple harmonic motion $\frac{d^2x}{dt^2} = -\mu^2 x$ is ______.

A) $C_1\cos\mu t - C_2\sin\mu t$ B) $C_1\cos\mu t + C_2\sin\mu t$
C) $C_1\cos t + C_2\sin t$ D) $C_1\cos t - C_2\sin t$

b. Solve $(D^2 - 6D + 13)y = 8e^{3x}\sin 4x + 2^x$. **(05 Marks)**

c. Solve $y'' - 2y' + y = xe^x \sin x$. **(05 Marks)**

d. Solve $(D + 3)x + (D + 1)y = e^t$ and $(D + 1)x + (D - 1)y = t$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) Particular solution of $(D^2-1)y=1$ using variation of parameters is y_p = ______ .

A) -1 B) -2 C) -3 D) -4

ii) The differential equation, $x^3y''' + x^2y'' = \log x$ reduces to the form when $x = e^t$ as,

A) $(D+1)^3 y = t$ B) $D(D-1)^2 y = t$ C) $D^3 y = 0$ D) $D^2 y = 0$

iii) The complementary function of, $(1+x)^2 y'' + (1+x)y' + y = 2\sin\log(x+1)$ with $(1+x)=e^t$ is y_c = ______ .

A) $C_1\cos t - C_2 \sin t$ B) $C_1\cos 2t + C_2\sin 2t$

C) $C_1\cos t + C_2\sin t$ D) $C_1\cos 2t - C_2\sin 2t$

iv) In $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$, if $P_0(x)=0$, then it has

A) Singular B) Regular singularity C) Exact D) Homogeneous

b. Solve $(D^2-3D+2)y = \dfrac{1}{1+e^{-x}}$ using variation of parameters. **(05 Marks)**

c. Solve $x^2y'' - x\dfrac{dy}{dx} + y = \log x$. **(05 Marks)**

d. Solve $xy'' + y' + xy = 0$ using Frobenius series solution method. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The partial differential equation of the relation $z = ax + a^2y^2 + b$ is q = ______ .

A) p^2y B) $2p^2y$ C) p^2y^2 D) $2py^2$

ii) The solution of $\dfrac{\partial^2 z}{\partial x^2} = z$ is z = ______ .

A) $C_1(x)e^y + C_2(x)e^{-y}$ B) $C_1(x)e^y - C_2(x)e^{-y}$

C) $C_1(y)e^x + C_2(y)e^{-x}$ D) $C_1(y)e^x - C_2(y)e^{-x}$

iii) The solution of $yq - xp = z$ by Lagrange's method is ______ $= 0$

A) $f\left(\dfrac{x}{y}, \dfrac{y}{z}\right)$ B) $f\left(\dfrac{y}{x}, \dfrac{y}{z}\right)$ C) $f\left(xyz, \dfrac{y}{z}\right)$ D) $f\left(xy, \dfrac{y}{z}\right)$.

iv) The solution of $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$ by separation of variables with K as the common solution is u = ______ .

A) $Ce^{K(x+y)}$ B) Ce^{Kxy} C) $Ce^{K(x+y)}$ D) Ce^{y}

b. Form the partial differential equation from the relation $f(x+y+z, x^2+y^2+z^2)=0$. **(05 Marks)**

c. Solve $y^2p - xyq = x(z-2y)$ using Lagrange's linear form. **(05 Marks)**

d. Solve $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0$ using separation of variables. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^1\int_0^6 xy\,dx\,dy$ is ______ .

A) 6 B) 7 C) 8 D) 9

ii) Area of the ellipse by double integration is = ______ .

A) $\pi(a+b)$ B) $\pi(a-b)$ C) πab D) $\pi(b-a)$

iii) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right) =$ ________ .

A) $\sqrt{\pi}$ B) π C) $\pi + 1$ D) $\frac{\pi}{2}$

iv) The value of $\Gamma\left(\frac{1}{4}\right)\times\Gamma\left(\frac{3}{4}\right) =$ ________

A) $\pi\sqrt{2}$ B) $2\sqrt{\pi}$ C) $\sqrt{2\pi}$ D) 2π

b. Change the order of integration in, $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate. **(05 Marks)**

c. Evaluate $\int_1^e \int_1^{\log_e y} \int_1^{e^x} \log z\,dz\,dx\,dy$. **(05 Marks)**

d. Define Beta and Gamma functions, derive the relation as $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\int_C F.d\vec{r} = 0$ then F is called,

A) Singular B) Irrotational C) Solenoidal D) Domain

ii) In Green's theorem $\iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial x}\right) dxdy =$ ________ .

A) $F_1 + F_2$ B) $\int_C (F_1 dx + F_2 dx)$ C) $\int_S F_1 dx + F_2 dy$ D) $\int_c (F_1 dx + F_1 dy)$

iii) In Stoke's theorem $\int_C F.dR =$ ________ .

A) $\int_C \text{curl}F.Nds$ B) $\int_C \text{div}F.Nds$ C) $\int_C \text{grad}F.Nds$ D) $\int_S \text{curl}F.Nds$

iv) If $\vec{F} = x^3 i + y^3 j + z^3 k$ then divF = ________

A) $x^2 + y^2 + z^2$ B) $2(x^2 + y^2 + z^2)$ C) $3(x^2 + y^2 + z^2)$ D) $3(x^2 i + y^2 j + z^2 k)$

b. If $F = (3x^2 + 6y)i - 14yzj + 20xz^2 k$, evaluate $\int_C F.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t, y = t^2, z = t^3$. **(05 Marks)**

c. Use Green's theorem to evaluate $\int_C (y - \sin x)dx + \cos x dy$, where C is the triangle in xy-plane bounded by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. **(05 Marks)**

d. Use Gauss divergence theorem to evaluate $\int_S F.Nds$ where $F = 4xyi + yzj - xzk$ and S is the surface of the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$. **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) $L\{e^{-t}\cosh t\} =$ ________

A) $\frac{s+1}{(s-1)^2+1}$ B) $\frac{s-1}{(s+1)^2+1}$ C) $\frac{s+1}{(s-1)^2-1}$ D) None of these

ii) $L\{t^{-1}f(t)\} =$ __________

A) $\int_s^\infty F(s)ds$ B) $\int_s^\infty f(t)dt$ C) $\int_t^\infty F(s)ds$ D) $\int_0^\infty f(t)dt$

iii) When T denotes period of the function f(t) then, $\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt =$ __________.

A) f(t) + C B) $L\{f(t)e^{-st}\}$ C) $L\{f(t)\}$ D) $L\{e^t\}$

iv) In unit step function if $u(t-a)=0$ then,

A) $t < a$ B) $t \geq a$ C) $t = a$ D) $t \leq a$

b. Find the Laplace transform of the function $f(t) = te^{-t}\sin^2 3t$. **(05 Marks)**

c. Find the Laplace transform of the function,

$$f(t) = \begin{cases} t, & \text{for } 0 < t \leq a \\ 2a - t, & \text{for } a < t < 2a \end{cases}, \text{ 2a is the period.}$$ **(05 Marks)**

d. Express f(t) in terms of unit step function and find the Laplace transform when,

$$f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4. \\ 8, & t > 4 \end{cases}$$ **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\} =$ __________.

A) $1-3t^2+2t$ B) $\frac{1+3t}{2t^2}$ C) $1-3t+2t^2$ D) $\frac{1+2t^2}{3t}$

ii) The value of $L^{-1}\left\{\frac{s}{(s-2)^2}\right\} =$ __________

A) $e^{2t}(1-2t)$ B) $e^{2t}(1+2t)$ C) $e^{2t}(2+2t)$ D) $2+2t$

iii) By convolution theorem $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\} =$ __________.

A) $\int_0^\infty e^{-t}e^{t-2}dt$ B) $\int_0^t e^{-t}e^{-(t^2-2)}dt$ C) $\int_0^t e^{-u}e^{-(t-u)(2)}du$ D) $\int_0^\infty e^{-2t}e^{t^2+1}dt$

iv) Laplace transform of $\frac{dy}{dt} + y = 0$ with y(0) = 1 is =__________

A) e^{-t} B) e^t C) te^t D) $\frac{e^t}{t}$

b. Find the inverse Laplace transform of, $F(s) = \frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. **(05 Marks)**

c. Find $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$ by convolution theorem. **(05 Marks)**

d. Solve by Laplace transform method, $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + 12e^{-t}$ with $y(0) = 6$, $y'(0) = -1$ **(06 Marks)**

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14MAT21

Second Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. Solve $(D^2 + 4D + 5)y = -2\cosh x$. **(06 Marks)**

b. Solve $\dfrac{d^4y}{dx^4} + m^4 y = 0$. **(07 Marks)**

c. Solve, by using method of variation of parameters, $y'' + 4y = \tan 2x$. **(07 Marks)**

2 a. Solve $y''' - 6y'' + 11y - 6y = 1 + x + \sin x$. **(06 Marks)**

b. Solve, by using method of undermined coefficients, $y'' - 3y' + 2y = 4x^2$. **(07 Marks)**

c. Solve, by using method of variation of parameters, $(D^2 + 2D + 1) = e^{-x} \log x$. **(07 Marks)**

Module-2

3 a. Solve, $x^3y''' + 2x^2y'' + 2y = 10\left(x + \frac{1}{x}\right)$. **(06 Marks)**

b. Solve, $x^2p^2 + xyp - 6y^2 = 0$ for p. **(07 Marks)**

c. Solve, $(px - y)(py + x) = 2p$ by substituting $X = x^2$, $Y = y^2$ and also find its singular solution. **(07 Marks)**

