Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

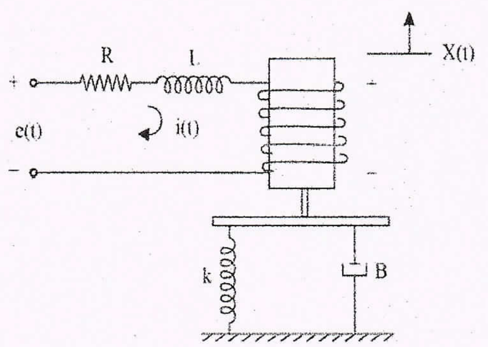
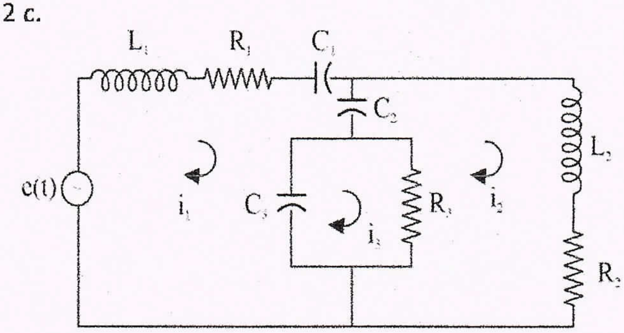
USN

Fourth Semester B.E. Degree Examination

Subject CONTROL SYSTEMS

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02.
03.

		Module -1	***Bloom's Taxonomy Level**	**Marks**
Q.01	a	What are the merits and demerits of Closed Loop control systems.	L1 CO1	04 M
	b	Define control system and explain the same with an example.	L1 CO1	04 M
	c	Find the transfer function for the given electro mechanical system shown in Fig 1(c).  $e_b(t) = k_b \frac{dx}{dt}$ k_b back emf constant V m^{-1} s^{-1} FIG 1(C)	L1, L2 CO1	12 M
		OR		
Q.02	a	What are the classification of control system.	L1 CO1	05 M
	b	Explain closed loop control system with an example.	L1 CO1	05 M
	c	Find the Translational mechanical system for the Force Voltage electrical circuit shown in Fig 2 c.  FIG 2 (C)	L1, L2 CO1	10 M

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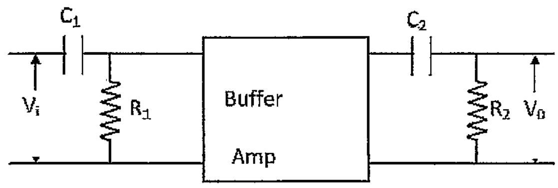
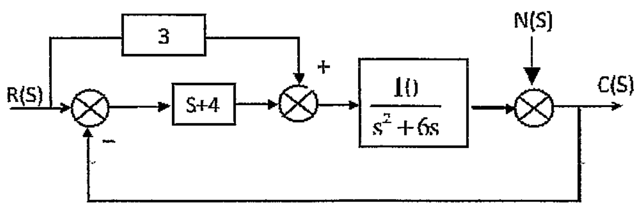
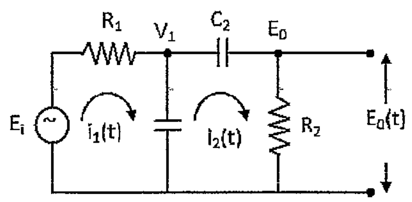
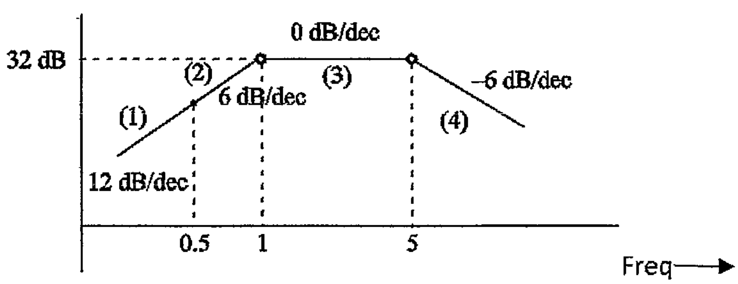
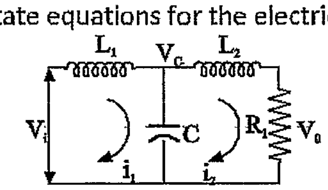
		Module-2		
Q. 03	a	Find the transfer function $\frac{V_0(s)}{V_i(s)}$ for the system shown with unity gain buffer amplifier shown in Fig 3(a)  FIG 3(a)	L1 CO2	06 M
	b	Compare Block diagram and signal flow graph method of finding the transfer function.	L1 CO2	04 M
	c	The system block diagram is given in Fig 3(c) below. Find $\frac{C(s)}{R(s)}$ if $N(s) = 0$.  FIG 3 (c)	L1, L2 CO2	10 M
		OR		
Q.04	a	What are the types of Loops and their respective loop gains in a signal flow graph?	L1 CO2	04 M
	b	Find the transfer function by constructing a Block diagram and reducing the same for the circuit shown in Fig 4(b).  Fig 4(b)	L1, L2 CO2	08 M
	c	Find the transfer function by constructing SFG and Mason's Gain formula for the circuit shown in Fig 4 b.	L1,L2 CO2	08 M
		Module-3		
Q. 05	a	Obtain expressions for specifications namely time constant, rise time, and settling time of first order system for a unit step input.	L1 CO3	04 M
	b	Derive an expression for C(t) of an under damped second order system for a unit step input.	L1, L2 CO3	08 M
	c	A unity feedback system is characterized by an open loop transfer function $G(S) = \frac{K}{S(S+10)}$ find the value of K so that the system will have the damping ratio of 0.5. for this value of K find M_p, t_p & t_s for a unit step input.	L1,L2 CO3	08 M
		OR		
Q. 06	a	Starting from the output equation C(t) derive expressions for: (i) Peak time (t_p) (ii) Peak overshoot (M_p) of an under damped second order system subjected to unit step input.	L1, L2 CO3	08 M

	b	A unity feedback system has $G(s) = \dfrac{K}{s(s+2)(s^2+2s+5)}$ (i) For a unit ramp input it is desired that $e_{ss} \le 0.2$. Find K. (ii) Find e_{ss} if $r(t) = 2 + 4t + \dfrac{t^2}{2}$	L3 CO3	08 M
	c	Write a short note on PID controllers.	L1 CO3	04 M
		Module-4		
Q. 07	a	Define stability and hence stable, unstable, marginally stable, and conditional stability of a unity feedback system.	L1 CO4	06 M
	b	In a unity feedback system find the range of K for stability and K_{mar}, ω_{mar} with $\boxed{G(s) = \dfrac{K}{s(1+0.4s)(1+0.25s)}}$	L1,L2 CO4	06 M
	c	Prove that part of root loci is a circle using angle condition and find the center as well as radius when $G(s)H(s) = \dfrac{K(s+2)}{s(s+1)}$	L2, CO4	08 M
		OR		
Q. 08	a	A – ve feedback control system is characterised by $G(s) = \dfrac{K}{s(s+\alpha)}$ H(s) = 1. Find value of K and α so that $M_r = 1.04$ and $\omega_r = 11.55$ rad/sec	L3 CO5	04 M
	b	Using RH criterion determine the stability of the system, the system is type one system with error constant of 10 sec^{-1} and poles at $S = -3$ and $S = -6$	L1 CO4	06M
	c	Find transfer function for the magnitude plot.  	L2,L3 CO5	10 M
		Module-5		
Q. 09	a	Compare transfer function method and state space approach in control systems.	L1,L2 CO5	04 M
	b	Find stability and range of K using Nyquist Plot $G(s)H(s) = \dfrac{K(s+1)}{s(s-1)}$	L1 CO5	10 M
	c	Write short note on Lead, Lag, lead lag compensators.	L1,CO5	06 M
		OR		
Q. 10	a	Define state, state variable, state space.	L1,CO5	04 M
	b	Obtain the state equations for the electrical network shown in fig 10 b.  FIG 10 (b)	L2, L3 CO5	08 M
	c	A system is given by the following vector matrix equation write $\phi(t)$	L2,L3	08 M

		$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$	CO5	

*Bloom's Taxonomy Level: Indicate as L1, L2, L3, L4, etc. It is also desirable to indicate the COs and POs to be attained by every bit of questions.

Model Question paper-2 (18EC43)

Module-1

Q1a. What are the merits and demerits of closed loop control systems

* **Merits of closed loop system**

1) The closed loop system are relatively insensitive to External and internal disturbance
2) Closed loop system are accurate and more reliable
3) Closed loop system reduce the effect of non-linearity & distortion

—02M—

* **Demerits of closed loop system**

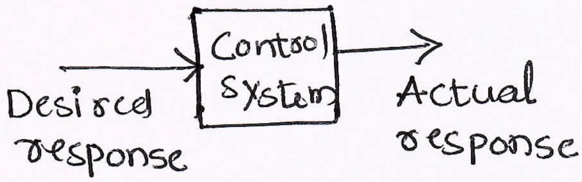
1) Closed loop system are complex in design
2) Closed loop system consume more power because of more no of components
3) Closed loop system are Expensive —02M—

Q1b. Define control system and Explain the same with an Example.

* A control system is an interconnection of components forming a system configuration that will provide a desired system response. —01M—

* Control system can be further classified as open loop control system and closed loop control system.

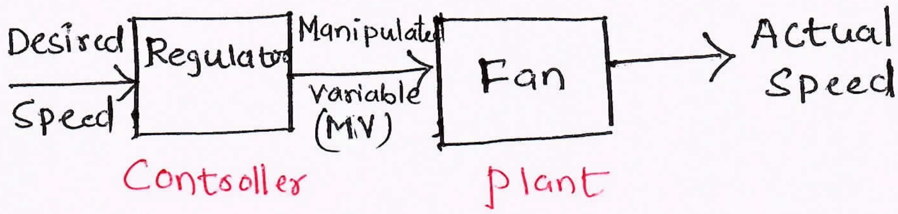
(Vinaychitare)



* Open loop control system :- The system in which the output has no effect on the control action is called open loop control system.

Ex Fan speed control

Objective :- To control the speed of fan

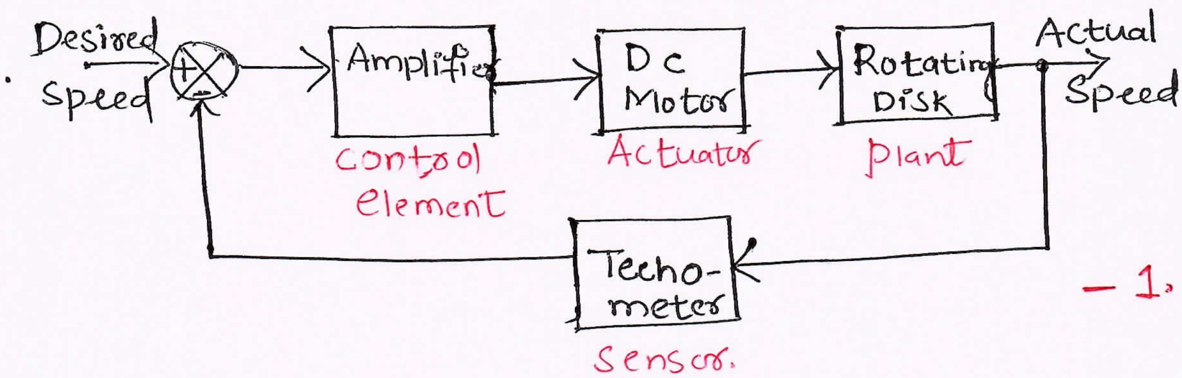


—1.5 M—

* Closed loop control system :- A system that maintains prescribed relationship between the output and the reference input by comparing them & using the difference as a means of control is called closed loop control system.

