FORMULA BOOK

KLS's VDIT -
Where Technology Meets Nature

KLS
VDIT

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VDIT : In the forefront of technological upgradation

Vast opportunities through

- IIT-Bombay Spoken Tutorial Project.
- NPTEL, SWAYAM Local Chapter- Govt. of India.
- HAL Management Academy, Bengaluru.
- IIRS (Indian Institute of Remote Sensing), Dehradoon- Outreach program.

Delivering the best to students

- Outcome-based education to ensure emergence of eminent engineers.

Idea to product realization

- MoU with Deshpande Startups- one of the biggest Startup incubators in India.
- The idea to product realization through New Age Incubation Network (NAIN).

Technical skills up-gradation

- World-class Coursera certifications through FUEL (Friends Union for Energizing Lives) NGO.
- E YANTRA – State of the art robotics lab across India, sponsored by IIT, Bombay & MHRD, New Delhi and funded by KLS VDIT

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Fundamental constants:

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	$9.81 ms^{-2}$
Gravitational constant	G	6.67×10^{-11} Nm^2 kg^{-2}
Avogadro's constant	N_A	6.02×10^{23} mol^{-1}
Gas constant	R	$8.31 JK^{-1}$ mol^{-1}
Boltzmann's constant	k_B	1.38×10^{-23} JK^{-1}
Stefan–Boltzmann constant	σ	$5.67 x 10^{-8}$ W m^{-2} K^{-4}
Coulomb constant	k	8.99×10^{9} Nm^2 C^{-2}
Permittivity of free space	ε_0	$8.85 x 10^{-12}$ C^2 N^{-1} m^{-2}
Permeability of free space	μ_0	$4\pi\times10^{-7}$ TmA^{-1}
Speed of light in vacuum	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} Js
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.110×10^{-31} kg = 0.000549u = $0.511 MeV$ c^{-2}
Proton rest mass	m_p	1.673×10^{-27} kg = 1.007276u = $938 MeV$ c^{-2}
Neutron rest mass	m_n	1.675×10^{-27} kg = 1.008665u = $940 MeV$ c^{-2}
Unified atomic mass unit	u	1.661×10^{-27} kg = $931.5 MeV$ c^{-2}
Solar constant	S	1.36×10^{3} W m^{-2}
Fermi radius	R_0	1.20×10^{-15} m

Chapter 1. Physics World

Science means organized knowledge. It is human nature to observe things and happenings around in the nature and then to relate them. This knowledge is organized so that it becomes well connected and logical. Then it is known as Science. It is a systematic attempt to understand natural phenomenon and use this knowledge to predict, modify and control phenomena.

Fundamental Forces in Nature : There is a large number of forces experienced or applied. These may be macroscopic forces like gravitation, friction, contact forces and microscopic forces like electromagnetic and inter-atomic forces. But all these forces arise from some basic forces called Fundamental Forces.

There are four Fundamental Forces in Nature.

1. Gravitational force:

- It is due to Mass of the two bodies.
- It is always attractive.
- It operates in all objects of universe.
- Its range is infinite. It's a weak force.

2. Electromagnetic Forces:

- It's due to stationery or moving Electrical charge.
- It may be attractive or repulsive.
- It operates on charged particles.
- Its range is infinite.
- Its stronger, 10^{36} times than gravitational force.

3. Strong nuclear force:

- Operate between Nucleons.
- It may be attractive or repulsive.
- Its range is very short, within nuclear size (10^{-15} m).
- Its strongest force in nature.

4. Weak Nuclear force:

- Operate within nucleons i.e. elementary particles like electron and neutrino.
- It appears during radioactive β decay.
- Has very short range 10^{-15}m.
- 10^{-13} times than Strong nuclear force.

Chapter 2. Units and Measurements

- **Units :** It is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.
- **Fundamental Units :** The physical quantities which are independent of each other and which can represent remaining physical quantities are called fundamental physical quantities and their units are called fundamental units. They are seven in number as mentioned below :

  Seven Fundamental physical quantities in SI system of units are :

(a) Mass - *kg* (Kilogram)
(b) Length - *m* (Meter)
(c) Time - *s* (Second)
(d) Temperature - K (Kelvin)
(e) Electric current - A (Ampere)
(f) Luminous Intensity - cd (Candela)
(g) Quantity of Matter - *mol* (Mole)

(ii) Derived Units : These are the units of measurement of all other physical quantities which can be obtained from fundamental units, e.g. Velocity - (*m/s*), Acceleration - m/s^2, Pressure - Pa, Force - N and so on.

Know the Formulae

- 1 AU = 1.496×10^{11} m.
- 1 ly = 9.46×10^{15} m.
- 1 par sec = 3.1×10^{16} m.
- 1 Å = 10^{-10} m; 1 nm = 10^{-9} m
  1 µm = 10^{-6} m, 1 mm = 10^{-3} m
- 60 seconds (of arc) = 1 min (arc)
- 60 min. (of arc) = 1 degree (of arc)
- 180 degree (of arc) = π radian
- Indirect methods for long and small distances :
  Angular diameter (θ) = $\frac{d/D}{}$

  d = diameter, D = distance, radius = r

Know the Terms

- **Dimensions of physical quantity** are the powers to which the symbols of fundamental quantities are raised to represent a derived unit of that quantity.

Dimensional formula of the given physical quantity is the expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity.

- **Dimensional constants** are the quantities whose values are constant and they possess dimensions *e.g*. universal gravitational constant G etc.
- **Dimensional variables** are the quantities whose values are variable and they possess dimensions *e.g*. area, volume, etc.
- **Dimensional less constants** are the quantities whose value are constant but they do not possess dimensions *e.g*. mathematical constants– π, e and numbers.
- **Dimensionless variables** are the quantities whose values are variable and they do not have dimensions *e.g.* angle, strain, etc.
- **Accuracy** is a measure of how close the measured value is to true value of quantity.
- **Precision** describes the limitation of a measuring instrument.

Dimensional formula of some of the Physical quantity:

Physical quantity	**Unit**	**Dimensional formula**
Acceleration	ms^{-2}	$[LT^{-2}]$
Density	Kgm^{-3}	$[ML^{-3}]$
Force	Newton (N)	$[MLT^{-2}]$
Work	Joule (J)(=N-m)	$[ML^2T^{-2}]$
Energy	Joule (J)(=N-m)	$[ML^2T^{-2}]$
Power	Watt (W) (=Js^{-1})	$[ML^2T^{-3}]$
Momentum	kg-ms^{-1}	$[MLT^{-1}]$
Angular momentum	kg-m^2s^{-1}	$[ML^2T^{-1}]$
Gravitational constant	N-m^2kg^{-2}	$[M^{-1}L^3T^{-2}]$
Moment of inertia	kg-m^2	$[ML^2]$
Torque	N-m	$[ML^2T^{-2}]$
Surface Tension	Nm^{-1}	$[MT^{-2}]$
Thermal conductivity	$Wm^{-1}K^{-1}$	$[MLT^{-3}K^{-1}]$

Chapter 3: Motion in Straight line

- Path length or distance, D = Speed × Time
- Displacement = Velocity × Time
- Speed $= \frac{\text{Distance}}{\text{Time}}$
- Velocity $= \frac{\text{Displacement}}{\text{Time}}$
- Average acceleration $= \frac{\text{Total change in velocity}}{\text{Total time taken}}$
- Suppose
  u = initial velocity of body,
  a = uniform acceleration of the body,
  v = velocity of the body after time t,
  s = distance travelled by body in time t,
  s_n = distance travelled by body in n^{th} second.

  The equation of motion for accelerated body are:

$v = u + at$ $\qquad$ $S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$ $\qquad$ $S_n = u + \frac{a}{2}(2n - 1)$

Vertical motion under gravity :

$S = ut \pm \frac{1}{2}gt^2$ $\qquad$ $v^2 = u^2 \pm 2gs$

$v = u \pm gt$ $\qquad$ $S_n = u \pm \frac{g}{2}(2n - 1)$

Chapter 4. Motion in a Plane

Angular displacement of the object moving around a circular path is defined as the angle traced out by radius vector at the centre of circular path in given time. It is vector quantity.

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

Uniform circular motion is the motion when a point object is moving on a circular path with a constant speed.

- Angle made by vertical component of u:

$$\theta = \tan\theta - \frac{x^2 \tan\theta}{R} \ or \ \tan^{-1}\left(\frac{u\ \sin\theta - gt}{u\cos\theta}\right)$$

Where u-initial velocity R-horizontal range g-gravitational acceleration t-time and x-X axis.

- Maximum height = $\frac{u^2 \sin^2\theta}{2g}$
- Time of flight= $2t$ or $\frac{2u\ \sin\theta}{2g}$
- Time taken to reach maximum height $t = \frac{u\ \sin\theta}{g}$
- Distance covered along x axis= $ucos\theta$ x t
- Maximum range =$\frac{u^2}{g}$
- Velocity at any instant (t), $v = \sqrt{(u\sin\theta - gt)^2 + (u\cos\theta)^2}$

Chapter 5. Newton's law of Motion

- Newton's second law$F = ma$
- Centripetal acceleration $F = \frac{mv^2}{R}$
- Angle of banking $\boldsymbol{v} = \sqrt{Rg\tan\theta}$
- Magnitude of resultant of 2 forces acting at point $F^2 = {F_1}^2 + F_2^2 + 2F_1F_2\cos\theta$ where θ angle between $F_1$and F_2
- Direction of resultant $\tan\alpha = \frac{F_2 sin\theta}{F_1 + F_2 cos\theta}$ where α angle between $F_1$and F

Chapter 6: Work Power and Energy

- Work done by a body $W = Fscos\theta$
- Power $P = \frac{w}{t}$
- Gravitational potential energy $PE = mgh$
- Kinetic energy $KE = \frac{1}{2}mv^2$
- For freely falling body $mgh = \frac{1}{2}mv^2$ or $v^2 = 2gh$

Chapter 7: System of Particles and Rotational motion

- Center of mass for system of two particles along X-axis $X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$
- Center of mass for system of n particles along X-axis $X = \frac{m_1x_1 + m_2x_2 \ldots\ldots\ldots\ldots}{m_1 + m_2 \ldots\ldots\ldots} = \frac{\Sigma m_i x_i}{m_i}$
- Relation between linear and angular quantities $s = r\theta, v = r\omega, a = r\alpha$

- Kinematics of rotational motion $\omega = \omega_0 + \alpha t,\ \theta = \omega_0 + \frac{1}{2}\alpha t^2,\ \omega^2 = {\omega_0}^2 + 2\alpha\theta$
- Vector product $\vec{A} \times \vec{B} = ABsin\theta\hat{n}$
- Torque $\tau = Frsin\theta$
- Angular momentum $l = rpsin\theta$
- Moment of inertia $I = MR^2$
- Conservation of angular momentum $I\omega = constant$

Chapter 8: Gravitation

- Gravitational force $F = G\frac{m_1 m_2}{d^2}$
- Relation between g and G $g = \frac{GM}{R^2}$
- variation of acceleration due to gravity with respect to altitude $g_h = g\left(1 - \frac{2h}{R}\right)$
- variation of acceleration due to gravity with respect to depth $g_d = g\left(1 - \frac{d}{R}\right)$
- Orbital velocity $v_o = \sqrt{\frac{GM}{(R+h)}}$
- Escape velocity $v_e = \sqrt{2gR}$
- Kepler's third law $\frac{T^2}{R^3} = constant$
- Gravitational potential of a body in earth's gravitational field $V = G\frac{Mm}{R^2}$

Chapter 9. Mechanical Properties of Solids.

- Normal stress (S) = F/A, $A = \pi r^2$
- Breaking force = Breaking stress × area of cross-section
- Longitudinal Strain= Δl/L
- Volumetric Strain= ΔV/V
- Shearing Strain,θ= Δl/L
- $Young's\ modulus\ of\ elasticity, Y = \frac{Normal\ Stress}{Longitudinal\ Strain}$
- $Bulk\ modulus\ of\ elasticity, K = \frac{Normal\ Stress}{Volumetric\ Strain}$
- $Modulus\ of\ Rigidity, \eta = \frac{Tangential\ Stress}{tangential\ Strain}$
- $Poission's\ ratio, \sigma = \frac{Lateral\ Strain(\beta)}{Longitudinal\ Strain(\alpha)}$

  Where β=dD/D

  And α=dl/L
- Relation between Y and α; Y=$\frac{1}{\alpha}$

- Relation between η,α and β; $\eta = \frac{1}{2(\alpha+\beta)}$
- Relation between Y,K and σ; Y=3K(1-2σ)
- Relation between Y,η and σ; Y=2η(1+σ)

Chapter 10. Mechanical Properties of Fluids.

- Relative density=$\frac{density\ of\ a\ substance}{density\ of\ water\ at\ 4^0C}$
- Surface tension S=F/l
- Surface energy, E=Work done
- Excess of pressure inside the liquid drop is P=$P_i - P_0 = \frac{2S}{r}; P_i =$ $Pressure\ inside\ the\ bubble$
- Ecess of pressure inside the soap bubble is P=$P_i - P_0 = \frac{4S}{r}; P_0 =$ $Pressure\ outside\ the\ bubble$
- Total pressure in the air bubble at a depth '*h'* below the surface of a liquid of density *ρ* is

$$P = P_o + h\rho g + \frac{2S}{r}$$

- Newton's viscous drag force; F=$\pm\eta A\frac{dv}{dx}$

  Where *η*= coefficiect of viscocity A=Area of layer of liquid
  dv/dx= velocity gradient.
- POiseuille's theorem; V=$\frac{\pi pr^4}{8\eta l}$

  p=pressure difference across length *l* of horizontal tube of radius *r*
- Stoke's law F=*6πηrv*
- Terminal velocity: V=$\frac{2r^2(\rho-\sigma)g}{9\eta}$

  Where *ρ*=density of spherical body
  σ= density of medium.
  r=radius of spherical body
  Ƞ=coefficient of viscocity.
- Bernoulli's theorem:

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = constant\ \ or\ \ \frac{P}{\rho g} + h + \frac{v^2}{2g} = constant$$

Chapter 11. Thermal Properties of Matter

- Specific heat, ΔQ=msΔT
- Molar specific heat C=m x s
- Latent heat, ΔQ=mL
- Specific heat of gases C_p-C_v=R

- Rate of reduction of heat, $\frac{\Delta Q}{\Delta T} = KA\frac{\Delta T}{\Delta x}$
  Where $\frac{\Delta T}{\Delta x} = temperature\ gradient$
- Thermal resistance , R_{th}=$\frac{T_1-T_2}{dQ/dt}$
- Emissive power e=$\int_0^\infty e_\lambda\, d\lambda$

Chapter 12. Thermodynamics

S. No.	Scale	Ice point	Steam point	No. of divisions	Smallest division
1.	**Centigrade scale**	**0°C**	**100°C**	**100**	**1°C**
2.	**Fahrenheit scale**	**32°F**	**212°F**	**180**	**1°F**
3.	**Reaumer scale**	**0°R**	**80°R**	**80**	**1°R**
4.	**Thermodynamical scale of Absolute Kelvin scale**	**273 K**	**373 K**	**100**	**1 K**

- Work done:
  In cyclic process $dQ=dW$
  In non cyclic process $dQ\neq dW$
- Change in Entropy $\Delta S = \frac{\Delta Q}{T} = \frac{Heat\ absrbed}{Absolute\ Temperature}$
- Efficiency of Heat Engine $\eta = \frac{W}{Q_1} = \frac{Q_1-Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$
  Where W=useful work done/cycle by the engine.
  Q_1=amount of heat energy absorbed/cycle from the source.
  Q_2=amount of heat rejected/cycle to the sink.
  If T_1 is the temperature of the source and T_2 is the temperature of the sink then,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Chapter 13. Kinetic Theory of Gases

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- **Assumptions of Kinetic Theory of Gases :**
  (a) A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for

different gases.

(b) The molecules of a gas are in a state of continuous, rapid and random motion.

(c) The volume occupied by the molecules is negligible in comparison to the volume of the gas.

(d) The molecules do not exert any force of attraction or repulsion on each other, except during collision.

- Boyle's Law : PV = constant
- Charles Law: V/T=constant
- Standard gas equation: PV=nRT

  where *n* is the number of moles contained in the given ideal gas of volume V, pressure P and temperature T.
- **Gas constant :**

  (i) R is a universal gas constant and *r* is a gas constant for 1 gram of a gas.

  (ii) The universal gas constant is defined as the work done by (or on) a gas per mole per Kelvin *i.e.*

$$R = \frac{PV}{nT}$$

  (iii) The value of R for every gas at S.T.P. = $8{\cdot}31$ J $mole^{-1}$ K^{-1} = 1.98 cal. mol^{-1} K^{-1}.

  (iv) Dimensional formula for R = $[ML^2T^{-2}K^{-1}\,mol^{-1}]$.
- Vander Wall's equation for one mole of a gas: $\left(P + \frac{a}{v^2}\right)(V - b) = RT$
- Pressure exerted by ideal gas: $P = \frac{1}{3}\frac{mnc^2}{V}$
- Average K.E.of translation of 1 mole: $\frac{3}{2}RT$
- Average K.E.of translation per molecule of a gas: $\frac{3}{2}k_bT$

Chapter 14: Oscillations

Periodic motion:

- Frequency: $\vartheta = \frac{1}{T}$
- Angular frequency $\omega = 2\pi\vartheta$
- Phase $=(\omega t + \phi)$

Simple Harmonic motion:

- Differential equation: $\frac{d^2y}{dx^2} + \omega^2 y = 0$

  Where $\omega^2 = \frac{k}{m}; here\ m\ is\ the\ mas\ of\ the\ body$
- General equation: $y = y_0 sin(\omega t + \phi)$
- Displacement :$y = Asin\omega t$

- Velocity: $v = \omega\sqrt{A^2 - y^2}$
- Acceleration: $a = \frac{dv}{dt} = -\omega^2 Asin\omega t$
- Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Oscillations:

- For loaded Spring:
  - Time period $T = 2\pi\sqrt{\frac{m}{k}}$; m-inertia factor and k-spring constant.
  - Frequency $\vartheta = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- For simple pendulum:
  - Time period $T = 2\pi\sqrt{\frac{l}{g}}$
  - Frequency $\vartheta = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$
- For loaded spring:
  - Two springs in parallel: $T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$

    If $k_1=k_2=k$, *then* $T = 2\pi\sqrt{\frac{m}{2k}}$
  - Two springs in series: $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 x\, k_2}}$

    If $k_1=k_2=k$, *then* $T = 2\pi\sqrt{\frac{2m}{k}}$
- Spring constant: $k=F/y$
- In parallel $k=k_1+k_2$
- In series $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

Chapter 15: Waves

- A wave can be represented by, $y = asin(\omega t \pm kx)$.
- A phase difference of 2π radians is equivalent to a path difference of λ and a time difference of time period T *i.e*, 2π= λ

  So phase difference, $\emptyset = \frac{2\pi}{\lambda}\ x\ path\ difference$
- Particle acceleration: $a(x,t) = -\omega^2 y$
- Standard equation for plane progressive wave:

$$y = rsin\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$$

  Where *y*-displacement; *r*-amplitude, *T*-time period, *x*-starting distance of the wave from origin and λ is the wavelength.

- Newton's corrected formula for velocity of sound: $v = \sqrt{\frac{\gamma P}{\rho}}$

- Doppler's effect: $v' = \frac{\{(v+v_m)-v_L\}v}{(v+v_m)-v_s}$

Where v'-apparent frequency of sound heard
v- Actual frequency of sound
v_m-Velocity of medium
v_s- Velocity of source
v_L- Velocity of the listener.

1. Electrostatics

Coulomb's inverse square law: Force between two static point charges is given by $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ where $\frac{1}{4\pi\varepsilon_0} = 9x10^9 Nm^2C^{-2}$ and $\varepsilon_0 = 8.852x10^{-12}Fm^{-1}$

Electric intensity (E): Force per unit charge; a vector; unit: V/m and $E = \frac{F}{q}$

Electric potential (V): Work done per unit charge in moving the test charge from infinity up to that point against the field.
$V = \frac{W}{q}$; a scalar; unit: Volt(V)

Relation between E and V: E=$-\frac{dV}{dx}$

Gauss theorem: Total flux across a closed surface=$\frac{1}{\varepsilon_0} x\ (algebraic\ sum\ of\ the\ charges\ within\ the\ sphere)$

Due to a charged conducting sphere:

$$\oint E.ds = \frac{1}{\varepsilon_0} xtotal\ charge\ within$$

a) Electric intensity $E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\ when\ r > R$
b) Electric intensity $E_{in} = 0\ when\ r < R$
c) Electric intensity on the surface $E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\ when\ r = R$
d) Electric potential $V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\ when\ r > R$
e) Electric potential $V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\ when\ r = R$
f) Electric potential $V_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = V_{surface}\ when\ r < R$

Coulomb's theorem: (Electric intensity near a charged conductor)
E=$\frac{1}{\varepsilon_0} x\ \sigma\ where\ \sigma = Q/A$

2. Capacitors

Capacity: Ability of the material to hold charge at a given potential.

Capacity of a conductor: Ratio of ' charge on conductor to the potential of the conductor. $C = \frac{Q}{V}$ Unit: 1Coulomb/1Volt

Capacitor: Arrangement consisting of a 'dielectric' sandwiched between two conductors which are charged equally and oppositely.

Capacity of-

a) Spherical capacitor: $C = 4\pi\varepsilon_0\frac{R_1 R_2}{(R_2 - R_1)}$
b) Parallel plate capacitor: $C = \frac{\varepsilon_0 A}{d}$
c) Cylindrical capacitor: $C = \frac{2\pi\varepsilon_0 L}{2.303\log\frac{R_2}{R_1}}$

Energy stored in a capacitor: E=$\frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$

Effect of dielectric: Decreases the field, potential difference but increases the capacitance.

Series Combination:

a) Arrangement having same charge; total potential difference =sum of potential difference (p.d's).

b) $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

c) $C_s = \frac{C}{n}$

Parallel Combination:

a) Arrangement having same p.d; total charge=sum of charges.

b) $C_p = C_1 + C_2 + C_3 + ...$

c) $C_p = nC$

Chapter 3: Current Electricity

- **Electric Current (I)**

The rate of flow of charge through any cross-section of a wire is called electric current flowing through it.

Electric current (I) = q / t. Its SI unit is ampere (A).

The conventional direction of electric current is the direction of motion of positive charge.

The current is the same for all cross-sections of a conductor of non-uniform cross-section.

Similar to the water flow, charge flows faster where the conductor is smaller in cross-section and slower where the conductor is larger in cross-section, so that charge rate remains unchanged.

(In a metallic conductor current flows due to motion of free electrons while in electrolytes and ionized gases current flows due to electrons and positive ions.)

