

Prepared by : Prof. A. S. Joshi  
Dept. of ECE

## Digital Communication

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15E1701

### Sixth Semester B.E. Degree Examination, June/July 2019 Digital Communication

Time: 3 hrs.

Max. Marks: 30

Note: Answer any 7 II & 3 full questions, choosing  
ONE full question from each module.

#### Module-1

1. a. Determine the Hilbert transform of the signal  $s(t) = \cos(\omega t)$ .  
b. Determine the real-envelope and complex envelope of the signal shown in Fig. Q1(b). (One Mark)

#### Ques. 1(b)

2. Explain the time-domain procedure for the complex representation of digital communication systems.

#### Ques. 2

3. a. For a binary sequence of bits, find the conversion of RZ bipolar format to Manchester format to HDB3 format. (6 Marks)  
Also mention the application of RZ, PSK and BPSK formats.  
b. Draw the power spectrum of a M-ary AM signal in NRZ polar format.  
c. Consider a bandpass signal  $s(t)$  which is expressed in terms of in-phase and quadrature components. Suggest a suitable scheme for:  
i) extracting the in-phase and quadrature components from the band pass signal  
ii) reconstructing the band pass signal from in-phase and quadrature components.

#### Module-2

3. a. For the signals with unit duty and unit power as shown in Fig. Q3, sketch their orthonormal basis functions using Gram-Schmidt orthogonalization procedure.

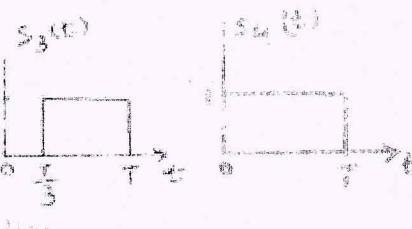


Fig. Q3(a)

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Head of the Department  
Dept. of Electronic & Communication Engg.  
KLS V.D.I.T., HALYAL (U.K.)

4. a. Explain the relevant rule for joint rate maximization theory and explain the conditions for its existence.  
b. Show that the noise variance of the channel output of a digital receiver is given by  $N_0/2$  and will increase exponentially with the number of bits in the digital receiver. (6 Marks)

Module-2

- a) Sketch the QPSK waveforms for the sequence of 101001. (06 Marks)  
 b) Obtain the expression for average probability of symbol error for BPSK using coherent detection. (06 Marks)  
 c) Obtain the constellation of QAM for  $M = 16$  and draw the signal space diagram. (04 Marks)

**OR**

- a) Explain the generation and coherent detection of DPSK system. (06 Marks)  
 b) The binary sequence 10101010101 is applied to the DPSK transmitter.  
   i. Sketch the resulting waveform from the transmitter output.  
   ii. Applying this waveform to the DPSK receiver, show that in the absence of noise, the original binary sequence is reconstructed at the receiver output. (06 Marks)  
 c) An FSK system transmits binary data at the rate of  $2 \times 10^3$  bps. During the course of transmission, AWGN of zero mean and two-sided PSD  $10^{-11} \text{ W/Hz}$  is added to the signal. The amplitude of the received sinusoidal wave for digit 1 or 0 is 1 mV. Determine the average probability of symbol error assuming coherent detection. (04 Marks)

Module-4

- a) Sketch the following terms with labeled waveform and diagrams with respect to base band transmission:  
   i. PAM and Manchester condition for  $b_1 = 1$   
   ii. Manchester signal pulse  
   iii. Modified Manchester signal pulse  
   iv. Parallel response signals  
   v. Raised cosine spectrum. (10 Marks)  
 b) Explain the need for precoder in a duobinary signaling. The binary sequence 10101010101 is the input to the precoder whose output is used to modulate a duobinary transmitting filter. Obtain the precoded sequence, transmitted amplitude and the received amplitude for the desired condition. (06 Marks)

**OR**

- a) With a neat diagram explain the concept of linear interpolation. (06 Marks)  
 b) Consider a channel distorted pulse  $s(t)$  as the input to the equalizer, given by  

$$s(t) = \frac{1}{2}e^{-\frac{|t|}{T}} + \frac{1}{2}e^{-\frac{|t-2T|}{T}}$$
 where  $T = 1$  is the symbol rate. The pulse is sampled at the rate of 1 and  
 quantized to a 4-bit binary equivalent. Determine the coefficients of a 3-tap zero-forcing  
 equalizer. (06 Marks)  
 c) State the equalizer and receiver. (04 Marks)

Module-5

- a) With a neat diagram explain the generation of DS sequences and state its properties. (06 Marks)  
 b) If the spread spectrum signal is designed so that the power ratio  $P_s/P_d$  is the intended power ratio. If the desired  $C/N_0$  is to be for acceptable performance, determine the minimum value of the spreading gain. (04 Marks)  
 c) Explain with neat block diagram a DS spread spectrum system. (06 Marks)

**OR**

- a) Discuss the generation and demodulation of DS spread spectrum signal  
 b) Write a note on applications of spread spectrum in mobile communication. (06 Marks)  
 c) Explain how a direct sequence spread spectrum system works. (06 Marks)

Subject: Digital Communication (15EC61) June/July 2019

Note: Answer any five full questions, choosing ONE full question from each module.

1 q)

$$g(t) = \text{sinc}(t) \quad G(f) \cdot A = 1 \quad \text{Total (04 m)}$$

$$\hat{g}(t) = \text{IFT} [G(f) H(f)]$$

$$\text{sinc}(t) \xleftrightarrow{\text{FT}} \text{rect}(f)$$

$$\hat{G}(f) = -j \text{sgn}(f) G(f)$$

$$\hat{g}(t) = \int_{-\infty}^{\infty} \hat{G}(f) e^{j2\pi f t} df$$

$$\begin{aligned} \hat{g}(t) &= \int_{f=-\frac{1}{2}}^{f=0} j e^{j2\pi f t} df + \int_{f=0}^{f=\frac{1}{2}} (-j) e^{j2\pi f t} df \\ &= \left[ \frac{j e^{j2\pi f t}}{j2\pi f t} \right]_{-\frac{1}{2}}^0 - \left[ \frac{j e^{j2\pi f t}}{j2\pi f t} \right]_0^{\frac{1}{2}} \end{aligned}$$

$$= \frac{1}{2\pi t} \left[ (1 - e^{-j\pi t}) - (e^{j\pi t} - 1) \right]$$

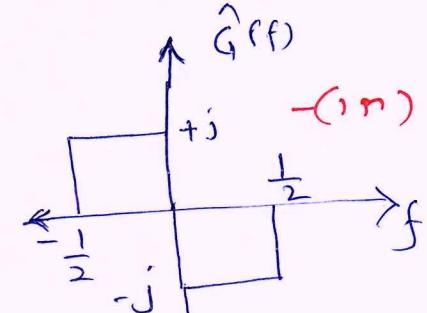
$$= \frac{1}{2\pi t} \left[ 2 - e^{-j\pi t} - e^{j\pi t} \right]$$

$$= \frac{1}{\pi t} \left[ 1 - \left( \frac{e^{j\pi t} + e^{-j\pi t}}{2} \right) \right]$$

$$\text{WKT } \cos(\pi t) = \frac{e^{j\pi t} + e^{-j\pi t}}{2}$$

$$\therefore \hat{g}(t) = \frac{1}{\pi t} \left[ 1 - \cos(\pi t) \right]$$

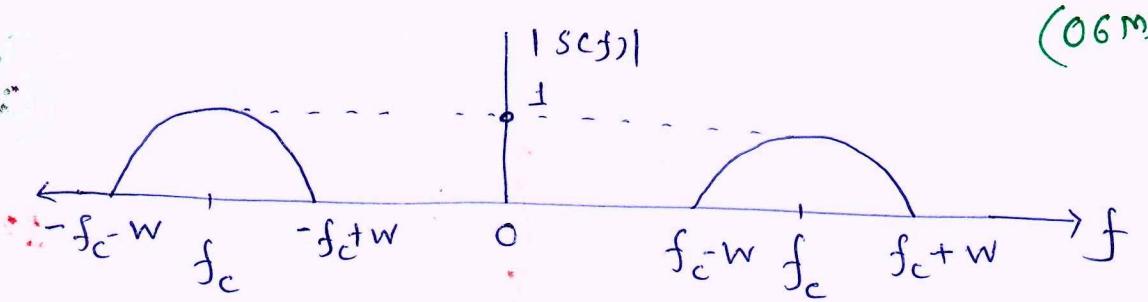
$$\text{or} \\ = \left[ \frac{\sin(\pi t/2)}{(\pi t/2)} \right] \times \sin(\frac{\pi t}{2})$$



$$\hat{g}(t) = \text{sinc}(\frac{t}{2}) \cdot \sin(\frac{\pi t}{2}) // \text{Ans} \quad -(1m)$$

Ans

1 b)



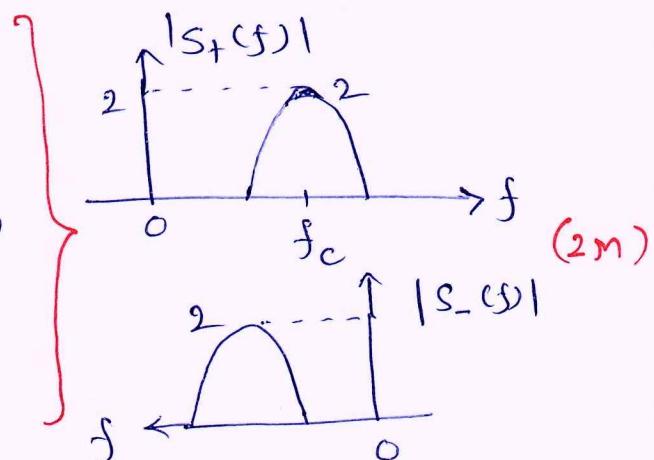
(06 M)

Pre-envelope is given by

$$S_+(f) = S(f) + j \hat{S}(f)$$

$$S_+(f) = \begin{cases} 2S(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

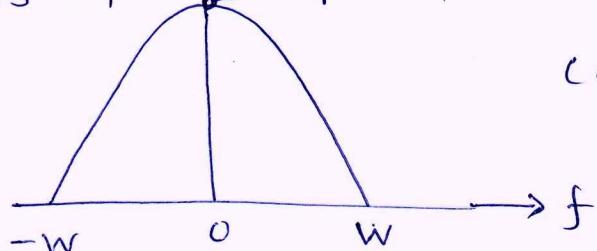
$$S_-(f) = \begin{cases} 0 & f > 0 \\ 2S(f) & f < 0 \end{cases}$$



(2 m)

Complex envelope is given by

$$\hat{S}(t) = S_+(t) e^{-j2\pi f_c t}$$



complex envelope

(2 m)

Bandpass system (06 m)

- i) Let  $s(t)$  is a bandpass signal //  $S(f) \rightarrow h(t) \rightarrow x(t)$   
 $S(f)$  is a Fourier Transform  $\hat{S}(t) \rightarrow \hat{h}(t) \rightarrow 2\hat{x}(t)$   
 $H(f)$  is a frequency response of the system  
 $x(t)$  is a output of bandpass system

in

Expressing  $x(t)$  in terms of its own low-pass complex envelope  $\hat{x}(t)$  as

Ans 2

$$x(t) = \operatorname{Re} [\hat{x}(t) e^{j\omega_{fc} t}] \quad \left. \begin{array}{l} \text{eqn } \textcircled{1} \\ \text{--- (1m)} \end{array} \right\}$$

$x(t)$  is related to  $s(t)$  by

$$\hat{x}(t) = \int_{-\infty}^{\infty} h(z) s(t-z) dz \quad \left. \begin{array}{l} \text{eqn } \textcircled{2} \\ \text{--- (1m)} \end{array} \right\}$$

In terms of pre-envelope, we have

$$h(t) = \operatorname{Re} [h_+(t)] \text{ and}$$

$$s(t) = \operatorname{Re} [s_+(t)]$$

- eqn  $\textcircled{3}$

- (1m)

Using equation  $\textcircled{3}$  in equation  $\textcircled{2}$  then

$$\hat{x}(t) = \int_{-\infty}^{\infty} \operatorname{Re} [h_+(t)] \operatorname{Re} [s_+(t-z)] dz \quad \left. \begin{array}{l} \text{eqn } \textcircled{4} \\ \text{--- (1m)} \end{array} \right\}$$

WKT

$$\int_{-\infty}^{\infty} \operatorname{Re} [h_+(t)] \operatorname{Re} [s_+(t-z)] dz = \frac{1}{2} \operatorname{Re} \left[ \int_{-\infty}^{\infty} s_+^*(t) h_+(t) dz \right]$$

WKT  $s(z) = s(-z)$  and WKT relationship  $\left. \begin{array}{l} \text{eqn } \textcircled{5} \\ \text{--- (1m)} \end{array} \right\}$

between pre-envelope and complex-envelope, we may express equation  $\textcircled{4}$  in the equivalent form as

$$x(t) = \frac{1}{2} \operatorname{Re} \left[ \int_{-\infty}^{\infty} \hat{h}(z) e^{j\omega_{fc} t} \hat{s}(t-z) e^{-j\omega_{fc} (t-z)} dz \right]$$

- eqn  $\textcircled{6}$

Comparing RHS of equation  $\textcircled{1}$  and  $\textcircled{6}$

$$\hat{x}(t) = \frac{1}{2} \int_{-\infty}^{\infty} \hat{h}(z) \hat{s}(t-z) dz \quad \left. \begin{array}{l} \text{--- eqn } \textcircled{7} \\ \text{--- (1m)} \end{array} \right\}$$

Complex envelope  $\hat{s}(t)$  in terms of inphase and quadrature component

$$\hat{s}(t) = s_I(t) + j s_Q(t) \text{ and} \quad \left. \begin{array}{l} \text{--- (1m)} \end{array} \right\}$$

$$\hat{h}(t) = h_I(t) + j h_Q(t)$$

Put these in equation (VII) then

$$2\hat{x}(t) = \hat{h}(t) * \hat{s}(t)$$

$$2\hat{x}(t) = [h_I(t) + j h_Q(t)] * [s_I(t) + j s_Q(t)] - \text{eqn}^{\text{viii}}$$

where  $*$  denotes convolution operation and is distributive in nature. we may write eqn (VIII) as

$$2\hat{x}(t) = [h_I(t) * s_I(t) - h_Q(t) * s_Q(t)] + j [h_Q(t) * s_I(t) + h_I(t) * s_Q(t)]$$

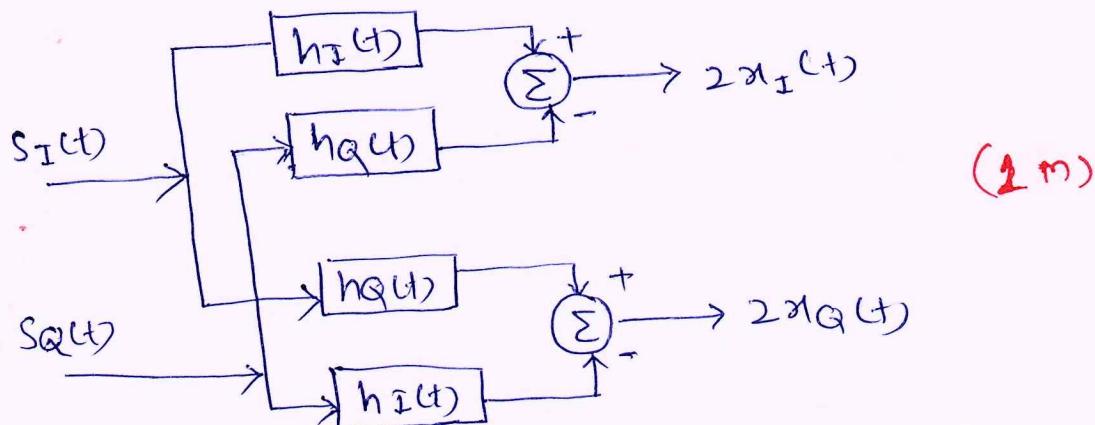
WKT  $\hat{s}(t) = x_I(t) + j x_Q(t)$  — eqn (IX)

