8th Sem
G.H. JOSHI.

CBCS SCHEME

USN | | | | | | | | | | |

15ME8

Eighth Semester B.E. Degree Examination, Aug./Sept.2020
Operation Research

Time: 3 hrs. Max. Marks: 80

Note: i) For Regular Students: Answer any FIVE full questions irrespective of modules.
ii) For Arrear Students : Answer any FIVE full questions, choosing ONE full question from each module.
iii) Use of normal distribution table is allowed.

Module-1

1 a. Define operation research and explain all phases of operation research. (05 Mar

b. Old hens can be brought at Rs.20 each and young one at Rs.50 each. The old hens lay 3 eg per week and young one lay 5 eggs per week. Each egg being worth of one rupee and thi paise. A hen cost Rs.4 per week to feed. He has only Rs.800 to spend for hens. How many each kind should be buy to give a profit of more than Rs.600 per week. Assuming that cannot handle more than 200 hens, formulate the above problem as LPP model. (11 Mar

2 a. Explain the limitations of operation research. (05 Mar

b. Solve the below given LPP graphically and find the value of 'Z'.

Minimize $Z = 1.5x_1 + 2.5x_2$

$$x_1 + 3x_2 \geq 3$$
$$x_1 + x_2 \geq 2$$
$$x_i \geq 0$$

(11 Mar

Module-2

3 a. Solve the below given LPP by Big-M method.

Maximize $Z = -2x_1 - x_2$

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_i \geq 0$$

(11

b. Define slack, surplus and artificial variable. (05

4 a. Explain how do you resolve degeneracy in simplex method. (0

b. Solve the below given LPP by two phase method.

Minimize $Z = \frac{15}{2}x_1 - 3x_2$

$$3x_1 - x_2 - x_3 \geq 3$$
$$x_1 - x_2 + x_3 \geq 2$$

Module-3

5 Solve the following transportation problem and find the minimum transportation cost.

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	

(16 Marks)

6 Solve the following assignment problem and find minimum time required to complete all jobs. Time each man would take to perform each task is given in the matrix.

Job → Men ↓	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(16 Marks)

Module-4

a. Explain the Fulkerson's rule for number of nodes. **(05 Marks)**

b. Time estimates in weeks for PERT net work is given below. Calculate the following:

(i) Total expected time for the critical path

(ii) Standard deviation and varience for the project

(iii) Probability of project completion atleast 4 weeks earlier than expected time

(iv) If the project due date is 19 weeks, what is the probability of not meeting the due date?

Activity	t_o	t_m	t_p
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

(11 Marks)

he queuing system description parameters. **(05 Marks)**

airman finds that the time spent on his jobs has an exponential distribution, with minutes. If he repairs set in the order in which they come in and if the arrival of roximately Poisson with an average rate of 10/8 hrs day, what is repairman's e time each day. How many jobs are ahead of average set just brought in? **(11 Marks)**

Module-5

9 a. Explain maximini and minimaxi principle and also explain characteristics of Game theory. (05 Marks)

b. Solve the following game graphically.

Player 'B'

		a	b	c	d	e
Player 'A'	I	–5	5	0	–1	8
	II	8	–4	–1	6	–5

(11 Marks)

10 a. Explain the assumptions made while solving sequencing problems. (05 Marks)

b. Find the sequence that minimizes the total elapsed time 'T' required to complete the following tasks. Each task can be processed in any two machines A, B and C in any order.

Tasks

		1	2	3	4	5	6	7
Machines	A	12	6	5	3	5	7	6
	B	7	8	9	8	7	8	3
	C	3	4	11	5	2	8	4

(11 Marks)

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QUESTION PAPER SOLUTIONS

Sub: Operation Research. Aug/Sep 2020.
(15ME81).

Q 1 (a) Operation Research:

It can be defined as the application of scientific methods to problems arising from operations involving integrated systems of Men, Machines and materials to provide managers of such systems with optimum solution.

The phases of operation Research.

1) The judgement phase
2. Research phase
3. Action phase

1) The judgement phase.

a) Determination of operation, In this steps variables involved such as men, machine and materials are Identified.

(b) Determination of effective measure: In this steps inter relationship between the variable are established and are further defined as a particular model based on the objective function:

(c) Formulation:

In this step, objective function & constraint equation are constructed.

2. Research phase:

In this step optimal solution is analysed and compatible variables are discussed.

