Sixth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Finite Element Method

Time: 3 hrs.

Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. List the type of elements with neat sketch. **(06 Marks)**

b. A simply supported beam subjected to point load at the centre. Derive an equation for maximum deflection using trigonometrically function by RR method. **(10 Marks)**

OR

2 a. List the advantages and disadvantages of FEM. **(03 Marks)**

b. Explain Elasticity matrix [D] for stress and plain strain. **(04 Marks)**

c. Explain simplex, complex and multiplex elements. **(09 Marks)**

Module-2

3 a. Derive the shape function, in natural coordinate system for:

(i) Constant strain triangle.

(ii) 1D bar element. **(08 Marks)**

b. Using two point Gaussian quadrature formula evaluate and compare with exact solution:

(i) $$I = \int_{1}^{+1} \left(1 + \xi + 2\xi^2 + 3\xi^3\right) d\xi$$

(ii) $$I = \int_{-2}^{-2} (4 - y)^2 dy$$ **(08 Marks)**

OR

4 a. For the stepped bar shown in Fig. Q4 (a), determine the nodal displacement, element stresses and reaction at supports.

$E_1 = 70$ GPa; $E_2 = 200$ GPa; $P = 200$ KN; $A_1 = 2400\ mm^2$; $A_2 = 600\ mm^2$ **(08 Marks)**

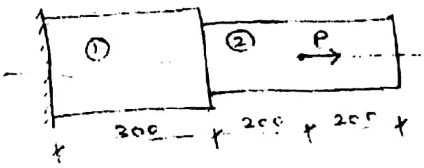


Fig. Q4 (a)

b. A plane truss shown in Fig. Q4 (b), determine nodal displacements, stresses in each element and reaction at supports.

$E = 200$ GPa ; $A_1 = 1200\ mm^2$; $A_2 = 1000\ mm^2$; $P = 50$ KN **(08 Marks)**

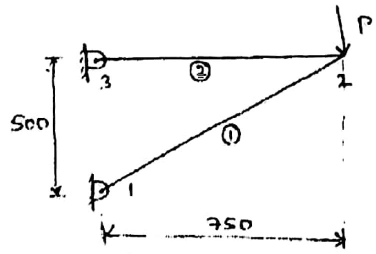


Fig. Q4 (b)

Module-3

5 a. Derive the Hermite function of a beam element. (08 Marks)

b. For the beam element shown in figure Q5 (b), determine the displacement and slope at the free end. Take E = 70 GPa, $I = 4 \times 10^{-4}$ m^4 (08 Marks)

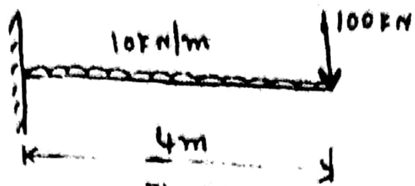
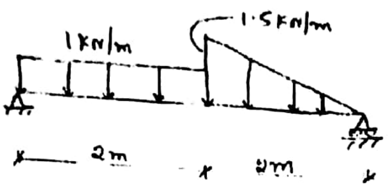


Fig. Q5 (b)

OR

6 a. Derive the stiffness matrix for a torsion element. (06 Marks)

b. Find the deflection and slopes at the nodes for the aluminium beam shown in Fig. Q6 (b). (10 Marks)



E = 70 GPa
$I = 2 \times 10^{-6}$ m^4

Fig. Q6 (b)

Module-4

7 a. With brief explanation obtain the rate equation that describes the rate of energy flow for the following conditions:

(i) Conduction (ii) Convection (iii) Radiation (06 Marks)

b. Derive the shape function of a 1 D bar element with temperature T_1 and T_2 at the nodes. (10 Marks)

OR

8 a. Determine the temperature distribution in the rectangular fin shown in Fig. Q8 (a). Neglect convection heat transfer and assume heat generated inside the fin as 500 W/m^3 (08 Marks)

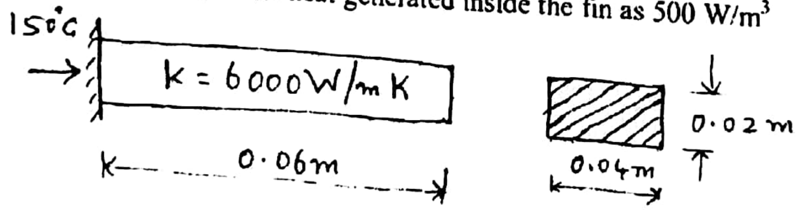


Fig. Q8 (a)

b. Derive the stiffness matrix for fluid flow in 1 D bar element. (08 Marks)

Module-5

9 Derive the shape function for axisymmetric triangular element. (16 Marks)

OR

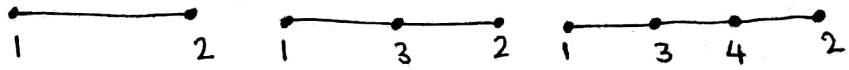
10 Derive the consistent mass matrix for the following:

(i) 1 D bar element.

(ii) 1 D truss element. (16 Marks)

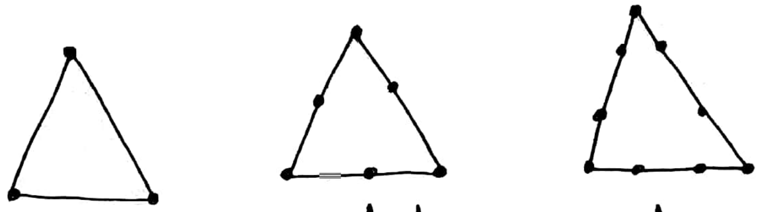
Q1 a Type of Elements

(i) 1-D elements



linear quadratic cubic

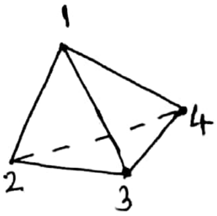
(ii) 2-D elements



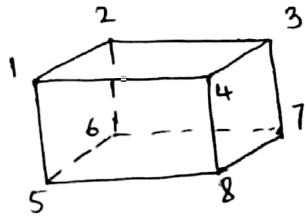
linear quadratic cubic

Quadrilateral Rectangle Parallelogram.

(iii) 3-D elements



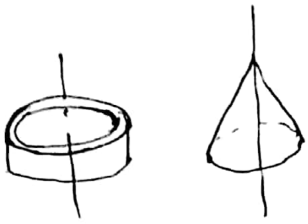
Tetrahedron



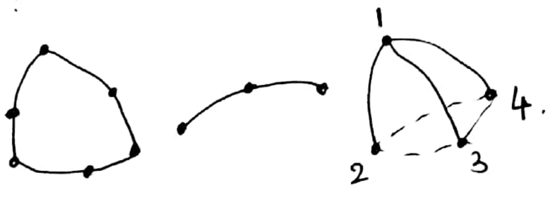
Rectangular prism or Brick element

Hexahedron

(iv) Axisymmetric elements



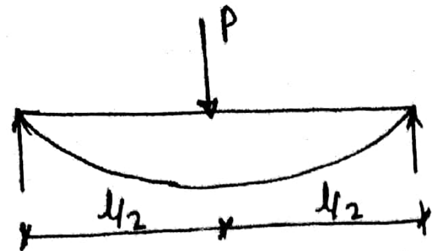
(v) Isoparametric elements or with curved boundaries



(vi) Simplex, complex & multiplex elements

(06 Marks)

Q1 b.