Ex Rotating Disk speed Control

Objective :- To control the speed of Motor



— 1.5 M—

(Vinaychitare)

Q1C. Find the transfer function for the given electromechanical system shown in Fig 1(c)

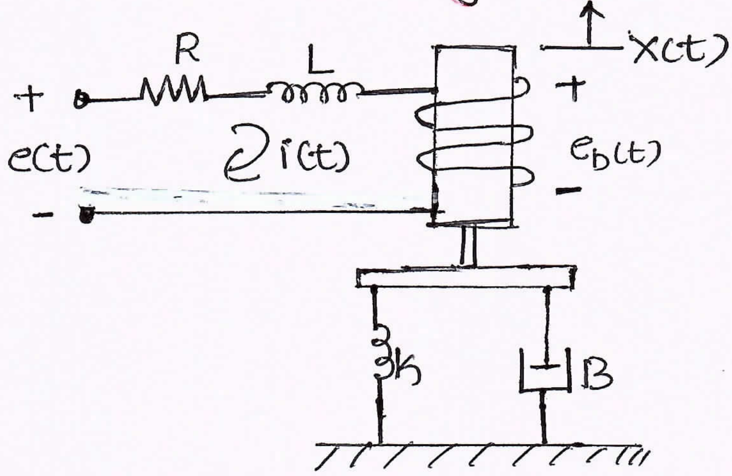


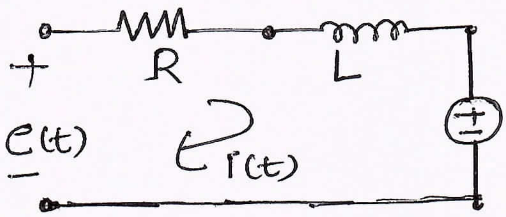
Fig 1(c)

$e_b(t) = K_b \dfrac{dx(t)}{dt}$

K_b: back emf constant

$V m^{-1} s^{-1}$

Step1 Electrical part of System



$e_b(t) = K_b \dfrac{dx(t)}{dt}$ —02M—

KVL to loop

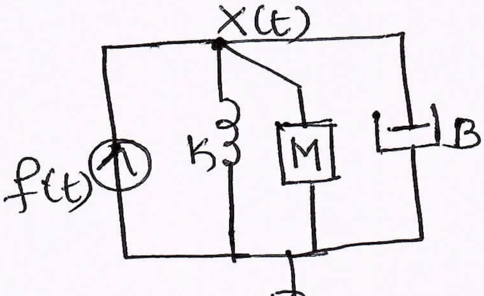
$$e(t) = i(t)R + L\frac{di(t)}{dt} + K_b \frac{dx(t)}{dt} \quad -(1)$$

Laplace transform of above eqn

$$E(S) = I(s)R + LS\,I(S) + K_b S X(S)$$ —02M—

$$I(S) = \frac{E(S) - K_b S X(S)}{R+SL} \quad -(2)$$

Step2 Mechanical part of system



—02M—

(Vinay chitame)

Newton's Law @ x(t) Node

$$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) \quad -③$$

Laplace transform above Eqn —02M—

$$F(S) = Ms^2X(s) + BsX(s) + KX(s) \quad -④$$

* $f(t) = K_f\, i(t)$ --⑤
Laplace transferring Eqn ⑤
$F(S) = K_f\, I(S)$ —⑥
—02M—

* Sub^n of Eqn ⑥ in Eqn ④

$$K_f I(S) = Ms^2X(s) + BsX(s) + KX(s)$$

$$I(S) = \frac{Ms^2X(s) + BsX(s) + KX(s)}{K_f}$$

* Equating Eqn ⑥ and ② we get

$$\frac{E(S) - K_b s X(s)}{R+sL} = \frac{(Ms^2+Bs+K)X(s)}{K_f}$$

$$\frac{X(S)}{E(S)} = \frac{K_f}{K_bK_f s + (R+sL)(Ms^2+Bs+K)} \quad -02M-$$

(Vinaychitore)

Q2a. What are the classification of control system

* Linear control system : A system is said to be linear if the principle of superposition holds true —01M—

* Non linear control system : A system is said to be Nonlinear if principle of superposition doesnot hold true. —01M—

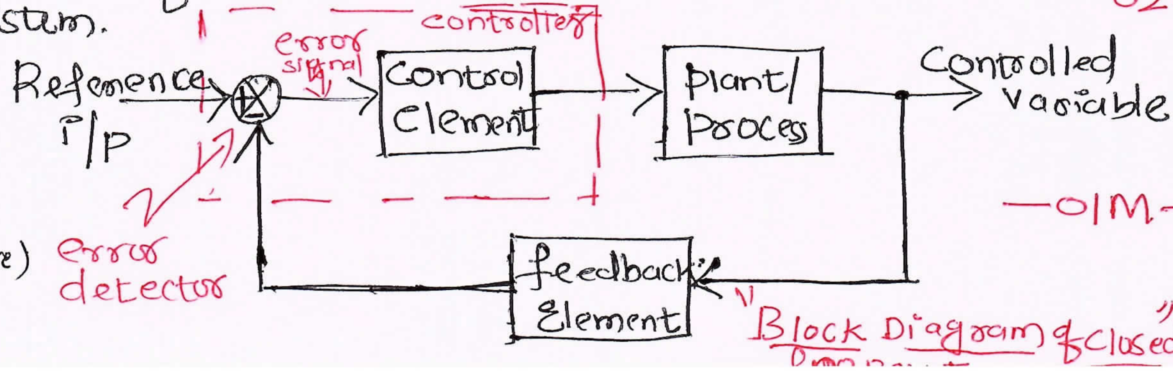
* Time invariant system : When the parameters of a control system are stationary with respect to time during the operation of system, the system is called time invariant —01M—

* Time varying system : when the parameters of a control system are varying w.r.t to time during the operation of system, the system is called time varying system. —01M—

* Single input single output system : A control system having single input and single output is called SISO system. —01M—

Q2b. Explain closed loop control system with an example

Closed loop control system : A system that maintains prescribed relationship between output and reference input by comparing them & using the difference as a means of control is called feed back control system. —02M—



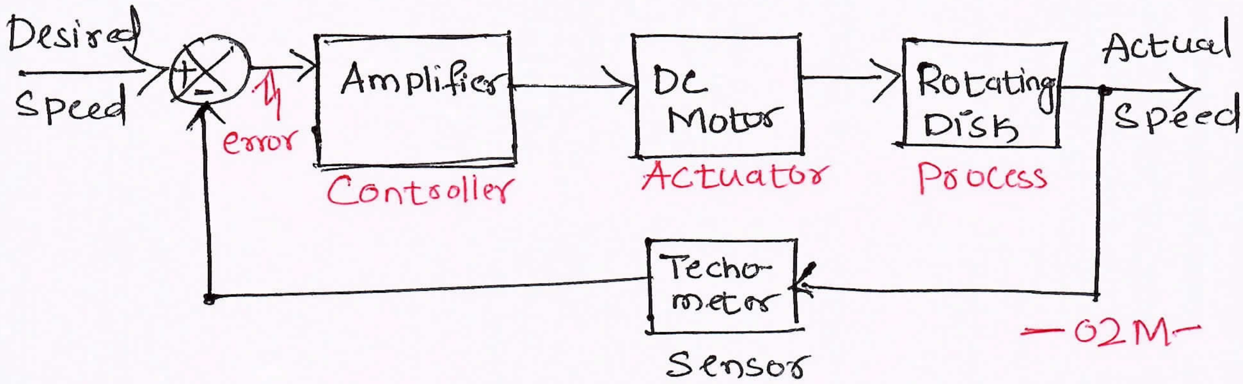
—01M—

(Vinaychitare)

"Block Diagram of closed loop ..."

Example

Rotating Disk Speed Control



—02M—

Q2c. Find the translation Mechanical system for the Force voltage electrical circuit shown in Fig 2(c)

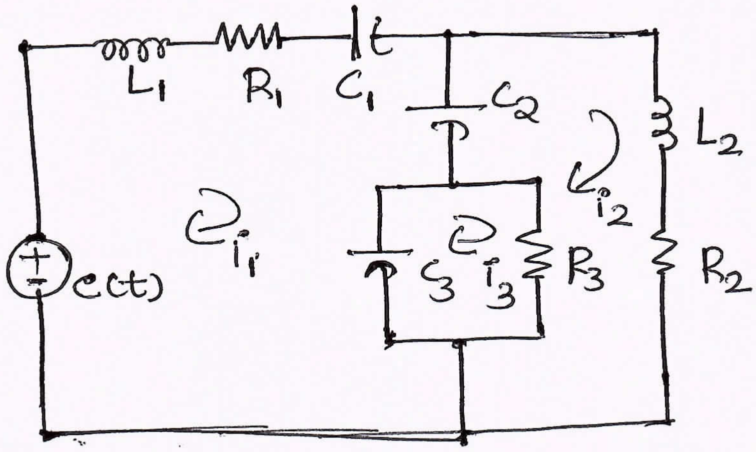


Fig 2(c)

Step 1 KVL to loop 1

$$-e(t) + L_1 \frac{di_1}{dt} + R_1 i_1(t) + \frac{1}{C_1}\int i_1\, dt + \frac{1}{C_2}\int (i_1 - i_2)\, dt + \frac{1}{C_3}\int (i_1 - i_3)\, dt = 0 \quad -①$$

KVL to loop 2

$$R_3(i_2 - i_3) + \frac{1}{C_2}\int (i_2 - i_1)\, dt + L_2 \frac{di_2}{dt} + R_2 i_2 = 0 \quad -②$$

KVL to loop 3

$$\frac{1}{C_3}\int (i_3 - i_1)\, dt + (i_3 - i_2) R_3 = 0 \quad -③-$$

—02M—

(Vinaychitore)