- **Types of Electric Current**

According to its magnitude and direction electric current is of two types

(i) **Direct Current (DC)** Its magnitude and direction do not change with time. A cell, battery or DC dynamo are the sources of direct current.

(ii) Alternating Current (AC) An electric current whose magnitude changes continuously and changes its direction periodically is called alternating current. AC dynamo is source of alternating current.

- **Current Density**

The electric current flowing per unit area of cross-section of conductor is called current density.

Current density (J) = $\frac{current}{Area} = \frac{I}{A}$

Its S1 unit is ampere metre^{-2} and dimensional formula is [AT^{-2}].

It is a vector quantity and its direction is in the direction of motion positive charge or in the direction of flow of current.

- **Thermal Velocity of Free Electrons**

Free electrons in a metal move randomly with a very high speed of the order of 10^5 ms^{-1}. This speed is called thermal velocity of free electron.

Average thermal velocity of free electrons in any direction remains zero.

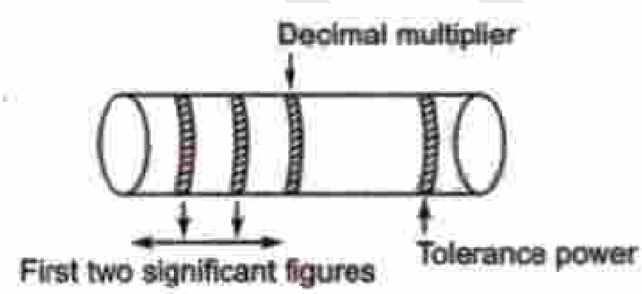
- **Drift Velocity of Free Electrons**
  When a potential difference is applied across the ends of a conductor, the free electrons in it move with an average velocity opposite to direction of electric field. which is called drift velocity of free electrons.
  Drift velocity vd = $\frac{eE\tau}{m} = \frac{eV\tau}{ml}$ where, τ = relaxation time, e = charge on electron,
  E = electric field intensity, 1 = length of the conductor, m = mass of electron.
  V = potential difference across the ends of the conductor
- **Relation between electric current and drift velocity is given by**
  Vd = $\frac{I}{An\,e}$
- **Mobility** (μ): The drift velocity of electron per unit electric field applied is mobility of electron. Mobility of electron (μ) = $\frac{V_d}{E}$
  Its SI unit is $m^2s^{-1}V^{-1}$ and its dimensional formula is $[M^{-1}T^2A]$.
- **Ohm's Law:**
  If physical conditions of a conductor such as temperature remains unchanged, then the electric current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends.
  $I \propto V$ or $V = IR$ where R is the electrical resistance of the conductor
- **Electrical Resistance:**
  The obstruction offered by any conductor in the path of flow of current is called its electrical resistance.
  Electrical resistance, R = $\frac{V}{I}$
  Its SI unit is ohm (Ω) and its dimensional formula is $[ML^2T^{-3}A^{-2}]$.
  Electrical resistance of a conductor R = $\frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} = \frac{\rho d l^2}{m}$
  where, l = length of the conductor, A = cross-section area and
  ρ = resistivity of the material of the conductor, d= density and m be mass.
- **Electrical Conductivity**
  The reciprocal of resistivity is called electrical conductivity.
  Electrical conductivity (σ) = $\frac{1}{\rho} = \frac{l}{RA} = \frac{ne^2\tau}{m}$
  Its SI units is $ohm^{-1}\,m^{-1}$ or mho m^{-1} or siemen m^{-1}.
- **Relation between current density (J) and electrical conductivity (σ)** is given by
  J = σ E where, E = electric field intensity.
- **Ohmic Conductors**
  Those conductors which obey Ohm's law are called ohmic conductors e.g., all metallic conductors are ohmic conductor.
  For ohmic conductors V – I graph is a straight line.
- **Non-ohmic Conductors**

Those conductors which do not obey Ohm's law, are called non-ohmic conductors. e.g., diode valve, triode valve, transistor , vacuum tubes etc.
For non-ohmic conductors V – I graph is not a straight line.

- **Superconductors**
  When few metals are cooled, then below a certain critical temperature their electrical resistance suddenly becomes zero. In this state, these substances are called **superconductors** and this phenomena is called**superconductivity**.
  Mercury become superconductor at 4.2 K, lead at 7.25 K and niobium at 9.2 K

- **Colour Coding of Carbon Resistors**
  The resistance of a carbon resistor can be calculated by the code given on it in the form of coloured strips.
- **Colour Coding**

Colour	Figure
Black	**0**
Brown	**1**
Red	**2**
Orange	**3**
Yellow	**4**
Green	**5**
Blue	**6**
Violet	**7**
Grey	**8**
White	**9**



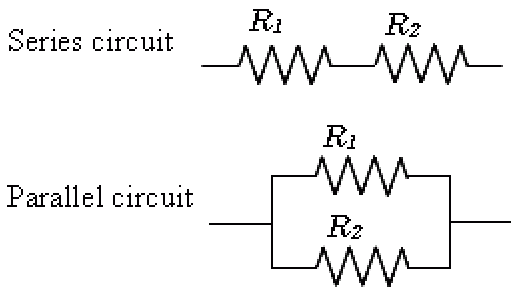
- **Tolerance power**
  Colour Tolerance

Colour	Tolerance
Gold	5%
Silver	10%
No colour	20%

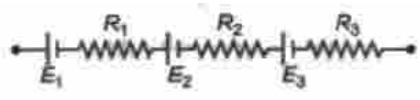
This colour coding can be easily learned in the sequence "**B B ROY** of **G**reat **B**ratain **V**ery **G**ood **W**ife".

Combination of Resistors

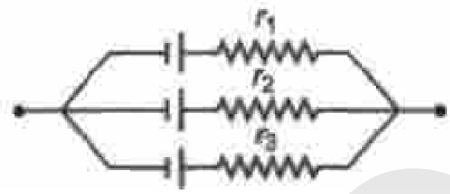
- **1.In Series**
  (i) Equivalent resistance, $R = R_1 + R_2 + R_3$
  (ii) Current through each resistor is same.
  (iii) Sum of potential differences across individual resistors is equal to the potential difference, applied by the source.



- **2. In Parallel**
  Equivalent resistance
  $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
  Potential difference across each resistor is same.
  Sum of electric currents flowing through individual resistors is equal to the be electric current drawn from the source.
- **Electric Cell**
  An electric cell is a device which converts chemical energy into electrical energy.
  Electric cells are of two types
  (i) **Primary Cells** Primary ceUs cannot be charged again. Voltic, Daniel and Leclanche cells are primary cells.
  (ii) **Secondary Cells** Secondary cells can be charged again and again. Acid and alkali accumulators are secondary cells.
- **Electro – motive – Force** (emf) **of a Cell**
  The energy given by a cell in flowing unit positive charge throughout the circuit completely one time, is equal to the emf of a cell.
  Emf of a cell (E) = W / q.
  Its SI unit is volt.
- **Terminal Potential Difference of a Cell**
  The energy given by a cell in flowing unit positive charge through till outer circuit one time from one terminal of the cell to the other terminal of the cell.
  Terminal potential difference (V) = W / q. Its SI unit is volt.
- **Internal Resistance of a Cell**
  The obstruction offered by the electrolyte of a cell in the path of electric current is called internal resistance (r) of the cell. Internal resistance of a cell
  (i) Increases with increase in concentration of the electrolyte.
  (ii) Increases with increase in distance between the electrodes.
  (iii) Decreases with increase in area of electrodes dipped in electrolyte.
  Relation between E. V and r :
  $E = V + Ir$
  $r = (E / V - 1) R$
  If cell is in charging state, then
  $E = V - Ir$
- **Grouping of Cells**
  (i) **In Series** If n cells, each of emf E and internal resistance r are connected in series to a resistance R. then equivalent emf

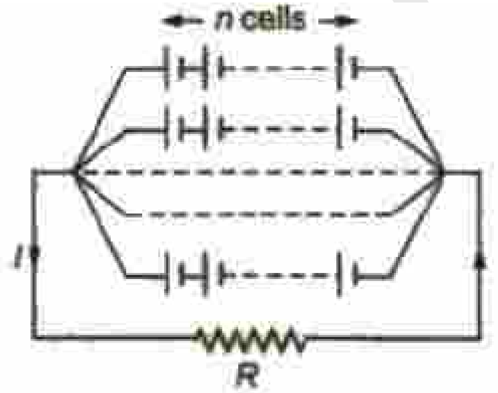


$E_{eq} = E_1 + E_2 + ... + E_n = nE$
Equivalent internal resistance $r_{eq} = r_1 + r_2 + ... + r_n = nr$
Current In the circuit $I = E_{eq} / (R + r_{eq}) = nE / (R + nr)$
(ii) **In Parallel** If n cells. each of emf E and internal resistance r are connected to in parallel.then equivalent emf. $E_{eq} = E$
Equivalent internal resistance



$1 / r_{eq} = 1 / r_1 + 1 / r_2 + ... + 1 / r_n = n / r$ or $r_{eq} = r / n$
Current In the circuit $I = E / (R + r / n)$

- **Mixed Grouping** of Cells If n cells, each of emf E and internal resistance r are connected in series and such m rows are connected in parallel, then



Equivalent emf, E_{eq} Equivalent Internal resistance r_{eq}
Current in the circuit, $I = nE / (R + nr / m)$
or $I = mnE / mR + nr$
Note: Current in this circuit will be maximum when external resistance is equal to the equivalent internal resistance, i.e.,
$R = nr / m \Rightarrow mR = nr$

- **Kirchhoff's Laws**
There are two Kirchhoff's laws for solving complicated electrical circuits
(i) **Junction Rule** The algebraic sum of all currents meeting at a junction in a closed circuit is zero, i.e., $\Sigma I = O$.
This law follows law of conservation of charge.

(ii) **Loop Rule** The algebraic sum of all the potential differences in any closed circuit is zero, i.e., ΣV = 0 ⇒ ΣE = ΣIR
This law follows law of conservation of energy.

- **Balanced Wheatstone Bridge**
  Wheatstone bridge is also known as a **metre bridge** or **slide wire bridge**.
  This is an arrangement of four resistance in which one resistance is unknown and rest known. The bridge is said to be balanced when deflection in galvanometer is zero,
  i.e., ig = O. $\frac{P}{Q} = \frac{R}{S}$
- **Principle of Wheatstone Bridge**
  The value of unknown resistance S can found. as we know the value of P,Q and R. It may be remembered that the bridge is most sensitive, when all the four resistances are of the same order.
- **Meter Bridge**
  This is the simplest form of Wheatstone bridge and is specially useful for comparing resistance more accurately. $\frac{R}{S} = \frac{L}{100-L}$
  where L is the length of wire from one end where null point is obtained.
- **Potentiometer**
  Potentiometer is an ideal device to measure the potential difference between two points. It consists of a long resistance wire AB of uniform cross section in which a steady direct current is set up by means of a battery.
  If R be the total resistance of potentiometer wire L its total length, then potential gradient, i.e., fall in potential per unit length along the potentiometer will be
  K = V / L = IR / L
  = Eo R / (Ro + R)L
  where, Eo = emf of battery and Ro = resistance inserted by means of rheostat Rh.
- **Determination of emf of a Cell using Potentiometer**
  If with a cell of emf E on sliding the contact point we obtain zero deflection in galvanometer G when contact point is at J at a length I from the end where positive terminal of cell have been joined. then fall in potential along length i is just balancing the emf of cell. Thus, we have $\frac{E_1}{E_2} = \frac{L_1}{L_2}$
  E1 / E2 = l1 / l2
- **Determination of Internal Resistance of a Cell using Potentiometer**
  Internal resistance of cell
  r = $\frac{E-V}{V}$, R = $\frac{L_1 - L_2}{L_2} R$
- **Important Points**

* Potentiometer is an ideal voltmeter.
* Sensitivity of potentiometer is increased by increasing length of potentiometer wire.

- **Electric Power P** = $\frac{Electric\ work\ done}{time\ taken}$, P = VI = $I^2R = \frac{V^2}{R}$

Chapter 4: Moving Charges and Magnetism

- Biot Savart's Law: dB = $\frac{I\,dl\sin\theta}{r^2}$
- The magnetic field B at appoint due to a straight wire of finite length carrying current I at a perpendicular distance r is
  $B = \frac{\mu_0}{4\pi r}[sin\alpha + sin\beta]$
- The magnetic field at the centreof circular coil of radius 'a' carrying current I is
  $B = \frac{\mu_0}{2a}I$
  If the circular coil consists of N turns , then $B = \frac{\mu_0}{2a}NI$
- The magnetic field at a point on the axis of the circular coil carrying current is
  $B = \frac{\mu_0}{4\pi r}\frac{2\pi NIa^2}{(a^{2+}x^2)^{3/2}}$
- Magnetic field at the centre of circular coil due to current carrying.
  $B = \frac{\mu_0 I\phi}{4\pi a}$
- Ampere's circuital law : $\oint B.dl = \mu_0 I$
- Magnetic field due to an infinitely long straight solid cylindrical wire of radius 'a', carrying current I.
  - Magnetic field at a point outside the wire. i.e. (r > a) is B= $\frac{\mu_0 I}{2\pi r}$
  - Magnetic field at a point inside the wire i.e. (r < a) is B= $\frac{\mu_0 Ir}{2\pi a^2}$
  - Magnetic field at appoint on the surface of a wire. i.e. (r = a) is B= $\frac{\mu_0 I}{2\pi a}$
- Force on a charged particle in a uniform electric field is F = qE
- Force on a charged particle in a uniform magnetic field. F = q(v x B)= qvBsinθ
- Motion of a charged particle in a uniform magnetic field.
  - Radius of circular path is R = $\frac{mv}{Bq} = \frac{\sqrt{2mK}}{qB}$
  - Time period of revolution is T = $\frac{2\pi R}{v} = \frac{2\pi m}{qB}$
  - The frequency is ν = $\frac{1}{T} = \frac{qB}{2\pi m}$
  - The angular frequency is ω = 2πυ = $\frac{qB}{m}$
- Cyclotron frequency is ω = $\frac{qB}{2\pi m}$
- Force on acurrent carrying conductor in a uniform magnetic field
  F = I (l x B) or F = I l B sinθ

- When two parallel conductors separated by a distance 'r' carrying current 'I_2' in the magnetic field one will exert a force on the other. The force per unit length on either conductor is F = $\frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$
- The force of attraction or repulsion acting on each conductor of length *l* due to currents in two parallel conductor is F = $\frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}\, l$
- When two charges q_1 and q_2 respectively moving with velocities v_1 and v_2 are at distance 'r' apart, then the force acting between them is

  F = $\frac{\mu_0}{2a}\frac{q_1 q_2 v_1 v_2}{r^2}$

- In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).

  $I \alpha \theta$ or $I = G\theta$ where $G = \frac{k}{NAB}$ = galvanometer constant.
- Current sensitivity: Is = $\frac{\theta}{T} = \frac{NAB}{k}$
- Conversion of galvanometer to ammeter: S = $\left(\frac{I_g}{I - I_g}\right) G$
- Conversion of galvanometer to voltmeter: R = $\frac{V}{I_g} - G$
- Magnetic dipole moment: M = m(2*l*)
- The magnetic field due to a bar magnet at any point on the axis line(end on position) is $B_{axis} = \frac{\mu_0}{4\pi r}\frac{2Mr}{\left(r^2 - l^2\right)^2}$

  For short magnet $l^2 << r^2$ Baxis = $\frac{\mu_0 2M}{4\pi r^3}$

  The direction of Baxis is along SN.
- The magnetic fields due to a bar magnet at any point on the equatorial line (board – side on position) of the bar magnet is

  Bequitorial = $\frac{\mu_0}{4\pi}\frac{M}{\left(r^2 + l^2\right)^{3/2}}$

  For short magnet, Bequitorial = $\frac{\mu_0}{4\pi}\frac{M}{r^3}$

  The direction of Bequitorial is parallel is NS.
- Torque on a current carrying coil placed in a uniform magnetic field

  τ = NIA Bsinθ = MB sinθ

  If α is the angle between plane of the coil and the magnetic field, then torque on the coil is τ = NIA Bcosα = MB cosα
- Potential energy of a magnetic dipole: U = M.B = – M B cosθ

Chapter 5: Magnetism and Matter

- Gauss's law for magnetism: $\phi = \sum_{all\ area\ elements\ \Delta S} B.\Delta S = 0$
- Horizontal component: $B_H = B.\cos\delta$
- Magnetic intensity: $B = \mu.H$
- Intensity of magnetization: $I = \frac{Magnetic\ moment}{Volume} = \frac{M}{V}$
- Magnetic susceptibility: $\chi_m = \frac{I}{H}$
- Magnetic permeability: $\mu = \frac{B}{H}$
- Relative permeability: $\mu_r = \frac{\mu}{\mu_0}$
- Relationship between magnetic permeability and susceptibility:
  $\mu_r = 1 + \chi_m$ with $\mu_r = \frac{\mu}{\mu_0}$
- Curie law: $\chi_m = \frac{C}{T}$ $\chi_m = \frac{C}{T - T_C}$ $(T > T_c)$

Chapter 6: ELECTROMAGNETIC INDUCTION

- Magnetic flux: $\Phi_B = B . A = BA\cos\theta$
- Faraday's law of EMI: The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit. Mathematically, the induced emf is given by
  $\varepsilon = -\frac{d\emptyset}{dt}$ The negative sign indicates the direction of ε and hence the direction of current in a closed loop.
- Self induced emf: $\varepsilon = -\frac{d\emptyset}{dt} = -L\frac{di}{dt}$
- Self inductance of a circular coil : $L = \frac{\mu_0 N^2 A}{l}$
- Let Ip be the current flowing through primary coil at any instant. If $\emptyset s$ is the flux linked with secondary coil then $\emptyset s$ α Ip or $\emptyset s$ = M.Ip where M is the coefficient of mutual inductance. The emf induced in the secondary coil is given by
  $\varepsilon_S = -M\frac{dI_p}{dt}$ where M is the coefficient of mutual inductance.
  $K = \frac{M}{\sqrt{L_1 L_2}}$
- The coefficient of mutual inductance of two long co-axial solenoids, each of length l, area of cross section A, wound on air core is
  $M = \frac{\mu_0 N_1 N_2\ A}{l}$
- Energy stored in an inductor $U = \frac{1}{2}LI^2$
- During the growth of current in a LCR circuit is

$I = Io(1 - e^{-Rt/L}) = q_0(1 - e^{-t/\tau})$ where I_0 is the maximum value of current
$\tau = L/R =$ time constant of LCR circuit.

- During the decay of current in a LCR circuit is

  $I = Io(1 - e^{-t/RC}) = q_0(1 - e^{-t/\tau})$ where q_o is the maximum value of charge.

  T = RC is the time constant of RC circuit.

- During discharging of capacitor through resistor

  $q = q_0\, e^{-t/RC} = q_0\, e^{-t/\tau}$

Chapter 7: ALTERNATING CURRENT

Alternating emf: Alternating emf is that emf which continuously changes in magnitude and periodically reverses its direction.

Alternating Current: Alternating current is that current which continuously changes in magnitude and periodically reverses its direction.

- Mean or average value of alternating current or voltage over one complete cycle.

$$I_m \text{ or } I = \frac{\int_0^T I_0 Sin\, \omega t.dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } V = \frac{\int_0^T V_0 Sin\, \omega t.dt}{\int_0^T dt} = 0$$

- Average value of alternating current for first half cycle is

$$I_m = \frac{\int_0^{T/2} I_0 Sin\, \omega t.dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

Similarly, for alternating voltage, the average value over first half cycle is

$$V_m = \frac{\int_0^{T/2} V_0 Sin\, \omega t.dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637\, V_0$$

- Average value of alternating current for second cycle is

$$I_m = \frac{\int_{T/2}^{T} I_0 Sin\, \omega t.dt}{\int_{T/2}^{T} dt} = \frac{2I_0}{\pi} = -0.637 I_0$$

Similarly, for alternating voltage, the average value over second cycle is

$$V_m = \frac{\int_{T/2}^{T} V_0 Sin\, \omega t.dt}{\int_{T/2}^{T} dt} = \frac{2V_0}{\pi} = -0.637\, V_0$$

- Mean value or average value of alternating current over any half cycle.

$$I_m = \frac{2I_0}{\pi} = 0.637 I_0$$

- Root mean square (rms) value of alternating current

  I_{rms} or $I_V = \frac{I_0}{\sqrt{2}}$ 0.707 I_0
- Root mean square (rms) value of alternating voltage

  $V_{rms} = \frac{V_0}{\sqrt{2}}$ 0.707 V_0
- Fermi factor = $\frac{I_{rms}}{I_{av}}$
- Inductive reactance: $X_L = \omega L = 2\pi fL$
- Capacitive reactance : $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$
- Impedence of the series LCR circuit.

