CBCS SCHEME

USN | | | | | | | | | | | 18MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020

Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, prove that for the curve $r = f(\theta)$, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(06 Marks)**

b. Find the radius of curvature at any point P(x, y) on the parabola $y^2 = 4ax$. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

OR

2 a. Find the pedal equation of the curve $\frac{2a}{r} = 1 - \cos\theta$. **(06 Marks)**

b. Find the radius of curvature of the tractrix $x = a[\cos t + \log\tan(t/2)]$, $y = a\sin t$. **(06 Marks)**

c. Show that the angle between the pair of curves: $r = 6\cos\theta$ and $r = 2(1 + \cos\theta)$ is $\pi/6$. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that
$\sqrt{1+\sin 2x} = 1 + x - x^2/2! - x^3/3! + x^4/4!$ **(06 Marks)**

b. Evaluate: i) $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x}$ ii) $\lim_{x \to 0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$ **(07 Marks)**

c. Examine the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for is extreme values. **(07 Marks)**

OR

4 a. If $U = f(x-y, y-z, z-x)$ show that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If $u = x\cos y\cos z$, $v = x\cos y\sin z$, $w = x\sin y$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -x^2\cos y$. **(07 Marks)**

c. Find the volume of the largest rectangular parallelopiped that can be inscribed in the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dz\,dy\,dx$ **(06 Marks)**

b. Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$ **(07 Marks)**

c. Show that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Change the order of Integration and hence evaluate

$$\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$$ **(06 Marks)**

b. Find the centre of gravity of the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

c. Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \cdot \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(06 Marks)**

b. Find the orthogonal trajectories of the family of cardioides $r = a(1 + \cos\theta)$ **(07 Marks)**

c. Solve: $[4x^3y^2 + y\cos(xy)]dx + [2x^4y + x\cos(xy)]dy = 0$ **(07 Marks)**

OR

8 a. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t. **(06 Marks)**

b. Solve : $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$ **(07 Marks)**

c. Solve : $x^2p^2 + xp - (y^2 + y) = 0$, where $p = \frac{dy}{dx}$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by applying elementary row operations. **(06 Marks)**

b. Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh's power method taking the initial Eigen vector as $[1, 1, 1]^T$. **(07 Marks)**

c. Apply Gauss-Jordon method to solve the system of equations:
$2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$. **(07 Marks)**

OR

10 a. Test for consistency and solve:
$5x_1 + x_2 + 3x_3 = 20$, $2x_1 + 5x_2 + 2x_3 = 18$, $3x_1 + 2x_2 + x_3 = 14$ **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Solve the system of equations
$5x + 2y + z = 12$
$x + 4y + 2z = 15$
$x + 2y + 5z = 20$
Using Gauss-Siedel method [carry out 4 iterations]. **(07 Marks)**

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CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks:100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at (2, 1, 2). **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at (1,–1, 1). **(07 Marks)**

c. Find the value of the constant a such that the vector field

$$\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2 j + (1-a)xz^2 k$$

is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\vec{F} = (3x^2 + 6y)i - 14yzj + 20xz^2 k$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t$, $y = t^2$ and $z = t^3$. **(06 Marks)**

b. Use Green's theorem to find the area between the parabolas $x^2 = 4y$ and $y^2 = 4x$. **(07 Marks)**

c. If $\vec{F} = 2xyi + yz^2 j + xzk$ and s is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0 and x = 2, y = 1, z = 3. Find the flux across S. **(07 Marks)**

Module-2

3 a. Solve $(D^2 + 3D + 2)y = 4\cos^2 x$. **(06 Marks)**

b. Solve $(D^2 + 1)y = \sec x \tan x$, by the method of variation of parameter. **(07 Marks)**

c. Solve $x^2y'' + xy' + 9 = 3x^2 + \sin(3\log x)$. **(07 Marks)**

OR

4 a. Solve $y'' + 2y' + y = 2x + x^2$. **(06 Marks)**

b. Solve $(2x + 1)^2y'' - 6(2x + 1)y' + 16y = 8(2x + 1)^2$. **(07 Marks)**

c. The current i and the charge q in a series circuit containing on inductance L, capacitance C, emf E satisfy the differential equation : $L\frac{di}{dt} + \frac{q}{c} = E$; $i = \frac{dq}{dt}$. Express q and i interms of t, given that L, C, E are constants and the value of i, q are both zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $\phi(xy + z^2, x + y + z) = 0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 if $y = (2n+1)\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional wave equation in the standard form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

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OR

6 a. Form the partial differential equation by eliminating the arbitrary function form $f\left(\frac{xy}{z}, z\right) = 0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. **(07 Marks)**

c. Find all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n, (x > 0)$. **(06 Marks)**

b. Solve the Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ interms of Legendre's polynomials. **(07 Marks)**

OR

8 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. **(06 Marks)**

b. If α and β are two distinct roots fo $J_n(x) = 0$. Prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$. If $\alpha \neq \beta$. **(07 Marks)**

c. Express $f(x) = x^3 + 2x^2 - x - 3$ interms of Legendre's polynomials. **(07 Marks)**

Module-5

9 a. Find the real root of the equation : $x^3 - 2x - 5 = 0$ using Regula Falsi method, correct to three decimal places. **(06 Marks)**

b. Use Lagrange's formula, find the interpolating polynomial that approximates the function described by the following data :

x	0	1	2	5
f(x)	2	3	12	147

c. Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule, taking seven ordinates and hence find $\log e^2$.

OR

10 a. Find the real root of the equation $xe^x - 2 = 0$ using Newton – Raphson method correct to three decimal places.

b. Use Newton's divided difference formula to find f(4) given the data :

x	0	2	3	6
f(x)	–4	2	14	158

c. Use Simpson's $\frac{3}{8}^{th}$ rule to evaluate $\int_1^4 e^{1/x} dx$.

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CBCS SCHEME

USN | | | | | | | | | | 18MAT31

Third Semester B.E. Degree Examination, Aug./Sept.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find $L\{e^{-2t}t\cos 2t\}$. **(06 Marks)**

b. Express the function in terms of unit step function and hence find Laplace transform of :

$$f(t)=\begin{cases}1 & 0\le t\le 1\\ t & 1<t\le 2\\ t^2 & t>2\end{cases}.$$ **(07 Marks)**

c. Solve the equation $y''(t)+3y'(t)+2y(t)=0$ under the condition $y(0)=1$, $y'(0)=0$. **(07 Marks)**

OR

2 a. Find :

i) $L^{-1}\left\{\dfrac{s+3}{s^2-4s+13}\right\}$ ii) $L^{-1}\left\{\log\dfrac{(s^2+1)}{s(s+1)}\right\}$. **(06 Marks)**

b. Find $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)^2}\right\}$ using convolution theorem. **(07 Marks)**

c. A periodic function of period 2a is defined by

$$f(t)=\begin{cases}E & 0\le t\le a\\ -E & a<t\le 2a\end{cases}$$

Where E is a constant and show that trim $L\{f(t)\}=\dfrac{E}{S}\tan h\left(\dfrac{as}{2}\right)$. **(07 Marks)**

Module-2

3 a. Express $f(x)=x^2$ as a Fourier series in the interval $-\pi<x<\pi$. Hence deduce that $\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}----=\dfrac{\pi^2}{12}$. **(07 Marks)**

b. Obtain the Fourier seires expression of $f(x)=\begin{cases}\pi x & 0<x<1\\ \pi(2-x) & 1<x<2\end{cases}$. **(07 Marks)**

c. Obtain the half range cosine series for the function $f(x)=(x-1)^2$ $0\le x\le 1$. **(06 Marks)**

OR

4 a. Obtain the Fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)$ $0 < x < 2\pi$. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\cdots=\frac{\pi}{4}$. **(07 Marks)**

b. Obtain the half range cosine series of $f(x) = x\sin x$ $0 \le x \le \pi$. **(07 Marks)**

c. Express f(x) as a Fourier series upto first harmonic.

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(06 Marks)

Module-3

5 a. Find the Fourier cosine transform of

$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ (2-x) & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}.$$ **(07 Marks)**

b. Find the Fourier transform by $f(x) = e^{-|x|}$. **(07 Marks)**

c. Obtain the inverse Z – transform by $u(z)=\frac{z}{(z-2)(z-3)}$. **(06 Marks)**

OR

6 a. Find the Fourier transform by

$$f(x)=\begin{cases} 1-|x| & |x|<1 \\ 0 & |x|>1 \end{cases}$$

and show that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. **(07 Marks)**

b. Find the z-transform of : i) $\cos n\theta$ ii) $\sin n\theta$. **(06 Marks)**

c. Solve using Z –transform $u_{n+2} - 4u_n = 0$ given that $u_0 = 0$ and $u_1 = 2$. **(07 Marks)**

Module-4

7 a. Using Taylor's series method solve y(x) = x + y, y(0) = 1 then find y at x = 0.1, 0.2 consider upto 4^{th} degree. **(07 Marks)**

b. Solve $y'(x)=1+\frac{y}{z}$, $y(1)=2$ then find y(1.2) with n = 0.2 using modified Euler's method. **(06 Marks)**

c. Solve $y'(x) = x - y^2$ and the data is y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762 then find y(0.8) by applying Milne's method and applying corrector formula twice. **(07 Marks)**

OR

8 a. Solve $y'(x) = 3x + \frac{y}{2}, y(0) = 1$ then find y(0.2) with n = 0.2 using modified Euler's method. **(06 Marks)**

b. Solve y(x) = $3e^x + 2y$, y(0) = 0 then find y(0.1) with h = 0.1 using Runge-Kutta method of fourth order. **(07 Marks)**

c. Solve $y'(x) = 2e^x - y$ and data is

x	0	0.1	0.2	0.3
y	2	2.010	2.040	2.090

Then find y(0.4) by using Adam's Bash forth method. **(07 Marks)**

Module-5

9 a. By applying Milne's predictor and corrector method to compute y(0.4) give the differential equation $\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx}$ and the following table by initial value. **(07 Marks)**

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y′	1	1.2103	1.4427	1.6990

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

c. Find the extremal of the functional $\int_{x_1}^{x_2} (y' + x^2 y'^2)dx$. **(07 Marks)**

OR

10 a. By Runge Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct to four decimal places. Using initial condition y(0) = 1, y′(0) = 0. **(07 Marks)**

b. Prove that the shortest distance between two points in a plane is a straight line. **(06 Marks)**

c. Find the curve on which the functional $\int_0^1 [y'^2 + 12xy]dx$ with y(0) = 0, y(1) = 1. **(07 Marks)**

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CBCS SCHEME

USN: | | | | | | | | | |

17MAT11

First Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. **(06 Marks)**

b. Find the angle intersection of the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(07 Marks)**

c. Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(07 Marks)**

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. Obtain the pedal equation of the curve,

$\frac{2a}{r} = (1+\cos\theta)$. **(07 Marks)**

c. Find the radius of curvature for the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-2

3 a. Obtain Taylor's series expansion of log(cosx) about the point $x = \frac{\pi}{3}$ upto the fourth degree term. **(06 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(07 Marks)**

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$. **(06 Marks)**

b. Obtain the Maclaurin's expansion of the function $\log(1+x)$ upto the term containing x^4. **(07 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves on the curve,
$x = 2t^2, \; y = t^2 - 4t, \; z = 3t - 5$
where t is the time. Find the components of velocity and acceleration at time t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl } \vec{A}) = 0$ **(07 Marks)**

OR

6 a. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(06 Marks)**

b. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$, where $\vec{F} = \text{grad}\left(x^3 + y^3 + z^3 - 3xyz\right)$. **(07 Marks)**

c. Prove that $\text{curl}(\text{grad}\phi) = 0$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^a x\sqrt{ax - x^2}\,dx$. **(06 Marks)**

b. Solve $\left(4xy + 3y^2 - x\right)dx + x(x+2y)dy = 0$ **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = a(1 + \sin\theta)$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(07 Marks)**

c. A body in air at 25°C cools from 100°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix,
$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$
(06 Marks)

b. Find the largest Eigen value and the corresponding Eigen vector of the matrix.
$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$
by power method taking the initial eigen vector as $[1 \quad 0 \quad 0]^T$ perform five iterations. **(07 Marks)**

c. Show that the transformation,
$y_1 = 2x + y + z, \; y_2 = x + y + 2z, \; y_3 = x - 2z$ is regular. Find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the following system of equations by Gauss-Siedel method.

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Carryout three iterations. **(06 Marks)**

b. Reduce the matrix,

$A = \begin{bmatrix} -1 & 3 \\ -1 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Reduce the following Quadratic form, $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. **(07 Marks)**

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CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivatives of $\dfrac{x^2-4x+1}{(x+2)(x^2-1)}$. **(06 Marks)**

b. Find the angle of intersection between the curves $r = ae^{\theta}$ and $re^{\theta} = b$ **(05 Marks)**

c. Obtain the pedal equation of the curve $r = a(1 + \cos\theta)$. **(05 Marks)**

OR

2 a. If $y = \sin^{-1}x$ prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ **(06 Marks)**

b. Find the pedal equation of the curve $r^n \operatorname{cosec} n\theta = a^n$. **(05 Marks)**

c. For the curve $y=\dfrac{ax}{a+x}$ show that $\left(\dfrac{2\rho}{a}\right)^{2/3}=\left(\dfrac{x}{y}\right)^2+\left(\dfrac{y}{x}\right)^2$. **(05 Marks)**

Module-2

3 a. Expand sinx in powers of $\left(x-\dfrac{\pi}{2}\right)$. Hence find the value of Sin91° correct to four decimal places. **(06 Marks)**

b. Evaluate $\underset{x\to\frac{\pi}{2}}{\text{Lt}} (\text{Sin}x)^{\tan x}$ **(05 Marks)**

c. If $u=\text{Sin}^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$ **(05 Marks)**

OR

4 a. Obtain the Maclaurin's expansion of the function $\log_e(1+x)$ up to fourth degree terms and hence find $\log_e(1-x)$. **(06 Marks)**

b. If $u = f(2x - 3y, 3y-4z, 4z - 2x)$ prove that $\dfrac{1}{2}\dfrac{\partial u}{\partial x}+\dfrac{1}{3}\dfrac{\partial u}{\partial y}+\dfrac{1}{4}\dfrac{\partial u}{\partial z}=0$ **(05 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$ show the $\dfrac{\partial(u\,v\,w)}{\partial(x\,y\,z)}=4$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Also find the velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. **(06 Marks)**

b. Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point (2, -1, 1) in the duration of the vector $\hat{i}-2\hat{j}+2\hat{k}$. **(05 Marks)**

c. Find constants a and b such that $\vec{F}=(axy+z^3)i+(3x^2-z)j+(bxz^2-y)k$ is irrotational. **(05 Marks)**

OR

6 a. Show that the vector
$\vec{v} = (x+3y)i + (y-3z)j + (x-2z)k$ is a solenoidal vector. Also find Curl $\vec{v}$. **(06 Marks)**

b. If $\vec{F} = (x+3y+1)i + j - (x+y)k$ show that $\vec{F}.\text{curl}\vec{F} = 0$ **(05 Marks)**

c. Prove that $\nabla \times (\phi \vec{A}) = \phi(\nabla \times \vec{A}) + \nabla\phi \times \vec{A}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\,dx$ and evaluate $\int_0^{\pi/2} \sin^n x dx$. **(06 Marks)**

b. Solve (ycosx + siny + y)dx + (sinx + xcosy + x)dy. **(05 Marks)**

c. Find the orthogonal trajectories of the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx \to 06$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ **(05 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C in one minute. Find the temperature of the body at the end of 3 minutes. **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss elimination method:
$x + y + z = 6$
$x - y + z = 2$
$2x - y + 3z = 9$ **(06 Marks)**

b. Use power method to find the largest eigen value and the corresponding eigen vector of the matrix A, taking [1, 0, 0]T as initial eigen vector. Perform three iterations.