$x=0$ $x = l/2$ $x = l$

$y=0$ $y = y_{max}$ $y=0$

$\frac{dy}{dx} \neq 0$ $\frac{dy}{dx} \neq 0$

Solution:

① Formulate Potential Energy functional

$$\Pi = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx - P y_{max}$$

② Assume displacement function

$$y = a\sin\left(\frac{\pi x}{l}\right)$$

B/s At $x=0$, $y=0$ & $x=l$ $y=0$

$$\frac{dy}{dx} = \frac{\pi a}{l}\cos\left(\frac{\pi x}{l}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-\pi^2 a}{l^2}\sin\left(\frac{\pi x}{l}\right)$$

At $x = l/2$ $y = y_{max}$ $y_{max} = a\sin\left(\frac{\pi \cancel{l}}{\cancel{l}2}\right) \Longrightarrow a \Rightarrow y_{max} = a.$

③ Substituting displacement function into PE functional.

$$\Pi = \frac{EI}{2}\int_0^l \left(-\frac{a\pi^2}{l^2}\sin\left(\frac{\pi x}{l}\right)\right)^2 dx - Pa$$

$$= \frac{EI}{2}\int_0^l \left[\frac{a^2\pi^4}{l^4}\sin^2\left(\frac{\pi x}{l}\right)\right]_0^l dx - Pa$$

$$= \frac{EI\pi^4 a^2}{4l^4}(l-0) - Pa = \frac{EI\pi^4 a^2}{4l^3} - Pa$$

~~Or~~

④ Minimizing Potential Energy functional.

$$\frac{\partial \pi}{\partial a} = 0 \qquad \frac{EI\, 2a\pi^4}{4l^3} - P = 0 \Rightarrow a = \frac{2Pl^3}{\pi^4 EI}$$

⑤ displacement function

$$y = a \sin\left(\frac{\pi x}{l}\right) = \frac{2Pl^3}{\pi^4 EI} \sin\left(\frac{\pi x}{l}\right)$$

Max deflection at $x = l/2$ $\quad y = y_{max}$.

$$y_{max} = \frac{2Pl^3}{\pi^4 EI} = \frac{Pl^3}{48.7\, EI}$$

(10 Marks)

2 (a) Advantages and Disadvantages of FEM.

(1) Irregular shaped bodies can be analyzed by FEM

(2) Non homogenity ~~an~~ in material and loading can be handled easily by FEM

(3) Several different types of boundary conditions can be handled easily.

(1) The cost of developing program is high.

(2) Very much ~~case~~ engineering knowledge is required to get sensible results from FEM, because it gives junk in junk out.

(03 Marks)

(b) Plane stress stress strain equations.

$\sigma_z = \tau_{xz} = \tau_{yz} = 0$ Plane stress.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\mu)(1-\mu)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\sigma\} = [D]\{\epsilon\}.$$

Plane stress

$$[D] = \text{Elasticity matrix} = \frac{E}{(1+\mu)(1-\mu)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

Plane strain $\epsilon_z = \tau_{xz} = \tau_{yz} = 0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & 1 & 0 \\ 1 & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \tau_{xy} \end{Bmatrix}$$

$$\{\sigma\} = [D]\{\epsilon\}$$

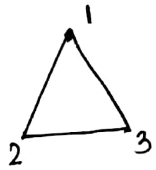
$$\therefore [D] = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & 1 & 0 \\ 1 & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix}$$

(04 Marks)

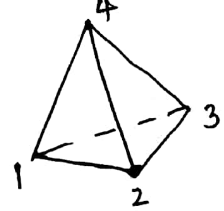
Q2(c) Simplex, Complex and Multiplex elements

Simplex elements: These elements are those elements for which the approximating polynomial consists of constant and linear terms

In 1-D $\phi(x) = a_1 + a_2 x$



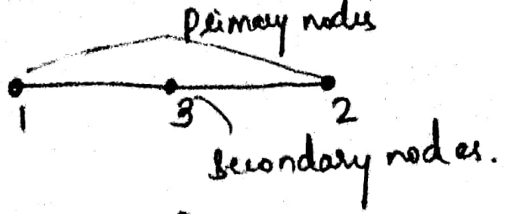
In 2-D $\phi(x) = a_1 + a_2 x + a_3 y$



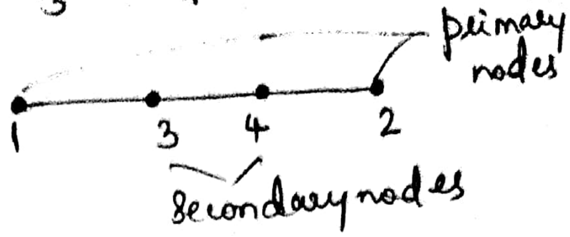
In 3-D $\phi(x) = a_1 + a_2 x + a_3 y + a_4 z$

Complex elements: These elements are those elements for which approximating polynomial consists of quadratic, cubic or higher order terms

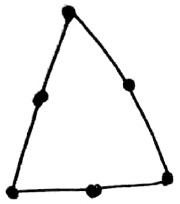
1-D $\phi(x) = a_1 + a_2 x + a_3 x^2$
quadratic



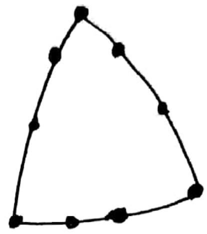
1-D cubic $\phi(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$



Similarly
In 2-D

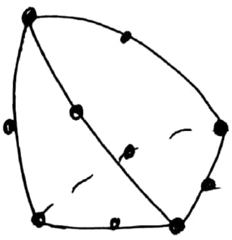


Quadratic

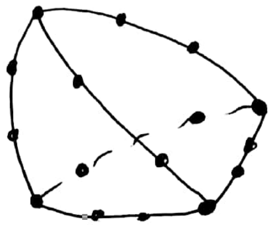


Cubic

In 3-D

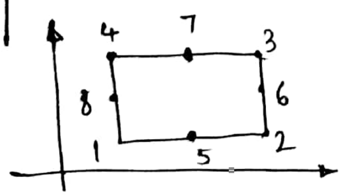


Quadratic



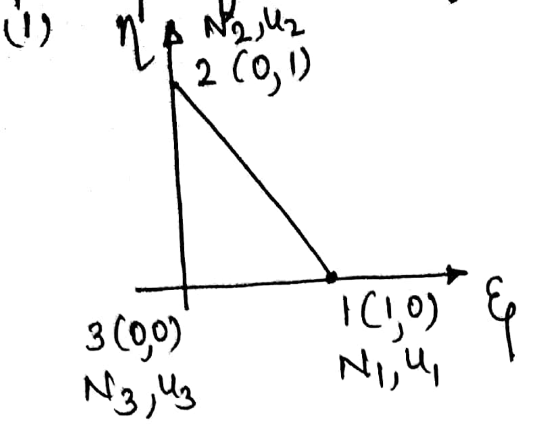
cubic

Multiplex elements: are those whose boundaries are parallel to the co-ordinate axes to achieve interelement connectivity and whose approximating polynomials contain higher order terms



Q3(a) Shape function for constant strain triangle.

(i)



Let $u = a_1 + a_2\xi + a_3\eta$

At node 1 $\quad u = u_1 = a_1 + a_2$

At node 2 $\quad u = u_2 = a_1 + a_3$

At node 3 $\quad u = u_3 = a_1$

$\therefore \quad a_1 = u_3$

$a_3 = u_2 - a_1 = u_2 - u_3$

$a_2 = u_1 - a_1 = u_1 - u_3$

Substituting these values in the above equation

$u = a_1 + a_2\xi + a_3\eta$

$= u_3 + (u_1 - u_3)\xi + (u_2 - u_3)\eta$

$= \xi u_1 + \eta u_2 + (1 - \xi - \eta) u_3$

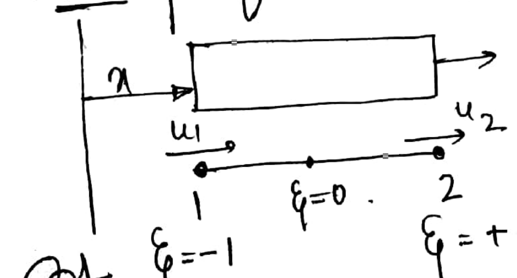
$u = N_1 u_1 + N_2 u_2 + N_3 u_3$

Comparing these two equations $N_1 = \xi$, $N_2 = \eta$, $N_3 = 1 - \xi - \eta$.