Step 2

* Writting above KVL eqn interms of system Variable, using $i = \frac{dq}{dt}$

$$e(t) = L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{q_1}{C_1} + \frac{1}{C_2}(q_1 - q_2) + \frac{1}{C_3}(q_1 - q_3)$$

$$R_3(\dot{q}_2 - \dot{q}_3) + \frac{1}{C_2}(q_2 - q_1) + L_2 \ddot{q}_2 + R_2 \dot{q}_2 = 0$$

$$\frac{1}{C_3}(q_3 - q_1) + R_3(\dot{q}_3 - \dot{q}_2) = 0$$

— 02M —

Step 3 Writting analogous Mechanical D.E using F-V analogy

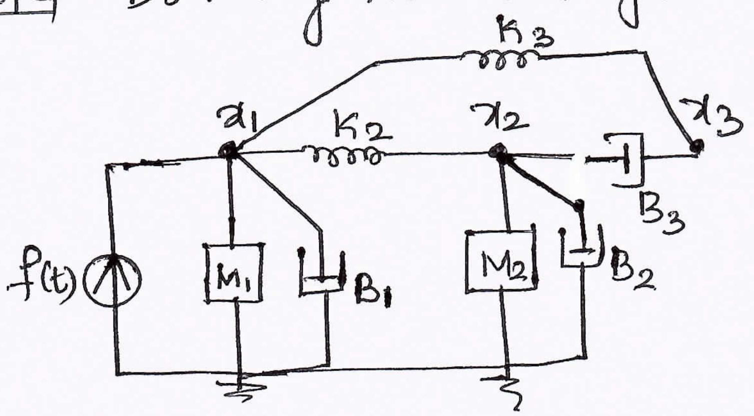
$$F = M_1 \ddot{x}_1 + B_1 \dot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) + K_3(x_1 - x_3)$$

$$B_3(\dot{x}_2 - \dot{x}_3) + K_2(x_2 - x_1) + M_2 \ddot{x}_2 + B_2 \dot{x}_2 = 0$$

$$K_3(x_3 - x_1) + B_3(\dot{x}_3 - \dot{x}_2) = 0$$

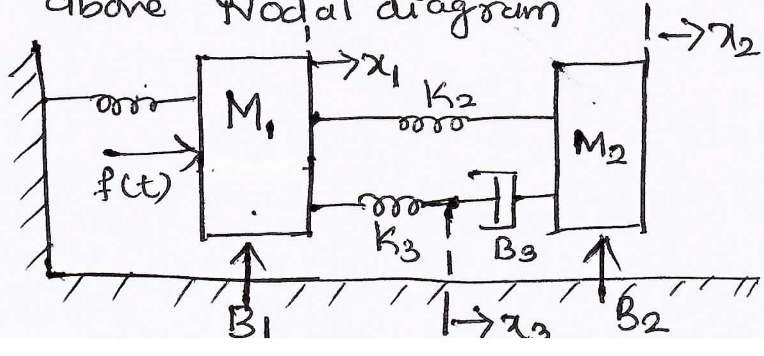
— 02M —

Step 4 Drawing Nodal diagram based on above eqn



— 02M —

Step 5 Drawing translation Mechanical system from above Nodal diagram



— 02M —

(Vinaychitare)

Q3a. Find the Transfer function $\frac{V_o(s)}{V_i(s)}$ for the system shown with unity gain buffer amplifier shown in Fig 3(a)

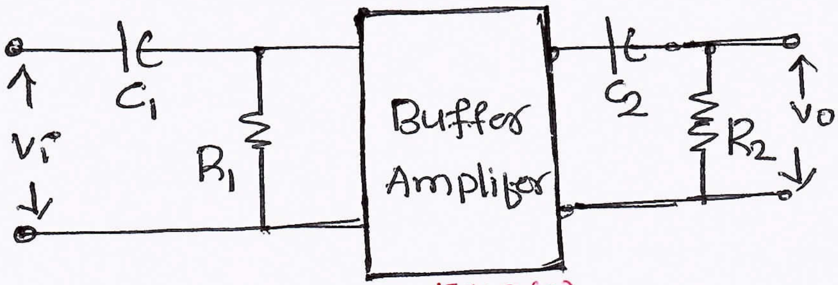
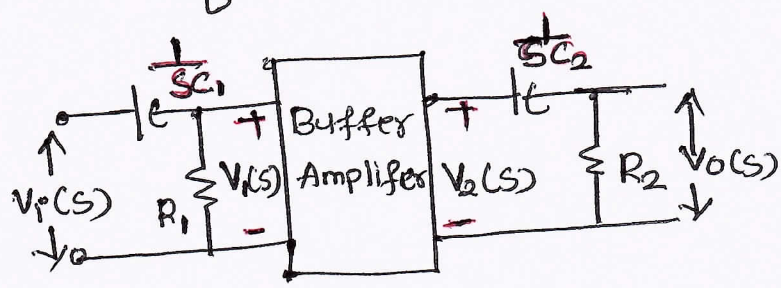


Fig 3(a)

Step 1 Transforming the above network in Laplace domain, let V_1 be i/p @ buffer amp & V_2 @ o/p of buffer amp



—02M—

Step 2 Applying voltage divider @ i/p & o/p

$$V_1(S) = \frac{V_i(S)\, R_1}{R_1 + \frac{1}{SC_1}} = \frac{V_i(S)\, SC_1R_1}{1 + SC_1R_1} \quad -①$$

$$V_o(S) = \frac{V_2(S)\, R_2}{R_2 + \frac{1}{SC_2}} = \frac{V_2(S)\, SC_2R_2}{1 + SC_2R_2} \quad -②$$

—02M—

Step 3 for Buffer amp $\boxed{V_2(S) = V_1(S)}$, Eqn ② can be written as, after subn of Eqn ① into ②

—01M—

$$V_o(S) = \frac{V_i(S)\,(SC_1R_1)(SC_2R_2)}{(1+SC_1R_1)(SC_2R_2+1)}$$

$$\therefore \boxed{\frac{V_o(S)}{V_i(S)} = \frac{S^2 C_1C_2R_1R_2}{S^2(C_1C_2R_1R_2) + S(C_1R_1 + C_2R_2) + 1}}$$

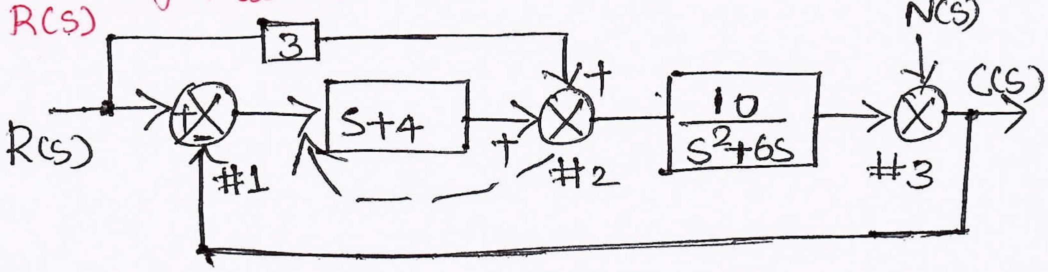
—01M—

(Vinaychitore)

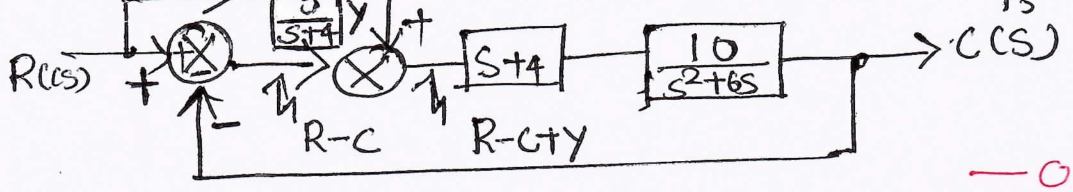
Q3b. Compare Block diagram and signal flow graph method of finding Transfer function

Block Diagram	Signal flow graph
1) It is a pictorial representation of the function performed by each component and the flow of signals	1) It is graphical representation of relationship between variables of a set of linear algebraic equation written in the form of cause and effect relations. —01M
2) It can be used to represent both linear & non-linear system	2) It can be used to represent only linear systems. —01M—
3) No direct formula is available to find the overall Transfer function of the system	3) Mason's gain formula is available to find the overall transfer function of the system. —01M—
4) It is not a systematic method	4) It is systematic method —01M—

Q3C. The system Block diagram is shown in Fig 3(c) below. Find $\frac{C(S)}{R(S)}$ if $N(S)=0$

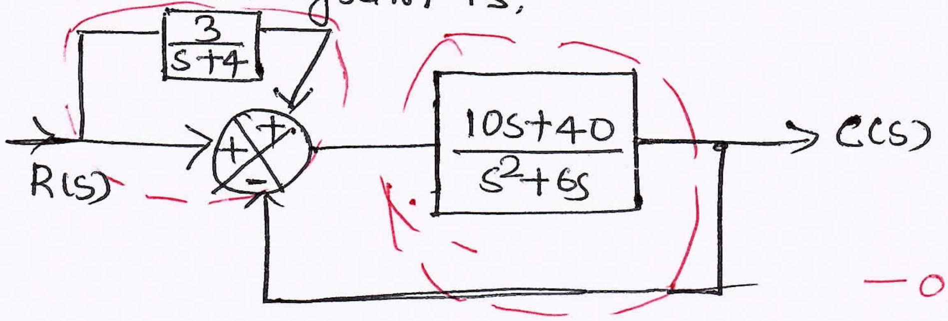


Step1 If $N(S)=0$, summer #3 is ineffective, Shifting Summer #2 in backward direction, the equivalent Block diagram is



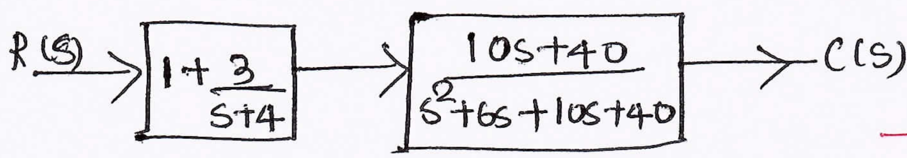
—03M—

Step 2 Combining two summer, the equivalent Block diagram is,



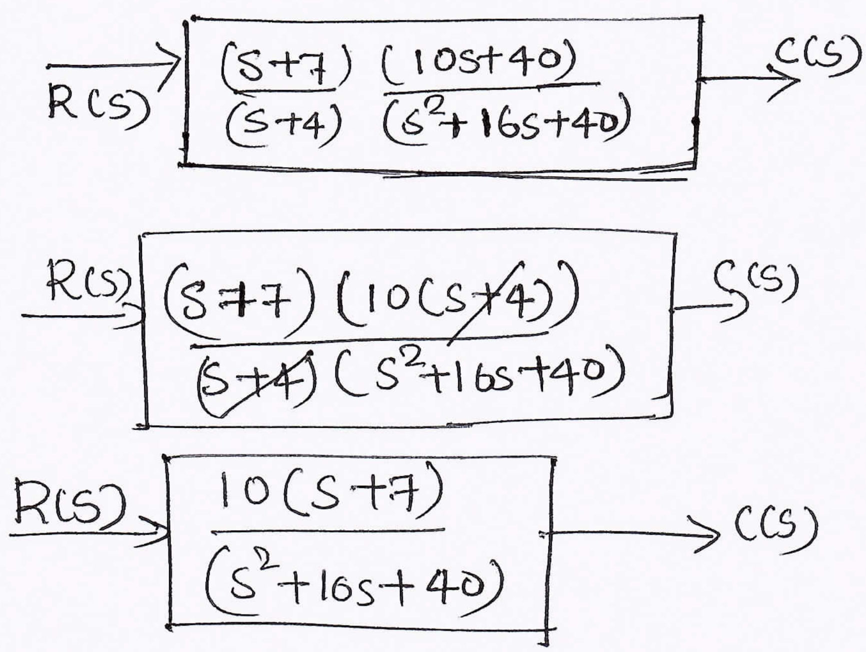
—02M—

Step 3 Combining Blocks in parallel, Eliminating -ve feedback, equivalent Block diagram is



— 02M —

step 4 : combining Blocks in cascade, Equivalent Block diagram is



—02M—

(Vinaychitare)

Q4a. What are the types of loops and their respective loop gains in a signal flow graph?

loop : A path which originates and terminate at the same node

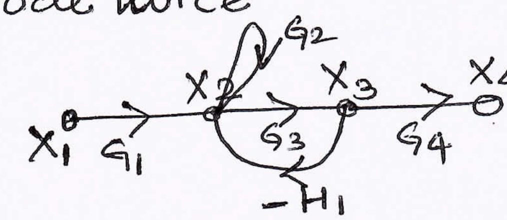
—01M—

Types of loop (i) Self loop (ii) feedback loop

self loop : A loop consisting of only one node is called self loop

feedback loop : A path which originates from particular node and terminating at the same node, travelling through at least one other node, without tracing any node twice

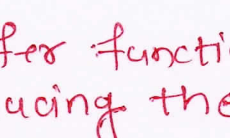
Ex



loop gains : The product of all the gains of the branches forming a loop is called loop gain.