  $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

  Admittance = $\frac{1}{Impedence}$ or $Y = \frac{1}{Z}$

  Susceptance = $\frac{1}{Reactance}$
- Inductive susceptance = $\frac{1}{Inductive\ Reactance}$

  Or $S_L = \frac{1}{X_L} = \frac{1}{\omega L}$
- Capacitive susceptance = $\frac{1}{Capacitive\ Reactance}$

  Or $S_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C$
- Resonant frequency : $F_r = \frac{1}{2\pi\sqrt{LC}}$
- Quality factor: $Q_r = \frac{1}{\sqrt{LC}}$

  $Q = \frac{X_L}{R} = \frac{Q_r L}{R}$ $Q = \frac{X_C}{R} = \frac{Q_r C}{R} = Q = \frac{1}{Q_r CR}$

  Therefore, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
- Average power(P_{av}) : $P_{av} = V_{rms} I_{rms} \cos\phi = \frac{V_0\ I_0}{2}\cos\phi$
- Apparent power: $P_V = V_{rms} I_{rms}\ \frac{V_0\ I_0}{2}$
- Efficiency of a transistor: $\eta = \frac{Output\ power}{Input\ power} = \frac{V_S\ I_S}{V_P\ I_P}$

Chapter 8: ELECTROMAGNETIC WAVES

- The displacement current : $I_d = \varepsilon_0 \frac{d\phi}{dt}$
- Four maxwell's equations:

1. Gauss's law for electrostatics: $\oint E \, . dS = \frac{q}{\varepsilon_0}$
2. Gauss's law for magneto statics: $\oint B \, . dS = 0$
3. Faraday's law of electromagnetic induction: $\oint E \, . dI = -\frac{dQ}{dt}$
4. Maxwell – Ampere's circuital law: $\oint B \, . dI = \mu_0 \left[1 + \varepsilon_0 \frac{d\phi}{dt}\right]$

- The amplitude of electric and magnetic fields in the space, in electromagnetic waves are related by

  $E_0 = cB_0$ or $B_0 = \frac{E_0}{c}$
- The speed of electromagnetic wave in the free space: $c = \frac{1}{\sqrt{\mu_0 \ \varepsilon_0}}$
- The speed of electromagnetic wave in medium: $v = \frac{1}{\sqrt{\mu\varepsilon}}$
- The energy density of magnetic field: $U_B = \frac{1}{2} \frac{B^2}{\mu_0}$
- Average energy density of electric field: $U_E = \frac{1}{4} \varepsilon_0 E_0^2$
- Average energy density of magnetic field: $U_B = \frac{1}{4} \frac{B^2}{\mu_0} = \frac{1}{4} \varepsilon_0 E_0^2$
- Average energy density of electromagnetic wave: $U_B = \frac{1}{2} \varepsilon_0 E_0^2$
- Intensity of electromagnetic wave: $I = U = c = \frac{1}{2} \varepsilon_0 E_0^2 C$
- Momentum of electromagnetic wave: $p = \frac{U}{c}$ (Complete absorption)

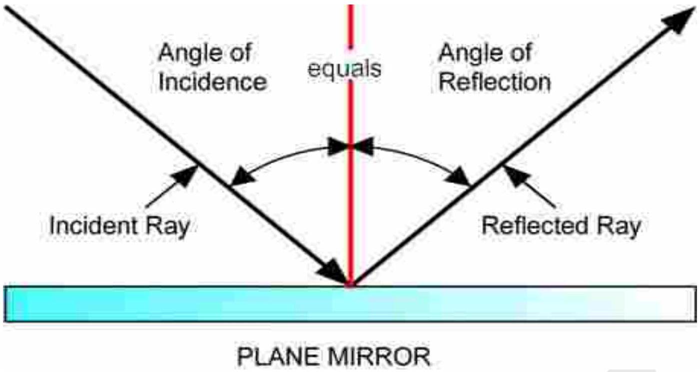
  $p = \frac{2U}{c}$ (Complete reflection)
- The pointing vector: $S = \frac{1}{\mu_0} (E \ x \ B)$

9. Geometrical optics

- **Reflection:** When light is incident on a surface, it is sent back by the surface in the same medium through which it had come. This phenomenon is called 'reflection of light' by the surface.
- **Laws of Reflection:** The reflection at a plane surface always takes place in accordance with the following two laws:

(i) The incident ray, the reflected ray and normal to surface at the point of incidence all lie in the same plane.

(ii) The angle of incidence i is equal to the angle of reflection r, i. e., $\hat{\imath} = \hat{r}$



- **Reflection of Light from Spherical Mirror:**

a) A spherical mirror is a part cut from a hollow sphere.
b) They are generally constructed from glass.
c) The reflection at spherical mirror also takes place in accordance with the laws of reflection.

- The focal length of a spherical mirror of Radius R is given by $f = \frac{R}{2}$
- Transverse linear magnification
  m=$\frac{size\ of\ image}{size\ of\ object} = \frac{v}{u}$
- Mirror formula: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$
- **Spherical Refracting Surfaces:**
  - A spherical refracting surface is a part of a sphere of refracting material.
  - A refracting surface which is convex towards the rarer medium is called convex refracting surface.
  - A refracting surface which is concave towards the rarer medium is called concave refracting surface.

- Laws of Refraction: $\mu_2^1 = \frac{\sin i}{\cos r}$
- Absolute refractive index: $\mu_2^1 = \frac{v_1}{v_2}$
- Lateral Shift: d= $t\frac{\sin(i-r)}{\cos r}$
- If there is a spot at the bottom of a glass slab, it appears to be raised by a distance
  d= $t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$
- Expression for 'object in rarer medium' is same for whether it is real or virtual image or convex or concave surface.

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

- Expression for 'object in denser medium' is same for whether it is real or virtual image or convex or concave surface.

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}$$

- Lens Makers formula: $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
- Thin lens formula: $\left(\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\right)$
- Linear Magnification: m=$\frac{size\ of\ image}{size\ of\ object}=\frac{v}{u}$
- Power of a lens: P=$\frac{1}{f}$=$\frac{1}{focal\ length\ in\ meters}$
- Combination of thin lenses in contact: $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\cdots$
- The total power of the combinations is given by: $P=P_1+P_2+P_3+$…..
- The total magnification of the combinations is given by: $m=m_1 \times m_2 \times m_3 \times$…
- When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d, the focal length of the combination is given by,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$$

- In terms of power $P= P_1+P_2-dP_1 P_2$
- The refractive index of the material of a prism is

$$\mu=\frac{\sin\left[\frac{(A+\delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

Where A is the angle of the prism and δ_m is the angle of minimum deviation.

- Mean deviation: $\delta=\frac{\delta_v+\delta_R}{2}$
- Dispersive power: $\omega=\frac{angular\ dispersion\ (\delta_v+\delta_R)}{mean\ deviation(\delta)}$
- $\omega=\frac{\mu_v+\mu_R}{(\mu-1)}$
- Magnifying power of a simple microscope: M=$\frac{angle\ subtended\ by\ image\ at\ the\ eye}{angle\ subtended\ by\ object\ at\ the\ eye}=\frac{\tan\beta}{\tan\alpha}=\frac{\beta}{\alpha}$
- When image formed at infinity: M=$\frac{D}{F}$
- When the image is formed at the least distance of distinct vision D (near point), M=1+$\frac{D}{F}$
- Magnifying power of a compound microscope: M=$m_a \times m_e$
- When the final image is formed at infinity (normal adjustment),

$$M = \frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$$

$$Length\ of\ the\ tube, L = v_0 + f_e$$

- When final image if=s formed at least distance of the distinct vision

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

Where u_0 and v_0 represent the distance of object and image from the objective lens, f_e is the focal length of the lens.

$$Length\ of\ the\ tube, L = v_0 + \frac{f_e D}{f_e + D}$$

- Astronomical telescope:
  Magnifying power $M = \frac{f_0}{f_e}$
  Length of the tube $L = v_0 + \frac{f_e D}{f_e + D}$

10. Theories of light

Corpuscular theory: Proposed by **Newton**; Conceives light as particles (Corpuscles); explains rectilinear propagation of light, reflection and refraction of light. Could not able to explain wave phenomenon.

Wave theory: Proposed by **Huygen**; Conceives light as waves; explains all phenomena except photoelectric effect and the like.

Electromagnetic theory: Proposed by **Maxwell**; Conceives light as transverse electromagnetic wave. Without electromagnetic theory polarization of light can't be explained.

Quantum theory: Proposed by **Max Planck;** Light emitted as packets of energy called quanta or photons; interact with matter and share the energy; explains photoelectric effect.

Schrȍdinger's wave theory: Extends the idea of dual nature of light to matter also.

Interference of light

Interference: Wave phenomenon in which redistribution of energy takes place at the locations of overlap of two progressive waves.

Coherent sources: Sources which maintain constant phase difference between them all the time.

Constructive interference: Interference where energy is reinforced (augmented) at the location where

a) Crest overlaps crest; trough overlaps trough.
b) Path difference= even integer $x\ \frac{\lambda}{2} = 2n\frac{\lambda}{2}$
c) Phase difference= even integer $x\ \pi = 2n\pi$

Destructive interference: Interference where energy is cancelled (annulled) at the location where

a) Crest overlaps trough or vice versa.
b) Path difference= odd integer $x \frac{\lambda}{2}$= (2n+1) $\frac{\lambda}{2}$
c) Phase difference= odd integer π=(2n+1)π

Intensity of a wave: If I_1 and I_2 be the individual intensities of the two waves at any point in the region of interference then the resultant intensity is

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}.\cos\emptyset$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \text{ if } I_1=I_2=I \text{ then } I_{max}=4I$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \text{ if } I_1=I_2=I \text{ then } I_{min}=0$$

Fringe width in double slit arrangement: $\beta=\frac{\lambda D}{d}$

Fringe width = $\frac{wavelength\ x\ screen\ to\ slit\ distance}{slit\ separation}$

Diffraction of light

Diffraction: Bending and spreading of waves around the edges of obstacles and apertures whose size is comparable to the wavelength of light.

Types of diffraction:

a) **Fresnel class:** Source of waves and screen are nearer to obstacles or apertures; wave fronts are spherical /cylindrical.
b) **Fraunhofer class:** Source of waves and screen are at infinity; wave fronts are plane.

Single slit diffraction pattern: Alternate bright and dark bands of unequal widths and unequal intensities.

Central maximum: Widest and brightest band in the pattern.

Condition for diffraction:

Maxima and minima

a) path difference=0 for central maximum.
b) path difference= $2n\frac{\lambda}{2}$ for n^{th} diffraction minimum.
c) path difference=(2n+1) $\frac{\lambda}{2}$ for nth diffraction maximum. n=1,2,3......

Half angular spread of central maximum: $\theta = \sin^{-1}\frac{\lambda}{d}$

Rayleigh criterion for resolution: Two objects are seen just separate when central maximum of diffraction pattern of one falls on the first minimum of the other and vice versa.

Resolving power: Ability to resolve two objects; reciprocal of limit of resolution.

Limit of resolution: Smallest distance (microscope) between objects or smallest angle that must be subtended at eye (for telescope) to see them just separate.

For microscope= $dx = \frac{\lambda}{2\mu\sin\theta}$ where μ=R.I of the medium that intervenes the objects and the objective of the microscope.

For telescope= $d\theta = \frac{1.22\lambda}{a}$ where *a*=width of the aperture (objective).

Resolving power = $\frac{1}{d\theta} = \frac{a}{1.22\lambda}$

Polarisation of light

Polarisation: Phenomenon exhibited by only transverse waves. It is the phenomenon in which light is confined to a single direction.

Ordinary light: Unpolarised light; It has vibration in all the directions.

Plane polarised light: Vibrations of light is confined to a single plane only.

Methods to produce plane polarised light:

1. Using dichroic crystals.
2. Reflecting at polarising angle.
3. Double refracting and eliminating one of the refracted rays.
4. Scattering.

Brewster's law: At the polarising angle (i_p), reflected and refracted rays are mutually perpendicular.

$$\mu = \tan(i_p)$$

Optical activity: Property of rotation of plane polarised light by certain substances.

Specific rotation: Angle of rotation of plane polarised light while passing throghan unit length of solution of unit concentration.

$$S = \frac{\theta}{lC}$$

Chapter 11: Dual nature of Matter and Radiation

- Photon: A packet or bundle of energy is called a photon. Energy of a photon is E = hν = $\frac{hc}{\lambda}$ where h is the Planck's constant, ν is the frequency of the radiation or photon, c is the speed of light (e.m. wave) and λ is the wavelength.
- Properties of photons:

  i) A photon travels at a speed of light c in vacuum. (i.e. 3 x 10^{-8} m/s)

  ii) It has zero rest mass. i.e. the photon can not exist at rest. H

  iii) The kinetic mass of a photon is, m $= \frac{m}{c^2} = \frac{h}{c\lambda}$

  iv) The momentum of a photon is, P $= \frac{E}{c}$

  v) Photons travel in a straight line.

  vi) Energy of a photon depends upon frequency of the photon; so the energy of the photon does not change when photon travels from one medium to another.

  vii) Wavelength of the photon changes in different media; so, velocity of a photon is different in different media.

viii) Photons are electrically neutral.

ix) Photons may show diffraction under given conditions.

x) Photons are not deviated by magnetic and electric fields.

- Einstein's Photoelectric Equation:
  $h\nu = \Phi + \frac{1}{2}mv^2_{max} = h\nu_0 + \frac{1}{2}mv^2_{max}$
  $\frac{1}{2}mv^2_{max} = h(\nu - \nu_0)$
- **de-Broglie wave:** According to de Broglie, a moving material particle can be associated with a wave. i.e. a wave can guide the motion of the particle. The waves associated with the moving material particles are known as de-Broglie waves or matter waves.
- **Expression for de Broglie wave:** According to quantum theory, the energy of the photon is $E = h\nu = \frac{hc}{\lambda} =$
  According to Einstein's theory, the energy of the photon is $E = mc^2$
  So, $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ where p = mc is momentum of a photon.
  If the rest mass of a particle is m_0, its de-Broglie wavelength is
  $$\lambda = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 v}$$
- In terms of kinetic energy K, the de-Broglie wavelength: $\lambda = \frac{h}{\sqrt{2mK}}$
- If a particle of charge q is accelerated through a potential difference V, its de-Broglie wavelength: $\lambda = \frac{h}{\sqrt{2mqV}}$
- For an electron: $\lambda = \frac{150}{\sqrt{V}} A^0$
- For a gas molecule of mass m at temperature T Kelvin, its de-Broglie wavelength $\lambda = \frac{h}{\sqrt{3mkT}}$ where k is Boltzmann's constant.

Chapter 12: ATOMS

- Rutherford's molecular model:
  $N(\theta) \; \alpha \; \frac{1}{sin^4(\theta/2)}$
  $N(\theta) = \frac{N_i nt Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 k^2 sin^4(\theta/2)}$
  The fraction of incident alpha particles scattered by angle θ or greater
  $f = \pi nt \left[\frac{Ze^2}{4\pi\varepsilon_0 K}\right] cot^2 \frac{\theta}{2}$
- The scattering angle θ of the particle and impact parameters b are related as

$b = \frac{Ze^2 \cot(\frac{\theta}{2})}{4\pi\varepsilon_0 K}$

- Distance of closest approach: $r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K}$
- Angular momentum of the electron in a stationary orbit is an integral multiple of h/2π. $L = \frac{nh}{2\pi}$ or $mvr = \frac{nh}{2\pi}$
- The frequency of a radiation from electrons makes a transition from higher to lower orbit, $\nu = \frac{E_2 - E_1}{h}$
- **Bohr's formula:**
  - Radius of n^{th} orbit: $R_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$ $R_n = \frac{0..53n^2}{Z} A^0$
  - Velocity of electron in the n^{th} orbit : $V_n = \left[\frac{e^2}{2h\varepsilon_0}\right]\frac{Z}{n} = \frac{2.2x10^6 Z}{n}$ m/s
  - The kinetic energy of the electron in the n^{th} orbit
    $K_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2R_n} = \frac{Ze^2}{8\varepsilon_0 R_n} = \frac{13.6\,Z^2}{n^2}$ eV
  - The potential energy of electron in n^{th} orbit
    $U = -\frac{Ze^2}{4\pi\varepsilon_0 R_n} = -\frac{27.2\,Z^2}{n^2}$ eV
  - Total energy of the electron in n^{th} orbit: $En = -\frac{Ze^2}{4\pi\varepsilon_0 R_n} = \frac{13.6\,Z^2}{n^2}$ eV
  - Frequency of electron in n^{th} orbit
    $\nu_n = \left[\frac{1}{4\pi\varepsilon_0}\right]\frac{4\pi^2Z^2e^4m}{n^3h^3} = \frac{6.2x10^{15}Z^2}{n^3}$
  - Wavelength of radiation in the transition from n_2 to n_1 is
    $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ where R is called Rydberg's constant.
    $R = \left[\frac{1}{4\pi\varepsilon_0}\right]^2\frac{2\pi^2\ \ me^2}{ch^3} = 1.097 \times 10^7\ m^{-1}$
- **Lyman series:** Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 2,3,4,…….∞) to first energy level (n_1=1) constitute Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$ where n_2 = 2,3,4,…….∞
- **Balmer series:** Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 3,4,5,…….∞) to first energy level (n_1=2) constitute Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$ where n_2 = 3,4,5…….∞
- **Paschen series:** Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 4,5.6,…….∞) to first energy level (n_1=3) constitute Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$ where n_2 = 4,5.6,…….∞

- **Brackett series:** Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 5.6,7……∞) to first energy level (n_1=4) constitute Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$ where n_2 = 5.6,7…….∞
- **Pfund series:** Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 6,7,8……∞) to first energy level (n_1=5) constitute Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$ where n_2 = 6,7,8……∞
- Number of spectral lines due to transition of electron from n^{th} orbit to lower orbit is $N = \frac{n(n-1)}{2}$
- Ionization energy = $\frac{13.6\,Z^2}{n^2}$ eV
- Ionization potential = $\frac{13.6\,Z^2}{n^2}$ Volt
- Energy quantization $E_n = \frac{n^2h^2}{8mL^2}$ where n = 1,2,3,……..
- Bragg's law: 2d sinθ = nλ
- X – rays : $\lambda_{min} = \frac{12400}{V}$ A^0

  Mosley's law : $\nu = a(Z - b)^2$

Chapter 13: NUCLEI

- **Nuclear radius**, $R = R_0A^{1/3}$ where R_0 is a constant & A is the mass number. where, Ro = 1.1 * 10^{-15} m is an empirical constant.
- **Nuclear density:** $\boldsymbol{\rho} = \frac{mass\ of\ nucleus}{volume\ of\ nucleus}$ $\rho = 3m / 4\pi R^3_o$

  where, m = average mass of a nucleon.
- **Atomic Mass Unit**: It is defined as 1 / 12th the mass of carbon nucleus. It is abbreviated as arnu and often denoted by u.

  Thus 1 amu = $1.992678*10^{-26}$ / 12 kg = $1.6 * 10^{-27}$ kg = 931 MeV
- **Mass Defect:** The difference between the sum of masses of all nucleons (M) mass of the nucleus (m) is called mass defect.

  Mass Defect (Δm) = M – m = $[Zm_p + (A - Z)m_n - m_n]$
- **Nuclear Binding:** Energy The minimum energy required to separate the nucleons up to an infinite distance from the nucleus, is called nuclear binding energy.

  Nuclear binding energy per nucleon = Nuclear binding energy / Total number of nucleons.

  Binding energy, Eb = $[Zm_p + (A - Z)\,m_n - m_N]c^2$
- Packing Fraction (P): p = (Exact nuclear mass) – (Mass number) / Mass number

  = M – A / M

- **Radioactive Decay law:** The rate of disintegration of radioactive atoms at any instant is directly proportional to the number of radioactive atoms present in the sample at that instant.
  Rate of disintegration $\left(-\frac{dN}{dt}\right) \propto N$ or $-\frac{dN}{dt} = \lambda N$
  where λ is the decay constant. The number of atoms present undecayed in the sample at any instant $\mathbf{N = No\ e^{-\lambda t}}$ where, N_o is number of atoms at time t =0 and N is number of atoms at time t.
- **Half-life of a Radioactive Element:** The time is which the half number of atoms present initially in any sample decays, is called half-life (T) of that radioactive element.
  Relation between half-life and disintegration constant is given by
  $T = \frac{log_e 2}{\lambda} = \frac{0.6931}{\lambda}$
- **Average Life or Mean Life(τ):** Average life or mean life (τ) of a radioactive element is the ratio of total life time of all the atoms and total number of atoms present initially in the sample. Relation between average life and decay constant $\tau = 1 / \lambda$ Relation between half-life and average life $\tau = 1.44\ T$
  The number of atoms left undecayed after n half-lifes is given by
  $N = No\ (1/2)^n = No\ (1/2)^{t/T}$ where, n = t / T, Here t = total time.

Chapter 14: SEMICONDUCTOR ELECTRONICS, MATERIALS, DEVICES AND SAMPLE CIRCUITS

- **Forbidden Band:** This band is completely empty. The minimum energy required to shift an electron from valence band to conduction band is called band gap (Eg). $E_g = h\nu = \frac{hc}{\lambda}$
- **Types of Semiconductor**:
  (i) **Intrinsic Semiconductor:** A semiconductor in its pure state is called intrinsic semiconductor.
  (ii) **Extrinsic Semiconductor:** A semiconductor doped with suitable impurity to increase its impurity, is called extrinsic semiconductor.
  n-type Semiconductor: Extrinsic semiconductor doped with pentavalent impurity like As, Sb, Bi, etc in which negatively charged electrons works as charge carrier, is called n-type semiconductor.
  Every pentavalent impurity atom donate one electron in the crystal, therefore it is called a doner atom
  p -type Semiconductor: Extrinsic semiconductor doped with trivalent impurity like Al, B, etc, in which positively charged holes works as charge carriers, is called p-type semiconductor.
  Every trivalent impurity atom have a tendency to accept one electron, therefore it is called an acceptor atom.

Electrical conductivity of extrinsic semiconductor is given by

$\sigma = 1/\rho = e(n_e \mu_e + n_h \mu_h)$ where ρ is resistivity, μ_e and μ_h are mobility of electrons and holes respectively.

Note: Energy gap for **Ge** is 0.72 eV and for **Si** it is 1.1 eV.