Compare equations (IX) and equation (X) then separating in real part and imaginary part

$$2x_I(t) = h_I(t) * s_I(t) - h_Q(t) * s_Q(t) - \text{Real}$$

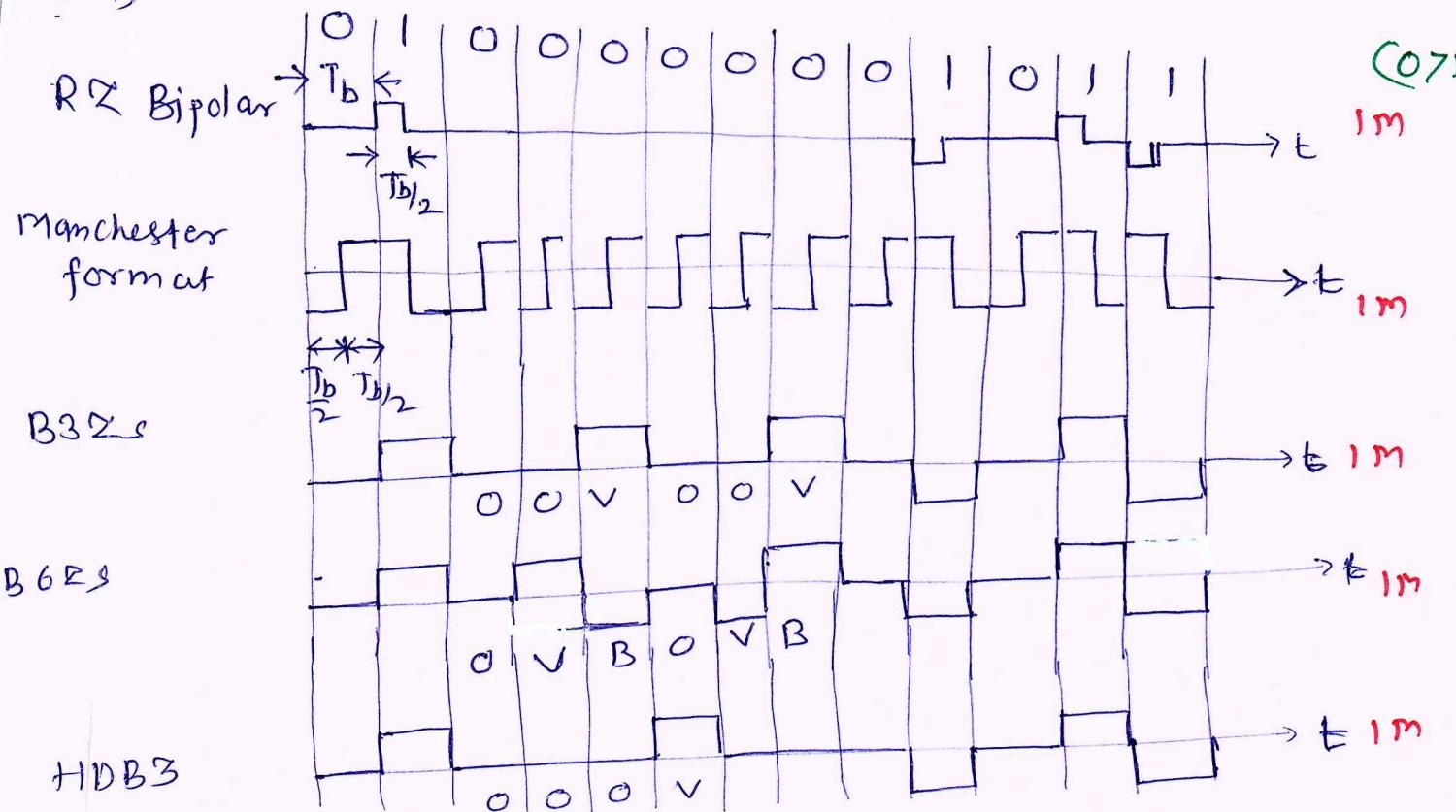
$$2x_Q(t) = h_Q(t) * s_I(t) - h_I(t) * s_Q(t) - \text{Imaginary}$$

Thus for the evaluation of inphase and quadrature components of  $\hat{s}(t)$  of the system output use following two-pass equivalent model shown below



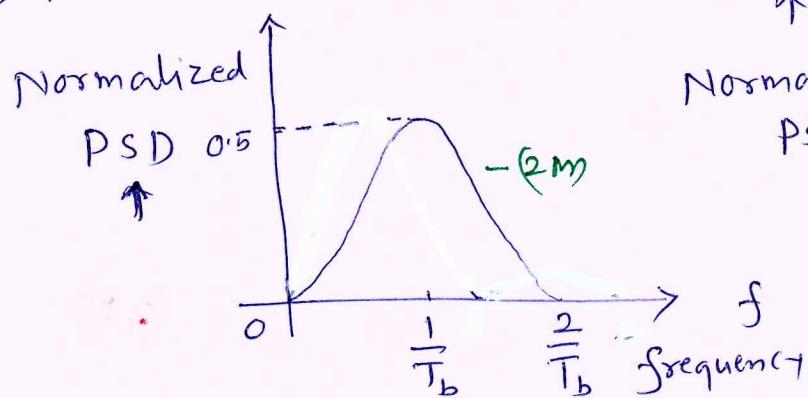
Joshi No. 2

2 a)

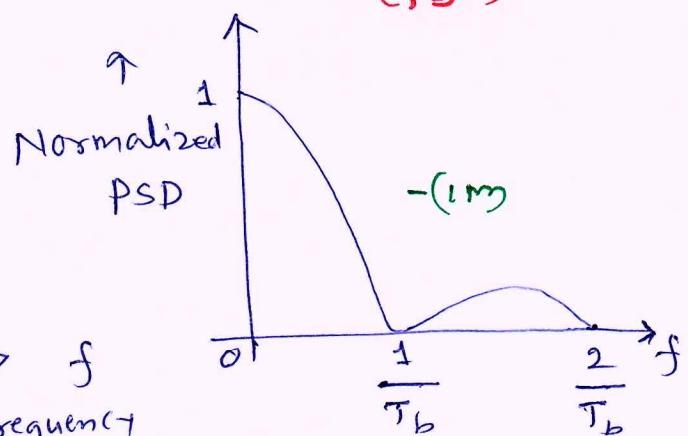


Applications of B3ZS - In DS3 signals of {  
digital telephone hierarchy  
B6ZS used in - DS2 carrier systems } 2m

b) power spectra of  
NRZ AMI (10.5M)



NRZ Polar (1.5M)



c) Let  $s(t)$  is a bandpass signal and  $(0.6M))$   
 $\hat{s}(t)$  is a complex envelope of  $s(t)$ .  
 $\hat{s}(t) = s_I(t) + j s_Q(t)$

Ans

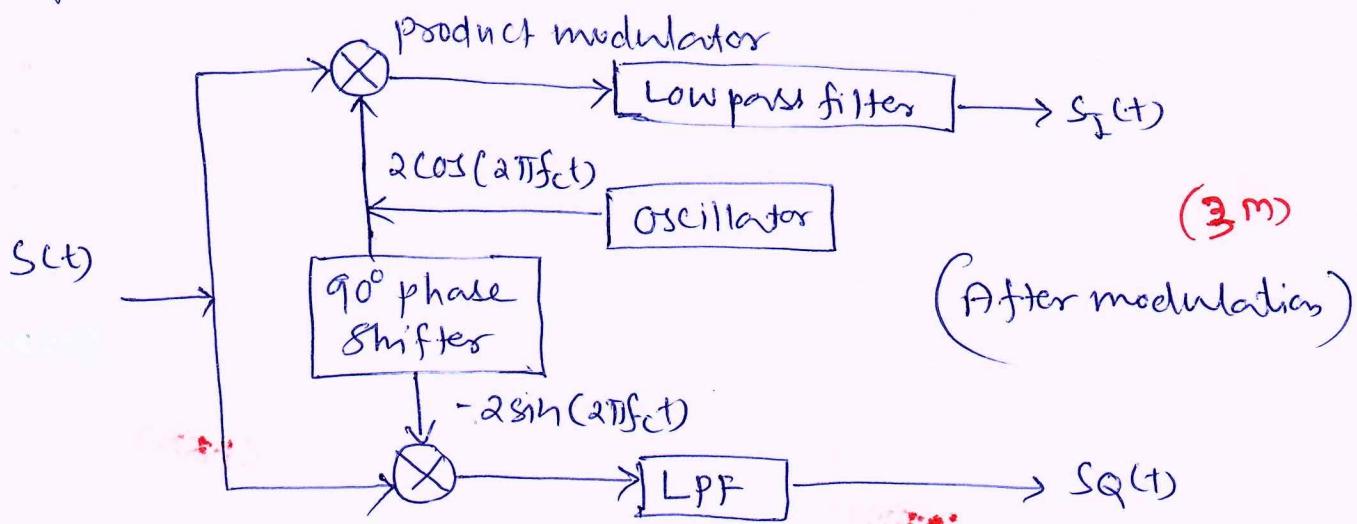
2c) Expressing  $s(t)$  in canonical form as

$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

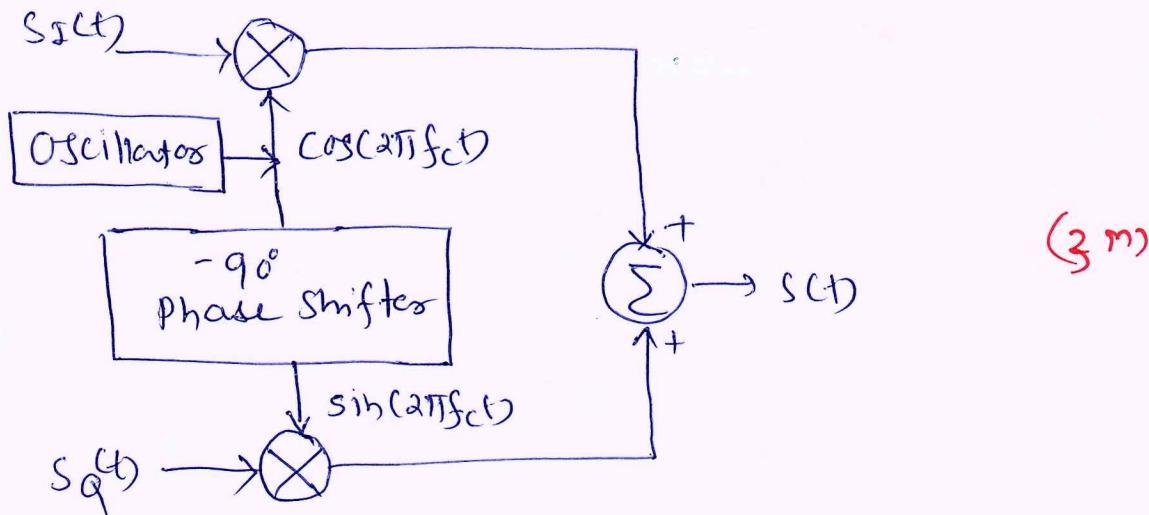
where  $s_I(t)$  - is in-phase component of the bandpass signal

$s_Q(t)$  - is quadrature component of the bandpass signal

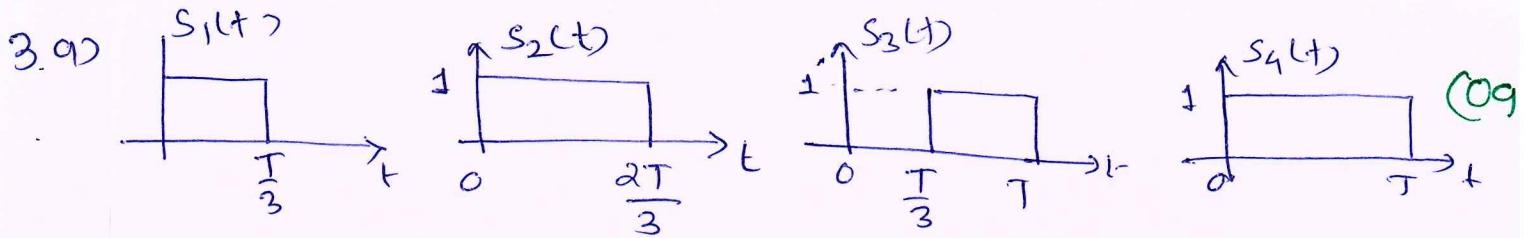
Scheme for extracting in-phase and quadrature component is



Scheme for reconstructing  $s(t)$  from  $s_I(t)$  &  $s_Q(t)$  is



*Joshi*



b) To obtain  $\phi_1(t)$

$$\text{Energy of } S_1(t) \text{ is } E_1 = \int_0^{T/3} S_1^2(t) dt = \frac{T}{3}$$

$$\therefore \phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} = \begin{cases} \sqrt{\frac{3}{T}} & 0 \leq t \leq \frac{T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

$\phi_1(t)$   
(2m)

2) To obtain  $\phi_2(t)$

$$S_{21} = \int_0^{T/3} S_2(t) \phi_1(t) dt = \int_0^{T/3} (1) \left( \sqrt{\frac{3}{T}} \right) dt$$

$$S_{21} = \sqrt{\frac{T}{3}}$$

$$S_{21} \phi_1(t) = \begin{cases} \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}} = 1 & 0 \leq t \leq \frac{T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

Intermediate signal  $g_2(t)$  is

$$g_2(t) = S_2(t) - S_{21} \phi_1(t) \quad - (1m)$$

$$= \begin{cases} 1 & \frac{T}{3} \leq t \leq \frac{2T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

Energy of  $g_2(t)$  will be

$$E_{g_2} = \int_0^{2T/3} g_2^2(t) dt = \int_0^{2T/3} (1)^2 dt = \frac{T}{3}$$

$$\therefore \phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}} = \begin{cases} \frac{1}{\sqrt{\frac{T}{3}}} = \sqrt{\frac{3}{T}} & \frac{T}{3} \leq t \leq \frac{2T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

(2m)

3) To obtain  $\phi_i(t)$   $j = i-1$

$$g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij} \phi_j(t), \quad i = 1, 2, 3, \dots, N$$

- (1m)

With  $N = 3$

$$g_3(t) = s_3(t) - \sum_{j=1}^2 s_{3j} \phi_j(t)$$

$$= s_3(t) - [s_{31} \phi_1(t) + s_{32} \phi_2(t)]$$

$\therefore s_{31} = \int_0^T s_3(t) \phi_1(t) dt = 0$  since there is no overlap between  $s_3(t)$  and  $\phi_1(t)$

Now,

$$s_{32} = \int_0^T s_3(t) \phi_2(t) dt = \int_{t=\frac{T}{3}}^{t=\frac{2T}{3}} (1) \times \left(\sqrt{\frac{3}{T}}\right) dt$$

$$s_{32} = \sqrt{\frac{T}{3}}$$

Here  $s_{31} \phi_1(t) = 0 \times \phi_1(t) = 0$  since  $s_{31} = 0$

$$s_{32} \phi_2(t) = \begin{cases} \left(\sqrt{\frac{T}{3}}\right) \left(\sqrt{\frac{3}{T}}\right) & \frac{T}{3} \leq t \leq \frac{2T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

$$= \begin{cases} 1 & \frac{T}{3} \leq t \leq \frac{2T}{3} \\ 0 & \text{elsewhere} \end{cases}$$

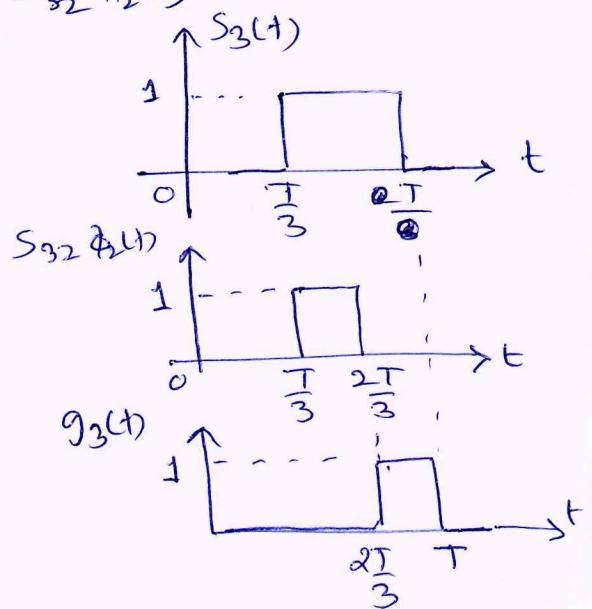
$$\therefore g_3(t) = s_3(t) - s_{31} \phi_1(t) - s_{32} \phi_2(t)$$

$$= s_3(t) - s_{32} \phi_2(t)$$

$$\therefore g_3(t) = \begin{cases} 1 & \frac{2T}{3} \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

Energy of signal  $g_3(t)$  is

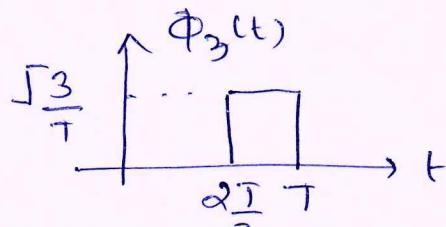
$$E_{g_3} = \int_0^T g_3^2(t) dt = \int_{\frac{T}{3}}^{2T/3} (1)^2 dt$$