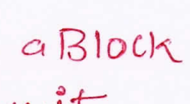
—02M—

feed back loop :



$\boxed{\text{feedback loop gain} = -G_3H_1}$

Self loop



—02M—

$\boxed{\text{Self loop gain} = G_2}$

Q4b. Find the transfer function by constructing a Block diagram & reducing the same for the circuit shown in Fig 4(b)

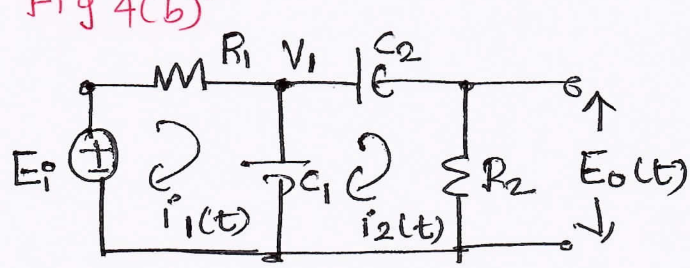
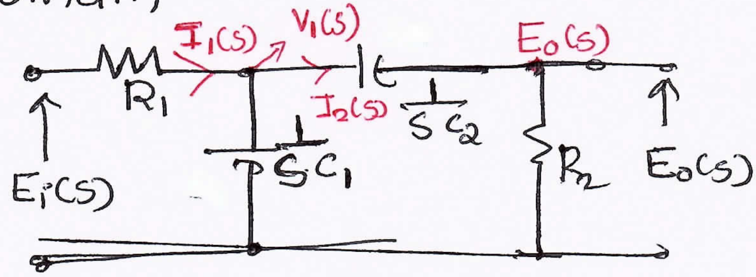


Fig 4(b)

(Vinaychitare)

Step 1 Transforming the network in Laplace domain



— 01M —

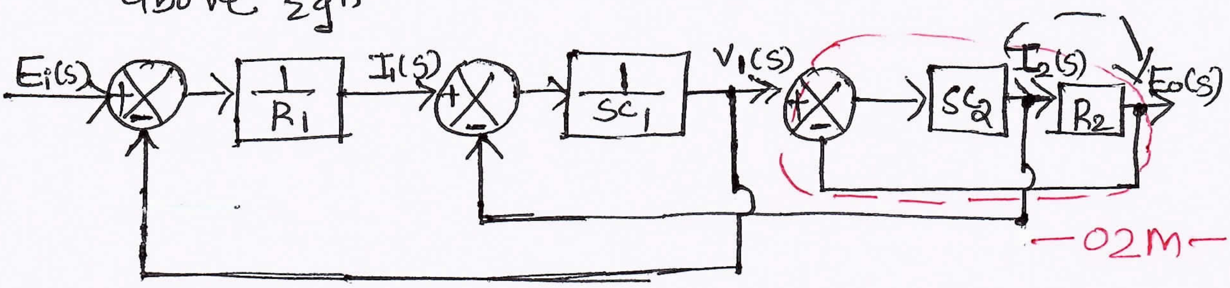
Step 2 :- writting eqn for defined variables

$$I_1(s) = \frac{E_i(s) - V_1(s)}{R_1} \quad \Big| \quad V_1(s) = \frac{(I_1(s) - I_2(s))}{sC_1}$$

— 01M —

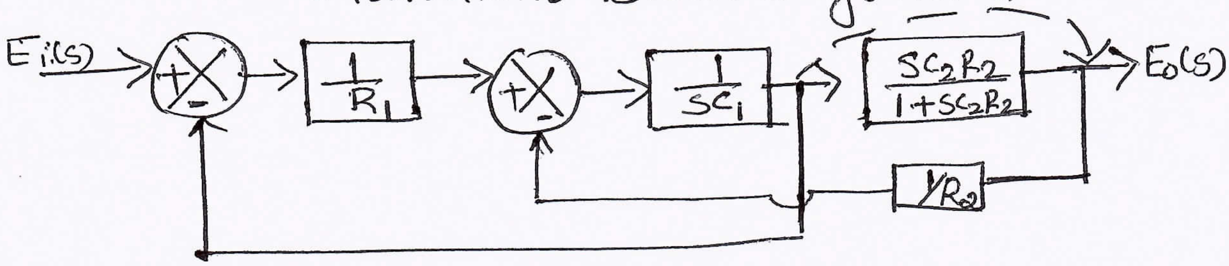
$$I_2(s) = (V_1(s) - E_0(s))\, sC_2 \quad \Big| \quad E_0(s) = I_2(s)\, R_2$$

Step 3 :- Constructing Block diagram with help of above eqn



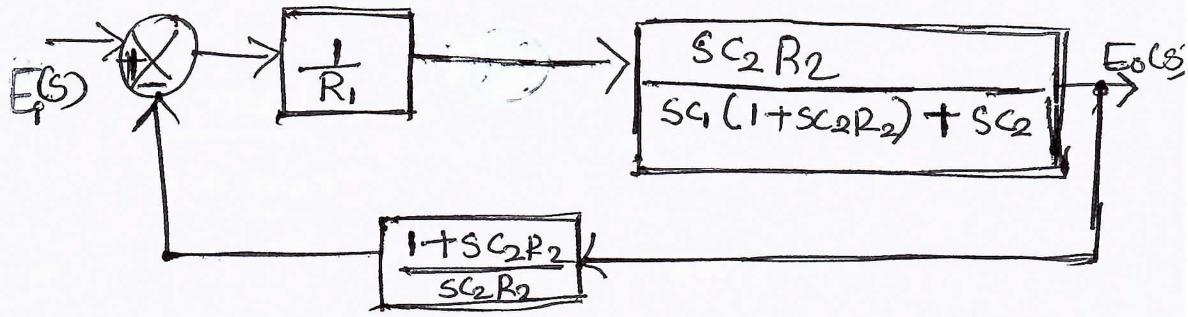
— 02M —

Step 4 :- Shifting takeoff point right side as shown in above fig & eliminating -ve feedback, the equivalent Block diagram is,



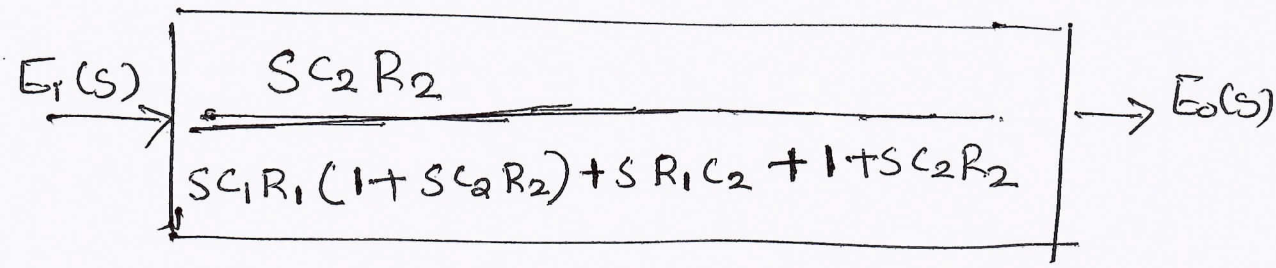
Step 5 Shifting takeoff point right side as shown in above fig, Combining blocks in cascade & Eliminating feedback after shifting

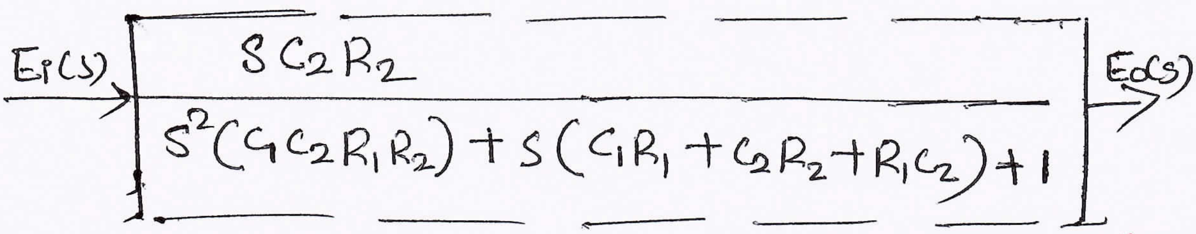
(Vinaychitore)



— 02M —

Step6 Eliminating -ve feedback





— 02M —

Q4c. Find the transfer function by constructing SFG and Mason's gain formula for the circuit shown in Fig 4(b)

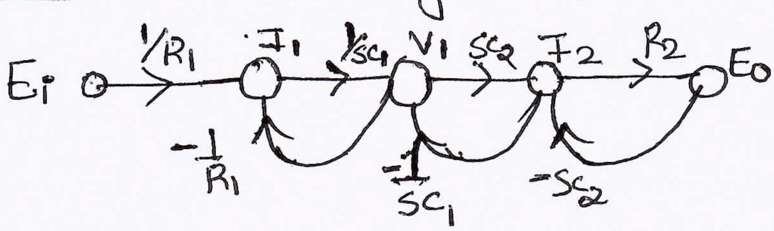
Step1 writting eqn's for defined variables

$$I_1(S) = \frac{E_i(S) - V_1(S)}{R_1} \quad \Big| \quad V_1(S) = \frac{I_1(S) - I_2(S)}{SC_1}$$

— 01M —

$$I_2(S) = (V_1(S) - E_o(S))SC_2 \quad \Big| \quad E_o(S) = I_2(S)R_2$$

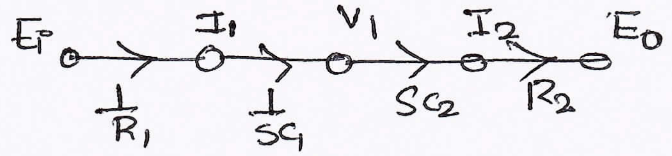
Step3 Constructing SFG from above eqn



— 01M —

(Vinaychitare)

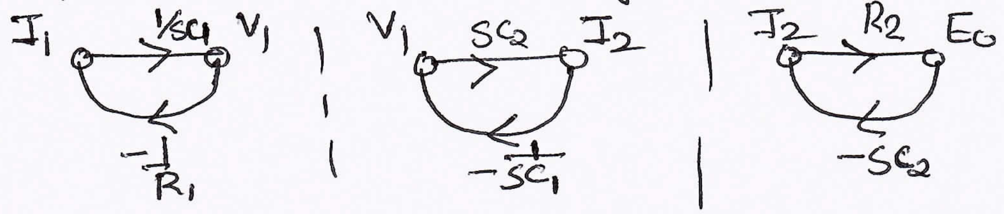
Step 3 : Identifying no. of forward path, with Path gain



