CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019

Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x}{(1+x)(1+2x)}$. **(06 Marks)**

b. Prove that the following curves cut orthogonally $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

2 a. If $\cos^{-1}(y/b) = \log(x/n)^n$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. **(06 Marks)**

b. Find the pedal equation of the curve $r^2 = a^2 \sec 2\theta$. **(07 Marks)**

c. Find the radius of curvature for the curve $y^2 = \dfrac{4a^2(2a-x)}{x}$, where the curve meets the x – axis. **(07 Marks)**

Module-2

3 a. Obtain the Taylor's expansion of $\log_e x$ about $x = 1$ upto the term containing fourth degree. **(06 Marks)**

b. If $u = \operatorname{cosec}^{-1}\left(\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{6}\tan u$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

OR

4 a. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left\{\dfrac{\sin 2x - 2\sin x}{x^3}\right\}$. **(06 Marks)**

b. Obtain the Maclaurin's expansion of the function $\log(1+x)$ upto 4^{th} degree terms. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve, $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. **(06 Marks)**

b. If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$, find a, b, c such that Curl $\vec{F} = \vec{O}$ and then find ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\operatorname{div}(\phi\, \bar{A}) = \phi\,(\operatorname{div} \bar{A}) + \operatorname{grad}\phi \cdot \bar{A}$. **(07 Marks)**

OR

6 a. The position vector of a particle at time t is $\vec{r} = \cos(t-1)\, i + \sinh(t-1)\, j + t^3 k$. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(07 Marks)**

c. Prove that Curl $(\phi \vec{A}) = \phi(\text{curl } \vec{A}) + \text{grad } \phi \times \vec{A}$. **(07 Marks)**

Module-4

7 a. Find the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. **(06 Marks)**

b. Solve $x^3 \frac{dy}{dx} - x^2 y = -y^4 \cos x$. **(07 Marks)**

c. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}} dx$. **(06 Marks)**

b. Solve $(y^2 e^{xy^2} + 4x^3)dx + (2xy\, e^{xy^2} - 3y^2)\, dy = 0$. **(07 Marks)**

c. A body in air at 25^0C cools from 100^0C to 75^0C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

(06 Marks)

b. Find the numerically largest eigen value and the corresponding eigen vector of the matrix by power method :

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as [1, 0.8, -0.8]'.

Perform 3 iterations. **(07 Marks)**

c. Show that the transformation :

$y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$ and $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(07 Marks)**

OR

10 a. Solve 20x + y – 2z = 17 ; 3x + 20y – z = -18 ; 2x – 3y + 20z = 25 by Gauss – Seidel method. **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1 x_3$ into Canonical form, using orthogonal transformation. **(07 Marks)**

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14MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019

Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $y = e^{ax}\sin(bx+c)$. (06 Marks)

b. Find the angle of intersection between the curves $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$. (07 Marks)

c. Find the radius of curvature of the curve $a^2y = x^3 - a^3$ at the point where the curve cuts the x-axis. (07 Marks)

2 a. If $y = \tan^{-1}x$ then prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. (06 Marks)

b. Find the pedal equation of $\frac{2a}{r} = 1+\cos\theta$. (07 Marks)

c. Find the radius of curvature of the curve $r = a(1-\cos\theta)$. (07 Marks)

Module-2

3 a. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\ldots$ (06 Marks)

b. If $u = \sin^{-1}\left[\frac{x^3+y^3}{x+y}\right]$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. (07 Marks)

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ find $J\left(\frac{u, v, w}{x, y, z}\right)$. (07 Marks)

4 a. If $Z = e^{ax+by}f(ax-by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. (06 Marks)

b. Evaluate $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ (07 Marks)

c. Find the extreme values of the function $f(x,y) = x^2 + 2xy + 2y^2 + 2x + y$. (07 Marks)

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the components of velocity and acceleration at time t = 1 in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$. (06 Marks)

b. Using differention under integral sign, evaluate $\int_0^\infty \frac{e^{-ax}\sin x}{x}dx$. (07 Marks)

c. Show that $\text{div}(\text{curl}\,\vec{A}) = 0$ (07 Marks)

6 a. If $\vec{v} = \vec{w}\times\vec{r}$, prove that $\text{curl}\,\vec{v} = 2\vec{w}$ where $\vec{w}$ is a constant vector. (06 Marks)

b. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)

c. Trace the curve $y^2(a-x) = x^3$, $a > 0$. (07 Marks)

Module-4

7 a. Obtain reduction formula for $\int \sin^n x dx$. **(06 Marks)**

b. Solve $(e^y + y\cos xy)dx + (xe^y + x\cos xy)dy = 0$. **(07 Marks)**

c. Find orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ where λ is the parameter. **(07 Marks)**

8 a. Evaluate $\int_0^1 x^5(1-x^2)^{5/2} dx$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(07 Marks)**

c. A body originally at 80°C cools down at 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ to canonical form. Hence find its rank, index and signature. **(07 Marks)**

10 a. Solve $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$ by Gauss elimination method. **(06 Marks)**

b. Show that, the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular transformation and find the inverse transformation. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Taking $[1\ 0\ 0]^T$ as initial eigen vector. Take five iterations. **(07 Marks)**

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15MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find y_n if $y = \dfrac{1}{x^2 - 5x + 6}$. (06 Marks)

b. Find the angle between the curves $r = a(1+\cos\theta)$ $r^2 = a^2 \cos 2\theta$ (05 Marks)

c. Find the radius of curvature for the curve $y^2 = \dfrac{4a^2(2a-x)}{x}$ where the curve meets x-axis. (05 Marks)

OR

2 a. If $x = \sin t$ $y = \cos mt$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ (06 Marks)

b. Find the Pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ (05 Marks)

c. Show that for the curve $r(1 - \cos\theta) = 2a$ ρ^2 varies as r^3. (05 Marks)

Module-2

3 a. Obtain the Taylor's expansion of $\tan^{-1}x$ in powers of $x - 1$ up to the term containing fourth degree. (06 Marks)

b. Evaluate $\lim\limits_{x \to 0}\left(\dfrac{1}{x^2} - \cot^2 x\right)$. (05 Marks)

c. If $z = x^2 \tan^{-1}\left(\dfrac{y}{x}\right) - y^2 \tan^{-1}\left(\dfrac{x}{y}\right)$ show that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{x^2 - y^2}{x^2 + y^2}$. (05 Marks)

OR

4 a. Using Maclaurin's series prove that $\sqrt{1 + \sin 2x} = 1 + x - \dfrac{x^2}{2} + \dfrac{x^4}{24}$ ...... (06 Marks)

b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. (05 Marks)

c. If $u = \sqrt{x_1 x_2}$ $v = \sqrt{x_2 x_3}$ $w = \sqrt{x_3 x_1}$ find $J\left(\dfrac{u, v, w}{x_1 x_2 x_3}\right)$. (05 Marks)

Module-3

5 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. (05 Marks)

b. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (05 Marks)

c. Show that $\vec{F} = (y + z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar potential such that $\vec{F} = \nabla\phi$. (06 Marks)

OR

6 a. If $\vec{F} = (3x^2y - z)i + (xz^3 + y^4)j - 2x^3z^2k$ find grad (div $\vec{F}$) at (2, –1, 0) **(06 Marks)**

b. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(05 Marks)**

c. Prove curl (grad ϕ) = 0 for any scalar function ϕ. **(05 Marks)**

Module-4

7 a. Obtain reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x\,dx$ using reduction formula. **(05 Marks)**

c. Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(05 Marks)**

OR

8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Obtain the orthogonal trajectory of the family of curves $r = a(1+\sin\theta)$ **(05 Marks)**

c. If the temperature of the air is 30°C and metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach temperature of 40°C . **(05 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(06 Marks)**

b. Solve by Gauss Jordan method $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$. **(05 Marks)**

c. Find the largest eigen value and the corresponding eigen vector by power method given that $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as $[1 \quad 0.8, -0.8]^1$. **(05 Marks)**

OR

10 a. Use Gauss seidel method to solve the equations
$x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. **(06 Marks)**

b. Reduce the matrix to diagonal form $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^4. **(05 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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17MAT21

Second Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $y''' - y'' + 4y' - 4y = \sin h(2x+3)$. **(06 Marks)**

b. Solve $y'' + 2y' + y = 2x + x^2$. **(07 Marks)**

c. Solve $(D^2+1)y = \tan x$ by method of variation of parameter. **(07 Marks)**

OR

2 a. Solve $(D^3 - 1)y = 3\cos 2x$, where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve $y'' - 6y' + 9y = 7e^{-2x} - \log 2$. **(07 Marks)**

c. Solve $y'' - 3y' + 2y = x^2 + e^x$ by the method of un-determined coefficients. **(07 Marks)**

Module-2

3 a. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$. **(06 Marks)**

b. Solve $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$. **(07 Marks)**

c. Solve $(px-y)(py+x) = 2p$ by reducing it into Cluiraut's form by taking $X = x^2$ and $Y = y^2$. **(07 Marks)**

OR

4 a. Solve $(3x+2)^2y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$. **(06 Marks)**

b. Solve $p^2 + 2py\cot x - y^2 = 0$. **(07 Marks)**

c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation and find its general and singular solution. **(07 Marks)**

Module-3

5 a. Form the partial differential equation of the equation $lx + my + nz = \phi(x^2 + y^2 + z^2)$ by eliminating the arbitrary function. **(06 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x^2} = x + y$. **(07 Marks)**

c. Derive the one dimensional heat equation $u_t = c^2.u_{xx}$ **(07 Marks)**

OR

6 a. Form the partial differential equation of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by eliminating arbitrary constants. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that $z = 0$ and $\frac{\partial z}{\partial y} = \sin x$ when $y = 0$. **(07 Marks)**

c. Obtain the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables for the positive constant. **(07 Marks)**

Module-4

7 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. **(06 Marks)**

b. Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Derive the relation between Beta and Gamma function as $\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma (m+n)}$ **(07 Marks)**

OR

8 a. Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} x^3 \cdot y\,dx\,dy$ **(06 Marks)**

b. Evaluate $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by changing into polar coordinates. **(07 Marks)**

c. Evaluate $\int_{0}^{\infty}\frac{dx}{1+x^4}$ by expressing in terms of beta function. **(07 Marks)**

Module-5

9 a. Find (i) $L[t\cos at]$ (ii) $L\left[\frac{\sin at}{t}\right]$. **(06 Marks)**

b. Find the Laplace transform of the full wave rectifier $f(t) = E\sin wt$, $0 < t < \frac{\pi}{w}$ with period $\frac{\pi}{w}$. **(07 Marks)**

c. Solve $y'' + k^2 y = 0$ given that $y(0) = 2$, $y'(0) = 0$ using Laplace transform. **(07 Marks)**

OR

10 a. Find Inverse Laplace transform of $\frac{s+2}{s^2(s+3)}$. **(06 Marks)**

b. Express the function

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$$

in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

c. Find Inverse Laplace transform of $\frac{1}{s(s^2+a^2)}$ using convolution theorem. **(07 Marks)**

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15MAT21

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Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$ by inverse differential operator method. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = e^x \cos x$ by inverse differential operator method. **(05 Marks)**

c. Solve $(D^2 + 1)y = \operatorname{cosec} x$ by the method of variation of parameters. **(05 Marks)**

OR

2 a. Solve $(D^3 - 5D^2 + 8D - 4)y = (e^x + 1)^2$ by inverse differential operator method. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - y = (1 + x^2)e^x$ by inverse differential operator method. **(05 Marks)**

c. Solve $(D^2 - 3D + 2)y = x^2 + e^{3x}$ by the method of undetermined coefficients. **(05 Marks)**

Module-2

3 a. Solve $x^2y'' + xy' + y = \sin^2(\log x)$ **(06 Marks)**

b. Solve $p^2 + p(x + y) + xy = 0$ **(05 Marks)**

c. Solve $p = \sin(y - xp)$. Also find its singular solution. **(05 Marks)**

OR

4 a. Solve $(1 + 2x)^2 y'' - 6(1 + 2x)y' + 16y = 8(1 + 2x)^2$ **(06 Marks)**

b. Solve $xp^2 - 2yp + x = 0$ **(05 Marks)**

c. Solve $y = 2px + y^2p^3$ **(05 Marks)**

Module-3

5 a. Form the partial differential equation from $z = f(x + ay) + g(x - ay)$ by eliminating arbitrary functions f and g. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$, given $\frac{\partial z}{\partial y} = -2\cos y$ when $x = 0$ and when y is odd multiple of π $z = 0$. **(05 Marks)**

c. Derive one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(05 Marks)**

OR

6 a. Obtain the partial differential equation by eliminating a, b, c from $z = ax^2 + bxy + cy^2$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$ when $y = 0$. **(05 Marks)**

c. Obtain the various possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of variables separable. **(05 Marks)**