3: Action phase

The managers who take involved in the integrated system will implement the solution and will analyse the outcome

Q 1(b).

Given

Rate of old hen = ₹ 20/-

Rate of young hen = Rs 50/-

Eggs laid by old hen = 3/week

Eggs laid by young hen = 5/week

Rate of egg = Rs 1.30

Feeding cost = Rs 4/week

Max. amount to spend = Rs 800/-

Profit per week = More than Rs 600/-

Max. hens can be handled = 200 Nos.

Let x_1 be the old hens.

x_2 be the no of young hens

Considering the profit.

$$Z = 1.3 (3x_1 + 5x_2) - 4(x_1 + x_2)$$

Considering investment

$$20x_1 + 50x_2 \leq 800$$

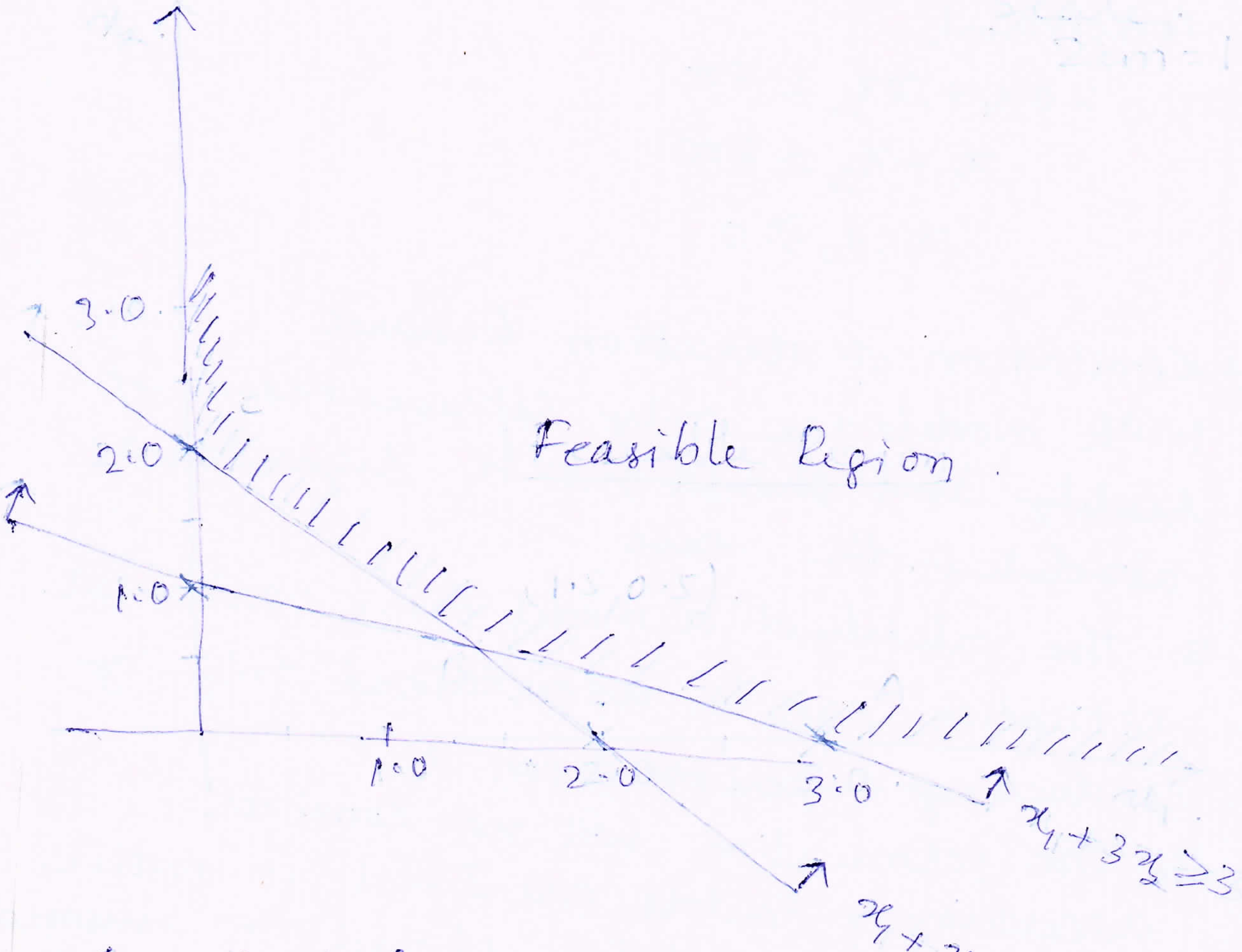
limitation of hens

$$x_1 + x_2 \leq 200$$

$x_1 + x_2 = 2$

when $x_1 = 0$ $\quad x_2 = 2 \Rightarrow (0,2)$

$x_2 = 0$. $\quad x_1 = 2 \Rightarrow (2,0)$



The feasible region boundary are

$A \equiv (3,1)$ $\quad B \equiv (0.5, 0.5)$ & $C\ (0,2)$

$Z_A \equiv 1.5 \times 3 + 2.5 \times 0 = 4.5$

$Z_B = 1.5 \times 0.5 + 2 \times 5 \times 0.5 = 4$

$Z_C = 1.5 \times 0 + 2.5 \times 2 = 5$

Min $(Z) = Z_B = 4$ units

Solution to the problem is

$(Z) = 4$ units $\quad x_1 = 0.5$ units

$x_2 = 0.5$ units.

∴ Mathematical model of the problem is
objective function:

$$\max (z) = -0.1x_1 + 2.5x_2$$

constraints,

$$20x_1 + 50x_2 \leq 800$$
$$x_1 + x_2 \leq 200$$
$$x_1, x_2 \geq 0$$

Q 2(a) Limitation of operation Research:

1. OR models are only representation of reality and should not be regarded as absolute in the case
2. The validity of a model for a particular situation can be ascertained only by conducting experimentaion it.
3. The formulation will not consider are Ideal in native ascertainly factors should be assumed.