4 a. Solve $(2x + 3)^2y'' - 2(2x + 3)y' - 12y = 6x$. **(06 Marks)**

b. Solve $y = 2px + y^2p^3$. **(07 Marks)**

c. Solve the following simultaneous linear equations :
$(D + 4)x + 3y = t$ and $2x + (D + 5)y = e^t$. **(07 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating arbitrary function, f from the relation :
$z = f\left(\dfrac{xy}{z}\right)$. **(06 Marks)**

b. Change the order of integration in $I = \displaystyle\int_{y=0}^{a}\int_{x=y}^{a} \frac{x}{x^2 + y^2}\,dxdy$ and hence evaluate the same. **(07 Marks)**

c. Solve the one dimensional wave equation $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$ by variable separable method. **(07 Marks)**

6 a. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that $z = e^x$, $\frac{\partial z}{\partial y} = e^{-x}$ at $y = 0$. **(06 Marks)**

b. Change into polar co-ordinates and evaluate : $I = \int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dydx$. **(07 Marks)**

c. Evaluate : $I = \int_{x=1}^{3} \int_{y=\frac{1}{x}}^{1} \int_{z=0}^{\sqrt{x}} xyz \, dzdydx$. **(07 Marks)**

Module-4

7 a. By using double integral, find the area bounded by the co-ordinate axes and the line $x + y = 2$. **(06 Marks)**

b. State and prove the relation between Beta and Gamma functions. **(07 Marks)**

c. Find the spherical polar co-ordinate system defined by $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$ and also prove that spherical polar co-ordinate system is orthogonal. **(07 Marks)**

8 a. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by using triple integral. **(06 Marks)**

b. Evaluate : $\int_0^a x^4\sqrt{a^2 - x^2}\,dx$ by using Beta and Gamma functions. **(07 Marks)**

c. Derive expression for div $\overline{A}$ in orthogonal curvilinear coordinates. **(07 Marks)**

Module-5

9 a. Find : $L(e^{t} \sin 6t + t\cos 3t)$. **(06 Marks)**

b. Find : $L^{-1}\left\{\frac{s-1}{s(s^2-2s+5)}\right\}$. **(07 Marks)**

c. Solve, by using Laplace transforms, $y''' + 2y'' - y' - 2y = 0$, where $y = 1$, $\frac{dy}{dt} = 2 = \frac{d^2y}{dt^2}$ at $t = 0$. **(07 Marks)**

10 a. Evaluate : $\int_0^{\infty} te^{-3t}\cos 2t \, dt$, by using Laplace transforms. **(06 Marks)**

b. If $f(t) = \begin{cases} E\sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ is periodic function then find $L(f(t))$. **(07 Marks)**

c. Find: $L^{-1}\left(\frac{s}{(s-1)(s^2+4)}\right)$ using convolution theorem. **(07 Marks)**

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CBCS Scheme

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15MAT21

Second Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve : $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$. **(05 Marks)**

b. Solve : $(D^2 - 4D + 3)y = e^{2x}.\cos 3x$. **(05 Marks)**

c. Apply the method of undetermined coefficients to solve $y'' - 3y' + 2y = x^2 + e^x$. **(06 Marks)**

OR

2 a. Solve : $(D^4 - 1)y = 0$. **(05 Marks)**

b. Solve : $(D^2 - 4D + 4)y = 8(e^{2x} + \sin 2x)$. **(05 Marks)**

c. By the method of variation of parameters solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$. **(06 Marks)**

Module-2

3 a. Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(05 Marks)**

b. Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **(05 Marks)**

c. Solve $(px - y)(py + x) = 2p$ by reducing it into the Clairauit's form by taking the substitution $X = x^2$, $Y = y^2$. **(06 Marks)**

OR

4 a. Solve : $(1+x^2)y'' + (1+x)y' + y = \sin\{\log(1+x)^2\}$. **(05 Marks)**

b. Obtain the general solution and the singular solution of the equation $p^2 + 4x^5p - 12x^4y = 0$. **(05 Marks)**

c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is a Clairauit's equation. Hence obtain the general solution and the singular solution. **(06 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating ϕ and ψ from the relation $z = x\phi(y) + y\psi(x)$. **(05 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} - a^2 z = 0$ under the conditions $z = 0$ when $x = 0$ and $\frac{\partial z}{\partial x} = a\sin y$ when $x = 0$. **(05 Marks)**

c. Derive an expression for the one dimensional heat equation. **(06 Marks)**

OR

6 a. Form a partial differential equation by eliminating ϕ from $\phi(x+y+z, xy+z^2) = 0$. **(05 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given that $\frac{\partial z}{\partial y} = -2\sin y$, when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **(05 Marks)**

c. Use the method of separation of variables to solve the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(06 Marks)**

Module-4

7 a. By changing the order of integration, evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$. **(05 Marks)**

b. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\,dx\,dy\,dz$. **(05 Marks)**

c. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ using definition of $\Gamma(n)$. **(06 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates. **(05 Marks)**

b. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{a^2 - x^2 - y^2 - z^2}}$. **(05 Marks)**

c. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta . \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(06 Marks)**

Module-5

9 a. Find the Laplace transform of,

$2^t + \frac{\cos 2t - \cos 3t}{t} + t\sin t$. **(05 Marks)**

b. A periodic function of period 2a is defined by, $f(t) = \begin{cases} E & \text{for } 0 \le t \le a \\ -E & \text{for } a < t \le 2a \end{cases}$ where E is a constant. Show that $L\{f(t)\} = \frac{E}{S}\text{Tanh}\left(\frac{aS}{2}\right)$. **(05 Marks)**

c. Find $L^{-1}\left\{\log\left[\frac{s^2+1}{s(s+1)}\right]\right\}$. **(06 Marks)**

OR

10 a. Express $f(t) = \begin{cases} \sin t, & 0 < t \le \frac{\pi}{2} \\ \cos t, & t > \frac{\pi}{2} \end{cases}$ in terms of unit step function and hence find its laplace transform. **(05 Marks)**

b. By using the convolution theorem find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$. **(05 Marks)**

c. By using Laplace transforms, solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{2t}$, $x(0) = 0$, $\frac{dx}{dt}(0) = -1$. **(06 Marks)**

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Fourth Semester B.E. Degree Examination, June/July 2017

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: *1 Answer FIVE full questions, selecting atleast TWO questions from each part.*
2. Use of standard normal tables permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using Taylor's series method, find y(0.1) given that $\frac{dy}{dx} = x^2y - 1$, y(0) = 1 considering up to 4^{th} degree terms. **(07 Marks)**

b. Apply Runge-Kutta method to find an approximate value of y for x = 0.2 insteps of 0.2 given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1. **(07 Marks)**

c. Calculate y(1.4) using Adam-Bashforth method if $\frac{dy}{dx} = x^2(1 + y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979. **(06 Marks)**

2 a. Prove that if f(z) = u + iv is analytic, then the family of curves u(x, y) = c_1 and v(x, y) = c_2 where c_1 and c_2 are constants, intersect each other orthogonally. **(07 Marks)**

b. Show that u = e^x(x cos y – y sin y) is harmonic and find its harmonic conjugate. Also determine the corresponding analytic function. **(07 Marks)**

c. Find the bilinear transformation which maps z = ∞, i, 0 into w = –1, –i, 1, respectively. Also find the fixed points of the transformation. **(06 Marks)**

3 a. State and prove Cauchy's theorem. **(07 Marks)**

b. Expand $f(z) = \frac{z+1}{(z+2)(z+3)}$ in Laurent's series valid for : i) | z | > 3 ii) 2 < | z | < 3. **(07 Marks)**

c. Evaluate $\int_c \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$ where c : | z | = 4 using Cauchy's residue theorem. **(06 Marks)**

4 a. Obtain the power series solution of the equation y″ + xy′ + y= 0. **(07 Marks)**

b. Obtain the series solution of Bessels differential equation in the form y = $AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

c. Using Rodrigue's formula for the Legendre polynomials obtain expressions for $P_0(x)$, $P_1(x)$, $P_3(x)$, $P_4(x)$. Hence express x^4 in terms of Legendre polynomial. **(06 Marks)**

PART – B

5 a. Fit a least square parabola to the following data : **(07 Marks)**

x	0.0	0.2	0.4	0.7	0.9	1.0
y	1.016	0.768	0.648	0.401	0.212	0.193

b. In a partially destroyed laboratory record of correlation data, the following results are only available. Variance of x is 9. Regression equations are $4x - 5y + 33 = 0$, $20x - 9y = 107$. Calculate :
i) The mean values of x and y
ii) Standard deviation of y
iii) Correlation coefficient between x and y. **(07 Marks)**

c. There are three bags : first containing 1 white, 2 red and 3 green balls ; second 2 white, 3 red, 1 green balls and the third 3 white 1 red and 2 green balls. A bag is chosen at random and two balls are drawn from it. These are found to be white and red. Find the probability that the balls so drawn came from the second bag. **(06 Marks)**

6 a. The probability distribution of X is given : **(07 Marks)**

X	–3	–2	–1	0	1	2
P(X)	0.01	0.06	0.4	R	0.3	0.02

Find R, E(X) and variance of X.

b. Show that the mean and the variance of the Poisson distribution are equal. **(07 Marks)**

c. The marks of 500 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks are :
i) less than 65 ii) more than 75 iii) between 65 and 75. **(06 Marks)**

7 a. Find how many heads in 64 tosses of a coin will ensure its fairness at 0.05 level of significance. **(07 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.
5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4
Can it be concluded that the stimulus will increase the blood pressure.
(Hint : $t_{0.05}$ for 11 d.f. = 2.201). **(07 Marks)**

c. Define the following :
i) Tests of significance and confidence limits ii) One tailed and two tailed tests. **(06 Marks)**

8 a. The joint distribution of two random variables X and Y is given by the following table :