Module-4

7 a. Evaluate $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz \, dz \, dy \, dx$ **(06 Marks)**

b. Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ and hence evaluate. **(05 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(05 Marks)**

OR

8 a. Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2}\, dy\,dx$ by changing into polar coordinates. **(06 Marks)**

b. Find by double integration the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(05 Marks)**

c. Prove that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **(05 Marks)**

Module-5

9 a. Find (i) $L\{ te^{-2t} \sin^2 t \}$ (ii) $L\left\{ \frac{e^{-at} - e^{-bt}}{t} \right\}$ **(06 Marks)**

b. Given $f(t) = t^2$, $0 < t < 2a$ and $f(t + 2a) = f(t)$, find $L\{f(t)\}$. **(05 Marks)**

c. Using Laplace transforms solve the differential equation
$y'' - 2y' + y = e^{2t}$ with $y(0) = 0$ and $y'(0) = 1$. **(05 Marks)**

OR

10 a. Find $L^{-1}\left\{ \frac{2s - 1}{s^2 + 2s + 17} \right\}$ **(06 Marks)**

b. Using convolution theorem find $L^{-1}\left\{ \frac{s}{(s^2 + a^2)^2} \right\}$ **(05 Marks)**

c. Express $f(t) = \begin{cases} \cos t & : \quad 0 < t \le \pi \\ \cos 2t & : \quad \pi < t \le 2\pi \\ \cos 3t & : \quad t > 2\pi \end{cases}$

interms of unit step function and hence find its Laplace transforms. **(05 Marks)**

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14MAT21

Second Semester B.E. Degree Examination, Dec.2018/Jan.2019

Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. Solve : $\frac{d^4y}{dx^4} + a^4y = 0$. **(06 Marks)**

b. Solve : $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$. **(07 Marks)**

c. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **(07 Marks)**

2 a. Solve $(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0$. **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2} + 4y - x\sin x + \sin 2x$. **(07 Marks)**

c. Solve by the method of undetermined coefficients $(D^2 - 2D)y = e^x \sin x$. **(07 Marks)**

Module-2

3 a. Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(06 Marks)**

b. Solve : $P^2 + 2\,p\,y\cot x = y^2$. **(07 Marks)**

c. Find the general and singular solution of the equation $\sin px \cos y = \cos px \sin y + p$. **(07 Marks)**

4 a. Solve : $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$. **(06 Marks)**

b. Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin\{\log(1+x)\}$. **(07 Marks)**

c. Solve : $y - 2px = \tan^{-1}(xp^2)$. **(07 Marks)**

Module-3

5 a. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **(06 Marks)**

b. Obtain the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(07 Marks)**

c. Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$. **(07 Marks)**

6 a. Form the partial differential equation by eliminating the arbitrary function form : $Z = y f(x) + x g(y)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$. **(07 Marks)**

c. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} x\,y\,dx\,dy$ and hence evaluate the same. **(07 Marks)**

Module-4

7 a. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, by double integration. **(06 Marks)**

b. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Express the vector zi – 2xj +yk in cylindrical coordinates. **(07 Marks)**

8 a. Find the volume generated by the revolution of the Cardioide $r = a(1 + \cos\theta)$ about the initial line. **(06 Marks)**

b. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

c. Prove that Spherical polar coordinate system is orthogonal. **(07 Marks)**

Module-5

9 a. Find $L\left\{e^{-t}\frac{\sin t}{t}\right\}$. **(06 Marks)**

b. Draw the graph of the periodic function : $f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$.

Where $f(t + 2\pi) = f(t)$ find $L\{f(t)\}$. **(07 Marks)**

c. Using convolution theorem, evaluate $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$. **(07 Marks)**

10 a. Find : $L\left\{\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3\right\}$. **(06 Marks)**

b. Express $f(t) = \begin{cases} \sin t, & 0 < t \le \frac{\pi}{2} \\ \cos t, & t > \frac{\pi}{2} \end{cases}$

interms of unit step function and hence find $L\{f(t)\}$. **(07 Marks)**

c. Solve : $y''' + 2y'' - y' - 2y = 0$, using Laplace transforming with $y(0) = y'(0) = 0$, $y''(0) = 6$. **(07 Marks)**

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CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Fourier series expansion for the periodic function f(x), if in one second $f(x)=\begin{cases}0; & -\pi<x<0\\ x; & 0<x<\pi\end{cases}$. **(08 Marks)**

b. Expand the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$ in half range Fourier cosine series. **(06 Marks)**

c. The following value of function y gives the displacement in inches of a certain machine part for rotations x of a flywheel. Expand y-in terms of Fourier series upto the second harmonic.

Rotations	x	0	π/6	2π/6	3π/6	4π/6	5π/6	π
Displacement	y	0	9.2	14.4	17.8	17.3	11.7	0

(06 Marks)

OR

2 a. Find the Fourier series expansion for the function :
$$f(x)=\begin{cases}\pi x; & 0\le x\le 1\\ \pi(2-x); & 1\le x\le 2\end{cases}$$
and deduce $\dfrac{\pi^2}{8}=\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}$. **(08 Marks)**

b. Expand in Fourier series $f(x) = (\pi - x)^2$ over the interval $0 \le x \le 2\pi$. **(06 Marks)**

c. The following table gives the variations of periodic current over a period T.

t (secs)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (Amps)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Expand the function (periodic current) by Fourier series and show that there is a direct current part of 0.75 amp and also obtain amplitude of first harmonic. **(06 Marks)**

Module-2

3 a. Find Fourier transform of $f(x)=\begin{cases}1-x^2; & |x|<1\\ 0; & |x|>1\end{cases}$

and hence evaluate $\displaystyle\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}dx$. **(08 Marks)**

b. Find Fourier Cosine transform of the function :
$$f(x)=\begin{cases}4x; & 0<x<1\\ 4-x; & 1<x<4\\ 0; & x>4\end{cases}$$
(06 Marks)

c. Find z-transforms of : i) $a^n \sin n\theta$ ii) $a^{-n}\cos n\theta$. **(06 Marks)**

OR

4 a. Find Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate : $\int_0^\infty \frac{x \sin mx}{1+x^2} dx, m > 0$. **(08 Marks)**

b. Find z-transform of $u_n = \cosh\left(\frac{n\pi}{2} + \theta\right)$. **(06 Marks)**

c. Solve the difference equation using z-transforms $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$. Given $u_0 = u_1 = 0$. **(06 Marks)**

Module-3

5 a. If θ - is the acute angle between the two regression lines relating the variables x and y, show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2\sigma_y^2}\right)$. **(08 Marks)**

Indicate the significance of the cases r = ±1 and r = 0.

b. Fit a straight line y = ax +b for the data.

x	12	15	21	25
y	50	70	100	120

(06 Marks)

c. Find a real root of the equation by using Newton-Raphson method near x = 0.5, $xe^x = 2$, perform three iterations. **(06 Marks)**

OR

6 a. Compute the coefficient of correlation and equation of regression of lines for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(08 Marks)

b. The Growth of an organism after x – hours is given in the following table :

x (hours)	5	15	20	30	35	40
y (Growth)	10	14	25	40	50	62

Find the best values of a and b in the formula $y = ae^{bx}$ to fit this data. **(06 Marks)**

c. Find a real root of the equation cos x = 3x – 1 correct to three decimals by using Regula – False position method, given that root lies in between 0.6 and 0.7. Perform three iterations. **(06 Marks)**

Module-4

7 a. Find y(8) from y(1) = 24, y(3) = 120, y(5) = 336, y(7) = 720 by using Newton's backward difference interpolation formula. **(08 Marks)**

b. Define f(x) as a polynomial in x for the following data using Newton's divided difference formula. **(06 Marks)**

x	–4	–1	0	2	5
f(x)	1245	33	5	9	1335

c. Evaluate the integral $I = \int_0^6 \frac{dx}{4x+5}$ using Simpson's $\frac{1}{3}$rd rule using 7 ordinates. **(06 Marks)**

OR

8 a. For the following data calculate the differences and obtain backward difference interpolation polynomial. Hence find f(0.35). **(08 Marks)**

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.0	2.28

b. Using Lagrange's interpolation find y when x = 10.

x	5	6	9	11
y	12	13	14	16

(06 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule considering seven ordinates. **(06 Marks)**

Module-5

9 a. Verify the Green's theorem in the plane for $\int_c (x^2+y^2)dx + 3x^2y\,dy$ where C is the circle $x^2+y^2 = 4$ traced in positive sense. **(08 Marks)**

b. Evaluate $\int_c (\sin z.dx - \cos x dy + \sin y dx)$ by using Stokes theorem. where C – is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and $z = 3$. **(06 Marks)**

c. Find the curve on which the functional : $\int_0^1 [y'^2 + 12xy]dx$ with y(0) = 0, y(1) = 1 can be extremised. **(06 Marks)**

OR

10 a. Given $f = (3x^2 - y)i + xzj + (yz - x)k$ evaluate $\int_c f \cdot dr$ from (0, 0, 0) to (1, 1, 1) along the paths $x = t$, $y = t^2$ and $z = t^3$. **(08 Marks)**

b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

c. Prove that the shortest distance between two points in a plane is a straight line. **(06 Marks)**

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CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. An alternating current after passing through a rectifier has the form.
$$I = \begin{cases} I_0 \sin x & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$$
where I_0 is the maximum current and the period is 2π. Express I as a Fourier series. **(08 Marks)**

b. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data: **(08 Marks)**

x^0	0	45	90	135	180	225	270	315
y	2	1.5	1	0.5	0	0.5	1	1.5

OR

2 a. Obtain the Fourier series expansion of the function, $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce that,
$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{8}$$ **(06 Marks)**

b. Find the Fourier series expansion of the function,
$$f(x) = \begin{cases} \pi x & \text{in } 0 \le x \le 1 \\ \pi(2 - x) & \text{in } 1 \le x \le 2 \end{cases}$$ **(05 Marks)**

c. The following table gives the variations of periodic current over a period.

t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A(amplitude)	1.98	1.30	1.05	1.3	−0.88	−0.25	1.98

Show by harmonic analysis that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of first harmonic. **(05 Marks)**

Module-2

3 a. Find the complex Fourier transform of the function $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. **(05 Marks)**

c. Compute the inverse z-transforms of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(05 Marks)**

OR

4 a. Find the z-transform of $e^{-an}n + \sin n\frac{\pi}{4}$. **(06 Marks)**

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z-transform. **(05 Marks)**

c. Find the Fourier cosine transform of, $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4 - x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$. **(05 Marks)**

Module-3

5 a. Find the Correlation coefficient and equations of regression lines for the following data:

x	1	2	3	4	5
y	2	5	3	8	7

(06 Marks)

b. Fit a straight line to the following data:

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(05 Marks)

c. Find a real root of the equation $xe^x = \cos x$ correct to three decimal places that lies between 0.5 and 0.6 using Regula-falsi method. **(05 Marks)**

OR

6 a. The following regression equations were obtained from a correlation table.