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$ **(05 Marks)**

c. Show that the transformation
$y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Find the inverse transformation. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss Seidal method
$10x + y + z = 12$
$x + 10y + z = 12$
$x + y + 10z = 12$ with $x_0 = y_0 = z_0 = 0$ **(06 Marks)**

b. Reduce the following matrix to the diagonal form
$$\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$ **(05 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ to the canonical form by orthogonal transformation. **(05 Marks)**

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USN											14MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. If $y = e^{m\cos^{-1}x}$, prove that $(1 - x^2)y_2 - xy_1 = m^2y$ and hence show that : $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$. **(07 Marks)**

b. Show that the pair of curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect each other orthogonally. **(06 Marks)**

c. Find the radius of curvature of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$. **(07 Marks)**

2 a. If $y = \sin^{-1}x$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. **(07 Marks)**

b. Find the pedal equation of the curve $r(1 - \cos\theta) = 2a$. **(06 Marks)**

c. Show that the radius of curvature for the catenary of uniform strength $y = a\log(\sec(x/a))$ is $a\sec(x/a)$. **(07 Marks)**

Module-2

3 a. Expand $\tan^{-1}x$ in ascending powers of x upto the term containing x^5. **(07 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(06 Marks)**

c. Find the extreme values of the function f(x, y) given as $x^3 + y^3 - 63(x + y) + 12xy$. **(07 Marks)**

4 a. If $u = \sin^{-1}\left(\frac{x^8 + y^8}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 7\tan u$. **(07 Marks)**

b. If $u = \frac{1}{r^2}\cos 2\theta$, prove that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} = 0$. **(06 Marks)**

c. Evaluate $\underset{x\to 0}{\text{Lt}}\frac{xe^x - \log(1+x)}{x^2}$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $R = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$ where t denotes time. Find the magnitudes of acceleration along the tangent and normal at time t = 2. **(07 Marks)**

b. Using differentiation under integral sign, evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx, \alpha \geq 0$. **(06 Marks)**

c. Trace the curve $y^2(a - x) = x^2(a + x)$. **(07 Marks)**

6 a. Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is a conservative force field. Find its scalar potential. **(07 Marks)**

b. Find the directional derivatives of the function xyz along the direction of the normal to the surface $x^2z + xy^2 + yz^2 = 3$ at (1, 1, 1). **(06 Marks)**

c. Find divF, CurlF, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x \, dx$. **(07 Marks)**

b. Solve $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(06 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

8 a. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

c. Find the orthogonal trajectories of the family $r^n \cos n\theta = a^n$. **(07 Marks)**

Module-5

9 a. Solve $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$, by Gauss elimination method. **(07 Marks)**

b. Diagonalize the matrix, $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(06 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$ into canonical form. Hence indicate its nature of rank and index. **(07 Marks)**

10 a. Solve the system of equations $27x + 6y - z = 85$, $x + y + 54z = 110$, $6x + 15y + 2z = 72$ by Gauss – Seidal method to obtain the solution correct to three decimal places. (Take 4 iterations). **(07 Marks)**

b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. **(06 Marks)**

c. Find the dominant eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method taking the initial values $[1, 1, 1]^T$. Obtain 5 iterations. **(07 Marks)**

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CBCS SCHEME

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17MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve: $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$ **(06 Marks)**

b. Solve: $y'' - 4y' + 13y = \text{Cos}2x$ **(07 Marks)**

c. Solve by the method of undetermined coefficients of the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$ **(07 Marks)**

OR

2 a. Solve: $\frac{d^2y}{dx^2} - 4y = \text{Cosh}(2x-1)$ **(06 Marks)**

b. Solve: $y'' + 3y' + 2y = 12x^2$ **(07 Marks)**

c. Solve by the method of variation of parameters: $y'' + a^2y = \sec ax$ **(07 Marks)**

Module-2

3 a. Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1+x)$ **(06 Marks)**

b. Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ **(07 Marks)**

c. Obtain the general solution and the singular solution of the equation $x^4p^2 + 2x^3py - 4 = 0$ **(07 Marks)**

OR

4 a. Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(x+1)]$ **(06 Marks)**

b. Solve: $p^2 + 2py\,\text{Cot}x = y^2$ **(07 Marks)**

c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitutions $X = x^2$, $Y = y^2$. **(07 Marks)**

Module-3

5 a. If $z = e^{ax+by} f(ax - by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions that $\frac{\partial z}{\partial x} = \log(1+y)$ when $x = 1$, and $z = 0$ when $x = 0$. **(07 Marks)**

c. Find the solution of one dimensional wave equation, using the method of separation of variables. **(07 Marks)**

OR

6 a. Form the PDE by eliminating the arbitrary functions ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$ **(06 Marks)**

b. Solve $\left[\frac{\partial^2 z}{\partial x \partial y}\right] = \sin x \sin y$ subject to the condition $\left[\frac{\partial z}{\partial y}\right] = -2\sin y$ when x = 0 and z = 0 if y is a odd multiple of $\pi/2$. **(07 Marks)**

c. Derive an one dimensional heat equation in the form $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

Module-4

7 a. Change the order of integration and hence evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy\, dy\, dx$ **(06 Marks)**

b. Evaluate $\int_0^a \int_0^x \int_0^{x+y} x + y + z\, dz\, dy\, dx$ **(07 Marks)**

c. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ with usual notations. **(07 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates. **(06 Marks)**

b. Find by double integration the area enclosed by the curve $r = a(1 + \cos\theta)$ between $\theta = 0$ and $\theta = \pi$. **(07 Marks)**

c. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ **(07 Marks)**

Module-5

9 a. Find the Laplace transform of $f(t) = t(\sin t)$. **(06 Marks)**

b. If $f(t) = t^2$, $0 < t < 2$ and $f(t + 2) = f(t)$ for $t > 2$, find $L\{f(t)\}$. **(07 Marks)**

c. Find the inverse Laplace transform of $\left[\frac{s+5}{s^2 - 6s + 13}\right]$ **(07 Marks)**

OR

10 a. Find the Laplace transform of the unit step function $f(t) = \begin{cases} \sin t, & 0 < t \le \frac{\pi}{2} \\ \text{Cos}t, & t > \frac{\pi}{2} \end{cases}$ **(06 Marks)**

b. Employ Laplace transform to solve the equation $y'' + 5y' + 6y = 5e^{2x}$ **(07 Marks)**

c. Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)^2}$, using convolution theorem. **(07 Marks)**

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CBCS SCHEME

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15MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics - II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $y''' + y'' + y' + y = e^{3x+4} + \sinh x$ by inverse differential operator method. **(06 Marks)**

b. Solve $y'' + 16y = x \sin 3x$ by inverse differential operator method. **(05 Marks)**

c. Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ by the method of variation of parameters. **(05 Marks)**

OR

2 a. Solve $y'' + 4y' + 4y = 3 \sin x + \cos 4x$ by inverse differential operator method. **(06 Marks)**

b. Solve $y'' + 2y = x^2 e^{3x} + e^x \cos 2x$ by inverse differential Operation method. **(05 Marks)**

c. Solve $y'' + y' - 2y = x + \sin x$ by the method of undetermined coefficients. **(05 Marks)**

Module-2

3 a. Solve $x^3 y''' + 2x^2 y'' + 2y = 10(x + \frac{1}{x})$. **(06 Marks)**

b. Solve $y = x(p + \sqrt{1+p^2})$ where $p = \frac{dy}{dx}$. **(05 Marks)**

c. Find the general and singular solution of the equation $y = xp + p^2$. **(05 Marks)**

OR

4 a. Solve $(2x+1)^2 y'' - 2(2x+1)\frac{dy}{dx} - 12y = 3(2x+1)$. **(06 Marks)**

b. Solve $y = 3px + 6p^2y^2$, solving for x. **(05 Marks)**

c. Find the general and singular solution of $y = px - \sqrt{1+p^2}$. **(05 Marks)**

Module-3

5 a. Obtain a partial differential equation by eliminating arbitrary constants in the equation $z = xy + y\sqrt{x^2 - a^2} + b$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = x + y$ given that $z = y^2$ when $x = 0$ and $\frac{\partial z}{\partial x} = 0$, when $x = 2$. **(05 Marks)**

c. Solve the one dimensional wave equation $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ by the method of separation of variables. **(05 Marks)**

OR

6 a. Form a partial differential equation by eliminating arbitrary function from the equation $xyz = f(x + y + z)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} + z = 0$, given that $z = \cos x$ and $\frac{\partial z}{\partial y} = \sin x$ when $y = 0$. **(05 Marks)**

c. Solve the one dimensional heat equation $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, by the method of separation of variables. **(05 Marks)**

Module-4

7 a. Evaluate $\int_0^1\int_0^2\int_1^2 xyz^2\ dx\ dy\ dz$. **(06 Marks)**

b. Evaluate by changing the order of integration $\int_0^1\int_x^{\sqrt{x}} xy\ dy\ dx$. **(05 Marks)**

c. Prove that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx,\ m > 0,\ n > 0$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_1^{\left(\frac{a^2-r^2}{a}\right)} r\ dz\ dr\ d\theta$. **(06 Marks)**

b. Change the order of integration and evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\ dx$. **(05 Marks)**

c. Prove that $\Gamma(n) = 2\int_0^\infty e^{-t^2}\ t^{2n-1}\ dt$. **(05 Marks)**

Module-5

9 a. Find the Laplace transform of

i) $t \sin t$ ii) $\left(\frac{\cos 6t - \cos 4t}{t}\right)$. **(06 Marks)**

b. Find $L[f(t)]$, if $f(t) = \begin{cases} t, & 0 < t \le a \\ (2a - t), & a < t \le 2a \end{cases}$, where $f(t + 2a) = f(t)$. **(05 Marks)**

c. Express $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$

in terms of unit step function and find its Laplace transform. **(05 Marks)**

OR

10 a. Find i) $L^{-1}\left[\frac{s}{(s-1)(s^2+4)}\right]$ ii) $L^{-1}[\tan^{-1} s]$. **(06 Marks)**

b. Using Convolution theorem find $L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$. **(05 Marks)**

c. Solve $y'' + 4y' + 3y = e^{-t}$ using Laplace transform, given that $y(0) = 1$, $y'(0) = 1$. **(05 Marks)**

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14MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

b. Solve $y'' - 4y' + 4y = e^x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters. **(07 Marks)**

2 a. Solve $(D^2 - 2D + 5)y = (\sin x)$. **(06 Marks)**

b. Solve $y'' + 2y' + y = 2x + x^2$. **(07 Marks)**

c. Solve by the method of undeterminant coefficient $(D^2 + 1)y = x^2$. **(07 Marks)**

Module-2

3 a. Solve $\frac{dx}{dt} + 2y = -\sin t$, $\frac{dy}{dt} - 2x = \cos t$. **(07 Marks)**

b. Solve $(1+x)^2 y'' + (1+x)y' + y = \sin 2[\log(1+x)]$. **(07 Marks)**

c. Solve $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$. **(06 Marks)**

4 a. Solve $x^3y''' + 3x^2y'' + xy' + 8y = 65\cos(\log x)$. **(07 Marks)**

b. Obtain the general solution of $x^2p^4 + 2xp - y = 0$. **(07 Marks)**

c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation. Hence obtain the general solution. **(06 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary functions ϕ and Ψ from the relation $Z = \phi(x + ay) + \psi(x - ay)$. **(06 Marks)**

b. Derive one dimensional heat equation. **(07 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dxdydz$. **(07 Marks)**

6 a. Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} = x + y$ by direct integration. **(06 Marks)**

b. Derive one dimensional wave equation. **(07 Marks)**

c. Evaluate $\iint_R (x^2 + y^2)dxdy$ where R is the region bounded by the lines y= 0, y = x and x = 1. **(07 Marks)**

Module-4

7 a. Using multiple integrals, find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (06 Marks)

b. Prove that $\beta(m,n)=\frac{\lfloor m \, \lfloor n}{\lfloor m+n}$ (07 Marks)

c. Express the vector $\vec{F}=2y\hat{i}-z\hat{j}+3x\hat{k}$ in cylindrical coordinates. (07 Marks)

8 a. Find the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ using double integration. (06 Marks)

b. Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$. Using Beta and Gamma functions. (07 Marks)

c. Prove that the cylindrical coordinate system orthogonal. (07 Marks)

Module-5

9 a. Find the Laplace transform of $\frac{\cos at-\cos bt}{t}$. (06 Marks)

b. Find the Laplace transform of

$$\begin{cases} 1, & 0<t\le 1 \\ t, & 1<t\le 2 \\ t^2, & t>2 \end{cases}$$

(07 Marks)

c. Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. (07 Marks)

10 a. Find the inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$. (06 Marks)

b. Given $f(t)=\begin{cases} E, & 0<t<a/2 \\ -E, & a/2<t<a \end{cases}$ where $f(t+a)=f(t)$, show that $L\{f(t)\}=\frac{E}{S}\tanh\left(\frac{as}{4}\right)$. (07 Marks)

c. Solve the differential equation $y''+4y'+3y=e^{-t}$ with $y(0) = 1 = y'(0)$ using Laplace transforms. (07 Marks)

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CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Fourier series to represent the periodic function $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. **(08 Marks)**

b. The following table gives the variations of periodic current over a period.

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amp.	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Expand A as a Fourier series upto first harmonic. Obtain the amplitude of the first harmonic. **(06 Marks)**

c. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. **(06 Marks)**

OR

2 a. Find the Fourier series of $f(x) = 2x - x^2$ in (0, 3). **(08 Marks)**

b. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier series expansion of y as given in the following table: **(06 Marks)**

x:	0	1	2	3	4	5	6
y:	9	18	24	28	26	20	9

c. Obtain the half-range sine series for the function,

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}.$$ **(06 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$. Hence deduce that

$$\int_0^\infty \frac{(\sin x - x\cos x)}{x^3} \cos\frac{x}{2}\, dx = \frac{3\pi}{16}.$$ **(08 Marks)**

b. Find the Z-transform of,
(i) $\cos n\theta$ and (ii) $\cosh n\theta$ **(06 Marks)**

c. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0 = y_1$, using z-transforms technique. **(06 Marks)**

OR

4 a. Find the Fourier cosine transform of e^{-ax}. Hence evaluate $\int_0^\infty \frac{\cos\lambda x}{x^2+a^2}dx$ **(08 Marks)**

b. Find the Z-transform of,
(i) $(n+1)^2$ (ii) $\sin(3n+5)$ **(06 Marks)**

c. Find the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. **(06 Marks)**

Module-3

5 a. Find the two regression lines and hence the correlation coefficient between x and y from the data. **(08 Marks)**

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	28	36	41	49	40	50

b. Fit a second degree parabola to the following data: **(06 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

c. Using Newton-Raphson method find the root of $x\sin x + \cos x = 0$ near $x = \pi$ corrected to 4 decimal places. **(06 Marks)**

OR

6 a. Two variables x and y have the regression lines $3x+2y=26$ and $6x+y=31$. Find the mean values of x and y and the correlation coefficient between them. **(08 Marks)**

b. Fit a curve of the form, $y = ae^{bx}$ to the following data: **(06 Marks)**

x:	5	15	20	30	35	40
y:	10	14	25	40	50	62

c. Using Regula-Falsi method find the root of $xe^x = \cos x$ in the interval (0, 1) carrying out four iterations. **(06 Marks)**

Module-4

7 a. Using Newton's forward and backward interpolation formulae, find f(1) and (10) from the following table: **(08 Marks)**

x	3	4	5	6	7	8	9
f(x)	4.8	8.4	14.5	23.6	36.2	52.8	73.9

b. Given that $f(5)=150$, $f(7)=392$, $f(11)=1452$, $f(13)=2366$, $f(17)=5202$. Using Newton's divided difference formulae find f(9). **(06 Marks)**

c. Using Simpson's $\frac{1}{3}^{rd}$ rule evaluate $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(06 Marks)**