X\Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of X and Y. Also, verify that X and Y are independent. **(07 Marks)**

b. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector. **(07 Marks)**

c. The transition matrix P of a Markov chain is given by $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ with the initial probability distribution $P^{(0)} = (\frac{1}{4}, \frac{3}{4})$. Define and find the following :
i) $P_{21}^{(2)}$ ii) $P^{(2)}$ iii) $P_1^{(2)}$ iv) the vector $P^{(0)}P^n$ approaches v) the matrix p^n approaches. **(06 Marks)**

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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Find y(0.1) by using Taylor's series method, given that $y' = \sqrt{x^2 + y}$, $y(0) = 0.8$. Consider upto third order derivative terms. **(06 Marks)**

b. Given : $\frac{dy}{dx} = \frac{1}{1+x^2} - 2y^2$, y(0) = 0. Find y(0.5), by taking h = 0.25, using Euler's modified method. **(07 Marks)**

c. If $y' = \frac{1}{x+y}$, $y(0) = 2.0000$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$, find y(0.8) by using Adams-Bash forth method. **(07 Marks)**

2 a. Using the Picard's method, obtain the 2nd order approximate solution of the problem at x = 0.2, $\frac{dy}{dx} = x + yz$; $\frac{dz}{dx} = y + zx$, $y(0) = 1$ and $z(0) = -1$. **(06 Marks)**

b. Using the R-K method, find the solution at x = 0.1 of an equation; $y'' - x^2y' - 2xy - 1 = 0$ with the conditions y(0) = 1, $y'(0) = 0$ and step size 0.1. **(07 Marks)**

c. Given that $y'' + xy = 0$, $y(0) = 1$, $y(0.1) = 1.0998$, $y(0.2) = 1.1987$, $y(0.3) = 1.2955$, $y'(0) = 1$, $y'(0.1) = 0.9946$, $y'(0.2) = 0.9773$, $y'(0.3) = 0.946$, find y(0.4), using Milne's method. (Apply corrector formula only once). **(07 Marks)**

3 a. Derive Cauchy-Riemann equations in the polar form. **(06 Marks)**

b. If $f(z) = u + iv$ is an analytic function, then prove that the family of curves; u(x, y) = C_1, v(x, y) = C_2, C_1 and C_2 being constants, interfect orthogonally. Is the converse true? Justify your answer. **(07 Marks)**

c. In a two dimensional fluid flow; if the velocity potential is $e^{-x}\cos y + xy$, find the stream function. **(07 Marks)**

4 a. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points w = i, 0, -i. Also find the invariant points. **(06 Marks)**

b. Discuss the transformation, $w = z + \frac{K^2}{z}$, where $z \neq 0$, $K \neq 0$. **(07 Marks)**

c. State and prove the Cauchy's theorem. **(07 Marks)**

PART – B

5 a. Obtain the series solution of Bessel's differential equation. **(07 Marks)**

b. Derive the Rodrigue's formula. **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(06 Marks)**

6 a. 'A' can hit a target 3 times in 5 shots, 'B' 2 times in 5 shots and 'C' 3 times in 4 shots. They fire a volley. Find the probability that (i) 2 shots hit (ii) at least 2 shots hit. **(06 Marks)**

b. If A and B are events with $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{4}$, $P(\overline{B}) = \frac{5}{8}$ find $P(A \cap B)$, $P(\overline{A} \cap \overline{B})$, $P(\overline{A} \cup \overline{B})$ and $P(\overline{A} \cap B)$. **(07 Marks)**

c. State and prove Baye's theorem. **(07 Marks)**

7 a. (i) Is the function defined as follows a density function? $f(x) = e^{-x}, x \geq 0$, $f(x) = 0$, $x < 0$.

(ii) If so, determine the probability that the variate having this density will fall in the interval (1, 2).

(iii) Also find the cumulative probability function F(2). **(06 Marks)**

b. Obtain the mean and standard deviation of the Poisson distribution. **(07 Marks)**

c. The life of an electric bulb is normally distributed with mean life of 200 hours and S.D. of 60 hours. Out of 2500 bulbs, find the number of bulbs which are likely to last between 1900 and 2100 hours. Given that $P(0 < Z < 1.67) = 0.4525$. **(07 Marks)**

8 a. Explain the following terms briefly: (i) Null hypothesis (ii) Type I and Type II errors (iii) Confidence limits. **(06 Marks)**

b. Two types of batteries are tested for their length of life and the following results are obtained:

Battery A : $n_1 = 10$, $\overline{x}_1 = 500$ hrs, $\sigma_1^2 = 100$

Battery B : $n_2 = 10$, $\overline{x}_2 = 560$ hrs, $\sigma_2^2 = 121$.

Find students 't' and test whether there is a significant difference in the two means. ($t_{0.05} = 2.10$ and $t_{0.01} = 2.88$). **(07 Marks)**

c. Genetic theory states that children having one parent of blood type M and the other of blood type N will always be one of the three types M, MN, N and that the proportions of these types will on an average be 1 : 2 : 1. A report states that out of 300 children having one M parent and one N parent, 30% are found to be of type M, 45% of type MN and the remainder of type N. Test the theory by ψ^2 (chi-square) test. **(07 Marks)**

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CBCS Scheme

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15MATDIP31

Third Semester B.E. Degree Examination, June/July 2017
Additional Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Express $\frac{3+4i}{3-4i}$ in the form $x + iy$. **(06 Marks)**

b. Express $\sqrt{3} + i$ in the polar form and hence find their modulus and amplitudes. **(05 Marks)**

c. Find the sine of the angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$. **(05 Marks)**

OR

2 a. Simplify **(06 Marks)**

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 + (\cos 5\theta + i\sin 5\theta)^{-4}}$$

b. If $\vec{a} = i + 2j - 3k$ and $\vec{b} = 3i - j + 2k$, then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal. **(05 Marks)**

c. Find the value of λ, so that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are co-planar. **(05 Marks)**

Module-2

3 a. If $y = \cos(m \log x)$ then prove that $x^2 y_{n+2} + (2n+1) x y_{n+1} + (m^2 + n^2) y_n = 0$. **(06 Marks)**

b. With usual notation prove that

$\tan\phi = \frac{rd\theta}{dr}$. **(05 Marks)**

c. If $u = \log_e\left(\frac{x^4 + y^4}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(05 Marks)**

OR

4 a. Find the Pedal equation of $r = a[1 - \cos\theta]$. **(06 Marks)**

b. Expand $\log_e(1 + x)$ in ascending powers of x as far as the term containing x^4. **(05 Marks)**

c. Find the total derivative of $Z = xy^2 + x^2 y$, where $x = at^2$ $y = 2at$. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi/6} \sin^6 3x \, dx$ using Reduction formula. **(06 Marks)**

b. Evaluate $\int_0^1 x^6 \sqrt{1 - x^2} \, dx$ – using Reduction formula. **(05 Marks)**

c. Evaluate $\int_1^2 \int_0^{2-y} xy\,dx\,dy$. **(05 Marks)**

OR

6 a. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi} x \cos^6 x \, dx$. **(05 Marks)**

c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z) \, dz \, dx \, dy$. **(05 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = (1-t^3)\hat{i} + (1+t^2)\hat{j} + (2t - 5)\hat{k}$. Determine the velocity and acceleration. **(06 Marks)**

b. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (2,-1, 1) in the direction of the vector i + 2j + 2k. **(05 Marks)**

c. Find the constant a, b, c. Such that the vector
$\vec{F} = (x + y + az)\ \hat{i} + (x + cy + 2z)\ \hat{k} + (bx + 2y - z)\hat{j}$ is irrotational. **(05 Marks)**

OR

8 a. Find the angle between the tangents to the curve $\vec{r} = t^2\ \hat{i} + 2t\hat{j} - t^3\hat{k}$ at the points $t = \pm 1$. **(06 Marks)**

b. Find the divergence and curl of the vector
$\vec{F} = (xyz + y^2z)\ \hat{i} + (3x^2 + y^2z)\ \hat{j} + (xz^2 - y^2z)\ \hat{k}$. **(05 Marks)**

c. If $\vec{F} = (ax + 3y + 4z)\ \hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\ \hat{k}$ is solenoidal, find a. **(05 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. **(05 Marks)**

c. Solve $\frac{dy}{dx} = \frac{x + 2y - 1}{x + 2y + 1}$. **(05 Marks)**

OR

10 a. Solve $(x^2 - y^2)\, dx = 2xy\, dy$. **(06 Marks)**

b. Solve $x \frac{dy}{dx} + y = x^3 y^6$. **(05 Marks)**

c. $(1 + xy)\, ydx + (1 - xy)\, xdy = 0$. **(05 Marks)**

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CBCS Scheme

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15MAT31

Third Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$ **(08 Marks)**

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

b. Obtain the constant term and first sine and cosine terms in the Fourier expansion of y from the following table. **(08 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

OR

2 a. Expand f(x) = |x| as a Fourier series in $-\pi \le x \le \pi$ and deduce that **(06 Marks)**

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

b. Obtain the half range cosine series for the function f(x) = x sin x in $0 < x < \pi$. **(05 Marks)**

c. The following table gives variations of periodic current over a period T. Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of first harmonic. **(05 Marks)**

t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A (amp)	1.98	1.3	1.05	1.3	-0.88	-0.25

Module-2

3 a. Find the Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$ **(06 Marks)**

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx$.

b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(05 Marks)**

c. Find the inverse Z – transform of

$\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(05 Marks)**

OR

4 a. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$. **(06 Marks)**

b. Find the Z – transform of i) $\cosh n\theta$ ii) n^2. **(05 Marks)**

c. Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$. **(05 Marks)**

Module-3

5 a. Find the coefficient of correlation and two regression lines for the following data : **(06 Marks)**