$y = 0.516x + 33.73$

$x = 0.516y + 32.52$

Find the value of (i) Correlation coefficient (ii) Mean of x's (iii) Mean of y's. **(06 Marks)**

b. Fit a second degree parabola to the following data:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(05 Marks)

c. Use Newton-Raphson's method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$, carry out three iterations. **(05 Marks)**

Module-4

7 a. The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in °C and P is the percentage of lead in the alloy:

P%	60	70	80	90
t	226	250	276	304

Find the melting point of the alloy containing 84% of lead, using Newton's interpolation formula. **(06 Marks)**

b. Apply Lagrange's interpolation formula to find a polynomial which passes through the points $(0, -20)$, $(1, -12)$, $(3, -20)$ and $(4, -24)$ **(05 Marks)**

c. Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's $\frac{3}{8}^{th}$ rule by dividing it into 6 equal parts. **(05 Marks)**

OR

8 a. From the following table :

x°	10	20	30	40	50	60
$\cos x$	0.9848	0.9397	0.8660	0.7660	0.6428	0.5

Calculate $\cos 25^\circ$ using Newton's forward interpolation formula. **(06 Marks)**

b. Use Newton's divided difference formula and find f(6) from the following data:

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

(05 Marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Weddle's rule by taking equidistant ordinates. **(05 Marks)**

Module-5

9 a. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ with the help of Green's theorem in a plane. **(06 Marks)**

b. Solve the variational problem $\delta\int_0^1 (12xy + y'^2)dx = 0$ under the conditions y(0) = 3, y(1) = 6. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them. **(05 Marks)**

OR

10 a. A cable hangs freely under gravity from the fixed points. Show that the shape of the curve is a catenary. **(06 Marks)**

b. Use Stoke's theorem to evaluate for $\vec{F} = (x^2 + y^2)i - 2xyj$ taken around the rectangle bounded by the lines $x = \pm a$, y = 0, y = b. **(05 Marks)**

c. Evaluate $\iint_S (yzi + zxj + xyk).\hat{n}ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. **(05 Marks)**

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CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Using Taylor's series method solve $\frac{dy}{dx} = x^2 + y^2$ with y(0) = 1 and hence find y(0.1) and consider upto 3rd degree. **(06 Marks)**

b. Using modified Euler's method solve $\frac{dy}{dx} = 1 + \frac{y}{x}$ with y(1) = 2 then find y(1.2) in two steps. **(05 Marks)**

c. Given $\frac{dy}{dx} = \frac{x+y}{2}$, give that y(0) = 2, y(0.5) = 2.636, y(1) = 3.595 and y(1.5) = 4.968 then find value of y at x = 2 using Milne's predictor and corrector formulae. **(05 Marks)**

OR

2 a. Using modified Euler's method solve $\frac{dy}{dx} = x + \sqrt{y}$, with y(0) = 1 then find y(0.2) with h = 0.2. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, with y(0) = 1 and hence find y(0.1) by taking one steps using Runge-Kutta method of fourth order. **(05 Marks)**

c. Given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, given that y(0) = 1, y(0.1) = 1.06. y(0.2) = 1.12 and y(0.3) = 1.21 then evaluate y(0.4) using Adam's – Bash forth method. **(05 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = \frac{2dy}{dx} - y$, y(0) = 1, y′(0) = 2, evaluate y(0.1) and y′(0.1) using Runge-Kutta method of fourth order. **(06 Marks)**

b. Solve the Bessel's differential equation : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ leading to $J_n(x)$. **(05 Marks)**

c. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(05 Marks)**

OR

4 a. Using Milne's method. obtain an approximate solution at the point x = 0.8 of the problem $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ using the following data :

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

(06 Marks)

b. If α and β are two distinct roots of $J_n(x) = 0$ then P-T $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \{0$ if $\alpha \neq \beta$. **(05 Marks)**

c. With usual notation, prove that $J_{+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(05 Marks)**

Module-3

5 a. State and prove Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find analytic function f(z) whose imaginary part is $v = \left(r - \frac{1}{r}\right)\sin\theta$. **(05 Marks)**

c. Discuss the transformation of $\omega = e^z$. **(05 Marks)**

OR

6 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Emulate $\oint_c \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is |z| = 3 using Cauchy's residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps z = –1, 0, 1 into ω = 0, i, 3i. **(05 Marks)**

Module-4

7 a. Derive mean and variance of the binomial distribution. **(06 Marks)**

b. A random variable x has the following probability mass function.

x	0	1	2	3	4	5
P(x)	k	3k	5k	7k	9k	11k

i) find k ii) find p(x < 3) iii) find p(3 < x ≤ 5). **(05 Marks)**

c. The joint distribution of two random variable x and y as follows :

x \ y	–4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute : i) E(x) and E(y) ii) E(xy) iii) cov(xy). **(05 Marks)**

OR

8 a. 2% of the fuses manufactured by a firm are found defective. Find the probability that a box containing 200 fuses contains. i) no defective fuses ii) 3 or more defective fuses. **(06 Marks)**

b. In a test on 2000 electric bulbs. It was found that the life of a particular brand was distributed normally with an average life of 2040 hours and S.D 60 hours. Estimate the number of bulbs likely to burn ($P(0 < z < 1.83) = 0.4664$ $P(1.33) = 0.4082$, $P(2) = 0.4772$) i) more than 2150 ii) less than 1960 iii) more than 1920 but less than 2160 hours. **(05 Marks)**

c. The joint probability distribution of two random variable X and Y given by the following table:

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find marginal distribution of X and Y and evaluate cov(XY). **(05 Marks)**

Module-5

9 a. Define: i) Null hypothesis ii) significance level iii) Type–I and Type–II error. **(06 Marks)**

b. Ten individual are chosen at random from a population and their height in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that mean height of the universe is 66 inches. Given that ($t_{0.05} = 2.262$ for 9d.f) **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix :

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}.$$ **(05 Marks)**

OR

10 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. **(06 Marks)**

b. Four coins are tossed 100 times and following results were obtained :

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit $\left(\chi^2_{0.05} = 9.49\right)$. **(05 Marks)**

c. A student's study habit are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night he is 60% sure not to study the next night. In the long run how often does he study? **(05 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | **17MATDIP31**

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$ **(08 Marks)**

b. Express $\sqrt{3}+i$ in the polar form and hence find its modulus and amplitude. **(06 Marks)**

c. Find the sine of the angle between vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+2\hat{k}$ **(06 Marks)**

OR

2 a. Express $\frac{3+4i}{3-4i}$ in the form x + iy. **(08 Marks)**

b. If the vector $2\hat{i}+\lambda\hat{j}+\hat{k}=0$ and $4\hat{i}-2\hat{j}-2\hat{k}$ are perpendicular to each other, find λ. **(06 Marks)**

c. Find λ, such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$, $3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar. **(06 Marks)**

Module-2

3 a. If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ **(08 Marks)**

b. With usual notations, prove that $\tan\phi=r\frac{d\theta}{dr}$. **(06 Marks)**

c. If $u=\log_e\frac{x^3+y^3}{x^2+y^2}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$. **(06 Marks)**

OR

4 a. Using Maclaurin's series, expand $\tan x$ upto the term containing x^5. **(08 Marks)**

b. Find the pedal equation of $r=a(1-\cos\theta)$. **(06 Marks)**

c. If $u=x+3y^2-z^3$, $v=4x^2yz$ and $w=2z^2-xy$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$, (n > 0). **(08 Marks)**

b. Evaluate $\int_0^a\frac{x^7}{\sqrt{a^2-x^2}}dx$ **(06 Marks)**

c. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$ **(06 Marks)**

OR

6 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$ **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dx\,dy\,dz$ **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$, where 't' is the time. Find its velocity and acceleration vectors and also magnitude of velocity and acceleration at $t = 1$. **(08 Marks)**

b. In which direction of the directional derivative of x^2yz^3 is maximum at $(2, 1, -1)$ and find the magnitude of this maximum. **(06 Marks)**

c. Show that $\vec{F} = (y + z)\hat{i} + (x + z)\hat{j} + (x + y)\hat{k}$ is irrotational. **(06 Marks)**

OR

8 a. If $\phi = xy^2z^3 - x^3y^2z$, find $\nabla\phi$ and $|\nabla\phi|$ at $(1, -1, 1)$. **(08 Marks)**

b. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$, show that $\vec{F}\cdot \text{Curl}\,\vec{F} = 0$. **(06 Marks)**

c. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of a curve, find the angle between the tangents at $t = 1$ and $t = 2$. **(06 Marks)**

Module-5

9 a. Solve : $\left(x\tan\frac{y}{x} - \frac{y}{x}\sec^2\frac{y}{x}\right)dx = x\sec^2\frac{y}{x}\,dy$ **(08 Marks)**

b. Solve : $xy(1 + xy^2)\frac{dy}{dx} = 1$ **(06 Marks)**

c. Solve : $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **(06 Marks)**

OR

10 a. Solve : $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$ **(08 Marks)**

b. Solve : $(1 + y^2)dx = (\tan^{-1}y - x)dy$ **(06 Marks)**

c. Solve : $(y\log y)dx + (x - \log y)dy = 0$. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

15MATDIP31

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of $\dfrac{(3-\sqrt{2}i)^2}{1+2i}$. **(06 Marks)**

b. Find the cube root of $(1-i)$. **(05 Marks)**

c. Prove that $\left(\dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(n\dfrac{\pi}{2}-n\theta\right)+i\sin\left(n\dfrac{\pi}{2}-n\theta\right)$. **(05 Marks)**

OR

2 a. For any three vector a, b, c show that
$\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ **(06 Marks)**

b. Find the value of λ so that the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+\lambda\hat{k}$ are coplanar. **(05 Marks)**

c. Find the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$ **(05 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\cos x\cos 2x\cos 3x$. **(06 Marks)**

b. If $y=a\cos(\log x)+b\sin(\log x)$, prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$. **(05 Marks)**

c. Find the angle between the radius vector and tangents for the curve $r^2\cos 2\theta = a^2$ **(05 Marks)**

OR

4 a. If $u=e^{ax+by}+(ax-by)$ prove that $b\dfrac{\partial u}{\partial x}+a\dfrac{\partial u}{\partial y}=2abu$. **(06 Marks)**

b. If $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x-y}\right)$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$. **(05 Marks)**

c. If $x=u(1-v)$, $y=uv$. Find $\dfrac{\partial(x,y)}{\partial(u,v)}$. **(05 Marks)**

Module-3

5 a. Obtain the reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$ (n>0). **(06 Marks)**

b. Evaluate $\int_0^1 x^6\sqrt{1-x^2}\,dx$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^y xyz\,dx\,dy\,dz$. **(05 Marks)**

OR

6 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$, $n > 0$. **(06 Marks)**

b. Evaluate $\int_0^a x^2(a^2 - x^2)^{\frac{3}{2}} dx$. **(05 Marks)**

c. Evaluate $\int_0^1 \int_0^{\sqrt{x}} xydydx$. **(05 Marks)**

Module-4

7 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Determine the component of velocity and acceleration vector at t = 0 in the direction of $\hat{i} + \hat{j} + \hat{k}$. **(08 Marks)**

b. Find the value of the constant a, b, such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. **(08 Marks)**

OR

8 a. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ show that $\vec{F} \cdot \text{curl}\,\vec{F} = 0$. **(06 Marks)**

b. If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at $(1, -1, 2)$. **(05 Marks)**

c. Find the directional derivative $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(05 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ **(05 Marks)**

c. $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$. **(05 Marks)**

OR

10 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(06 Marks)**

b. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xyz)dy = 0$ **(05 Marks)**

c. Solve $(1 + y^2)dx + (x - \tan^{-1} y)dy = 0$ **(05 Marks)**

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CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ (05 Marks)

b. Solve by Gauss elimination method:
$2x + y + 4z = 12$ $4x + 11y - z = 33$ $8x - 3y + 2z = 20$ (05 Marks)

c. Find all the eigen values of the matrix
$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (06 Marks)

OR

2 a. Find the values of K, such that the matrix A may have the rank equal to 3:
$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & K \\ 1 & 4 & 10 & K^2 \end{bmatrix}$ (05 Marks)

b. Solve by Gauss elimination method
$x_1 - 2x_2 + 3x_3 = 2$ $3x_1 - x_2 + 4x_3 = 4$ $2x_1 + x_2 - 2x_3 = 5$ (05 Marks)

c. Find all the eigen values and corresponding eigen vectors of the matrix
$A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ (06 Marks)

Module-2

3 a. Find C.F of $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. (05 Marks)

b. Solve the initial value problem $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 29x = 0$
Subject to the conditions $x(0) = 0$, $\frac{dx}{dt}(0) = 15$. (05 Marks)

c. Using the method of undetermined coefficients, solve $(D^2 - 4D + 3)y = 20\cos x$ (06 Marks)

OR

4 a. Solve $(D^2 - 2D + 4)y = e^x \cos x$. (05 Marks)

b. Solve $(D^2 + 4)y = x^2 + 2^{-x}$. (05 Marks)

c. Using the method of variation of parameters, find the solution of $(D^2 - 2D + 1)y = e^x / x$. (06 Marks)