(ii) Shape function for 1-D bar element

$\xi = \frac{2x}{l} - 1$

$d\xi = \frac{2}{l} dx \qquad dx = \frac{l}{2} d\xi.$



Q3 b (i) $I = \int_{-1}^{+1} (1+\xi+2\xi^2+3\xi^3)\,d\xi$.

<u>Two point formula</u>. $w_1 = w_2 = 1$ $\quad \xi_1 = \frac{-1}{\sqrt{3}}$ $\quad \xi_2 = \frac{1}{\sqrt{3}}$

$$I = W_1(3\xi_1^3 + 2\xi_1^2 + \xi_1 + 1) + W_2(3\xi_2^3 + 2\xi_2^2 + \xi_2 + 1)$$

$$I = 1\left[3\left(-\frac{1}{\sqrt{3}}\right)^3 + 2\left(\frac{-1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right) + 1\right] + 1\left[3\left(\frac{1}{\sqrt{3}}\right)^3 + 2\left(\frac{+1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right) + 1\right]$$

$$= 0.51 + 2.82 = 3.33$$

<u>Exact solution</u>

$$I = \int_{-1}^{+1} 1\,d\xi + \int_{-1}^{+1} \xi\,d\xi + 2\int_{-1}^{+1} \xi^2\,d\xi + 3\int_{-1}^{+1} \xi^3\,d\xi$$

$$[\xi]_{-1}^{+1} + \left[\frac{\xi^2}{2}\right]_{-1}^{+1} + 2\left[\frac{\xi^3}{3}\right]_{-1}^{1} + 3\left[\frac{\xi^4}{4}\right]_{-1}^{+1}$$

$$1+1=2 \quad \frac{1}{2} - \frac{1}{2} = 0 \quad + 2\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{4}{3} \quad + 3\left(\frac{1}{4}-\frac{1}{4}\right) = 0$$

$$2 + 0 + \frac{4}{3} + 0 = \frac{10}{3} = 3.33$$

(ii) $\int_{-2}^{+2} (4-y)^2\,dy = \int_{-2}^{2} (16 - 8y + y^2)\,dy = \int_{-2}^{2} (y^2 - 8y + 16)\,dy$

limits from -2 to $+2$ to -1 to $+1$

$$y = \left(\frac{b-a}{2}\right)\xi + \left(\frac{b+a}{2}\right)$$

$$= \left(\frac{2+2}{2}\right)\xi + \left(\frac{2-2}{2}\right)$$

$$y = 2\xi \qquad dy = 2\,d\xi$$

$$\int_{-1}^{+1} (4\xi^2 - 16\xi + 16)\,2\,d\xi = \int_{-1}^{+1} (8\xi^2 - 32\xi + 32)\,d\xi.$$

$$I = W_1\left(8\xi_1^2 - 32\xi_1 + 32\right) + W_2\left(8\xi_2^2 - 32\xi_2 + 32\right)$$

$$= 1\left[8\left(\frac{-1}{\sqrt{3}}\right)^2 - 32\left(\frac{-1}{\sqrt{3}}\right) + 32\right] + 1\left[8\left(\frac{+1}{\sqrt{3}}\right)^2 - 32\left(\frac{1}{\sqrt{3}}\right) + 32\right]$$

$$= 53.13 + 16.20 = 69.33 \,//.$$

Exact solution

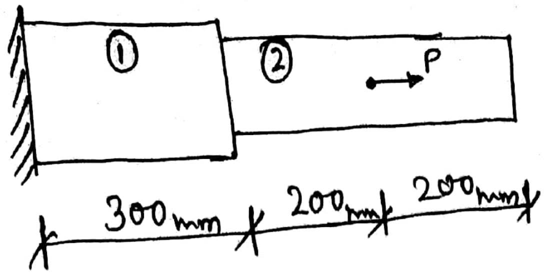
$$\int_{-1}^{+1}\left(8\xi^2 - 32\xi + 32\right)d\xi$$

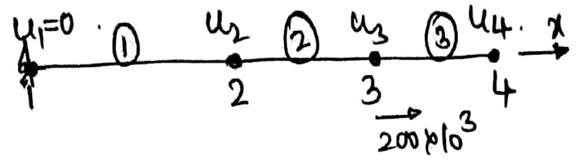
$$8\int_{-1}^{+1}\xi^2 d\xi - 32\underbrace{\int_{-1}^{+1}\xi\, d\xi}_{0} + 32\int_{-1}^{+1} d\xi$$

$$8\,\frac{4}{3} \quad - \quad 0 \quad + \quad 64$$

$$= \frac{32}{3} + 64 = 74.66 \,//.$$

Q4(a)





Given

$E_1 = 70\ GPa = 70\times10^3\ N/mm^2$

$E_2 = 200\ GPa = 200\times10^3\ N/mm^2$

$P = 200\ kN = 200\times10^3$

$A_1 = 2400\ mm^2$

$A_2 = 600\ mm^2$

Solution For 1

Step 1: $K_1 = \dfrac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \dfrac{2400\times70\times10^3}{300}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = 10^5 \begin{array}{cc} 1 & 2 \\ \end{array}\begin{bmatrix}5.6 & -5.6\\ -5.6 & 5.6\end{bmatrix}$

For 2

$K_2 = K_3 = \dfrac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \dfrac{600\times200\times10^3}{200}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = 10^5 \begin{array}{cc} 3 & 4 \\ 2 & 3 \end{array}\begin{bmatrix}6 & -6\\ -6 & 6\end{bmatrix}$

$K_1 + K_2 + K_3 = K \qquad [K]\{q\} = \{F\}$

$$10^5 \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$
$$10^5\begin{bmatrix} \cancel{5.6} & \cancel{-5.6} & \cancel{0} & 0 \\ -5.6 & 5.6+6 & -6 & 0 \\ 0 & -6 & 6+6 & -6 \\ 0 & 0 & -6 & 6 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 200\times10^3 \\ 0 \end{Bmatrix}$$

Applying B.C $u_1 = 0$

Reduced matrix

$$10^5\begin{bmatrix} 11.6 & -6 & 0 \\ -6 & 12 & -6 \\ 0 & -6 & 6 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix} 10^5$$

Solving above by Gauss elimination

~~Bt~~

$R_{II} - \left(\frac{-6}{11.6}\right)$ I Row

$-6 - \left(\frac{-6}{11.6}\right) \times 11.6 = 0.$

$12 - (-0.517) \times -6 = 8.8$

$2 - (-0.57) \times 0 = 2.$

$R_{III} - \left(\frac{-6}{8.8}\right) \times$ II Row.

$= 0.$

$6 - \left(\frac{-6}{8.8}\right) \times -6 = 1.95$

$0 - \left(\frac{-6}{8.8}\right) \times 2 = 1.36.$

$$\begin{bmatrix} 11.6 & -6 & 0 \\ 0 & 8.8 & -6 \\ 0 & 0 & 1.95 \end{bmatrix} \begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 11.6 & -6 & 0 \\ 0 & 8.8 & -6 \\ 0 & 0 & 1.95 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \\ 1.36 \end{Bmatrix}$$

$\therefore$ nodal displacements $\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0.36 \\ 0.7 \\ 0.70 \end{Bmatrix}$ mm.