Ans 2

$$E_{g_3} = \frac{T}{3}$$

$$\therefore \phi_3(t) = \frac{g_3(t)}{\sqrt{E_{g_3}}} = \begin{cases} \frac{1}{\sqrt{T/3}} = \sqrt{\frac{3}{T}} & 0 \leq t \\ 0 & \text{elsewhere} \end{cases}$$



(2m)

Here  $s_4(t) = s_1(t) + s_3(t)$ , therefore the number of orthonormal basis functions required are  $\underline{N = 3}$  - (1m)

3b) Consider a linear time-invariant filter with impulse response of  $h_i(t)$ , with the received signal  $x(t)$  operating as input. Then the resulting filter output is defined by the convolution integral

$$z = \infty$$

$$y_i(t) = \int_{z=-\infty}^{z=\infty} x(z) h_i(t-z) dz \quad - \text{eqn } ①$$

- (2m)

Consider the  $0 \leq t \leq T$  interval over which symbol is transmitted, with time  $t$  restricted in this manner, we may replace variable  $z$  with  $t$  and then

$$y_i(T) = \int_{t=0}^{t=T} x(t) h_i(T-t) dt \quad - \text{eqn } ②$$

- (2m)

The output of  $j^{\text{th}}$  correlator is

$$x_j = \int_{t=0}^{T=1} x(t) \phi_j(t) dt \quad - \text{eqn } ③$$

For  $y_j(t) = x_j$  we find from equation ② and equation ③ that this condition is satisfied provided that we choose

$$h_j(T-t) = \phi_j(t) \quad 0 \leq t \leq T \quad - \text{eqn } ④$$

$j = 1, 2, \dots, M \quad - (1m)$

Equivalently,

we may express condition imposed on the desired impulse response of the filter as

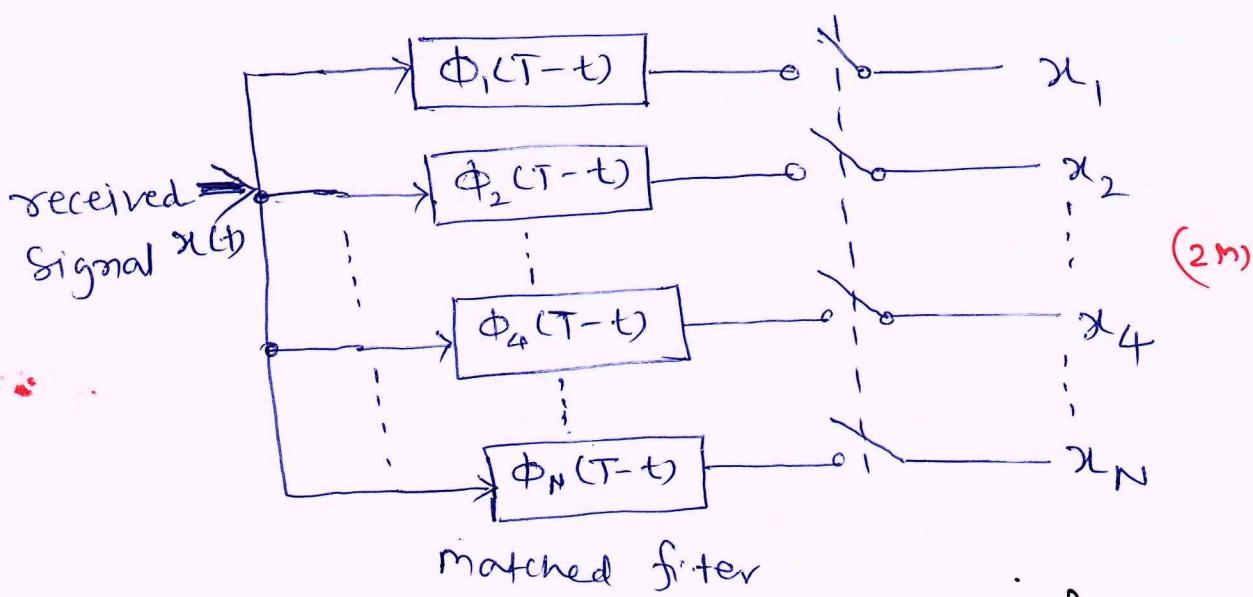
$$h_j(t) = \phi_j(T-t) \quad 0 \leq t \leq T \quad - \text{eqn } ⑤$$

$j = 1, 2, \dots, M \quad - (1m)$

Given a pulse signal  $\phi(t)$  occupying the interval  $0 \leq t \leq T$ , a linear time invariant filter is said to be matched to the signal  $\phi(t)$  if its impulse response  $h(t)$  satisfies the condition

$$h(t) = \phi(T-t) \quad 0 \leq t \leq T \quad - \text{eqn } ⑥$$

$- (1m)$



*Yashwant*

Q. Obtain the decision rule for maximum likelihood decoding and explain the correlation received. (8m)

Ans: For an AWGN channel, a sample function of the received random process  $x(t)$  is given by

$$x(t) = s_i(t) + w(t) \quad 0 \leq t \leq T, i = 1, 2, \dots, M \quad \text{Eqn } ①$$

where  $w(t)$  = sample function of white Gaussian noise process  $w(t)$ , with zero mean and PSD =  $N_0/2$

The set of outputs of the correlations constitutes a vector  $\mathbf{x}$

$$\mathbf{x} = S_i \mathbf{t} + \mathbf{w} \quad i = 1, 2, \dots, M \quad \text{Eqn } ②$$

$$\text{where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}, \quad S_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \\ \vdots \\ s_{iN} \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad (1m)$$

→ Vectors  $\mathbf{x}$  &  $\mathbf{w}$  are sample values of the random vectors  $\mathbf{x}$  and  $\mathbf{w}$  respectively.

$$P_c = P(m_i \text{ sent} | \mathbf{x} \text{ received}) \quad \text{Eqn } ③ \quad (1m)$$

→ The average probability of symbol error  $P_e$  is given by

$$P_e = 1 - P(m_i \text{ sent} | \mathbf{x} \text{ received}) \quad \text{Eqn } ④$$

The estimate of  $R_x^{\text{ed}}$  symbols

$$\hat{m} = m_i \text{ if, } P(m_i \text{ sent} | \mathbf{x} \text{ received}) \geq P(m_k \text{ sent} | \mathbf{x} \text{ received})$$

$$\text{for all } k = 1, 2, \dots, M \quad k \neq i \quad \text{Eqn } ⑤ \quad (1m)$$

→ The decision rule given by Eqn ⑤ is called max. a posteriori probability.

→ The likelihood function  $f(x | m_i)$  is the conditional PDF of the received vector  $\mathbf{x}$  given that  $m_i$  was transmitted.

→ The decision rule is to choose estimate

$$\hat{m} = m_i \text{ if}$$

$f_x(x|m_k)$  is maximum for  $k=i$  — Eqn ⑥

set  $\hat{m} = m_i$  if  $L(m_k)$  is maximum for  $k=i$

Observation vector  $x$  lies in region  $Z_i$  if  $L(m_k)$

is maximum for  $k=i$

Observation vector  $x$  lies in region  $Z_i$  if

$$\sum_{j=1}^N (x_j - s_{ij})^2 \text{ is minimum for } k=i$$

Observation vector  $x$  lies in region  $Z_i$  if

$$\|x - s_{ii}\| \text{ is minimum for } k=i$$

Observation vector  $x$  lies in region  $Z_i$  if

$$\sum_{j=1}^N x_j s_{kj} - \frac{1}{2} E_k \text{ is maximum for } k=i \quad (1m)$$

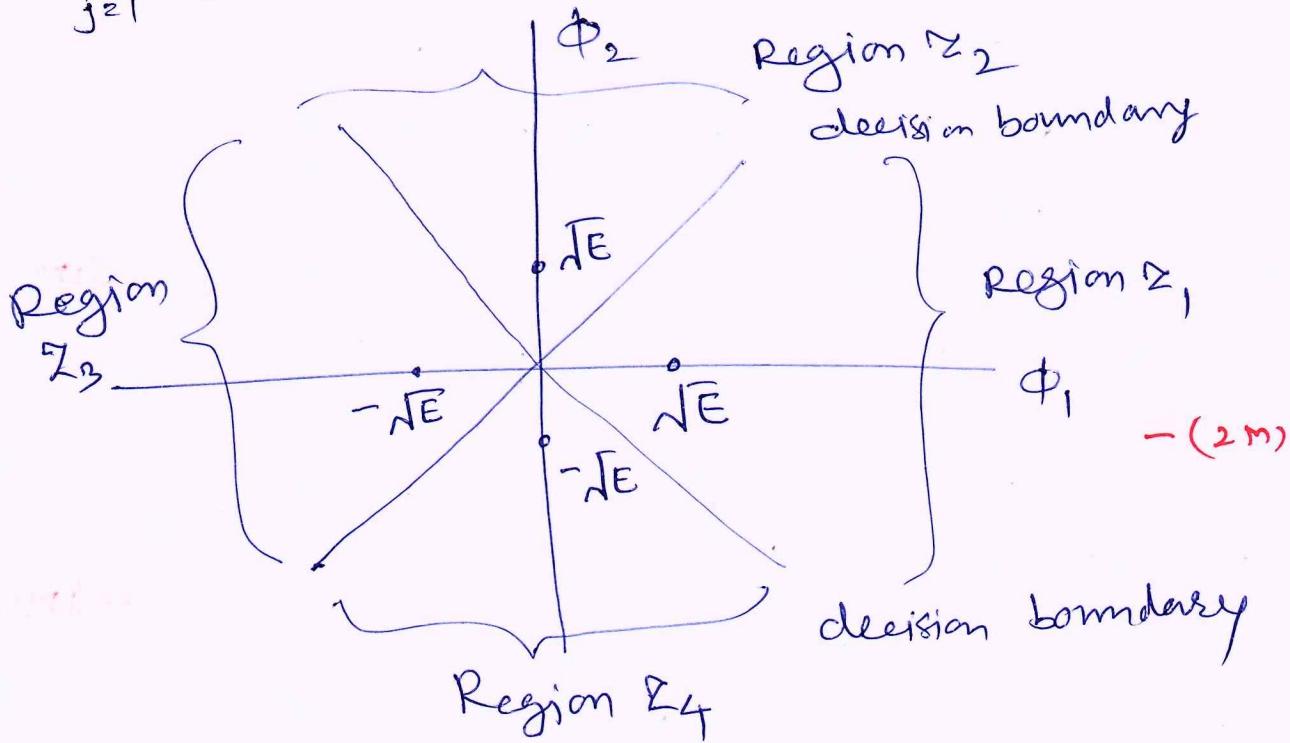


Figure illustrates partitioning of the observation space into decision regions for the case when  $N=2$   $M=4$ .

- 4b. Show that for a noisy input the mean value of the  $j^{\text{th}}$  correlator output  $x_j$  depends only on  $s_{ij}$  and all the correlators outputs  $x_j$ ,  $j = 1, 2, \dots, N$  have a variance equal to  $\text{PSD} = N\sigma_e^2$  of the additive noise process  $w(t)$  (8M)

**Ans** Here we wish to develop a statistical characterization of the set of  $N$  correlator outputs.  
 → Let  $x(t)$  — denote the stochastic process and  $x(t)$  is sample function of stochastic process  $x(t)$ .  
 → Correspondingly,

Let  $x_j$  — denote the random variable whose sample value is represented by the correlator outputs  $x_j$ ,  $j = 1, 2, \dots, N$

→ Accordingly to AWGN model, the stochastic process  $x(t)$  is a Gaussian process.

→ It follows, therefore that  $x_j$  is a Gaussian random variable for all  $j$  in accordance with property 1.

[If a Gaussian process  $x(t)$  is applied to a stable linear filter, then the stochastic process  $y(t)$  developed at the output of the filter is also Gaussian].

→ Hence  $x_j$  is characterized completely by its mean & variance. (3M)

Let  $w_j$  — denote the random variable represented by the sample value  $w_j$  produced by the  $j^{\text{th}}$  correlator in response to white Gaussian noise component  $w(t)$ .

→ The random variable  $w_j$  has zero mean because the channel noise process  $w(t)$  ~~is not~~ <sup>is</sup> white

represented by  $w(t)$  in the AR(1) model has zero mean by definition.

→ As a result, mean of  $x_j$  depends only on  $s_{ij}$  as shown by

$$\begin{aligned} E[x_j] &= E[x_j] \\ &= E[s_{ij} + w_j] \\ &= s_{ij} + E[w_j] \quad \dots \text{Eqn } ① \\ &= s_{ij} \end{aligned}$$

— (1m)

→ To find the variance of  $x_j$  we start with definition

$$\begin{aligned} \sigma^2_{x_j} &= \text{var}[x_j] \\ &= E[(x_j - s_{ij})^2] \quad [ \because x_j = s_{ij} + w_j ] \quad — (1m) \\ &= E[w_j^2] \quad \text{Eqn } ② \end{aligned}$$

→ Here  $x_j + w_j$  replaced by  $x_j + w_j$

→ WKT random variable  $w_j$  is defined by

$$w_j = \int_0^T w(t) \phi_j(t). dt \quad \text{Eqn } ③$$

use Eqn ③ in Eqn ② then

$$\begin{aligned} \sigma^2_{x_j} &= E \left[ \int_0^T w(t) \phi_j(t). dt \int_0^T w(u) \phi_j(u). du \right] \quad — (1m) \\ &= E \left[ \int_0^T \int_0^T \phi_j(t) \phi_j(u) w(t) w(u). dt. du \right] \quad \dots \text{Eqn } ④ \end{aligned}$$

For the linear operations we can interchange the order of summation & expectation then

$$\begin{aligned} \sigma^2_{x_j} &= \int_0^T \int_0^T \phi_j(t) \phi_j(u) E[w(t). w(u)]. dt. du \quad — (1m) \\ &= \int_0^T \int_0^T \phi_j(t) \phi_j(u) R_w(t, u). dt. du \quad \dots \text{Eqn } ⑤ \end{aligned}$$

where

$R_w(t, u)$  = Auto correlation function of the noise process  $w(t)$

70/100?