OR

8 a. Using Newton's Backward difference interpolation formula find f(105) from, **(08 Marks)**

x	80	85	90	95	100
f(x)	5026	5674	6362	7088	7854

b. If $f(1)=-3$, $f(3)=9$, $f(4)=30$, $f(6)=132$ find Lagrange's interpolation polynomial that takes the same value as f(x) at the given point. **(06 Marks)**

c. Evaluate $\int_4^{5.2}\log_e x\,dx$ by Simpson's $\frac{3}{8}^{th}$ rule with h = 0.1. **(06 Marks)**

Module-5

9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2dy$ where C is bounded by $y = x$ and $y = x^2$. **(08 Marks)**

b. Using Gauss divergence theorem evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$,

where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallel piped $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$. **(06 Marks)**

c. With usual notations derive Euler's equation, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

OR

10 a. If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{i}$, evaluate $\oint_C \vec{F}.d\bar{r}$ along the curve C in the xy-plane, $y = x^3$ from (1, 1) to (2, 8). **(08 Marks)**

b. Find the extremals of the functional with y(0) = 0 and y(1) = 1. **(06 Marks)**

c. Show that Geodesics on a plane arc straight lines. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

15MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the Fourier series of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$ and hence deduce that :
$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ **(08 Marks)**

b. Express y as a Fourier series upto the second harmonics given :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(08 Marks)

OR

2 a. Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2$. **(06 Marks)**

b. Expand the function $f(x) = x\sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. **(05 Marks)**

c. Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 \le x < 1$. **(05 Marks)**

Module-2

3 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

And hence deduce that $\int_0^{10} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$ **(06 Marks)**

b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & \text{else where} \end{cases}.$$ **(05 Marks)**

c. Find the z – transform of : i) $\cos n\theta$ ii) $\sin n\theta$. **(05 Marks)**

OR

4 a. Obtain the Fourier transform of $f(x) = x e^{-|x|}$. **(06 Marks)**

b. If $u(z) = \frac{2z^2 + 3z}{(z+2)(z-4)}$, find the inverse z-transform. **(05 Marks)**

c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z – transforms. **(05 Marks)**

Module-3

5 a. Compute the co-efficient of correlation and equation of lines of regression for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

b. Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data :

x	1	2	3	4	5
y	10	12	13	16	19

(05 Marks)

c. Use the Regula – Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places. (05 Marks)

OR

6 a. Find the co-efficient of correlation for the following data :

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(06 Marks)

b. Fit a least square geometric curve $y = ae^{bx}$ for the following data :

x	0	2	4
y	8.12	10	31.82

(05 Marks)

c. Use Newton – Raphson method to find a real root of the equation : $x \log_{10}^{x} = 1.2$ correct to four decimal places that is near to 2.5. (05 Marks)

Module-4

7 a. From the following table find the number of students who have obtained :
i) Less than 45 marks
ii) Between 40 and 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

(06 Marks)

b. Find the Legrange's interpolation polynomial for the following values y(1) = 3, y(3) = 9, y(4) = 30 and y(6) = 132. (05 Marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. (05 Marks)

OR

8 a. Give $u_{20} = 24.37$, $u_{22} = 49.28$, $u_{29} = 162.86$ and $u_{32} = 240.5$ find u_{28} by Newton's divided difference formula. (06 Marks)

b. Extrapolate for 25.4 given the data using Newton's backward formula :

x	19	20	21	22	23
y	91	100.25	110	120.25	131

(05 Marks)

c. Evaluate : $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule taking seven ordinates. (05 Marks)

Module-5

9 a. Verify Green's theorem for $\oint_C (xy+y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by y = x and y = x^2. **(06 Marks)**

b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0$. **(05 Marks)**

c. If $\vec{F} = xyi + yzj + zxk$ evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve represented by x = t, y = t^2, z = t^3, $-1 \le t \le 1$. **(05 Marks)**

OR

10 a. Verify Green's theorem in the plane for $\int_C (x^2+y^2)dx + 3x^2y\,dy$ where C is the circle $x^2 + y^2 = 4$ traced in the positive sence. **(06 Marks)**

b. Evaluate $\int_C (xydx + xy^2dy)$ by Stoke's theorem C is the square in the x-y plane with the vertices (1, 0), (–1, 0), (0, 1) and (0, 1). **(05 Marks)**

c. Prove that the geodesics on a plane are straight lines. **(05 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | 17MAT41

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note:1) ***Answer any FIVE full questions, choosing ONE full question from each module.***
2) ***Use of Statistical tables allowed.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Use Taylor's series to obtain approximate value of y at x = 0.1 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. **(06 Marks)**

b. Apply Runge Kutta method of fourth order to find an approximate value of y when x = 0.2 for the equation $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ taking $h = 0.2$. **(07 Marks)**

c. Using Milne's predictor – corrector method, find y when x = 0.8 given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. **(07 Marks)**

OR

2 a. Given that $\frac{dy}{dx} = \log(x + y)$ and y(1) = 2, then find y(1.2) in step of 0.2 using modified Euler's method carry out two iterations. **(06 Marks)**

b. Using fourth order Runge-Kutta method to find y at x = 0.2 equation given that $\frac{dy}{dx} = x + y$, y(0) = 1 and h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1 + y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adam's-Bashforth predictor-corrector method. **(07 Marks)**

Module-2

3 a. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y^2$ for x = 0.2, correct to three decimal places, with initial conditions $y(0) = 1$, $y'(0) = 0$. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, then $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

OR

4 a. Apply Milne's predictor-corrector method to compute y(0.4) given the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following initial values:
$y(0) = 1, y(0.1) = 1.1103, y(0.2) = 1.2427, y(0.3) = 1.399$
$y'(0) = 1,\ y'(0.1) = 1.2103,\ y'(0.2) = 1.4427,\ y'(0.3) = 1.699$ **(06 Marks)**

b. With usual notation, show that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$ **(07 Marks)**

c. With usual notation, derive the Rodrigue's formula $P_n(x) = \frac{1}{(2^n)n!} \frac{d^n}{dx^n}(x^2 - 1)^n$. **(07 Marks)**

Module-3

5 a. Find the bilinear transformation which map the points z = 0, 1, ∞ into the points $w = -5, -1, 3$ respectively. **(06 Marks)**

b. Derive Cauchy-Riemann equations in Cartesian form. **(07 Marks)**

c. Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where $C : |z| = 2.5$ by residue theorem. **(07 Marks)**

OR

6 a. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(06 Marks)**

b. Discuss the transformation $W = Z^2$. **(07 Marks)**

c. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)}$, where C is the circle $|z| = 3$, using Cauchy residue theorem. **(07 Marks)**

Module-4

7 a. The probability density function of a variate x given by the following table:

X	−3	−2	−1	0	1	2	3
P(X)	K	2K	3K	4K	3K	2K	K

Find the value of K, mean and variance. **(06 Marks)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for, (i) more than 2150 hours, (ii) less than 1950 hours, (iii) more than 1920 hours and but less than 2160 hours.
Given : $A(0 < z < 1.83) = 0.4664$, $A(0 < z < 1.33) = 0.4082$ and $A(0 < z < 2) = 0.4772$ **(07 Marks)**

c. A joint probability distribution is given by the following table:

X \ Y	−3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Determine the marginal probability distributions of X and Y. Also find COV(X, Y). **(07 Marks)**

OR

8 a. Derive mean and variance of the Poisson distribution. **(06 Marks)**

b. In a certain town the duration of a shower is exponentially distributed within mean 5 minute. What is the probability that a shower will last for,
(i) less than 10 minutes (ii) 10 minutes or more
(iii) between 10 and 12 minutes. **(07 Marks)**

c. Given,

X \ Y	0	1	2	3
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

(i) Find Marginal distribution of X and Y.
(ii) Find E(X), E(Y) and E(XY). **(07 Marks)**

Module-5

9 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. **(06 Marks)**

b. Five dice were thrown 96 times and number 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as follows:

No. of dice showing 1, 2 or 3 :	5	4	3	2	1	0
Frequency :	7	19	35	24	8	3

Test the hypothesis that the data follow a binomial distribution at 5% level of significance $\left(\chi^2_{0.05} = 11.07 \text{ for d.f is } 5\right)$. **(07 Marks)**

c. A student's study habits are as follows:
If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night he is 60% sure not to study the next night. In the long run how often does he study? **(07 Marks)**

OR

10 a. If $p = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$, find the fixed probabilities vector. **(06 Marks)**

b. A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this supports the hypothesis that the population mean of I.Q's is 100 at 5% level of significance? ($t_{0.05} = 2.262$ for 9 d.f) **(07 Marks)**

c. Explain : (i) Transient state (ii) Absorbing state (iii) Recurrent state. **(07 Marks)**

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CBCS SCHEME

USN 2VD17ME424

15MAT41

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Using the Taylor's series method, solve $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider the series upto third degree terms. **(05 Marks)**

b. By using the modified Euler's method, solve $\frac{dy}{dx} = \log_e(x + y)$, $y(1) = 2$ at the point $x = 1.2$. Take $h = 0.2$ and carry out two modifications. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ at $x = 0.8$, using Adams - Bashforth method, given that $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. **(06 Marks)**

OR

2 a. Employ the Taylor's series method to find y(4.1) given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$, $y(4) = 4$. Consider terms upto third degree. **(05 Marks)**

b. Given $\frac{dy}{dx} = 3e^x + 2y$ and $y(0) = 0$. Find y(0.1) using the Range-Kutta method. Take step length $h = 0.1$ **(05 Marks)**

c. Given $5x\frac{dy}{dx} + y^2 - 2 = 0$ and the set of values of (x, y) given in the following table, find y at x = 4.5 using the Milne's method.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

(06 Marks)

Module-2

3 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute y(0.2) and y′(0.2) using fourth order Runge-Kutta method. **(05 Marks)**

b. Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Apply Milne's method to compute y(0.4) given that $y'' + xy' + y = 0$ and the table

x	0	0.1	0.2	0.3
y	1	0.995	0.9801	0.956
y′	0	-0.0995	-0.196	-0.2867

(05 Marks)

b. Explain $J_{-\frac{1}{2}}(x)$ in terms if cos x. **(05 Marks)**

c. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n \cdot n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$ **(06 Marks)**

Module-3

5 a. Define analytic function and obtain Cauchy – Riemann equations in Polar form. **(05 Marks)**

b. Evaluate $\int_c \frac{dz}{z^2-4}$ in the cases where 'C' is the circle |z+2| = 1 using Cauchy's integral formula. **(05 Marks)**

c. Discuss the transformation $w = z^2$. **(06 Marks)**

OR

6 a. Given $u = 3x^2y - y^3$. Find the analytic function f(z). **(05 Marks)**

b. Evaluate $\int_c \frac{e^z}{(z-1)(z-5)^2}dz$, where C is the circle | z | = 8 using Cauchy's Residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps the points z = 0, i, ∞ on to the points w = 1, –i, –1 respectively. **(06 Marks)**

Module-4

7 a. A random variable x has the following probability function for various values of 'x'

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k (ii) Evaluate $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$ **(05 Marks)**

b. Obtain Mean and Standard Deviation of the Exponential Distribution. **(05 Marks)**

c. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) one error per micro second (iii) atleast one error per micro second (iv) two errors (v) atmost two errors. **(06 Marks)**

OR

8 a. The pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that
(i) exactly 2 will be defective
(ii) atleast 2 will be defective
(iii) none will be defective **(05 Marks)**

b. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D. of the distribution. (S.D = Standard deviations $\phi(0.5) = 0.1915$, $\phi(1.4) = 0.4192$). **(05 Marks)**

c. The joint probability distribution table for two random variables X and Y as follows:

X \ Y	–2	–1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal probability distributions of X and Y. Also compute (i) Expectations of X, Y and XY (ii) Covariance of X and Y. **(06 Marks)**

Module-5

9 a. Certain tubes manufactured by a company have mean life time of 800 hours and standard deviation of 60 hours. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time
(i) between 790 hours and 810 hours
(ii) less than 785 hours
(iii) more than 820 hours
(iv) between 770 hours and 830 hours. **(05 Marks)**

b. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights (lbs).

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly regarding their effect on increase in weight. ($t_{0.05}$ for 16 d.f = 2.12). **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

(06 Marks)

OR

10 a. A random sample for 1000 workers in company has mean wage of Rs. 50 per day and standard deviation of Rs. 15. Another sample of 1500 workers from another company has mean wage of Rs. 45 per day and standard deviation of Rs. 20. Does the mean rate of wages varies between the two companies? Find the 95% confidence limits for the difference of the mean wages of the population of the two companies. **(05 Marks)**

b. Five dice were thrown 96 times and the numbers 1,2 or 3 appearing on the face of the dice follows the frequency distribution as below.

Number of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution $(\chi^2_{0.05} = 11.07 \text{ for } 5 \text{ d.f.})$
(05 Marks)

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(06 Marks)**

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USN | | | | | | | | | | 10MAT41

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO full questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using Taylor's series method, solve the differential equation $\frac{dy}{dx} = x^2y + 1$ with y(0) = 0 at x = 0.4. Consider terms up to fourth degree. **(06 Marks)**

b. Solve the differential equation $\frac{dt}{dx} = -xy^2$ under the initial condition y(0) = 2, by using the modified Euler's method at x = 0.1 and x = 0.2. Take the step size h = 0.1 and carryout two modifications at each step. **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $\frac{dy}{dx} = x^2(1+y)$ given y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4). **(07 Marks)**

2 a. Solve the differential equations:
$\frac{dy}{dx} = 1 + xz$, $\frac{dz}{dx} = -xy$ for x = 0.3 using fourth order Runge-Kutta method. Initial values are x = 0, y = 0, z = 1, Take h = 0.3. **(06 Marks)**

b. Apply Picard's method upto third approximation to find y and z for the equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ given y(0) = 1 = y′(0). **(07 Marks)**

c. Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following initial values y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795 y(0.6) = 0.1762, y′(0) = 0, y′(0.2) = 0.1996, y′(0.4) = 0.3937, y′(0.6) = 0.5689. **(07 Marks)**

3 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Find the analytic function whose imaginary part is e^x(xsiny + ycosy). **(07 Marks)**

c. If f(z) is an analytic function then prove that
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$
(07 Marks)

4 a. Discuss the Transformation $W = e^z$. **(06 Marks)**

b. Find the bilinear transformation which maps the points 1, i, -1 onto the points i, 0, -1 respectively. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$ where C is the circle $|z| = 3$, using Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Find the solution of the Laplace's equation in cylindrical system leading to Bend's differential equation. **(06 Marks)**

b. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$ **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(07 Marks)**

6 a. A five figure number is formed by the digits 0, 1, 2, 3, 4 without repitition. Find the probability that the number is divisible by 4. **(06 Marks)**

b. If A and B are any two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$ find $P(A/B)$, $P(B/A)$, $P(A/\overline{B})$, $P(\overline{A}/\overline{B})$, $P(\overline{B}/\overline{A})$. **(07 Marks)**

c. The contents of three boxes are 1 white, 2 red, 3 green balls, 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from the box chosen at random. These are found to be one white and one green. Find the probability that the balls are from the third box. **(07 Marks)**

7 a. The probability distribution of a random variable X is given by the following table:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

Find: $P(X < 5)$, $P(X \geq 6)$, $P(3 < X \leq 6)$. Also find mean and variance. **(06 Marks)**

b. Find mean and variance of Binomial distribution. **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation. Give A(0.5) = 0.19, A(1.4) = .42 where A(z) is the area under the standard normal curve from 0 to z. **(07 Marks)**

8 a. Explain the following terms:

i) Null hypothesis
ii) Significances level
iii) Confidence limits. **(06 Marks)**

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times on the assumption of random throwing, do the data indicate an unbiased die. **(07 Marks)**

c. A Machinist is making engine parts with axle diameter of 0.7inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior? Given for $\rho = 9$, $T_{0.05} = 2.262$. **(07 Marks)**

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USN [| | | | | | | | |]