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

b. Fit a curve of the form $y = ae^{bx}$ for the following data : **(05 Marks)**

x	5	6	7	8	9	10
y	133	55	23	7	2	2

c. Use Newton – Raphson method to find a real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. **(05 Marks)**

OR

6 a. In a partially destroyed lab record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and coefficient of correlation between x and y. **(06 Marks)**

b. Fit a second degree parabola to the following data : **(05 Marks)**

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

c. Use the regula – falsi method to obtain a root of the equation $2x - \log_{10}x = 7$ which lies between 3.5 and 4. Carryout 2 iterations. **(05 Marks)**

Module-4

7 a. The population of a town is given by the table **(06 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using Newton's forward and backward interpolation formula, calculate the increase in the population from the year 1955 to 1985.

b. Use Lagrange's interpolation formula to find y at x = 10, given **(05 Marks)**

x	5	6	9	11
y	12	13	14	16

c. Given the values

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

Construct the interpolating polynomial using Newton's divided difference interpolation formula. **(05 Marks)**

OR

8 a. From the following table, estimate the number of students who obtained marks between 40 and 45. **(06 Marks)**

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

b. Apply Lagrange's formula inversely to obtain the root of the equation f(x) = 0, given f(30) = -30 , f(34) = -13 , f(38) = 3 , f(42) = 18. **(05 Marks)**

c. Use Simpson's $\frac{1}{3}$ rule to find $\int_0^{0.6} e^{-x^2}\,dy$ by taking 7 ordinates. **(05 Marks)**

Module-5

9 a. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,i + (2xz - y)j + z\,k$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from x = 0 to x = 2. **(06 Marks)**

b. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xy\,j$ around the rectangle $x = \pm a$, y = 0 , y = b. **(05 Marks)**

c. Solve the Euler's equation for the functional $\int_{x_0}^{x_1} (1 + x^2 y^1) y^1 dx$. **(05 Marks)**

OR

10 a. Verify Green's theorem for $\int_c (xy + y^2)dx + x^2 dy$, where c is bounded by y = x and $y = x^2$. **(06 Marks)**

b. Evaluate the surface integral $\iint_s \vec{F}\,.\,N ds$ where $\vec{F} = 4xi - 2y^2 j + z^2 k$ and s is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. **(05 Marks)**

c. Show that the shortest distance between any two points in a plane is a straight line. **(05 Marks)**

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CBCS Scheme

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15MAT41

Fourth Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics–IV

Time: 3 hrs. Max. Marks: 80

Note: **1.** ***Answer FIVE full questions, choosing one full question from each module.***
2. ***Use of statistical tables are permitted.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find by Taylor's series method the value of y at x = 0.1 from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ (upto 4^{th} degree term). **(05 Marks)**

b. The following table gives the solution of $5xy' + y^2 - 2 = 0$. Find the value of y at x = 4.5 using Milne's predictor and corrector formulae. **(05 Marks)**

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

c. Using Euler's modified method. Obtain a solution of the equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, with initial conditions y = 1 at x = 0, for the range $0 \le x \le 0.4$ in steps of 0.2. **(06 Marks)**

OR

2 a. Using modified Euler's method find y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ taking $h = 0.2$. **(05 Marks)**

b. Given $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) =1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adams-Bashforth method. **(05 Marks)**

c. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at x = 0.2 by taking h = 0.2 **(06 Marks)**

Module-2

3 a. Obtain the solution of the equation $2\frac{d^2y}{dx^2} = ux + \frac{dy}{dx}$ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data: **(05 Marks)**

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

b. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(05 Marks)**

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ **(06 Marks)**

OR

4 a. By Runge-Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2. Correct to four decimal places using the initial conditions y = 1 and $y' = 0$ at x = 0, h = 0.2. **(05 Marks)**

b. Prove that $J_{+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ **(05 Marks)**

c. Prove the Rodrigues formula,

$$\rho_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}$$ **(06 Marks)**

Module-3

5 a. State and prove Cauchy's-Riemann equation in polar form. **(05 Marks)**

b. Discuss the transformation $W = e^z$. **(05 Marks)**

c. Evaluate $\int_C \left\{\frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)^2(z-2)}\right\}dz$

using Cauchy's residue theorem where 'C' is the circle $|z| = 3$ **(06 Marks)**

OR

6 a. Find the analytic function whose real part is, $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(05 Marks)**

b. State and prove Cauchy's integral formula. **(05 Marks)**

c. Find the bilinear transformation which maps z = ∞, i, 0 into ω = -1, -i, 1. Also find the fixed points of the transformation. **(06 Marks)**

Module-4

7 a. Find the mean and standard deviation of Poisson distribution. **(05 Marks)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for,

(i) more than 2150 hours.

(ii) less than 1950 hours

(iii) more than 1920 hours and less than 2160 hours.

[A(1.833) = 0.4664, A(1.5) = 0.4332, A(2) = 0.4772] **(05 Marks)**

c. The joint probability distribution of two random variables x and y is as follows:

x/y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine:

(i) Marginal distribution of x and y.

(ii) Covariance of x and y

(iii) Correlaiton of x and y. **(06 Marks)**

OR

8 a. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured what is the probability that, (i) Exactly 2 are defective (ii) at least 2 are defective (iii) none of them are defective. **(05 Marks)**

b. Derive the expressions for mean and variance of binomial distribution. **(05 Marks)**

c. A random variable X take the values -3, -2, -1, 0, 1, 2, 3 such that $P(x = 0) = P(x < 0)$ and $P(x = -3) = P(x = -2) = P(x = -1) = P(x = 1) = P(x = 2) = P(x = 3)$. Find the probability distribution. **(06 Marks)**

Module-5

9 a. In 324 throws of a six faced 'die' an odd number turned up 181 times. Is it reasonable to think that the 'die' is an unbiased one? **(05 Marks)**

b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results:

Horse A:	28	30	32	33	33	29	34
Horse B:	29	30	30	24	27	29	

Test whether you can discriminate between the two horses.($t_{0.05}$=2.2 and $t_{0.02}$=2.72 for 11 d.f) **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ **(06 Marks)**

OR

10 a. Define the terms: (i) Null hypothesis (ii) Type–I and Type–II error (iii) Confidence limits. **(05 Marks)**

b. Prove that the Markov chain whose t.p.m $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector. **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball. (ii) B has the ball. (iii) C has the ball. **(06 Marks)**

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CBCS Scheme

USN 2VD16CV401 **15MATDIP41**

Fourth Semester B.E. Degree Examination, June/July 2017
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix :
$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by elementary row transformations. **(06 Marks)**

b. Solve the following system of equations by Gauss elimination method :
$2x + y + 4z = 12$
$4x + 11y - z = 33$
$8x - 3y + 2z = 20$. **(05 Marks)**

c. Find all the eigen values and eigen vector corresponding to largest eigen value of the matrix :
$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(05 Marks)**

OR

2 a. Solve the following system of equations by Gauss elimination method :
$x + y + z = 9$
$2x + y - z = 0$
$2x + 5y + 7z = 52$. **(06 Marks)**

b. Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ into its echelon form and hence find its rank. **(05 Marks)**

c. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley – Hamilton theorem. **(05 Marks)**

Module-2

3 a. Solve $(D^2 - 4D + 13)y = \cos 2x$ by the method of undetermined coefficients. **(06 Marks)**

b. Solve $(D^2 + 2D + 1)y = x^2 + 2x$. **(05 Marks)**

c. Solve $(D^2 - 6D + 25)y = \sin x$. **(05 Marks)**

OR

4 a. Solve $(D^2 + 1)y = \tan x$ by the method of variation of parameters. **(06 Marks)**

b. Solve $(D^3 + 8)y = x^4 + 2x + 1$. **(05 Marks)**

c. Solve $(D^2 + 2D + 5)y = e^{-x} \cos 2x$. **(05 Marks)**

Module-3

5 a. Find the Laplace transforms of :

i) $e^{-t}\cos^2 3t$ ii) $\dfrac{\cos 2t - \cos 3t}{t}$. **(06 Marks)**

b. Find:

i) $L\left[t^{-5/2} + t^{5/2}\right]$ ii) $L[\sin 5t \cdot \cos 2t]$. **(05 Marks)**

c. Find the Laplace transform of the function : $f(t) = E\sin\left(\frac{\pi t}{\omega}\right)$, $0 < t < \omega$, given that $f(t+\omega) = f(t)$. **(05 Marks)**

OR

6 a. Find :

i) $L\left[t^2 \sin t\right]$ ii) $L\left[\dfrac{\sin 2t}{t}\right]$. **(06 Marks)**

b. Evaluate : $\int_0^{\infty} \dfrac{\cos 6t - \cos 4t}{t}\, dt$ using Laplace transform. **(05 Marks)**

c. Express $f(t) = \begin{cases} \sin 2t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$, in terms of unit step function and hence find $L[f(t)]$. **(05 Marks)**

Module-4

7 a. Solve the initial value problem $\dfrac{d^2y}{dx^2} + \dfrac{5dy}{dx} + 6y = 5e^{2x}$, $y(0) = 2$, $y'(0) = 1$ using Laplace transforms. **(06 Marks)**

b. Find the inverse Laplace transforms : i) $\dfrac{3(s^2-1)^2}{2s^2}$ ii) $\dfrac{s+1}{s^2+6s+9}$. **(05 Marks)**

c. Find the inverse Laplace transform : $\log\left[\dfrac{s^2+4}{s(s+4)(s-4)}\right]$. **(05 Marks)**

OR

8 a. Solve the initial value problem :