- 15
- Since this noise is stationary  $R_{w(t,u)}$  depends only on the time difference  $t-u$
  - wkt  $w(t)$  is white Gaussian noise with  $\text{PSD} = N$ , therefore we may express

$$R_w(t,u) = \left(\frac{N_0}{2}\right)\delta(t-u) \quad \dots \text{Eqn } 6 \quad - (1m)$$

Put Eqn 6 in Eqn 5 and then using shifting property of the delta fun  $\delta(t)$ , we get

$$\begin{aligned}\sigma_{x_j}^2 &= \frac{N_0}{2} \int_0^T \int_0^T \phi_j(t) \phi_j(u) R_w(t-u) dt du \\ &= \frac{N_0}{2} \int_0^T \phi_j^2(t) dt\end{aligned}$$

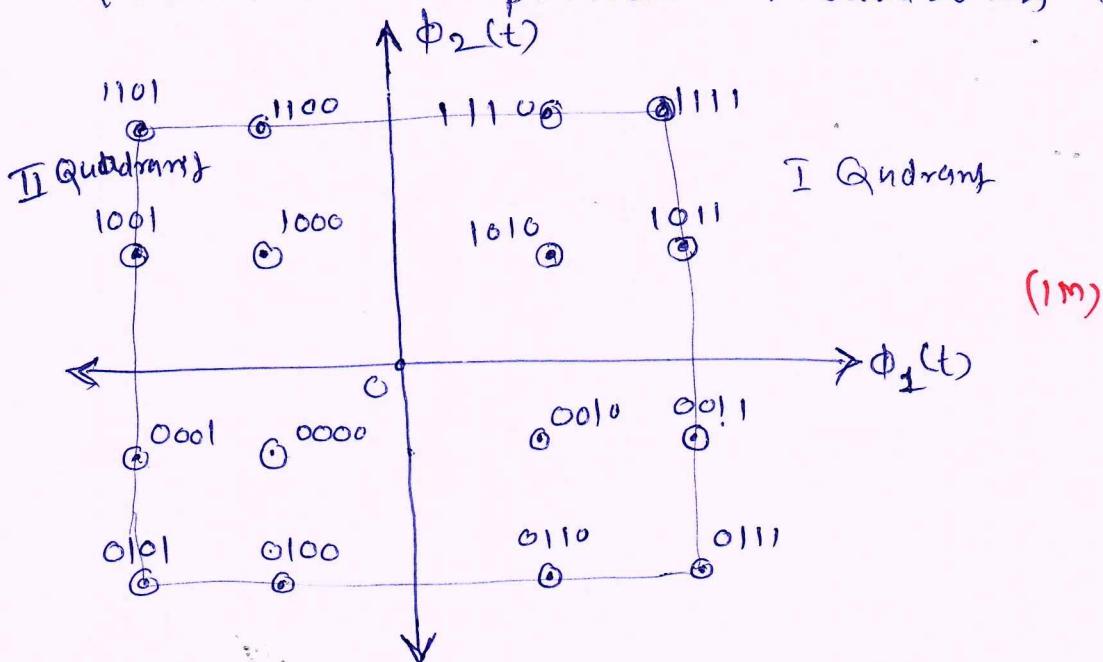
- Since  $\phi_j(t)$  have unit energy, by definition, the expression for noise variance  $\sigma_{x_j}^2$  reduce to

$$\sigma_{x_j}^2 = \frac{N_0}{2} \quad \text{for all } j \quad \dots \text{Eqn } 7 \quad - (1m)$$

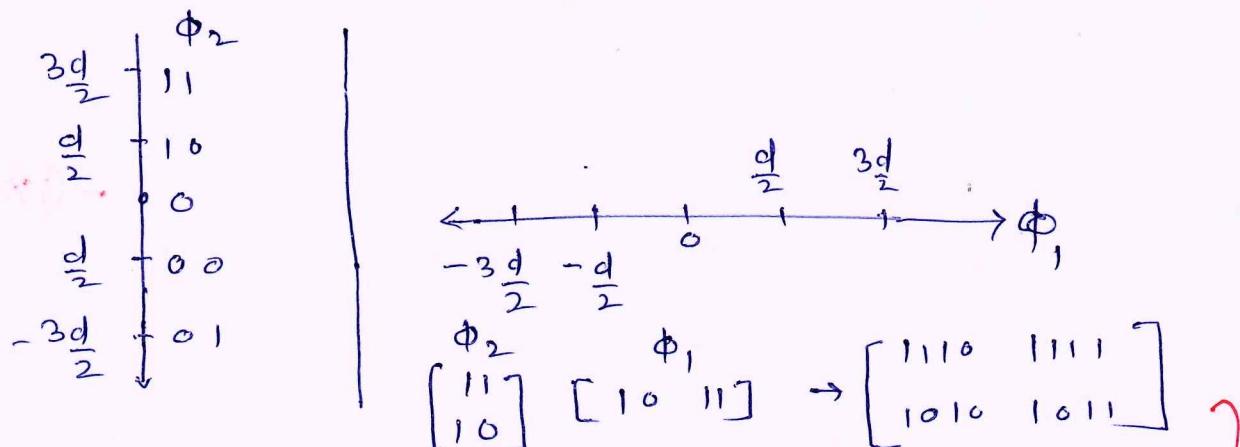
- 5c) Obtain the Constellation of QAM for  $M=16$   
and draw signal space diagram (4m)

M-any QAM is nothing but M-any

Quadrature Amplitude Modulation scheme



### Step 1 First Quadrant Constellation



### Step 2 Second Quadrant Constellation

$$\begin{bmatrix} 11 \\ 10 \end{bmatrix} \begin{bmatrix} 0100 \end{bmatrix} \rightarrow \begin{bmatrix} 1101 & 1100 \\ 1001 & 1000 \end{bmatrix}$$

### Step 3 Third Quadrant Constellation

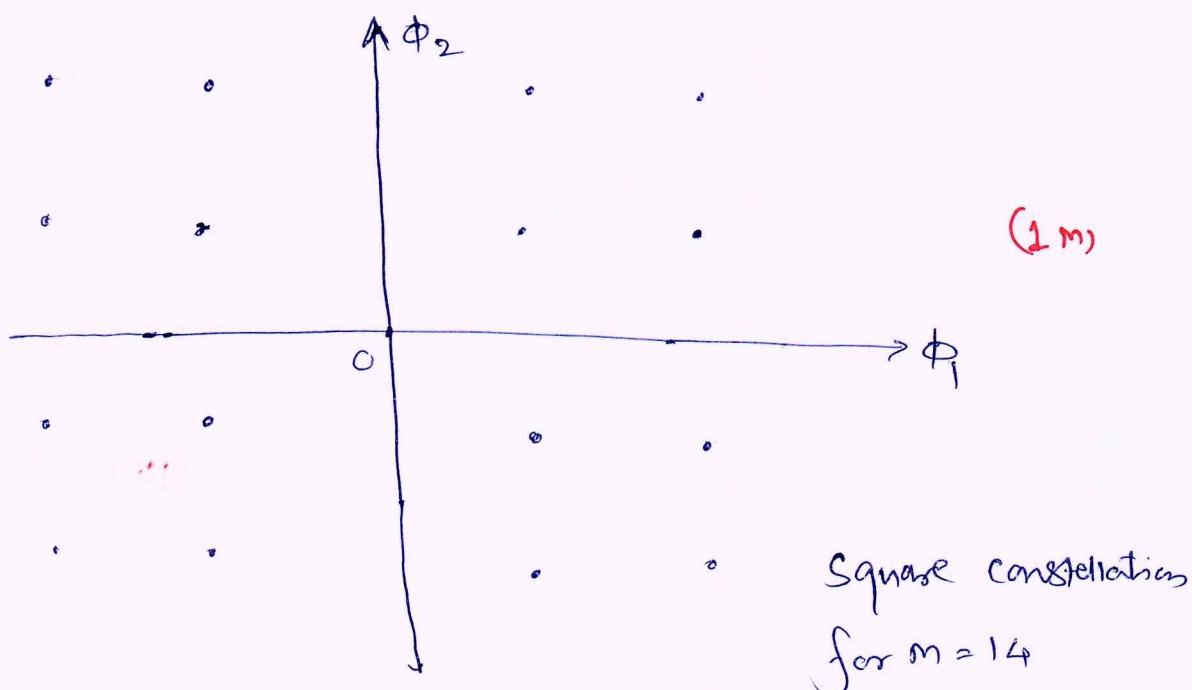
$$\begin{bmatrix} 00 \\ 01 \end{bmatrix} \begin{bmatrix} 0100 \end{bmatrix} \rightarrow \begin{bmatrix} 0001 & 0000 \\ 0101 & 0100 \end{bmatrix}$$

(2m)

### Step 4 Fourth Quadrant Constellation

$$\begin{bmatrix} 00 \\ 01 \end{bmatrix} \begin{bmatrix} 1011 \end{bmatrix} \rightarrow \begin{bmatrix} 0010 & 0011 \\ 0110 & 0111 \end{bmatrix}$$

(1m)



708162

S @

(06m)

i) Input binary sequence

ii) odd numbered sequence

Its polar NRZ O representation

iii)  $s_{i1} \phi_1(t)$

iv) Even numbered sequence

v)  $s_{i2} \phi_2(t)$

$$vi) s(t) = s_{i1}(t)\phi_1(t)$$

$$+ s_{i2}(t)\phi_2(t)$$

$$\frac{ST}{4}$$

$$\frac{\pi}{4}$$

$$\frac{3\pi}{4}$$

(2m)

5 b) Obtain the expression for average probability of symbol error for BPSK using coherent detection. (06m)

Ans: Signal space representation of BPSK

$$\text{Basic function: } \phi_i(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$s_1(t) = \sqrt{E_b} \phi_1(t)$$

$$s_2(t) = -\sqrt{E_b} \phi_2(t)$$

- (1m)

$$s_{11} = \sqrt{E_b} \quad s_{21} = -\sqrt{E_b}$$

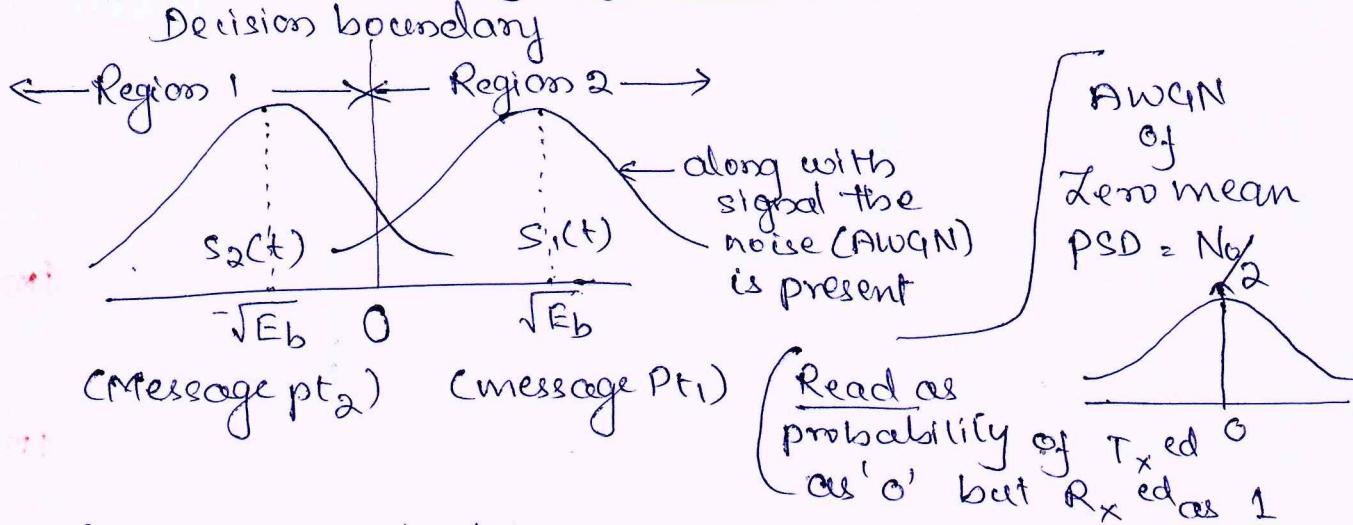
$$\leftarrow z_2 \rightarrow i \leftarrow 2, \rightarrow$$

$$-\sqrt{E_b} \quad 0 \quad \sqrt{E_b} \quad \phi_1$$

decision boundary

Ans: ? CAPT

## BPSK - Probability of Error Calculation :



Average probability of error =  $P_{e1} + P_{eo}$

The observation vector  $X_1$  is related to the received signal  $x(t)$  by

$$X_1 = \int_0^T x(t) \phi_1(t) dt \quad \rightarrow (1m)$$

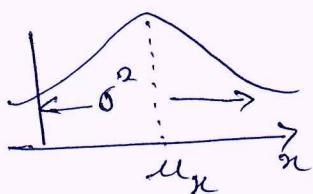
The error is of two types

- (1)  $P_{e0}/1$ ) i.e. transmitted ( $T_x^ed$ ) as '1' but received as '0'
- (2)  $P_{e1}/0$ ) i.e. transmitted as '0' but received as '1'

We assume that in

- most of the cases we have the areas  $A_1$  and  $A_2$  are same.
- Now we assume Gaussian noise.

Any Gaussian fm is given by



mean value  $\mu$

variance  $\sigma^2$

random variable  $x$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx \quad -(1m)$$

$$P_{e1}/0 = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x_i-4)^2}{2\sigma^2}\right] dx_i$$

$\mu$  = mean value (expected value) =  $-\sqrt{E_b}$  for the  $T_x^u$  of symbol '0'

$\sigma^2$  = variance =  $N_0/2$  for AWGN

~~notable~~

Value =  $T_b = 0$  [indicates lower limit in integration]  
 of AWGN → of zero shifted to

Mean value  $(C + \sqrt{E_b})$  or  $\sqrt{E_b}$  derates because of noise

$$\therefore P_{e0} = P_e(1|0) = \frac{1}{\sqrt{2\pi(\frac{N_0}{2})^2}} \int_0^\infty \exp\left[-\frac{(x_1 - (C - \sqrt{E_b}))^2}{2\pi(\frac{N_0}{2})^2}\right] dx_1,$$

$$P_{e0} = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{[\sqrt{N_0}]^2 = N_0}\right] dx_1, \quad - (1m)$$

It is not in standard form. The two standard forms are

(i) erfc      (ii)  $\phi$  Form

$$\therefore P_{e0} \circ Z = \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}} \quad dx_1 = \sqrt{N_0} \cdot dz$$

$$\therefore P_{e0} = \frac{1}{\sqrt{\pi}} \int_0^\infty \exp[-z^2] dz$$

$$Z = \sqrt{E_b/N_0}$$

$$\therefore P_{e0} = \frac{1}{2} \operatorname{erfc}\left(\frac{\sqrt{E_b}}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{\sqrt{\text{Energy of the signal}}}{\text{Noise power}}\right) \quad - (1m)$$

$$\left( \frac{1}{2} \operatorname{erfc}(u) = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-z^2} dz \right)$$

$$u = \sqrt{E_b/N_0}$$

$$\text{Similarly } P_e(0|1) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_\infty^\infty \exp\left[-\frac{(x_1 - u)^2}{2\sigma^2}\right] dx_2$$

$$P_e(0|1) = \frac{1}{2} \operatorname{erfc}\sqrt{\frac{E_b}{N_0}}$$

$$\therefore \text{Total probability of error } P_e = P_e(1|0)P_{e0} + P_e(0|1)P_e(1)$$

Assuming probability of 1's & 0's are equal

$$P_e = \frac{1}{2} [P_e(1|0) + P_e(0|1)]$$

$$P_e = \frac{1}{2} \operatorname{erfc}\sqrt{\frac{E_b}{N_0}} \quad - (1m)$$

G. a. Explain the generation and coherent declaration of BFSK system. (06m)

Ans:

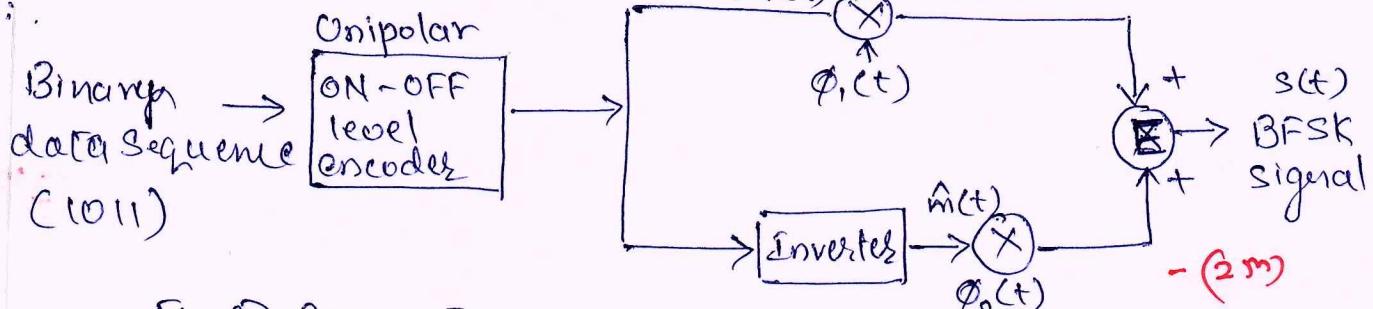


Fig ④ BFSK Tx

→ Here we are using unipolar encoders or ON-OFF encoders. which represents symbol '1' by a positive pulse of amplitude  $\sqrt{E_b}$  and duration  $T_b$  also symbol '0' is represented by no pulse i.e. its amplitude is 0 volts for duration,  $T_b$

Unipolar NRZ encoder O/p →

→ pair of oscillator whose frequencies  $f_1$  and  $f_2$  differ by an integer multiple of the bit rate  $\frac{1}{T_b}$

→ By using an inverter in the lower path, we ensure that when the input symbol is 1, the oscillator with frequency  $f_1$  in the upper path is switched on while the oscillator with frequency  $f_2$  in the lower path switched off.

→ Therefore, when the input symbol is 1, the output of the summer is sinewave of frequency  $f_1$ .

→ Conversely,

when input symbol is 0

lower oscillator is switched on having frequency  $f_2$   
upper oscillator is switched off

- Therefore, when the input symbol is '0', the output of the summer is sine wave of frequency  $f_2$ .
- Summarising, we find that a sine wave  $\phi_1(t)$  having frequency  $f_1$  appears at the output of the summer when the input symbol is '1' and a sine wave  $\phi_2(t)$  of frequency  $f_2$  appears at the output of the summer when the input symbol is '0'. (3m)
- With phase continuity as a requirement, the two oscillators are synchronized with each other. If VCO is used phase continuity is automatically satisfied.