—01—

$$P_1 = \frac{1}{R_1} \cdot \frac{1}{\cancel{S}C_1} \cdot \cancel{S}C_2 \cdot R_2 = \frac{C_2 R_2}{R_1 C_1}$$

step 4 : Individual loop with gain



$$L_1 = \frac{-1}{SC_1R_1} \qquad L_2 = -\frac{C_2}{C_1} \qquad L_3 = -SR_2C_2$$

—01—

steps : Nontouching loop taken two @ a time

$$L_1L_3 = \frac{\cancel{S}R_2C_2}{\cancel{S}C_1R_1} = \frac{R_2C_2}{C_1R_1}$$ —01—

* Non touching loop taken three @ a time
—NiL—

Step 6 $\Delta_1 = 1$, all loops are touching P_1 Path

—01—

Step 7 $\Delta = 1 - [L_1 + L_2 + L_3] + L_1L_3$

$$= 1 - \left[\frac{-1}{SC_1R_1} - \frac{C_2}{C_1} - SR_2C_2\right] + \frac{R_2C_2}{C_1R_1}$$

$$= 1 - \left[\frac{-C_1 - SC_1C_2R_1 - S^2R_1R_2C_2C_1^2}{(SC_1R_1)C_1}\right] + \frac{R_2C_2}{C_1R_1}$$

aychitame)

$$\Delta = 1 + \frac{C_1(1 + SC_2R_1 + S^2R_1R_2C_2C_1)}{SC_1R_1C_1} + \frac{R_2C_2}{C_1R_1}$$

Step 8 $T.F = \dfrac{\sum_{K=1}^{N} P_1 \Delta_1}{\Delta} = \dfrac{P_1 \Delta_1}{\Delta}$ $\boxed{N=1}$

$$= \frac{C_2 R_2}{R_1 C_1 \left[1 + \dfrac{\left(1 + sC_2R_1 + s^2 R_1 R_2 C_2 C_1\right)}{sC_1R_1} + \dfrac{R_2C_2}{C_1R_1}\right]}$$

$$= \frac{C_2 R_2}{R_1C_1 + \dfrac{1 + sC_2R_1 + s^2 R_1R_2C_2C_1}{s} + R_2C_2}$$

$$= \frac{sR_2C_2}{sR_1C_1 + 1 + sC_2R_1 + s^2 R_1R_2C_2C_1 + sR_2C_2}$$

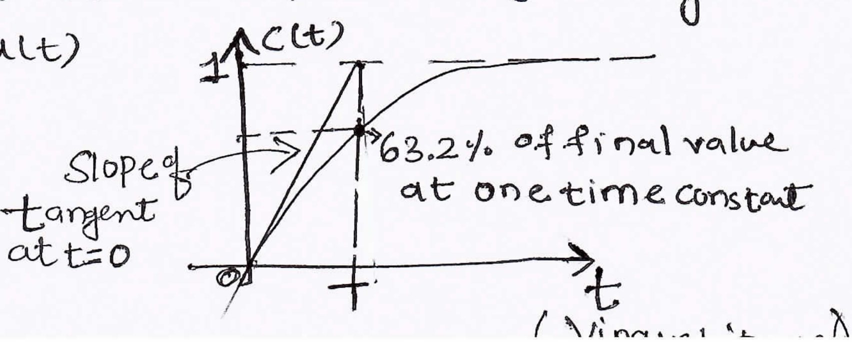
$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{sR_2C_2}{s^2(R_1R_2C_1C_2) + s(R_1C_1 + R_2C_2 + C_2R_1) + 1}} \quad -02-$$

Q5a. Obtain Expressions for Specifications namely time-Constant, risetime, and settling time of firstorder system for a unitstep input

* Unit Step response of firstorder system is given by

$c(t) = \left(1 - e^{-t/T}\right) u(t)$

* T = Time constant of response



* Slope of tangent @ $t=0$

$$\left.\frac{dc(t)}{dt}\right|_{t=0} = \left.\frac{1}{T} e^{-\frac{t}{T}}\right|_{t=0} = \frac{1}{T} = \frac{1}{\text{Time constant}}$$

* T = Time constant

* $\frac{1}{T}$ specifies speed of response

$\therefore$ $\boxed{\text{speed of response} = \frac{1}{T}}$ —01M—

* Time constant is the time it takes for the step response to rise 63% of its final value

---continued in next page--

Q5b. Derive an Expression for $c(t)$ of an underdamped Second order system for a unit step input.

Step1 : Transfer function of underdamped system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad \left[\begin{array}{c} i/p \\ u(t) \leftrightarrow \frac{1}{s} \end{array}\right]$$

o/p : $C(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \cdot \frac{1}{s}$ —01M—

Step2 Applying paratial fraction expansion

$$C(s) = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+2\zeta\omega_n s+\omega_n^2}$$ —01M—

finding K_1, K_2, K_3 by equating coefficients

$K_1 + K_2 = 0$; $2\zeta\omega_n K_1 + K_3 = 0$

$K_1\omega_n^2 = \omega_n^2$

$\boxed{K_1 = 1}$; $K_2 = -K_1$

$$\boxed{\begin{array}{l} K_1 = 1 \\ K_2 = -1 \\ K_3 = -2\zeta\omega_n \end{array}}$$ —02M—

(Vinaychitore)

*

$$C(s) = \frac{1}{s} + \frac{(-s - 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Step 3 Denominator can be written as

$$C(s) = \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{(s + \zeta\omega_n)^2 + (\omega_n^2 - \zeta^2\omega_n^2)} \rightarrow \omega^2(1-\zeta^2) = \omega_d^2$$

$$= \frac{1}{s} - \left[\frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2}\right]$$ —02M—

$$= \frac{1}{s} - \left[\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2}\right]$$

Step 4 Taking Inverse Laplace

$$c(t) = u(t) - \left[e^{-\zeta\omega_n t}\cos\omega_d t + \frac{\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t\right]u(t)$$

—01M—

$$= \left\{1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]\right\}u(t)$$

$$= \left\{1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos\omega_d t + \zeta\sin\omega_d t\right]\right\}u(t)$$

Step 5 : above Exp can be written as in the form

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

$$\sin b = \sin\theta = \sqrt{1-\zeta^2} \; ; \; \cos b = \cos\theta = \zeta$$

$$\text{OR } \theta = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

$$\therefore \boxed{c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sin\omega_d t + \theta\right]}$$ —01M—

(Vinaychitore)

5a continued

* **Risetime :** According to defn of Rise time

for $t = 0.1$

$$c(0.1) = 1 - e^{-(0.1)/T}$$

$$T_{r_1} = 0.11T$$

for $t = 0.9$

$$c(0.9) = 1 - e^{-(0.9)/T}$$

$$T_{r_2} = 2.33T$$

$$T_r = 2.33T - 0.11T$$

$$\boxed{T_r = 2.2T}$$

—1.5M—

* **Settling time :** According to defn of Settling time

for 2%

$$c(ts) = 0.98$$

$$c(ts) = 1 - e^{-ts/T} = 0.98$$

$$e^{-ts/T} = 0.02$$

$$-ts/T = \ln(0.02)$$

$$\boxed{ts = 4T}$$

—1.5M—

(Vinaychitare)

Q5c. A unity feedback system is characterized by open loop transfer function $G(s) = \frac{K}{s(s+10)}$ find the value of K so that the system will have $\zeta = 0.5$. for this value of 'K' find M_p, t_p & t_s for unit step.

Step 1: Transfer function of unity f/b system is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K}{s^2+10s+K}$$

Comparing above T.F with standard second order T.F $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\boxed{\omega_n^2 = K}$ — ①, $\boxed{2\zeta\omega_n = 10}$ — ② — 02M —

Step 2: Given $\zeta = 0.5$. from eqn ②

$\omega_n = \frac{10}{2\times 0.5} = 4$; $\boxed{\omega_n = 4\ rad/s}$ — 01M —

$\therefore \boxed{K = 4^2 = 16}$ — 01M —

Step 3 (i) $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

$\boxed{M_p = 16.3\%}$ — 02M —

(ii) $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$\boxed{t_p = 0.906s}$ — 01M —

(iii) $t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5\times 4} = 2$; $\boxed{t_s = 2s}$

— 01M

(Vinaychitare)

Q6a. Starting from the output equation c(t) derive expressions for (i) Peak time (t_p) (ii) Peak overshoot (M_p) of underdamped second order system subjected to unit step input.

Peak time: output of underdamped second order system is $c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$

* from response curve @ $t = t_p$ slope of tangent is zero, $\left.\frac{d\,c(t)}{dt}\right|_{t=t_p} = 0$

* $$\left.\frac{d\,c(t)}{dt}\right|_{@t=t_p} = 0 - \frac{e^{-\zeta\omega_n t_p}(-\zeta\omega_n)\sin(\omega_d t_p + \theta)}{\sqrt{1-\zeta^2}} - \frac{e^{-\zeta\omega_n t_p}\,\omega_d \cos(\omega_d t_p + \theta)}{\sqrt{1-\zeta^2}}$$

* $$\left.\frac{d\,c(t)}{dt}\right|_{@t=t_p} = 0 \Rightarrow \frac{\cancel{e^{-\zeta\omega_n t_p}}(-\zeta\omega_n)\sin(\omega_d t_p + \theta)}{\cancel{\sqrt{1-\zeta^2}}} = \frac{\cancel{e^{-\zeta\omega_n t_p}}}{\cancel{\sqrt{1-\zeta^2}}}\,\omega_d \cos(\omega_d t_p + \theta)$$

— 02M —

* we know that $\sin\theta = \sqrt{1-\zeta^2}$, $\cos\theta = \zeta$

$$-\zeta\omega_n \sin(\omega_d t_p + \theta) = \omega_d \cos(\omega_d t_p + \theta)$$

$$-\zeta \sin(\omega_d t_p + \theta) = \frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n} \cos(\omega_d t_p + \theta)$$

* $\cos\theta \sin(\omega_d t_p + \theta) - \sin\theta \cos(\omega_d t_p + \theta) = 0$

* $\sin(\omega_d t_p + \theta - \theta) = \sin(\omega_d t_p) = 0$

* $\omega_d t_p = \sin^{-1} 0 = n\pi$

$\omega_d t_p = n\pi$, $n = 1$ for first instant

$$\therefore \boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$ — 02M —

(Vinaychitare)

*(ii) <u>Peak overshoot</u> :- According to defn

$M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$, for unit step i/p

$C(\infty) = 1$

$\therefore M_p = \frac{C(t_p) - 1}{1}$

$\therefore C(t_p) = 1 - \frac{e^{-\zeta \omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \theta)$

$M_p = C(t_p) - 1 = -\frac{e^{-\zeta \omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \theta)$

— 02M —

But $t_p = \frac{\pi}{\omega_d}$, subn of t_p in above Expression

$M_p = -\frac{e^{-\frac{\zeta \omega_n \pi}{\omega_n \sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d \frac{\pi}{\omega_d} + \theta\right)$

$= -\frac{e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \sin(\pi + \theta)$ | $\sin(\pi + \theta) = -\sin\theta$ | $\sin\theta = \sqrt{1-\zeta^2}$

$M_p = \frac{e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}}}{\cancel{\sqrt{1-\zeta^2}}} (\cancel{\sqrt{1-\zeta^2}})$

$M_p = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}}$ | $\boxed{M_p = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \times 100}$

— 02M —

(Vinaychitare)

Q6b

A unity feedback system has $G(s) = \frac{K}{s(s+2)(s^2+2s+5)}$

(i) for a unit ramp input it is desired $e_{ss} \leq 0.2$. Find K.