$\dfrac{d^2y}{dt^2} + \dfrac{4dy}{dt} + 3y = e^{-t}$ with $y(0) = 1 = y'(0)$ using Laplace transforms. **(06 Marks)**

b. Find the inverse Laplace transform : i) $\dfrac{1}{s\sqrt{5}} + \dfrac{3}{s^2\sqrt{5}} - \dfrac{8}{\sqrt{5}}$ ii) $\dfrac{3s+1}{(s-1)(s^2+1)}$. **(05 Marks)**

c. Find the inverse Laplace transform : $\dfrac{2s-1}{s^2+4s+29}$. **(05 Marks)**

Module-5

9 a. State and prove Baye's theorem. (06 Marks)

b. A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that i) two shots hit ii) atleast two shots hit? (05 Marks)

c. Find P(A), P(B) and $P(A \cap \overline{B})$, if A and B are events with $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{5}{8}$. (05 Marks)

OR

10 a. Prove that $P(A \cup B) = P(A) + (B) - P(A \cap B)$, for any two events A and B. (06 Marks)

b. Show that the events $\overline{A}$ and $\overline{B}$ are independent, if A and B are independent events. (05 Marks)

c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. (05 Marks)

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USN | | | | | | | | | | | **MATDIP301**

Third Semester B.E Degree Examination, June/July 2017
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express : $\frac{1}{(2+i)^2}-\frac{1}{(2-i)^2}$ in the form of a + i b. **(07 Marks)**

b. Find the modulus and amplitude of the complex number $1-\cos\alpha+i\sin\alpha$. **(06 Marks)**

c. Express the complex number $\sqrt{3}+i$ in the polar form. **(07 Marks)**

2 a. Find the n^{th} derivative of log (ax + b). **(07 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. **(06 Marks)**

c. If $y=\sin^{-1}x$, prove that : $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-n^2y_n=0$. **(07 Marks)**

3 a. Using Taylor's theorem, expand sin x in power of $(x-\pi/2)$. **(07 Marks)**

b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ up to the term containing x^4. **(06 Marks)**

c. State and prove Euler's theorem. **(07 Marks)**

4 a. Find the total derivative of $z=xy^2+x^2y$ where x = at, y = 2at, and also verify the result by direct substitution. **(07 Marks)**

b. If u = f(y – z, z – x, x – y) prove that : $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(06 Marks)**

c. if x = u(1 – v) and y = uv, find $J=\frac{\partial(x,y)}{\partial(u,v)}$ and $J'=\frac{\partial(u,v)}{\partial(x,y)}$ and also verify $J\cdot J'=1$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int\cos^n x\cdot dx$. **(07 Marks)**

b. Evaluate : $\int_0^2\frac{x^4}{\sqrt{4-x^2}}\cdot dx$. **(06 Marks)**

c. Evaluate : $\int_1^2\int_1^3 xy^2\,dx\;dy$. **(07 Marks)**

6 a. Evaluate : $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} x\,y\,z\,dz\,dy\,dx$. **(07 Marks)**

b. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(06 Marks)**

c. Prove that $\beta(m,n)=\frac{\Gamma_m\Gamma_n}{\Gamma(m+n)}$. **(07 Marks)**

7 a. Solve : $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$. **(07 Marks)**

b. Solve $x^2y\,dx - (x^3 + y^3)\,dy = 0$. **(06 Marks)**

c. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(07 Marks)**

8 a. Solve : $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$. **(05 Marks)**

b. Solve $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{-4x}$. **(05 Marks)**

c. Solve : $y'' + 2y' + y = e^{-x} + \cos 2x$. **(05 Marks)**

d. Solve : $\frac{d^2y}{dx^2} - 4y = x\sin 2x$. **(05 Marks)**

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USN

14MAT11

First Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module – 1

1 a. Find the n^{th} derivative of $\dfrac{x}{(x-1)(x^2+x-2)}$. **(07 Marks)**

b. Find the angle of interaction of the curves, $r = a(1+\cos\theta)$ and $r^2 = a^2\cos 2\theta$. **(06 Marks)**

c. Derive an expression for the radius of curvature in polar form. **(07 Marks)**

OR

2 a. If $\sin^{-1} y = 2\log(x+1)$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. **(07 Marks)**

b. Find the pedal equation, $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

c. If ρ be the radius of curvature at any point P(x, y) on the parabola $y^2 = 4ax$ show that ρ^2 varies as $(SP)^3$ where S is the focus of the parabola. **(07 Marks)**

Module – 2

3 a. Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x = \dfrac{\pi}{3}$ upto the fourth degree term. **(07 Marks)**

b. If $u = f(r)$ where $r = \sqrt{x^2+y^2+z^2}$, show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = f''(r) + \dfrac{2}{r}f'(r)$. **(06 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2} - \cot^2 x\right]$. **(07 Marks)**

b. If u be a homogeneous function of degree n in x and y, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu$. **(06 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

Module – 3

5 a. Find the unit tangent vector and unit normal vector to the curve $\vec{r} = \cos 2t\, i + \sin 2t\, j + tk$ at $x = \dfrac{1}{\sqrt{2}}$. **(07 Marks)**

b. Using differentiation under the integral sign, show that $\displaystyle\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}dx = \pi\sin^{-1} a$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$ **(07 Marks)**

OR

6 a. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

b. Show that $\text{curl}\left(\phi \vec{A}\right) = \phi(\text{curl}\,\vec{A}) + \text{grad}\phi \times \vec{A}$. **(06 Marks)**

c. Use general rules to trace the curve, $r = a\cos 2\theta$ (four leaved rose). **(07 Marks)**

Module – 4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. **(07 Marks)**

b. Solve $y(1 + xy + x^2y^2)dx + x(1 - xy + x^2y^2)dy = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = 4a\sec\theta\tan\theta$. **(07 Marks)**

OR

8 a. Evaluate (i) $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$ (ii) $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}}dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x\cos^2 x}{y^2}$ **(06 Marks)**

c. The R-L series circuit differential equation acted on by an electromotive force $E\sin\omega t$ satisfies the differential equation, $L\frac{di}{dt} + Ri = E\sin\omega t$. If there is no current in the circuit initially, obtain the value of current at any time 't'. **(07 Marks)**

Module – 5

9 a. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$ by Gauss elimination method. **(07 Marks)**

b. Diagonalize the matrix, $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

c. Determine the largest eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ using Rayleigh's power method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method, $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$. **(07 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(06 Marks)**

c. Reduce the quadratic form, $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 6x_1x_3 + 2x_2x_3$ into canonical form by orthogonal transformation. Indicate the orthogonal transformation. **(07 Marks)**

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USN | | | | | | | | | | | **15MAT11**

First Semester B.E. Degree Examination, Dec.2016/Jan.2017

Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. If $y = e^{-3x}\cos^3 x$, find y_n. **(06 Marks)**

b. Find the angle between the curves

$$r = \frac{a}{1+\cos\theta} \text{ and } r = \frac{b}{1-\cos\theta}.$$ **(05 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(05 Marks)**

OR

2 a. If $x = \tan(\log y)$, find the value of $(1+x^2)y_{n+1} + (2nx-1)y_n + (n)(n-1)y_{n-1}$. **(06 Marks)**

b. Find the Pedal equation of $\frac{2a}{r} = 1 + \cos\theta$. **(05 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

Module-2

3 a. Explain log(cos x) about the point $x = \frac{\pi}{3}$ upto 3rd degree terms using Taylor's series. **(06 Marks)**

b. Evaluate $\underset{x\to 0}{\text{Limit}}\left(\frac{\tan x}{x}\right)^{1/x^2}$. **(05 Marks)**

c. State Euler's theorem and use it to find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ when $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$. **(05 Marks)**

OR

4 a. Expand $\frac{e^x}{1+e^x}$ using Maclaurin's series upto and including 3rd degree terms. **(06 Marks)**

b. Find $\frac{du}{dt}$ when $u = x^3y^2 + x^2y^3$ with $x = at^2$, $y = 2at$. Use Partial derivatives. **(05 Marks)**

c. If $u = \frac{x_2x_3}{x_1}$, $v = \frac{x_1x_3}{x_2}$, $w = \frac{x_1x_2}{x_3}$, find the value of Jacobian $J\left(\frac{u, v, w}{x_1, x_2, x_3}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time find the components of velocity and acceleration at time t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. Find the divergence and curl of the vector $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)K$ at the point (2, -1, 1). **(05 Marks)**

c. A vector field is given by $\vec{A} = (x^2 + xy^2)\, i + (y^2 + x^2y)j$, show that the field is irrotational and find the scalar potential. **(05 Marks)**

OR

6 a. Find grad ϕ when $\phi = 3x^2y - y^3z^2$ at the point (1, -2, -1). **(06 Marks)**

b. Find a for which $f = (x + 3y)i + (y - 2z)j + (x + az)k$ is solenoidal. **(05 Marks)**

c. Prove that $\text{Div}(\text{curl } \vec{V}) = 0$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula of $\int \sin^m x \cos^n x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x\sqrt{2ax - x^2}\, dx$. **(05 Marks)**

c. Solve $(2x \log x - xy)\, dy + 2y\, dx = 0$. **(05 Marks)**

OR

8 a. Obtain the reduction formula of $\int \cos^n x \, dx$. **(06 Marks)**

b. Obtain the Orthogonal trajectory of the family of curves $r^n \cos n\theta = a^n$. Hence solve it. **(05 Marks)**

c. A body originally at 80^0C cools down at 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original? **(05 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ **(06 Marks)**

b. Solve by Gauss – Jordan method the system of linear equations
$2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$. **(05 Marks)**

c. Find the largest eigen value and the corresponding Eigen vector by power method given that

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$ (Use $[1\ 0\ 0]^T$ as the initial vector). (Apply 4 iterations). **(05 Marks)**