Conductivity of intrinsic semiconductor is given by

$\sigma = n_i e(\mu_e + \mu_h)$ where $n_e = n_h = n_i$

- **Conductivity of n-type semiconductor:** $\sigma = e n_e \mu_e$
- **Conductivity of p-type semiconductor:** $\sigma = e n_h \mu_h$
- **p-n Junction:** An arrangement consisting a p -type semiconductor brought into a close contact with n-type semiconductor, is called a p -n junction.
  The current in a p-n junction is given by $\mathbf{k_B = Io\,(e^{eV/kBT} - 1)}$
  where Io is reverse saturation current, V is potential difference across the diode, and k_B is the Boltzmann constant.
- **Dynamic resistance,** $\mathbf{r_d} = \frac{\Delta V}{\Delta I}$
- **Half wave rectifier:** peak value of current is $I_m = \frac{V_m}{r_f + R_L}$
  Where, r_f is the forward diode resistance, R_L is the load resistance and V_m is the peak value of the alternating voltage.
- rms value of current is $I_{rms} = \frac{I_m}{2}$
- DC value of current is $I_{dc} = \frac{I_m}{\pi}$
- Peak inverse voltage is P.I.V = Vm
- DC value of voltage is $V_{dc} = I_{dc} \quad R_L = \frac{I_m}{\pi} R_L$
- **Full wave rectifier:**
- peak value of current is $I_m = \frac{V_m}{r_f + R_L}$
  Where, r_f is the forward diode resistance, R_L is the load resistance and V_m is the peak value of the alternating voltage.
- rms value of current is $I_{rms} = \frac{I_m}{\sqrt{2}}$
- DC value of current is $I_{dc} = \frac{2I_m}{\pi}$
- Peak inverse voltage is P.I.V = 2Vm
  DC value of voltage is $V_{dc} = I_{dc} R_L = \frac{2I_m}{\pi} R_L$
- Ripple frequency : $r = \frac{rms\ value\ of\ the\ components\ of\ wave}{average\ or\ dcvalue} = \sqrt{\left[\frac{I_{rms}}{I_{dc}}\right]^2 - 1}$
- For half wave rectifier:
  $I_{rms} = \frac{I_m}{2} \quad I_{dc} = \frac{I_m}{\pi}$

$r = = \sqrt{\left[\frac{I_{m/2}}{I_{m/\pi}}\right]^2 - 1}$ = 1.21

- For Full wave Rectifier: $I_{rms} = \frac{I_m}{\sqrt{2}}$, $I_{dc} = \frac{2I_m}{\pi}$ $r = \sqrt{\left[\frac{I_{m/\sqrt{2}}}{2I_{m/\pi}}\right]^2 - 1}$ = 0.482
- Rectification efficiency **η**:

$$\eta = \frac{\textit{dc power delivered to load}}{\textit{ac input power from transformer secondary}}$$

- **For a half wave rectifier**,

dc power delivered to the load is $P_{dc} = I_{dc}^2 R_L = \left[\frac{I_m}{\pi}\right]^2 R_L$

Input ac power is $P_{ac} = I_{dc}^2 (r_f + R_L)$

Rectification Efficiency (**η**) = $\frac{40.6}{1+r_f/R_L}$ %

- **For a Full wave rectifier**,

dc power delivered to the load: $P_{dc} = I_{dc}^2 R_L = \left[\frac{2I_m}{\pi}\right]^2 R_L$

Input ac power is $P_{ac} = I_{rms}^2 (r_f + R_L) = \left[\frac{I_m}{\sqrt{2}}\right]^2 (r_f + R_L)$

Rectification Efficiency (**η**) = $\frac{P_{dc}}{P_{ac}} = \frac{\left[\frac{I_m}{\pi}\right]^2 R_L}{\left[\frac{I_m}{\sqrt{2}}\right]^2 (r_f/R_L)}$ x 100 % $= \frac{81.2}{1+r_f/R_L}$%

If $r_f << R_L$, Maximum rectification efficiency, **η = 81.2%**

- **Form factor:**
- **For half wave rectifier:**

$I_{rms} = \frac{I_m}{2}$ $I_{dc} = \frac{I_m}{\pi}$

Form factor = $\frac{\frac{I_m}{2}}{\frac{I_m}{\pi}} = \frac{\pi}{2}$ = 1.57

- **For Full wave rectifier:**

$I_{rms} = \frac{I_m}{\sqrt{2}}$, $I_{dc} = \frac{2I_m}{\pi}$

Form factor = $\frac{\frac{I_m}{\sqrt{2}}}{\frac{2I_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.11$

- **Common emitter amplifier:**

dc current gain $\beta_{dc} = \frac{I_C}{I_B}$

ac current gain $\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$

Power gain, $A_P = \frac{output\ power(Pout)}{Input\ power(Pin)}$

Voltage gain(in db) = 20 log $_{10} \frac{V_0}{V_i}$ = 20 log $_{10}$ Av

Power gain(in db) = 10 log $_{10} \frac{P_0}{P_i}$

- **Common Base Amplifier**

dc current gain $\alpha_{dc} = \frac{I_C}{I_B}$

ac current gain $\alpha_{ac} = \frac{\Delta I_C}{\Delta I_B}$

Power gain, $A_P = \frac{output\ power(Pout)}{Input\ power(Pin)} = \alpha_{ac} \times A_c$

Voltage gain(in db) Av = $\frac{V_0}{V_i} = \alpha_{ac} \times \frac{R_0}{R_i}$

- **Relation between α and β:**

$\beta = \frac{\alpha}{1-\alpha}$ **and** $\alpha = \frac{\beta}{1-\beta}$

- **Light Emitting Diodes (LED)**

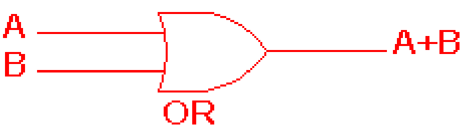
It is forward biased p-ri junction diode which emits light when recombination of electrons and holes takes place at the junction.
If the semiconducting material of p-ri junction is transparent to light, the light is emitting and the junction becomes a light source, i.e.,
Light Emitting Diode (LED). The colour of the light depends upon the types of material used in making the semiconductor diode.

(i)Gallium – Arsenide (Ga-As) – Infrared radiation
(ii) Gallium – phosphide (GaP) – Red or green light
(iii) Gallium – Arsenide – phosphide (GaAsP) – Red or yellow light

- **Logic Gate:** A digital circuit which allows a signal to pass through it, only when few logical relations are satisfied, is called a logic gate.
- **Truth Table:** A table which shows all possible input and output combinations is called a truth table.

Basic Logic Gates:

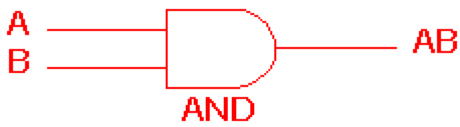
(i) **OR Gate:** It is a two input and one output logic gate.



2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

The OR gate is an electronic circuit that gives a high output (1) if **one or more** of its inputs are high. A plus (+) is used to show the OR operation.

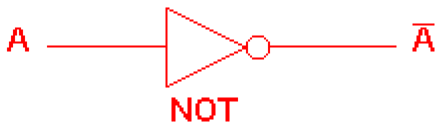
(ii) **AND Gate**: It is a two input and one output logic gate.



2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high. A dot (.) is used to show the AND operation i.e. A.B. Bear in mind that this dot is sometimes omitted i.e. AB

(iii) **NOT Gate:** It is a one input and one output logic gate.

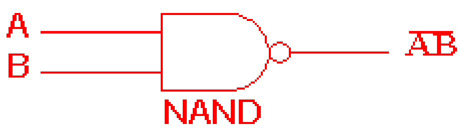


NOT gate	
A	$\overline{A}$
0	1
1	0

The NOT gate is an electronic circuit that produces an inverted version of the input at its output. It is also known as an *inverter*. If the input variable is A, the inverted output is known as NOT A. This is also shown as A', or A with a bar over the top, as shown at the outputs. The diagrams below show two ways that the NAND logic gate can be configured to produce a NOT gate. It can also be done using NOR logic gates in the same way.

Combination of Gates:

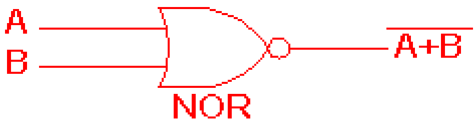
(i) NAND Gate When output of AND gate is applied as input to a NOT gate, then it is called a NAND gate.



2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

This is a NOT-AND gate which is equal to an AND gate followed by a NOT gate. The outputs of all NAND gates are high if **any** of the inputs are low. The symbol is an AND gate with a small circle on the output. The small circle represents inversion.

(ii) **NOR Gate:** When output of OR gate is applied as input to a NOT gate, then it is called a NOR gate.



2 Input NOR gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

This is a NOT-OR gate which is equal to an OR gate followed by a NOT gate. The outputs of all NOR gates are low if **any** of the inputs are high.
The symbol is an OR gate with a small circle on the output. The small circle represents inversion.

Chapter 15: COMMUNICATION SYSTEM

- **Critical Frequency** For a given layer, it is the highest frequency that will return down to earth by that layer.
  $f_C = 9(N_{max})^{1/2}$ where N_{max} the maximum number density of electrons per m^3.
- Maximum usable frequency: MUF $= \frac{U_C}{\cos i}$ = Uc seci
- **Skip Distance:** It is the shortest distance from a transmitter measured along the surface of earth at which a sky wave of fixed frequency c more than f_c will be returned to earth.
  $D_{skip} = 2h\sqrt{\left[\frac{\nu_0}{\nu_c}\right] - 1}$ where h is the height ofreflecting layer of atmosphere. ν_0 – maximum frequency of electromagnetic waves used and
  ν_C – is the critical frequency for that layer.
- If h is the highest of the transmitting antenna, then the distance to the horizontal given by $d = \sqrt{2hR}$ where R is the radius of the earth.
  For TV signal: Area covered = $\pi d^2 = \pi\ 2hR$
  Population covered = population density x area covered
- The maximum line of sight distance d_M between two antennas having heights h_T and h_R above the earth is given by
  $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
  where h_T is the height of the transmitting antenna and h_R is the height of the receiving antenna and R is the radius of the earth.
- The amplitude modulated signed contains three frequencies, viz
  ν_C, $\nu_C + \nu_m$ and $\nu_C - \nu_m$. the first frequency is the carrier frequency. Thus, the process of modulation does not change the original carrier frequency, but produces two new frequencies ($\nu_C + \nu_m$) and ($\nu_C - \nu_m$) which are known as sideband frequencies.

Frequency of lower side band $\nu_{LSB} = \nu_C - \nu_m$
Frequency of higher side band $\nu_{USB} = \nu_C + \nu_m$
Bandwidth of AM signal = $\nu_{USB} + \nu_{LSB} = 2\nu_m$
Average power per cycle in the carrier wave is
$Pc = \frac{A^2}{2R}$ where R is the resistance
Total power per cycle in the modulated wave: $P_1 = Pc\left[1 + \frac{\mu^2}{2}\right]$

- If I_t is rms values of total modulated current and I_C is the rms value of un modulated carrier current, then $\frac{I_t}{I_c} = \sqrt{1 + \frac{\mu^2}{2}}$

- For detection of AM wave, the essential condition is $\frac{I}{\boldsymbol{\nu_c}} << RC$

- The instantaneous frequency of the frequency modulated wave is
  $\nu(t) = \boldsymbol{\nu_c} + \boldsymbol{k}\frac{I_m}{2\pi}\, sin\omega_m t$ where k is proportionality constant.
- The maximum and minimum values of the frequency is
  $\nu_{max} = \boldsymbol{\nu_c} + \frac{kV_m}{2\pi}$ and $\nu_{min} = \boldsymbol{\nu_c} - \frac{kV_m}{2\pi}$

CHAPTER-1
SETS

SETS:

- Definition:- It is a collection of "well defined objects", where objects may be anything like numbers, Letters, Books, Persons etc.
  The sets are usually denoted by the capital letters A, B, X, Y etc., and its elements by small letters x, y, a, b etc.,

Some important results

- $n\,(A \cup B) = n(A) + n(B) - n\,(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(c \cap A) + n\,(A \cap B \cap C)$
- $n\,(A - B) = n\,(A) - n(A \cap B)$
- $n(A' \cup B') = n(A \cap B)'$
- $n(A' \cap B') = n(A \cup B)'$
- $n(A') = n(U) - n(A)$ where U is the Universal Set
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive law)
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- Ordered Pairs:- An orders pair consists of 2 elements say a and b, where a is called *first element* and b is called *second element* and it is denoted by (a, b)
- Cartesian product:-The Cartesian product of two sets A and B is the set of all ordered pair (a,b) such that $a \in A \text{ and } b \in B$ and it is denoted by $A \times B$ i.e. $A \times B = \{(a, b) |\, a \in A \text{ and } b \in B\}$
- Note:
  1) If $n(A) = m \text{ and } n(B) = n$ then $n(A \times B) = mn$
  2) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  3) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

CHAPTER 2
RELATIONS & FUNCTIONS

- Definition:- Given any two non-empty sets A and B a relation R from A to B is defined as sub set of $A \times B$. i.e. $R = \{(x, y) |\, x \in A \text{ and } y \in B\}$.
- Equality of two ordered pairs: $(a, b) = (c, d) \Leftrightarrow a = c \text{ and } b = d$
- Cartesian product of two sets A and B is $A \times B = \{(a, b): a \in A \text{ and } b \in B\}$

- If $n(A) = m$ and $n(B) = n$ then $n(A \times B) = mn$
- A relation R from A to B is denoted by $R: A \to B$ & is defined as a subset of A × B.
  i.e $R = \{(a, b): a \in A, b \in B \; and \; aRb\}$
- $(a, b) \in R \Leftrightarrow aRb$ means a is related to b by the relation R.
- If $n(A) = m$ and $n(B) = n$ then the total number of possible relations from A to B is 2^{mn}.
- A relation R from A to itself is known as relation on A & denoted by $R: A \to A$
- Let A & B be non-empty sets. The function f from A to B is denoted by $f: A \longrightarrow B$ & is defined as a relation which relates every element of A with only one element of B by the rule f.
- If $f: A \longrightarrow B$ is a function from A to B.then for $every \; x \in A$, there exist unique $y \in B$ such that $(x, y) \in f$
- $(x, y) \in f \Leftrightarrow xfy \Leftrightarrow y = f(x)$ Where $y = f(x)$ is called image of x under f. x is called pre-image of y under f.
- For the function $f: A \longrightarrow B$, The set A is called domain of $f = D_f$. Set B is called codomain of f. $f(A) = \{f(x): \forall x \in A\}$ is range of $f = R_f$.
- Domain & codomain of function f are subsets of set of real numbers then f is known as real valued function of real variable.
- When function is defined by an equation $y = f(x)$,

  Domain of f = Set of real numbers for which $f(x)$ is well defined. i.e $D_f = \{x \in R: f(x) \; is \; well \; defined\}$

  Range of: Solve for x in terms of y then set of values of y for which x is well defined. $R_f = \{y \in R: x = g(y) \; is \; well \; defined\}$
- Graph of the function $f: A \longrightarrow B$ is the set of all points $(x, y) \in A \times B$ where $x \in A$ and $y = f(x) \in B$.

Function	Domain	Range
Identity function $f(x) = x$	R	R
Constant function $f(x) = k$	R	R
Squaring function $f(x) = x^2$	R	R_+ or $R^+ \cup \{0\}$
Cubing function $f(x) = x^3$	R	R

Function	Domain	Range
Signum function $f(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$	R	$\{-1,0,1\}$
Modulus function $f(x) = \lvert x \rvert = \begin{cases} x & if\ x > 0 \\ 0 & if\ x = 0 \\ -x & if\ x < 0 \end{cases}$	R	$R_+ = [0, \infty)$
Greatest integer function $f(x) = [x] = n$ If $n \le [x] < n + 1$, for $n \in Z$	R	Z
Fractional part function $f(x) = \{x\}$ $x = [x] + \{x\}$	R	$[0, 1)$
Circle branch $y = \sqrt{a^2 - x^2}$	$\lvert x \rvert \le a$	$[0, a]$
Hyperbola branch $y = \sqrt{x^2 - a^2}$	$\lvert x \rvert \ge a$	$[0, \infty)$
Sine function $y = sinx$	R	$[-1,1]$
Cosine function $y = cosx$	R	$[-1,1]$
Tangent function $y = tanx$	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$, $n \in Z$	R
Cotangent function $y = cotx$	$R - \{n\pi : n \in Z\}$	R
Secant function $y = secx$	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$,	$(-\infty, -1] \cup [1, \infty)$

	$n \in Z$	
Cosecant function $y = cosecx$	$R - \{n\pi : n \in Z\}$	$(-\infty, -1] \cup [1, \infty)$
Exponential function $y = e^x$	R	$R^+ = (0, \infty)$
Logarithmic function $y = logx$	$R^+ = (0, \infty)$	R
Reciprocal function $f(x) = \frac{1}{x}$	$R - \{0\}$	$R - \{0\}$
$f(x) = \frac{1}{x^2}$	$R - \{0\}$	$R^+ = (0, \infty)$

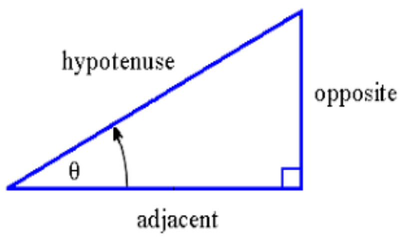
- Even function: $f(-x) = f(x)$, $0, k, x^{2n}, cosx$,etc
- Odd function:$f(-x) = -f(x)$ x, x^3,$sinx, tanx$,etc
- Periodic function: $f(x + T) = f(x)$

$f(x)$	$sinx$	$cosx$	$tanx$	$cotx$	$secx$	$cosecx$
Period	2π	2π	π	π	2π	2π

CHAPTER 3
TRIGONOMETRY

- Measurements of angles: Degree(°) system, radian(^{c})system
- Degree system: one revolution = 360°, $1° = 60'$; $1' = 60''$.
- Relationship between radian & degree: $\pi^c = 180^\circ$
- Trigonometric Functions

In a right angled triangle ΔOPM,



$$sin\theta = \frac{PM}{OP} = \frac{Opposite\ side}{Hypotenuse}$$

$$cos\theta = \frac{OM}{OP} = \frac{Adjacent\ side}{Hypotenuse}$$

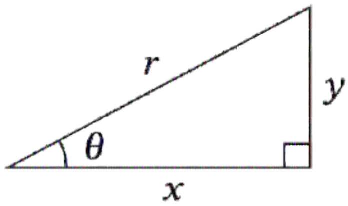
$$tan\theta = \frac{PM}{OM} = \frac{Opposite\ side}{Adjecent\ side}$$

- Trigonometric functions for general angles

$sin\theta = \frac{y}{r}$;

$cos\theta = \frac{x}{r}$;

$tan\theta = \frac{y}{x}$;



- Relationship of trigonometric functions

I. Reciprocally related

$cosec\theta = \frac{1}{sin\theta}$; $sec\theta = \frac{1}{cos\theta}$; $cot\theta = \frac{1}{tan\theta}$

II. Quotiently related

$tan\theta = \frac{sin\theta}{cos\theta}$ $cot\theta = \frac{cos\theta}{sin\theta}$

III. Squarely related

$sin^2\theta + cos^2\theta = 1; sin^2\theta = 1 - cos^2\theta; \ cos\theta = \pm\sqrt{1 - sin^2\theta}$

$sec^2\theta = 1 + tan^2\theta; sec^2\theta - tan^2\theta = 1; sec^2\theta - 1 = tan^2\theta$; $\frac{1}{sec\theta - tan\theta} = sec\theta + tan\theta$

$cosec^2\theta = 1 + cot^2\theta; cosec^2\theta - cot^2\theta = 1; cosec^2\theta - 1 = cot^2\theta; \frac{1}{cosec\theta - cot\theta} = cosec\theta + cot\theta$

IV. Co-ratio

$ratio(90° - \theta) = co - ratio(\theta)$: complementary angle formulae

$sin(90° - \theta) = cos\theta$; $tan(90° - \theta) = cot\theta$; $cosec(90° - \theta) = sec\theta$

θ°	0°	15°	30°	45°	60°	75°	90°	$180°$
θ^c	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	π
sin	0	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	1	0
cos	1	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0	-1

tan	0	$2-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$2+\sqrt{3}$	ND	0

- $\cos\left(any\ odd\ \frac{\pi}{2}\right) = 0$; $\cos(odd\ \times \pi) = -1$; $\cos(even \times \pi) = 1$
- $\sin(any\ \ \pi) = 0$; $\sin\left[(4n+1)\ \times \frac{\pi}{2}\right] = 1$; $\sin\left[(4n+3)\ \times \frac{\pi}{2}\right] = -1$
- ASTC rule:

  A: All t-ratios are positive for the I-Quadrant angles.

  S: Sin and Cosine are only positive for II- Quadrant angles.

  T: Tan and cot are positive for III- Quadrant angles.

  C: Cos and sec are positive for IV- quadrant angles.
- Co-functions

  $sine \rightleftharpoons cosine$

  $tangent \rightleftharpoons cotangent$

  $secant \rightleftharpoons cosecant$
- Allied angle Formulae

  In general,

$$T - function(n \cdot 90^\circ \pm \theta) = \begin{cases} \pm(\text{according to ASTC rule})co - function(\theta) & if\ n\ is\ odd \\ \pm(\text{according to ASTC rule})function(\theta) & if\ n\ is\ even \end{cases}$$

- Trigonometric-Functions of Compound angles:

  1. $sin(A \pm B) = sinA\ cosB \pm cosAsinB$

  2. $cos(A \pm B) = cosA\ cosB \mp sinA\ sinB$

  3. $tan(A \pm B) = \frac{tanA \pm tanB}{1 \mp tanAtanB}$; $tan\left(\frac{\pi}{4} \pm A\right) = \frac{1 \pm tanA}{1 \mp tanA}$

  4. $cot(A \pm B) = \frac{1 \mp cotAcotB}{cotB \pm cotA}$

  5. $sin(A+B)sin(A-B) = sin^2A - sin^2B = cos^2B - cos^2A$

6. $cos(A+B)cos(A-B) = cos^2A - sin^2B$

7. $\tan A \pm \tan B = tan(A \pm B)(1 \mp \tan A tan\ B)$

8. If $A = B + C$ then $\tan A - \tan B - \tan C = \tan A tan\ B tan\ C$

9. $\sin(A+B+C) = \sin A cos\ B cos\ C + \sin B cos\ C \cos A + sinC cos\ A cos\ B - \sin A sin\ B \sin C$

10. $\cos(A+B+C) = \cos A cos\ B cos\ C - \cos \mathrm{A} \sin B\ sin\ C - \cos B \sin C \sin A - \cos C\ sinA \sin B$

11. $tan(A+B+C) = \frac{\tan A + \tan B + \tan C - tanAtanBtanC}{1 - tanAtanB - tanBtanC - tanCtanA}$

❖ **Trigonometric- functions of multiple angles**

I. Double angle Formulae:

$$sin2A = 2sinAcosA = \frac{2tanA}{1+tan^2A}$$

$$cos2A = cos^2A - sin^2B = \frac{1-tan^2A}{1+tan^2A}$$

$$= 2\cos^2\mathrm{A} - 1$$

$$= 1 - 2sin^2A$$

$$2\cos^2\mathrm{A} = 1 + \cos 2\mathrm{A} \qquad ; \sin^2\mathrm{A} = \frac{1-\cos 2\mathrm{A}}{2}$$

$$tan2A = \frac{2tanA}{1-tan^2A}$$

Useful results

$$1 + cos(double) = 2cos^2(single) \quad \Leftrightarrow \quad cos^2(half) = \frac{1+cos(Full)}{2}$$

$$1 - cos2\theta = 2sin^2\theta \Leftrightarrow sin^2\frac{A}{2} = \frac{1-cosA}{2}$$

II. Triple angle Formulae:

$$sin3A = 3sinA - 4sin^3A. \qquad \Leftrightarrow Sin^3\theta = \frac{3Sin\theta - Sin3\theta}{4}$$

$$cos3A = 4cos^3A - 3cosA. \qquad \Leftrightarrow Cos^3\theta = \frac{3Cos\theta + Cos3\theta}{4}$$

$$tan3A = \frac{3tanA - tan^3A}{1-3tan^2A}$$

❖ **Transformation Formulae**

I. Sum/Difference into(⟹) Product

$$sinC + sinD = 2sin\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$$

$$sinC - sinD = 2cos\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$$

$$cosC + cosB = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$$

$$cosC - cosB = -2sin\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$$

II. Product into(⟹) Sum/Difference

$$2sinAcosB = sin(A+B) + sin(A-B)$$

$$2cosAsinB = sin(A+B) - sin(A-B)$$

$$2cosAcosB = cos(A+B) + cos(A-B)$$

$$-2sinA\,sinB = cos(A+B) - cos(A-B)$$

❖ **General equation of T-equations:**

Let α be the principle solution of T-equation $\Leftrightarrow \quad 0 \le \alpha < 2\pi$

If $sinx = sin\alpha$ then $x = n\pi + (-1)^n\alpha, n \in Z$

If $cosx = cos\alpha$ then $x = 2n\pi \pm \alpha, n \in Z.$

If $tanx = tan\alpha$ then $x = n\pi + \alpha, n \in Z$

CHAPTER 4
PRINCIPLE OF MATHEMATICAL INDUCTION

❖ ℕ is the smallest Inductive set.