(ii) Find e_{ss} if $r(t) = 2 + 4t + \frac{t^2}{2}$

Step 1: for Ramp i/p $e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{s \to 0} (s\,G(s))$$

$$\therefore K_v = \lim_{s \to 0} \frac{K\cancel{s}}{\cancel{s}(s+2)(s^2+2s+5)} = \frac{K}{(2)(5)} = \frac{K}{10}$$

—02M—

$$\boxed{e_{ss} = \frac{1}{K_v} = 10/K}$$

Step 2: ∵ According to Given condn

$$e_{ss} \leq 0.2 \quad \Big| \quad \frac{10}{K} \leq 0.2 \quad \Big| \quad \boxed{K \geq 50}$$

—02M—

(ii) $K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{K}{s(s+2)(s^2+2s+5)} = \infty$

Step 1:

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{Ks}{s(s+2)(s^2+2s+5)} = \frac{50}{10} = 5$$

$$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} \frac{Ks^{\cancel{2}}}{\cancel{s}(s+2)(s^2+2s+5)} = 0$$

—02M—

Step 2:

for Given i/p $e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a}$

$$= \frac{1}{1+\infty} + \frac{4}{5} + \frac{1}{0}$$

$$= 0 + 4/5 + \infty$$

—02M—

$$\boxed{e_{ss} = \infty}$$

(Vinaychitare)

Q6C. Write a short note on PID Controllers

* Proportional-Integral-Derivative Controller (PID)
* The controller with PID control action the relation b/w o/p of controller & actuating error signal is given by

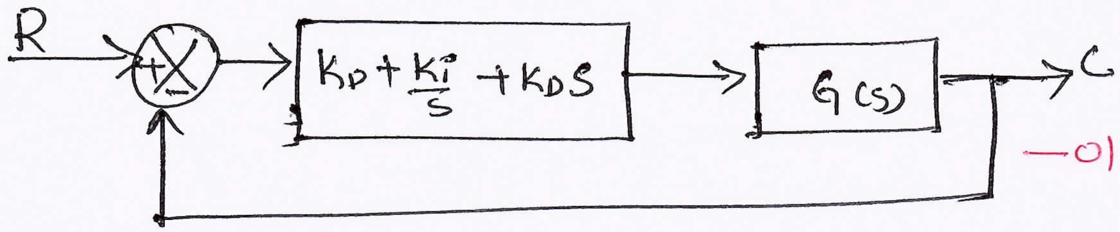
$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_D \frac{de(t)}{dt}$$

Laplace domain of above eqn

$$U(s) = K_p E(s) + \frac{K_i}{s} E(s) + K_D s E(s)$$

$$\boxed{\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_D s}$$ — 03M —

PID Controller is represented by Block with Block gain as $K_p + \frac{K_i}{s} + K_D s$



—01M—

Q7a. Define stability and hence, stable, unstable, marginally stable and conditional stablity of a Unity feedback system.

* Stability in a system implies that small change in the system input, initial conditions or in system parameters; do not result in large change in system output.

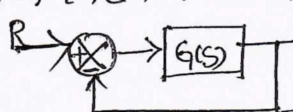
— 01M —

(Vinaychitare)

* A system is stable if for every Bounded input signal the system response is Bounded
* A system in unstable if for any bounded i/p the system response is Unbounded — 02M —

* A system is marginally stable if the tends to Oscillate about its equilibrium state sy subjected to initial condition. — 01M —

* If a system output is stable for limited range of variation of its parameter, then the system is called conditionally stable —01M—



C.E = $1+G(s)=0$ —02M—

* If roots of C.E are in LHP → stable
* If any roots of C.E are in RHP → unstable
* If root C.E are in LHP for range of 'k' → conditionally stable

| * If roots of C.E are on jω axis, nonrepeated marginally stable

Q7b. In Unity feed back system find the range of 'k' for stability and K_{mar}, ω_{mar} with $G(s) = \dfrac{K}{s(1+0.4s)(1+0.25s)}$

Step 1 Characteristic Eqn, $1+G(s)H(s)=0$

$$1+\frac{K}{s(1+0.4s)(1+0.25s)}=0$$

$$\therefore\ 0.1s^3+0.65s^2+s+K=0 \quad \text{— 02M —}$$

Step 2 R-H table

s^3	0.1	1
s^2	0.65	K
s^1	$\dfrac{0.65-0.1K}{0.65}$	0
s^0	K	

for system to be stable

$K>0$ -- from s^0

from s^1

$$\frac{0.65-0.1K}{0.65}>0 \quad \text{—02M—}$$

$$K<\frac{0.65}{0.1} \quad \boxed{K<6.5}$$

(Vinaychitare)

* Range of 'K' is $0 < K < 6.5$

* for K_{max} we need row of zero, from s^1.

$$\frac{0.65 - 0.1K}{0.65} = 0$$

$$\boxed{K = K_{max} = 6.5}$$

—01M—

* $A(s) = 0.65s^2 + 6.5 = 0$

$\therefore 0.65s^2 + 6.5 = 0$

$s^2 = -10$

$s^2 = \pm j\sqrt{10} = \pm j3.162$

$\therefore s \pm j3.162$

$\boxed{\omega = 3.162 \text{ rad/s}}$ -- frequency of oscillation.

—01M—

Q7C Prove that part of root loci is circle using angle condition and find the center as well as radius when $G(s)H(s) = \frac{K(s+2)}{s(s+1)}$

*Step1 Angle condition is $\angle G(s)H(s) = \pm 180^\circ(2q+1)$

* let $S_0 = \alpha + j\beta$ be point on circle

$$G(s)H(s)\Big|_{s=s_0} = \frac{K(\alpha + j\beta + 2)}{(\alpha + j\beta)(\alpha + j\beta + 1)}$$ —01M—

* if this point on root locus it should satisfy angle condition

$$\angle G(s)H(s)\Big|_{s=s_0} = \frac{\tan^{-1}\left(\frac{\beta}{\alpha+2}\right)}{\tan^{-1}\left(\frac{\beta}{\alpha}\right) + \tan^{-1}\left(\frac{\beta}{\alpha+1}\right)} = 180^\circ$$ —01M—

Step2 from trig identity $\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left[\frac{A \pm B}{1 \mp AB}\right]$

—01M—

(Vinaychitare)

$$* \quad \frac{\tan^{-1}\left[\frac{\beta}{\alpha+2}\right]}{\tan^{-1}\left[\frac{\frac{\beta}{\alpha}+\frac{\beta}{\alpha+1}}{1-\frac{\beta^2}{\alpha^2+\alpha}}\right]} = 180 \quad -①-$$

$$* \quad \frac{\tan^{-1}\left[\frac{\beta}{\alpha+2}\right]}{\tan^{-1}\left[\frac{\beta\alpha+\beta+\beta\alpha}{\alpha^2+\alpha-\beta^2}\right]} = 180 \quad -②-$$

$$\tan^{-1}\left[\frac{\beta}{\alpha+2}\right] - \tan^{-1}\left[\frac{\beta+2\alpha\beta}{\alpha^2+\alpha-\beta^2}\right] = 180^{\circ} \quad -③$$

* Taking tangent on both side we get

$$\frac{\beta}{\alpha+2} - \frac{\beta+2\alpha\beta}{\alpha^2+\alpha-\beta^2} = 0 \quad \text{— 02M —}$$

$$\alpha^2\beta + \alpha\beta - \beta^3 - [\alpha\beta + 2\alpha^2\beta + 2\beta + 4\alpha\beta] = 0$$

$$\alpha^2 + \cancel{\alpha} - \beta^2 - \cancel{\alpha} - 2\alpha^2 - 2 - 4\alpha = 0$$

$$-\alpha^2 - \beta^2 - 4\alpha - 2 = 0$$

$$\alpha^2 + \beta^2 + 4\alpha + 2 = 0 \quad \text{— 01M —}$$

* The above eqn can be written as

$$\alpha^2 + \beta^2 + 4\alpha + 2 + 2 - 2 = 0$$

$$|\alpha^2 + 4\alpha + 4| + \beta^2 - 2 = 0$$

$$\boxed{(\alpha+2)^2 + \beta^2 = (\sqrt{2})^2}$$

* The above is eqn of cle with center ~~radius~~ $(0, -2)$ & radius $\sqrt{2}$ // —— 02M —

(Vinaychitore)

Q8a A -ve feedback control is characterised by $G(s) = \frac{K}{s(s+\alpha)}$, $H(s) = 1$. Find values of K and α so that $M_r = 1.04$ and $\omega_r = 11.55$ rad/s

Step1: Closed loop T.F $= \frac{G(s)}{1+G(s)} = \frac{K}{s^2+as+K}$

Comparing with standard T.F $s^2+2\zeta\omega_n s+\omega_n^2$

$\omega_n^2 = K$ — ① | $2\zeta\omega_n = a$ — ②

Step2: We have correlation between frequency resp spec and time resp spec

$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.04$

$(2\zeta)^2(1-\zeta^2) = \left(\frac{1}{1.04}\right)^2$

$\zeta^2(1-\zeta^2) = 0.23$ — ③

let $\zeta^2 = x$, subⁿ of ζ^2 in eqⁿ ③

$-x^2+x-0.23 = 0$

$x = 0.35, 0.64$

$\zeta^2 = x = 0.35, 0.64$

$\boxed{\zeta = 0.6, 0.8}$

* We known that M_r does not exist for $\zeta > 0.707$

$\boxed{\zeta = 0.6}$ — 02M —

* $\omega_r = \omega_n\sqrt{1-\zeta^2}$ | $\boxed{\omega_n = 21.8 \text{ rad/s}}$

* subⁿ of ζ & ω_n in eqⁿ ① & ②

$\omega_n^2 = (21.8)^2 = K$ | $\boxed{K = 475}$ — 01M —

$a = 2 \times 0.6 \times 21.8$ | $\boxed{a = 26.16}$ — 01M —

(Vinay Chitare)

Q8b. Using RH criterion determine the stability of the system, the system is type one system with error constant of 10 sec^{-1} and pole at $S=-3$ and $S=-6$.