OR

10 a. Use Gauss – Seidel method to solve the equations **(06 Marks)**
$20x + y - 2x = 17$
$3x + 20y - z = 18$
$2x - 3y + 20z = 25$. Carry out 2 iterations with $x_0 = y_0 = z_0 = 0$.

b. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. **(05 Marks)**

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USN

14MAT21

Second Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module – 1

1 a. Solve $\frac{d^4y}{dx^4}+5\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}-4\frac{dy}{dx}-8y=0$. (06 Marks)

b. Solve $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x$. (07 Marks)

c. Solve by the method of undetermined coefficient. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=2x^2+3e^{-x}$. (07 Marks)

2 a. Solve $4\frac{d^4y}{dx^4}-8\frac{d^3y}{dx^3}-7\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$ (06 Marks)

b. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=1+3x+x^2$. (07 Marks)

c. Solve by the method of variation of parameter $y''+a^2y=\sec ax$. (07 Marks)

Module – 2

3 a. Solve $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+13y=\log x+x^2$. (06 Marks)

b. Solve $x^2p^2+3xyp+2y^2=0$. (07 Marks)

c. Find the general and singular solution of, $(x^2-1)p^2-2xyp+y^2-1=0$. (07 Marks)

4 a. Solve the system of equations,
$\frac{dx}{dt}=3x-4y$, $\frac{dy}{dt}=x-y$. (06 Marks)

b. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\log(1+x)$. (07 Marks)

c. Solve $y=2px-yp^2$ (07 Marks)

Module – 3

5 a. Form a partial differential equation by eliminating arbitrary function.
$f(x+y+z, x^2+y^2+z^2)=0$ (06 Marks)

b. Derive one dimensional wave equation. (07 Marks)

c. Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} x^3y\,dxdy$. (07 Marks)

6 a. Form a P.D.E by eliminating arbitrary constants,
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (06 Marks)

b. Evaluate $\int_0^4\int_0^{2\sqrt{z}}\int_0^{\sqrt{4z-x^2}} dydxdz$. (07 Marks)

c. Solve one dimensional heat equation by separation of variables. Given $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. (07 Marks)

Module – 4

7 a. For m > 0, n > 0 show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)

b. Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \cdot \int_0^1 \frac{1}{\sqrt{1+x^4}} dx = \frac{\pi}{4\sqrt{2}}$. (07 Marks)

c. Prove that cylindrical co-ordinate system is orthogonal. (07 Marks)

8 a. Find the volume of the sphere. $x^2 + y^2 + z^2 = a^2$ using triple integral. (06 Marks)

b. For m and n positive prove that.

$$\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx.$$ (07 Marks)

c. Express the vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical co-ordinates. (07 Marks)

Module – 5

9 a. Find the Laplace transform of. (i) $e^{3t}t^4$ (ii) $\sin t \sin 2t \sin 3t$ (06 Marks)

b. A periodic function of period $\frac{2\pi}{W}$ is defined by,

$$f(t) = \begin{cases} E \sin \omega t & \text{for } 0 \le t \le \frac{\pi}{W} \\ 0 & \text{for } \frac{\pi}{W} \le t \le \frac{2\pi}{W} \end{cases}$$ where E and W are positive constants. Show that

$$L\{f(t)\} = \frac{EW}{(s^2 + w^2)\left(1 - e^{\frac{-\pi s}{W}}\right)}.$$ (07 Marks)

c. Find the inverse Laplace transform. $\frac{1}{s(s+1)(s+2)}$. (07 Marks)

10 a. Find $L\left(\frac{\cos 2t - \cos 3t}{t}\right)$. (06 Marks)

b. Express $f(t) = \begin{cases} t^2 & 1 < t \le 2 \\ 4t & t > 2 \end{cases}$

in terms of unit step function and hence find $L[f(t)]$. (07 Marks)

c. Solve using Laplace transform method.

$y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$ (07 Marks)

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CBCS Scheme

USN | | | | | | | | | | | **15MAT21**

Second Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics - II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $(D-2)^2 y = 8(e^{2x} + x + x^2)$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^2 - 4D + 3)\, y = e^x \cos 2x$, by inverse differential operator method. **(05 Marks)**

c. Solve by the method of variation of parameters $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$. **(05 Marks)**

OR

2 a. Solve $(D^2 - 1)y = x \sin 3x$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^3 - 6D^2 + 11D - 6)y = e^{2x}$ by inverse differential operator method. **(05 Marks)**

c. Solve $(D^2 + 2D + 4)\, y = 2x^2 + 3\, e^{-x}$ by the method of undetermined coefficient. **(05 Marks)**

Module-2

3 a. Solve $x^3 y''' + 3x^2 y'' + xy' + 8y = 65 \cos(\log x)$. **(06 Marks)**

b. Solve $xy\, p^2 + p(3x^2 - 2y^2) - 6xy = 0$. **(05 Marks)**

c. Solve the equation $y^2(y - xp) = x^4 p^2$ by reducing into Clairaut's form, taking the substitution $x = \dfrac{1}{x}$ and $y = \dfrac{1}{y}$. **(05 Marks)**

OR

4 a. Solve $(2x + 3)^2 y'' - (2x + 3)\, y' - 12y = 6x$. **(06 Marks)**

b. Solve $p^2 + 4x^5 p - 12x^4 y = 0$. **(05 Marks)**

c. Solve $p^3 - 4xy\, p + 8y^2 = 0$. **(05 Marks)**

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function. $Z = f(x + at) + g(x - at)$. **(06 Marks)**

b. Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\dfrac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$, when y is an odd multiple of $\pi/2$. **(05 Marks)**

c. Find the solution of the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(05 Marks)**

OR

6 a. Obtain the partial differential equation by eliminating the arbitrary function $lx + my + nz = \phi(x^2 + y^2 + z^2)$. **(06 Marks)**

b. Solve $\dfrac{\partial^2 z}{\partial y^2} = z$, given that, when $y = 0$, $z = e^x$ and $\dfrac{\partial z}{\partial y} = e^{-x}$. **(05 Marks)**

c. Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(05 Marks)**

Module-4

7 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$. **(06 Marks)**

b. Evaluate $\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dy\, dx$ by changing the order of integration. **(05 Marks)**

c. Evaluate $\int_{0}^{4} x^{3/2} (4-x)^{5/2}\, dx$ by using Beta and Gamma function. **(05 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar co-ordinates. Hence show that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi/2}$. **(06 Marks)**

b. Find by double integration, the area lying inside the circle $r = a \sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$. **(05 Marks)**

c. Obtain the relation between beta and gamma function in the form $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(05 Marks)**

Module-5

9 a. Find i) $L\{e^{-3t}(2\cos 5t - 3\sin 5t)\}$ ii) $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. **(06 Marks)**

b. If a periodic function of period 2a is defined by

$$f(t) = \begin{cases} t & \text{if } 0 \le t \le a \\ 2a - t & \text{if } a \le t \le 2a \end{cases}$$

then show that $L\{f(t)\} = \frac{1}{s^2} \tan h\left(\frac{as}{2}\right)$. **(05 Marks)**

c. Solve the equation by Laplace transform method. $y''' + 2y'' - y' - 2y = 0$. Given $y(0) = y'(0) = 0$, $y''(0) = 6$. **(05 Marks)**

OR

10 a. Find $L^{-1}\left\{\frac{s+3}{s^2 - 4s + 13}\right\}$. **(06 Marks)**

b. Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ by using Convolution theorem. **(05 Marks)**

c. Express

$$f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t < 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases}$$

in terms of unit step function and hence find its Laplace transforms. **(05 Marks)**

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USN | | | | | | | | | | | **10MAT31**

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Obtain the Fourier series in $(-\pi, \pi)$ for $f(x) = x\cos x$. **(07 Marks)**

b. Obtain the Fourier half range sine series,

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}.$$ **(07 Marks)**

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. **(06 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2 a. Find the Fourier transforms of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ and hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$$ **(07 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. **(07 Marks)**

c. Find the inverse Fourier sine transform of $\hat{f}_s(\alpha) = \frac{e^{-a\alpha}}{\alpha}$, $a > 0$. **(06 Marks)**

3 a. Solve the wave equation $u_{tt} = c^2 u_{xx}$ given that $u(0,t) = 0 = u(2l, t)$, $u(x, 0) = 0$ and $\frac{\partial u}{\partial t}(x,0) = a\sin^3\frac{\pi x}{2l}$ **(07 Marks)**

b. Solve the boundary value problem $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ $0 < x < l$, $\frac{\partial u}{\partial x}(0,t) = 0$, $\frac{\partial u}{\partial x}(l,t) = 0$, $u(x, 0) = x$. **(07 Marks)**

c. Obtain the D'Almbert's solution of the wave equation, $u_{tt} = C^2 u_{xx}$ subject to the conditions $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(06 Marks)**

4 a. Fit a parabola $y = a + bx + cx^2$ for the data: **(07 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

b. Solve by using graphical method the L.P.P.
Minimize $z = 30x + 20y$
Subject to the constraints: $x - y \leq 1$
$x + y \geq 3$, $y \leq 4$
and $x \geq 0$, $y \geq 0$ **(07 Marks)**

c. Maximize $z = 3x + 4y$
subject to the constraints $2x + y \leq 40$, $2x + 5y \leq 180$,
$x \geq 0, y \geq 0$ using simplex method. **(06 Marks)**

PART – B

5 a. Find the fourth root of 12 correct to three decimal places by using regula Falsi method. **(07 Marks)**

b. Solve $9x-2y+z=50$, $x+5y-3z=18$, $-2x+2y+7z=19$ by relaxation method obtaining the solution correct to two decimal places. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of, $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using power method by taking initial vector as $[1 \quad 1 \quad 1]^T$. **(06 Marks)**

6 a. The table gives the values of $\tan x$ for $0.10 \le x \le 0.30$ **(07 Marks)**