Module-3

5 a. Find the Laplace transform of $\frac{\cos 3t - \cos 4t}{t}$. (05 Marks)

b. Find $L\{t\sin^2 t\}$ (05 Marks)

c. Express the following function interms of Heaviside unit step function and hence find the Laplace transform where

$$f(t) = \begin{cases} t^2 & 0 < t \le 2 \\ 4t & t > 2 \end{cases}$$ (06 Marks)

OR

6 a. Find $L\left[\frac{e^{-t}.\sin t}{t}\right]$. (05 Marks)

b. Using Laplace transform evaluate $\int_0^\infty e^{-t} t \sin^2 3t dt$. (05 Marks)

c. If $f(t) = \begin{cases} t & 0 \le t \le a \\ 2a - t & a \le t \le 2a \end{cases}$ $f(t+2a) = f(t)$, show that $L[f(t)] = \frac{1}{s^2}\tan h\left(\frac{as}{2}\right)$. (06 Marks)

Module-4

7 a. Find inverse Laplace transform of $\frac{s+5}{s^2 - 6s + 13}$. (05 Marks)

b. Find inverse Laplace transform of $\log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$. (05 Marks)

c. Solve by using Laplace transform method $y''(t) + 4y(t) = 0$, given that $y(0) = 2$, $y'(0) = 0$. (06 Marks)

OR

8 a. Find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$. (05 Marks)

b. Find $L^{-1}\left[\frac{(s+2)e^{-s}}{(s+1)^4}\right]$ (05 Marks)

c. Solve by using Laplace transform method $y'' + 5y' + 6y = 5e^{2x}$, $y(0) = 2$, $y'(0) = 1$. (06 Marks)

Module-5

9 a. There are 10 students of which three are graduates. If a committee of five is to be formed, what is the probability that there are (i) only 2 graduates (ii) atleast 2 graduates? (05 Marks)

b. In a school 25% of the students failed in the first language, 15% of the students failed in second language and 10% of the students failed in both. If a student is selected at random find the probability that :

i) He failed in first language if he had failed in the second language.

ii) He failed in second language if he had failed in the first language. (05 Marks)

c. In a bolt factory there are four machines A, B, C and D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3% and 2% are defective. If a bolt drawn at random was found defective what is the probability that it was manufactured by A or D. (06 Marks)

OR

10 a. From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is a positive number? **(05 Marks)**

b. Three students A, B, C write an entrance examination. Their chances of passing are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that (i) atleast one of them passes (ii) all of them passes. **(05 Marks)**

c. Three major parties A, B, C are contending for power in the elections of a state and the chance of their winning the election is in the ratio 1:3:5. The parties A, B, C respectively have probability of banning the online lottery $\frac{2}{3}$, $\frac{1}{3}$, $\frac{3}{5}$. What is the probability that there will be a ban on the online lottery in the state? What is the probability that the ban is from the party 'C'? **(06 Marks)**

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CBCS SCHEME

USN

18MAT21

Second Semester B.E. Degree Examination, June/July 2019
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find div $\vec{F}$ and curl $\vec{F}$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, −1, 2). **(07 Marks)**

c. Find the value of a, b, c such that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational, also find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. Find the total work done in moving a particle in the force field $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2+1$, $y = 2t^2$, $z = t^3$ from t = 1 to t = 2. **(06 Marks)**

b. Using Green's theorem, evaluate

$\int_C (xy + y^2)dx + x^2dy$, where C is bounded by y = x and $y = x^2$. **(07 Marks)**

c. Using Divergence theorem, evaluate $\int_S \vec{F} ds$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. **(07 Marks)**

Module-2

3 a. Solve $(D^2 - 3D + 2)y = 2x^2 + \sin 2x$. **(06 Marks)**

b. Solve $(D^2 + 1)y = \sec x$ by the method of variation of parameter. **(07 Marks)**

c. Solve $x^2y'' - 4xy' + 6y = \cos(2 \log x)$ **(07 Marks)**

OR

4 a. Solve $(D^2 - 4D + 4)y = e^{2x} + \sin x$. **(06 Marks)**

b. Solve $(x+1)^2y'' + (x+1)y' + y = 2\sin[\log_e(x+1)]$ **(07 Marks)**

c. The current i and the charge q in a series containing an inductance L, capacitance C, emf E, satisfy the differential equation $L\frac{d^2q}{dt^2} + \frac{q}{C} = E$, Express q and i interms of 't' given that L, C, E are constants and the value of i and q are both zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z,\ x^2 + y^2 + z^2) = 0$ **(06 Marks)**

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ **(07 Marks)**

c. Derive one dimensional heat equation in the standard form as $\frac{\partial U}{\partial t} = C^2 \frac{\partial^2 U}{\partial x^2}$. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ such that $z = e^y$ where x = 0 and $\frac{\partial z}{\partial x} = 1$ when x = 0. **(06 Marks)**

b. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - \ell z)\frac{\partial z}{\partial y} = \ell y - mx$ **(07 Marks)**

c. Find all possible solutions of one dimensional wave equation $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Discuss the nature of the series $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$. **(06 Marks)**

b. With usual notation prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_3 + bP_2\ cP_1 + dP_0$, find a, b, c and d using Legendre's polynomial. **(07 Marks)**

OR

8 a. Discuss the nature of the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{3.4} + \ldots\ldots\ldots$$ **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation in terms of $P_n(x)$

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$ **(07 Marks)**

c. Express $x^4 - 3x^2 + x$ interms of Legendre's polynomial. **(07 Marks)**

Module-5

9 a. Find the real root of the equation xsinx + cosx = 0 near $x = \pi$ using Newton-Raphson method. Carry out 3 iterations. **(06 Marks)**

b. From the following data, find the number of students who have obtained (i) less than 45 marks (ii) between 40 and 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 - 80
No. of Students	31	42	51	35	31

(07 Marks)

c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ using Simpson's $\frac{3}{8}^{th}$ rule by taking 7 ordinates. **(07 Marks)**

OR

10 a. Find the real root of the equation $x \log_{10} x = 1.2$ which lies between 2 and 3 using Regula-Falsi method. **(06 Marks)**

b. Using Lagrange's interpolation formula, find y at x = 4, for the given data:

x	0	1	2	5
y	2	3	12	147

(07 Marks)

c. Evaluate $\int_4^{5.2} \log_e x\, dx$ using Weddle's rule by taking six equal parts. **(07 Marks)**

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CBCS SCHEME

USN

18MAT11

First Semester B.E. Degree Examination, June/July 2019
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$. **(06 Marks)**

b. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts the x-axis. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

OR

2 a. Prove that the pedal equation of the curve $r^n = a^n\cos n\theta$ is $a^n.p = r^{n+1}$. **(06 Marks)**

b. Show that for the curve $r(1-\cos\theta) = 2a$, ρ^2 varies as r^3. **(06 Marks)**

c. Find the angle between the polar curves $r = a(1-\cos\theta)$ and $r = b(1+\cos\theta)$. **(08 Marks)**

Module-2

3 a. Expand $\log(1+\cos x)$ by Maclaurin's series up to the term containing x^4. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left\{\frac{a^x + b^x + c^x}{3}\right\}^{1/x}$ **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

OR

4 a. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\cdot\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y} + z\cdot\frac{\partial u}{\partial z} = 0$ **(06 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point (1, -1, 0). **(07 Marks)**

c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum. **(07 Marks)**

Module-3

5 a. Evaluate by changing the order of integration

$$\int_0^a \int_0^{2\sqrt{ax}} x^2 \cdot dy \cdot dx, \quad a > 0$$ **(06 Marks)**

b. Find the area bounded between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$. **(07 Marks)**

c. Prove that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz.dy.dx$ **(06 Marks)**

b. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration. **(07 Marks)**

c. Show that $\int_{0}^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_{0}^{\pi/2}\sqrt{\sin\theta}.d\theta=\pi$ **(07 Marks)**

Module-4

7 a. Solve $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ **(06 Marks)**

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. **(07 Marks)**

c. Solve $yp^2+(x-y)p-x=0$. **(07 Marks)**

OR

8 a. Solve $\frac{dy}{dx}+y\cdot\tan x=y^3\cdot\sec x$ **(06 Marks)**

b. Find the orthogonal trajectory of the family of the curves $r^n\cdot\cos n\theta=a^n$, where a is a parameter. **(07 Marks)**

c. Solve the equation $(px-y)\cdot(py+x)=2p$ by reducing into Clairaut's form taking the substitution $X=x^2$, $Y=y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A=\begin{pmatrix}1&2&-2&3\\2&5&-4&6\\-1&-3&2&-2\\2&4&-1&6\end{pmatrix}$$ by applying elementary Row transformations. **(06 Marks)**

b. Solve the following system of equations by Gauss-Jordan method:
$x+y+z=9,\ 2x+y-z=0,\ 2x+5y+7z=52$ **(07 Marks)**

c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vector of the matrix $A=\begin{pmatrix}2&0&1\\0&2&0\\1&0&2\end{pmatrix}$ with $X^{(0)}=(1,0,0)^T$ as the initial eigen vector carry out 5 iterations. **(07 Marks)**

OR

10 a. For what values of λ and μ the system of equations.
$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ may have
i) Unique solution ii) Infinite number of solutions iii) No solution. **(06 Marks)**

b. Reduce the matrix $A=\begin{pmatrix}-1&3\\-2&4\end{pmatrix}$ into diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss-Seidel method
$20x+y-2z=17,\ 3x+20y-z=-18,\ 2x-3y+20z=25$. Carry out 3 iterations. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | |

17MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\sin 2x \ \sin 3x$. **(06 Marks)**

b. Find the angle between the two curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$ **(07 Marks)**

c. Find the radius of curvature for the curve $x^3 + y^3 = 3xy$ at (3/2, 3/2). **(07 Marks)**

OR

2 a. If $y = \cos(m \log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$. **(06 Marks)**

b. With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$. **(07 Marks)**

c. Find the pedal equation of the curve $r^m = a^m \cos m\theta$. **(07 Marks)**

Module-2

3 a. Find the Taylor's series of log(cosx) in powers of $(x - \pi/3)$ upto fourth degrees terms. **(06 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ by using Euler's theorem. **(07 Marks)**

c. If $u = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{xy}{z}$ then find $J = \frac{\partial(uvw)}{\partial(xyz)}$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$. **(06 Marks)**

b. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - + - - - -$. **(07 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the cuvre $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$. Find the components of velocity and acceleration in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$ at t = 0. **(06 Marks)**

b. Find the constant a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational and find scalar potential function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{curl}(\phi\vec{A}) = \phi\,\text{curl}\,\vec{A} + \text{grad}\phi \times \vec{A}$. **(07 Marks)**

OR

6 a. Show that vector field $F = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(06 Marks)**

b. If $\vec{F} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$ then prove that $\vec{F} = \text{curl}\,\vec{F} = 0$. **(07 Marks)**

c. Show that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(07 Marks)**

Module-4

7 a. Obtain reduction formula for $\int \sin^n x\,dx (n > 0)$. **(06 Marks)**

b. Solve the differential equation $\frac{dy}{dx} + y\cot x = \cos x$. **(07 Marks)**

c. Find the orthogonal trajectory of the curve $r = a(1 + \sin\theta)$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^6\theta\, d\theta$. **(06 Marks)**

b. Solve the differential equation : $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(07 Marks)**

c. If the temperature of air is 30°C and the substance cools from 100°C to 70°C in 15 mins. Find when the temperature will be 40°C. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ by reducing to Echelon form. **(06 Marks)**

b. Find the largest eigen value and egien vector of the matrix : $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking initial vector as $[\,1\ 1\ 1]^T$ by using Rayleigh's power method. Carry out five iteration. **(07 Marks)**

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form, using orthogonal transformation. **(07 Marks)**

OR

10 a. Solve the system of equations

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

by using Gauss-Seidel method. Carry out three iterations. **(06 Marks)**

b. Diagonalise the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(07 Marks)**

c. Show that the transformation

$y_1 = x_1 + 2x_2 + 5x_3$

$y_2 = 2x_1 + 4x_2 + 11x_3$

$y_3 = -x_2 + 2x_3$

is regular. Write down inverse transformation. **(07 Marks)**

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CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{7x+6}{2x^2+7x+6}$ (05 Marks)

b. Find the angle between the radius vector and the tangent for the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. (05 Marks)

c. Show that the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos(\theta/2)$ (06 Marks)