❖ Let $P(n)$ be a statement(result) involving positive integer n.

❖ Principle of mathematical induction
If $P(1)$ is true and $P(m)\ true \implies P(m+1)\ is\ also\ true$ then $P(n)$ is true for all $n \in N.$

Some Standard result:

1. Sum of first n natural numbers is $\sum n = \frac{n(n+1)}{2}$

2. Sum of squares of first n natural numbers is $\sum n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{2}\frac{(2n+1)}{3}$

3. Sum of cubes first n natural numbers is $\sum n^3 = \frac{n^2(n+1)^2}{4} = \left(\frac{n(n+1)}{2}\right)^2$

4. Sum of first n odd natural numbers is $\sum(2n-1) = n^2$

CHAPTER 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

❖ Complex number $Z = x + iy$, where $x \to real\ part\ of\ Z = Re(Z)$

$y \to Imaginary\ part\ of\ (Z) = Im(Z)$

❖ $i = \sqrt{-1}$, $i^{4n} = 1, i^{4n+1} = i, i^{4n+2} = -1, i^{4n+3} = -i, \frac{1}{i} = -i$

❖ Conjugate of Z is $\bar{Z} = x - iy$. $Z + \bar{Z} = 2Re(Z)$; $Z - \bar{Z} = 2iIm(Z)$

❖ Multiplicative inverse of $Z = Z^{-1} = \frac{1}{Z} = \frac{\bar{Z}}{|Z|^2} = \frac{x-iy}{x^2+y^2}$

❖ Geometrically $Z = x + iy$ is represented by a point (x, y) in Argand Plane.

❖ Modulus of Z is $r = |Z| = \sqrt{x^2 + y^2} = \sqrt{(Re(Z))^2 + (Im(Z))^2}$

❖ $Z\bar{Z} = |Z|^2$, $|Z_1Z_2| = |Z_1||Z_2|$, $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$

❖ Amplitude (Argument) of $Z = x + iy$ is $amp(Z)$.

❖ First define dummy amplitude as $\tan\alpha = \left|\frac{y}{x}\right|$

Quadrant	$\theta = amp(Z)$
I Quadrant $(+,+)$	$\theta = \alpha$
II Quadrant $(-,+)$	$\theta = \pi - \alpha$
III Quadrant $(-,-)$	$\theta = -(\pi - \alpha)$
IV quadrant $(+,-)$	$\theta = -\alpha$

- $Arg(Z_1 Z_2) = Arg(Z_1) + Arg(Z_2)$
- $Arg\left(\frac{Z_1}{Z_2}\right) = Arg(Z_1) - Arg(Z_2)$.

Quadratic Equations: $ax^2 + bx + c = 0$

- Shridharacharya formulae: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant $\Delta = b^2 - 4ac$

Discriminant	Nature of roots
$\Delta > 0$	Real but distinct
$\Delta = 0$	Real & equal
$\Delta < 0$	Complex & conjugate

- Sum of roots $= -\frac{b}{a}$, Product of roots $= \frac{c}{a}$
- If m & n are roots of quadratic equation then quadratic equation is $x^2 - (m+n)x + mn = 0$
- Cube roots of unity $1, \omega, \omega^2$; where $\omega = \frac{-1+i\sqrt{3}}{2}$, $1 + \omega + \omega^2 = 1$, $\omega^3 = 1$
- Fourth roots of unity are $1, -1, i, -i$;$i^4 = 1$
- Square roots of $a + ib = x + iy$ then $x^2 - y^2 = a$ & $2xy = b$. Solving for x & y.

CHAPTER 6

LINEAR INEQUALITIES

- Strict inequality: Less than (<), greater than (>),
- Slack inequality: less than or equal (≤) ,greater than or equal to (≥)
- Linear inequalities in two variables x & y are of the form $ax + by < c, ax + by \leq c, ax + by > c, ax + by \geq c$
- For any two real numbers a & b, we have

  i. $ab = 0 \Longrightarrow either\ a = 0\ or\ b = 0$

ii. $ab > 0 \Rightarrow a > 0 \;\&\; b > 0 \;\; or \; a < 0 \;\&\; b < 0$

iii. $ab < 0 \Rightarrow only\ either\ a\ or\ b\ positive$ but not both

❖ For any positive real number a

i. $|x| = 0 \Leftrightarrow x = 0$

ii. $|x| = a \Leftrightarrow either\ x = a\ or\ x = -a$

iii. $|x| < a \Leftrightarrow -a < x < a \equiv x \in (-a, a)$

iv. $|x| \leq a \Leftrightarrow -a \leq x \leq a \equiv x \in [-a, a]$

v. $|x| \geq a \Leftrightarrow x \leq -a\ or\ x \geq a \equiv x \in (-\infty, -a] \cup [a, \infty)$

Basic rules of inequality:

❖ $ax + by < (\leq) c$ represents half plane (set of pts) lying below (& also on) the line $ax + by = c$

❖ $ax + by > (\geq) c$ represents a half plane (set of pts) lying above (& also on) the line $ax + by = c$.

CHAPTER 7

PERMUTATIONS AND COMBINATIONS

❖ Permutation is an arrangement of given objects in a definite order.

❖ No. of arrangements of n objects taken r at a time is

$^nP_r = n(n-1)(n-2) \ldots \ldots (n-r+1)$

❖ n Factorial: $n! = n(n-1)(n-2) \ldots \ldots 3 \times 2 \times 1$.

❖ $^nP_r = \frac{n!}{(n-r)!}$

❖ Distinct objects & without repetition

i. No. of permutations of n objects taking r at a time is ^nP_r

ii. No. of permutations of n objects taken all at a time is $^nP_n = n!$

❖ Distinct objects & with repetition

No. of permutations of n objects taking r objects <u>one by one</u> with repetition is n^r

- Objects are not distinct

  Among n objects, p_1 are of one kind, p_2 are of second kind, ………, p_k are of k^{th} kind then No. of permutation $\frac{n!}{p_1!p_2!p_3!....p_k!}$

- Combination (selections)

  A combination is a selection of some or all of given different objects (where order of selection is not important)

  Number of selection of r objects from the given n objects ${}^nC_r = \frac{n!}{(n-r)!} = {}^nP_r \div r!$

  ${}^nC_r = {}^nC_{r-1}$ $\quad$ $n.{}^{n-1}C_r = (n-r-1)\ {}^nC_r$

  ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

CHAPTER 8
BINOMIAL THEOREM

Binomial Theorem

- If n is a positive integer, then
  $(x + a)^n = {}^nC_a\ x^n + {}^nC_1\ x^{n-1} a + {}^nC_2\ x^{n-2} a^2 + \ldots + {}^nC_n\ a^n$

- **Observation from the binomial theorem**

(a) The number of terms in the expansion is one greater than the index if (x + a). That is there are (n + 1) terms.

(b) In the expansion of $(x + a)^n$, the power of ‘x’ in each term goes on decreases by one and whereas the power of ‘a’ goes on increases by one.

(c) In the expansion of $(x + a)^n$, the sum of the powers of x and a in each term is equal to n.

(d) The power of x in any terms is equal to difference of upper and lower suffixes of c. For example, the power x in third term is n – 2, which is the difference of n and 2, of ${}^nC_2$.

(e) The power of a in any term is equal to lower suffix of C. For example, the power of a in the third term is 2, which is the lower suffix of ${}^nC_2$.

❖ The general term of binomial expansion

The $(r + 1)^{th}$ term of the binomial expansion of $(x + a)^n$ is called the general term. This is given by $T_{r+1} = {}^nC_r\ x^{n-r} a^r$

By putting r = 0,1,2,…..we get different of $(x + a)^n$.

❖ Middle term (or terms) of the expansion

(a) If n is even in $(x + a)^n$ then there will be only one middle term in the expansion of $(x + a)^n$. The middle term will be $\left(\frac{n}{2} + 1\right)^{th}$ terms. This is given by $T\left(\frac{n}{2} + 1\right) = {}^nC_{\frac{n}{2}} \cdot x^{\frac{n}{2}} \cdot a^{\frac{n}{2}}$

(b) If n is odd then ther will be two middle terms in the expansion of $(x + a)^n$, The middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ term. These are given by

$${}^nC_{\frac{n+1}{2}} \cdot x^{\frac{n+1}{2}} \cdot a^{\frac{n-1}{2}} \text{ and } {}^nC_{\frac{n+1}{2}} \cdot x^{\frac{n-1}{2}} \cdot a^{\frac{n+1}{2}}$$

❖ Binomial coefficients

The coefficient ${}^nC_0,\ {}^nC_1,\ {}^nC_2,\ldots,\ {}^nC_n$ in the binomial expansion of $(x + a)^n$ are called binomial coefficient. these are denoted by $C_0, C_1, C_2,\ldots, C_n$. here C_r denotes ${}^nC_r$.

❖ Properties of binomial coefficients

a) $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$
b) $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
c) $C_0^{\,2} + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{(2n)!}{(n!)}$
d) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$
e) $C_0 + \frac{C_2}{3} + \frac{C_4}{5} + \ldots = \frac{2^n}{n+1}$
f) $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \ldots = \frac{2^n - 1}{n+1}$
g) $a \cdot C_0 + (a + d) \cdot C_1 + (a + 2d) \cdot C_2 + \ldots + (a + nd) \cdot C_n = (2a + nd) \cdot 2^{n-1}$
h) $C_1 + 2 \cdot C_2 + 3 \cdot C_3 + \ldots + n \cdot C_n = n \cdot 2^{n-1}$
i) $C_0 + 2 \cdot C_1 + 3 \cdot C_2 + \ldots + (n+1) \cdot C_n = (n+2) \cdot 2^{n-1}$
j) $C_0 + 3 \cdot C_1 + 5 \cdot C_2 + \ldots + (2n+1) \cdot C_n = (n+1) \cdot 2^{n-1}$

CHAPTER 9
SEQUENCE AND SERIES

- **Sequence**
  A succession of numbers arranged in a definite order according to a given certain rule is called sequence. A sequence is either finite or infinite depending upon the number of terms in a sequence.

- **Series**
  If $a_1, a_2, a_3, \ldots\ldots, a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + a_4 + \ldots + a_n$ is called series.

- **Progression**
  A sequence whose terms follow certain patterns are more often called progression.

- **Arithmetic Progression (AP)**
  A sequence in which the difference of two consecutive terms is constant, is called Arithmetic progression (AP).

- **Properties of Arithmetic Progression (AP)**
  If a sequence is an A.P. then its nth term is a linear expression in n i.e. its nth term is given by An + B, where A and S are constant and A is common difference.

- n^{th} term of an AP: If a is the first term, d is common difference and l is the last term of an AP then

  nth term is given by $a_n = a + (n-1)d$
  nth term of an AP from the last term is $a'_n = a_n - (n-1)d$
  $a_n + a'_n$ = constant
  Common difference of an AP i.e. $d = a_n - a_{n-1}$, ∀ n > 1.

- If a constant is added or subtracted from each term of an AR then the resulting sequence is an AP with same common difference.

- If each term of an AP is multiplied or divided by a non-zero constant, then the resulting sequence is also an AP.

- If a, b and c are three consecutive terms of an A.P then, 2b = a + c.

- Any three terms of an AP can be taken as (a – d), a, (a + d) and any four terms of an AP can be taken as (a – 3d), (a – d), (a + d), (a + 3d).

- Sum of n Terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a_1 + a_n)$

- A sequence is an AP If the sum of n terms is of the form $An^2 + Bn$, where A and B are constant and A = half of common difference i.e. 2A = d.

- $a_n = S_n - S_{n-1}$

- Arithmetic Mean
  If a, A and b are in A.P then $A = \frac{a+b}{2}$ is called the arithmetic mean of a and b.
  If $a_1, a_2, a_3, \ldots, a_n$ are n numbers, then their arithmetic mean is given by

$$A = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

- Geometric Progression (GP)
  A sequence in which the ratio of two consecutive terms is constant is called geometric progression. The constant ratio is called common ratio(r).
  i.e. $r = \frac{a_n+1}{a_n}$, ∀ n > 1

- Properties of Geometric Progression
  If a is the first term and r is the common ratio, then the general term or nth term of GP is $a_n = ar^{n-1}$.

- nth term of a GP from the end is $a'_n = \frac{1}{r^{n-1}}$, l = last term.
- If all the terms of GP be multiplied or divided by same non-zero constant, then the resulting sequence is a GP with the same common ratio.

- The reciprocal terms of a given GP form a GP.

- If each term of a GP be raised to some power, the resulting sequence also forms a GP

- If a, b and c are three consecutive terms of a GP then, $b^2 = ac$.

- Any three terms can be taken in GP as $\frac{a}{r}$, a and ar and any four terms can be taken in GP as $\frac{a}{r^3}, \frac{a}{r}$, ar and ar^3.
- Sum of n terms of G.P.

$$S_n = \begin{cases} a\dfrac{1-r^n}{1-r}, & if\ |r|<1 \\ a\dfrac{(r^n-1)}{r-1}, & if\ |r|>1 \\ a_n, & if\ |r|=1 \end{cases}$$

- Sum of an infinite G.P. is given by $S_\infty = \frac{a}{1-r}, |r| < 1$
- Geometric mean : If a, G and b are in GR then G is called the geometric mean of a and b and is given by $G = \sqrt{ab}$

  If $a, G_1, G_2 \dots\dots G_n, b$ are in GP then $G_1, G_2, \dots . G_n$ are in GM's between a and b, then common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

  If $a_1, a_2 \dots . a_n$ are n numbers are non zero and non negative, then their GM is given by $GM = (a_1 . a_2 . a_3 \dots\dots a_n)^{\frac{1}{n}}$

  Product of $n \times GM$ is $G_1 . G_2 \dots\dots G_n = G_n = (ab)^{\frac{n}{2}}$
- Sum of first n natural numbers is $\sum n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
- Sum of squares of first n natural numbers is $\sum n^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers is $\sum n^3 = 1^3 + 2^3 + \cdots . + n^3 = \left(\frac{n(n+1)(2n+1)}{6}\right)^2$.

CHAPTER 10
STRAIGHT LINES

- **Slope or gradient of a straight line**

  Making an angle θ with +ve direction of x-axis is $m = tan\theta$

  Passing through two points $A(x_1, y_1)$ & $B(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$

  Having equation $ax + by + c = 0$ is $m = \frac{-a}{b}$

  Slope of horizontal line(parallel to x-axis) is zero

Slope of vertical line(parallel to y-axis) is ND

- **Intercepts**

If a line meets the x-axis at $A(a,0)$ then OA(with proper sign)

is called $x - intercept$, denoted by a

If a line meets the y-axis at $B(0,b)$ then OB(with proper sign)

is called $y - intercept$, denoted by b

If equation of the line is $ax + by + c = 0$ then

$x - intercept = \frac{-c}{a}$ & $y - intercept = \frac{-c}{b}$

If intercepts are a & b then the line passes through the points $(a,0)$ &$(0,b)$

- **Different forms of equation of a straight line**

Slope point form:$(y - y_1) = m(x - x_1)$

Two- point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

Slope- intercept:$y = mx + c$

Double intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

Normal form: $xcos\omega + ysin\omega = p$

General form: $ax + by + c = 0$

- The equation of x-axis is $y = 0$;the equation of any line *parallel to x-axis* is of the form $y = k$
- The equation of y-axis is $x = 0$;the equation of any line *parallel to y-axis* is of the form $x = k$
- If θ is the acute angle between two lines with slopes m_1& m_2 then $tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$
- Condition for parallelism: $m_1 = m_2$
- Condition for perpendicularity: $m_1m_2 = -1$
- Any line parallel to the line $ax + by + c = 0$ is of the

form $ax + by + k = 0$.

- Any line perpendicular to the line $ax + by + c = 0$ is of the form $bx - ay + k = 0$.
- The point of intersection of two lines is obtained by solving the two equations.
- Equation of a line through the intersection of $L_1 = 0$ & $L_2 = 0$ is

  of the form $L_1 + kL_2 = 0$
- Length of the distance of a point (x_1, y_1) to a line $ax + by + c = 0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$
- Distance between the parallel lines $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$

  is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$
- Line and Points

  Sign of $ax + by + c$:

  - Two points $A(x_1, y_1)$ & $B(x_2, y_2)$ lie on the same side or opposite sides of the line $ax + by + c = 0$ according as $(ax_1 + by_1 + c)\ and\ (ax_2 + by_2 + c)$ are of same sign or opposite signs.
  - The ratio in which $ax + by + c = 0$ divides the join of $A(x_1, y_1)$ & $B(x_2, y_2)$ is $-\frac{ax_1+by_1+c}{ax_2+by_2+c}$.

CHAPTER 11

CONIC SECTIONS

- **Circles**

Circle is a path traced by a point in a plane such that distance of a point from fixed point is always constant.

Fixed point is center & fixed distance is radius

Equation of the circle with centre at (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

General equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Its centre $(-g, -f)$ & radius $r = \sqrt{g^2 + f^2 - c}$

❖ Parabola

Standard equation: $y^2 = 4ax,\ a > 0$

Symmetric about X-axis.

vertex $(0,0)$

Focus $(a, 0)$ on +ve X - axis

Open towards right side (+ direction of X-Axis)

Equation of latus rectum is $x = a$.

Equation of directrix is $x = -a$.

Length of latus rectum = 4a units.

Locus pt of parabola always equidistant from focus & directrix

Note: when vertex (h, k) equation will be $(y - k)^2 = 4a(x - h)$

Standard equation	$x^2 = 4ay$	$y^2 = -4ax$	$x^2 = -4ay$
Vertex	$(0,0)$	$(0,0)$	$(0,0)$
Symmetric about	Y-axis	X-axis	Y-axis
Open	Upward	Left side	Downward
Focus	$(0, a)$ on +ve Y	$(-a, 0)$ on –ve X	$(0, -a)$ on –ve Y
Eqn of directrix	$y = -a$	$x = a$	$y = a$
Latus Rectum	4a	4a	4a
Eqn of latus rectum	$y = a$	$x = -a$	$y = -a$
Vertex (h, k)	$(x - h)^2 = 4a(y - k)$	$(y - k)^2 = -4a(x$	$(x - h)^2 = -4a(y$
Note : Every coordinates & equations will be obtained by adding h to x coordinates & k to y coordinates			

❖ Ellipse

Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$

Symmetric about both axes.
Vertex $(0,0)$

Since $a > b$, Major axis is X-axis and minor axis is Y-axis

Equation of major axis is $y = 0$ and eqn of minor axis is $x = 0$.

Length of major axis =2a; Length of minor axis =2b

Distance between directrix = $\frac{2a}{e}$

Equation of directrix are $x = \pm\frac{a}{e}$

Eccentricity e always less than 1

$b^2 = a^2(1-e^2)$, $e < 1$, $ae = \sqrt{a^2-b^2}$

Focus: $S(ae,0), S'(-ae,0)$ on X-axis.

Distance between foci = $2ae$

Length of latus rectum $= \frac{2b^2}{a}$.

End points of latus rectum are $L\left(ae,\frac{b^2}{a}\right)$, $L'\left(ae,-\frac{b^2}{a}\right)$, $T\left(-ae,\frac{b^2}{a}\right), T'\left(-ae,-\frac{b^2}{a}\right)$

Equations of latus rectum are $x = \pm ae$

Note: when vertex (h,k) then equation will be $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

- **Hyperbola**

Standard equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $a > b$

Symmetric about both axes.

Vertex $(0,0)$

Focus: $S(ae,0), S'(-ae,0)$ on X-axis.

Since $a > b$, Transverse axis is X-axis and Conjugate axis is Y-axis

Eqn of transverse axis is $y = 0$ and eqn of conjugate axis is $x = 0$.

Length of transverse axis =2a

Distance between directrice = $\frac{2a}{e}$

Eqn of directrice are $x = \pm\frac{a}{e}$

Eccentricity e always greater than 1

$b^2 = a^2(e^2-1)$, $e > 1$, $ae = \sqrt{a^2+b^2}$

Focus: $S(ae,0), S'(-ae,0)$ on X-axis.

Distance between foci = $2ae$

Length of latus rectum $= \frac{2b^2}{a}$

End points of latus rectum are $L\left(ae, \frac{b^2}{a}\right), L'^{\left(ae, -\frac{b^2}{a}\right)}$, $T\left(-ae, \frac{b^2}{a}\right), T'^{\left(-ae, -\frac{b^2}{a}\right)}$

Equations of latus rectum are $x = \pm ae$

Rectangular hyperbola $x^2 - y^2 = a^2$ its eccentricity $e = \sqrt{2}$

Note: when vertex (h, k) then equation will be $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

CHAPTER 12

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

Three mutually perpendicular real number lines constitute a three dimensional rectangular coordinate system. Those three lines are $X - axis, Y - axis$ & $Z - axis$.

A pair of coordinate axis determine a plane, called coordinate plane. The coordinate-plane determined by $X - axis$ & $Y - axis$ is called $XY\ plane$, simillarly $YZ\ plane, ZX\ plane$.

The three coordinate planes divide the whole space into 8 equal parts, called ***Octants***.

Every point P in the 3-D space has three coordinates $P(x, y, z)$

(i) x -coordinate of P = Directed distance of pt P from $YZ - plane$

(ii) y -coordinate of P = Directed distance of pt P from $ZX - plane$

(iii) z -coordinate of P = Directed distance of pt P from $XY - plane$

Any point in $XY - plane$ is of the form $(x, y, 0)$

Any point in $YZ - plane$ is of the form $(0, y, z)$

Any point in $ZX - plane$ is of the form $(x, 0, z)$

Any point on $X - axis$ is of the form $(x, 0,0)$

Any point on $Y - axis$ is of the form $(0, y, 0)$

Any point on $Z - axis$ is of the form $(0,0, z)$

Distance between two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Three points A, B & C are said to be collinear iff $AB + BC = AC$

Section formulae: $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$

Internally in the ratio $m{:}n$ is $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Externally: $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

For $A(x_1, y_1, z_1)$ *and* $B(x_2, y_2, z_2)$, mid –point formulae $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Centroid of the triangle with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is

$$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

CHAPTER 13

LIMITS AND DERIVATIVES

1) Limit of a function $f(x)$ as x tends to a, is denoted by $\lim_{x\to a} f(x) = l$.