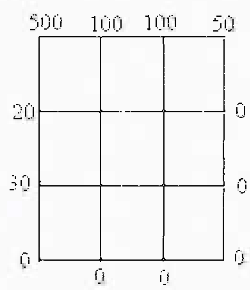
x	0.10	0.15	0.20	0.25	0.30
tanx	0.1003	0.1511	0.2027	0.2553	0.3093

b. Using Newton's forward and backward interpolation formula, calculate the increase in population from the year 1955 to 1985. The population in a town is given by. **(07 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce the value of $\log_e 2$. **(06 Marks)**

7 a. Solve the Laplace's equation $u_{xx} + u_{yy} = 0$, given that **(07 Marks)**



b. Solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0,t)=0$; $u(4,t)=0$; $u(x,0)=x(4-x)$. Take h = 1, K = 0.5 upto Four steps. **(07 Marks)**

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x, 0) = \sin \pi x$, $0 \le x \le 1$, $u(0,t) = u(1,t) = 0$ using Schmidt's method. Carry out computations for two levels, taking $h = \frac{1}{3}$, $K = \frac{1}{36}$. **(06 Marks)**

8 a. Find the z-transform of, (i) $\cosh n\theta$ (ii) $\sinh n\theta$ **(07 Marks)**

b. Obtain the inverse z-transform of, $\frac{4z^2-2z}{z^3-5z^2+8z-4}$. **(07 Marks)**

c. Solve the difference equation,
$y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = y_1 = 0$ using z-transforms. **(06 Marks)**

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CBCS Scheme

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Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Expand $f(x) = x - x^2$ as a Fourier series in the interval $(-\pi, \pi)$. **(08 Marks)**

b. Obtain the half-range cosine series for the function $f(x) = x(l - x)$ in the interval $0 \le x \le l$. **(08 Marks)**

OR

2 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$ **(06 Marks)**

b. Find the half-range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in} \quad 0 < x < 1/2 \\ x - \frac{3}{4} & \text{in} \quad 1/2 < x < 1 \end{cases}.$$ **(05 Marks)**

c. Compute the constant term and the coefficient of the 1st sine and cosine terms in the Fourier series of y as given in the following table:

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

(05 Marks)

Module-2

3 a. If $f(x) = \begin{cases} 1 - x^2; & |x| < 1 \\ 0; & |x| \ge 1 \end{cases}$. Find the Fourier transform of f(x) and hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3} dx$. **(06 Marks)**

b. Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}.$$ **(05 Marks)**

c. Solve using Z-transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0$, $y_1 = 2$. **(05 Marks)**

OR

4 a. Obtain the inverse Fourier sine transform of $F_S(\alpha) = \frac{e^{-a\alpha}}{\alpha}$, $a > 0$. **(06 Marks)**

b. Find the Z-transform of $2n + \sin\left(\frac{n\pi}{4}\right) + 1$. **(05 Marks)**

c. If $U(z) = \frac{z}{z^2 + 7z + 10}$, find the inverse Z-transform. **(05 Marks)**

Module-3

5 a. Obtain the coefficient of correlation for the following data:

x :	10	14	18	22	26	30
y :	18	12	24	6	30	36

(06 Marks)

b. By the method of least square find the straight line that best fits the following data:

x :	1	2	3	4	5
y :	14	27	40	55	68

(05 Marks)

c. Use Newton-Raphson method to find a root of the equation $\tan x - x = 0$ near $x = 4.5$. Carry out two iterations. **(05 Marks)**

OR

6 a. Find the regression line of y on x for the following data:

x :	1	3	4	6	8	9	11	14
y :	1	2	4	4	5	7	8	9

Estimate the value of y when x = 10. **(06 Marks)**

b. Fit a second degree parabola to the following data:

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(05 Marks)

c. Solve $xe^x - 2 = 0$ using Regula – Falsi method. **(05 Marks)**

Module-4

7 a. From the data given in the following table. Find the number of students who obtained less than 70 marks.

Marks :	0-19	20-39	40-59	60-79	80-99
Number of students :	41	62	65	50	17

(06 Marks)

b. Find the equation of the polynomial which passes through the points (4, -43), (7, 83), (9, 327) and (12, 1053). Using Newton's divided difference interpolation. **(05 Marks)**

c. Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\,dx$ using Simpson's $\frac{3}{8}^{th}$ rule taking six parts.

(05 Marks)

OR

8 a. Using Newton's backward interpolation formula find the interpolating polynomial for the function given by the following table:

x :	10	11	12	13
f(x) :	22	24	28	34

Hence fine f(12.5). **(06 Marks)**

b. The following table gives the premium payable at ages in years completed. Interpolate the premium payable at age 35 completed. Using Lagrange's formula.

Age completed :	25	30	40	60
Premium in Rs. :	50	55	70	95

(05 Marks)

c. Evaluate $\int_{4}^{5.2} \log_e x \; dx$ taking 6 equal strips by applying Waddles rule. **(05 Marks)**

Module-5

9 a. Verify Green's theorem for $\oint_c (xy + y^2)dx + x^2 dy$ where c is the closed curve of the region bounded by y = x and y = xz. **(06 Marks)**

b. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xy\,j$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. **(05 Marks)**

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary. **(05 Marks)**

OR

10 a. Use divergence theorem to evaluate $\iint_S \vec{F}\,\hat{n}\, ds$ over the entire surface of the region above XoY plane bounded by the cone $z^2 = x^2 + y^2$, the plane z = 4 where $\vec{F} = 4xz^1\hat{i} + xyz^2\hat{j} + 3z\hat{k}$. **(06 Marks)**

b. Find the extremal of the functional $\int_{x_1}^{x_2} \left[(y^1)^2 - y^2 + 2y \sec x\right] dx$. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them. **(05 Marks)**

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CBCS Scheme

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15MATDIP31

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Simplify $\dfrac{(\cos 3\theta - i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^5}{(\cos\theta + i\sin\theta)^3(\cos 2\theta - i\sin 2\theta)^4}$. **(06 Marks)**

b. Determine λ such that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar. **(05 Marks)**

c. Find sine angle of two vectors $4\hat{i} + 3\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + 2\hat{k}$. **(05 Marks)**

OR

2 a. Express $\dfrac{1}{2+i} - \dfrac{(1+i)^2}{3+i}$ in the form a + ib. **(06 Marks)**

b. Find modulus and amplitude of $1 + \cos\theta + i\sin\theta$. **(05 Marks)**

c. If $\vec{a} = 3\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} + 5\hat{j} + 10\hat{k}$ find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. **(05 Marks)**

Module-2

3 a. If y = a cos(logx) + b sin (logx) show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. With usual notation prove that $\tan\varphi = r\dfrac{d\theta}{dr}$. **(05 Marks)**

c. If $u = e^{ax+by} f(ax - by)$ prove that $b\dfrac{\partial u}{\partial x} + a\dfrac{\partial u}{\partial y} = 2abu$. **(05 Marks)**

OR

4 a. Find n^{th} derivative of $y = e^x \sin 4x \cos x$ **(06 Marks)**

b. Find pedal equation of $r = a(1 + \cos\theta)$. **(05 Marks)**

c. If $u = f(x - y, y - z, z - x)$ show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} \sin^5(x/2)dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$. **(05 Marks)**

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$. **(05 Marks)**

OR

6 a. Evaluate $\int_0^a \frac{x^3 dx}{\sqrt{a^2 - x^2}}$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx \, dy$. **(05 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$. **(05 Marks)**

Module-4

7 a. A particle moves along the curve c : $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where t denotes time. Find velocity and acceleration at t = 2. **(06 Marks)**

b. Find unit normal vector to surface $Q = x^2yz + 4xz^2$ at (1, -2, -1). **(05 Marks)**

c. Show that $\vec{f} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational. **(05 Marks)**

OR

8 a. A particle moves along the curve c : $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where 't' is the time. Find the components of velocity and acceleration at t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). **(05 Marks)**

c. If $\phi = 2x^3y^2z^4$ find div(gradϕ). **(05 Marks)**

Module-5

9 a. Solve : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. **(06 Marks)**

b. Solve : $x^2y \, dx - (x^3 + y^3) \, dy = 0$. **(05 Marks)**

c. Solve : $(y^3 - 3x^2y) \, dx - (x^3 - 3xy^2) \, dy = 0$. **(05 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)dx + xy \, dy = 0$. **(05 Marks)**

c. Solve : $\frac{dy}{dx} + y \cot x = \cos x$. **(05 Marks)**

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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2016/Jan.2017

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Using Taylor series method, solve $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ at $x = 0.2$. **(06 Marks)**

b. Using Runge – Kutta method of fourth order solve for $y(0.1)$, $y(0.2)$ given that $\frac{dy}{dx} = y(x + y)$, $y(0) = 1$. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1 + y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Milne's Predictor – Corrector method. **(07 Marks)**

2 a. Approximate y and z at x = 0.1, using Picard's method for the solution of the equations $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y + z)$, given that $y(0) = 1$ and $z(0) = ½$. **(06 Marks)**

b. Using Runge – Kutta fourth order method to solve $y'' = xy' - y$, $y(0) = 3$, $y'(0) = 0$, find y and z at x = 0.1. **(07 Marks)**

c. Apply Milne's method to compute y(0.4) given that $y'' + xy' + y = 0$ and the values $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9801$, $y(0.3) = 0.956$, $y'(0) = 0$, $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2867$. **(07 Marks)**

3 a. Prove that the C – R equations in polar form. **(06 Marks)**

b. Show that $f(z) = z^n$, where n is a positive integer is analytic and hence find its derivative. **(07 Marks)**

c. If $\phi + i\Psi$ represents the complex potential of an electrostatic field where $\Psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, find ϕ. **(07 Marks)**