4. Suitable probability

Q 2(b)

Given,

$$\min(z) = 1.5x_1 + 2.5x_2$$
$$x_1 + 3x_2 \geq 3$$
$$x_1 + x_2 \leq 2$$
$$x_1 \geq 0$$

Determining the co-ordinates for each constraint equations.

$$x_1 + 3x_2 = 3$$

when $x_1 = 0$ $x_2 = 1 \Rightarrow (0, 1)$

$x_2 = 0$ $x_1 = 3 \Rightarrow (3, 0)$

Q 3 (a).

Max (Z) = $-2x_1 - x_2$

Subj. to.

$3x_1 + x_2 = 3.$

$4x_1 + 3x_2 \geq 6.$

$x_1 + 2x_2 \leq 4.$

$x_1, x_2 \geq 0.$

Soln.

Let us introduce slack, surplus & Artificial variables

$\max(Z) = -2x_1 - x_2 + 0S_1 + 0S_2 + MA_1 + MA_2.$

$3x_1 + x_2 + A_1 = 3.$

$4x_1 + 4x_2 + S_1 = 6.$

$x_1 + 2x_2 + S_2 + A_2 = 4.$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0.$

step1. Constructing the matrix.

C_B.	x_B.	x_1	x_2	S_1	S_2	A_1	A_2	b.	b_i
M.	A_1	3	1	0	0	1	0	3	1
0	S_1	4	4	1	0	0	0	6	6/4
M.	A_2	(1.)	2	0	1	0	1.	4.	4 ←
	C_j	-2	-1	0	0	M	M.		
$\Sigma x_i c_j$		4M.	3M	0	M.	M.	M.		
$C_j - \Sigma x_i c_j$		(-2-4M) ↑	(-1-3M)	0	-M	0	0.		

∴ x_1 be the entering & A_2 be the leaving variable.

~~C_B. x_B C_j~~

~~x_1~~

C_j

C_B	x_B	x_1	x_2	S_1	S_2	A_1	A_2	b
	A_1	0	$1/3$	0	$-4/3$	1	$-4/3$	[illegible]
	S_1	0	-4	1	-4	0	-4	$\cancel{4}$ -10
	x_1	1	2	0	1	0	1	4

Q.4(a) Degeneracy in Simplex Method.

Degeneracy in LPP is said to occur when one or more basic variables have zero value. At the initial stage of finding basic feasible solution.

It may generate at subsequent iteration stages.

It can be solved as follows:

1. Devide each element of the tied row by +ve coeff. of key element.
2. Compare the resulting ratio column by column from left to right of the Matrix.
3. The smallest algebraic ratio will become outgoing variable.