Element stresses.

$\sigma_1 = E\epsilon_1 = \frac{E\,(u_2 - u_1)}{l} = \frac{70 \times 10^3}{300}(0.7 - 0) = 163.33$ MPa.

$\sigma_2 = E\epsilon_2 = \frac{200 \times 10^3}{200}(0.7 - 0.36) = 340$ MPa.

$\sigma_3 = E\epsilon_3 = \frac{200 \times 10^3}{200}(0.7 - 0.7) = 0$ MPa.

Reactions.

$R_1 = 10^5 \left[-5.6 \times 0.36\right] = -2.016 \times 10^5 = -201$ KN //.

(08 Marks)

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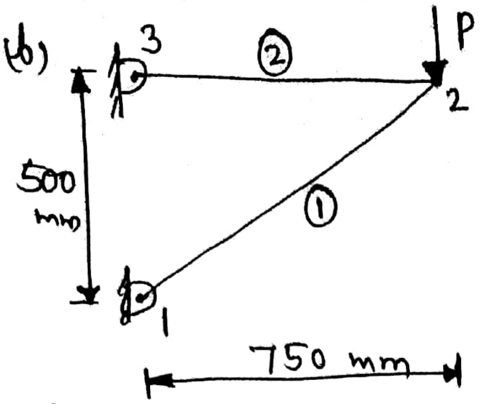
Q4(b)



Given!

$E = 200\,GPa = 200 \times 10^3\ N/mm^2$

$A_1 = 1200\ mm^2$

$A_2 = 1000\ mm^2$

$P = 50kN = 50 \times 10^3\ N$

Solution.

Sl No	Element	Nodes	le	l	m	l^2	m^2	lm
1	1	1-2	901.38 mm	0.83	0.55	0.68	0.3	0.45
2	2	2-3	750 mm	-1	0	1	0	0

$l_1 = \frac{750-0}{901.4} = 0.83,\ m_1 = \frac{500-0}{901.38} = 0.55,\ l_2 = \frac{0-750}{750} = -1,\ m_2 = \frac{500-500}{750} = 0$

$$K_1 = \frac{AE}{l}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} = \frac{1200 \times 2 \times 10^5}{901.34}\begin{bmatrix} 0.68 & 0.45 & -0.68 & -0.45 \\ 0.45 & 0.30 & -0.45 & -0.3 \\ -0.68 & -0.45 & 0.68 & 0.45 \\ -0.45 & -0.3 & 0.45 & 0.3 \end{bmatrix}$$

$$= 10^5\begin{bmatrix} 1.8 & 1.19 & -1.8 & -1.19 \\ 1.19 & 0.8 & -1.19 & -0.8 \\ -1.8 & -1.19 & 1.8 & 1.19 \\ -1.19 & -0.8 & 1.19 & 0.80 \end{bmatrix}$$

$$K_2 = \frac{1000 \times 2 \times 10^5}{750}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 10^5\begin{bmatrix} 2.67 & 0 & -2.67 & 0 \\ 0 & 0 & 0 & 0 \\ -2.67 & 0 & 2.67 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$10^5 \begin{bmatrix} 1.8 & 1.19 & -1.8 & -1.19 & 0 & 0 \\ 1.19 & 0.80 & -1.19 & -0.8 & 0 & 0 \\ -1.8 & -1.19 & 4.47 & 1.19 & -2.67 & 0 \\ -1.19 & -0.8 & 1.19 & 0.8 & 0 & 0 \\ 0 & 0 & -2.67 & 0 & 2.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -50\times10^3 \\ 0 \\ 0 \end{Bmatrix}$$

Apply Boundary conditions $q_1 = q_2 = q_5 = q_6 = 0$

$$10^5 (4.47\, q_3 + 1.19\, q_4) = 0$$

$$10^5 (1.19\, q_3 + 0.80\, q_4) = -50\times10^3$$

Results in $q_3 = 0.28$ mm, $q_4 = -1.04$ mm

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.28 \text{ mm} \\ -1.04 \text{ mm} \\ 0 \\ 0 \end{Bmatrix}$$

Stresses.

$$\sigma_1 = \frac{2\times10^5}{901.4} [-0.83 \; -0.55 \;\; 0.83 \;\; 0.55] \begin{Bmatrix} q_1 = 0 \\ q_2 = 0 \\ q_3 = 0.28 \\ q_4 = -1.04 \end{Bmatrix} = -75.35 \text{ MPa}$$

$$\sigma_2 = \frac{2\times10^5}{750} [1 \;\; 0 \; -1 \;\; 0] \begin{Bmatrix} q_3 = 0.28 \\ q_4 = -1.04 \\ q_5 = 0 \\ q_6 = 0 \end{Bmatrix} = 74.67 \text{ MPa}$$

Reactions.

$R_1 = 10^5[-1.8\times0.28 + (-1.19)\times -1.04] = 73.36$ kN.

$R_2 = 10^5[-1.19\times0.28 + -0.8\times-1.04] = 49.88 \approx 50$ kN.

$R_5 = 10^5[-2.67\times0.28] = -74.76$ kN.

$R_6 = 0$

(08 Marks)

$$u(\xi) = a_1 + a_2\xi$$

At $\xi=-1$ $\quad u_1 = a_1 - a_2$

At $\xi=+1$ $\quad u_2 = a_1 + a_2$

$$\therefore\ a_1 = \frac{u_1+u_2}{2} \quad \text{and} \quad a_2 = \frac{u_2-u_1}{2}$$

$$\therefore\ u = \frac{u_1+u_2}{2} + \left(\frac{u_2-u_1}{2}\right)\xi$$

$$= \left(\frac{1-\xi}{2}\right)u_1 + \left(\frac{1+\xi}{2}\right)u_2$$

$$= N_1 u_1 + N_2 u_2$$

$$\therefore\ N_1 = \frac{1-\xi}{2} \quad \& \quad N_2 = \frac{1+\xi}{2}.$$

Q5(a) Hermite shape functions of a beam element

The hermite shape functions are governed by equation

$$H_i = a_1 + a_2\xi + a_3\xi^2 + a_4\xi^3 \qquad i = 1, 2, 3, 4$$

The following conditions must be satisfied.

	H_1	H_1'	H_2	H_2'	H_3	H_3'	H_4	H_4'
$\xi=-1$	1	0	0	1	0	0	0	0
$\xi=+1$	0	0	0	0	1	0	0	1

The coefficients a_1, a_2, a_3, a_4 are obtained by imposing the above conditions.