OR

2 a. If $x = \sin t$ and $y = \cos mt$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. (05 Marks)

b. Find the pedal equation of the curve $r^2 = a^2 \sec 2\theta$. (05 Marks)

c. Prove with usual notation $\tan\phi = \dfrac{rd\theta}{dr}$. (06 Marks)

Module-2

3 a. Expand $e^{\sin x}$ using Maclaurin's series upto third degree term. (05 Marks)

b. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right]$. (05 Marks)

c. If $u = e^{(ax+by)} f(ax - by)$, prove that $b\dfrac{\partial u}{\partial x} + a\dfrac{\partial u}{\partial y} = 2abu$ (06 Marks)

OR

4 a. Expand sin x in ascending power of $\pi/2$ upto the term containing x^4. (05 Marks)

b. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, show that $x u_x + y u_y = \sin 2u$. (05 Marks)

c. If $u = \dfrac{yz}{x}, v = \dfrac{zx}{y}, w = \dfrac{xy}{z}$. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. (06 Marks)

Module-3

5 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$. (05 Marks)

b. Show that $\vec{F} = (y+z)i + (x+z)j + (x+y)k$ is irrotational. Also find a scalar function φ such that $\vec{F} = \nabla\phi$. (05 Marks)

c. Prove that $\nabla\cdot(\phi\vec{A}) = \phi(\nabla\cdot\vec{A}) + \nabla\phi\cdot\vec{A}$. (06 Marks)

OR

6 a. Prove that Curl $(\phi\vec{A}) = \phi(\text{Curl}\,\vec{A}) + \text{grad}\phi \times \vec{A}$ (05 Marks)

b. A particle moves along the curve C ; $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where 't' denotes the time. Find the component of acceleration at t = 2 along the tangent. (05 Marks)

c. Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is a conservative force field. Find its scalar potential. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\,dx$. **(05 Marks)**

b. Solve $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of r = a (1+sinθ). **(06 Marks)**

OR

8 a. Evaluate $\int_0^2 x\sqrt{2x - x^2}\,dx$ **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. **(05 Marks)**

c. A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F. After half an hour, water cooled to 61°F

i) What is the temperature of the mineral water in another half an hour?

ii) How long will it take to cool to 50°F? **(06 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$ **(05 Marks)**

b. Find the largest eigen value and corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ by power method taking $X^{(0)} = [1, 1, 1]^1$ **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(06 Marks)**

10 a. Use Gauss elimination method to solve

$2x + y + 4z = 12$

$4x + 11y - z = 33$

$8x - 3y + 2z = 20$ **(05 Marks)**

b. Find the inverse transformation of the following linear transformation.

$y_1 = x_1 + 2x_2 + 5x_3$

$y_2 = 2x_1 + 4x_2 + 11x_3$

$y_3 = -x_2 + 2x_3$ **(05 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ to the Cannonical form. **(06 Marks)**

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14MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If x = tan (logy) then prove that $(1+x^2)y_{n+2}+[2(n+1)x-1]y_{n+1}+n(n+1)y_n=0$. **(06 Marks)**

b. Prove that the angle of intersection between the curves r = a (1 – cos θ) and r = 2a cosθ as 1/2 (π + cos^{-1} (1/3)). **(07 Marks)**

c. Define the curvature and radius of curvature of a curve. Derive the expression for radius of curvature in polar form. **(07 Marks)**

OR

2 a. State Leibnitz theorem for nth derive of product of two functions. Find the nth derivative of $y = x^2 \log x$. **(06 Marks)**

b. Find the angle of intersection between the curves $\frac{2a}{r} = 1+\cos\theta$ and $\frac{2a}{r} = 1-\cos\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(07 Marks)**

Module-2

3 a. Expand logx in the powers of (x-1) upto and including $(x-1)^3$ and hence compute log (1.1). **(06 Marks)**

b. Define homogeneous function. Give suitable example.

If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3axy$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x \to 0}\left(\frac{e^x \sin x - x^2 - x}{x^2 + x\log(1-x)}\right)$. **(06 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. **(07 Marks)**

c. If x = r sin θ cosϕ , y = rsin θ sin ϕ, z = r cos θ, find the value of $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$. **(07 Marks)**

Module-3

5 a. Find a, b and c such that $\vec{F} = (axy - z^3)i + (bx^2 + z)j + (bxz^2 + cy)k$ is irrotational and find the scalar potential. **(06 Marks)**

b. Prove that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x)}dx = \frac{\pi}{2}\log(1+a)$, a≥ 0 using differentiation under integration. **(07 Marks)**

c. Apply the general rule to trace the curve r = a (1 + cosθ). **(07 Marks)**

OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2i - j - 2k$. **(06 Marks)**

b. Evaluate $\int_0^{\alpha} \frac{\log(1+\alpha x)}{1+x^2}dx$ and hence prove that $\int_0^1 \frac{\log(1+x)}{1+x^2} = \frac{\pi}{8}\log 2$. **(07 Marks)**

c. Apply the general rule to trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. **(07 Marks)**

Module-4

7 a. Establish the reduction formula for $\int \sin^m x \cos^n x\, dx$ and hence find $\int_0^{\pi/2} \sin^6 x \cos^4 dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{-(y\cos x + \sin y + y)}{\sin x + x\cos y + x}$. **(07 Marks)**

c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

8 a. Establish the reduction formula of $\int \sin^n x\, dx$ and evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}dx$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(07 Marks)**

c. A 12 volt battery is connected to a series circuit in which the inductance is 1/2 Henry and the resistance is 10 ohms. Determine the current I if the initial current is zero. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ using elementary transformations. **(06 Marks)**

b. Use Gauss-Seidal iterative method to solve the system of equations $20x + y - 2z = 17$, $3x + 20y - z = -18$ and $2x - 3y + 20z = 25$. (Use three iterations). Take initial values for {x, y, z} as {0, 0, 0}. **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. (Use $x^{(0)} = [1 \quad 0 \quad 0]^T$. Take 5 iterations. **(07 Marks)**

OR

10 a. Solve by Gauss elimination method the system of equations,
$x + y + z + t = 2$, $2x - y + 2z - t = -5$, $3x + 2y + 3z + 4t = 7$ and $x - 2y - 3z + 2t = 5$. **(06 Marks)**

b. Reduce the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to canonical form. **(07 Marks)**

c. Test whether the transformation (x_1, x_2, x_3) to $(2x_1 + x_2 + x_3, x_1 + x_2 + 2x_3, x_1 - 2x_3)$ is non-singular. If so write the inverse transformation. **(07 Marks)**

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17MAT21

Second Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $(D^2+1)y = 3x^2+6x+12$. **(06 Marks)**

b. Solve $(D^3+2D^2+D)y = e^{-x}$. **(07 Marks)**

c. By the method of undetermined coefficients, solve $(D^2+D-2)y = x+\sin x$. **(07 Marks)**

OR

2 a. Solve $(D^2-6D+9)y = 6e^{3x}+7e^{-2x}$. **(06 Marks)**

b. Solve $(D^3-D)y = (2x+1)+4\cos x$. **(07 Marks)**

c. By the method of variation of parameters, solve $(D^2+1)y = \operatorname{cosec} x$. **(07 Marks)**

Module-2

3 a. Solve $x^2y''-3xy'+4y = 1+x^2$. **(06 Marks)**

b. Solve $xyp^2-(x^2+y^2)p+xy = 0$. **(07 Marks)**

c. Solve $(px-y)(py+x) = a^2p$ by taking $x^2 = x$ and $y^2 = y$. **(07 Marks)**

OR

4 a. Solve $(2+x)^2y''+(2+x)y'+y = \sin(2\log(2+x))$. **(06 Marks)**

b. Solve $yp^2+(x-y)p-x = 0$. **(07 Marks)**

c. Obtain the general and singular solution of the equation $\sin px\cos y = \cos px\sin y+p$. **(07 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating arbitrary function
$lx+my+nz = \phi(x^2+y^2+z^2)$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions $\frac{\partial z}{\partial x} = \log(1+y)$ when $x = 1$ and $z = 0$ when $x = 0$. **(07 Marks)**

c. Derive an expression for the one dimensional wave equation. **(07 Marks)**

OR

6 a. Form a partial differential equation $z = f(y+2x)+g(y-3x)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. **(07 Marks)**

c. Find all possible solutions of heat equation $u_t = c^2u_{xx}$ by the method of separation of variables. **(07 Marks)**

Module-4

7 a. Evaluate $\iint r\sin\theta \, dr \, d\theta$ over the cardioids $r = a(1-\cos\theta)$ above the initial line. **(06 Marks)**

b. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$. **(07 Marks)**

c. Derive the relation between Beta and Gamma function as $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

OR

8 a. Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y} dydx$. **(06 Marks)**

b. Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **(07 Marks)**

c. Show that $\int_0^{\pi/2} \sqrt{\cot\theta} \, d\theta = \frac{1}{2} \left|\overline{\left(\frac{1}{4}\right)} \right|\overline{\left(\frac{3}{4}\right)}$ **(07 Marks)**

Module-5

9 a. Find the Laplace transform of $\left(t\cos 2t + \dfrac{1-e^{3t}}{t}\right)$. **(06 Marks)**

b. Find the Laplace transform of $f(t) = E\sin\omega t$, $0 < t < \dfrac{\pi}{\omega}$ having the period $\dfrac{\pi}{\omega}$. **(07 Marks)**

c. Solve $y'' - 3y' + 2y = 2e^{3t}$, $y(0) = y'(0) = 0$ by using Laplace transforms. **(07 Marks)**

OR

10 a. Find the inverse Laplace transforms of $\dfrac{s+1}{s^2+2s+2} + \log\left(\dfrac{s+a}{s+b}\right)$. **(06 Marks)**

b. By using convolution theorem, find $L^{-1}\left[\dfrac{s}{(s^2+1)(s-1)}\right]$. **(07 Marks)**

c. Express $f(t) = \begin{cases} \sin t, & 0 < t \le \pi/2 \\ \cos t, & \pi/2 < t \le \pi \\ 1, & \pi < t \end{cases}$ in terms of unit step functions and hence find its Laplace transform. **(07 Marks)**

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CBCS SCHEME

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15MAT21

Second Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $(D^2-4D+4)y = e^{2x} + \cos 2x + 4$ by inverse differential operator method. **(06 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 5y = e^{2x}\sin x$ by inverse differential operator method. **(05 Marks)**

c. Using the method of undetermined coefficients, solve $y'' - 3y' + 2y = x^2 + e^x$. **(05 Marks)**

OR

2 a. Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$ by inverse differential operator method. **(05 Marks)**

c. Solve $y'' - 2y' + y = \dfrac{e^x}{x}$ by method of variation of parameters. **(05 Marks)**

Module-2

3 a. Solve $(2x-1)^2\dfrac{d^2y}{dx^2} + (2x-1)\dfrac{dy}{dx} - 2y = 8x^2 - 2x + 3$. **(06 Marks)**

b. Solve $xy\left(\dfrac{dy}{dx}\right)^2 - (x^2+y^2)\dfrac{dy}{dx} + xy = 0$ **(05 Marks)**

c. Solve $x^2(y-px) = p^2y$ by reducing into Clairaut's form and using the substation $X = x^2$ and $Y = y^2$. **(05 Marks)**

OR

4 a. Solve $x^2y'' - xy' + 2y = x\sin(\log x)$. **(06 Marks)**

b. Obtain the general solution of the differential equation $p^2 + 4x^5p - 12x^4y = 0$. **(05 Marks)**

c. Obtain the general and singular solution of $y = 2px + p^2y$. **(05 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from the relation $Z = y\,f(x) + x\,g(y)$. **(06 Marks)**

b. Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\dfrac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 when y is an odd multiple of $\pi/2$. **(05 Marks)**

c. Derive one dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2\dfrac{\partial^2 u}{\partial x^2}$. **(05 Marks)**

OR

6 a. Form a partial differential by eliminating the arbitrary function ϕ from the relation $\phi(x^2+y^2+z^2, z^2-2xy)=0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2}+4z=0$, given that when x = 0, $z=e^{2y}$ and $\frac{\partial z}{\partial x}=2$ **(05 Marks)**

c. Determine the solution of the heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial K^2}$ by the method of separation of variables for the constant K is positive. **(05 Marks)**