Q 4(b). Two phase Method.

$$\min(Z) = \frac{15}{2}x_1 - 3x_2.$$

Subjected to,

$$3x_1 - x_2 - x_3 \geq 3$$
$$x_1 - x_2 + x_3 \geq 2$$

Phase I.

Let us construct new constraint equations with surplus & artificial variables.

$$3x_1 - x_2 - x_3 + S_1 + A_1 = 3.$$
$$x_1 - x_2 + x_3 + S_2 + A_2 = 2$$

$$\max(w) = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 + A_1 + A_2.$$

Step 2. Basic feasible solution.

	C_j	0	0	0	0	0	1	1	b	
C_B.	B.V.	x_1	x_2	x_3	S_1	S_2	A_1	A_2	3	−3
1	A_1	3	−1	−1	1	0	1	0		
1	A_2	1	−1	1	0	1	0	1.	1	−1.
	Z_j.	4	−2	0	1	1	1	1		
	$(C_j - Z_j)$	−4	2 ↑	0	−1	−1	0	0		

Since the variable ratio is −ve the problem is said to be unbound solution.

Q 5. Given,

Warehouse

		1	2	3	4	Capacity
Factory	1	10	30	50	10	7
	2	70	30	40	60	9
	3	40	8	70	20	18
Req.		5	8	7	14	34 / 34

Step 1. Since total capacity is equal to requirement of warehouses, the given model is a balanced transportation model.

Step 2. Basic feasible solution using VAM method,

10 [5]	30	50	10 [2]	7/2/0	(20) (20) (40) (40)
70	30	40 [7]	60 [2]	9/2/0	(10) (10) (20) (20)
40	8 [8]	70	20 [10]	18/10/0	(12) (12) (50) ←
5/0	8/0	7/0	14/4/2/0		
(30) ↑	(22)	(10)	(10)		
	(22) ↑	(10)	(10)		
		(10)	(10)		
		(10)	(50) ↑		

Step 3: Optimality test

No. of allocated cell = 6.

Condition for degeneracy $(m+n-1)$

$= (3+4-1) = 6.$ $\quad m = 3, \; n = 4.$

$\therefore$ No. of Allocated cell $= (m+n-1)$

$\therefore$ The problem is non-degeneracy.

By MODI method,

Determining dual variables.

10 $\boxed{5}$			10 $\boxed{2}$	$u_1 = 0$
		40 $\boxed{7}$	60 $\boxed{2}$	$u_2 = 50$
	8 $\boxed{8}$		20 $\boxed{10}$	$u_3 = 10$.
$v_1 = 10$	$v_2 = -2$	$v_3 = -10$	$v_4 = 10$	

the constraint equations are

$u_1 + v_1 = 10$, $u_1 + v_4 = 10$, $u_2 + v_3 = 40$, $u_2 + v_4 = 60$.

$u_3 + v_2 = 8$ $u_3 + v_4 = 20$.

Let $u_1 = 0$

$0 + v_1 = 10 \Rightarrow v_1 = 10$; $0 + v_4 = 10$ $v_4 = 10$

$u_2 + 10 = 60$ $u_2 = 50$; $50 + v_3 = 40$ $v_3 = -10$

$u_3 + 10 = 20$ $u_3 = 10$; $10 + v_2 = 8$ $v_2 = -2$

Cell evaluation Matrix.

	30 $\boxed{32}$	50 $\boxed{60}$		0
70 $\boxed{10}$	30 $\boxed{-18}$			50
40 $\boxed{20}$		70 $\boxed{70}$		10
10	-2	-10	10	

Cell (2,2) has -ve evaluation variable

ie the bfs is not optimal.

Iterate towards optimality by assigning a new unit 'δ' to cell (2,2).

Constructing a loop throught (2,2).

Iteration towards optimality

10 $\boxed{5}$	30	50	10 $\boxed{2}$
70	30 $\boxed{8}$	40 $\boxed{7}$	$-\delta$ 60 $\boxed{2}$
40	$-\delta$ 8 $\boxed{8}$	70	$+\delta$ 20 $\boxed{10}$

The max value of 'δ' = 2.

<u>2nd basic feasible solution</u>

10 $\boxed{5}$			10 $\boxed{2}$	$u_1 = 0$
	30 $\boxed{2}$	40 $\boxed{7}$		$u_2 = 32$
	8 $\boxed{6}$		20 $\boxed{12}$	$u_3 = 10$
$v_1 = 10$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$	

Cell evaluation Matrix.