For example $H_1 = a_1 + a_2\xi + a_3\xi^2 + a_4\xi^3$

At node 1. $H_1 = 1$ $\xi = -1 \Rightarrow 1 = a_1 - a_2 + a_3 - a_4$ —(1)

At node 1 $\frac{dH_1}{d\xi} = 0$ $\xi = -1 \Rightarrow 0 = a_2 + 2a_3\xi + 3a_4\xi^2 = \frac{dH_1}{d\xi}$ —(2)

$= a_2 - 2a_3 + 3a_4$ —(2)

At node 2 $H_1 = 0$ $\xi = +1 \Rightarrow 0 = a_1 + a_2 + a_3 + a_4$

$\frac{dH_1}{d\xi} = 0$ $\xi = +1 \Rightarrow 0 = a_2 + 2a_3 + 3a_4.$

Add (1) & (3) $1 = 2a_1 + 2a_3 \Rightarrow a_1 + a_3 = 1/2$ —(5)

Add (2) & (4) $0 = 2a_2 + 6a_4 \Rightarrow 2a_2 = -6a_4 \rightarrow a_2 = -3a_4$ —(6)

From (3) $a_1 + a_2 + a_3 + a_4 = 0$

$a_1 - 3a_4 + a_3 + a_4 = 0$

$a_1 + a_3 - 2a_4 = 0$ —(7)

From (5) $a_1 + a_3 = 1/2$ $\therefore$ $1/2 - 2a_4 = 0 \Rightarrow a_4 = 1/4$ —(8)

Substitute $a_4 = \frac{1}{4}$ in $a_2 = -3a_4 = -3/4$ —(9)

From (4) $a_2 + 2a_3 + 3a_4 = 0$

$-3/4 + 2a_3 + 3 \times \frac{1}{4} = 0.$

$2a_3 = 0 \Longrightarrow a_3 = 0.$

From (3) $a_1 + a_2 + a_3 + a_4 = 0$

$$a_1 - \frac{3}{4} + 0 + \frac{1}{4} = 0 \Rightarrow a_1 = 1/2 \quad \text{—(11)}$$

Substitute the values $a_1 = 1/2$, $a_2 = -3/4$, $a_3 = 0$ & $a_4 = 1/4$ in H_1

$$H_1 = \frac{1}{2} - \frac{3\xi}{4} + 0 + \frac{1}{4}\xi^2 = \frac{1}{2} - \frac{3\xi}{4} + \frac{\xi^3}{4} = 1/4\,(2 - 3\xi + \xi^3)$$

$$= \frac{1}{4}(1-\xi)^2 (2+\xi)$$

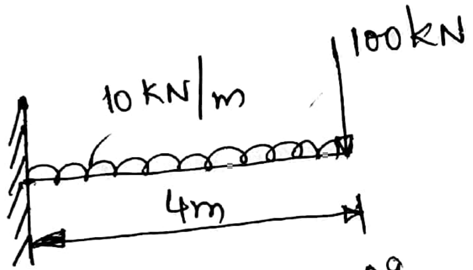
Similarly we can find H_2, H_3 & H_4 Hermite shape functions

$$H_2 = \frac{1}{4}(1 - \xi - \xi^2 + \xi^3) \quad \text{OR} \quad \frac{1}{4}(1-\xi)^2(\xi+1)$$

$$H_3 = \frac{1}{4}(2 + 3\xi - \xi^3) \quad \text{OR} \quad 1/4\,(1+\xi)^2(2-\xi)$$

$$H_4 = \frac{1}{4}(-1 - \xi + \xi^2 + \xi^3) \quad \text{OR} \quad \frac{1}{4}(1+\xi)^2(\xi - 1)$$

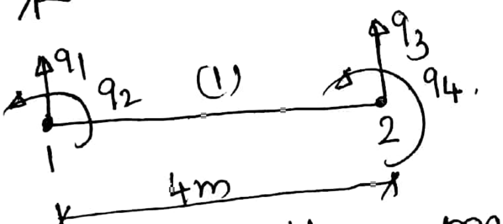
Q5 (b)



Given

$E = 70\,GPa = 70 \times 10^9\ N/m^2$

$I = 4 \times 10^{-4}\ m^4$



The element stiffness matrix for the element is given by.

$$K = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} = \frac{70\times10^9 \times 4\times10^{-4}}{4^3}\begin{bmatrix} 12 & 24 & -12 & 24 \\ 24 & 64 & -24 & 32 \\ -12 & -24 & 12 & -24 \\ 24 & 32 & -24 & 64 \end{bmatrix}$$

$$= 4.375\times10^5 \begin{bmatrix} 12 & 24 & -12 & 24 \\ 24 & 64 & -24 & 32 \\ -12 & -24 & 12 & -24 \\ 24 & 32 & -24 & 64 \end{bmatrix}$$

Load Vector

$$\{F\} = \begin{Bmatrix} -Pl/2 \\ -\frac{Pl^2}{12} \\ -Pl/2 \\ +Pl^2/12 \end{Bmatrix} = \begin{Bmatrix} -\frac{10\times4}{2}\times10^3 \\ -\frac{10\times4^2}{12}\times10^3 \\ -\frac{10\times4}{2}\times10^3 \\ +\frac{10\times4^2}{12}\times10^3 \end{Bmatrix} = \begin{Bmatrix} -20\times10^3 + 0 \\ -13.33\times10^3 + 0 \\ -20\times10^3 - 100\times10^3 \\ 13.33\times10^3 + 0 \end{Bmatrix}$$

$[K]\{q\} = \{F\}$

$$4.375\times10^5 \begin{bmatrix} 12 & 24 & 12 & 24 \\ 24 & 64 & 24 & 32 \\ -12 & -24 & 12 & -24 \\ 24 & 32 & -24 & 64 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} -20\times10^3 \\ -13.33\times10^3 \\ -120\times10^3 \\ 13.33\times10^3 \end{Bmatrix}$$

Apply Boundary conditions $q_1 = q_2 = 0$. We get.

$$4.375\times10^5 \begin{bmatrix} 12 & -24 \\ -24 & 64 \end{bmatrix} \begin{Bmatrix} q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} -120\times10^3 \\ 13.33\times10^3 \end{Bmatrix}.$$

By Solving We get.

$q_3 = -87.625\times10^{-3}$ m — displacement

$q_4 = -32.38\times10^{-3}$ rad — slope.

$[52.5 q_3 - 105 q_4 = -1.2 \times 10^5 \Rightarrow 5512.5 q_3 - 11025 q_4 = -126$

$-105 q_3 + 280 q_4 = +0.133 \times 52.5 \quad - \; 5512.5 q_3 + 14700 q_4 = 6.98$

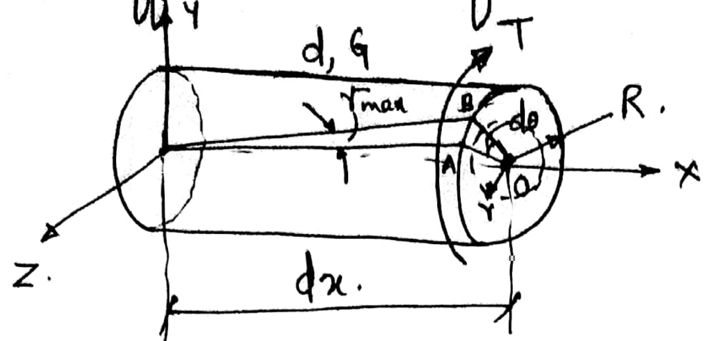
$3675 q_4 = -119.02$

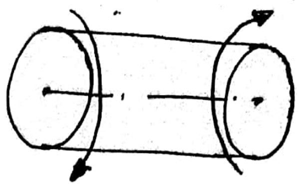
$\therefore q_4 = -0.03238$ rad. $= -32.38\times10^{-3}$ rad.

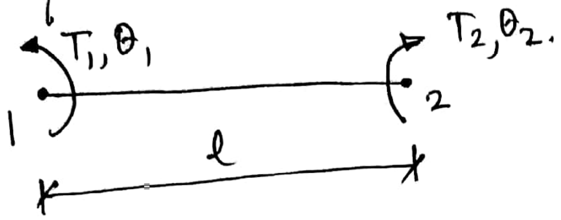
$52.5 q_3 - 105 \times -0.03238 = -1.2 \Rightarrow q_3 = 0.0876$ m $]$.