* **Step1** : let us find open loop transfer function with Given information

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

Type1 : Pole @ origin s^1 —01M—

* for type one system K_v exist (finite value) $[K_p=\infty, K_a=0]$ —01M—

$$K_v = \lim_{s\to 0}\left[s\,G(s)\right]$$

$$= \lim_{s\to 0} \frac{\cancel{s}K}{\cancel{s}(s+3)(s+6)} = \frac{K}{(3)(6)}$$

∴ Given that $K_v = 10$

∴ $\frac{K}{18} = 10$ | $\boxed{K=180}$ —01M—

Step2 : C.E with obtained open loop T.F

$$C.E = 1 + G(s) = 0$$

$$\therefore 1 + \frac{180}{s(s+3)(s+6)} = 0$$

$$\therefore (s^2+3s)(s+6) + 180 = 0$$

$$s^3+6s^2+3s^2+18s+180=0$$

$$s^3+9s^2+18s+180=0 \quad \text{—01—}$$

RH Table

s^3	1	18
s^2	9	180
s^1	-18	0
s^0	180	0

No sign Change

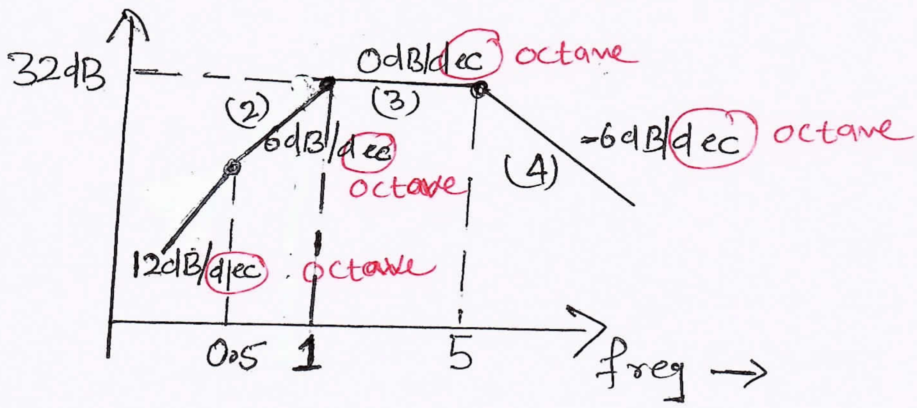
sign Change

sign Change

—01M—

* from RH table we observe that two sign change which implies that two roots are in RHP therefore system is Unstable —01M— (Vinayshitha...)

Q8C. Find transfer function for magnitude plot



Note: from above graph, slope changes; when frequency double, so it is 12dB/octave, 6dB/octave, 0dB/octave & -6dB/octave

Step1: Intial slope is 12dB/octave interms of decade it is 40dB/decade, which implies that there two zeros @ origin.

Step2 Eqn of line after $\omega = 0.5$ is

$M = 20\log\omega + C$ | $32 = 20\cancel{\log 1} + C$ ($\log 1 = 0$)

@ $\omega = 1$, $M = 32dB$ ∴ | ∴ $\boxed{C = 32}$ — 02M —

@ $\omega = 0.5$, $M = 20\log 0.5 + 32$ } $\omega = 0.5$ is also on initial line of slope 40dB/decade

$\boxed{M = 26dB}$ — 01M —

Step3 Eqn of line with slope of 40dB/decade

$M = 40\log\omega + C_1$ | for $K = 1$, initial line would have $M = 0dB$ @ $\omega = 1$, But in given graph, initial line @ $\omega = 1$, $M = 40\log 1 + 38.0412$

@ $\omega = 0.5$ $26 = 40\log 0.5 + C_1$

— 01M — $\boxed{C_1 = 38.0412dB}$

$\boxed{M = 38.0412}$ — 01M —

* This is due to contribution of K ($M = 38.0412$)

* $20\log K = 38.0412$

$\boxed{K = 79.8}$ —01M—

* Step 4 at $\omega = \omega_{c_1} = 0.5$, Slope changes by -20 (6dB/octave = 20dB/decade)

this implies there is Simple pole factor

$\frac{1}{(1+T_1 s)}$, where $T_1 = \frac{1}{\omega_{c_1}} = \frac{1}{0.5} = 2.$

$\therefore \boxed{\frac{1}{(1+2s)}}$ —01M—

* at $\omega = \omega_{c_2} = 1$, slope changes by -20

this implies there is Simple Pole factor

$\frac{1}{1+T_2 s}$, where $T_2 = \frac{1}{\omega_{c_2}} = 1$

$\therefore \boxed{\frac{1}{1+s}}$ —01M—

* at $\omega = \omega_{c_3} = 5$, slope changes by -20,

this implies there is Simple pole factor

$\frac{1}{1+T_3 s}$, where $T_3 = \frac{1}{\omega_{c_3}} = \frac{1}{5} = 0.2$

$\boxed{\frac{1}{1+0.2s}}$ —01M—

* The Transfer function with above obtained information

$$\boxed{T(s) = \frac{79.8\, s^2}{(1+2s)(1+s)(1+0.2s)}}$$ —01M—

(Vinaychitore)

Q9a. Compare transfer function method and state space approach in control system.

Transfer function	State Space
1) It is applicable for LTI system, it is generally limited SISO system	1) It is applicable for Linear, Nonlinear, Time varying Time invariant, SISO, MIMO wide range of system —01M—
2) It is frequency domain approach	2) It is time domain approach —01M—
3) In Transfer function method, initial conditions are considered to be zero	3) Initial condition are considered in state space approach —01M—
4) Transfer function provide rapidly the information about stability & transient response (calculation are faster)	4) State space approach require lot of calculation befor the physical interpretation of the model is apparent. —01M—

Q9b. Find stability and range of K using Nyquist plot

$$G(s)H(s) = \frac{K(s+1)}{s(s-1)}$$

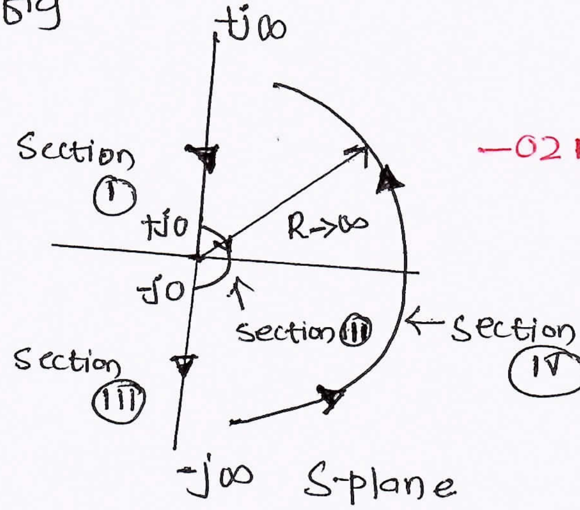
<u>Step1</u>:- P=1 as one open loop in is RHP of s-plane

Step2 N = −P = −1 is the criterion for stability that Nyquist plot must encircle −1+j0 point once in clockwise direction for stability

—01M—

(Vinaychitore

* **Step 3:** One pole at origin, hence Nyquist path is as shown in below fig



—02 M—

* **Step 4:** $s = j\omega$ in $G(s)H(s)$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega)}{j\omega(j\omega-1)}$$

* **Section I:** $s = +j\infty \rightarrow s = +j0$

Starting point	$\omega \to +\infty$	$0\angle\frac{0^\circ\ 90^\circ}{90^\circ\ 90^\circ} = 0\angle -90^\circ$
Terminating point	$\omega \to +0$	$\infty\angle\frac{0^\circ\cdot 0^\circ}{90^\circ\ 180^\circ} = \infty\angle -270^\circ$

$-270^\circ - (-90^\circ)$
$= -270 + 90^\circ$
$= -180^\circ$
Clockwise rotation

—01M—

* **Section II:** $s = +j0 \rightarrow s = -j0$

Starting point	$\omega \to +0$	$\infty\angle -270$
Terminating point	$\omega \to -0$	$\infty\angle\frac{0\cdot 0}{-90+180} = \infty\angle -90^\circ$

$-90 - (-270^\circ)$
$+180^\circ$
anticlockwise rotation

—01M—

* **Section III:** Mirror image of section I about real axis.

* **Step 5:** Intersection with -ve real axis

Rationalizing $G(j\omega)H(j\omega)$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega)\ (+j\omega+1)(-j\omega)}{j\omega(j\omega-1)(-j\omega)(+j\omega+1)}$$

$$= \frac{-Kj\omega(1-\omega^2+2j\omega)}{\omega^2(-\omega^2-1)} = \frac{+K2\omega^2}{\omega^2(-\omega^2-1)} - \frac{Kj\omega(1-\omega^2)}{\omega^2(-\omega^2-1)}$$

$$= \frac{-K2}{(\omega^2+1)} + \frac{Kj\omega(1-\omega^2)}{\omega^2(\omega^2+1)}$$

(Vinaychitoral

* equating Img part to zero

$$\frac{Kj\omega(1-\omega^2)}{\omega^2(\omega^2+1)} = 0$$

$1-\omega^2 = 0$ | $\omega^2 = 1$ | $\boxed{\omega = 1\,rad/s}$

sub^n of $\omega = 1$ in real part

$$Q = \frac{-K2}{(1+1)} = \frac{-\cancel{2}K}{\cancel{2}}$$ | $\boxed{Q = -K}$ —02M—

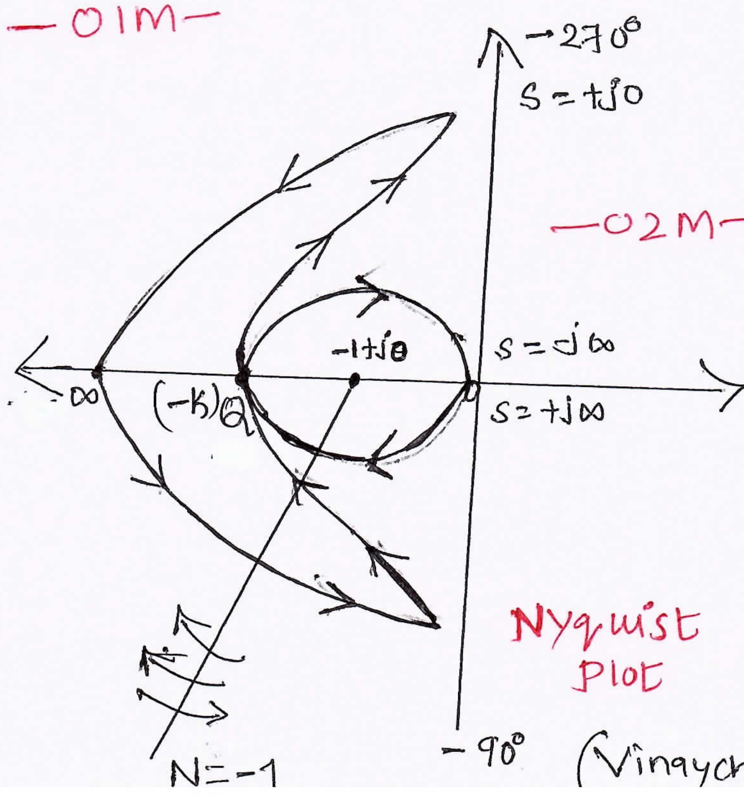
* step 6 : Nyquist plot is shown in below fig.

* step 7 : Net encirclements are $N = -1$ if the critical point $-1+j0$ is lying to the right of point Q. so for stability

$|OQ| > |-1+j0|$

$K > 1$

* Hence the range of values of K for stability is $1 < K < \infty$ —01M—

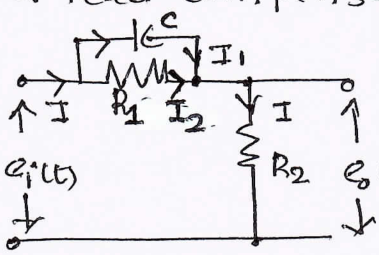


Nyquist plot

(Vinaychitore)

Q9c. Write short note on lead, Lag, Lead lag compensators

* **lead compensator:-** An electric network which acts as a lead compensator is shown in below fig

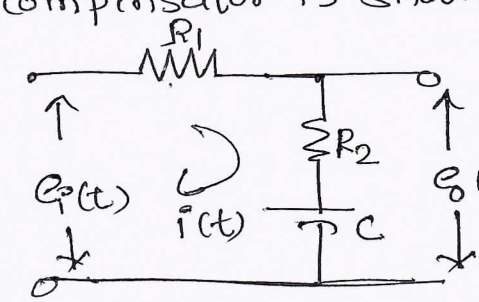


$$T.F = \frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)}$$

* where $T = R_1C$ and $\alpha = \frac{R_2}{R_1+R_2} < 1$
* zeros at $s = -\frac{1}{T}$ * polse at $s = -\frac{1}{\alpha T}$
* Effects of lead compensation
  * It improves the phase margin of the closed loop system
  * The steady state error does not get effected
* Limitations of lead compensation
  * The compensated system may have a large undershoot than overshoot. So tendency to over compensate system may lead to conditionally stable system.
  * the maximum lead angle available from a single lead network is about 60°. Thus if lead of more than 70° to 90° is required a multistage lead compensators are required.