	30 $\boxed{32}$	50 $\boxed{42}$		0
70 $\boxed{28}$	8		60 $\boxed{18}$	32
40 $\boxed{20}$		70 $\boxed{52}$		10
10	-2	8	10	

Since All variables are positive, the above solution is optimal.

∴ Optimal Transportation Cost

$$Z = 10 \times 5 + 10 \times 2 + 30 \times 2 + 40 \times 7 + 8 \times 6 + 20 \times 12$$

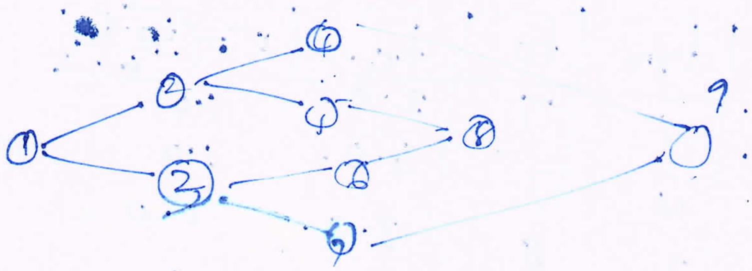
$$= 698/-$$

Q6. Assignment problem is not there in 2017 syllabus.

Q7. (a).

Fulkerson's rule for Numbering the nodes.

Let us assume a network.

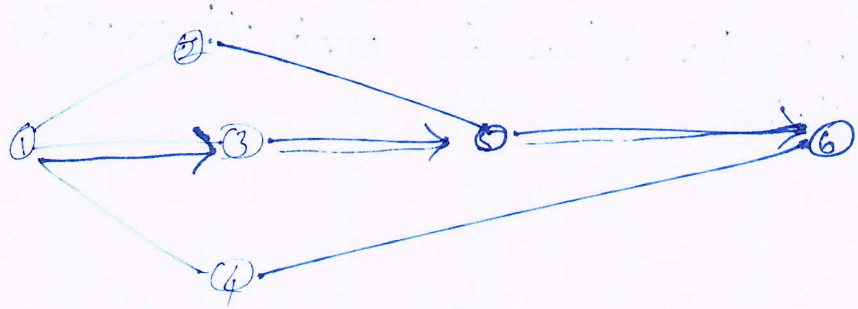


1. Number the starting node as 1.
2. Successively one need to number the node from each activity.
3. The largest No. should be given to the last activity and node.

(b). Given,

Activity	t_o	t_m	t_p
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

Step 1: Construction of the network



step 2. Determination of expected time

Activity	t_o	t_m	t_p	t_e	$\sigma = \left(\frac{t_p - t_o}{6}\right)$
1-2	1	1	7	2.0	1.0
1-3	1	4	7	4.0	1.0
1-4	2	2	8	3.0	1.0
3-5	1	1	1	1.0	0
4-6	2	5	14	6.0	2.0
5-6	2	5	8	5.0	1.0
	3	6	15	7.0	2.0

$$t_e = \left(\frac{t_o + 4t_m + t_p}{6}\right)$$

step 2 CPM.

Activity	te	Early start	Early finish	Latest start	Latest finish	Float
1-2	2	0	2	7	9	7
1-3	4	0	4	0	4	0
1-4	3	0	3	9	12	9
2-5	1	2	3	9	10	7
3-5	6	4	10	4	10	0
4-6	5	3	8	12	17	9
5-6	7	10	17	10	17	0

Critical activities are
1-3, 3-5, 5-6.

Q(8) (a) Queueing System:

Queuing model is constructed for a service system. Various factors are considered for the representation of the system.

(a) Input – arrival distribution

– No. of customers, jobs etc arriving at the service counter are defined and are assumed to be distributed with Poisson's theory.

(b) Output – departure distribution

No. of customers or jobs which are leaving the service counter considered and assumed to be with Binomial distribution

(c) Service channel.

The actual service counters are considered in the system.

(d) Service desciplino:

The order at which the service is provided like First come first serve etc are defined for the system.

(f) Max. no. of Customers in the system

The strength of service is defined in the model as a definite no. or infinite

(g) Population of Queue:

The length of queue defines the population of the queue.

8

Q 8 (b). Given,

Mean = 30 min. = ~~λ~~.