MATDIP301

Third Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If n is positive integer prove that :

$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$. (06 Marks)

b. Sum the series : $1 + x\cos\theta + \frac{x^2}{2!}\cos 2\theta + \frac{x^3}{3!}\cos 3\theta + ----+\infty$. (07 Marks)

c. Put the complex number $\left(\frac{2+i}{3-i}\right)^2$ into polar form. (07 Marks)

2 a. Find the n^{th} derivative $e^{ax}\cos(bx+c)$. (07 Marks)

b. Find the n^{th} derivative of $\frac{x}{(2x+1)(x-2)}$. (06 Marks)

c. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (07 Marks)

3 a. With usual notations prove that $\tan\varphi = \frac{rd\varphi}{dr}$. (07 Marks)

b. Find the pedal equation : $r^m = a^m \cos m\theta$. (06 Marks)

c. Expand $\log(1+\sin^2 x)$ in powers of x as for as the term in x^6. (07 Marks)

4 a. If $Z = e^{ax+by} f(ax-by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2ab\,z$. (06 Marks)

b. If $Z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. (07 Marks)

c. If $u = \frac{xy}{z}, v = \frac{yz}{x}, w = \frac{zx}{y}$. Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. (07 Marks)

PART – B

5 a. Derive the reduction formula for $I_n = \int_0^{\pi/2} \cos^n x \, dx$. **(07 Marks)**

b. Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$. **(06 Marks)**

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dx \, dy$. **(07 Marks)**

6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$. **(07 Marks)**

b. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$ in teams of gamma functions. **(06 Marks)**

c. Prove that $\beta(m, \tfrac{1}{2}) = 2^{2m-1} \beta(m, m)$. **(07 Marks)**

7 a. Solve $(x + y + 1)^2 \frac{dy}{dx} = 1$. **(07 Marks)**

b. Solve $(x \tan \frac{y}{x} - y \sec^2 \frac{y}{x}) dx + x \sec^2 \frac{y}{x} dy = 0$. **(06 Marks)**

c. Solve $\frac{dy}{dx} - \frac{\tan y}{1 + x} = (1 + x) e^x \sec y$. **(07 Marks)**

8 a. Solve $\frac{d^4 y}{dx^4} + y = \sin 2x \sin x$. **(07 Marks)**

b. Solve $\frac{d^2 y}{dx^2} + 4y = x^4$. **(06 Marks)**

c. Solve $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$. **(07 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the angles between any two diagonals of a cube. **(06 Marks)**

b. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are the direction cosines of two lines then angle θ between the lines is $\theta = \cos^{-1}(\ell_1 \ell_2 + m_1 m_2 + n_1 n_2)$. **(07 Marks)**

c. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, show that : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$. **(07 Marks)**

2 a. Find the equation of the plane through (1, –2, 2), (–3, 1, –2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

b. Find the equation of the line passing through the points (1, 2, –1) and (3, –1, 2). At what point does it meet the yz – plane. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect. Find the point of intersection and the equation of the plane in which they lie. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} + 4\hat{k}$ from a right–angle triangle. **(06 Marks)**

b. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. **(07 Marks)**

c. Find the constant a so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} - a\hat{j} - 5\hat{k}$ are coplanar. **(07 Marks)**

4 a. If $\frac{d\vec{A}}{dt} = \vec{W} \times \vec{A}$, $\frac{d\vec{B}}{dt} = \vec{W} \times \vec{B}$, show that $\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{W} \times (\vec{A} \times \vec{B})$. **(06 Marks)**

b. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at time t = 1 in the direction $2\hat{i} - 3\hat{j} - 6\hat{k}$. **(07 Marks)**

c. Find the angle between the surfaces $x^2yz + 3xz^2 = 5$ and $x^2y^3 = 2$ at (1, –2, –1). **(07 Marks)**

5 a. Find unit vector normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2). **(06 Marks)**

b. Prove that $\text{curl}(\phi\vec{A}) = (\text{grad}\phi) \times \vec{A} + \phi\,\text{curl}\,\vec{A}$. **(07 Marks)**

c. Prove that $\nabla^2(r)^n = n(n+1)r^{n-2}$, where $r = |x\,i + y\,j + z\,k|$. **(07 Marks)**

6 a. Find Laplace transform of coshat. **(06 Marks)**

b. If $f(t) = \begin{cases} 2t & \text{for} \quad 0 \le t \le 5 \\ 1 & \text{for} \quad t > 5 \end{cases}$, find L[f(t)]. **(07 Marks)**

c. Find $L\left[\frac{\cos 2t - \cos 3t}{t}\right]$. **(07 Marks)**

7 a. By using the convolution theorem find the inverse Laplace transforms of $\frac{1}{s^2(s+5)}$. **(06 Marks)**

b. Find $L^{-1}\left[\frac{(3s+7)}{s^2+2s-3}\right]$. **(07 Marks)**

c. Find the inverse Laplace transform of $\log\left(1+\frac{a^2}{s^2}\right)$. **(07 Marks)**

8 a. Using Laplace transform solve :

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, y(0) = 0 = y'(0)$. **(10 Marks)**

b. Solve the system of equations by the method of Laplace transform

$(D-2)x + 3y = 0, 2x + (D-1)y = 0$

Where $D = \frac{d}{dt}$, given that x = 8, y = 3 at t = 0. **(10 Marks)**

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CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that $(1+i)^n + (1-i)^n = 2^{n/2+1}\cos\frac{n\pi}{4}$ **(08 Marks)**

b. Expression the complex number $(2+3i)+\frac{1}{1-i}$ in the form a + ib. **(06 Marks)**

c. Find the modulus and amplitude of the complex number $1-\cos\alpha + i\sin\alpha$. **(06 Marks)**

OR

2 a. If $\vec{A} = i+2j-3k$, $\vec{B} = 3i-j+2k$ show that $\vec{A}+\vec{B}$ is perpendicular to $\vec{A}-\vec{B}$. Also find the angle between $2\vec{A}+3\vec{B}$ and $\vec{A}+2\vec{B}$. **(08 Marks)**

b. Show that the vectors $i-2j+3k$, $2i+j+k$, $3i+4j-k$ are coplanar. **(06 Marks)**

c. Find the sine of the angle between $\vec{A} = 4i-j+3k$ and $\vec{B} = -2i+j-2k$. **(06 Marks)**

Module-2

3 a. Obtain the Maclaurin's series expansion of sin x upto term containing x^4. **(08 Marks)**

b. If $u = \sin^{-1}\left[\frac{x^2+y^2}{x-y}\right]$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

c. If $u = f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$. **(06 Marks)**

OR

4 a. Prove that $\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots\ldots$ by using Maclaurin's series. **(08 Marks)**

b. If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$. **(06 Marks)**

c. If $z = e^{ax+by} f(ax-by)$ then show that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

Module-3

5 a. Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point $(2,-1,2)$ **(08 Marks)**

b. Find the unit vector normal to the surface $x^2y+2xz = 4$ at $(2,-2,3)$. **(06 Marks)**

c. Show that the vector $(-x^2+yz)i+(4y-z^2x)j+(2xz-4z)k$ is solenoidal. **(06 Marks)**

OR

6 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at t = 1 in the direction i + j + 3k. **(08 Marks)**

b. Find the values of a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. **(06 Marks)**

c. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, $n > 0$. **(08 Marks)**

b. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1 - x^2}}\,dx$ **(06 Marks)**

c. Evaluate $\iint xy(x + y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$. **(06 Marks)**

OR

8 a. Obtain the reduction formula for

$\int_0^{\pi/2} \sin^n x\,dx$, $n > 0$. **(08 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1 - x^2)^{7/2}}\,dx$ **(06 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$ **(06 Marks)**

Module-5

9 a. Solve $y(\log y)dx + (x - \log y)dy = 0$ **(08 Marks)**

b. Solve $x.\frac{dy}{dx} + y = x^3y^6$ **(06 Marks)**

c. Solve $(xy^2 - e^{1/x^3})dx - x^2y\,dy = 0$ **(06 Marks)**

OR

10 a. Solve $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ **(08 Marks)**

b. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ **(06 Marks)**

c. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)\,dy = 0$ **(06 Marks)**

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CBCS SCHEME

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Third Semester B.E. Degree Examination, Aug./Sept. 2020
Additional Mathematics – I

Time: 3 hrs.   Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of,
$1+\cos\alpha + i\sin\alpha$ **(06 Marks)**

b. Express the complex number $\dfrac{(1+i)(2+i)}{(3+i)}$ in the form a + ib. **(07 Marks)**

c. Find a unit vector normal to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. **(07 Marks)**

OR

2 a. Show that $\left[\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right]^n = \cos n\left(\dfrac{\pi}{2}-\theta\right) + i\sin n\left(\dfrac{\pi}{2}-\theta\right)$. **(06 Marks)**

b. If $\vec{A} = i - 2j - 3k$, $\vec{B} = 2i + j - k$, $\vec{C} = i + 3j - k$
find (i) $\left(\vec{A}\times\vec{B}\right)\times\left(\vec{B}\times\vec{C}\right)$ (ii) $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$ **(07 Marks)**

c. Show that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$. **(07 Marks)**

Module-2

3 a. If $y = (x^2-1)^n$ then prove that $(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(07 Marks)**

c. Show that the following curves intersect orthogonally $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$. **(07 Marks)**

OR

4 a. Show that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24}$...... using Maclaurin's series expansion. **(06 Marks)**

b. If $u = e^{ax+by}f(ax-by)$, prove that $b\dfrac{\partial u}{\partial x} + a\dfrac{\partial u}{\partial y} = 2abu$. **(07 Marks)**

c. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. **(07 Marks)**

Module-3

5 a. Obtain a reducation formula for $\int \cos^n x\,dx$. **(06 Marks)**

b. Evaluate $\displaystyle\int_0^2 \frac{x^4}{\sqrt{4-x^2}}dx$. **(07 Marks)**

c. Evaluate $\displaystyle\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$. **(07 Marks)**

OR

6 a. Obtain a reducation formula for $\int \sin^n x\, dx$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y\, dxdy$. **(07 Marks)**

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dzdydx$. **(07 Marks)**

Module-4

7 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$.
(i) Determine its velocity and acceleration.
(ii) Find the components of velocity and acceleration at t = 1 in the direction $2i + j + 2k$. **(06 Marks)**

b. Find the directional derivative of, $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

c. If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ find a, b, c such that curl $\vec{F} = 0$ and then find ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

8 a. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ prove that $\nabla(r^n) = nr^{n-2}.\vec{r}$ **(06 Marks)**

b. If $\vec{F} = (x + y + 1)i + j - (x + y)k$ show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(07 Marks)**

c. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

Module-5

9 a. Solve : $\frac{dy}{dx} = \frac{y - x}{y + x}$. **(06 Marks)**

b. Solve : $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. **(07 Marks)**

c. Solve : $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(07 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

b. Solve : $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$. **(07 Marks)**

c. Solve : $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$. **(07 Marks)**

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CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express $\frac{5+2i}{5-2i}$ in the form $xi + iy$. **(06 Marks)**

b. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$ **(05 Marks)**

c. If $\vec{a} = (3, -1, 4)$, $\vec{b} = (1, 2, 3)$, $\vec{c} = (4, 2, -1)$ find $\vec{a} \times (\vec{b} \times \vec{c})$ **(05 Marks)**

OR

2 a. Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cdot\cos\frac{n\theta}{2}$. **(06 Marks)**

b. Find the sine of angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$ **(05 Marks)**

c. Find the value of λ, so that the vector $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are coplanar. **(05 Marks)**

Module-2

3 a. If $y = \tan^{-1}x$, prove that

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$ **(06 Marks)**

b. Find the angle between the radius vector and tangent to the curve $r = a(1 - \cos\theta)$ **(05 Marks)**

c. If $u = \sin^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. **(05 Marks)**

OR

4 a. Find the pedal equation of the curve $r = 2(1 + \cos\theta)$ **(06 Marks)**

b. Find the total derivative of $u = x^3y^2$, where $x = e^t$, $y = \log t$. **(05 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\sin x$. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\cos^6 x\, dx$ **(06 Marks)**

b. Evaluate $\int_0^1\int_0^3 x^3y^3 dx\, dy$ **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y+z) dx\, dy\, dz$ **(05 Marks)**

OR

6 a. Evaluate $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$ using Reduction formula. **(06 Marks)**

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$ **(05 Marks)**

c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz \, dx \, dy \, dz$ **(05 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$. Determine the velocity and acceleration at t = 2. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$. **(05 Marks)**

c. Find the constants a and b, such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. **(05 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at t = 1 and t = 2. **(06 Marks)**

b. Find div$\vec{F}$ and curl$\vec{F}$ where $\vec{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$ **(05 Marks)**

c. Find 'a' for which $\vec{F} = (x + 3y)i + (y - 2z)j + (x + az)k$ is solenoidal. **(05 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ **(06 Marks)**

b. Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$ **(05 Marks)**

c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$ **(05 Marks)**

OR

10 a. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ **(06 Marks)**

b. Solve $x \frac{dy}{dx} = y + x \cos^2\left(\frac{y}{x}\right)$ **(05 Marks)**

c. Solve $(x^4 + y^2)dy = 4x^3y \, dx$ **(05 Marks)**

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CBCS SCHEME

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17MATDIP41

Fourth Semester B.E. Degree Examination, Aug./Sept. 2020
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(07 Marks)**

b. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ using Cayley-Hamilton theorem. **(07 Marks)**

c. Find the Eigen values of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(06 Marks)**

OR

2 a. Solve the system of equation by Gauss elimination method,

$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20$$

(07 Marks)

b. Using Cayley-Hamilton theorem find A^{-1}, given

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **(07 Marks)**

c. Find the rank of the matrix by reducing in to row echelon form, given

$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(06 Marks)**

Module-2

3 a. Solve by method of undetermined co-efficient $y'' - 4y' + 4y = e^x$. **(07 Marks)**

b. Solve $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$. **(07 Marks)**

c. Solve $y'' + 2y' + y = 2x$. **(06 Marks)**

OR

4 a. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by method of variation of parameter. **(07 Marks)**

b. Solve $y'' - 4y' + 13y = \cos 2x$. **(07 Marks)**

c. Solve $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$. **(06 Marks)**

Module-3

5 a. Express the following function into unit step function and hence find $L[f(t)]$ given $f(t)=\begin{cases} t, & 0<t<4 \\ 5, & t>4 \end{cases}$. **(07 Marks)**

b. Find $L\left[\dfrac{1-e^{-at}}{t}\right]$. **(07 Marks)**

c. Find $L[t.\cos at]$. **(06 Marks)**

OR

6 a. Find $L[\sin 5t.\cos 2t]$. **(07 Marks)**

b. Find $L[e^{t}\cos^2 3t]$. **(07 Marks)**

c. Find $L[\cos 3t.\cos 2t.\cos t]$. **(06 Marks)**

Module-4

7 a. Employ Laplace transform to solve the equation $y''+5y'+6y=5e^{2x}$ given $y(0)=2,\ y'(0)=1$. **(07 Marks)**

b. Find $L^{-1}\left[\dfrac{1}{s(s+1)(s+2)(s+3)}\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\dfrac{s+5}{s^2-6s+13}\right]$. **(06 Marks)**

OR

8 a. Using Laplace transforms solve $y''+4y'+4y=e^{-t}$ given $y(0)=0,\ y'(0)=0$. **(07 Marks)**

b. Find $L^{-1}\left[\log\left(\dfrac{s+a}{s+b}\right)\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\dfrac{2s-5}{4s^2+25}\right]+L^{-1}\left[\dfrac{8-6s}{16s^2+9}\right]$. **(06 Marks)**

Module-5

9 a. State and prove Baye's theorem. **(07 Marks)**

b. A shooter can hit a target in 3 out of 4 shots and another shooter can hit the target in 2 out of 3 shots. Find the probability that the target is being hit.

(i) When both of them try.