(08 Marks)

Q 6(a) Stiffness matrix for torsion element







Let T_1 & T_2 are the end torques at node 1 & 2 resply
θ_1 & θ_2 are the end twists at node 1 & 2 resply
Let us assume linear angle of twist variation along the x-axis of the bar such that

$$\theta = a_1 + a_2 x$$

$$\theta = [N_1 \ N_2] \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$$

where $N_1 = \dfrac{x_2 - x}{x_2 - x_1}$ and $N_2 = \dfrac{x - x_1}{x_2 - x_1}$

The relationship between shear strain γ and angle of twist θ can be obtained by considering the torsional deformation of the bar segment Assume all radial lines remain straight during twisting (torsional deformation)

$$\text{Arc } AB = \gamma_{max}\, dx = R\, d\theta$$

$$\therefore \gamma_{max} = R \frac{d\theta}{dx}$$

Similarly at any radial position 'r' from similar triangles OAB & OCD we have.

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$$\gamma = r\frac{d\theta}{dx} = \frac{r}{L}(\theta_2-\theta_1)$$

Also for linear elastic isotropic materials the shear stress (τ) and shear strain (γ) is given by

$$\frac{\tau}{\gamma} = G$$

Also from torsion equation

$$\frac{T}{J} = \frac{\tau}{R} \implies T = \frac{\tau J}{R} = \frac{GJ\gamma}{R}$$

where $J =$ Polar M I

$$= \frac{GJ}{\cancel{R}}\frac{\cancel{R}}{L}(\theta_2-\theta_1)$$

$$T = \frac{GJ}{l}(\theta_2-\theta_1)$$

At node 1 $\quad T = -T_1 \qquad T_1 = \frac{GJ}{l}(\theta_1-\theta_2)$

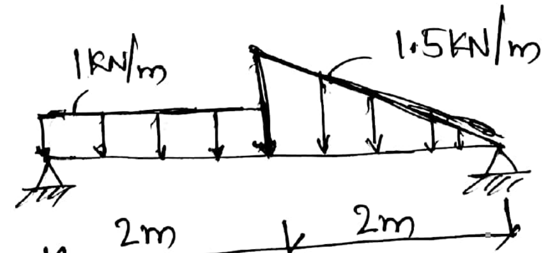
||ly at node 2 $\quad T = T_2 \qquad T_2 = \frac{GJ}{l}(\theta_2-\theta_1)$

Writing above equations in matrix form.

$$\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \frac{GJ}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}.$$

$$\{T\} = [K]\{\theta\}. \quad \therefore [K] = \frac{GJ}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} //.$$

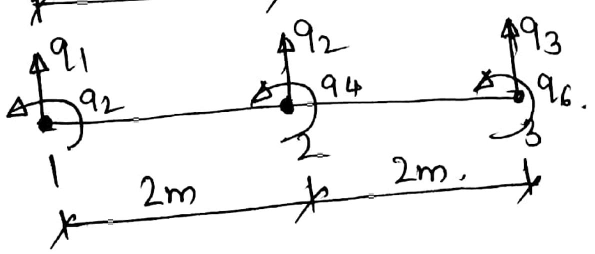
Q6(b)



Given.

$E = 70\,GPa = 70 \times 10^9\, N/m^2$

$I = 2 \times 10^{-6}\, m^4.$



Element stiffness matrix

$$K_1 = K_2 = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} = \frac{70\times10^9 \times 2\times10^{-6}}{2^3}\begin{bmatrix} 12 & 12 & -12 & 12 \\ 12 & 16 & -12 & 8 \\ -12 & -12 & 12 & -12 \\ 12 & 8 & -12 & 16 \end{bmatrix}$$

$$(17.5\times10^3)$$

Load vector on element ①.

$$\{F_1\} = \begin{Bmatrix} -Pl/2 \\ -Pl^2/12 \\ -Pl/2 \\ Pl^2/12 \end{Bmatrix} = \begin{Bmatrix} -\frac{1\times10^3\times2}{2} \\ -\frac{1\times10^3\times2^2}{12} \\ -\frac{1\times10^3\times2}{2} \\ \frac{1\times10^3\times2^2}{12} \end{Bmatrix} = \begin{Bmatrix} -1 \\ -0.33 \\ -1 \\ 0.33 \end{Bmatrix} 10^3$$

Load vector on element ②

$$\{F_2\} = \frac{Pl}{60}\begin{Bmatrix} -21 \\ -3l \\ -9 \\ 2l \end{Bmatrix} = \frac{1.5\times10^3\times2}{60}\begin{Bmatrix} -21 \\ -6 \\ -9 \\ +4 \end{Bmatrix} = \begin{Bmatrix} -1050 \\ -300 \\ -450 \\ +200 \end{Bmatrix}$$

Global force vector. $\{F\} = \begin{Bmatrix} -1000 \\ -333.33 \\ -2050 \\ 33.33 \\ -450 \\ 200 \end{Bmatrix}$

Global stiffness matrix is given by,

$$[K] = 17.5\times10^3 \begin{bmatrix} 12 & 12 & -12 & 12 & 0 & 0 \\ 12 & 16 & -12 & 8 & 0 & 0 \\ -12 & -12 & 24 & 0 & -12 & 12 \\ 12 & 8 & 0 & 32 & -12 & 8 \\ 0 & 0 & -12 & -12 & 12 & -12 \\ 0 & 0 & 12 & 8 & -12 & 16 \end{bmatrix}$$

The Equilibrium Equation is

$$17.5\times10^3 \begin{bmatrix} 12 & 12 & -12 & 12 & 0 & 0 \\ 12 & 16 & -12 & 8 & 0 & 0 \\ -12 & -12 & 24 & 0 & -12 & 12 \\ 12 & 8 & 0 & 32 & -12 & 8 \\ 0 & 0 & -12 & -12 & 12 & -12 \\ 0 & 0 & 12 & 8 & -12 & 16 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{Bmatrix} -1000 \\ -333.33 \\ -2050 \\ 3333 \\ -450 \\ 200 \end{Bmatrix}$$

Apply BCs $q_1 = q_5 = 0$ We get.
$q_3 = -9.226\times10^{-3}$ m $q_4 = -2.113\times10^{-3}$ rad, $q_6 = 8.6904\times10^{-3}$ rad.

(08 Marks)

Q.7 (a) i, Conduction.

The rate of heat flow in x direction by conduction (q) given by Fourier's law of heat conduction. rate of heat transfer is directly proportional to gradient of heat transfer and area perpendicular to the heat flow direction

$$\therefore q_x = -KA\frac{dT}{dx}.$$

(ii) Convection: Convection involves conduction (by diffusion) and bulk motion of fluid (i.e advection)

The convection is governed by Newton's law of cooling where rate of heat transfer is directly proportional to convective surface area & difference between media and convective surface $(h_s - h_\infty)$

$$q = hA_s(T_s - T_\infty)$$

(iii) Radiation: Radiation heat transfer is the process by which thermal energy is exchanged between surfaces obeying the laws of electromagnetics

The rate of heat-transfer is governed by Stefan Boltzman's law of radiation and equation is

$$q = \sigma \epsilon A(T^4 - T_\infty^4)$$

(06 Marks)

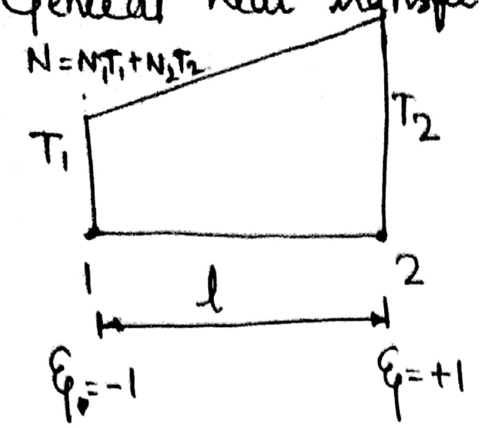
ϵ – Emissivity of surface.