(Mean service)

~~μ~~ = Arrival rate = 10 jobs per 8 hours.

$\therefore$ Arrival rate $= \lambda = \dfrac{10}{8\times 60} = \dfrac{1}{48}$ units/min.

Service rate $= \mu = \dfrac{1}{30}$ units/min.

(a) Number of jobs ahead = Av. No. of jobs in the System

$$= \frac{\lambda}{(\mu-\lambda)}$$

$$= \frac{1/48}{1/30 - 1/48} = 1.67.$$

(b) No. of hours for which the repairman remain busy in 8 hours per day.

$$= 8.\frac{\lambda}{\mu} = 8 \times \frac{1/48}{1/30} = 2.5 \text{ hours}.$$

$\therefore$ Idle time for the repairman

$$= (8-2.5) = 5.5 \text{ hours}.$$

&

Q 9 (a) <u>Maximin / minimax principle</u>

The strategies for player A & B and the payoff by each player will be represented in the form of matrix.

		B	
		P	Q
A	A1		
	A2		

It is clear from the matrix that, the cell (1,1) indicates that, when player A select strategy 1, and player B also select strategy 'P'. the value in the cell (1,1) represents gain for player A & loss for player B.

The maximin theory for player A states, the value indicates the max gain of the player A.

The minimax indicates the minimum loss of player B. This value explains the range of gain or loss for the game.

If both values are equal, the game is said to be pure strategy. This condition is called saddle point. The condition is called zero sum game because it has no gain or loss.

[signature]

Q 9

(b). Given,

player B.

player A

	a	b	c	d	e
I	-5	5	0	-1	8
II	8	-4	-1	6	-5

strategy e dominates over other strategies, let us eliminate the strategy

		a	b	c	d
x_1	I	-5	5	0	-1
$(1-x_1)$	II	8	-4	-1	6.

Representing in graph with player A along x axis.

B's pure strategies.

1 $\Rightarrow -5x_1 + 8(1-x_1) = -13x_1 + 8$

2 $\Rightarrow 5x_1 - 4(1-x_1) \Rightarrow 9x_1 - 4$.

3 $\Rightarrow 0x_1 - (1-x_1) \Rightarrow x_1 - 1$.

4 $\Rightarrow -x_1 + 6(1-x_1) \Rightarrow -7x_1 + 6$.

5 $\Rightarrow 8x_1 - 5(1-x_1) = 13x_1 - 5$.

Q 10.

(a). Assumption made in Sequencing Problem.

1. Only one operation is carried out on a machine at a particular time.

(2) Each operation once started, must be completed

(3) An operation must be completed before its succeeding operation.

(4) Only one machine of each type is available

(5) Processing time are independent of order of performing the operation

(6) Job movement time is neglected.

(7) Cost of in-process inventory is assumed to be zero.

10(b). Given,

		Task 1	2	3	4	5	6	7
M/c	A	12	6	5	3	5	7	6
	B	7	8	9	8	7	8	3
	C	3	4	11	5	2	8	4.