(ii) By only one shooter. **(07 Marks)**

c. If A and B are any two mutually exclusive events of S, then show that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. **(06 Marks)**

OR

10 a. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective out put of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item non produced by machine C. **(07 Marks)**

b. Prove the following : (i) $P(\phi)=0$ (ii) $P(\overline{A})=1-P(A)$ **(07 Marks)**

c. If A and B are events with $P(AUB)=\dfrac{7}{8}$, $P(A\cap B)=\dfrac{1}{4}$ and $P(\overline{A})=\dfrac{5}{8}$ find P(A), P(B) and $P(A\cap\overline{B})$. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | |

15MATDIP41

Fourth Semester B.E. Degree Examination, Aug./Sept. 2020
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix,

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

By reducing it to the echelon form. **(05 Marks)**

b. Solve the following system of equations by Gauss Elimination method.

$4x + y + z = 4$

$x + 4y - 2z = 4$

$3x + 2y - 4z = 6$ **(05 Marks)**

c. Find all the eigen values and the eigen vector corresponding to the least eigen value of the matix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(06 Marks)**

OR

2 a. Find the rank of the matrix,

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

By applying elementary row transformations. **(05 Marks)**

b. Solve the following system of equations, by Gauss-Elimination method:

$x + 2y + z = 3$,

$2x + 3y + 3z = 10$,

$3x - y + 2z = 13$ **(05 Marks)**

c. Using Cayley-Hamilton theorem, find the inverse of the matrix,

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$ **(06 Marks)**

Module-2

3 a. Solve : $(D^2 - 6D + 9)y = e^x + e^{3x}$ **(05 Marks)**

b. Solve : $(D^2 + 3D + 2)y = 1 + 3x + x^2$ **(05 Marks)**

c. Using the method of variation of parameters, solve :
$(D^2 + 1)y = \sec x \tan x$. **(06 Marks)**

OR

4 a. Solve : $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$. **(05 Marks)**

b. Solve : $(D^2 - 2D + 4)y = e^x \cos x$. **(05 Marks)**

c. By the method of undetermined coefficients, solve :
$(D^2 - D - 2)y = 10\sin x$. **(06 Marks)**

Module-3

5 a. Find the Laplace transform of,
(i) $\sin^2 2t$ (ii) $e^{-t}(3\sinh 2t - 2\cosh 3t)$ **(05 Marks)**

b. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. **(05 Marks)**

c. If $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$. Find $\alpha\{f(t)\}$. **(06 Marks)**

OR

6 a. Find $L\{\sin t \sin 2t \sin 3t\}$. **(05 Marks)**

b. Find (i) $L\{te^{-t}\sin 4t\}$ (ii) $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$. **(05 Marks)**

c. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit-step function and hence find $L\{f(t)\}$. **(06 Marks)**

Module-4

7 a. Find the inverse Laplace transform of :
(i) $\frac{3s-4}{16-s^2}$ (ii) $\frac{s}{s^2-a^2}$ **(06 Marks)**

b. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ **(05 Marks)**

c. Solve the equation, $y'' + 4y' + 3y = e^{-t}$, with $y(0) = 1$, $y'(0) = 1$, using Laplace transforms. **(05 Marks)**

OR

8 a. Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **(06 Marks)**

b. Find $L^{-1}\left\{\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$. **(05 Marks)**

c. Solve the equation $y'' + 6y' + 9y = 12t^2e^{-3t}$, with $y(0) = y'(0) = 0$, using Laplace transforms. **(05 Marks)**

Module-5

9 a. For any two events A and B, prove that
(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(ii) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ **(05 Marks)**

b. Given $P(A) = 0.4$, $P\left(\frac{B}{A}\right) = 0.9$ and $P\left(\frac{\overline{B}}{\overline{A}}\right) = 0.6$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{A}{\overline{B}}\right)$. **(06 Marks)**

c. State and prove Bayes's theorem. **(05 Marks)**

OR

10 a. Let A and B be events with $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{4}$, $P(\overline{B})=\frac{5}{8}$. Find $P(A\cap B)$, $P(\overline{A}\cap\overline{B})$, $P(\overline{A}\cup\overline{B})$ and $P(B\cap\overline{A})$. **(06 Marks)**

b. In a certain engineering college, 25% of First semester students have failed in Mathematics, 15% have failed in Chemistry and 10% have failed in both Mathematics and Chemistry. A student is selected at random.

(i) If he has failed in Chemistry, what is the probability that he has failed in Mathematics?

(ii) If he has failed in Mathematics, what is the probability that he has failed in Chemistry? **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of total number of items in a factory. Percentage of defective output of these machines are respectively 2%, 3% and 4%. An item selected at random is found to be defective. Find the probability that it is produced by machine C. **(05 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | | **17MAT11**

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – I

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\sin 2x \cos x$. **(06 Marks)**

b. Prove that the following curves cuts orthogonally $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(07 Marks)**

c. Find the radius of the curvature of the curve $r = a\sin n\theta$ at the pole. **(07 Marks)**

OR

2 a. If $\tan y = x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. With usual notations, prove that $\tan\phi = \frac{\tau d\theta}{dr}$. **(07 Marks)**

c. Find the radius of curvature for the curve $n^2y = a(x^2+y^2)$ at (–2a, 2a). **(07 Marks)**

Module-2

3 a. Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + ...$ **(06 Marks)**

b. If $U = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = -\frac{1}{4}\sin 2U$. **(07 Marks)**

c. Find the Jacobian of $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$. **(06 Marks)**

b. Find the Taylor's sense of log(cos x) about the point $x = \frac{\pi}{3}$ upto the third degree. **(07 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of a curve then, find velocity and acceleration at t = 1. **(06 Marks)**

b. Find the constants a and b such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(07 Marks)**

OR

6 a. Find the component of velocity and acceleration for the curve $\vec{r} = 2t^2 i + (t^2 - 4t) j + (3t - 5)k$ at the points t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. If $\vec{t} = \nabla(xy^3z^2)$, find div $\vec{t}$ and curl $\vec{t}$ at the point (1, −1, 1). **(07 Marks)**

c. Prove that curl(grad ϕ) = 0. **(07 Marks)**

Module-4

7 a. Prove that $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} dx = 3\pi$ using reduction formula. **(06 Marks)**

b. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(07 Marks)**

c. Find the orthogonal trajectory of $r^n = a \sin n\theta$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int \cos^n xdy$ and hence evaluate $\int_0^{\pi/2} \cos^n xdx$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. **(07 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing to row echelon form. **(06 Marks)**

b. Find the largest eigen and the corresponding eigen vector for $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation as $[1, 0.8, -0.8]^T$ by using power method. Carry out four iterations. **(07 Marks)**

c. Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular. Find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$ by using Gauss Seidal method. Carryout three iterations taking the initial approximation to the solution as (1, 0, 3). **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | | **15MAT11**

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – I

Time: 3 hrs.   Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. If $y = e^{-2x}\cos^3 x$, find y_n. **(05 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

c. Prove that the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$ cut orthogonally. **(05 Marks)**

OR

2 a. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

b. Find the pedal equation of $r = 2(1+\cos\theta)$. **(06 Marks)**

c. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2x+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(05 Marks)**

Module-2

3 a. Expand $\log\cos x$ in powers of $\left(x - \frac{\pi}{3}\right)$ using Taylor's series. **(05 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. **(06 Marks)**

c. If $\sin u = \frac{x^2y^2}{x+y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. **(05 Marks)**

OR

4 a. Using Maclaurin's series, expand $\log(1+e^x)$ in ascending powers of x. **(05 Marks)**

b. If u = f(x, y) and x = r cos θ, y = r sin θ, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial v}\right)^2$. **(06 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = x + y + z$, $w = xy + yz + zx$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$, determine the components of velocity and acceleration at t = 1 in the direction 2i + j + 2k. **(05 Marks)**

b. Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at the point (1, –2, –1) along the direction of 2i – j – 2k. **(06 Marks)**

c. Prove that $\text{div}(\text{curl}\,\vec{F}) = 0$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. If $\vec{F} = (3x^2 3yz)\hat{i} + (3y^2 - 3zx)\hat{j} + (3z^2 - 3xy)\hat{k}$, find (i) div F (ii) curl F. **(05 Marks)**

b. If $F = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational, find a, b, c. **(06 Marks)**

c. Prove that $\text{curl}(\phi A) = \phi(\text{curl } A) + \nabla\phi \times A$ **(05 Marks)**

Module-4

7 a. Find the reduction formula for $\int \sin^n x dx$ **(05 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ **(06 Marks)**

c. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1 - x^2}} dx$. **(05 Marks)**

OR

8 a. Find the orthogonal trajectory of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter. **(05 Marks)**

b. Solve $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(06 Marks)**

c. A body in air at 25°C cools from 100°C to 75° in one minute. Find the temperature of the body at the end of three minutes. **(05 Marks)**

Module-5

9 a. Find the Rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(05 Marks)**

b. Apply Gauss-elimination method, to solve the system of equations $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$. **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to diagonal form. **(05 Marks)**

OR

10 a. Find the largest Eigen value and the corresponding Eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $X = (1\ 0\ 0)^1$ as initial vectors. **(05 Marks)**

b. Solve the system of equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$. Carry out the 4th iterations, using Gauss-Seidal method. **(06 Marks)**

c. Reduce the quadratic form of $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz$ into canonical form. **(05 Marks)**

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USN | | | | | | | | | | |

14MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. If $Y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

b. Find the angle of intersection of the curves $r = a\cos\theta$ and $2r = a$. **(06 Marks)**

c. Derive an expression to find the radius of curvature in Polar form. **(07 Marks)**

OR

2 a. If $\sin^{-1} y = 2\log(x+1)$. Prove that $(x+1)^2y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. **(07 Marks)**

b. Find the Pedal equation for $r = a\,\text{cosec}^2\,\theta/2$. **(06 Marks)**

c. Show that the radius of curvature at any point θ on the Cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ is $4a\cos(\theta/2)$. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series, expand $\log(\sec x)$ upto x^4. **(07 Marks)**

b. If $Z = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

c. If $u = f(x/y,\ y/z,\ z/x)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. **(07 Marks)**

b. If $\cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-\cot u}{2}$. **(06 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Using differentiation under the integral sign, show that $\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x}\,dx = \pi\sin^{-1} a$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

b. Show that div (Curl $\vec{A}$) = 0. **(06 Marks)**

c. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find ϕ such that $\vec{F} = \nabla \phi$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$, where m and n are positive integers. **(07 Marks)**

b. Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of family of curves $r = 4a \sec\theta \tan\theta$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = y^2 \sec x$. **(06 Marks)**

c. If the air is maintained at 30^0 C and the temperature of the body cools from 80^0C to 60^0C in 12 mins, find the temperature of the body after 24 mins. **(07 Marks)**

Module-5

9 a. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$ by Gauss elimination method. **(07 Marks)**

b. Diagonalize the Matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

c. Determine the largest eigen value and the corresponding eigen vector of

$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ using Rayleigh's Power Method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$. **(07 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(06 Marks)**

c. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 6x_1x_3 + 2x_2x_3$ into canonical form by orthogonal transformation. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

18MAT21

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020

Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) along $2\bar{i} - 3\bar{j} + 6\bar{k}$. **(06 Marks)**

b. If $\bar{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find div $\bar{f}$ and curl $\bar{f}$. **(07 Marks)**

c. Find the constants a and b such that $\bar{F} = (axy + z^3)\bar{i} + (3x^3 - z)\bar{j} + (bxz^2 - y)\bar{k}$ is irrotational. Also find a scalar potential ϕ if $\bar{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\bar{F} = xy\bar{i} + yz\bar{j} + zx\bar{k}$ evaluate $\int_C \bar{F}.d\bar{r}$ where C is the curve represented by x = t, y = t^2, $z = t^3$, $-1 \le t \le 1$. **(06 Marks)**

b. Using Stoke's theorem Evaluate $\oint_C \bar{F}.d\bar{r}$ if $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(07 Marks)**

c. Using divergence theorem, evaluate $\iint_S \bar{F}.\hat{n}\,ds$ if $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken around $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. **(07 Marks)**

Module-2

3 a. Solve $(4D^4 - 8D^3 - 7D^3 + 11D + 6)y = 0$ **(06 Marks)**

b. Solve $(D^2 + 4D + 3)y = e^{-x}$ **(07 Marks)**

c. Using the method of variation of parameter solve $y'' + 4y = \tan 2x$. **(07 Marks)**

OR

4 a. Solve $(D^3 - 1)y = 3\cos 2x$ **(06 Marks)**

b. Solve $x^2y'' - 5xy' + 8y = 2\log x$ **(07 Marks)**

c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2} + W_0^2x = F_0\text{Sin}nt$, where W_0 and F_0 are constants. Also initially x = 0, $\frac{dx}{dt} = 0$ solve it. **(07 Marks)**

Module-3

5 a. Find the PDE by eliminating the function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given $\frac{\partial z}{\partial y} = -2\sin y$, when x = 0 and z = 0, when y is odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one-dimensional wave equation in usual notations. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$ $\frac{\partial z}{\partial x} = a \sin y$ and $z = 0$. **(06 Marks)**

b. Solve $x(y - z)\, p + y\,(z - x)\, q = z\,(x - y)$. **(07 Marks)**

c. Find all possible solution of $U_t = C^2\, U_{xx}$ one dimensional heat equation by variable separable method. **(07 Marks)**

Module-4

7 a. Test for convergence for

$$1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \ldots\ldots\ldots$$ **(06 Marks)**

b. Find the series solution of Legendre differential equation $(1 - x^2)y'' - 2xy' + n(n + 1) = 0$ leading to $P_n(x)$. **(07 Marks)**

c. Prove the orthogonality property of Bessel's function as

$$\int_0^1 x\, \bar{j}_n(\alpha x)\, \bar{j}_n(\beta x)dx = 0 \quad \alpha \neq \beta$$ **(07 Marks)**

OR

8 a. Test for convergence for

$$\sum\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$ **(06 Marks)**

b. Find the series solution of Bessel differential equation $x^2y'' + xy' + (n^2 - x^2)\, y = 0$ Leading to $\bar{j}_n(x)$. **(07 Marks)**

c. Express the polynomial $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(07 Marks)**

Module-5

9 a. Using Newton's forward difference formula find f(38). **(06 Marks)**

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

b. Find the real root of the equation $x \log_{10} x = 1.2$ by Regula falsi method between 2 and 3 (Three iterations). **(07 Marks)**

c. Evaluate $\int_4^{5.2} \log x\, dx$ by Weddle's rule considering six intervals. **(07 Marks)**

OR

10 a. Find f(9) from the data by Newton's divided difference formula:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(06 Marks)

b. Using Newton – Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. **(07 Marks)**

c. By using Simpson's $\left(\frac{1}{3}\right)$ rule, evaluate $\int_0^6 \frac{dx}{1 + x^2}$ by considering seven ordinates. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | **17MAT21**

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – II

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$ **(06 Marks)**

b. Solve $(D^2 - 4)y = \text{Cosh}\,(2x - 1) + 3^x$ **(07 Marks)**

c. Solve $(D^2 + 1)y = \text{Sec}x$ by the method of variation of parameters. **(07 Marks)**

OR

2 a. Solve $D^3 - 9D^2 + 23D - 15)y = 0$ **(06 Marks)**

b. Solve $y'' - 4y' + 4y = 8\,(\text{Sin}2x + x^2)$ **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2$ by the method of undetermined coefficients. **(07 Marks)**

Module-2

3 a. Solve $(x^2D^2 + xD + 1)y = \sin\,(2\log x)$ **(06 Marks)**

b. Solve $x^2p^2 + 3xyp + 2y^2 = 0$ **(07 Marks)**

c. Find the general and singular solution of Clairaut's equation $y = xp + p^2$. **(07 Marks)**