σ – Stefan Boltzman constant $(\sigma = 5.67 \times 10^{-8}\, W/m^2K^4)$

T – Absolute temp of the body (°C or K)

T_∞ – Absolute surrounding temp

Q7(b) General heat transfer element with endnodes 1 & 2



The linear temperature variation in the element is represented by linear interpolation function

$$T(\xi) = a_1 + a_2 \xi \quad \text{--- (1)}$$

At $\xi = -1$ $\quad T_1 = a_1 - a_2$ --- (2)

At $\xi = +1$ $\quad T_2 = a_1 + a_2$ --- (3)

From (2) & (3) $\quad a_1 = \frac{T_1 + T_2}{2}$ & $a_2 = \frac{T_2 - T_1}{2}$

Substituting a_1 & a_2 values in eqn. (1)

$$T = a_1 + a_2 \xi$$

$$= \frac{T_1 + T_2}{2} + \left(\frac{T_2 - T_1}{2}\right)\xi$$

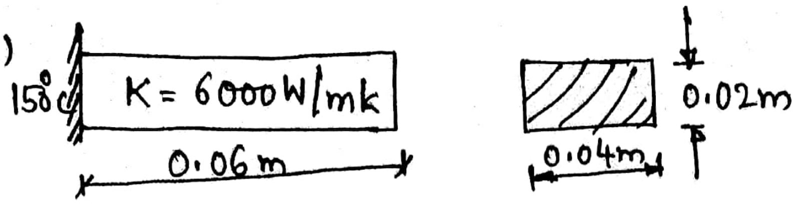
$$= \left(\frac{1-\xi}{2}\right) T_1 + \left(\frac{1+\xi}{2}\right) T_2$$

$$= N_1 T_1 + N_2 T_2$$

$$\therefore \boxed{N_1 = \frac{1-\xi}{2}} \quad \& \quad \boxed{N_2 = \frac{1+\xi}{2}}$$

(08 Marks)

Q8 (a)



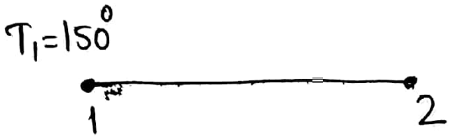
Given:

$Q = 500\ W/m^3$

$K = 6000\ W/m°K$

$T_1 = 150+273 = 423\ K$

No Convection $[H_T] = 0$



Conductivity matrix

$$[K] = \frac{KA}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{6000\times 8\times 10^{-4}}{0.06}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}80 & -80\\ -80 & 80\end{bmatrix}$$

Element heat rate vector

$$\{R\} = \frac{QlA}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = \frac{500\times 0.06\times 8\times 10^{-4}}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = \begin{Bmatrix}12\times 10^{-3}\\ 12\times 10^{-3}\end{Bmatrix}$$

The governing equation

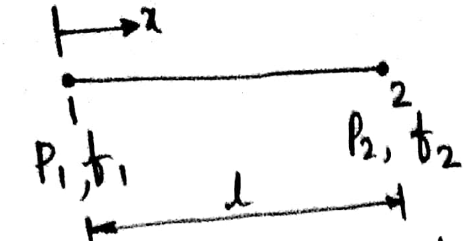
$$[K]\{T\} = \{R\}$$

$$\begin{bmatrix}80 & -80\\ -80 & 80\end{bmatrix}\begin{Bmatrix}T_1\\T_2\end{Bmatrix} = \begin{Bmatrix}12\times 10^{-3}\\ 12\times 10^{-3}\end{Bmatrix}$$

Applying BC $T_1 = 423K$.

$T_2 = 422.98°K = 423K$.

Q8(b) Consider 1-D element with two end nodes



P_1 & P_2 are fluidheads or potentials at node 1 & 2 resply

The potential shape function is given by

$$\phi = N_1 P_1 + N_2 P_2$$

shape functions $N_1 = 1 - \frac{x}{l}$ $\quad N_2 = \frac{x}{l}$

$$[N] = \left[1 - \frac{x}{l}, \frac{x}{l}\right]$$

The hydraulic gradient matrix ~~is~~ $\{g\}$ is given by

$$\{g\} = \left\{\frac{\partial \phi}{\partial x}\right\} = [B]\{P\}$$

where $[B] = \left[\frac{dN_1}{dx} \quad \frac{dN_2}{dx}\right] = \left[-\frac{1}{l} \quad \frac{1}{l}\right]$ & $\{P\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$

The velocity gradient relationship based on Darcy's law is given by $V_x = -[D]\{g\}$

where material property matrix is given by $[D] = [K_x]$

For ideal flow through pipes/over a solid body $K_x = 1$

$\therefore Q = VA \Rightarrow -[D]\{g\}A = -[K_x]\{g\}A$

$\therefore Q = -K_x A g$ $\quad$ Q - Volumetric flowrate in m^3/s

Hydraulic gradient matrix $\{g\} = [B]\{P\} = \left[-\frac{1}{l} \ \frac{1}{l}\right]\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$

$$g = \frac{P_2 - P_1}{l}$$

Substituting in the above equation we get

$$Q_1 = -K_x A\left(\frac{P_2 - P_1}{l}\right), \quad Q_2 = K_x A\left(\frac{P_2 - P_1}{l}\right)$$

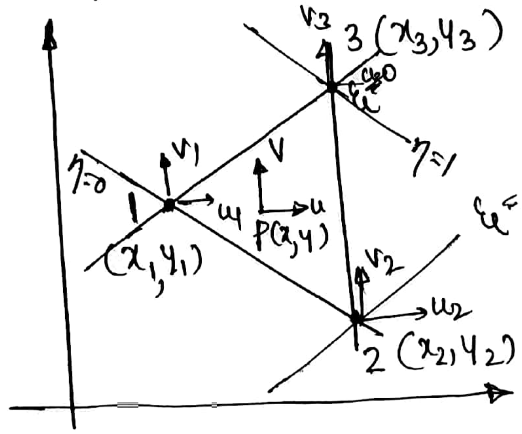
In matrix form

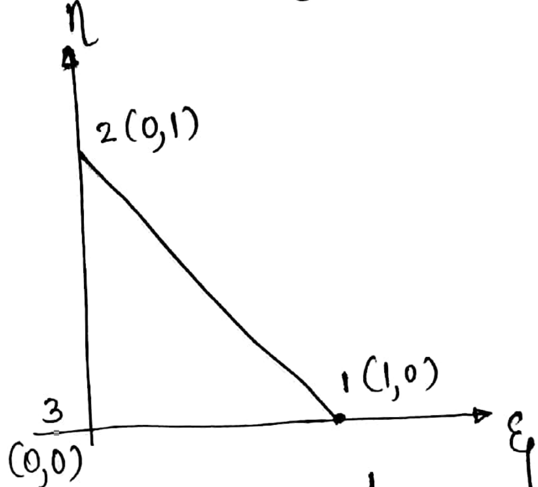
$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \frac{AK_x}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

Therefore the stiffness matrix is given by

$$K = \frac{AK_x}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Q9. Shape function for axisymmetric triangular element





The displacement model as linear polynomials are given by

$$u(\xi, \eta) = a_1 + a_2\xi + a_3\eta$$
$$v(\xi, \eta) = a_4 + a_5\xi + a_6\eta$$

Let $u = a_1 + a_2\xi + a_3\eta$ ——— (1)