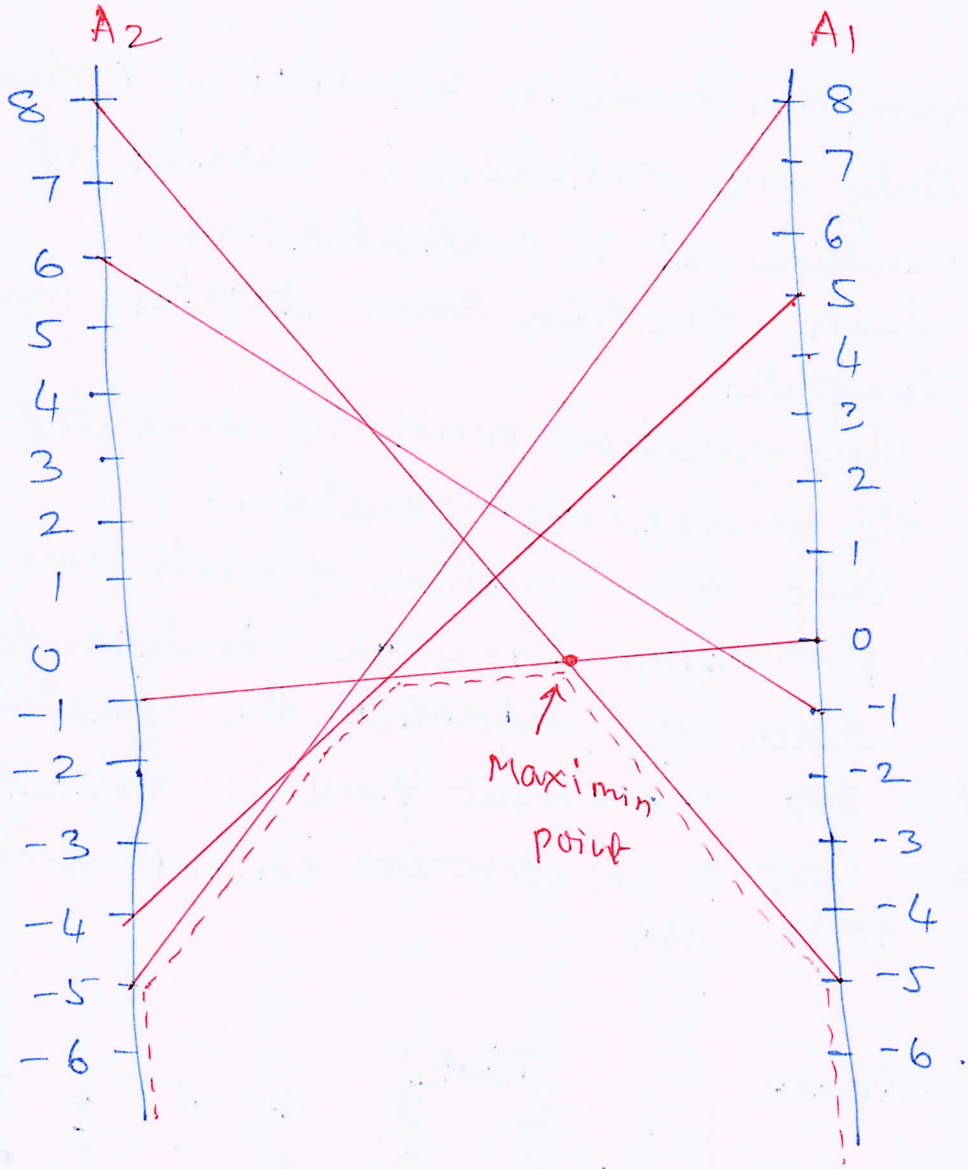
The tasks can be completed on any two M/c's at any order.

There are six possibilities of machining order.

(1) order AB.

4	5	3	2	6	1.	7

Sd/-



Considering the maximin point

	1	3		
1	-5	0	9	9/14
2	8	-1	5	5/14
	1	13		
	1/14	13/14.		

optimum strategy are.

$A(9/14, 5/14)$. $B(1/14, 0, \frac{13}{14}, 0, 0)$

Value of the game

$$V = \frac{-5\times 1 + 0\times 13}{1+13.} = \frac{-5}{14}.$$

Processing time.

order AB	M/c A in	M/c A out	M/c B in	M/c B out	order BA	M/c B in	M/c B out	M/c A in	M/c A out
4	0	3	3	11	7	0	3	3	9
5	3	8	11	18	1	3	10	10	22
3	8	13	18	27	6	10	18	22	29
2	13	19	27	35	2	18	26	29	35
6	19	26	35	43	3	29	35	35	40
1	26	38	43	50	5	35	42	42	47
7	38	44	50	53.	4	42	50	50	53.

(2) Sequence AC & CA.

4	3	6	2	7	1	5

order AC	M/c A in	M/c A out	M/c C in	M/c C out
4	0	3	3	8
3	3	8	8	19
6	8	15	19	27
2	15	21	27	31
7	21	27	31	35
1	27	39	39	42
5	39	44	44	46.

∴ Total Elapsed time = 46.

(3) Order BC. or CB.

5	7	6	3	4	2	1

order	M/c B in	M/c B out	M/c C in	M/c C out
5	0	~~8~~ 7	7	9
7	7	10	19	14
6	10	18	18	24
3	18	27	27	38
4	27	35	38	43
2	35	43	43	47
1	43	50	50	53.

It is clear from the above calculation that order A C or C A leads to optimal sequence

∴ The sequence is

4	3	6	2	7	1	5.

Total Elapsed time = 46 hours.
Idle time for M/c A = 2 hours
Idle time for M/c C = 07 hours.

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