OR

4 a. Solve $(2x + 1)^2\, y'' - 2\,(2x + 1)\, y' - 12y = 6x$ **(06 Marks)**

b. Solve $p^2 + 2py\cot x - y^2 = 0$ **(07 Marks)**

c. Find the general solution of $(p - 1)e^{3x} + p^3\, e^{2y} = 0$ by using the substitution $X = e^x$, $Y = e^y$. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the function from $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x\,\sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional wave equation $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the function from $f(x+y+z, x^2+y^2+z^2)=0$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2}+z=0$ given that $z=\cos x$ and $\frac{\partial z}{\partial y}=\sin x$ when $y=0$. **(07 Marks)**

c. Obtain the variable separable solution of one dimensional heat equation $\frac{\partial U}{\partial t}=C^2\frac{\partial^2 U}{\partial x^2}$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^2\int_1^2 (x^2+y^2)\,dx\,dy$ **(06 Marks)**

b. Evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Derive the relation between Beta and Gamma function as $B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

OR

8 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$ **(06 Marks)**

b. Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$. **(07 Marks)**

c. Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}=\pi$ **(07 Marks)**

Module-5

9 a. Find the Laplace transform of $\left[\frac{\cos at-\cos bt}{t}\right]$. **(06 Marks)**

b. Express the function $f(t)=\begin{cases}\sin t & 0<t\le\frac{\pi}{2}\\ \cos t & t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find Laplace transform. **(07 Marks)**

c. Find $L^{-1}\left(\frac{s+2}{s^2-2s+5}\right)$. **(07 Marks)**

OR

10 a. Find the Laplace transform of the periodic function $f(t)=t^2$, $0<t<2$. **(06 Marks)**

b. Using convolution theorem obtain the Inverse Laplace transform of $\frac{1}{s^3(s^2+1)}$. **(07 Marks)**

c. Solve by using Laplace transform $y''+4y'+4y=e^{-t}$. Given that $y(0)=0$, $y'(0)=0$. **(07 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | |

15MAT21

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020

Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $\frac{dy^2}{dx^2} - 4y = \text{Cosh}(2x-1) + 3^x$ by inverse differential operators method. **(06 Marks)**

b. Solve $(D^3 - 1)y = 3\,\text{Cos}\,2x$ by inverse differential operators method. **(05 Marks)**

c. Solve $(D^2 + a^2)\,y = \text{Sec}\,(ax)$ by the method of variation of parameters. **(05 Marks)**

OR

2 a. Solve $(D^2 - 2D + 5)\,y = e^{2x}\,\text{Sin}\,x$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^3 + D^2 + 4D + 4)\,y = x^2 - 4x - 6$ by inverse differential operator method. **(05 Marks)**

c. Solve $y'' - 2y' + 3y = x^2 - \text{Cos}\,x$ by the method of undetermined coefficients. **(05 Marks)**

Module-2

3 a. Solve $x^3y''' + 3x^2y'' + xy' + 8y = 65\,\text{Cos}(\log x)$ **(06 Marks)**

b. Solve $xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$ **(05 Marks)**

c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form taking the substitution $X = x^2$, $Y = y^2$. **(05 Marks)**

OR

4 a. Solve $(2x-1)^2y'' + (2x-1)y' - 2y = 8x^2 - 2x + 3$ **(06 Marks)**

b. Solve $y = 2px + p^2y$ by solving for 'x'. **(05 Marks)**

c. Find the general and singular solution of equation $xp^2 - py + kp + a = 0$. **(05 Marks)**

Module-3

5 a. Obtain partial differential equation by eliminating arbitrary function.
Given $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(06 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x\,\partial t} = e^{-t}\,\text{Cos}\,x$, given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ at $x = 0$. **(05 Marks)**

c. Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = C^2\frac{\partial^2 u}{\partial x^2}$ **(05 Marks)**

OR

6 a. Obtain partial differential equation of
$f(x^2 + 2yz,\ y^2 + 2zx) = 0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that when $x = 0$, $z = 0$ and $\frac{\partial z}{\partial x} = a\sin y$. **(05 Marks)**

c. Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(05 Marks)**

Module-4

7 a. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx\,dy\,dx}{(1+x+y+z)^3}$. **(06 Marks)**

b. Evaluate integral $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration. **(05 Marks)**

c. Obtain the relation between Beta and Gamma function in the form $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$ **(05 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar co-ordinates. **(06 Marks)**

b. If A is the area of rectangular region bounded by the lines x = 0, x = 1, y = 0, y = 2 then evaluate $\int_A (x^2 + y^2) dA$. **(05 Marks)**

c. Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ using Beta and Gamma functions. **(05 Marks)**

Module-5

9 a. Find Laplace transition of i) $t^2 e^{2t}$ ii) $\frac{e^{-at} - e^{-bt}}{t}$. **(06 Marks)**

b. If a periodic function of period 2a is defined by $f(t) = \begin{cases} t & \text{if} \quad 0 \le t \le a \\ 2a - t & \text{if} \quad a \le t \le 2a \end{cases}$

Then show that $L\{f(t)\} = \frac{1}{s^2} \tan h\left(\frac{as}{2}\right)$. **(05 Marks)**

c. Solve $y''(t) + 4y'(t) + 4y(t) = e^t$ with $y(0) = 0$ $y'(0) = 0$. Using Laplace transform. **(05 Marks)**

OR

10 a. Find $L^{-1}\left[\frac{7s}{(4s^2 + 4s + 9)}\right]$ **(06 Marks)**

b. Find $L^{-1}\left[\frac{s}{(s-1)(s^2+4)}\right]$ using convolution theorem. **(05 Marks)**

c. Express the following function interms of Heaviside unit step function and hence its Laplace transistor $f(t) = \begin{cases} t^2, & 0 < t \le 2 \\ 4t, & t > 2 \end{cases}$ **(05 Marks)**

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USN | | | | | | | | | | **14MAT21**

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Solve : $y'' + 3y' + 2y = 1 - 2e^x + e^{2x}$. **(06 Marks)**

b. Solve : $y'' + 2y = x^2$. **(07 Marks)**

c. Solve : $y'' + y = \operatorname{cosec} x$ by method of variation of parameter. **(07 Marks)**

OR

2 a. Solve $(D^3 - 6D^2 + 11D - 6)y = 0$. **(06 Marks)**

b. Solve : $(D^2 - 1)y = \sin 2x$. **(07 Marks)**

c. Solve by the method of undetermined coefficient $(D^2 + D - 2)y = x$. **(07 Marks)**

Module-2

3 a. Solve $x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x^2 + 1$. **(06 Marks)**

b. Solve for P, given that $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$. **(07 Marks)**

c. Solve the equation (px – y) (x – py) = 2p. Reducing it into Clairauts form by taking a substitution $U = x^2$ and $V = y^2$. **(07 Marks)**

OR

4 a. Solve : $(1 + x)^2 y'' + (1 + x)y' + y = \sin 2[\log(1 + x)]$. **(06 Marks)**

b. Solve the system of equations $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dx} = x - y$. **(07 Marks)**

c. Find the general and singular solution for $\sin px \cos y = \cos px \sin y + p$. **(07 Marks)**

Module-3

5 a. Form partial differential equation by eliminating arbitrary function from $f(x^2 + y^2, z - xy) = 0$. **(06 Marks)**

b. Evaluate : $\int_0^a \int_0^b \int_0^c xyz \, dx\, dy\, dz$. **(07 Marks)**

c. Obtain the solution of one dimensional wave equation by variable separable method. **(07 Marks)**

OR

6 a. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$. **(06 Marks)**

b. Evaluate : $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy dx$ by changing the order of integration. **(07 Marks)**

c. Derive one dimensional heat equation in the form $u_t = c^2 u_{xx}$. **(07 Marks)**

Module-4

7 a. Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **(06 Marks)**

b. Obtain the relation a between Beta and Gamma function on the form $\beta(m,n) = \frac{\Gamma m . \Gamma n}{\Gamma m+n}$. **(07 Marks)**

c. Express the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. **(07 Marks)**

OR

8 a. Show that $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1} dx = 2^{p+q-1}\beta(p,q)$. **(06 Marks)**

b. Find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ **(07 Marks)**

c. Express divergence of $\vec{F}$ where $\vec{F} = x\hat{i} - y\hat{j} + z\hat{k}$ in spherical polar co-ordinates. **(07 Marks)**

Module-5

9 a. Find i) $L\{e^{2t} + 4t^3 + 3\cos 3t\}$ ii) $L\left\{\frac{\sin t}{t}\right\}$. **(06 Marks)**

b. Find the Laplace transfer of the triangular wave of period 2a given by $f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$. **(07 Marks)**

c. Solve y′ + y = t by using Laplace transformation, given y(0) = 0. **(07 Marks)**

OR

10 a. Find the inverse Laplace transforms of :

i) $\log\left(\frac{s+1}{s-1}\right)$ ii) $\frac{2s-4}{4s^2+25}$. **(06 Marks)**

b. Express $f(t) = \begin{cases} 0 & 0 < t < 1 \\ t-1 & 1 < t < 2 \\ 1 & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transformation. **(07 Marks)**

c. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$. **(07 Marks)**

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CBCS SCHEME

USN 2VD19CV401 **18MAT31**

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Laplace transform of:

(i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (ii) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ (iii) $t\cos at$. **(10 Marks)**

b. The square wave function f(t) with period 2a defined by $f(t)=\begin{cases}1 & 0\le t<a\\ -1 & a\le t<2a\end{cases}$. Show that $\left(\frac{1}{s}\right)\tanh\left(\frac{as}{2}\right)$. **(05 Marks)**

c. Employ Laplace transform to solve $\frac{d^2y}{dt^2}-\frac{dy}{dt}=0$, $y(0)=y_1(0)=3$. **(05 Marks)**

OR

2 a. Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $L^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\}$ **(10 Marks)**

b. Find the inverse Laplace transform of, $\frac{1}{s(s^2+1)}$ using convolution theorem. **(05 Marks)**

c. Express $f(t)=\begin{cases}2 & \text{if } 0<t<1\\ \frac{t^2}{2} & \text{if } 1<t<\frac{\pi}{2}\\ \cos t & t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find its Laplace transformation. **(05 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x)=\begin{cases}2 & -2<x<0\\ x & 0<x<2\end{cases}$. **(08 Marks)**

b. Find the half range cosine series of, $f(x)=(x+1)$ in the interval $0\le x\le 1$. **(06 Marks)**

c. Express $f(x)=x^2$ as a Fourier series of period 2π in the interval $0<x<2\pi$. **(06 Marks)**

OR

4 a. Compute the first two harmonics of the Fourier Series of f(x) given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

b. Find the half range size series of e^x in the interval $0 \le x \le 1$. **(06 Marks)**

c. Obtain the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ valid in the interval $(-\pi \quad \pi)$ **(06 Marks)**

Module-3

5 a. Find the Infinite Fourier transform of $e^{-|x|}$. **(07 Marks)**

b. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$. **(06 Marks)**

c. Solve $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$, given $u_0 = u_1 = 0$. **(07 Marks)**

OR

6 a. If $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$, find the infinite transform of f(x) and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

b. Obtain the Z-transform of $\cosh n\theta$ and $\sinh n\theta$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4^{th} degree terms and find the value of $y(1.1)$. **(07 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at x = 1.1 given $y(1) = 3$ (Take h = 0.1) **(06 Marks)**

c. Apply Milne's predictor-corrector formulae to compute y(0.4) given $\frac{dy}{dx} = 2e^x y$, with **(07 Marks)**

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

8 a. Given $\frac{dy}{dx} = x + \sin y$; y(0) = 1. Compute y(0.4) with h = 0.2 using Euler's modified method. **(07 Marks)**

b. Apply Runge-Kutta fourth order method, to find y(0.1) with h = 0.1 given $\frac{dy}{dx} + y + xy^2 = 0$; y(0) = 1. **(06 Marks)**

c. Using Adams-Bashforth method, find y(4.4) given $5x\left(\frac{dy}{dx}\right) + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

(07 Marks)

Module-5

9 a. Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct 4 decimal places, using initial conditions $y(0) = 1$, $y'(0) = 0$, $h = 0.2$. **(07 Marks)**

b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(06 Marks)**

c. Find the extramal of the functional, $\int_{x_1}^{x_2} y^2 + (y')^2 + 2ye^x dx$. **(07 Marks)**

OR

10 a. Apply Milne's predictor corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y′	1	1.2103	1.4427	1.6990

(07 Marks)

b. Find the extramal for the functional, $\int_0^{\frac{\pi}{2}} [y^2 - y'^2 - 2y\sin x]dx$; $y(0) = 0$; $y\left(\frac{\pi}{2}\right) = 1$. **(06 Marks)**

c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**

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USN | | | | | | | | | | |

17MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Fourier series expansion of $f(x) = x - x^2$ in $(-\pi, \pi)$, hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - - - -$. **(08 Marks)**

b. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 \le x \le 1$. **(06 Marks)**

c. Express y as a Fourier series upto first harmonics given :

x	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(06 Marks)

OR

2 a. Obtain the Fourier series for the function :

$$f(x) = \begin{cases} 1 + \frac{4x}{3} & \text{in } \frac{-3}{2} < x \le 0 \\ 1 - \frac{4x}{3} & \text{in } 0 \le x < \frac{3}{2} \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - -$. **(08 Marks)**

b. If $f(x) = \begin{cases} x & \text{in } 0 < x < \pi/2 \\ \pi - x & \text{in } \pi/2 < x < \pi \end{cases}$

Show that the half range sine series as

$$f(x) = \frac{4}{\pi}\left[\sin x - \frac{\sin 3x}{3^2} - \frac{\sin 5x}{5^2} - - - -\right].$$ **(06 Marks)**

c. Obtain the Fourier series upto first harmonics given :

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

(06 Marks)

Module-2

3 a. Find the complex Fourier transform of the function :

$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(08 Marks)**

b. Find the Fourier cosine transform of e^{-ax}. **(06 Marks)**

c. Solve by using z – transforms $u_{n+2} - 4u_n = 0$ given that $u_0 = 0$ and $u_1 = 2$. **(06 Marks)**

OR

4 a. Find the Fourier sine and Cosine transforms of :
$$f(x)=\begin{cases} x & 0<x<2 \\ 0 & \text{elsewhere} \end{cases}.$$ **(08 Marks)**

b. Find the Z – transform of : i) n^2 ii) ne^{-an}. **(06 Marks)**

c. Obtain the inverse Z – transform of $\dfrac{2z^2+3z}{(z+2)(z-4)}$. **(06 Marks)**

Module-3

5 a. Obtain the lines of regression and hence find the co-efficient of correlation for the data :

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(08 Marks)

b. Fit a parabola $y = ax^2 + bx + c$ in the least square sense for the data :

x	1	2	3	4	5
y	10	12	13	16	19

(06 Marks)

c. Find the root of the equation $xe^x - \cos x = 0$ by Regula – Falsi method correct to three decimal places in (0, 1). **(06 Marks)**

OR

6 a. If $8x - 10y + 66 = 0$ and $40x - 18y = 214$ are the two regression lines, find the mean of x's, mean of y's and the co-efficient of correlation. Find σ_y if $\sigma_x = 3$. **(08 Marks)**

b. Fit an exponential curve of the form $y = ae^{bx}$ by the method of least squares for the data :

No. of petals	5	6	7	8	9	10
No. of flowers	133	55	23	7	2	2

(06 Marks)

c. Using Newton–Raphson method, find the root that lies near x = 4.5 of the equation $\tan x = x$ correct to four decimal places. **(06 Marks)**

Module-4

7 a. From the following table find the number of students who have obtained marks :
i) less than 45 ii) between 40 and 45.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

(06 Marks)

b. Using Newton's divided difference formula construct an interpolating polynomial for the following data :

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

and hence find f(8). **(08 Marks)**

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. **(06 Marks)**

OR

8 a. In a table given below, the values of y are consecutive terms of a series of which 23.6 is the 6^{th} term. Find the first and tenth terms of the series by Newton's formulas.