### Detection of BFSK Signal

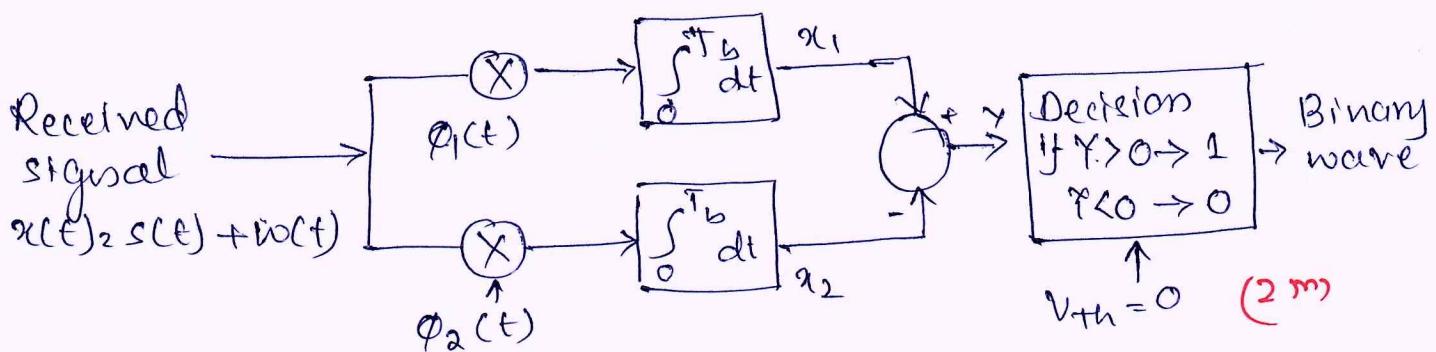


Fig ② BFSK receiver

→ Let the received signal  $x(t)$  be represented by a vector  $x_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  where,

$$x_1 = \int_0^{T_b} x(t) \phi_1(t) dt \quad \text{and} \quad x_2 = \int_0^{T_b} x(t) \phi_2(t) dt \quad \left. \right\} \text{eqn ①}$$

→ When symbol '1' is transmitted  $x(t) = s_1(t) + w(t)$  and when symbol '0' is transmitted  $x(t) = s_2(t) + w(t)$  where  $w(t)$  - sample function of AWGN process

of Zero mean and PSD =  $N_0/2$

→ To coherently detect the original binary sequence we may use the receiver shown in fig ②

→ The BFSK receiver consists of two correlators with a common input  $x(t)$ , which are supplied with locally generated coherent reference signals  $\phi_1(t)$  and  $\phi_2(t)$ .

→ The received signal  $x(t)$  is cross-correlated with the signals  $\phi_1(t)$  and  $\phi_2(t)$  to get components  $x_1$ ,  $x_2$  respectively and they are given in eqn ①

→ The correlator outputs are then subtracted

$$\text{i.e. } y = x_1 - x_2 \quad (1m)$$

→ The difference  $y = (x_1 - x_2)$  is used in making a decision as follows:

Choose symbol 1, if  $y > 0$ . (that is,  $x_1 > x_2$ )  
 $y = \sqrt{E_b} \text{ (tve)}$

Choose symbol 0, if  $y < 0$  (that is,  $x_1 < x_2$ )  
 $y = -\sqrt{E_b} \text{ (-ve)}$

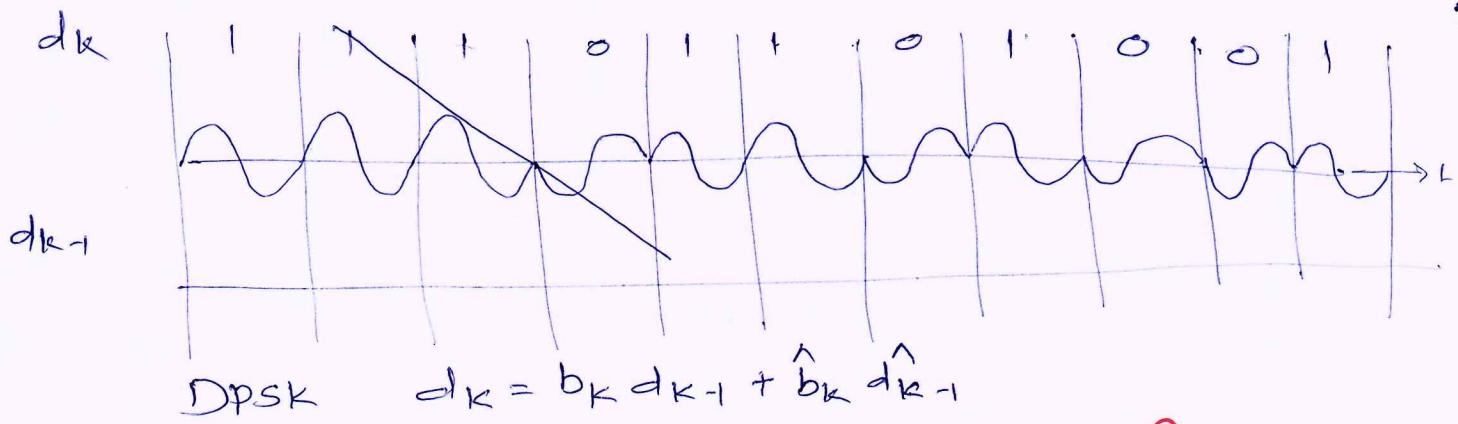
(If  $y=0$  either 1 or 0 can be chosen.)

Q. b. The binary sequence 1100100010 is applied to the DPSK transmitter (6m)

(i) Sketch the resulting waveform at the transmitter output.

(ii) Applying this waveform to the DPSK receiver, show that in the absence of noise, the original binary sequence is reconstructed at the receiver output.

|      |                                            |                                     |
|------|--------------------------------------------|-------------------------------------|
| Ans: | i) Input $\{b_k\}$                         | 1 1 0 0 1 0 0 0 1 0                 |
|      | Sequence                                   |                                     |
|      | ii) encoded $\{d_k\}$                      | 1 1 0 1 1 0 1 0 0 1                 |
|      | Sequence                                   |                                     |
|      | iii) Transmitted $\theta$ Sequence (phase) | 0 0 $\pi$ 0 0 $\pi$ 0 $\pi$ $\pi$ 0 |
|      |                                            |                                     |
|      | iv) detected output                        | 1 1 0 D 1 0 0 0 1 0                 |
|      |                                            |                                     |
|      | v) $d_{k+1}$                               | 1 1 1 0 & 1 0 0 0 1                 |



$b_k$  : 1 1 0 0 1 0 0 0 1 0  
 $d_{k-1}$  : 1 1 1 0 1 1 0 1 0 0  
 $d_k$  : 1 1 1 0 1 1 0 1 0 0 1  
 (Encoded Sequence)      (3m)

Transmitted Phase (degrees)  
 0 0 0  $\pi$  0 0  $\pi$  0  $\pi$   $\pi$  0



(c)  $P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{-E_b}{2N_0}\right)$  (1m)

 $E_b = \frac{A^2 T_B}{2} = 0.25 \times 10^{-18} \text{ J} \quad (1m)$ 
 $\frac{N_0}{2} = 10^{-20} \text{ W/Hz}$ 
 $\therefore P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{-0.25 \times 10^{-18}}{4 \times 10^{-20}}\right) = 0.965 \times 10^{-3} \quad (2m)$

Ans

7 a) i) ISI and Nyquist criterion for zero ISI. 23  
(Tom)

When a pulse of short duration  $T_b$  is transmitted on a limited channel, frequency components of the pulse are differentially attenuated due to the frequency response of channel, causing dispersion of pulse over the interval greater than  $T_b$ .

$$y(iT_b) = \underbrace{\frac{1}{T} \int_{-T/2}^{T/2} x(t) dt}_{\text{This is desired}} + u \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b) \quad - (1m)$$

Output for  $i^{\text{th}}$  sampling bit

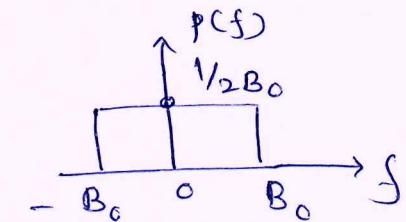
extra pulses  
i.e. residual effect  
called as ISI

Nyquist criterion for zero ISI

FOR

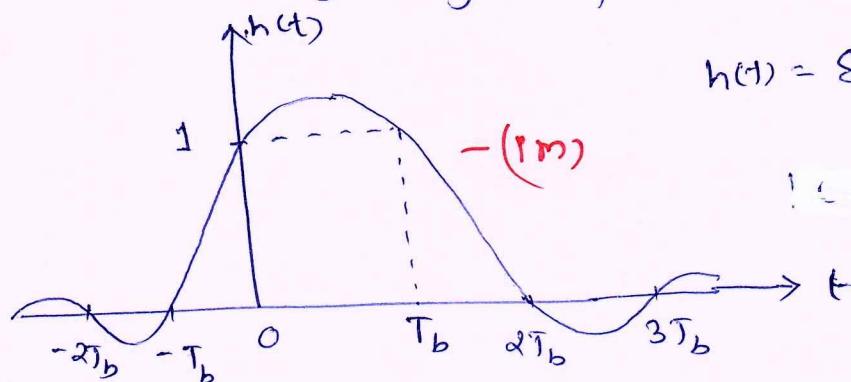
$$x(nT) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$p(f) = \begin{cases} \frac{1}{2B_0} & |f| < B_0 \\ 0 & |f| > B_0 \end{cases}$$



$$\sum_{n=-\infty}^{\infty} p(f-nR_b) = T_b \quad \text{has } p(iT_b - kT_b) = \begin{cases} 1 & i=k \\ 0 & i \neq k \end{cases} \quad - (1m)$$

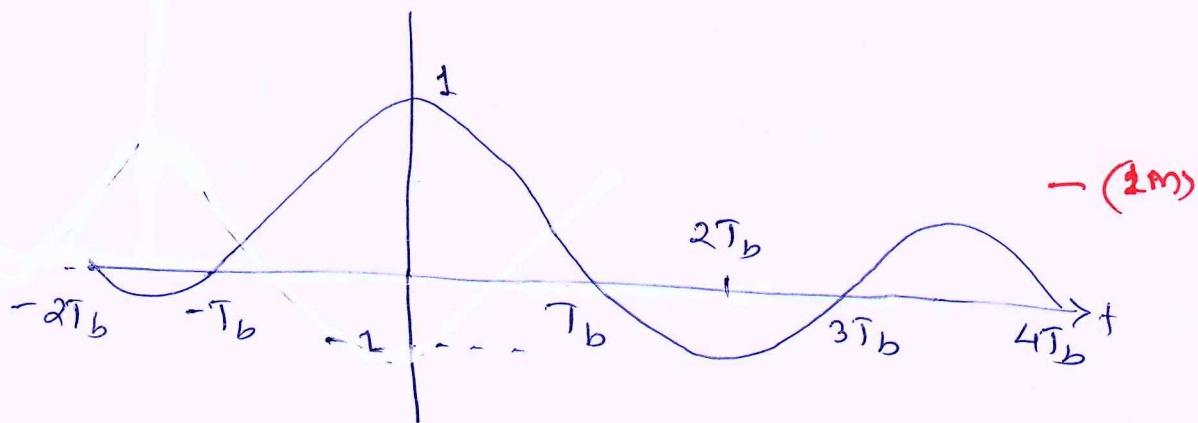
ii) Nonbinary Signal pulse



$$h(t) = \operatorname{sinc}\left(\frac{\pi t}{T_b}\right) + \operatorname{sinc}\left(\frac{\pi(t-T_b)}{T_b}\right) \quad - (1m)$$

!.....  
!.....  
!.....  
!.....

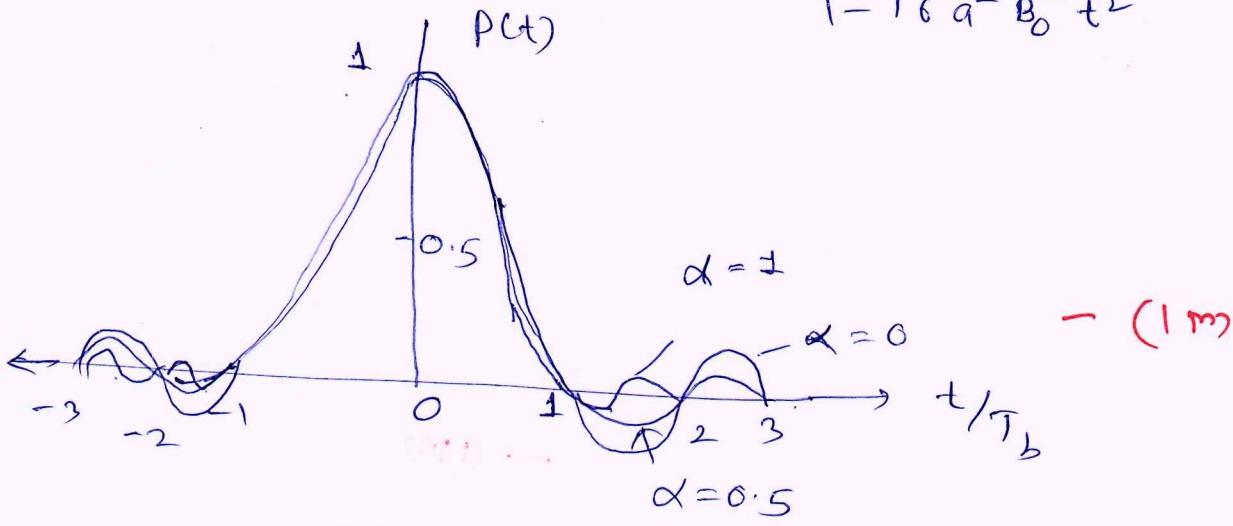
III) modified dual binary pulse



$$h(t) = \operatorname{sinc}\left(\frac{\pi t}{T_b}\right) + \operatorname{sinc}\left[\frac{\pi(t-2T_b)}{T_b}\right] \quad - (1m)$$

(IV) ~~Pulse Response~~ Raised Cosine  
Partial response signal

$$p(t) = \sin(2B_0 t) \frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 B_0^2 t^2} \quad - (1m)$$



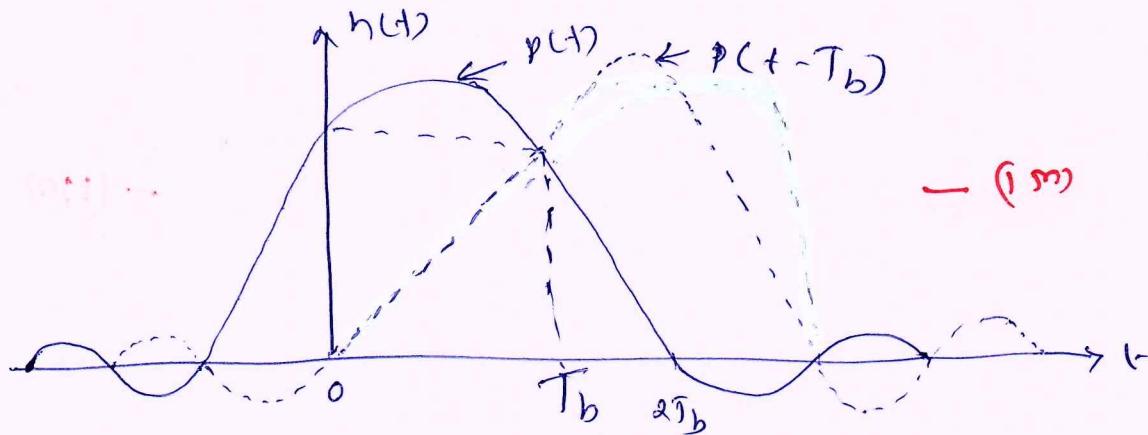
or

$$x(t) = \frac{\sin 2\pi n (t - n/2\omega)}{2\pi n (t - n/2\omega)}$$

Joshua D.