Module-4

7 a. Evaluate $\int_1^2\int_3^4 (xy+e^y)\,dydx$. **(06 Marks)**

b. Evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ by changing the order of integration. **(05 Marks)**

c. Obtain the relation between the beta and gamma function in the form

$$\beta(m,n)=\frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$$ **(05 Marks)**

OR

8 a. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates. **(06 Marks)**

b. Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}dzdydx$. **(05 Marks)**

c. Using beta and gamma function, prove that $\int_0^1\frac{x^2}{\sqrt{1-x^4}}dx\times\int_0^1\frac{x^2}{\sqrt{1+x^4}}=\frac{\pi}{4\sqrt{2}}$. **(05 Marks)**

Module-5

9 a. Find $L\left[\frac{\cos 2t-\cos 3t}{t}+t\sin t\right]$. **(06 Marks)**

b. If $f(t)=\begin{cases} t & 0\le t\le\pi \\ 2\pi-t & \pi<t\le 2\pi\end{cases}$, where $f(t+2\pi)=f(t)$, then prove that $L[f(t)]=\frac{1}{s^2}\tanh\left[\frac{\pi s}{2}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ using convolution theorem. **(05 Marks)**

OR

10 a. Express $f(t)=\begin{cases} 1 & 0<t<1 \\ t & 1<t<2 \\ t^2 & t>2\end{cases}$ in term of unit step function and hence find its Laplace transform. **(06 Marks)**

b. Find $L^{-1}\left[\frac{s+5}{s^2-6s+13}\right]$. **(05 Marks)**

c. Employ the Laplace transform to solve the differential equation $y''(t)+4y'(t)+4y(t)=e^{-t}$ with the initial condition y(0) = 0 and y′(0) = 0. **(05 Marks)**

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14MAT21

Second Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module – 1

1 a. Solve $(D^2-3D+2)y=(e^{3x}+\sin x)$. (06 Marks)

b. Solve $(D^2+2D+1)y=(x^2+3x+2)$. (07 Marks)

c. By the method of undetermined coefficients solve $(D^2+D+1)y=6e^x+\cos x$. (07 Marks)

2 a. Solve $(D^3-6D^2+11D-6)y=e^{-x}$. (06 Marks)

b. Solve $(D^2-1)y=x^2$. (07 Marks)

c. By the method of variation of parameters solve $(D^2-2D+1)y=e^x$. (07 Marks)

Module – 2

3 a. Solve the simultaneous equations,

$\frac{dx}{dt}=5x+y$, $\frac{dy}{dt}=y-4x$ (06 Marks)

b. Solve $x^2y''-xy'+y=\log x$. (07 Marks)

c. Solve $xyp^2-(x^2+y^2)p+xy=0$. (07 Marks)

4 a. Solve $(1+x)^2y''+(1+x)y'+y=\sin[2\log(1+x)]$. (06 Marks)

b. Solve $y+px=x^4p^2$. (07 Marks)

c. Find the general and singular solution of the equation, $\sin px\cos y=\cos px\sin y+p$. (07 Marks)

Module – 3

5 a. Form the partial differential equation by eliminating arbitrary function from,
$F(x+y+z,\ x^2+y^2+z^2)=0$ (06 Marks)

b. Derive one dimensional heat equation. (07 Marks)

c. Evaluate $\int_0^1\int_1^2\int_2^3(x+y+z)\,dx\,dy\,dz$. (07 Marks)

6 a. Solve $\frac{\partial^2 z}{\partial x\partial y}=\frac{x}{y}+a$. (06 Marks)

b. Evaluate by changing the order of integration, $\int_0^1\int_x^{\sqrt{x}}xy\,dy\,dx$. (07 Marks)

c. Find all possible solutions of one dimensional wave equation, $u_{tt}=C^2u_{xx}$ by the method of separation of variables. (07 Marks)

Module – 4

7 a. Evaluate $\int_0^\infty e^{-4x} x^2 dx$ using gamma function. **(06 Marks)**

b. Prove that spherical polar co-ordinate system is orthogonal. **(07 Marks)**

c. Find the area enclosed by the curve $r = a(1+\cos\theta)$ above the initial line. **(07 Marks)**

8 a. Show that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

b. Express the vector, $F = zi - 2xj + yk$ in cylindrical co-ordinates. **(07 Marks)**

c. Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. **(07 Marks)**

Module – 5

9 a. Find the Laplace transform of $t\cos 2t + e^{-2t}t^3$. **(06 Marks)**

b. Express $f(t) = \begin{cases} \sin t, & 0 < t \le \frac{\pi}{2} \\ \cos t, & \frac{\pi}{2} < t < \pi \\ 0, & t > \pi \end{cases}$

in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

c. Solve $y'' - 3y' + 2y = e^{-t}$, $y(0) = y'(0) = 0$ by Laplace transform method. **(07 Marks)**

10 a. Find the inverse Laplace transforms of $\dfrac{3s+7}{s^2-2s-3} + \log\left(\dfrac{s-a}{s+b}\right)$. **(06 Marks)**

b. Using convolution theorem find the inverse Laplace transform of $\dfrac{1}{s(s^2+a^2)}$. **(07 Marks)**

c. Find the Laplace transform of the periodic function, $f(t) = \begin{cases} K, & 0 \le t \le a \\ -K, & a < t \le 2a \end{cases}$, period is 2a. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

17MAT31

Third Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the fourier series of the function $f(x) = x - x^2$ in $-\pi \le x \le \pi$ and hence deduce $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots$ **(08 Marks)**

b. Obtain the Half Range Fourier cosine series for the $f(x) = \sin x$ in $[0, \pi]$. **(06 Marks)**

c. Obtain the constant term and the coefficients of first sine and cosine terms in the fourier expansion of y given

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(06 Marks)

OR

2 a. Obtain the fourier series of $f(x) = \frac{\pi - x}{2}$ in $[0, 2\pi]$ and hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$ **(08 Marks)**

b. Find the fourier half range cosine series of the function $f(x) = 2x - x^2$ in $[0, 3]$. **(06 Marks)**

c. Express y as a fourier series upto first harmonic given

x :	0	30	60	90	120	150	180	210	240	270	300	330
y :	1.8	1.1	0.30	0.16	1.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

(06 Marks)

Module-2

3 a. Find the fourier transform of $f(x) = \begin{cases} a^2 - x^2 ; & |x| \le a \\ 0 ; & |x| > a \end{cases}$ and hence deduce $\int_0^{\infty} \frac{\sin x - x\cos x}{x^3} dx = \frac{\pi}{4}$ **(08 Marks)**

b. Find the fourier sine transform of $e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin ax}{1 + x^2} dx \; ; a > 0$ **(06 Marks)**

c. Obtain the z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

OR

4 a. Find the fourier transform of $f(x) = xe^{-|x|}$. **(08 Marks)**

b. Find the fourier cosine transform of f(x) where

$$f(x) = \begin{cases} x ; & 0 < x < 1 \\ 2 - x ; & 1 < x < 2 \\ 0 ; & x > 2 \end{cases}$$ **(06 Marks)**

c. Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z-transform. **(06 Marks)**

Module-3

5 a. Fit a straight line $y = ax + b$ for the following data by the method of least squares.

x :	1	3	4	6	8	9	11	14
y :	1	2	4	4	5	7	8	9

(08 Marks)

b. Calculate the coefficient of correlation for the data:

x :	92	89	87	86	83	77	70	63	53	50
y :	86	83	91	77	68	85	54	82	37	57

(06 Marks)

c. Compute the real root of $x\log_{10}x - 1.2 = 0$ by the method of false position. Carry out 3 iterations in (2, 3). **(06 Marks)**

OR

6 a. Fit a second degree parabola to the following data $y = a + bx + cx^2$.

x :	1	1.5	2	2.5	3	3.5	4
y :	1.1	1.3	1.6	2	2.7	3.4	4.1

(08 Marks)

b. If θ is the angle between two regression lines, show that

$$\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$; explain significance of $r = 0$ and $r = \pm 1$. **(06 Marks)**

c. Using Newton Raphson method, find the real root of the equation $3x = \cos x + 1$ near $x_0 = 0.5$. Carry out 3 iterations. **(06 Marks)**

Module-4

7 a. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

(08 Marks)

b. Use Newton's dividend formula to find f(9) for the data:

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

(06 Marks)

c. Find the approximate value of $\int_0^{\pi/2}\sqrt{\cos\theta}\,d\theta$ by Simpson's $\frac{1}{3}^{rd}$ rule by dividing $\left[0, \frac{\pi}{2}\right]$ into 6 equal parts. **(06 Marks)**

OR

8 a. The area A of a circle of diameter d is given for the following values:

d :	80	85	90	95	100
a :	5026	5674	6362	7088	7854

Calculate the area of circle of diameter 105 by Newton's backward formula. **(08 Marks)**

b. Using Lagrange's interpolation formula to find the polynomial which passes through the points (0, –12), (1, 0), (3, 6), (4, 12). **(06 Marks)**

c. Evaluate $\int_4^{5.2}\log_e x\,dx$ taking 6 equal parts by applying Weddle's rule. **(06 Marks)**

Module-5

9 a. If $\vec{F} = 3xy\,\hat{i} - y^2\,\hat{j}$, evaluate $\int_C \vec{F}.d\vec{r}$ where 'C' is arc of parabola $y = 2x^2$ from (0, 0) to (1, 2) **(08 Marks)**

b. Evaluate by Stokes theorem $\oint_C (\sin z\,dx - \cos x\,dy + \sin y\,dz)$, where C is the boundary of the rectangle $0 \le x \le \pi$; $0 \le y \le 1,\ z = 3$ **(06 Marks)**

c. Prove that the necessary condition for the $I = \int_{x_1}^{x_2} f(x, y, y')\,dx$ to be extremum is $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ **(06 Marks)**

OR

10 a. Using Green's theorem evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by the lines x = 0, y = 0, x + y = 1. **(08 Marks)**

b. Find the external value of $\int_0^{\pi/2} \left[(y')^2 - y^2 + 4y\cos x\right]dx$. Given that y(0) = 0, $y\left(\frac{\pi}{2}\right) = 0$. **(06 Marks)**

c. Prove that the shortest distance between two points in a plane is along a straight line joining them. **(06 Marks)**

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CBCS SCHEME

USN 2VD17EC408

15MAT31

Third Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the Fourier series for the function :

$$f(x)=\begin{cases}-\pi & \text{in } -\pi<x<0\\ x & \text{in } 0<x<\pi\end{cases}$$

Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$. **(08 Marks)**

b. Express y as a Fourier series up to the second harmonics, given :

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

(08 Marks)

OR

2 a. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. **(08 Marks)**

b. Obtain the constant term and the first two coefficients in the only Fourier cosine series for given data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(08 Marks)

Module-2

3 a. Find the Fourier transform of $xe^{-|x|}$. **(06 Marks)**

b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}, a>0$. **(05 Marks)**

c. Obtain the z – transform of $\sin n\theta$ and $\cos n\theta$. **(05 Marks)**

OR

4 a. Find the inverse cosine transform of $F(\alpha)=\begin{cases}1-\alpha, & 0\le\alpha\le1\\ 0, & \alpha>1\end{cases}$.

Hence evaluate $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$. **(06 Marks)**

b. Find inverse Z – transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$ **(05 Marks)**

c. Solve the difference equation $y_{n+2}+6y_{n+1}+9yl=2^n$ with $y_0=0, y_1=0$, using z – transforms. **(05 Marks)**

Module-3

5 a. Find the lines of regression and the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

b. Fit a second degree polynomial to the data :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(05 Marks)

c. Find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$, by using Newton – Raphson method upto four decimal places. (05 Marks)

OR

6 a. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\overline{x}, \overline{y}$ and the coefficient of correlation between x and y. (06 Marks)

b. Fit a curve of the type $y = ae^{bx}$ to the data :

x	5	15	20	30	35	40
y	10	14	25	40	50	62

(05 Marks)

c. Solve $\cos x = 3x - 1$ by using Regula – Falsi method correct upto three decimal places, (Carryout two approximations). (05 Marks)

Module-4

7 a. Give f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304. Find f(38) using Newton's forward interpolation formula. (06 Marks)

b. Find the interpolating polynomial for the data :

x	0	1	2	5
y	2	3	12	147

By using Lagrange's interpolating formula. (05 Marks)

c. Use Simpson's $\frac{3}{8}$th rule to evaluate $\int_0^{0.3} (1 - 8x^3)^{1/2} dx$ considering 3 equal intervals.