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(08 Marks)

b. Fit an interpolating polynomial of the form x = f(y) for data and hence find x(5) given :

x	2	10	17
y	1	3	4

(06 Marks)

c. Use Simpson's $\frac{1}{3}^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking 6 sub-intervals. **(06 Marks)**

Module-5

9 a. Verify Green's theorem in the plane for $\phi_C(3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the closed curve bounded by $y = \sqrt{x}$ and $y = x^2$. **(08 Marks)**

b. Evaluate $\int_c xydx + xy^2dy$ by Stoke's theorem where C is the square in the x – y plane with vertices (1, 0)(–1, 0)(0, 1)(0, –1). **(06 Marks)**

c. Prove that Catenary is the curve which when rotated about a line generates a surface of minimum area. **(06 Marks)**

OR

10 a. If $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3 evaluate $\iint_s \vec{F}\cdot\hat{n}\,ds$. **(08 Marks)**

b. Derive Euler's equation in the standard form viz $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(06 Marks)**

c. Find the external of the functional $I = \int_0^{\pi/2}(y^2 - y'^2 - 2y\sin x)dx$ under the end conditions y(0) = y(π/2) = 0. **(06 Marks)**

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CBCS SCHEME

USN | | | | | | | | | | **15MAT31**

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – III

Time: 3 hrs.   Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the Fourier expansion of the function f(x) = x over the interval $(-\pi, \pi)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ **(08 Marks)**

b. The following table gives the variations of a periodic current A over a certain period T:

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. **(08 Marks)**

OR

2 a. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. **(06 Marks)**

b. Represent the function

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2 & \text{for } \pi/2 < x < \pi \end{cases}$$

in a half range Fourier sine series. **(05 Marks)**

c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

x°	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(05 Marks)

Module-2

3 a. Find the complex Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$ Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ show that $u_0 = 0 \quad u_1 = 0 \quad u_2 = 2 \quad u_3 = 11$. **(05 Marks)**

c. Obtain the Fourier cosine transform of the function

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0 & x > 4 \end{cases}$$ **(05 Marks)**

OR

4 a. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx \quad m > 0$. **(05 Marks)**

c. Solve by using Z-transforms $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$. **(05 Marks)**

Module-3

5 a. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at x = 6. **(06 Marks)**

x	1	2	3	4	5
y	10	12	13	16	19

b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(05 Marks)

c. Use Newton-Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$. Carryout the iterations upto four decimal places of accuracy. **(05 Marks)**

OR

6 a. Show that a real root of the equation $\tan x + \tanh x = 0$ lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. **(06 Marks)**

b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(05 Marks)

c. Fit a curve of the form $y = ae^{bx}$ for the data:

x	0	2	4
y	8.12	10	31.82

(05 Marks)

Module-4

7 a. From the following table find the number of students who have obtained:

i) Less than 45 marks

ii) Between 40 and 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

(06 Marks)

b. Construct the interpolating polgnomial for the data given below using Newton's general interpolation formula for divided differences and hence find y at x = 3.

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

(05 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule. Taking seven ordinates. Hence find $\log_e 2$. **(05 Marks)**

OR

8 a. Use Lagrange's interpolation formula to find f(4) given below. **(06 Marks)**

x	0	2	3	6
f(x)	-4	2	14	158

b. Use Simpson's 3/8th rule to evaluate $\int_1^4 e^{1/x}dx$. **(05 Marks)**

c. The area of a circle (A) corresponding to diameter (D) is given by

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(05 Marks)**

Module-5

9 a. Evaluate Green's theorem for $\oint_c (xy + y^2)\,dx + x^2dy$ where c is the closed curve of the region bounded by y = x and $y = x^2$. **(06 Marks)**

b. Find the extremal of the functional $\int_a^b (x^2 + y^2 + 2y^2 + 2xy)dx$. **(05 Marks)**

c. Varity Stoke's theorem for $\bar{F} = (2x - y)\,\bar{i} - yz^2\bar{j} - y^2z\,\bar{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ C is its boundary. **(05 Marks)**

OR

10 a. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial yl}\right) = 0$. **(06 Marks)**

b. If $\bar{F} = 2xy\hat{i} + y^2z\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3. Evaluate $\iint_S \bar{F}.\hat{n}\,ds$. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. **(05 Marks)**

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CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. From Taylor's series method, find y(0.1), considering upto fourth degree term if y(x) satisfying the equation $\frac{dy}{dx} = x - y^2$, y(0) = 1. **(06 Marks)**

b. Using Runge-Kutta method of fourth order $\frac{dy}{dx} + y = 2x$ at x = 1.1 given that y = 3 at x = 1 initially. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find y(0.4) correct upto four decimal places by using Milne's predictor-corrector formula. **(07 Marks)**

OR

2 a. Using modified Euler's method find y at x = 0.2 given $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with y(0) = 1 taking h = 0.1. **(06 Marks)**

b. Given $\frac{dy}{dx} + y + zy^2 = 0$ and y(0) = 1, y(0.1) = 0.9008, y(0.2) = 0.8066, y(0.3) = 0.722. Evaluate y(0.4) by Adams-Bashforth method. **(07 Marks)**

c. Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, y(0) = 1 taking h = 0.2. **(07 Marks)**

Module-2

3 a. Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

(06 Marks)

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ leading to $J_n(x)$. **(07 Marks)**

OR

4 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$ using fourth order Runge-Kutta method. **(06 Marks)**

b. Prove $J_{-1/2}(k) = \sqrt{\frac{2}{\pi x}} \cos x$. **(07 Marks)**

c. Prove the Rodfigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n y}{dx^n}(x^2 - 1)^n$ **(07 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Discuss the transformation $w = z^2$. **(07 Marks)**

c. By using Cauchy's residue theorem, evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)} dz$ if C is the circle $|z| = 3$. **(07 Marks)**

OR

6 a. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. **(07 Marks)**

Module-4

7 a. Find the mean and standard of Poisson distribution. **(06 Marks)**

b. In an examination 7% of students score less than 35 marks and 89% of the students score less than 60 marks. Find the mean and standard deviation if the marks are normally distributed given A(1.2263) = 0.39 and A(1.4757) = 0.43 **(07 Marks)**

c. The joint probability distribution table for two random variables X and Y is as follows:

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine:

i) Marginal distribution of X and Y

ii) Covariance of X and Y

iii) Correlation of X and Y **(07 Marks)**

OR

8 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find K and evaluate $P(x \geq 6)$, $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that

i) Exactly 2 are defective

ii) Atleast two are defective

iii) None of them are defective. **(07 Marks)**

c. The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made

i) Ends in less than 5 minutes

ii) Between 5 and 10 minutes. **(07 Marks)**

Module-5

9 a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the dia cannot be regarded as an unbiased die. **(06 Marks)**

b. A group of 10 boys fed on diet A and another group of 8 boys fed on a different disk B for a period of 6 months recorded the following increase in weight (lbs):

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly $t.05 = 2.12$ at 16df. **(07 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$ **(07 Marks)**

OR

10 a. Define the terms:

i) Null hypothesis

ii) Type-I and Type-II error

iii) Confidence limits **(06 Marks)**

b. The t.p.m. of a Markov chain is given by $P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$. Find the fined probabilities vector. **(07 Marks)**

c. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing ball from one to another. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run how often does each receive the ball? **(07 Marks)**

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CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – IV

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Time: 3 hrs. Max. Marks: 80

Note: 1. ***Answer FIVE full questions, choosing ONE full question from each module.***
2. ***Use of statistical table can be provided.***

Module-1

1 a. Using Taylor's series method find, y(0.1) given that $\frac{dy}{dx} = x - y^2$, y(0) = 1 by considering upto third degree terms. **(05 Marks)**

b. Apply Runge Kutta method of fourth order to find an approximate value of y when x = 0.5 given that $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(0.4) = 1$. Take h = 0.1. **(05 Marks)**

c. Evaluate y(0.4) by Milne's Predictor-Corrector method given that $\frac{dy}{dx} = \frac{y^2(1+x^2)}{2}$ and y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21. Apply the corrector formula twice. **(06 Marks)**

OR

2 a. Solve by Euler's modified method $\frac{dy}{dx} = \log_e(x+y)$; y(0) = 2 to find y(0.2) with h = 0.2. Carryout two modifications. **(05 Marks)**

b. Using Runge-Kutta method of fourth order find y(0.2) to four decimal places given that $\frac{dy}{dx} = 3x + \frac{y}{2}$; $y(0) = 1$. Take $h = 0.2$. **(05 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$; y(1) =1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) to four decimal places using Adam's-Bashforth predictor corrector method. Apply the corrector formula twice. **(06 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx}$ with y(0) = 1, $y'(0) = 0$. Evaluate y(0.2) using Runge Kutta method of fourth order. Take h = 0.2. **(05 Marks)**

b. With usual notation prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. Express $f(x) = 2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomial. **(06 Marks)**

OR

4 a. Apply Milnes predictor corrector method to compute y(0.4) given that $\frac{d^2y}{dx^2} = 6y - 3x\frac{dy}{dx}$ and the following values: **(05 Marks)**

x	0	0.1	0.2	0.3
y	1	1.03995	1.138036	1.29865
y′	0.1	0.6955	1.258	1.873

b. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. **(05 Marks)**

c. If α and β are the two roots of the equation $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equation in Cartesian form. **(05 Marks)**

b. Evaluate using Cauchy's residue theorem, $\int_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$ where C is the circle $|z| = 2$. **(05 Marks)**

c. Find the bilinear transformation which maps the points $-1, i, 1$ onto the points 1, i, −1 respectively. **(06 Marks)**

OR

6 a. Find the analytic function, $f(z) = u + iv$ if $v = r^2\cos 2\theta - r\cos\theta + 2$. **(05 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$ using Cauchy integral formula. **(05 Marks)**

c. Discuss the transformation $\omega = e^z$. **(06 Marks)**

Module-4

7 a. Find the constant C such that the function,

$$f(x) = \begin{cases} Cx^2 & \text{for } 0 < x < 3 \\ 0 & \text{Otherwise} \end{cases}$$ is a probability density function.

Also compute $P(1<X<2)$, $P(X \leq 1)$, $P(X>1)$. **(05 Marks)**

b. Out of 800 families with five childrens each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at most 2 girls, assume equal probabilities for boys and girls. **(05 Marks)**

c. Given the following joint distribution of the random variables X and Y.

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find (i) E(X) (ii) E(Y) (iii) E(XY) (iv) COV(X, Y) (v) ρ(X,Y) **(06 Marks)**

OR

8 a. Obtain the mean and standard deviation of Poisson distribution. **(05 Marks)**

b. In a test on electric bulbs it was found that the life time of bulbs of a particular brand was distributed normally with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for,
(i) More than 2100 hours (ii) Less than 1950 hours (iii) Between 1900 and 2100 hours.
Given that $\phi(1.67) = 0.4525$, $\phi(0.83) = 0.2967$ **(05 Marks)**

c. A fair coin is tossed thrice. The random variables X and Y are defined as follows:
X = 0 or 1 according as head or tail occurs on the first toss.
Y = number of heads
Determine (i) The distribution of X and Y (ii) Joint distribution of X and Y. **(06 Marks)**

Module-5

9 a. In a city A 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant. **(05 Marks)**

b. The nine items of a sample have the following values : 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ from the assumed mean 47.5. Apply student's t – distribution at 5% level of significance ($t_{0.05} = 2.31$ for 8 d.f) **(05 Marks)**

c. Find the unique fixed probability vector of the regular stochastic matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. **(06 Marks)**

OR

10 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40,650 kms with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000 kms (use 0.05 level of significance) Establish 99% confidence limits within which the mean life of tyres is expected to lie, (given $Z_{0.05} = 1.96$, $Z_{0.01} = 2.58$) **(05 Marks)**

b. In the experiments of pea breeding the following frequencies of seeds were obtained.

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9 : 3 : 3 : 1. Examine the correspondence between theory and experiment.
$(\chi^2_{0.05} = 7.815 \text{ for } 3 \text{ d.f})$ **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B is to A. If C was the first person to throw the ball find the probabilities that after the three throws.
(i) A has the ball (ii) B has the ball (iii) C has the ball. **(06 Marks)**

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express the following complex number in the form of $x+iy$: $\dfrac{(1+i)(1+3i)}{1+5i}$. **(06 Marks)**

b. Prove that $\left(\dfrac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4=\cos 8\theta+i\sin 8\theta$. **(07 Marks)**

c. If $\vec{a}=(3,-1,4)$, $\vec{b}=(1,2,3)$ and $\vec{c}=(4,2,-1)$, find $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$. **(07 Marks)**

OR

2 a. Find the angle between the vectors, $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(06 Marks)**

b. Prove that $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$ **(07 Marks)**

c. Find the fourth roots of $-1+i\sqrt{3}$ and represent them on the argand diagram. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's expansion of $\log_e(1+x)$. **(06 Marks)**

b. If $u=\sin^{-1}\left[\dfrac{x^3+y^3}{x+y}\right]$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2\tan u$. **(07 Marks)**

c. If $u=x(1-y)$, $v=xy$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

OR

4 a. Obtain the Maclauvin's series expansion of the function $\log_e \sec x$. **(06 Marks)**

b. If $u=x^2-2y$; $v=x+y$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

c. If $u=f(x-y,y-z,z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(07 Marks)**

Module-3

5 a. Find the velocity and acceleration of a particle moves along the curve, $\vec{r}=e^{-2t}\hat{i}+2\cos 5t\hat{j}+5\sin 2t\hat{k}$ at any time t. **(06 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F}=\nabla\left(x^3+y^3+z^3-3xyz\right)$. **(07 Marks)**

c. Show that $\vec{F}=(2xy+z^2)\hat{i}+\left(x^2+2yz\right)\hat{j}+\left(y^2+2xz\right)\hat{k}$ is conservative force field and find the scalar potential. **(07 Marks)**

OR

6 a. Show that the vector field, $\vec{F} = (3x+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}$ is solenoidal. **(06 Marks)**

b. Find the directional derivative of $\phi = \dfrac{xz}{x^2+y^2}$ at $(1, -1, 1)$ in the direction of $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$. **(07 Marks)**

c. Find the constant 'a' such that the vector field $\vec{F} = 2xy^2z^2\hat{i} + 2x^2yz^2\hat{j} + ax^2y^2z\hat{k}$ is irrotational. **(07 Marks)**

Module-4

7 a. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^3 x^3y^3 dxdy$. **(07 Marks)**

c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z) dzdxdy$. **(07 Marks)**

OR

8 a. Evaluate : $\int_0^{\frac{\pi}{6}} \sin^6(3x) dx$. **(06 Marks)**

b. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} xy \, dydx$. **(07 Marks)**

c. Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz dzdydx$. **(07 Marks)**

Module-5

9 a. Solve : $\dfrac{dy}{dx} + y\cot x = \sin x$. **(06 Marks)**

b. Solve : $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. **(07 Marks)**

c. Solve : $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. **(07 Marks)**

OR

10 a. Solve : $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. **(06 Marks)**

b. Solve : $\dfrac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(07 Marks)**

c. Solve : $[1 + (x+y)\tan y]\dfrac{dy}{dx} + 1 = 0$. **(07 Marks)**

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CBCS SCHEME

USN 2VD18ME425

17MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of $\frac{3+i}{2+i}$ **(07 Marks)**

b. If $x = \cos\theta + i\sin\theta$, then show that $\frac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$. **(07 Marks)**

c. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$ **(06 Marks)**

OR

2 a. Find the sine of the angle between $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Find the value of λ, so that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{k}$ are coplanar. **(07 Marks)**

c. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$. **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$. **(07 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

OR

4 a. Find the pedal equation of $r^n = a^n \cos n\theta$. **(07 Marks)**

b. Expand $\log_e(1 + x)$ in ascending powers of x as for as the term containing x^4. **(07 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^1 \int_{y^2}^{y} (1 + xy^2)\,dx\,dy$ **(07 Marks)**

b. Evaluate $\int_0^{2\pi} \sin^4 x \cos^6 x \; dx$ **(07 Marks)**

c. Evaluate $\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\,dx$ **(06 Marks)**