## IV Partial response signal

(25)



- (1m)

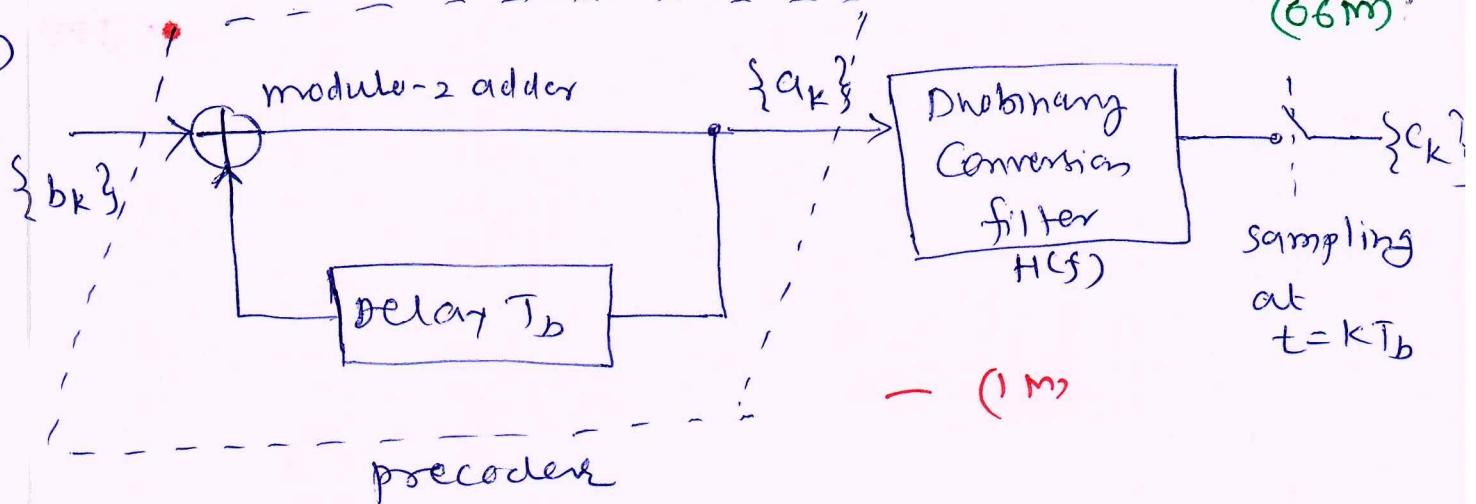
$$n = 0, 1 \quad - (n)$$

$$\text{for all other } n$$

$$p(hT_b) = \begin{cases} 1 \\ 0 \end{cases}$$

7 b)

(66m)



- (1m)

→ In duobinary coding without precoder, if the error is made in the previous estimate, then the error propagates

→ A precoding means avoiding this error propagation. For this precoding of binary Sequence is performed before duobinary encoding

(1M)

Start bit is assumed as 0

(2)

binary sequence  $\{b_k\}$

1 1 1 0 1 0 0 1 0 0 0 1 1 0 1

Precoded

Sequence

$$a_k = 0$$

$$a_k = b_k \oplus a_{k-1}$$

polar representation  
of precoded sequence

$$a_k$$

-1 +1 -1 +1 +1

1 0 1 1 0 0 0 1 1 1 1 0 1 1 0  
- (1m)

Dicode binary Coder

output

$$c_k = a_k + a_{k-1} \quad 0 0 0 2 0 -2 -2 0 2 2 2 0 0 +2 0$$

- (1m)

Decoding decision rule

If  $c_k = \pm 2$  decide  $b_k = 0$

If  $c_k = 0$  decide  $b_k = 1$

Received Sequence

0 0 0 2 0 -2 -2 0 2 2 2 0 0 2 0

Decoded binary Sequence

$\{b_k\}$

1 1 1 0 1 0 0 1 0 0 0 1 1 0 1

- (1m)

Yashika

8 Q. With a neat diagram, explain the concept of linear traversal filter. (06m)

Ans: we know that in real channels, the ISI is limited to a finite number of unequalized.

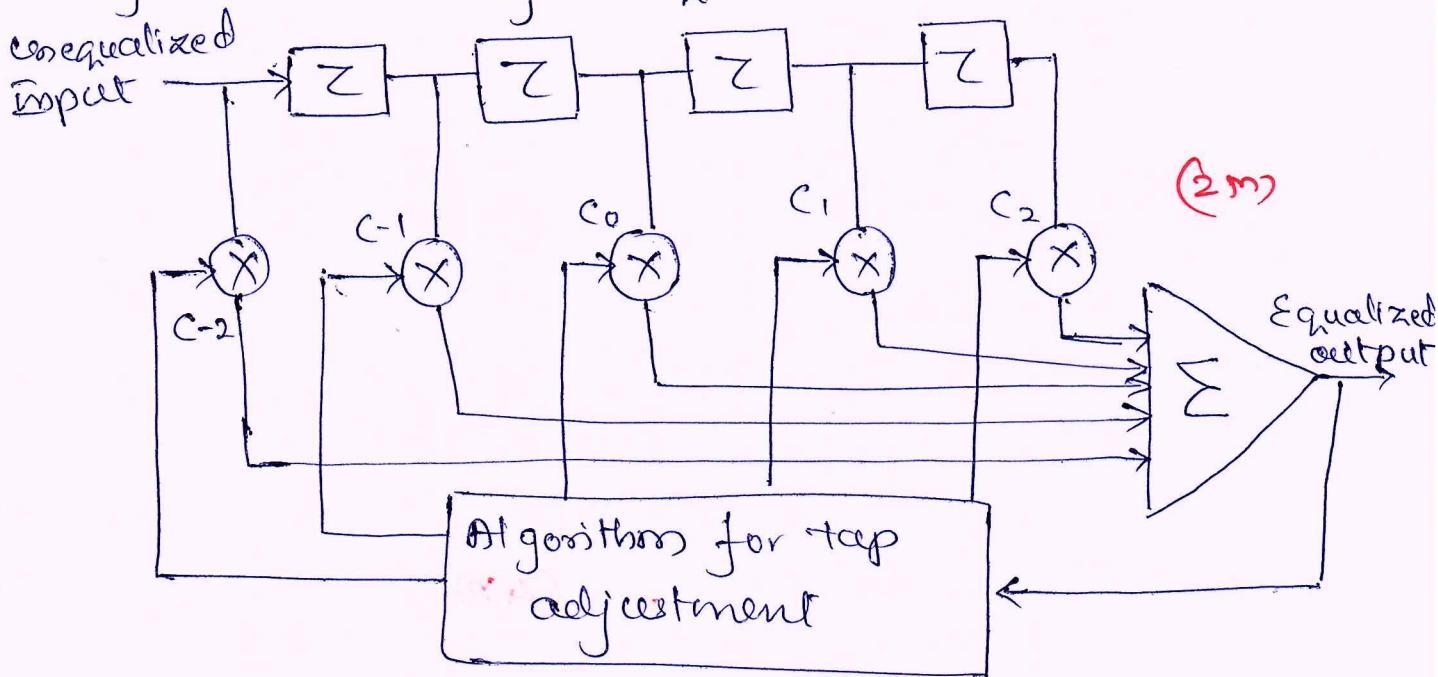


fig: Linear Transversal Filter

→ We know that in real channels, the ISI is limited to a finite no. of samples L samples.

→ As a result, in practice for example, the channel equalizer is approximated by a finite duration impulse response (FIR) filter or transversal filter, with adjustable tap coefficients  $\{c_n\}$ .

→ The time delay  $\tau$  between adjacent taps may be selected as large as  $T$ , the symbol interval in which case the FIR equalizer is called a symbol spaced equalizer.

→ In this case the input to the equalizer is the sampled sequence given by

$$x_m = x_{0,m} + \sum_{n=-\infty}^{\infty} x_{m-n} + w_m$$

→ However, we note that when  $\frac{1}{T} < 2w$ , frequencies in the received signal that are above the folding frequency  $\frac{1}{T}$  are aliased into frequencies below  $\frac{1}{T}$ .

→ In this case the equalizer compensates for the aliased channel - distorted signal.

→ When the time delay  $Z$  between the adjacent taps is such that  $\frac{1}{Z} \geq 2w \geq \frac{1}{T}$  no aliasing occurs hence the inverse channel equalizer compensates for the true channel distortion.

→ Since  $Z < T$ ,

channel equalizer is said to have fractionally spaced taps, and it is called a fractionally spaced equalizer.

→ In practice  $Z$  is often selected as  $Z = \frac{T}{2}$ , in this case the sampling rate at the output of the filter  $GR(f)$  is  $\frac{2}{T}$  — (4m)

8b) According to condition (06m)

$$q(mT) = \sum_{n=-N}^N c_n x(mT - nT) = \begin{cases} 1 & m=0 \\ 0 & m = \pm 1, \pm 2 \\ \dots, 0 & \pm N \end{cases}$$

the ZF equalizer must satisfy the equations

$$q(mT) = \sum_{n=-2}^{n=2} c_n x(mT - nT/2) = \begin{cases} 1 & m=0 \\ 0 & m = \pm 1, \pm 2 \end{cases} - (1m)$$

The matrix  $X$  with elements  $x(mT - nT/2)$  is given

as

$$X = \begin{bmatrix} \frac{1}{5} & \frac{1}{10} & \frac{1}{17} & \frac{1}{26} & \frac{1}{37} \\ 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{10} & \frac{1}{17} \\ \frac{1}{5} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{17} & \frac{1}{10} & \frac{1}{5} & \frac{1}{2} & 1 \\ \frac{1}{37} & \frac{1}{26} & \frac{1}{17} & \frac{1}{10} & \frac{1}{5} \end{bmatrix} \quad (2m)$$

*Zohreh*

The coefficient vector  $c$  and vector  $q$  are given as

$$c = \begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} \quad q = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad - (1m)$$

Linear matrix  $X.c = q$  can be solved by inverting the matrix  $X$ .

$$\text{Optimum} = X^{-1}q = \begin{bmatrix} -2.2 \\ 4.9 \\ -3.0 \\ 4.9 \\ -2.2 \end{bmatrix} \quad - (1m)$$

8c) Eye diagram is an experimental method of knowing the status of received signal. (04m)

→ It is an experimental method that contains all the information concerning the development of quality. Therefore careful analysis of the eye pattern is important in analyzing degradation mechanism.

- (1m)

→ Eye pattern can be observed using oscilloscope

→ To observe eye pattern the -

received signal is applied to the vertical deflection plates of an oscilloscope and

sawtooth wave signal at a rate equal to the

transmitted symbol rate is applied to the horizontal deflection plates. - (1m)

→ Resulting display is eye pattern.

It resembles the eye opening of human eye therefore it is called as eye pattern (1m)

→ The internal region of eye pattern is called as eye opening -(1m)

→ Let us consider an example

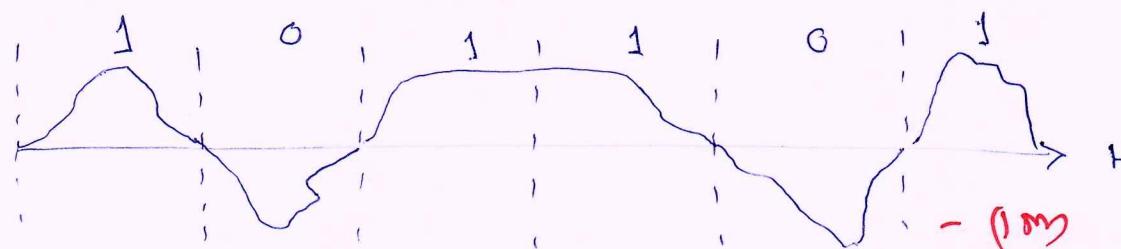


fig: received signal which is corrupted due to ISI

→ we get superposition of successive symbol intervals to produce eye pattern as shown below

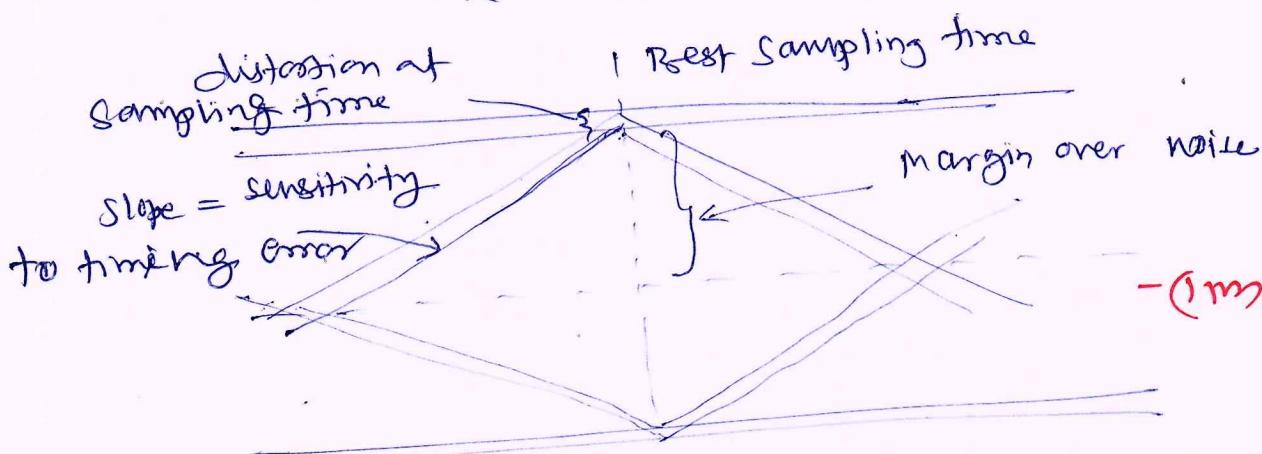
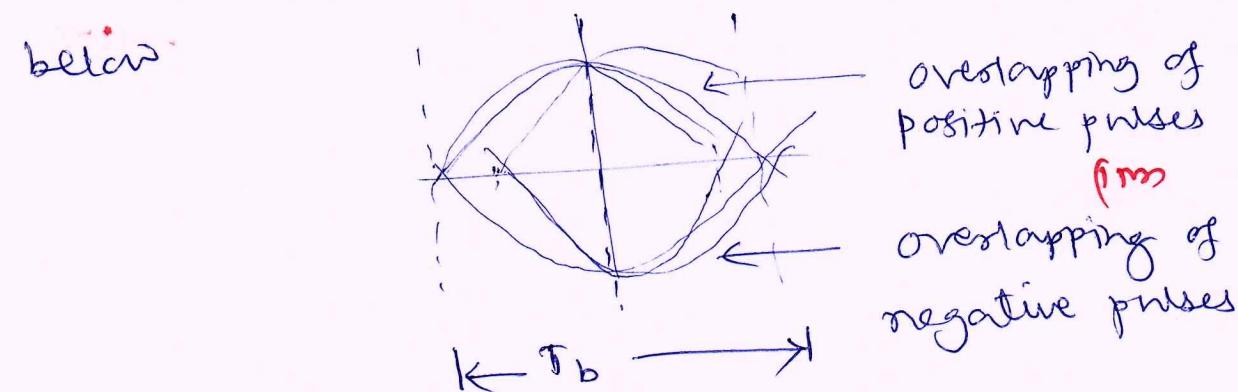


fig: interpretation of eye pattern

(Q. a. with a neat diagram, explain the generation of PN sequences and state its properties.)

Ans: Pseudo Noise Sequence:

- Are class of sequences of 1s and 0s which are periodic and processes autocorrelation property. - (2m)
- PN sequences are of much greater length since it is intended for BW spreading
- Main benefits of using this PN sequence are
- Protection against interference. - coding enables a BW Trade for processing gain against interfering signals.
- Provision for privacy - coding enables protection of signals (High length code) - (1m)
- Noise - effect reduction - code can reduce the effects of noise and interference.
- Maximal length codes are commonly used PN codes.
- These PN sequence which are maximal length codes are generated by using shift registers.
- Here we are making use of no. of stages of shift registers of even lengths in conjunction with the appropriate logic, which feedback a logical combination of the state of two or more of its stages to its input.
- The binary shift register, the maximum length sequence is

$$N = 2^m - 1 \text{ chips}$$

Zeshan

where

$m$  = no. of stages of flip-flops in the shift register.

$N$  = is the period.