At node 1 $\quad u = u_1 = a_1 + a_2 \qquad \because \xi = 1 \ \eta = 0$

At node 2 $\quad u = u_2 = a_1 + a_3 \qquad \because \xi = 0, \eta = 1$

At node 3 $\quad u = u_3 = a_1$

$\therefore \ a_1 = u_3$

$a_3 = u_2 - a_1 = u_2 - u_3$

$a_2 = u_1 - a_1 = u_1 - u_3$

Substituting these values in the above equation (1)

$$u = u_3 + (u_1 - u_3)\xi + (u_2 - u_3)\eta$$

$$= u_3 + u_1\xi - u_3\xi + u_2\eta - u_3\eta$$

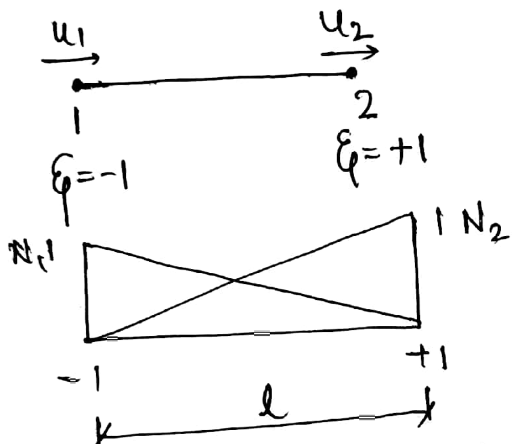
$$u = \xi u_1 + \eta u_2 + (1 - \xi - \eta) u_3$$

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3$$

Comparing these two equations we can write

$N_1 = \xi$, $N_2 = \eta$ $N_3 = 1 - \xi - \eta$ (16 Marks)

Q10(a) Consistent mass matrix of 1-D bar element



Shape function matrix $[N] = [N_1 N_2] = \left[\frac{1-\xi}{2}, \frac{1+\xi}{2}\right]$

$\xi = \frac{2x}{l} - 1$ $d\xi = \frac{2}{l}dx$ $dx = \frac{l}{2}d\xi$

Consistent element mass matrix

$$m^e = \int_e \rho N^T N \, dV$$

$$= \rho \int_e N^T N A \, dx \qquad \because dV = A\,dx$$

$$= \rho \int_{-1}^{+1} N^T N A \frac{l}{2} d\xi$$

$$\frac{\rho A l}{2}\int_{-1}^{+1}\begin{pmatrix}\frac{1-\xi}{2}\\ \frac{1+\xi}{2}\end{pmatrix}\left(\frac{1-\xi}{2}\quad\frac{1+\xi}{2}\right)$$

$$\frac{\rho A l}{2}\,\frac{1}{4}\begin{bmatrix}\int_{-1}^{+1}(1-2\xi+\xi^2)\,d\xi & \int_{-1}^{+1}(1-\xi^2)\,d\xi\\ \int_{-1}^{+1}(1-\xi^2)\,d\xi & \int_{-1}^{+1}(1+2\xi+\xi^2)\,d\xi\end{bmatrix}$$

$$\int_{-1}^{+1}1\,d\xi = 2,\qquad \int_{-1}^{+1}\xi\,d\xi = 0\qquad \int_{-1}^{+1}\xi^2\,d\xi = 2/3$$

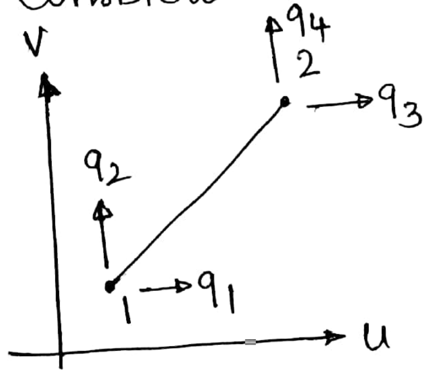
$$= \frac{\rho A l}{2}\times\frac{1}{4}\begin{bmatrix}2-0+2/3 & 2-2/3\\ 2-2/3 & 2+0+2/3\end{bmatrix}$$

$$= \frac{\rho A l}{2}\,\frac{1}{4}\begin{bmatrix}8/3 & 4/3\\ 4/3 & 8/3\end{bmatrix}$$

$$= \frac{\rho A l}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} //.$$

(08 Marks)

(ii) Consistent mass matrix of 1-D truss element



The shape function matrix for truss element

$$[N] = \begin{bmatrix}N_1 & 0 & N_2 & 0\\ 0 & N_1 & 0 & N_2\end{bmatrix}$$

~~Or~~

$N_1 = \frac{1-\xi}{2} \quad N_2 = \frac{1+\xi}{2} \quad dx = \frac{l}{2} d\xi$

& ξ — -1 to $+1$

Consistent ~~Mass~~ element mass matrix

$$m^e = \rho \int_e N^T N \, dv$$

$$= \rho \int_{-1}^{+1} N^T N \frac{Al_e}{2} d\xi$$

$$= \frac{\rho A l}{2} \int_{-1}^{+1} N^T N \, d\xi.$$

$$= \frac{\rho A l}{2} \int_{-1}^{+1} \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix} d\xi$$

$$\frac{\rho A l}{2} \int_{-1}^{+1} \begin{bmatrix} \frac{1-\xi}{2} & 0 \\ 0 & \frac{1-\xi}{2} \\ \frac{1+\xi}{2} & 0 \\ 0 & \frac{1+\xi}{2} \end{bmatrix} \begin{bmatrix} \frac{1-\xi}{2} & 0 & \frac{1+\xi}{2} & 0 \\ 0 & \frac{1-\xi}{2} & 0 & \frac{1+\xi}{2} \end{bmatrix} d\xi$$

$$\frac{\rho A l}{2} \times \frac{1}{4} \begin{bmatrix} \int_{-1}^{+1} (1-2\xi+\xi^2) d\xi & 0 & \int_{-1}^{+1} (1-\xi^2) d\xi & 0 \\ 0 & \int_{-1}^{+1} (1-2\xi+\xi^2) d\xi & 0 & \int_{-1}^{+1} (1-\xi^2) d\xi \\ \int_{-1}^{+1} (1-\xi^2) d\xi & 0 & \int_{-1}^{+1} (1+2\xi+\xi^2) d\xi & 0 \\ 0 & \int_{-1}^{+1} (1-\xi^2) d\xi & 0 & \int_{-1}^{+1} (1+\xi)^2 d\xi. \end{bmatrix}$$

$$\int_{-1}^{+1}(1-\xi^2)\,d\xi = \int_{-1}^{+1}1\,d\xi - \int_{-1}^{+1}\xi^2\,d\xi = 2-2/3 = 4/3$$

$$\int_{-1}^{+1}(1-\xi)^2\,d\xi = \int_{-1}^{+1}1\,d\xi - \int_{-1}^{+1}2\xi\,d\xi + \int_{-1}^{+1}\xi^2\,d\xi = 2+2/3 = 8/3$$

$$\int_{-1}^{+1}(1+\xi)^2\,d\xi = \int_{-1}^{+1}1\,d\xi + \int_{-1}^{+1}2\xi\,d\xi + \int_{-1}^{+1}\xi^2\,d\xi = 2+2/3 = 8/3$$

$$= \frac{\rho A l_e}{2}\times\frac{1}{4}\begin{bmatrix} 8/3 & 0 & 4/3 & 0 \\ 0 & 8/3 & 0 & 4/3 \\ 4/3 & 0 & 8/3 & 0 \\ 0 & 4/3 & 0 & 8/3 \end{bmatrix}$$

$$= \frac{\rho A l}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

(08 Marks)

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