OR

6 a. Evaluate $\int_1^2\int_3^4 (xy + e^y)\, dydx$ **(07 Marks)**

b. Evaluate $\int_0^{\pi} x \sin^8 x \, dx$ **(07 Marks)**

c. Evaluate $\int_1^2 \int_0^1 \int_{-1}^1 (x^2 + y^2 + z^2)\, dxdydz$ **(06 Marks)**

Module-4

7 a. If particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the velocity and acceleration at $t = 1$. **(07 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the point $t = \pm 1$. **(07 Marks)**

c. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$ find grad(div $\vec{F}$) at (2, –1, 0). **(06 Marks)**

OR

8 a. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) along $2\hat{i} - 3\hat{j} + 6\hat{k}$ **(07 Marks)**

b. Find the unit normal to the surface $x^2y + 2xz = 4$ at (2, –2, 3). **(07 Marks)**

c. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \sin(x + y)$ **(07 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$ **(07 Marks)**

c. Solve $(x - y + 1)dy - (x + y - 1)dx = 0$ **(06 Marks)**

OR

10 a. Solve $(1 + e^{x/4})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(07 Marks)**

b. Solve $(x^3\cos^2 y - x\sin 2y)\, dx = dy$. **(07 Marks)**

c. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ **(06 Marks)**

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CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find modulus and amplitude of $1-\cos\theta+i\sin\theta$. **(05 Marks)**

b. Express $\dfrac{3+4i}{3-4i}$ in $a+ib$ form. **(05 Marks)**

c. Find the value of 'λ' so that the points A(−1, 4, −3), B(3, 2, −5), C(−3, 8, −5) and D(−3, λ, 1), may lie on one plane. **(06 Marks)**

OR

2 a. Find the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(05 Marks)**

b. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$. **(05 Marks)**

c. Find the real part of $\dfrac{1}{1+\cos\theta+i\sin\theta}$. **(06 Marks)**

Module-2

3 a. Obtain the n^{th} derivative of $\sin(ax+b)$. **(05 Marks)**

b. Find the pedal equation of $r^n=a^n\cos n\theta$. **(05 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$, show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}=4$. **(06 Marks)**

OR

4 a. If $u=\log\left(\dfrac{x^4+y^4}{x+y}\right)$ show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$. **(05 Marks)**

b. If $u=f(x-y,\ y-z,\ z-x)$, show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(05 Marks)**

c. If $y=a\cos(\log x)+b\sin(\log x)$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$ **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi}x\sin^8 x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1 x^2(1-x^2)^{3/2}dx$. **(05 Marks)**

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

c. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}\,dx$. **(06 Marks)**

Module-4

7 a. If $\vec{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}$, find the angle between the tangents at t = 1 and t = 2. **(05 Marks)**

b. If $\vec{r} = e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$, find the velocity and acceleration at any time t, and also their magnitudes at t = 0. **(05 Marks)**

c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function 'φ' such that $\vec{F} = \nabla\phi$. **(06 Marks)**

OR

8 a. Find the unit normal vector to the surface $x^2y + 2xz = 4$ at (2, −2, 3). **(05 Marks)**

b. If $\vec{F} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$ at (1, −1, 1). **(05 Marks)**

c. If $\frac{d\vec{a}}{dt} = \vec{w}\times\vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w}\times\vec{b}$, then show that $\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{w}\times(\vec{a}\times\vec{b})$ **(06 Marks)**

Module-5

9 a. Solve $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$. **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. **(05 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(06 Marks)**

OR

10 a. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(05 Marks)**

b. Solve $x^2y\,dx - (x^3 + y^3)dy = 0$ **(05 Marks)**

c. Solve $y(x+y)dx + (x+2y-1)dy = 0$ **(06 Marks)**

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CBCS SCHEME

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17MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix:
$$A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$ by elementary row transformations. **(08 Marks)**

b. Solve by Gauss elimination method
$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20$$
(06 Marks)

c. Find all the eigen values for the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **(06 Marks)**

OR

2 a. Reduce the matrix
$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$ into its echelon form and hence find its rank. **(06 Marks)**

b. Applying Gauss elimination method, solve the system of equations
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$
$$x + y + z = 9$$
(06 Marks)

c. Find all the eigen values for the matrix $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ **(08 Marks)**

Module-2

3 a. Solve $\frac{d^4y}{dx^4} - \frac{2d^3y}{dx^3} + \frac{d^2y}{dx^2} = 0$ **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 5e^{-2x}$ **(06 Marks)**

c. Solve $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters. **(08 Marks)**

OR

4 a. Solve $\frac{d^3y}{dx^3} + y = 0$ **(06 Marks)**

b. Solve $y'' + 3y' + 2y = 12x^2$ **(06 Marks)**

c. Solve by the method of undetermined coefficients :

$$y'' - 4y' + 4y = e^x$$ (08 Marks)

Module-3

5 a. Find the Laplace transforms of $\sin 5t \cos 2t$ (06 Marks)

b. Find the Laplace transforms of $(3t+4)^3$ (06 Marks)

c. Express $f(t) = \begin{cases} \sin 2t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$,

in terms of unit step function and hence find $L[f(t)]$. (08 Marks)

OR

6 a. Find the Laplace transforms of $\dfrac{\sin^2 t}{t}$ (06 Marks)

b. Find the Laplace transform of $2^t + t \sin t$ (06 Marks)

c. If $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$, for $t > 2$, find $L[f(t)]$. (08 Marks)

Module-4

7 a. Find the Laplace Inverse of

$$\frac{1}{(s+1)(s-1)(s+2)}$$ (08 Marks)

b. Find the inverse Laplace transform of $\dfrac{3s+7}{s^2-2s-3}$. (06 Marks)

c. Solve $y'' + 2y' - 3y = \sin t$, $y(0) = 0$, $y'(0) = 0$. (06 Marks)

OR

8 a. Find the inverse Laplace transform of

$$\log\left(\frac{s+a}{s+b}\right)$$ (06 Marks)

b. Find the inverse Laplace transform of $\dfrac{4s-1}{s^2+25}$ (06 Marks)

c. Find the inverse Laplace of $y'' - 5y' + 6y = e^t$ with $y(0) = y'(0) = 0$. (08 Marks)

Module-5

9 a. State and prove Addition theorem on probability. (05 Marks)

b. A student A can solve 75% of the problems given in the book and a student B can solve 70%. What is the probability that A or B can solve a problem chosen at random. (06 Marks)

c. Three machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random, what is the probability that it is defective? If a selected item is defective, what is the probability that it is from machine A? (09 Marks)

OR

10 a. Find the probability that the birth days of 5 persons chosen at random will fall in 12 different calendar months. (05 Marks)

b. A box A contains 2 white balls and 4 black balls. Another box B contains 5 white balls and 7 black balls. A ball is transferred from box A to box B. Then a ball is drawn from box B. Find the probability that it is white. (06 Marks)

c. State and prove Baye's theorem. (09 Marks)

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CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix by
$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by applying elementary row transformations. **(06 Marks)**

b. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Caylery-Hamilton theorem. **(05 Marks)**

c. Solve the following system of equations by Gauss elimination method.
$2x + y + 4z = 12, \quad 4x + 11 - z = 33, \quad 8x - 3y + 2z = 20$ **(05 Marks)**

OR

2 a. Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to echelon form. **(06 Marks)**

b. Find the eigen values of $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ **(05 Marks)**

c. Solve by Gauss elimination method: $x + y + z = 9, \quad x - 2y + 3z = 8, \quad 2x + y - z = 3$ **(05 Marks)**

Module-2

3 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$ **(05 Marks)**

b. Solve $y'' - 4y' + 13y = \cos 2x$ **(05 Marks)**

c. Solve by the method of undetermined coefficients $y'' + 3y' + 2y = 12x^2$ **(06 Marks)**

OR

4 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$ **(05 Marks)**

b. Solve $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$ **(05 Marks)**

c Solve by the method of variation of parameter $\frac{d^2y}{dx^2} + y = \tan x$ **(06 Marks)**

Module-3

5 a. Find the Laplace transform of
i) $e^{-2t}\sin h\,4t$ ii) $e^{-2t}(2\cos 5t - \sin 5t)$ **(06 Marks)**

b. Find the Laplace transform of $f(t) = t^2 \quad 0 < t < 2$ and $f(t + 2) = f(t)$ for $t > 2$. **(05 Marks)**

c. Express $f(t)=\begin{cases} t & 0<t<4 \\ 5 & t>4 \end{cases}$ interms of unit step function and hence find L[f(t)]. **(05 Marks)**

OR

6 a. Find the Laplace transform of i) t cosat ii) $\dfrac{\cos at - \cos bt}{t}$ **(06 Marks)**

b. Given $f(t)=\begin{cases} E & 0<t<a/2 \\ -E & a/2<t<a \end{cases}$ where f(t + a) = f(t). Show that $L[f(t)] = \dfrac{E}{S}\tan h\left(\dfrac{as}{4}\right)$. **(05 Marks)**

c. Express $f(t)=\begin{cases} 1 & 0<t<1 \\ t & 1<t\le 2 \\ t^2 & t>2 \end{cases}$ interms of unit step function and hence find L[f(t)]. **(05 Marks)**

Module-4

7 a. Find the inverse Laplace transform of i) $\dfrac{2s-1}{s^2+4s+29}$ ii) $\dfrac{s+2}{s^2+36}+\dfrac{4s-1}{s^2+25}$ **(06 Marks)**

b. Find the inverse Laplace transform of $\log\sqrt{\dfrac{s^2+1}{s^2+4}}$ **(05 Marks)**

c. Solve by using Laplace transforms $y''+4y'+4y=e^{-t}$, given that y(0) = 0, y′(0) = 0. **(05 Marks)**

OR

8 a. Find the inverse Laplace transform of $\dfrac{1}{(s+1)(s+2)(s+3)}$. **(06 Marks)**

b. Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{s+a}{b}\right)$. **(05 Marks)**

c. Using Laplace transforms solve the differential equation $y'''+2y''-y'-2y=0$ given y(0) = y′(0) = 0 and y″(0) = 6. **(05 Marks)**

Module-5

9 a. State and prove Baye's theorem. **(06 Marks)**

b. The machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine "C". **(05 Marks)**

c. The probability that a team wins a match is 3/5. If this team play 3 matches in a tournament, what is the probability that i) win all the matches ii) lose all the matches. **(05 Marks)**

OR

10 a. If A and B are any two events of S, which are not mutually exclusive then $P(A\cup B) = P(A) + P(B) - P(A\cap B)$. **(06 Marks)**

b. If A and B are events with $P(A\cup B) = 7/8$, $P(A\cap B) = 1/4$, $P(\overline{A}) = 5/8$. Find P(A), P(B) and $P(A\cap \overline{B})$. **(05 Marks)**

c. The probability that a person A solves the problem is 1/3, that of B is 1/2 and that of C is 3/5. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved? **(05 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express $\frac{(2+3i)^2}{(1+i)^2}$ in the form of complex number a + ib. **(06 Marks)**

b. Prove that $(1+i)^4 + (1-i)^4 = -8$. **(07 Marks)**

c. Find the cube root of $(\sqrt{3} - i)$. **(07 Marks)**

2 a. Find n^{th} derivative of sin(ax + b). **(06 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$. Show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. Find n^{th} derivative of $\log_e\left(\frac{2x+3}{2-3x}\right)^{\frac{1}{10}}$. **(07 Marks)**

3 a. Find the angle between the curves. $r = a(\sin\theta + \cos\theta)$ and $r = 2a\cos\theta$. **(06 Marks)**

b. Find the pedal equation for the curve $r^2 = a^2\sec(2\theta)$. **(07 Marks)**

c. Expand y = Log (cos x) using Maclaurin's series upto 4^{th} degree term. **(07 Marks)**

4 a. If $u = \sin^{-1}\left[\frac{x^3+y^3+z^3}{ax+by+cz}\right]$ show that $xu_x + yu_y + zu_z = 2\tan u$. **(06 Marks)**

b. If u = f(r, s, t) where $r = \frac{x}{y}$ $s = \frac{y}{z}, t = \frac{z}{x}$. Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. **(07 Marks)**

c. If $u = x + y + z$, $v = y + z$, $w = z$ find $\frac{\partial(uvw)}{\partial(xyz)}$. **(07 Marks)**

5 a. Obtain reduction formula for $\int \sin^n x \, dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate : $\int_0^1 x^9\sqrt{1-x^2}\, dx$. **(07 Marks)**

c. Evaluate : $\int_0^1\int_1^2 (x^2+y^2)dxdy$. **(07 Marks)**

6 a. Evaluate : $\int_0^1\int_0^2\int_1^2 x^2yz \, dx\, dy\, dz$. **(06 Marks)**

b. Prove that $\beta(m,n) = \beta(n,m)$. **(07 Marks)**

c. Evaluate : $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$. **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = e^{-y}(e^x + x^2)$. **(06 Marks)**

b. Solve $(x^2 + y^2)dx = 2xy\,dy$. **(07 Marks)**

c. Solve $\frac{dx}{dy} = \frac{x}{y} + 2y^2$. **(07 Marks)**

8 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$. **(07 Marks)**

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USN

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If $[l_1, m_1, n_1]$ and $[l_2, m_2, n_2]$ be the direction cosines of two lines subtending an angle θ between them then prove that $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. **(06 Marks)**

b. Find the angle between two lines whose direction cosines satisfy the relations $l + m + n = 0$ and $2lm + 2nl - mn = 0$ **(07 Marks)**

c. Find the co-ordinates of the foot of the perpendicular from A(1,1,1) to the line joining B(1,4,6) and C(5,4,4). **(07 Marks)**

2 a. Find the equation of the plane which bisects the line joining (3, 0, 5) and (1, 2, −1) at right angles. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, −1, −1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the shortest distance and the equations of the line of shortest distance between the lines: $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{b} = 5\hat{i} + 6\hat{j} + 4\hat{k}$ and $\vec{c} = 6\hat{i} + 4\hat{j} + 5\hat{k}$ form an isosceles triangle. **(06 Marks)**

b. Prove that the points with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $\hat{j} + \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar. **(07 Marks)**

c. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$ where t is the time t. Find the components of velocity and acceleration in the direction of the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 1$. **(07 Marks)**

4 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 - z^2 = 3$ at $(2, -1, 2)$. **(06 Marks)**

b. Find the directional derivatives of the function $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2\hat{i} - \hat{j} - 2\hat{k}$ **(07 Marks)**

c. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at the point $(1, -1, 1)$ where $\vec{F} = \nabla(xy^3z^2)$. **(07 Marks)**

5 a. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ then prove that,

(i) $\nabla(r^n) = nr^{n-2}\,\vec{r}$ (ii) $\nabla.\left(r^n.\vec{r}\right) = (n+3)r^n$ **(06 Marks)**

b. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Find the value of the constant 'a' such that $\vec{A} = y(ax^2 + z)\hat{i} + x(y^2 - z^2)\hat{j} + 2xy(z - xy)\hat{k}$ is Solenoidal. For this value of 'a' show that $\text{curl}\,\vec{A}$ is also solenoidal. **(07 Marks)**

6 a. Find the Laplace transform of, (i) $\sin 5t \cos 2t$ (ii) $(3t+2)^2$ **(06 Marks)**

b. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. **(07 Marks)**

c. Find the Laplace transform of $t^2 \sin at$. **(07 Marks)**

7 a. Find the inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$. **(06 Marks)**

b. Find $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$. **(07 Marks)**

c. Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$. **(07 Marks)**

8 a. Using convolution theorem find the Laplace transform of $\frac{1}{s(s^2+a^2)}$. **(10 Marks)**

b. Solve the differential equation, $y''+5y'+6y=5e^{2x}$ under the condition $y(0) = 2$, $y'(0)=1$ using Laplace transform. **(10 Marks)**

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