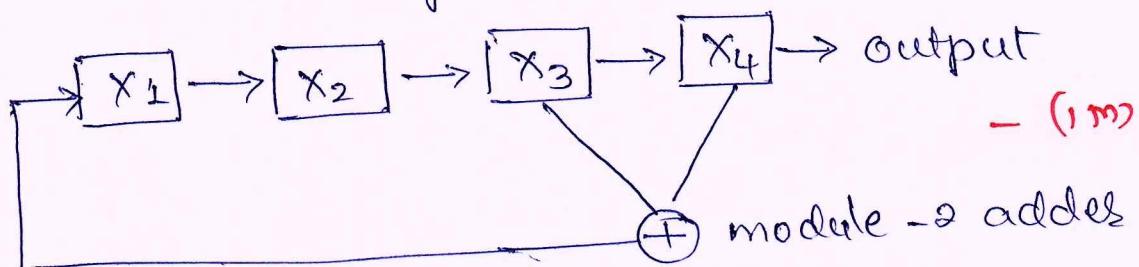
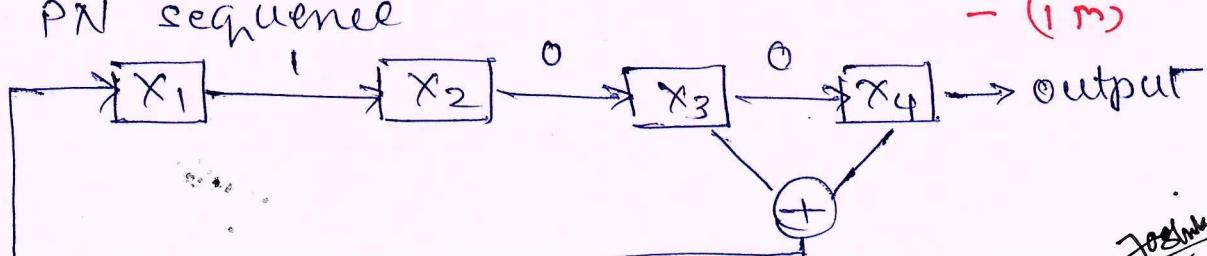


Fig: Linear Feedback shift Register with modulo-2 adder

At each clock pulse

- contents of registers shift one bit right
- contents of required stages are modulo 2 added and fed back
- The fig shows binary linear feedback shift register with four flip-flops  $X_1, X_2, X_3 + X_4$  + feedback mechanism i.e. modulo 2 adder. This feeds back the addition of  $X_3 + X_4$  stage back to the input. output is obtained at the last stage
- Contents of shift register at any moment of time depending upon clock applied shifted to the right by 1 bit. This is indicated by the arrow shown in fig.
- Let us take an example of generation of PN sequence



Let us consider initial state of shift register be 1000

|                       | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|-----------------------|-------|-------|-------|-------|
| CLOCK 1 $\rightarrow$ | 1     | 0     | 0     | 0     |
| CLOCK 2 $\rightarrow$ | 0     | 1     | 0     | 0     |
| .                     | 0     | 0     | 1     | 0     |
| ;                     | 1     | 0     | 0     | 1     |
| ,                     | 1     | 1     | 0     | 0     |
| :                     | 0     | 1     | 1     | 0     |
| ,                     | 1     | 0     | 1     | 1     |
| :                     | 0     | 1     | 0     | 1     |
| )                     | 1     | 0     | 1     | 0     |
| :                     | 1     | 1     | 0     | 1     |
| )                     | 1     | 1     | 1     | 0     |
| :                     | 1     | 1     | 1     | 1     |
| ,                     | 0     | 1     | 1     | 1     |
| ,                     | 0     | 0     | 1     | 1     |
| ,                     | 0     | 0     | 0     | 1     |

16<sup>th</sup> clock  $\rightarrow$  1 0 0 0 0

Jaswanth

- We can see for shift registers of length m.
- At each clock the change in state of flip-flop is shown.
- Feedback function is modulo two of  $x_3 + x_4$
- After 15 clock pulses the sequence repeats
- Output sequence is

000100110101111

### Properties of PN Sequence — (3m)

#### ① Balance property

In each period of the sequence, no. of binary ones differ from binary zeros by at most one digit.

Consider output of shift register

000100110101111

Here there are seven zeros & eight ones — meets balance condition.

#### ② Run length property

Among the runs of ones and zeros in each period, it is desirable that about one half the runs of each type are of length 1, one-fourth are of length 2 and one-eighth are of length 3 & so on.

Consider opp of shift register

No. of runs = 8

$$\begin{array}{r} 0 \ 0 \ 0 \quad 1 \ 0 \ 0 \quad 1 \ 1 \quad 0 \quad 1 \quad 0 \quad 1 \ 1 \ 1 \\ \hline 3 \qquad 1 \qquad 2 \qquad 2 \qquad 1 \qquad 1 \qquad 2 \qquad 4 \end{array}$$

Runs meaning grouping of similar bits

Zoheb  
(ASJ)

### (3) Auto correlation property

→ Auto correlation function of a maximal length sequence is periodic and binary valued. Auto correlation will indicate how well the sequence is correlated with shift in itself.

→ Auto correlation sequence of binary sequence in polar format is given by

$$R_c(k) = \frac{1}{N} \sum_{n=1}^{N=N} c_n c_{n+k}$$

where  $c(k)$  - PN sequence

$N$  - length or period of the sequence

$k$  - lag of autocorrelation

(It indicates what will be the shift i.e. 1 bit or 2 bits etc.)

$$\text{Auto correlation } R_c(k) = \begin{cases} 1 & \text{if } k=1 \\ -\frac{1}{N} & \text{if } k \neq 1 \end{cases}$$

where 1 is any integer

Or we can state autocorrelation function as

$$R_c(k) = \frac{1}{N} \left\{ \text{No. of agreements} - \text{No. of disagreements in comparison of one full period} \right\}$$

Consider output of shift registers for  $l=1$

0 0 0 1 0 0 1 1 0 1 0 1 0 1 1 1  
 1 0 0 0 1 0 0 1 1 0 1 0 1 1 1  
 ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
 d a a d d a d a d d d a a a

$$R_c(k) = \frac{1}{15} \{ 7 - 8 \}$$

Yours?

$$R_c(K) = -\frac{1}{15}$$

a. b. write a note on application of sp (04m)

A DS spread spectrum signal is designed so that the power ratio  $P_R/P_N$  at the intended receiver is  $10^{-2}$ . If the desired  $E_b/N_0 = 10$  for acceptable performance, determine the minimum value of the processing gain.

Ans.

$$\frac{E_b}{N_0} = \left( \frac{P_R}{P_N} \right) L_c \quad \therefore L_c = 1000 \quad - (04m)$$

q.c.: Explain with neat block diagram FH spread spectrum systems. (08m)

Ans:

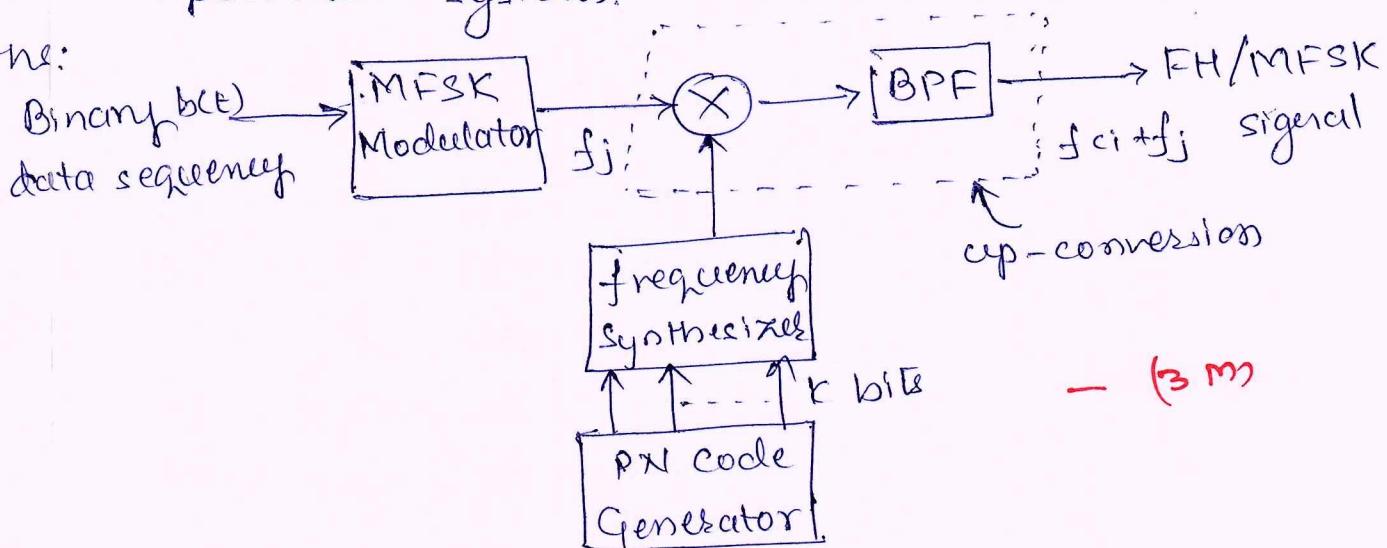


Fig ① FH/MFSK transmitter.

→ The first stage of an FH/MFSK transmitter is the frequency modulator and second stage is frequency mixer.

→ The incoming binary sequence is applied to a serial-to-parallel converter to get blocks of k-bit.

Therefore because of large FH bandwidths, coherent detection is possible only within each hop, since frequency synthesizers are incapable of maintaining phase coherence over successive hops.

→ As a result, non-coherent MFSK detection is used or FSK modulations with noncoherent demodulations is usually used in FHSS systems.

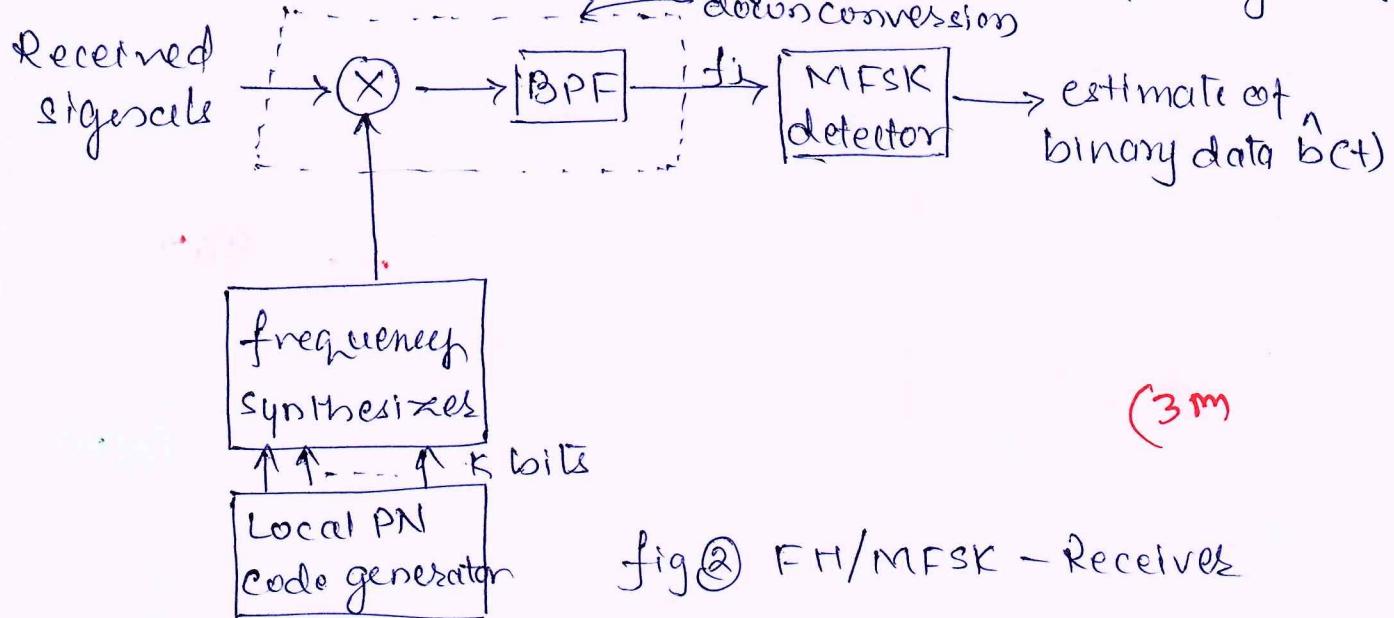


fig ② FH/MFSK - Receiver

- In the first stage, mixing operation (down conversion) removes the frequency hopping.
- The mixer inputs are the received signals and the output of a local frequency synthesizer that is in synchronization with that of transmitter.
- The output of mixer is passed through a BPF which selects the difference frequency component from the mixer.
- The output of the BPF is the MFSK signal, which is demodulated using noncoherent MFSK detector.
- The noncoherent MFSK detector consists of a bank of  $M$ .
- Depending on  $K$ -bit binary pattern, any one of the  $2^K$  discrete amplitude levels of  $M$ -ary PAM signal is obtained.

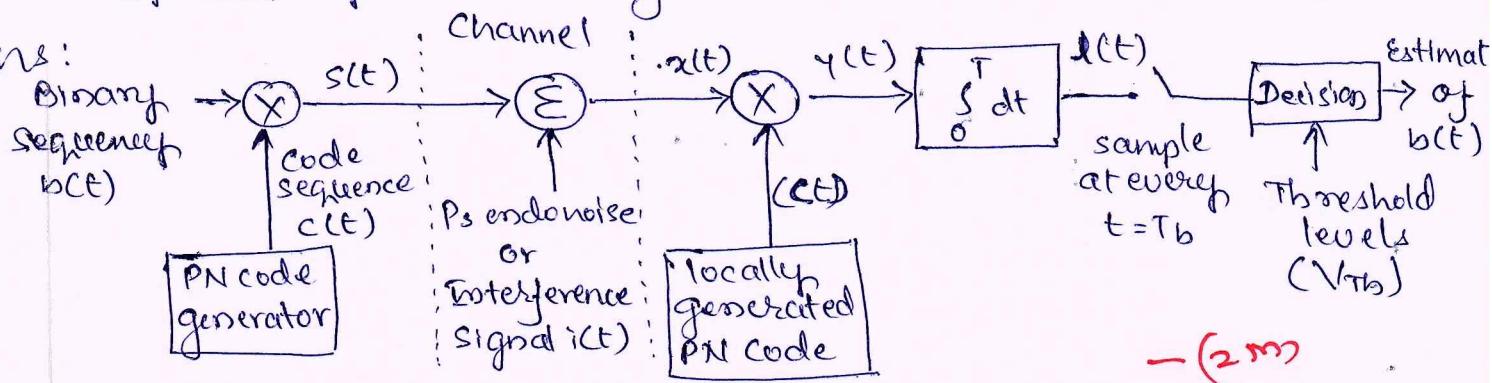
Yashika

- This M-ary PAM signal is applied to a voltage controlled oscillator (VCO)
- For every amplitude level in the M-ary PAM signal, the VCO produces one of the  $2^k$  discrete frequencies.
- Thus,  $s(t)$  is an MFSK wave.
- The output of MFSK modulator is then mixed with the output of a frequency synthesizer.
- The frequency synthesizer output is one of  $4 = 2^k$  values, where  $k$  - equals no. of bits of the PN sequence generator output.
- As a result, frequency hops over  $2^k$  distinct values.
- The BPF (Band Pass filter) passes the sum frequency for the transmitter and rejects the difference frequency components.

*Joshi*

10(a) Explain the generation and demodulation of DS spread spectrum signal. 39  
6cm

Ans:



- (2m)

Fig : Baseband DS-SSS system

→ The input binary sequence  $b(t)$  is the baseband signal in NRZ polar format. In which symbol 1 is represented by +1 volt and symbol 0 by -1 volt, with each bit occupying a time of  $T_b$  seconds.

→  $c(t)$  is a wideband code sequence which has noise like properties.

This sequence  $c(t)$ , also spreading sequence, is generated by a feedback shift register.

→ Like  $b(t)$ ,  $c(t)$  is also expressed in NRZ polar format so that it has two levels, +1 volt and -1 volt.

→ It is important to note that, the duration of each bit  $T_c$  in  $c(t)$  is the period of the clock pulse that is applied to the feedback shift register used for generating  $c(t)$ .

→ Normally,  $T_c \ll T_b$  and hence bit rate  $R_c$  of the  $c(t)$  is much greater than input bit rate  $R_b = \frac{1}{T_b}$ .

→ Often,  $R_c$  is also called chip rate while  $T_c$  is called as chip interval.