(05 Marks)

OR

8 a. The area of a circle (A) corresponding to diameter (D) is given below :

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105, using an appropriate interpolation formula. (06 Marks)

b. Given the values :

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula. (05 Marks)

c. Evaluate $\int_0^1 \frac{x}{1 + x^2} dx$ by Weddle's rule taking seven ordinates. (05 Marks)

Module-5

9 a. Using Green's theorem, evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the triangle formed by the lines x = 0, y = 0 and x + y = 1. **(06 Marks)**

b. Verify Stoke's theorem for $\vec{f} = (2x - y)i - yz^2 j - y^2 zk$ for the upper half of the sphere $x^2 + y^2 + z^2 = 1$. **(05 Marks)**

c. Find the external of the functional $\int_{x_1}^{x_2} \left\{ y^2 + (y^1)^2 + 2ye^x \right\} dx$. **(05 Marks)**

OR

10 a. Using Gauss divergence theorem, evaluate $\int_S \vec{f} \cdot \hat{n}\, ds$, where $\vec{f} = 4xzi - y^2 j + yzk$ and s is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **(05 Marks)**

b. A heavy cable hangs freely under the gravity between two fixed points. Show that the shape of the cable is a Catenary. **(06 Marks)**

c. Find the external of the functional $\int_0^{\pi/2} \left\{ (y^1)^2 - y^2 + 4y\cos x \right\} dx$, give that y = 0 = y(π/2). **(05 Marks)**

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CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If $y' + y + 2x = 0$, $y(0) = -1$ then find $y(0.1)$ by using Taylor's series method. Consider upto third order derivative term. **(06 Marks)**

b. Find $y(0.2)$ by using modified Euler's method, given that $y' = x + y$, $y(0) = 1$. Take $h = 0.1$ and carry out two modifications at each step. **(07 Marks)**

c. If $y' = \dfrac{1}{x + y}$, $y(0) = 2$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$ then find $y(0.8)$ by Milne's method. **(07 Marks)**

OR

2 a. Use Taylor's series method to find $y(0.1)$ from $y' = 3x + y^2$, $y(0) = 1$. Consider upto fourth derivative term. **(06 Marks)**

b. Use Runge – Kutta method to find $y(0.1)$ from $y' = x^2 + y$, $y(0) = -1$. **(07 Marks)**

c. Use Adam – Bashforth method to find $y(0.4)$ from $y' = \frac{1}{2}xy$, $y(0) = 1$, $y(0.1) = 1.0025$, $y(0.2) = 1.0101$, $y(0.3) = 1.0228$. **(07 Marks)**

Module-2

3 a. Express $x^3 - 5x^2 + 6x + 1$ in terms of Legendre polynomials. **(06 Marks)**

b. Find $y(0.1)$, by using Runge – Kutta method , given that $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$. **(07 Marks)**

c. Solve Bessel's operation leading to $J_n(x)$. **(07 Marks)**

OR

4 a. Prove that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$. **(06 Marks)**

b. Find $y(0.4)$ by using Milne's method, given that $y(0) = 1$, $y'(0) = 1$, $y(0.1) = 1.0998$, $y'(0.1) = 0.9946$, $y(0.2) = 1.1987$, $y'(0.2) = 0.9773$, $y(0.3) = 1.2955$, $y'(0.3) = 0.946$. **(07 Marks)**

c. State and prove Rodrigue's formula. **(07 Marks)**

Module-3

5 a. Derive Cauchy – Riemann equations in Cartesian coordinates. **(06 Marks)**

b. Find an analytic function $f(z) = u + iv$ in terms of z , given that $u = e^{2x}(x\cos 2y - y\sin 2y)$. **(07 Marks)**

c. Evaluate $\displaystyle\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$, c is $|z| = 3$ by residue theorem. **(07 Marks)**

OR

6 a. Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (06 Marks)

b. Discuss the transformation $W = Z^2$. (07 Marks)

c. Find a bilinear transformation that maps the points ∞, i, o in Z – plane into -1, -i, 1 in W – plane respectively. (07 Marks)

Module-4

7 a. In a sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2, out of 1000 such samples, how many would be expected to contain atleast 3 defective parts? (06 Marks)

b. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that i) $26 \le X \le 40$ ii) $X > 45$ iii) $|X - 30| > 5$.
Given that $\phi(0.8) = 0.288$, $\phi(2.0) = 0.4772$, $\phi(3) = 0.4987$, $\phi(1) = 0.3413$. (07 Marks)

c. The joint density function of two continuous random variables X and Y is given by

$$f(x, y) = \begin{cases} K\ xy, & 0 \le x \le 4, \quad 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find i) K ii) E(x) iii) E(2x + 3y). (07 Marks)

OR

8 a. Derive mean and standard deviation of the Poisson distribution. (06 Marks)

b. The joint probability distribution for two random variables X and Y as follows :

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.3	0

Find i) Expectations of X, Y, XY ii) SD of X and Y iii) Covariance of X, Y iv) Correlation of X and Y. (07 Marks)

c. In a certain town the duration of shower has mean 5 minutes. What is the probability that shower will last for i) 10 minutes or more ii) Less than 10 minutes iii) Between 10 and 12 minutes. (07 Marks)

Module-5

9 a. A group of boys and girls were given in Intelligence test. The mean score, SD score and numbers in each group are as follows : (06 Marks)

	Boys	Girls
Mean	74	70
SD	8	10
X	12	10

Is the difference between the means of the two groups significant at 5% level of significance? Given that $t_{0.05} = 2.086$ for 20 d.f.

b. The following table gives the number of accidents that take place in an industry during various days of the week. Test if accidents are uniformly distributed over the week.

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	18	12	11	15	14

Given that $X^2 = 11.09$ at 5% level for 5 d.f. (07 Marks)

c. Find the unique fixed probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$ **(07 Marks)**

OR

10 a. Define the following terms :

i) Type I error and type II error.

ii) Transient state.

iii) Absorbing state. **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following increases of blood pressure : 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will be general be accompanied by an increase in blood pressure. Given that $t_{0.05} = 2.2$ for 11 d.f. **(07 Marks)**

c. If $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$. Find the corresponding stationary probability vector. **(07 Marks)**

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CBCS SCHEME

USN

15MAT41

Fourth Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Employ Taylor's series method, find y(0.1) considering upto third degree term if y(x) satisfies the equation $\frac{dy}{dx} = x - y^2$, y(0) = 1. **(05 Marks)**

b. Using Runge-Kutta method of fourth order, find y(0.1) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.1. **(05 Marks)**

c. Apply Milne's method to compute y(1.4) correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data : y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. **(06 Marks)**

OR

2 a. Use Taylor's series method to find y(4.1) given that $(x^2 + y)y' = 1$ and y(4) = 4. **(05 Marks)**

b. Find y at x = 0.8, given $y' = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Using Adams – Bashforth method. Apply the corrector formula. **(05 Marks)**

c. Using Modified Euler's method find y at x = 0.1 given $y' = 3x + \frac{y}{2}$ with y(0) = 1 taking h = 0.1. **(06 Marks)**

Module-2

3 a. Obtain the solution of the equation 2y″ = 4x + y′ with initial conditions y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514 and y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725, y′(1.3) = 3.0657 by computing y(1.4) applying Milne's method. **(05 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if α ≠ β. **(05 Marks)**

c. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ **(06 Marks)**

OR

4 a. Given $y'' - xy' - y = 0$ with the initial conditions y(0) = 1, y′(0) = 0. Compute y(0.2) and y′(0.2) by taking h = 0.2 using Runge - Kutta method of fourth order. **(05 Marks)**

b. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ then, find the values of a, b, c, d. **(05 Marks)**

c. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$ **(06 Marks)**

Module-3

5 a. State and prove Cauchy-Reimann equation in polar form. **(05 Marks)**

b. Discuss the transformation $w = z^2$. **(05 Marks)**

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = 2, i, -2$. **(06 Marks)**

OR

6 a. Find the analytic function whose real part is

$$\frac{x^4 - y^4 - 2x}{x^2 + y^2}$$ **(05 Marks)**

b. State and prove Cauchy Integral formula. **(05 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is the circle : $|z| = 3$ using Cauchy's Residue theorem. **(06 Marks)**

Module-4

7 a. The probability function of a variate x is :

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k (ii) Evaluate $p(x < 6)$, $p(x \geq 6)$ and $p(3 < x \leq 6)$. **(05 Marks)**

b. Obtain mean and standard deviation of Binomial distribution. **(05 Marks)**

c. The joint distribution of two discrete variables x and y is $f(x, y) = (2x + y)$ where x and y are integers such that $0 \leq x \leq 2$; $0 \leq y \leq 3$.
Find : (i) Marginal distribution of x and y.
(ii) Are x and y independent. **(06 Marks)**

OR

8 a. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be
(i) less than 65 (ii) more than 75 (iii) between 65 and 75 [Given $\phi(1) = 0.3413$] **(05 Marks)**

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction. **(05 Marks)**

c. The joint distribution of the random variables X and Y are given. Find the corresponding marginal distribution. Also compute the covariance and the correlation of the random variables X and Y. **(06 Marks)**

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Module-5

9 a. Explain the terms: (i) Null hypothesis (ii) type-I and type-II errors (iii) Significance level **(05 Marks)**

b. In 324 throws of a six faced 'die', an odd number turned up 181 times. Is it reasonable to think that 'die' is an unbiased one? **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball. **(06 Marks)**

OR

10 a. Find the unique fixed probability vector for the matrix

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$ **(05 Marks)**

b. A random sample for 1000 workers in company has mean wage of Rs. 50 per day and standard deviation of Rs. 15. Another sample of 1500 workers from another company has mean wage of Rs. 45 per day and standard deviation of Rs. 20. Does the mean rate of wages varies between the two companies? **(05 Marks)**

c. A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution.

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2. **(06 Marks)**

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CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(08 Marks)**

b. Express the complex number $\dfrac{(1+i)(1+3i)}{1+5i}$ in the form a + ib. **(06 Marks)**

c. Find the modulus and amplitude of $\dfrac{(1+i)^2}{3+i}$. **(06 Marks)**

OR

2 a. Show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cdot\cos^n\left(\dfrac{\theta}{2}\right)\cdot\cos\left(\dfrac{n\theta}{2}\right)$. **(08 Marks)**

b. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = 8\hat{i} - 4\hat{j} + \hat{k}$, then prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a} \times \vec{b}|$. **(06 Marks)**

c. Determine λ such that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar. **(06 Marks)**

Module-2

3 a. If $y = \cos(m\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$. **(08 Marks)**

b. Find the angle of intersection of the curves $r^2\sin 2\theta = a^2$ and $r^2\cos 2\theta = b^2$. **(06 Marks)**

c. Find the pedal equation of the curve $r = a(1+\sin\theta)$. **(06 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of $\log\sec x$ up to the terms containing x^6. **(08 Marks)**

b. If $u = \operatorname{cosec}^{-1}\left(\dfrac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)$, prove that $xu_x + yu_y = -\dfrac{1}{6}\tan u$. **(06 Marks)**

c. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x+y+z$, $v = y+z$, $w = z$. **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, (n > 0). **(08 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ **(06 Marks)**

OR

6 a. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. **(08 Marks)**

b. Evaluate $\int_0^\infty \frac{x^6}{(1+x^2)^{9/2}} dx$. **(06 Marks)**

c. Evaluate $\iint_A xy dx dy$ where A is the area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$. Find the components of velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \hat{k}$. **(08 Marks)**

b. Find divergence and curl of the vector $\vec{F} = (xyz + y^2 z)\hat{i} + (3x^2 + y^2 z)\hat{j} + (xz^2 - y^2 z)\hat{k}$. **(06 Marks)**

c. Find the directional derivative of $\phi = x^2 y z^3$ at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at $t = 2$ and $t = 3$. **(08 Marks)**

b. Find $\text{curl}(\text{curl}\,\vec{A})$ where $\vec{A} = xy\hat{i} + y^2 z\hat{j} + z^2 y\hat{k}$. **(06 Marks)**

c. Find the constants a, b, c such that the vector field $(\sin y + az)\hat{i} + (bx\cos y + z)\hat{j} + (x + cy)\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(08 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \sin x$. **(06 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(06 Marks)**