4 a. Find the Bilinear transformation which maps the points 1, i – 1 into 0, 1, ∞. **(06 Marks)**

b. State and prove the Cauchy's integral formula. **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)} dz$, where c : | z | = 3. **(07 Marks)**

PART – B

5 a. Find the solution of the Laplace's equation in cylindrical system leading to Bessel's differential equation. **(06 Marks)**

b. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n.$$ **(07 Marks)**

c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms Legendre polynomials. **(07 Marks)**

6 a. Define the Empherical and Axiomatic definition of probability and give an example of each. **(06 Marks)**

b. Of the cigarette smoking population 70% are men and 30% are women, 10% of these men and 20% of these women smoke wills. What is the probability that person seen smoking a wills will be a man? **(07 Marks)**

c. The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnose is 40% and the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was correctly diagnosed? **(07 Marks)**

7 a. Derive the mean and variance of Binomial distribution. **(06 Marks)**

b. If x is an exponential distribution with mean 4, evaluate i) $P(0 < x < 1)$ ii) $P(x > 2)$ and iii) $P(-\infty < x < 10)$. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) More than 75 and iii) between 65 and 75. **(07 Marks)**

8 a. Define the following terms :
i) Type I – error and Type II – error ii) Level of significance. **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following :
Change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4, can it be concluded that the stimulus will increase the blood pressure? (t. 05 for 11 d.f = 2.201). **(07 Marks)**

c. The theory predicts the proportion of beans in the four groups G_1, G_2, G_3, G_4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (at 5% LOS for 3 d.f = 7.815). **(07 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Advanced Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

1 a. Express the $\frac{3}{1+i} - \frac{1}{2-i} + \frac{1}{1-i}$ in the form of a + ib. **(06 Marks)**

b. Find the cube roots of 1 - i. **(07 Marks)**

c. Prove that $\left(\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n = \cos n\theta + i\sin n\theta$. **(07 Marks)**

2 a. Find the nth derivative of $e^{ax}\cos(bx + c)$. **(07 Marks)**

b. Find the nth derivative of $\frac{x}{(x-1)(2x+3)}$. **(06 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$ prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

3 a. With usual notations P.T $\tan\phi = \frac{rd\theta}{dr}$. **(06 Marks)**

b. Find the angle between the pairs of curves
$r = a\log\theta \quad r = \frac{a}{\log\theta}$. **(07 Marks)**

c. Find the Pedal equation to the curve $r = a(1+\sin\theta)$. **(07 Marks)**

4 a. State and prove Euler's theorem of Homogeneous functions. **(06 Marks)**

b. If u = f(x-y , y-z , z-x)
P.T $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $u = \tan^{-1}x + \tan^{-1}y$, $V = \frac{x+y}{1-xy}$
S.T $\frac{\partial(u,v)}{\partial(x,y)} = 0$. **(07 Marks)**

5 a. Obtain the Reduction formula for $\int \sin^m x \cos^n x \, dx$. Where m, n are positive integers. **(07 Marks)**

b. Evaluate $\int_1^2 \int_0^{2-y} xy \, dx\, dy$. **(06 Marks)**

c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z)\, dz\, dx\, dy$. **(07 Marks)**

6 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Prove that $\int_0^\infty x^2 e^{-x^4} dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. **(07 Marks)**

c. Evaluate the Integral $\int_0^1 x^5(1-x)^6\,dx$. **(07 Marks)**

7 a. Solve $(D^3 - 3D - 2)y = 0$. **(06 Marks)**
b. Solve $(y'' + y) = e^{-x} + \cos x + x^3$. **(07 Marks)**
c. Solve $y'' - 2y' + y = xe^x \sin x$. **(07 Marks)**

8 a. Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$. **(06 Marks)**
b. Solve $x \log x \frac{dy}{dx} + y = 2 \log x$. **(07 Marks)**
c. Solve $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$. **(07 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2016/Jan.2017
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. The direction cosines of three mutually perpendicular lines are l_1, m_1, n_1 l_2, m_2, n_2 and l_3, m_3, n_3. Show that the line with direction cosines $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ is equally inclined to the above lines. **(07 Marks)**

c. Find the equations of the plane passing through the points (1, 2, 3) (0, 1, 4) and (0, 0, 1). **(07 Marks)**

2 a. Derive the equation to the plane in the intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(06 Marks)**

b. Find the angle between the lines $\frac{x-1}{1}=\frac{y-5}{0}=\frac{z+1}{2}$ and $\frac{x+3}{3}=\frac{y}{5}=\frac{z-5}{2}$. **(07 Marks)**

c. Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2}=\frac{y-3}{3}=-z$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2i-j+k$, $i-3j-5k$, $3i-4j-4k$ form a right angled triangle. **(06 Marks)**

b. Find a vector of magnitude 12 units which is perpendicular to the vectors $\vec{a}=4i-j+3k$ and $\vec{b}=-2i+j-2k$. **(07 Marks)**

c. Find λ so that the points $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ and $D(-3,\lambda,1)$ are coplanar. **(07 Marks)**

4 a. Find the unit tangent vector of the space curve $x=1+t^3$, $y=2t^3$, $z=2-t^3$ at $t=1$. **(06 Marks)**

b. Find the angle between the tangents to the curve $\vec{r}=\left(t-\frac{t^2}{2}\right)i+t^2j+\left(t+\frac{t^2}{2}\right)k$ at $t=\pm1$. **(07 Marks)**

c. A particle moves along the curve whose parametric equations are $x=t-\frac{t^3}{3}$, $y=t^2$ and $z=t+\frac{t^3}{3}$, where 't' is the time. Find the velocity and acceleration at any time 't'. Also find their magnitudes at t = 3. **(07 Marks)**

5 a. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at $(2,-1,2)$. **(06 Marks)**

b. Find the constants a, b, c such that the vector. $\vec{F}=(x+y+az)i+(bx+2y-z)j+(x+cy+2z)k$ is irrotational. **(07 Marks)**

c. If $\vec{A}=\text{grad}(x^3+y^3+z^3-3xyz)$ then find div $\vec{A}$ and curl $\vec{A}$. **(07 Marks)**

6 a. Find the expression for $L[\sin at]$. **(05 Marks)**

b. Find $L[t \sin at]$. **(05 Marks)**

c. Find $L\left[\frac{1-e^{at}}{t}\right]$. **(05 Marks)**

d. Find $L[e^{t}\cos^{2} 2t]$. **(05 Marks)**

7 a. Find $L^{-1}\left[\frac{s}{(s+2)(s^{2}+1)}\right]$. **(06 Marks)**

b. Find $L^{-1}\left[\frac{s+2}{s^{2}+2s+2}\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\log\left[\frac{s^{2}+1}{s(s-1)}\right]\right]$. **(07 Marks)**

8 a. Using Laplace transform solve:
$y''-2y'+y=e^{2t}$ with $y(0)=0$ and $y'(0)=1$. **(10 Marks)**

b. Solve using Laplace transformation, method $y''+2y'-3y=\sin t$, $y(0)=y'(0)=0$. **(10 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2017
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. Find the angle between two lines whose direction cosines are given by $\ell + 3m + 5n = 0$ and $2mn - 6n\ell - 5\ell m = 0$. **(07 Marks)**

c. Find the coordinates of the foot of the perpendicular from A(1, 1, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4). **(07 Marks)**

2 a. Find the equation of the plane through (2, -1, 6) and (1, -2, 4) and perpendicular to the plane $x - 2y - 2z + 9 = 0$. **(06 Marks)**

b. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines. $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$. **(07 Marks)**

c. Find the angle between the planes $x - y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$. **(07 Marks)**

3 a. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors then prove that
$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$$
(06 Marks)

b. If $\vec{A} = 4i + 3j + k$, $\vec{B} = 2i - j + 2k$ find a unit vector N perpendicular to the vectors $\vec{A}$ and $\vec{B}$ also show that $\vec{A}$ is not perpendicular to $\vec{B}$. **(07 Marks)**

c. Find the value of λ so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, λ, 1) lie on the same plane. **(07 Marks)**

4 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is time. Find the components of its velocity and acceleration in the direction of the vector $i - 3j + 2k$ at t = 1. **(06 Marks)**

b. Find the angle between tangents to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at t = 1 and t = 2. **(07 Marks)**

c. Find the directional derivative of $x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2i - j - 2k$. **(07 Marks)**

5 a. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(06 Marks)**

b. Find the divergence and curl of the vector.
$\vec{F} = (xyz + y^2z)i + (3x^2y + y^2z)j + (xz^2 - y^2z)k$ **(07 Marks)**

c. Find the constants a, b, c so that the vector,
$\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 Find : (05 Marks)

a. $L[\sin 5t \sin 3t]$ (05 Marks)

b. $L[te^{8t}\cos 2t]$ (05 Marks)

c. $L\left[\frac{1-e^{at}}{t}\right]$ (05 Marks)

d. $L\left[\int_0^t e^{2t}\frac{\sin at}{t}dt\right]$ (05 Marks)

7 a. Find $L^{-1}\left[\frac{2s-1}{s^2+2s+17}\right]$. (05 Marks)

b. Find $L^{-1}\left[\frac{s+1}{(s-1)^2(s+2)}\right]$. (05 Marks)

c. Find $L^{-1}\left[\cot^{-1}\left(\frac{s}{a}\right)\right]$. (05 Marks)

d. Using convolution theorem evaluate $L^{-1}\left[\frac{s}{(s+2)(s^2+9)}\right]$. (05 Marks)

8 a. Using Laplace transforms, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given $y(0)=y'(0)=0$. (10 Marks)

b. Using Laplace transforms, solve $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$, given x = 2, y = 0 when t = 0. (10 Marks)

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