2) $\lim_{x\to a} f(x) = l$ means $f(x)$ is close to l when x is close to a.

3) Algebra of limits

Limit of constant is itself.

$$\lim_{x\to a}\big(f(x) \pm g(x)\big) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

Limits of sum = Sum of limits

$$\lim_{x\to a}\big(f(x).\, g(x)\big) = \lim_{x\to a} f(x). \lim_{x\to a} g(x)$$

Limits of product = product of limits

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ provided } \lim_{x\to a} g(x) = 0$$

4) Standard results

1. $\lim_{n\to\infty} \frac{1}{n^p} = 0\,, p > 0$; $\lim_{x\to\infty} \frac{1}{x} = 0$

2. $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1},\ n \in Q$

3. $\lim_{x\to 0} \frac{sinx}{x} = 1$ and $\lim_{x\to 0} \frac{cosx}{x} = 1$

4. $\lim_{x\to 0} \frac{tanx}{x} = 1$

5. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$, $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

6. $\lim_{x\to 0}\frac{1}{x} log(1+x) = 1$, $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

7. $\lim_{x\to a} \frac{a^x - 1}{x} = loga$, $\lim_{x\to a} \frac{e^x - 1}{x} = 1$

5) One sided limits

I. Left- hand Limit of f at $x = a$:

LHL = $f(a-) = \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h)$

II. Right- hand Limit of f at $x = a$:

RHL = $f(a+) = \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$

Existence of Limit:

Limit of f at $x = a$ exist **iff $LHL = RHL$ at $x = a$**

Differentiation

Let $y = f(x)$ be a continuous function then derivative of $y = f(x)$ with respect to x is denoted by $\frac{dy}{dx}$ $or\ f'(x)$ and is given by $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

$f(x)$ is differentiable at $x = a$ and given by $f'(a)$ or $\left(\frac{dy}{dx}\right)_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

1. $\lim_{n\to\infty}\frac{1}{n^p} = 0\,, p > 0$; $\lim_{x\to\infty}\frac{1}{x} = 0$

2. $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1},\ n \in Q$

3. $\lim_{x\to 0}\frac{sinx}{x} = 1$ and $\lim_{x\to 0}\frac{cosx}{x} = 1$

4. $\lim_{x\to 0}\frac{tanx}{x} = 1$

5. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$, $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

6. $\lim_{x\to 0}\frac{1}{x}log(1+x) = 1$, $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

7. $\lim_{x\to a}\frac{a^x-1}{x} = loga$, $\lim_{x\to a}\frac{e^x-1}{x} = 1$

5) One sided limits

I. Left- hand Limit of f at $x = a$:

$$\text{LHL} = f(a-) = \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h)$$

II. Right- hand Limit of f at $x = a$:

$$\text{RHL} = f(a+) = \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$$

Existence of Limit:

Limit of f at $x = a$ exist **iff $LHL = RHL$ at $x = a$**

Differentiation

Let $y = f(x)$ be a continuous function then derivative of $y = f(x)$ with respect to x is denoted by $\frac{dy}{dx}$ $or\ f'(x)$ and is given by $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

$f(x)$ is differentiable at $x = a$ and given by $f'(a)$ or $\left(\frac{dy}{dx}\right)_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

Derivative of $f(x)$ wrt x

$y = f(x)$	$\frac{dy}{dx}$ or $f'(x)$
Algebraic function	
x^n , $n \in \boldsymbol{Q}$	nx^{n-1}
x	1
x^2	$2x$
x^3	$3x^2$
x^{power}	$power\ (x)^{one\ less\ than\ power}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\frac{1}{x}$	$\frac{-1}{x^2}$
$\frac{1}{x^n}$	$\frac{-n}{x^{n+1}}$
Trigonometric Function	
$sinx$	$cosx$
$cosx$	$-sinx$
$tanx$	sec^2x
$cotx$	$-cosec^2x$
$secx$	$secx\ tanx$
$cosecx$	$-cosecx\ cotx$
Inverse Trigonometric Function	
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$\frac{-1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$cosec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
Exponential & Logarithmic Function	
$a^x, a > 0$	$a^x loga$
e^x	e^x
$logx$, $x > 0$	$\frac{1}{x}$

Rules of differentiation

1. $\frac{d}{dx}(constant) = 0$

2. $\frac{d}{dx}(kf) = k\frac{df}{dx}$

= Derivative (constant × function) = constant × Derivative(function)

3. $\frac{d}{dx}(f_1 \pm f_2) = \frac{df_1}{dx} \pm \frac{df_2}{dx}$ i.e. Derivative of sum = Sum of derivatives

4. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

5. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{d}{dx}\left(\frac{Nr}{Dr}\right) = \frac{(Dr)\frac{d(Nr)}{dx} - (Nr)\frac{d(Dr)}{dx}}{(Dr)^2}$

where, Dr is the Denominator and Nr is the Numerator

Laws of indices

1. $a^m a^n = a^{m+n}$,(while product,indices are added when base is same)
2. $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$ i.e. (while division, indices are added when base is same)
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Logarithmic Function

$log_b : R^+ \to R$ is defined as $log_b(x) = y$ iff $b^y = x$.

Simple log means $log_e = ln$

Properties of logarithm:

$log(mn) = logm + logn$,

log **of Product is same as sum of each log**

$log\left(\frac{m}{n}\right) = logm - logn$

log **of quotient is same as difference of each log**

$loga^m = mloga$ **, (power becomes factor)**

$a^{log_a X} = X$ **, for** $a > 0, X > 0$**. (both bases are same)**

IDENTITIES:

1. $(a \pm b)^2 = a^2 \pm 2ab + b^2$
2. $(a + b)(a - b) = a^2 - b^2$
3. $(x - a)(x - b) = x^2 - x(a + b) + ab$
4. $(x - a)(x - b)(x - c) = x^3 - x^2(a + b + c) + x(ab + bc + ca) - abc$
5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
8. $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

Factorizing Identities:

1. $a^2 - b^2 = (a + b)(a - b)$
2. $a^2 \pm 2ab + b^2 = (a \pm b)^2$
3. $x^2 + x(a + b) + ab = (x + a)(x + b)$
4. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)$$

7. $(a+b)^2 = (a-b)^2 + 4ab$

8. $(a-b)^2 = (a+b)^2 - 4ab$

9. $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$

Conditional Identity:

1. **If $\boldsymbol{a+b+c=0}$ then** $a^3+b^3+c^3 = 3abc$
2. **If $\boldsymbol{a+b+c=0}$ then** $a^2+b^2+c^2 = -2(ab+bc+ca)$

Partial Fraction:

The process of expressing given proper fraction into sum of two or more proper fractions.

Rules of Partial fractions of rational function $\frac{p(x)}{q(x)}$

In all cases, express denominator $q(x)$ as product of factors (factors may be linear/ repeated linear/quadratic/repeated quadratic)

Case 1: When Factors of Dr are ***linear*** but ***not repeated***

$$\frac{px+q}{(ax\pm b)(cx\pm d)(mx\pm n)} = \frac{A}{(ax\pm b)} + \frac{B}{(cx\pm d)} + \frac{C}{(mx\pm d)}$$

Case 2: When Factors of Dr are **linear** and ***repeated***

$$\frac{px+q}{(ax\pm b)^3} = \frac{A}{(ax\pm b)} + \frac{B}{(ax\pm b)^2} + \frac{C}{(ax\pm b)^3}$$

Using both case 1 & case 2

$$\frac{px+q}{(ax\pm b)(cx\pm d)(mx\pm n)^2} = \frac{A}{(ax\pm b)} + \frac{B}{(cx\pm d)} + \frac{C}{(mx\pm n)} + \frac{D}{(mx\pm n)^2}$$

Case 3: When Factors are non reducible **quadratic** but ***not repeated.***

$$\frac{px+q}{(ax^2+b)(cx^2+d)} = \frac{Ax+B}{ax^2+b} + \frac{Cx+D}{(cx^2+d)}$$

where $(\mathrm{a, b, c} \ \& \ d \ are\ of\ same\ sign)$

Note: $(x^2 + a^2), (x^2 + x + 1), (ax^2 + bx + c: b^2 - 4ac < 0)$ are some non-reducible quadratic.

CHAPTER 14

MATHEMATICAL REASONING

1) **Statements:** A statement is a sentence which either true or false, but not both simultaneously.
2) **Negaiton of a statement**: Negation of a statement p: If p denote a statement, then the negation of p is denoted by ~p.
3) **Compound statement**: A statement is a compound statement if it is made up of two or more smaller statements. The smaller statements are called component statements of the compound statement.
4) The Compound statements are made by:
   (i) **Connectives**: "AND", "OR"
   (ii) **Quantifiers**: "There exists", "For every"
   (iii) **Implications:** The meaning of implications "If ", "only if ", " if and only if ".
5) (a) "p⇒q" : p is sufficient condition for q or p implies q.
   q is necessary condition for p.
   The converse of a statement p ⇒ q is the statement q ⇒ p.
   p⇒q together with its converse, gives p if and only if q.
   (b) "p⇔q"
   A sentence with if p, then q can be written in the following ways.
   p implies q (denoted by p ⇒ q)
   p is a sufficient condition for q
   q is a necessary condition for p
   p only if q
   ~q implies ~p
6) **Contrapositive**: The contrapositive of a statement p ⇒ q is the statement ~ q ⇒~p.
7) **Contradiction**: If to check whether p is true we assume negation p is true.
8) **Validating statements**: Checking of a statement whether it is true or false. The validity of a statement depends upon which of the special.

The following methods are used to check the validity of statements:

(i) direct method

(ii) contrapositive method

(iii) method of contradiction

(iv) using a counter example.

CHAPTER 15

STATISTICS

1) **Mean:** $\bar{x} = \frac{x_1+x_2+\cdots\ldots\ldots\ldots+x_n}{2}$

2) **Median**: If the number of observations n is odd, then median is $\left(\frac{n+1}{2}\right)^{th}$ observation and if the number of observations n is even, then median is the mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n+1}{2}\right)^{th}$ observations.

3) **Measures of Dispersion, Range and Mean Deviation**

Range, Quartile deviation, mean deviation, variance, standard deviation are measures of dispersion.

Range = Maximum Value – Minimum Value

Mean deviation for ungrouped data

$$\text{M. D. } (\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

Mean Deviation from Median for ungrouped data

$$\text{M. D. } (M) = \frac{\sum |x_i - M|}{n}$$

Mean deviation for grouped data

$$\text{M. D. } (\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

Mean Deviation from Median for grouped data

$$\text{M. D. } (M) = \frac{\sum f_i |x_i - M|}{N}$$

Variance and standard deviation for ungrouped data

$$\text{Variance } (\sigma^2) = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$\text{Standard Deviation } \quad \sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

Variance and standard deviation of a discrete frequency distribution

$$\text{Variance } (\sigma^2) = \frac{1}{N}\sum (x_i - \bar{x})^2$$

$$\sigma = \sqrt{\frac{1}{N}\sum (x_i - \bar{x})^2}$$

Variance and Standard Deviation of a continuous frequency distribution

(i) If $\frac{x_i}{f_i}$, $i = 1,2,3, \ldots \ldots, n$ is a continuous frequency distribution of a variate X, then $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$

(ii) If $x_1, x_2, \ldots \ldots \ldots \ldots, x_n$ be the given n observations with respective frequencies $f_1, f, \ldots \ldots \ldots \ldots, f_n$, then, $\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$ where, $N = \sum f_i$.

(iii) If $d_i = x_i - A$, where A is assumed mean, then

$$\sigma^2 = \frac{1}{N}\left[\sum f_i {d_i}^2 - \left(\frac{\sum f_i d_i}{N}\right)^2\right]$$

(iv) If $u_i = \frac{x_i - A}{h}$, where h is the common difference of values of x, then

$$\sigma^2 = \frac{1}{N}\left[\sum f_i {u_i}^2 - \left(\frac{\sum f_i u_i}{N}\right)^2\right]$$

4) **Analysis of frequency distribution with equal means but different variances**: If the S.D. of group A < the S.D. of group B, then group A is considered more consistent or uniform.

5) **Ananlysis of frequency distribution with unequal means:** In this case we compare the coefficient of variation [Coefficient of variation (C.V. $=\frac{100\times S.D.}{Mean}$). The series having greater coefficient of variation is said to be more variable than the other.

6) Variance of the combined two series: $\sigma^2 = \frac{1}{n_1+n_2}\left[n_1\left({\sigma_1}^2 + {d_1}^2\right) + n_2\left({\sigma_2}^2 + {d_2}^2\right)\right]$ where n_1 and n_2 are the sizes of two groups, σ_1 and σ_2 are the S.D. of two groups, $d_1 = \bar{a} - \bar{x}$, $d_2 = \bar{b} - \bar{x}$ and $\bar{x} = \frac{n_1\bar{a} + n_2\bar{b}}{n_1+n_2}$.

CHAPTER 16

PROBABILITY

1) **Coin**: On tossing a coin there are two possibilities either head may come up or tail may come up.

2. **Die**: A die is a well balanced cube with its six faces marked with numbers (dots) from 1 to 6, one number on the one face. The plural of die is dice.

3. **Cards**: A pack of cards consists of four suits i.e.., Spades, Hearts, Diamonds and Clubs. Each suit consists of 13 cards, nine cards numbered 2, 3, 4, ..., 10 and an Ace, a King, a Queen and a Jack or Knave. Colour of Spades and Clubs is

black and that of Hearts and Diamonds is red. Ace, King, Queen and Jack cards are called Face cards.

4. **Random Experiments:** An experiment, whose outcomes cannot be predicted in advance is called a Random experiment. For example, on tossing a coin, we cannot predict whether head will come up or tail will come up.
5. **Event**: Every subset of a sample space is called an Event.

6. **Types of Events:**

Simple Event: Single element of the sample space is called a Simple event. It is denoted by S.

Compound Event: Compound event is the joint occurrence of two or more events.

Sure Event: In a sure event, a set of all the favorable outcomes is the sample event itself. Its probability is always 1.

Impossible Event: If E is an impossible event, then S E = and the probability of impossible event is 0.

Equally Likely Events: Two events are said to be equally likely, if none of them is expected to occur in preference to the other. For example, if we toss a coin, each outcome head or tail is equally likely to occur.

Mutually Exclusive Event: Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cup E_2 = \emptyset$. On tossing a coin two events are possible, (i) coming up a head excludes coming of a tail, (ii) coming up a tail excludes coming of a head. Coming of a head and coming of a tail are mutually exclusive events.

Independent Events: Occurrence of one event does not depend on the occurrence of other. For example, on tossing two coins simultaneously occurrence of one toss does not depend upon the occurrence of the second one.

Exhaustive Events: Exhaustive events consist of all possible outcomes.

Complement of an Event: The complement of an event E with respect to the sample space S is the set of all elements of S, which are not in E. The compliment of E is denoted by E' or $\bar{E}$.

$E \cap E' = \emptyset$ or $E \cup \bar{E} = \emptyset$ and $P(\bar{E}) = 1 - P(E)$

Probability of an Event: $P(A) = \frac{Number\ of\ outcomes\ favourable\ to\ A}{Total\ number\ of\ possible\ outcomes}$

Probability of an event $P(A) = \frac{n(A)}{n(S)}$

where $n(A)$ = number of elements in the set A, $n(S)$= number of elements in the set S.

Probability: Number $P(\omega_i)$ associated with sample point ω_i such that

(i) $0 \leq P(\omega_i) \leq 1$

(ii) $\sum P(\omega_i)$ for all $\omega_i \in S = 1$

(iii) $P(A) = \sum P(\omega_i)$ for all $\omega_i \in A$

Odds: If an event E occurs in the m ways and does not occur in n ways, then

(i) odds in the favour of the events $= \frac{m}{n}$

(ii) odds against the event $= \frac{n}{m}$

(iii) $P(E) = \frac{m}{m+n}$

Addition law of probability:

If A and B are any two events associated with an experiment, then

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

equivalently,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

and $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(A \cap B \cap C)$

Multiplication law of probability:

$$P(A \cap B) = P(A) \times P(B)$$

*********END*********

CHAPTER-1
SETS RELATIONS AND FUNCTIONS

SETS:

- **Definition:-** It is a collection of "well defined objects". Where objects may be anything like numbers, Letters, Books, Persons etc.
  The sets are usually denoted by the capital letters A, B, X, Y etc., and its elements by small letters x, y, a, b etc.,

Some important results

- $n\,(A\cup B) = n(A) + n(B) - n\,(A\cap B)$
- $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(c\cap A) + n\,(A\cap B\cap C)$
- $n\,(A - B) = n\,(A) - n(A\cap B)$
- $n(A'\cup B') = n(A\cap B)'$
- $n(A'\cap B') = n(A\cup B)'$
- $n(A') = n(U) - n(A)$ where U is the Universal Set
- $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ (Distributive law)
- $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

- **Ordered Pairs:-** An orders pair consists of 2 elements say a and b, where a is called *first element* and b is called *second element* and it is denoted by (a, b)
- **Cartesian product:-**The Cartesian product of two sets A and B is the set of all ordered pair (a,b) such that $a\in A\ and\ b\in B$ and it is denoted by $A\times B$ **i.e.** $A\times B = \{(a,b)|\ a\in A\ and\ b\in B\}$
- **Note:**
  1) If $n(A) = m\ and\ n(B) = n$ then $n(A\times B) = mn$
  2) $A\times(B\cup C) = (A\times B)\cup(A\times C)$
  3) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

RELATIONS:

- **Definition:-** Given any two non empty sets A and B a relation R from A to B is defined as sub set of$A\times B$. **i.e.** $R = \{(x,y)|\ x\in A\ and\ y\in B\}$.
- **Empty Relation:** Let, $\emptyset\subseteq A\times B$ and R be the relation from A to B if $R = \{\,\}$ then R is called empty relation.

- **Universal Relation:-**$A \times B \subseteq A \times B$ and R be the relation from A to B if $R = A \times B$ then R is called universal relation.
- **Domain and Range of a relation:** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called domain of R and while the set of all second components or coordinates of the ordered pairs belonging to R is called range of R.
  Thus, Domain of $(R) = \{a: (a, b) \in R\}$ and Range of $(R) = \{b: (a, b) \in R\}$
- **Inverse Relation:** The inverse of the relation R, is $R^{-1} = \{(b, a): (a, b) \in R\}$.

Types of Relation

1) **Reflexive relation:-** A relation R on set A is said to be reflexive if $(a, a) \in R$ for all $a \in A$.
2) **Identity relation:-** A relation R on set A is said to be identity if $(a, a) \in R$ for all $a \in A$ and $(a, b) \notin R$.
3) **Symmetric relation:-** A relation R on set A is said to be symmetric if $(a, b) \in R \Longrightarrow (b, a) \in R$.
4) **Anti-symmetric relation:** A relation R on set A is said to be anti-symmetric if $(a, b) \in R$ &$(b, a) \in R \Longrightarrow a = b$ or $(a, b) \in R$ &$(b, a) \notin R \Longrightarrow a \neq b$.
5) **Transitive relation:-** A relation R on set A is said to be transitive if $(a, b) \in R$ and $(b, c) \in R \Longrightarrow (a, c) \in R$.
6) **Equivalence relation:-** A relation R on set A is said to be equivalence if R is reflexive, symmetric and transitive.

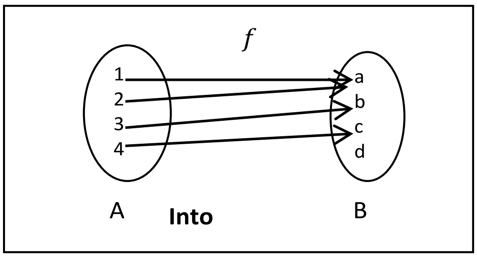
- **Equivalence classes of an equivalence relation:** Let R be equivalence relation on a non-empty set A. Let $a \in A$. Then the equivalence class of a denoted by $[a]$ or $\{\bar{a}\}$ is defined as the set of all points of A which are related to a under the relation R. Thus $[a] = \{x \in A: xRa\}$.

FUNCTIONS

- **Definition:-** Given two non empty sets A and B a function $f: A \to B$ (read it as f from A to B) is a rule which associates every element of the set A with a unique element of B.
- **Domain, Co-Domain and Range of the function:-** In a function $f: A \to B$, the set A is called **Domain**, the set B is called **co-domain** and the set of all images is called **range** of f.

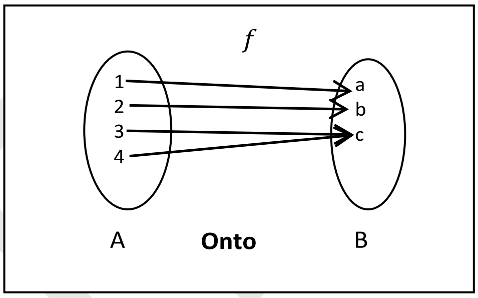
Different types of Functions:-

1) **Into Function:-** A function is said to be into function if some element of co-domain are not a images. i.e. the range of $f(A)$ is Proper subset of co-domain B. In the diagram the function $f: A \rightarrow B$ is an into function. Because the range $\{a, b, c\}$ is the proper subset of B.



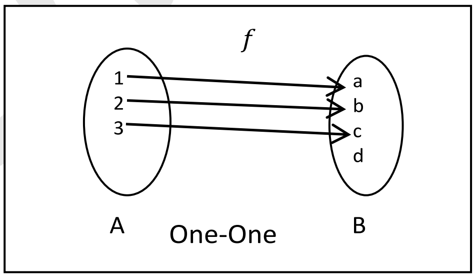
2) **Onto (Surjective) Function:-** A function is said to be onto function if all the elements of co-domain are images. i.e. $f(A) = B$.
In the diagram the function $f: A \rightarrow B$ is an onto function, because all the elements of B are images. i.e. $f(A) = \{a, b, c,\} = B$.

Note: If $f : A \rightarrow B$ is onto function then $n(A) \geq n(B)$.

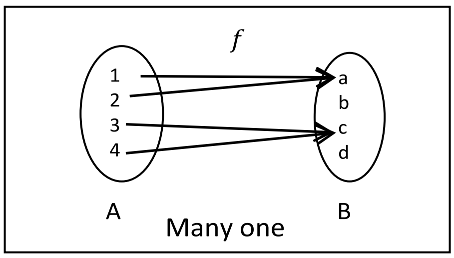


3) **One –One (injective) Function:-** A function is said to be one-one function if different elements of domain have different images. i.e. $f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$ or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
In the diagram the function $f: A \rightarrow B$ is an one-one function, because different elements of A have different images in B.

Note: If $f: A \rightarrow B$ is one-one function then $n(A) \leq n(B)$.