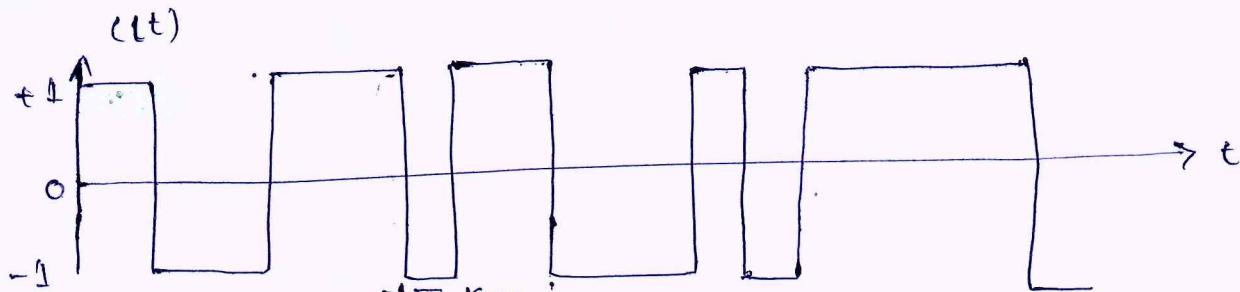


fig (a) PN signal

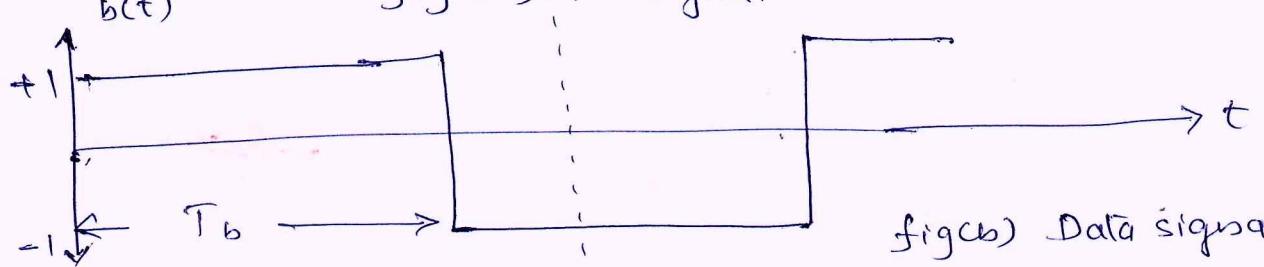


fig (b) Data signal

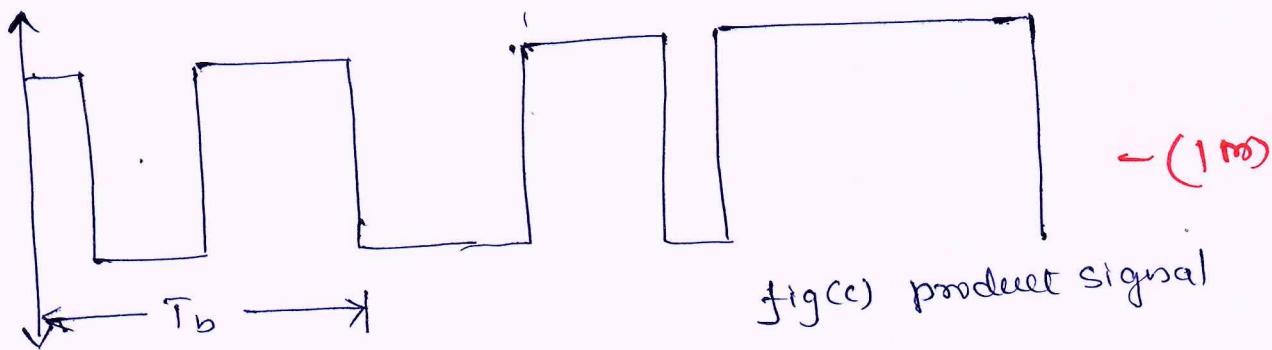


fig (c) product signal

fig ② Generation of a DSSS signal

Fig ② shows the basic method for accomplishing the spreading

→ The information bearing baseband signal  $b(t)$  is multiplied by the signal from the PN sequence generator i.e.  $c(t)$

→ This multiplication operation serves to spread the BW of the information bearing signal (whose BW is approximately  $R_b$  Hz) into the wider BW occupied by PN generator signal  $c(t)$  (whose BW is approximately  $1/T_c$ ) (1m)

*J. Shrivastava*

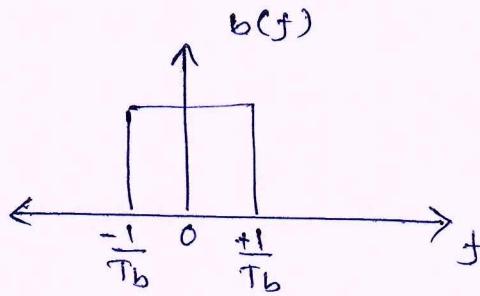


fig (d)

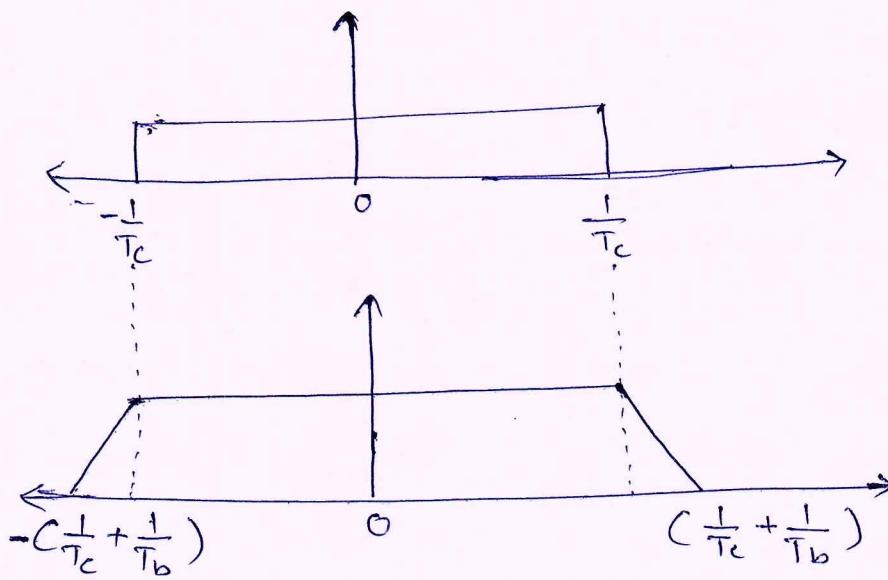


fig (e)

- (im)

fig (3): Convolution of power spectra of  $b(f)$  (data signal) and  $c(f)$  (spreading or PN sequence)

→ The Fig (3) shows spectrum spreading where input binary sequence  $b(f)$  is narrowband and PN sequence  $c(t)$  is wideband, the BW of the spectrum  $s(f)$  is nearly equal that of  $c(f)$ .

→ Therefore PN sequence performs the role of a spreading code

→ Let the signal  $R_x^{ed}$  at the Rx be  $x(t)$ .

Then

$$x(t) = s(t) + i(t) \quad \text{equ}(1)$$

where  $i(t)$  — denotes an interference signal or a jamming signal aimed at disrupting the information

→ The first step in the detection process is to multiply the  $R_x^{ed}$  signal  $x(t)$  by a locally generated PN sequence which is the exact replica of that used at the Tx.

→ This multiplication is performed by a product modulator shown at the receiver (Fig (2))

The output of product modulator is given by

$$\begin{aligned}
 y(t) &= s(t) \cdot c(t) \\
 \therefore y(t) &= [s(t) + i(t)] \cdot c(t) \\
 &= [b(t) \cdot c(t) + i(t) \cdot c(t)] \cdot c(t) \\
 &= b(t) c^2(t) + i(t) \cdot c(t) \quad \xrightarrow{\text{eqn 2}}
 \end{aligned}$$

Since  $c^2(t) = 1$  for all  $t$

$$\therefore y(t) = b(t) + i(t) \cdot c(t) \quad \xrightarrow{\text{eqn 3}}$$

Since  $c^2(t) = 1$

Thus from eqn 3 it is clear that the spectrum of  $s(t)$  is despread, resulting in spectrum of  $b(t)$ .

→ On multiplication by code sequence  $c(t)$  the spectrum of  $i(t)$  is spread but spectral height of  $i(t) \cdot c(t)$  is small.

→ Next  $y(t)$  is applied to the integrator as shown in fig 8.

→ During every bit interval  $T_b$ , the component  $b(t)$  of  $y(t)$  generates a positive ramp if  $b(t)=1$  or negative ramp if  $b(t)=-1$  - (1m)

→ Since  $i(t) \cdot c(t)$  are uncorrelated, the component  $i(t) \cdot c(t)$  integrates out to a very small value.

→ Thus the effect of jamming signal  $i(t)$  is substantially reduced.

→ The integrator output  $l(t)$  is sampled every  $T_b$  seconds to get  $l(T_b)$  and integrator is reset to zero after every sampling.

→  $l(T_b)$  is compared with threshold  $V_{Th} = 0$

→ If  $l(T_b) > 0$  the decision is in favour of data symbol 1

Otherwise the decision is in favour of data symbol 0 [ $l(T_b) < 0$ ]

*Anscombe*

10.b. write a note on application of spread spectrum in wireless LAN's. (04 m)

- Ans: → Spread spectrum signals have been used in the IEEE wireless LAN standards 802.11 & 802.11b which operate in the 2.4 GHz ISM (Industrial, Scientific and Medical) unlicensed frequency band.
- The available BW is subdivided into 14 overlapping 22MHz channels, although not all channels are used in all countries.  
 In USA only channels 1 through 11 are used.
- In the 802.11 standard one 11 chip Barker sequence is modulated and transmitted at a chip rate of 11 MHz i.e. chip duration is 0.909 usec
- The 11 chip Barker sequence is  
 $\{1, -1, 1, 1, -1, 1, 1, 1, -1, -1, -1\}$
- This sequence is desirable because its auto-correlation has sidelobes of less than or equal to 1 compared with the peak autocorrelation value of 11.
- Its future values can only be derived by a statistical model. (04 m)
- However a pseudorandom signal is not random at all.
- It is deterministic, periodic signal that is known to both the Tx and Rx
- Even though it is deterministic, it appears to have the statistical properties of sampled white noise.
- It appears to an unauthorized listener, as a truly random signal.
- Such pseudorandom noise (PN) sequences or pseudorandom binary sequences (PRBS) are binary sequences that are generated using shift registers

having feedback connections.

→ Using an  $n$ -stage shift register having an appropriate linear feedback signals, it is possible to generate a periodic sequence with a period equal to  $2^n - 1$  bits.

→ Such sequences are also called maximum length (ML) sequences.

→ An  $n$ -stage shift register consists of  $n$  clocked D-type F/F's. D-type F/F's are also called delay F/F's.

→ The output  $Q$  of the F/F after a clock pulse is equal to the input  $D$  of the F/F just before the clock pulse.

→ The Barker sequence is modulated either with BPSK or QPSK.

When BPSK is used with 11 chips/bit, a data rate of 1 Mbps is achieved.

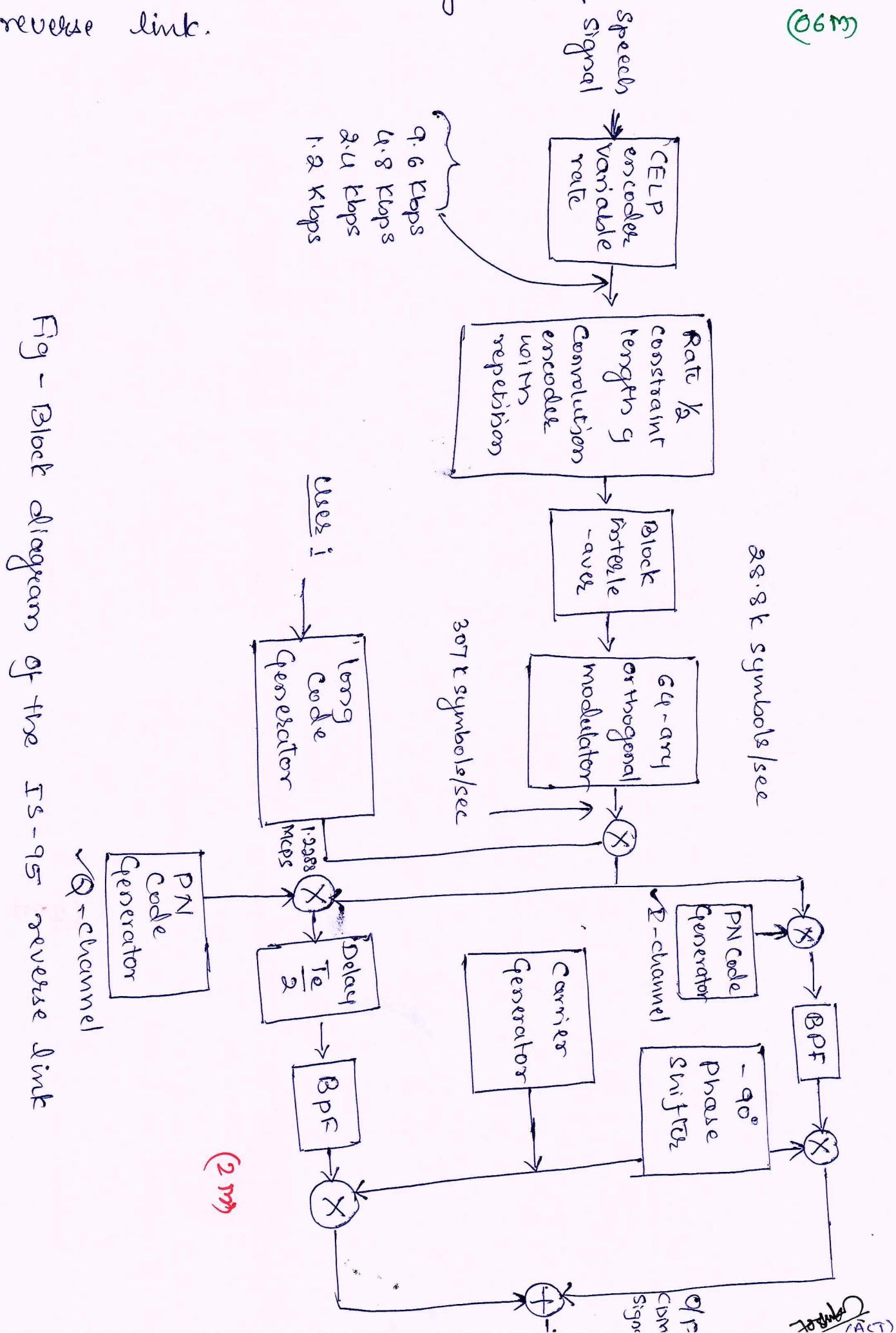
→ DSSS is also used in the higher speed (2nd generation) IEEE 802.11 b wireless LAN standard which operates in the same 2.4 GHz ISM band.

→ In 802.11 b, the 11 MHz chip rate is maintained but the Barker sequence is replaced by a set of 8 chip wolf sequences, called complementary code shift keying (CCK), which can be viewed as DSSS modulation with multiple spreading sequences.

→ The use of CCK modulation results in a data rate 11 Mbps.

*Yashwant*

Q. c. with a neat block diagram, explain the IS-95 reverse link. (06m)



- The receive link modulator from a mobile transmitter ( $T_x^{ext}$ ) to a base station is different from the forward link modulator.
- A block diagram of modulator is shown.
- The signals transmitted from various to the base station are asynchronous resulting in interference from other users.
- Battery operated mobile transmitters result in a power limited transmissions.
- Effect of channel noise is reduced by using rate  $\frac{1}{3}$  constraint lengths 9 convolutional code in the reverse link
- Coded bit rate is 28.8 kbps.
- For low rate speech output, bits from convolutional encoder are repeated approx by 2 or 4 or 8 times.
- As show in figure  
 The Q channel signal is delayed by  $\frac{T_c}{2}$  relative to I-channel signal producing an offset QPSK signal. (2m)
- Demodulator uses non-coherent demodulation of the 64 orthogonal Walsh sequences to recover the encoded data bits.  
 Basic parameters in the CDMA IS-95 sys are given in Table 1.

*forwards*

Table 1. Basic parameters in IS-95 systems

| <u>System parameters</u>                           | <u>Specifications</u>                                                   |
|----------------------------------------------------|-------------------------------------------------------------------------|
| 1) Uplink frequency band (U)<br>(mobile to base)   | 824-849 MHz                                                             |
| 2) Downlink frequency band (D)<br>(base to mobile) | 869-894 MHz                                                             |
| 3) Bandwidth/carrier                               | 1.25 MHz                                                                |
| 4) No. of carriers/band                            | 20                                                                      |
| 5) No. of user carriers                            | 60                                                                      |
| 6) Chip rate                                       | 1.2288 Mcps                                                             |
| 7) Speech codes                                    | variable rate CELP                                                      |
| 8) Speech rate                                     | 9600, 4800, 2400, 1200 bps                                              |
| 9) Channel encoders                                | Rate $\frac{1}{2}$ constraint lengths $q(D)$                            |
|                                                    | Rate $\frac{1}{3}$ " $\frac{q(D)}{q(U)}$                                |
| 10) Interleaves                                    | Block                                                                   |
| 11) Modulation                                     | BPSK with QPSK spreading (D)<br>64-ary orthogonal with QPSK spreading   |
| 12) Demodulation                                   | RAKE matched filter with maximal ratio combining.                       |
| 13) Signature sequences                            | Walsh (Hadamard) sequences of length 64                                 |
| 14) PN sequences                                   | $N = 2^{42} - 1$ — long code<br>$N = 2^{15}$ — spreading code.<br>- FMS |

— end —