OR

10 a. Solve $x^2 y dx - (x^3 + y^3) dy = 0$. **(08 Marks)**

b. Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$. **(06 Marks)**

c. Solve $\left[y\left(1 + \frac{1}{x}\right) + \cos y\right]dx + \left[x + \log x - x\sin y\right]dy = 0$. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | |

15MATDIP31

Third Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express the complex number $\frac{(1+i)(1+3i)}{1+5i}$ in the form a + ib. **(05 Marks)**

b. Find the modulus and amplitude of $1+\cos\theta+i\sin\theta$. **(05 Marks)**

c. Show that $(a+ib)^n+(a-ib)^n = 2(a^2+b^2)^{n/2}\cos\left(n\tan^{-1}\left(\frac{b}{a}\right)\right)$ **(06 Marks)**

OR

2 a. If $\vec{A} = i-2j+3k$ and $\vec{B} = 2i+j+k$, find the unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. **(05 Marks)**

b. Show that the points $-6i+3j+2k$, $3i-2j+4k$, $5i+7j+3k$ and $-13i+17j-k$ are coplan. **(05 Marks)**

c. Prove that $\left[\vec{B}\times\vec{C},\ \vec{C}\times\vec{A},\ \vec{A}\times\vec{B}\right] = \left[\vec{A}\vec{B}\vec{C}\right]^2$ **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. **(05 Marks)**

b. Find the angle of intersection of the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(05 Marks)**

c. Obtain the Maclourin series expansion of the function sin x upto the term containing x^4. **(06 Marks)**

OR

4 a. Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = 2u\log u$ where $\log u = \frac{x^3+y^3}{3x+4y}$. **(05 Marks)**

b. If $u = f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, −1, 0). **(06 Marks)**

Module-3

5 a. Obtain the reduction formula for $\int \sin^n x\,dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^7}dx$. **(05 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\,dx\,dy\,dz$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^{2a}\int_0^{x^2/4a} xy\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

c. Evaluate $\int_0^a \frac{x^7 dx}{\sqrt{a^2-x^2}}$ by using reduction formula. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+3$ where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of $i+j+3k$. **(05 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F}=\operatorname{grad}(x^3+y^3+z^3-3xyz)$. **(05 Marks)**

c. Prove that $\operatorname{div}(\operatorname{curl}\vec{F})=0$. **(06 Marks)**

OR

8 a. Find the directional derivative of $f(x,y,z)=xy^3+yz^3$ at (2, −1, 1) in the direction of $i+2j+2k$. **(08 Marks)**

b. Prove that $\nabla^2\left(\frac{1}{r}\right)=0$ where $r=\sqrt{x^2+y^2+z^2}$. **(08 Marks)**

Module-5

9 a. Solve $(x^2-y^2)dx-xy\,dy=0$. **(05 Marks)**

b. Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+(x+\log x-x\sin y)dy=0$. **(05 Marks)**

c. Solve $\frac{dy}{dx}-\frac{y}{1+x}=e^{3x}(x+1)$. **(06 Marks)**

OR

10 a. Solve $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$. **(08 Marks)**

b. Solve $(3y+2x+4)dx-(4x+6y+5)dy=0$. **(08 Marks)**

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CBCS SCHEME

35

USN 2VD18CV405

17MATDIP41

Fourth Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix}$ by elementary row operations. **(08 Marks)**

b. Test for consistency and solve $x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$. **(06 Marks)**

c. Solve the system of equations by Gauss elimination method :
$x+y+z=9$ $\quad x-2y+3z=8$ $\quad 2x+y-z=3$ **(06 Marks)**

OR

2 a. Find all the eigen values and the corresponding eigen vectors of the matrix
$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ **(08 Marks)**

b. Solve by Gauss elimination method $x_1-2x_2+3x_3=2$, $3x_1-x_2+4x_3=4$, $2x_1+x_2-2x_3=5$. **(06 Marks)**

c. If $A=\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ find A^{-1} by Cayley Hamilton theorem. **(06 Marks)**

Module-2

3 a. Solve $\frac{d^3y}{dx^2}-2\frac{d^2y}{dx^2}+4\frac{dy}{dx}-8y=0$. **(08 Marks)**

b. Solve $6\frac{d^2y}{dx^2}+17\frac{dy}{dx}+12y=e^{-x}$. **(06 Marks)**

c. Solve $y''-4y'+13y=\cos 2x$. **(06 Marks)**

OR

4 a. Solve $\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$. **(08 Marks)**

b. Solve $y''+2y+y=\frac{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}{2}$. **(06 Marks)**

c. Solve $y''+2y'+y=2x+x^2$. **(06 Marks)**

Module-3

5 a. Find $L[\cosh at]$. **(08 Marks)**

b. Find $L[e^{-2t}\sinh 4t]$ **(06 Marks)**

c. Find $R\{t\sin 2t\}$. **(06 Marks)**

OR

6 a. Show that $\int_0^t t^3 e^{-st} \sin t\,dt = 0$. **(08 Marks)**

b. If $f(t) = t^2$, $0 < t < 2$ and $f(t + 2) = f(t)$ for $t > 2$, find $L[f(t)]$. **(06 Marks)**

c. Express $f(t)=\begin{cases} t, & 0<t<4 \\ 5, & t>4 \end{cases}$ in terms of unit step function and hence find their Laplace Transforms. **(06 Marks)**

Module-4

7 a. Find the inverse Laplace Transform of $\frac{3}{s^2}+\frac{2e^{-s}}{s^3}-\frac{3e^{-2s}}{s}$. **(08 Marks)**

b. Find $L^{-1}\left[\frac{s^3+6s^2+12s+8}{s^6}\right]$. **(06 Marks)**

c. Find the inverse Laplace Transform of $\frac{s+5}{s^2-6s+13}$. **(06 Marks)**

OR

8 a. Solve by using Laplace Transform $\frac{d^2y}{dt^2}+k^2y=0$, given that $y(0) = 2$, $y'(0) = 0$. **(08 Marks)**

b. Find inverse Laplace Transform of $\frac{1}{(s+1)(s+2)(s+3)}$. **(06 Marks)**

c. Find $L^{-1}\left[\frac{s+1}{s^2+6s+9}\right]$. **(06 Marks)**

Module-5

9 a. Find the probability that a leap year selected at random will contain 53 Sundays. **(08 Marks)**

b. A six faced die on which the numbers 1 to 6 are marked is thrown. Find the probability of (i) 3 (ii) an odd number coming up. **(06 Marks)**

c. State and prove Bayee's theorem. **(06 Marks)**

OR

10 a. A problem is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved. **(08 Marks)**

b. For any three events A, B, C, prove that $P\{(A\cup B)/C\}=P(A/C)+P(B/C)-P\{(A\cap B)/C\}$. **(06 Marks)**

c. Three machines A, B and C produce respectively 60%, 30% and 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

15MATDIP41

Fourth Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$ by elementary row operation. **(06 Marks)**

b. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ using Cayley - Hamilton theorem. **(05 Marks)**

c. Find all eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **(05 Marks)**

OR

2 a. Solve the system of equation by Gauss - Elimination method.

$$x + y + z = 9$$
$$x - 2y + 3z = 8$$
$$2x + y - z = 3$$
(06 Marks)

b. Using Cayley – Hamilton theorem find A^{-1}, given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into row echelon form and hence find its rank. **(05 Marks)**

Module-2

3 a. Solve by the method of undetermined co-efficient $y'' - 4y' + 4y = e^x$. **(06 Marks)**

b. Solve $(D^3 + 6D^2 + 11D + 6)y = 0$. **(05 Marks)**

c. Solve $y'' + 2y' + y = 2x$. **(05 Marks)**

OR

4 a. Solve by the method of variation of parameter $y'' + a^2y = \sec ax$. **(06 Marks)**

b. Solve $y'' - 4y' + 13y = \cos 2x$. **(05 Marks)**

c. Solve $(D^2 - 1)y = e^{2x}$. **(05 Marks)**

Module-3

5 a. If $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$, find $L[f(t)]$. **(06 Marks)**

b. Find $L[\cos t \,.\, \cos 2t \,.\, \cos 3t]$ **(05 Marks)**

c. Find $L[e^{-2t}(2\cos 5t - \sin 5t)]$ **(05 Marks)**

OR

6 a. Find $L[e^{-t} \,.\, \cos^2 3t]$ **(06 Marks)**

b. Express the following function into unit step function and hence find $L[f(t)]$ given

$$f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$$ **(05 Marks)**

c. Find $L[t \,.\, \cos at]$ **(05 Marks)**

Module-4

7 a. Using Laplace transforms solve the differential equation $y'' + 4y' + 4y = e^{-t}$ given $y(0) = 0$, $y'(0) = 0$. **(06 Marks)**

b. Find $L^{-1}\left[\dfrac{2s-5}{4s^2+25}\right] + L^{-1}\left[\dfrac{8-6s}{16s^2+9}\right]$ **(05 Marks)**

c. Find $L^{-1}\left[\dfrac{1}{s(s+1)(s+2)(s+3)}\right]$ **(05 Marks)**

OR

8 a. Employ Laplace transform to solve the equation

$y'' + 5y' + 6y = 5e^{2x}$, $y(0) = 2$, $y'(0) = 1$. **(06 Marks)**

b. Find $L^{-1}\left[\dfrac{s+5}{s^2-6s+13}\right]$ **(05 Marks)**

c. Find $L^{-1}\left[\log\left(\dfrac{s+a}{s+b}\right)\right]$ **(05 Marks)**

Module-5

9 a. If A and B are any two mutually exclusive events of S, then show that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(06 Marks)**

b. Prove the following :

(i) $P(\phi) = 0$ (ii) $P(\overline{A}) = 1 - P(A)$ **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(05 Marks)**

OR

10 a. State and prove Bay's theorem. **(06 Marks)**

b. If A and B are events with $P(A \cup B) = \dfrac{7}{8}$, $P(A \cap B) = \dfrac{1}{4}$ and $P(\overline{A}) = \dfrac{5}{8}$ find P(A), P(B) and $P(A \cap \overline{B})$. **(05 Marks)**

c. A shooter can hit a target in 3 out of 4 shots and another shooter can hit the target in 2 out of 3 shots. Find the probability that the target is being hit.

(i) when both of them try (ii) by only one shooter. **(05 Marks)**

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USN

MATDIP301

Third Semester B.E. Degree Examination, June/July 2019

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express square root of $1 - i$ in the form of $x + iy$. **(07 Marks)**

b. Find the modulus and amplitude of the following and express each in polar form.

(i) $1 - i\sqrt{3}$ (ii) $\frac{1-i}{1+i}$ **(07 Marks)**

c. Expand $\cos^6\theta$ in series of multiples of θ. **(06 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(x+1)(x-2)}$. **(07 Marks)**

c. If $y = \log\left(x + \sqrt{1+x^2}\right)$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y = 0$. **(07 Marks)**

3 a. Find the angle between radius vector and the tangent of the curve $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Find the Taylor's series expansion of the function e^x about $x = 1$. **(07 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\log_e(1 + x)$ up to third degree terms. **(07 Marks)**

4 a. If $\cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$. **(06 Marks)**

b. If $x = r\cos\theta$ and $y = r\sin\theta$, prove that $JJ' = 1$. **(07 Marks)**

c. If $x^y + y^x = c$, where c is a constant, find $\frac{dy}{dx}$. **(07 Marks)**

5 a. Obtain the reduction formula $I_n = \int \sin^n x\, dx$, where n is a positive integer. **(06 Marks)**

b. Evaluate : $\int_0^1\int_0^{\sqrt{x}} xy(x+y)\,dx\,dy$ **(07 Marks)**

c. Evaluate : $\int_0^1\int_0^{1-z}\int_0^{1-z-y}(x+y+z)\,dx\,dy\,dz$ **(07 Marks)**

6 a. Prove the following :

$\beta(m, n) = \beta(n, m)$ **(06 Marks)**

b. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ **(07 Marks)**

c. Using Gamma function, evaluate the integral $\int_0^1 \frac{1}{\sqrt{1-x^4}}\,dx$ **(07 Marks)**