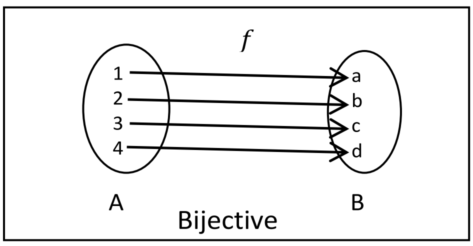


4) **Many One Function:-** A function is said to be Many-one function if 2 or more elements of domain associates with one element of co-domain.
In the diagram the function $f: A \rightarrow B$ is an many-one Function. Because '1' and '2' of domain associates with 'a' of co-domain and '3' and '4' of domain associates with 'c' of co-domain.



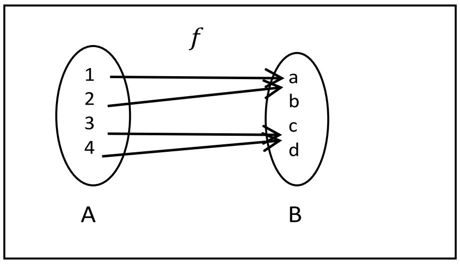
5) **Bijective (one-one and onto):-** A function is said to be Bijective function if it is both one-one and onto. In the diagram the function $f: A \rightarrow B$ is Bijective function. Because $f: A \rightarrow B$ is both one-one and onto

Note: If $f: A \rightarrow B$ is bijective function then $n(A) = n(B)$.



6) **Real Value Function:-** A function is said to be real value function if its domain and range are subsets of the set of real numbers.

❖ **Inverse of a element:-** Let $A = \{1,2,3,4\}$ and $B = \{a, b, c, d\}$ now a function $f: A \to B$ is defined



by $f(1) = a, f(2) = a, f(3) = c \ and \ f(4) = c$
Now 1, 2 are called inverse element of 'a' in symbol it is written as $f^{-1}(a) = \{1,2\}$. Similarly, $f^{-1}(c) = \{3,4\}$, $f^{-1}(b) = \phi$ and $f^{-1}(d) = \phi$.

7) **Inverse Function:-** The inverse of the function exists if and only if the function is Bijective (one-one and onto) i.e. If $f: A \to B$ is a Bijective function, then and then inverse function i.e. $g: B \to A$ exist.

8) **Composite Function:-** Let A, B, C be three non-empty sets and $f: A \to B$ and $g: A \to C$ be the two functions, let $x \in A$ and let it be associated with $y \in B$ under function f then, $y = f(x)$ ……… (1). Let, $y \in B$ associated with $z \in C$ under a function g then $z = g(y)$ ………… (2).
Now, from (1) and (2) $z = g(y) = g(f(x))$, so z is the image of x under a new function which is called composite function and it is denoted by gof.
Thus $gof: A \to C$ is a composite function is defined by $gof(x) = g(f(x))$
Similarly, a composite function fog is defined by $fog(x) = f(g(x))$.

❖ **Algebra of Functions:**

1) Scalar multiplication of a function: $(cf)(x) = c(f(x))$, where c is any constant.
2) Addition/Subtraction of a function: $(f \pm g)(x) = f(x) \pm g(x)$
3) Multiplication of function: $(fg)(x) = f(x)g(x)$
4) Division of function: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

❖ **Domain and Range of some standard functions:**

Function	Domain	Range
Polynomial Function $\boldsymbol{f(x) = a_n x^n + \ldots\ldots + a_1 x + a_0}$	R	R
Identity Function($\boldsymbol{I_x = x}$ **)**	R	R
Constant Function $\boldsymbol{(f(x) = k)}$	R	$\{k\}$
Reciprocal function $\boldsymbol{\left(f(x) = \frac{1}{x}\right)}$	$R - \{0\}$	$R - \{0\}$
Singnum Function:	R	$\{-1,0,1\}$

$f(x) = \begin{cases} 1 \; if \; x > 0 \\ 0 \; if \; x = 0 \\ -1 \; if \; x < 0 \end{cases} = \begin{cases} \frac{\lvert x \rvert}{x} \; if \; x \neq 0 \\ 0 \; if \; x = 0 \end{cases}$		
Modulus function: $f(x) = \lvert x \rvert = \begin{cases} x \; if \; x \geq 0 \\ -x \; if \; x < 0 \end{cases}$	R	$R^+ \cup \{0\}$
Greatest Integer Function: $f(x) = [x]$	R	Z
Exponential Function: $f(x) = a^x$	R	R^+
$f(x) = log\, x$	R^+	R
$f(x) = sin\, x$	R	$-1 \leq x \leq 1$
$f(x) = cos\, x$	R	$-1 \leq x \leq 1$
$f(x) = tan\, x$	$R - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\ldots\right\}$	R
$f(x) = cot\, x$	$R - \{0, \pm\pi, \pm 2\pi, \ldots\ldots\}$	R
$f(x) = sec\, x$	$R - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\ldots\right\}$	$x \geq 1, x \leq -1$
$f(x) = cosec\, x$	$R - \{0, \pm\pi, \pm 2\pi, \ldots\ldots\}$	$x \geq 1, x \leq -1$
$f(x) = x^2$	R	$R^+ \cup \{0\}$
$f(x) = \sqrt{x}$	$R^+ \cup \{0\}$	$R^+ \cup \{0\}$
$f(x) = x - [x]$	R	$[0,1)$
$f(x) = sinh\, x$	R	R
$f(x) = cosh\, x$	R	$[1, \infty)$
$f(x) = tanh\, x$	R	$(-1,1)$
$f(x) = coth\, x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, -1) \cup (1, \infty)$
$f(x) = sech\, x$	R	$(0,1]$
$f(x) = cosech\, x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$

- **Even Function:** A function $f(x)$ is said to be even function iff $f(-x) = f(x)$.
- **Odd Function:** A function $f(x)$ is said to be odd function iff $f(-x) = -f(x)$
- **Binary Operations:-** On a non-empty set A, if $\forall\, a, b \in A, a * b \in$ A and $a * b$ is unique then $*$ is called a binary operation.
- If S be a non-empty set and $*$ be a binary operation on it then
  1) Closure: $a * b \,\epsilon\, S \;\; \forall\, a, b \,\epsilon\, S$
  2) Commutative: $a * b = b * a$, for all $a, b \in S$.
  3) Associativity: $(a * b) * c = a * (b * c), \forall\, a, b, c \,\epsilon\, S$
  4) Existence of identity element: There exists an element $e \in S$ such that $a * e = e * a = a$

CHAPTER-II
INVERSE TRIGONOMETRIC FUNCTIONS

1. **Inverse Function:-** If f is a function from A to B i.e. $f: A \to B$, then the inverse function $f^{-1}: B \to A$ exist iff, f is one-one and onto (Bijective).
2. Domain and range of the inverse trigonometric function as follows

Functions	Domain	Range
$y = sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = tan^{-1}x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = sec^{-1}x$	$x \le -1, x \ge 1$	$0 \le y \le \pi \left(y \ne \frac{\pi}{2}\right)$
$y = cosec^{-1}x$	$x \le -1, x \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2} (y \ne 0)$
$y = cot^{-1}x$	$-\infty < x < \infty$	$0 < y < \pi$

3. **Properties of Inverse Trigonometric Function:-**

1. $sin\, sin^{-1} x = x, -1 \le x \le 1$ and $sin^{-1}sinx = x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$
2. $\cos cos^{-1} x = x, -1 \le x \le 1$ and $cos^{-1}cosx = x, 0 \le x \le 2\pi$
3. $\tan \tan^{-1} x = x, -\infty < x < \infty$ and $\tan^{-1}\tan x = x, -\frac{\pi}{2} < x, < \frac{\pi}{2}$
4. $sin^{-1}(-x) = -sin^{-1}x, -1 \le x \le 1$
5. $cos^{-1}(-x) = \pi - cos^{-1}x , -1 \le x \le 1$
6. $tan^{-1}(-x) = -tan^{-1}x , -\infty < x < \infty$
7. $sec^{-1}(-x) = \pi - sec^{-1}x,\ x \le -1, x \ge 1$
8. $cosec^{-1}(-x) = -cosec^{-1}x,\ x \le -1, x \ge 1$
9. $cot^{-1}(-x) = \pi - cot^{-1}x,\ -\infty < x < \infty$
10. $\sin^{-1} x = cosec^{-1}\frac{1}{x}. -1 \le x \le 1\ \&\ x \ne 0$
11. $\cos^{-1} x = sec^{-1}\frac{1}{x}. -1 \le x \le 1\ \&\ x \ne 0$
12. $\sec^{-1} x = cos^{-1}\frac{1}{x}\ ,\ x \le -1, x \ge 1$

13. $\text{cosec}^{-1} x = sin^{-1}\frac{1}{x}$, $x \leq -1, x \geq 1$
14. $\tan^{-1}x = cot^{-1}\frac{1}{x}$, $if\ x > 0$ and $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right) -$ $, if\ x < 0$
15. $\cot^{-1}x = tan^{-1}\frac{1}{x}$, $if\ x > 0$ and $\cot^{-1}x =$ $+ \tan^{-1}\left(\frac{1}{x}\right)$ $, if\ x < 0$
16. $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$ $, -1 \leq x \leq 1$
17. $sec^{-1}x + cosec^{-1}x = \frac{\pi}{2}$ $, x \leq -1, x \geq 1$
18. $tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$ $, -\infty < x < \infty$

4. Standard Formulae:-

1. $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $x \geq 0, y \geq 0\ and\ xy < 1$.
2. $tan^{-1}x + tan^{-1}y = \pi + tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $x \geq 0, y \geq 0\ and\ xy > 1$.
3. $2tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $0 \leq x \leq 1$.
4. $2tan^{-1}x = \pi + tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $|x| > 1$.
5. $tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $x \geq 0, y \geq 0$.
6. $2\ sin^{-1}x = sin^{-1}\left(2x\sqrt{1-x^2}\right)$
7. $2\ cos^{-1}x = cos^{-1}(2x^2 - 1)$
8. $sin^{-1}x + sin^{-1}y = cos^{-1}\left(\sqrt{1-x^2}\sqrt{1-y^2} - xy\right)$ if $x \geq 0, y \geq 0$.
9. $cos^{-1}x + cos^{-1}y = cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ if $x \geq 0, y \geq 0$.
10. $sin^{-1}x + cos^{-1}y = cos^{-1}\left(y\sqrt{1-x^2} - x\sqrt{1-y^2}\right)$ if $x \geq 0, y \geq 0$.
11. $sin^{-1}x - sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ if $x \geq 0, y \geq 0$.
12. $cos^{-1}x - cos^{-1}y = sin^{-1}\left(y\sqrt{1-x^2} - x\sqrt{1-y^2}\right)$ if $x \geq 0, y \geq 0$.
13. $sin^{-1}x - cos^{-1}y = sin^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ if $x \geq 0, y \geq 0$
14. If $tan^{-1}x + tan^{-1}y = \frac{\pi}{4}$ then $x + y + xy = 1$
15. If $tan^{-1}x + tan^{-1}y = \frac{\pi}{2}$ then $xy = 1$
16. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$ then $x + y + z = xyz$
17. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$ then $xy + yz + zx = 1$
18. If $x^2 + y^2 + z^2 = r^2$, then $\tan^{-1}\frac{xy}{zr} + \tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{zx}{yr} = \frac{\pi}{2}$
19. If $sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ then $x^2 + y^2 = 1$
20. If $cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi$ then $x^2 + y^2 + z^2 + 2xyz = 1$
21. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \frac{\pi}{2}$ then $x^2 + y^2 + z^2 + 2xyz = 1$
22. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \pi$ then $x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$

23. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then

$$x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$$

24. $If\ \cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3$ then $x = y = z = -1$

25. $If\ \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3}{2}$ then $x = y = z = 1$

26. $\cos^{-1}\sqrt{x} + \cos^{-1}\sqrt{1-x} = \frac{\pi}{2}, \forall\, 0 < x < 1$

27. $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$ then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = sin^2\alpha$

28. $sin^{-1}\frac{x}{a} + \sin^{-1}\frac{y}{b} = \alpha$then $\frac{x^2}{a^2} + \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = sin^2\alpha$

29. $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$

30. $\sin^{-1}\frac{1}{\sqrt{n}} - \sin^{-1}\frac{1}{\sqrt{n+1}} = \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}$

CHAPTER-III

MATRICES

1. A matrix is said to have an ordered rectangular array of functions or numbers. A matrix of order $m \times n$ consists of m rows and n columns.
2. An $m \times n$ matrix will be known as a square matrix when $m = n$.
3. $A = [a_{ij}]_{m \times m}$ will be known as diagonal matrix if $a_{ij} = 0$, when $i \neq j$.
4. $A = [a_{ij}]_{n \times n}$ is a scalar matrix if $a_{ij} = 0$, when $i \neq j$, $a_{ij} = k$, (where k is some constant); and $i = j$.
5. $A = [a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = 1$, when $i = j$ and $a_{ij} = 0$, when $i \neq j$.
6. A zero matrix will contain all its element as zero.
7. $A = [a_{ij}] = [b_{ij}] = B$ if and only if:

   (i) A and B are of the same order

   (ii) $a_{ij} = b_{ij}$ for all the certain values of i and j

8. Some basic operations of matrices:

   (i) $kA = k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$

   (ii) $-A = (-1)A$

   (iii) $A - B = A + (-1)B$

   (iv) $A + B = B + A$

(v) (A + B) + C = A + (B + C); where A, B and C all are of the same order

(vi) k(A + B) = kA + kB; where A and B are of the same order; k is constant.

(vii) (k + l)A = kA + lA; where k and l are the constant.

9. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$, then
$AB = C = [c_{ik}]_{m \times p}$

(i) A.(BC) = (AB).C

(ii) A(B + C) = AB + AC

(iii) (A + B)C = AC + BC

10. If $A= [a_{ij}]_{m \times n}$, then A' or AT = $[a_{ji}]_{n \times m}$

(i) (A')' = A

(ii) (kA)' = kA'

(iii) (A + B)' = A' + B'

(iv) (AB)' = B'A'

11. A is said to known as a symmetric matrix if A′ = A

12. A is said to be the skew symmetric matrix if A′ = –A

CHAPTER -IV

DETRMINANTS

1. The determinant of a matrix $A = [a_{11}]_{1 \times 1}$ can be given as: $|a_{11}| = a_{11}$.
2. For any square matrix A, the |A| will satisfy the following properties:

(i) |A′| = |A|, where A′ = transpose of A.

(ii) If we interchange any two rows (or columns), then sign of determinant changes.

(iii) If any two rows or any two columns are identical or proportional, then the value of the determinant is zero.

(iv) If we multiply each element of a row or a column of a determinant by constant k, then the value of the determinant is multiplied by k.

3. Determinant of a matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ can be expresses as

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

4. Area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is:

$$\Delta = \frac{1}{2}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

5. Cofactor of aij of given by $A_{ij} = (-1)^{i+j} M_{ij}$

6. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $adj\ A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ where

A_{ij} is the cofactor of a_{ij}.

7. Inverse of a matrix A is , $A^{-1} = \frac{1}{|A|}(adj\ A)$

8. For a square matrix A in matrix equation AX = B

(i) $|A| \neq 0$, there exists unique solution

(ii) $|A| = 0$ and (adj A) B $\neq 0$, then there exists no solution

(iii) $|A| = 0$ and (adj A) B = 0, then the system may or may not be consistent.

CHAPTER V

CONTINUITY AND DIFFERENTIABILITY

1) A function is said to be continuous at a given point if the limit of that function at the point is equal to the value of the function at the same point.

2. Properties related to the functions

(i) $(f \pm g)(x) = f(x) \pm g(x)$ is continuous.

(ii) $(f.g)(x) = f(x).g(x)$ is continuous.

(iii) $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$ is continuous.

3) Mean value theorem: If $f: [a, b] \to R$ is continuous on $[a, b]$ and differentiable on

(a, b) then there exists some c in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

4) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

5) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

6) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

7) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

8) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{1-x^2}}$

9) $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

10) $\frac{d}{dx}(e^x) = e^x$

11) $\frac{d}{dx}(logx) = \frac{1}{x}$

12) The equation of the tangent at (x_0, y_0) to the curve y=f(x) is $y - y_0 = f'(x_0)(x - x_0)$

13) Slope of the tangent $\frac{dy}{dx} = tan\theta$.

14) The equation normal to the curve y=f(x) at (x_0, y_0) is $(y - y_0)f'(x_0) + (x - x_0) = 0$

15) Slope of the normal = $\frac{-1}{slope\ of\ the\ tangent}$

CHAPTER-VI

INTEGRALS

1) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

2) $\int cosx\, dx = sinx + c$

3) $\int sinx\, dx = -cosx + c$

4) $\int \sec^2 x\, dx = tanx + c$

5) $\int cosec^2x\, dx = -cotx + c$

6) $\int secx\, tanx\, dx = secx + c$

7) $\int cosecx\, cotx\, dx = -cosecx + c$

8) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

9) $\int \frac{1}{\sqrt{1-x^2}} dx = -\cos^{-1} x + c$

10) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

11) $\int \frac{1}{1+x^2} dx = -cot^{-1} x + c$

12) $\int e^x dx = e^x + c$

13) $\int a^x dx = \frac{a^x}{loga} + c$

14) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

15) $\int x \frac{1}{\sqrt{x^2-1}} dx = -\text{cosec}^{-1} x + c$

16) $\int \frac{1}{x} dx = \log|x| + c$

17) $\int tanx\ dx = \log|secx| + c$

18) $\int cotx\ dx = \log|sinx| + c$

19) $\int secx\ dx = \log|secx + tanx| + c$

20) $\int cosecx\ dx = \log|cosecx - cotx| + c$

21) $\int \frac{1}{x^2-a^2}\ dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

22) $\int \frac{1}{a^2-x^2}\ dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

23) $\int \frac{1}{x^2+a^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

24) $\int \frac{1}{\sqrt{x^2-a^2}}\ dx = \log\ |x + \sqrt{x^2 - a^2}| + c$

25) $\int \frac{1}{\sqrt{x^2+a^2}}\ dx = \log\ |x + \sqrt{x^2 + a^2}| + c$

26) $\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

27) $\sqrt{x^2 - a^2}\ dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

28) $\sqrt{x^2 + a^2}\ dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

29) $\sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

CHAPTER-VII
APPLICATION OF INTEGRALS

1) Area bounded by the curve y=f(x); $x -$ axis and the lines $x = a$ and $x = b$ is given by the formula $Area = \int_a^b y\ dx.$
2) Area of the region bounded by the curve x=g(y); y-axis and the lines y=c, y=d is given by $Area = \int_c^d x\ dy.$
3) The area enclosed in between the two given curves $y = f(x)$, $y = g(x)$ and the lines $x = a, x = b$ is given by $Area = \int_a^b [f(x) - g(x)]dx$ where $f(x) \geq g(x)$ in $[a, b]$.
4) If $f(x) \geq g(x)\ in\ [a, c]$ and $f(x) \leq g(x)\ in\ [c, b],\ a < c < b$ then,
   $Area = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$

CHAPTER VIII
DIFFERENTIAL EQUATUONS

1) Differential equation is an equation involving derivatives of dependent variable with respect to independent variable.

2) An equation of the form $\frac{dy}{dx} + Py = Q$ where P and Q are the functions of x only is called a linear differential equation in y. Solution of this differential equation is $y(I.F) = \int Q(I.F)dx + c$, where I.F.$= e^{\int pdx}$.

3) An equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are the functions of y only is called a linear differential equation in x. Solution of this differential equation is $x(I.F) = \int Q(I.F)dy + c$, where I.F.$= e^{\int pdy}$.

CHAPTER IX
VECTOR ALGEBRA

1) The position vector of a point $P(x, y, z)$ is given by $\overrightarrow{OP} = \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

2) The scalar product of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them Is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$.

3) If two vectors $\vec{a}$ and b are given in its component forms as $\vec{a} = a_1 i + a_2 j + a_3 k$ and $\vec{b} = b_1 i + b_2 j + b_3 k$ and λ as the scalar part then

(i) $\vec{a} + \vec{b} = (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$

(ii) $\lambda\vec{a} = \lambda a_1 i + \lambda a_2 j + \lambda a_3 k$

(iii) $\vec{a}.\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

(iv) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(v) $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a}.\vec{b} = 0$

4) Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is given by $\vec{a}.\frac{\vec{b}}{|\vec{b}|}$

5) The vector product of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|sin\theta\hat{n}$

6) Two vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} \times \vec{b} = 0$

CHAPTER X
THREE DIMENSIONAL GEOMETRY

1) If l, m, n are direction cosines of a line then $l^2 + m^2 + n^2 = 1$

2) The direction cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$
where $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

3) Equation of a line through a point (x_1, y_1, z_1) and having direction cosines l, m, n is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

4) The vector equation of a line which passes through two points whose position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

5) The distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $\left|\frac{(\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}\,|}\right|$

6) Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 X \vec{b}_2) = 0$

CHAPTER XI
PROBABILITY

1) The conditional probability of an event E holds the value of the occurrence of the event F as $P\left(\frac{E}{F}\right) = \frac{E\cap F}{P(F)}$, $P(F) \neq 0$.

2) Baye's theorem: If $E_1, E_2, \ldots\ldots\ldots . E_n$ are the events constituting in a sample space S then, $P\left(\frac{E_i}{A_i}\right) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$

3) Mean $E(X) = \sum_{i=1}^{n} x_i p_i$, Variance $V = E(X^2) - \left(E(X)\right)^2$

4) Binomial distribution $B(n,p), P(X = x) = n_{C_x}\, q^{n-x} p^x$ where $x = 0,1,2, \ldots\ldots\ldots$ and $q = 1 - p$.

*********END*********

UNIT 1 SOME BASICS OF CHEMISTRY

1. $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

2. $\text{Mass \% of an element} = \dfrac{\text{Mass of that element in the compound} \times 100}{\text{Molar mass of the compound}}$

3. $\text{Mass percent} = \dfrac{\text{Mass of solute}}{\text{Mass of solution}} \times 100$

4. $\text{Mole fraction of A} = \dfrac{\text{No. of moles of A}}{\text{No. moles of solution}}$

5. $\text{Molarity} = \dfrac{\text{No. of moles of solute}}{\text{Volume of solution in litres}}$

6. $\text{Molality} = \dfrac{\text{No. of moles of solute}}{\text{Mass of solvent in kg}}$

UNIT 2 STRUCTURE OF ATOM

1. Atomic number (Z) = Number of protons in the nucleus of an atom or Number of electrons in a neutral atom

2. Mass number(A) = Number of protons (Z) + Number of neutrons (n)

3. $\text{Speed of light (C)} = \dfrac{\text{Frequency } (v)}{\text{Wavelentgh } (\lambda)}$

4. Energy of quantum radiation (E) = Planck's constant (h) × Frequency (v)

5. Heisenberg's Uncertainty Principle

   It states that it is impossible to determine simultaneously, the exact position and exact momentum (or velocity) of an electron.

   $$\Delta X \times \Delta p_x = \frac{h}{4\Pi}$$

   Where, ΔX = Uncertainty in position, Δp_x = Uncertainty in momentum of particle

UNIT 5 STATES OF MATTER

1. Boyle's Law
   At constant temperature, the pressure of a fixed amount of gas varies inversely with its volume.

   $p = k_1 \frac{1}{V}$ Where, k_1 is proportionality constant, p is pressure, V is volume

2. Charles' Law
   Pressure remaining constant, the volume of fixed mass of a gas is directly proportional to its absolute temperature.