— 02M —

* **Lag compensator:-** An electric network which acts as a lag compensator is shown in below fig



$$T.F = \frac{E_o(s)}{E_i(s)} = \frac{1}{\beta}\frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)}$$

(Vinaychitore)

* where $T = R_2 C$, $\beta = \frac{R_1+R_2}{R_2} > 1$

* It has zeros @ $s = -\frac{1}{T}$ and poles at $s = \frac{-1}{\beta T}$

* Effects of Lag compensator

* It is basically low pass filter. Hence it improves the steady state performance

* The attenuation characteristic is used for the compensation, it shifts gain crossover frequency to a lower frequency point. Thus BW gets reduced.

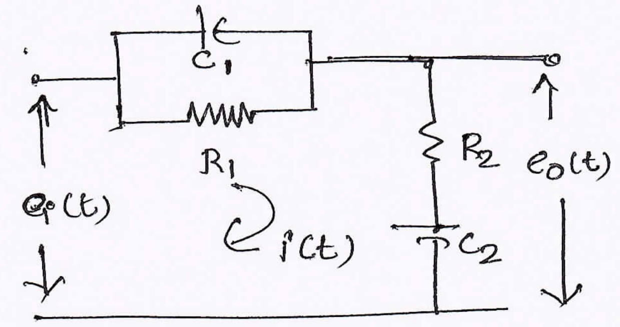
* Limitation of Lag compensator

  * The system become more sensitive to the parameter variations

  * Due to reduced bandwidth, rise time, settling time are usually longer. The transient respons lasts for longer time.

— 02M —

* Lag-lead compensator : A lag-lead compensator is combination of lag and lead compensators. An electrical network which acts as lag-lead compensator is shown in below fig



$$\frac{E_o(s)}{E_i(s)} = \frac{(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_1}{\beta} s\right)(1+T_2 \beta s)}$$

* where $T_1 = R_1 C_1$, $T_2 = R_2 C_2$, $\alpha\beta = 1$

(Vinaychitare)

* Phase lead portion involving T_1, adds phase lead angle

* Phase lag portion involving T_2, adds attenuation near and above gain crossove frequency

* poles are at $s = \frac{-\beta}{T_1}, \frac{-1}{\beta T_2}$

* Zeros are at $s = \frac{-1}{T_1}, \frac{-1}{T_2}$

* Effects of leag-lead Compensator

  * lag-lead compensator is used when both fast response and good static accuracy are desired

  * It increases bandwidth of system making system response very fast.

— 02M —

Q10a. Define state, state variable, state space

* state :
The state of a system is a set of variables (state-variables) so that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, determines the behavior of the system for any time $t \geq t_0$.

—02M—

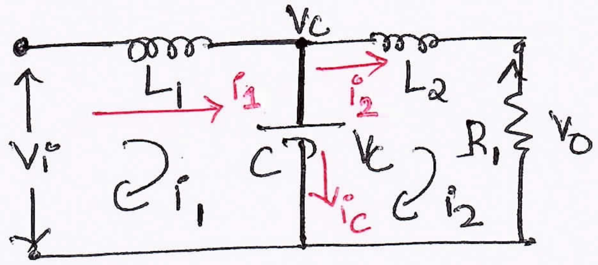
* State variables : The state variables of a system are the variables making up the smallest set of variables that determine the state of the system.

—01M—

* State Space : The n-dimensional space whose coordinates axes consist of the x_1 axis, x_2 axis, x_3 axis, — — → x_n axis, where $x_1, x_2, \ldots, x_n$ are state variables, is called a state space.

—01M—

Q10b. Obtain the state equation for the electrical network shown in fig 10b.



Step1 : Label all branch current in the given n/w.

Step2 : writing derivative eqn for storage elements

$$C \frac{dV_c}{dt} = i_c \quad -③-$$

$$L_1 \frac{di_1}{dt} = V_{L_1} \quad --①$$

$$L_2 \frac{di_2}{dt} = V_{L_2} \quad -②$$

—02M—

(Vinaychitore)

* State variable are i_{L_1}, i_{L_2}, V_c

* State eqn is linear combn of state variable & i/p
So writting i_c, V_{L_1}, V_{L_2} as linear combn of i_1 i_2, and V_c

Step 3 from given n/w KCL @ node 2

* $\boxed{i_c = i_1 - i_2}$ — (4) —01m—

* KVL @ Loop 1

$-V_i + V_{L_1} + V_c = 0$

$\boxed{V_{L_1} = V_i - V_c}$ — (5) —01m—

* KVL @ loop 2

$-V_c + V_{L_2} + V_o = 0$ | But $V_o = R_1 i_2$

$V_{L_2} = V_o - V_c$

$\boxed{V_{L_2} = V_c - i_2 R_1}$ — (6) —01m—

* Subn of 4, 5, & 6 into eqn (1), (2) & (3)

$L_1 \frac{di_1}{dt} = V_i - V_c$ — (7) — | $L_2 \frac{di_2}{dt} = V_c - i_2 R_1$ — (8) —

$c \frac{dV_c}{dt} = i_1 - i_2$ — (9) —

Step 4 :- output eqn

$\boxed{V_o = i_2 R_1}$ — (10)

$\frac{di_1}{dt} = \frac{V_i}{L_1} - \frac{V_c}{L_1}$ — (7a) | $\frac{di_2}{dt} = \frac{V_c}{L_2} - \frac{i_2 R_1}{L_2}$ — (8a) —

$\frac{dV_c}{dt} = \frac{i_1}{c} - \frac{i_2}{c}$ — (9a) —

(Vinaychitare)

* Representing Eqⁿ 7a, 8a, 9a, (10) in vector matrix form

$$\underbrace{\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{V}_C \end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix} 0 & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_1}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}}_{B} \underbrace{V_i}_{u}$$

-- State Eqⁿ -

— 02M —

$$\underbrace{V_O}_{y} = \underbrace{\begin{bmatrix} 0 & R_1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix}}_{X}$$

-- output Eqⁿ

— 01M —

Q10c A system is given by the following vector matrix equation write $\phi(t)$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

Step1 : let us find state transition matrix using Laplace transform method

$$\phi(t) = \mathcal{L}^{-1}\left[(SI-A)^{-1}\right]$$

* let us find $SI = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix}$

$$SI-A = \begin{bmatrix} S & -1 \\ 4 & S+5 \end{bmatrix}$$

Step2 $[SI-A]^{-1} = \dfrac{Adj[SI-A]}{|SI-A|}$

$$Adj[SI-A] = \begin{bmatrix} S+5 & 1 \\ -4 & S \end{bmatrix}$$

$$|SI-A| = S(S+5)-(-1)(4)$$
$$= S^2+5S+4$$
$$= (S+4)(S+1)$$

— 02M —

(Vinaychitore)

$$[SI-A]^{-1} = \frac{\begin{bmatrix} S+5 & 1 \\ -4 & S \end{bmatrix}}{(S+4)(S+1)} = \begin{bmatrix} \frac{S+5}{(S+4)(S+1)} & \frac{1}{(S+4)(S+1)} \\ \frac{-4}{(S+4)(S+1)} & \frac{S}{(S+4)(S+1)} \end{bmatrix}$$

Step 3 $\phi(t) = \mathcal{L}^{-1} \begin{bmatrix} \underbrace{\frac{S+5}{(S+4)(S+1)}}_{\phi_1(S)} & \underbrace{\frac{1}{(S+4)(S+1)}}_{\phi_2(S)} \\ \underbrace{\frac{-4}{(S+4)(S+1)}}_{\phi_3(S)} & \underbrace{\frac{S}{(S+4)(S+1)}}_{\phi_4(S)} \end{bmatrix}$ — 01M —

$\phi_1(t) = \mathcal{L}^{-1}\{\phi_1(S)\}$ | $\phi_2(t) = \mathcal{L}^{-1}\{\phi_2(S)\}$ | $\phi_3(t) = \mathcal{L}^{-1}\{\phi_3(S)\}$

$\phi_4(t) = \mathcal{L}^{-1}\{\phi_4(S)\}$

$\phi_1(S) = \frac{k_1}{S+4} + \frac{k_2}{S+1}$ | $k_1 = \frac{-4+5}{-4+1} = \frac{1}{-3}$ | $\boxed{k_1 = -1/3 \quad k_2 = 4/3}$

$k_2 = \frac{-1+5}{-1+4} = 4/3$

$$\boxed{\mathcal{L}^{-1}\phi_1(S) = -\frac{1}{3}e^{-4t} + \frac{4}{3}e^{-t} = \phi_1(t)}$$ — 01M —

$\phi_2(S) = \frac{k_1}{S+4} + \frac{k_2}{S+1}$ | $k_1 = \frac{1}{-4+1} = \frac{1}{-3}$ | $k_2 = \frac{1}{-1+4}$

$\boxed{k_1 = -1/3}$ $\boxed{k_2 = \frac{1}{3}}$

$$\boxed{\mathcal{L}^{-1}\phi_2(S) = -\frac{1}{3}e^{-4t} + \frac{1}{3}e^{-t} = \phi_2(t)}$$ — 01M —

$\phi_3(S) = \frac{k_1}{S+4} + \frac{k_2}{S+1}$ | $k_1 = \frac{-4}{-4+1} = 4/3$ | $k_2 = \frac{-4}{-1+4}$

$\boxed{k_1 = 4/3}$ | $\boxed{k_2 = -4/3}$

— 01M —

$$\boxed{\phi_3(t) = \mathcal{L}^{-1}\phi_3(S) = \frac{4}{3}e^{-4t} - \frac{4}{3}e^{-t}}$$

(Vinaychitore)

* $\phi_4(s) = \dfrac{K_1}{(s+4)} + \dfrac{K_2}{s+1}$

$K_1 = \dfrac{-4}{-4+1} = 4/3$

—01M—

$K_2 = \dfrac{-1}{-1+4} = \dfrac{-1}{3}$

$$\boxed{\phi_4(t) = \mathcal{L}^{-1}\{\phi_4(s)\} = \frac{4}{3}e^{-4t} - \frac{1}{3}e^{-t}}$$

$$\Phi(t) = \begin{bmatrix} -\frac{1}{3}e^{-4t} + \frac{4}{3}e^{-t} & -\frac{1}{3}e^{-4t} + \frac{1}{3}e^{-t} \\ \frac{4}{3}e^{-4t} - \frac{4}{3}e^{-t} & \frac{4}{3}e^{-4t} - \frac{1}{3}e^{-t} \end{bmatrix}$$

—01M—

(Vinaychitore)