   $V = K_2 T$ where, K_2 is constant, V is volume and T is temperature

3. Gay Lussac's Law
   At constant volume, pressure of a fixed amount of a gas varies directly with the temperature.

   $P = K_3 T$ where, K_3 is constant, P is pressure and T is temperature

4. Avogadro's law
   It states that equal volumes of all gases under the same conditions of temperature and pressure contain equal number of molecules

   $V = K_4 n$ where, K_4 is constant, V is volume of gas, n is number of moles of gas

5. Ideal gas equation or Universal gas constant
   $pV = nRT$

Where, p is pressure, T is temperature, V is volume, n is number of moles of gas and R is gas constant

UNIT 6. THERMODYNAMICS

1. First law of thermodynamics
   The energy of an isolated system is constant

   $\Delta U = q + W$

2. Enthalpy
   $\Delta H = \Delta U + \Delta pV$

3. Gibbs free energy
   $\Delta G = \Delta H - T\Delta S$ where, ΔG is change in Gibbs free energy, ΔH is change in enthalpy, ΔS is change in entropy and T is system temperature.

UNIT 7. EQUILBRIUM

1. pH scale
   Acidic solution has pH < 7
   Basic solution has pH > 7
   Neutral solutions has pH = 7
   $pK_w = pH + pOH = 14$

2. Ionization constant of weak acids

   $K_a = \frac{c\alpha^2}{1-\alpha}$ Where, K_a is dissociation constant, c is initial concentration of undissociated acid HX at time, t = 0. α = degree of ionization of acid
   $K_a = [H^+][X^-]/[HX]$

   $pK_a = -\log(K_a)$

   Ionization of weak bases
   $K_b = [M^+][OH^-]/[MOH]$

$K_b = \frac{c\alpha^2}{1-\alpha}$ where , K_b dissociation constant of base, α = degree of ionization of base

$pK_a + pK_b = pK_w = 14$

3. Hydrolysis of salts
   $pH = 7 + ½ (pK_a – pK_b)$

UNIT 8. REDOX REACTIONS

1. Oxidation : Loss of electron(s) by any species
2. Reduction: Gain of electron(s) by any species
3. Oxidizing agent : Acceptor of electron(s)
4. Reducing agent : Donor of electron(s)

PUC 2nd year formula

UNIT 2 SOLUTIONS

1. Mass percentage of component= $\frac{\text{Mass of the component in the solution}}{\text{Total mass of the solution}} \times 100$

2. Mass percentage of component = $\frac{\text{Mass of the component in the solution}}{\text{Total mass of the solution}} \times 100$

3. Volume percentage of component = $\frac{\text{Volume of the component}}{\text{Total volume of the solution}} \times 100$

4. Parts per million = $\frac{\text{Number of parts of the component}}{\text{Total number of parts of all components of the solution}} \times 10^6$

5. Mole fraction of component = $\frac{\text{Number of moles of component}}{\text{Total number of moles of all components}}$

6. Molarity = $\frac{\text{Mass of the solute}}{\text{volume of solution in litre}}$

7. Molality = $\frac{\text{Moles of solute}}{\text{Mass of solvent in kg}}$

8. Henry's law states that "the partial pressure of the gas in vapour phase (*p*) is proportional to the mole fraction of the gas (*x*) in the solution"

   Henry's law (p) = $K_H \times x$ where, K_H is the Henry's law constant

9. Elevation of boiling point (ΔT_b) = $T_b - T_b^o$
   where, T_b^o is the boiling point of pure solvent, T_b is the boiling point of the solution.

10. Depression of freezing point (ΔT_f) = $T_f - T_f^0$
    where, T_f^o is the freezing of pure solvent, T_b is the freezing point of the nonvolatile solute.

11. Elevation of boiling point (ΔT_b) for dilute solutions = $K_b \times m$
    Where, K_b is elevation in boiling point constant, m is molality

12. Depression of freezing point (ΔT_f) for dilute solutions = $K_f \times m$
    Where, K_b is depression in freezing point constant, m is molality

13. Van't Hoff factor (i) = $\frac{\text{Total number of moles of particles after association/dissociation}}{\text{Number of moles of particles before association/dissociation}}$

UNIT 3. ELECTRO CHEMISTRY

1. The potential difference between the two electrodes of a galvanic cell is called the *cell potential* and is measured in volts. The cell potential is the difference between the electrode potentials (reduction potentials) of the cathode and anode.
   $E_{Cell} = E_R - E_L$
   where, E_R = Electrode potential of catode, E_L = Electrode potential of anode

2. $E_{M^{n+}/M} = E^o{}_{M^{n+}/M} - \frac{2.303\ RT}{nF} \log \frac{1}{[M^{n+}]}$
   Where, R is gas constant (8.314 JK^{-1} mol^{-1}),
   F is Faraday constant (96487 C mol^{-1}), *T* is temperature in kelvin and
   $[M^{n+}]$ is the concentration of the species, Mn+. E= electrode potential, E^o= Std. electrode potential.

UNIT 5. SURFACE CHEMISTRY

1. Freundlich adsorption isotherm $\frac{x}{m} = kP^{1/n}$ where n> 1
   where x is the mass of the gas adsorbed on mass m of the adsorbent at pressure P, k and n are constants which depend on the nature of the adsorbent and the gas at a particular temperature.

UNIT 6. GENERAL PRINCIPLES AND PROCESS OF ISOALTION OF ELEMENTS

1. $\Delta G = \Delta H - T\Delta S$
   Where, ΔH is the enthalpy change and ΔS is the entropy change for the process. T= temperature, ΔG = free energy change.

UNIT 10. HALOALKANES AND HALOARENES

1. Synthesis of Haloalkanes

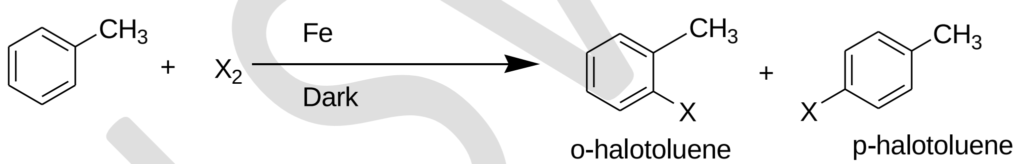
$$\text{R-OH} + \text{HCl} \xrightarrow{ZnCl_2} \text{R-Cl} + H_2O$$

$$\text{R-OH} + \text{NaBr} + H_2SO_4 \longrightarrow \text{R-Br} + NaHSO_4 + H_2O$$

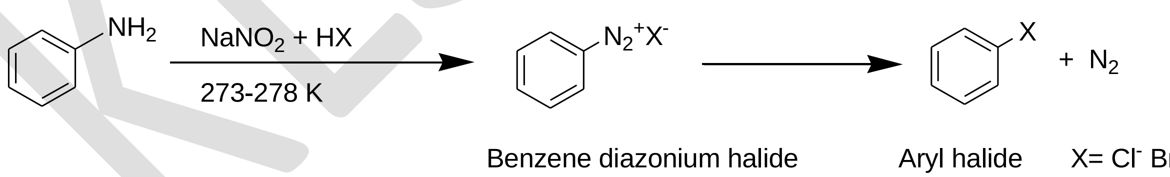
$$\text{R-OH} + SOCl_2 \longrightarrow \text{R-Cl} + SO_3 + \text{HCl}$$

2. Preparation of Haloarenes

From hydrocarbons by electrophilic substitution



From Amines by Sandmeyer's reaction

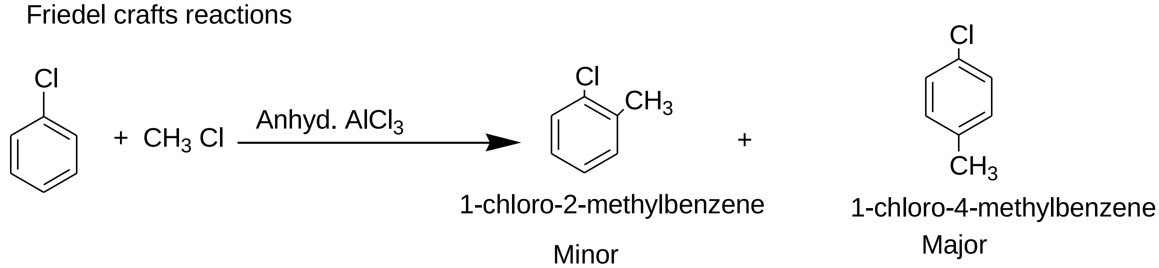


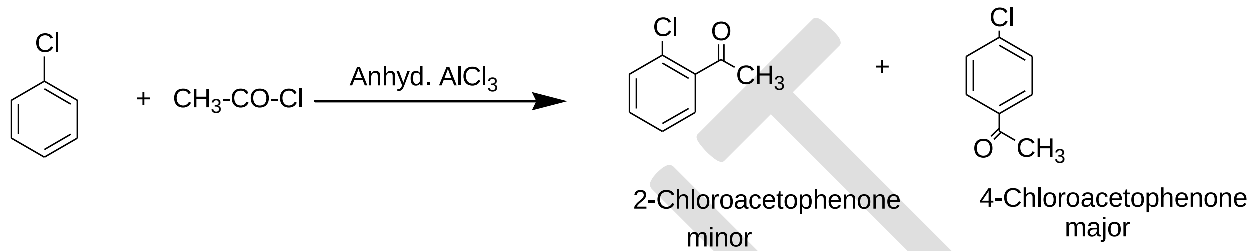
3. Grignard reagent

$$\underset{\text{Haloalkane}}{\text{CH3CH2Br}} + \text{Mg} \xrightarrow{\text{Dry ether}} \underset{\text{Grignard reagent}}{\text{CH3CH2MgBr}}$$

4. Friedel-crafts reaction

Friedel crafts reactions

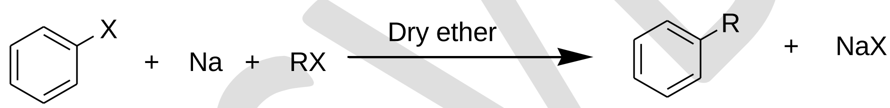




5. Wurtz-Fittig Reaction

A mixture of an alkyl halide and aryl halide gives an alkylarene when treated with sodium in dry ether and is called Wurtz-Fittig reaction.

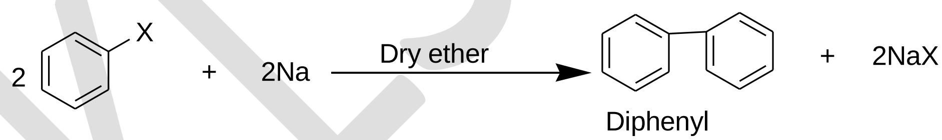
Wurtz-Fittig Reaction



6. Fittig reaction

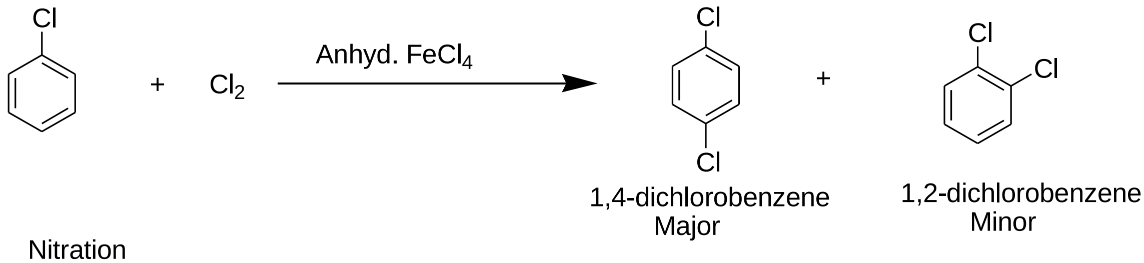
Aryl halides also give analogous compounds when treated with sodium in dry ether, in which two aryl groups are joined together. It is called Fittig reaction.

Fittig reaction

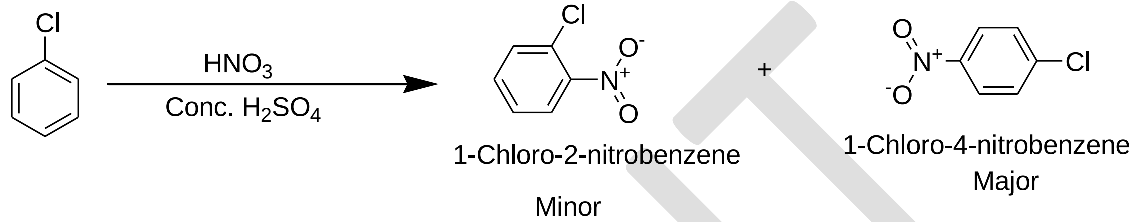


7. Electrophilic substitution reactions

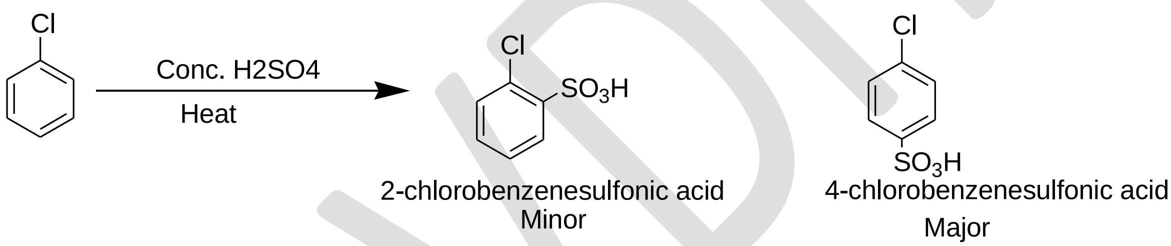
Halogenation



Nitration



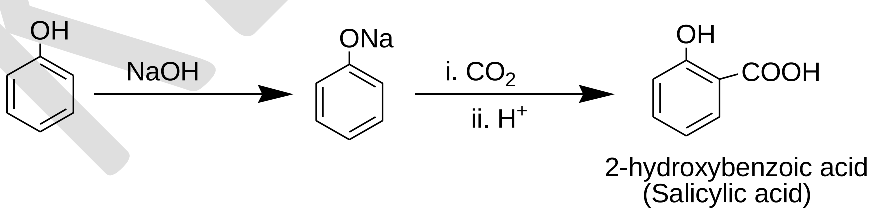
Sulphonation



UNIT 11. ALCOHOLS, PHENOLS AND ETHERS

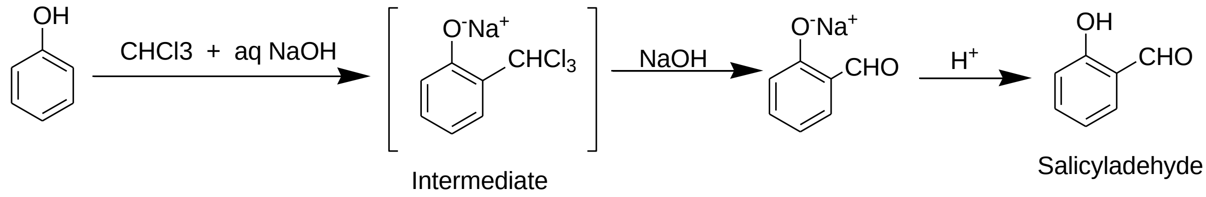
1. Kolbe's reaction

Phenoxide ion generated by treating phenol with sodium hydroxide is even more reactive than phenol towards electrophilic aromatic substitution. Hence, it undergoes electrophilic substitution with carbon dioxide, a weak electrophile. *Ortho* hydroxybenzoic acid is formed as the main reaction product.



2. Reimer-Tiemann reaction

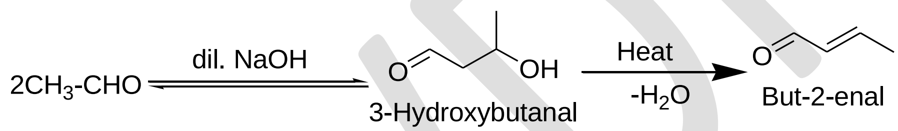
On treating phenol with chloroform in the presence of sodium hydroxide, a –CHO group is introduced at *ortho* position of benzene ring. This reaction is known as *Reimer - Tiemann reaction*. The intermediate substituted benzal chloride is hydrolysed in the presence of alkali to produce salicylaldehyde.



UNIT 12. ALDEHYDES KETONES AND CARBOXYLIC ACIDS

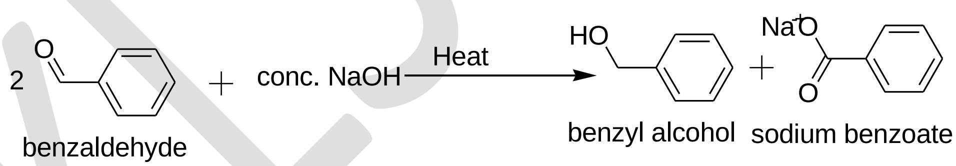
1. Aldol condensation

   Aldehydes and ketones having at least one a-hydrogen undergo a reaction in the presence of dilute alkali as catalyst to form b-hydroxy aldehydes (aldol) or b-hydroxy ketones (ketol), respectively. This is known as Aldol reaction



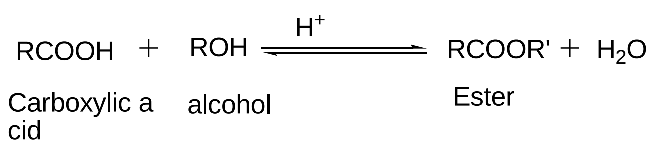
2. Cannizzaro reaction

   Aldehydes which do not have an a-hydrogen atom, undergo self-oxidation and reduction (disproportionation) reaction on heating with concentrated alkali. In this reaction, one molecule of the aldehyde is reduced to alcohol while another is oxidized to carboxylic acid salt.



3. Esterification

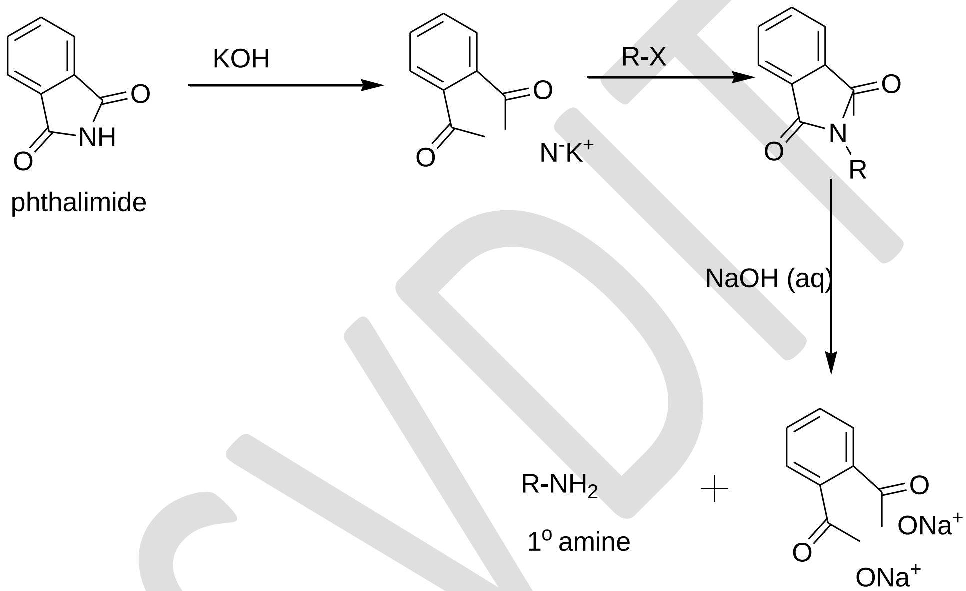
   Carboxylic acids are esterified with alcohols or phenols in the presence of a mineral acid such as concentrated H_2SO_4 or HCl gas as a catalyst.



UNIT 13. AMINES

1. Gabriel phthalimide synthesis

   Gabriel synthesis is used for the preparation of primary amines. Phthalimide on treatment with ethanolic potassium hydroxide forms potassium salt of phthalimide which on heating with alkyl halide followed by alkaline hydrolysis produces the corresponding primary amine. Aromatic primary amines cannot be prepared by this method because aryl halides do not undergo nucleophilic substitution with the anion formed by phthalimide



2. Hoffmann bromamide degradation reaction

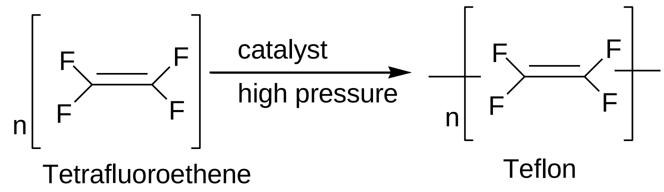
   Hoffmann developed a method for preparation of primary amines by treating an amide with bromine in an aqueous or ethanolic solution of sodium hydroxide. In this degradation reaction, migration of an alkyl or aryl group takes place fromarbonyl carbon of the amideto the nitrogen atom. The amine so formed contains one carbon less than that present in the amide.

$$R\text{-}CO\text{-}NH_2 \;+\; Br2+ \;\; 4NaOH \longrightarrow R\text{-}NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$$

UNIT 15. PLOYMERS

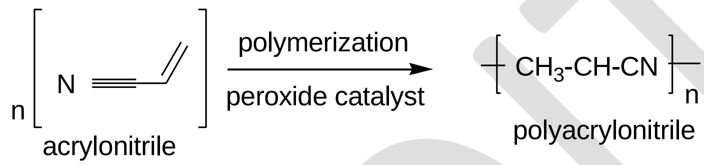
1. Polytetrafluoroethene (Teflon)

   Teflon is manufactured by heating tetrafluoroethene with a free radical or persulphate catalyst at high pressures. It is chemically inert and resistant to attack by corrosive reagents. It is used in making oil seals and gaskets and also used for non – stick surface coated utensils.



2. Polyacrylonitrile

The addition polymerisation of acrylonitrile in presence of a peroxide catalyst leads to the formation of polyacrylonitrile



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