Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

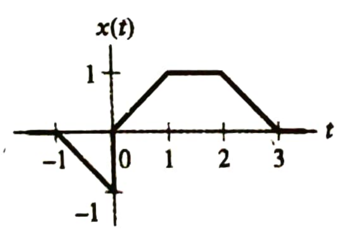
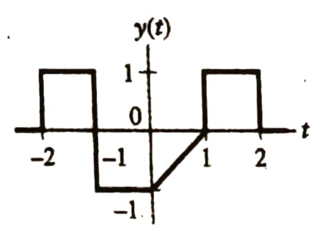
USN | | | | | | | | | | |

Fourth Semester B.E. Degree Examination

Signals and Systems

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02. Short forms used take usual meaning.
03. Missing data may be suitably assumed.

		Module -1	*Bloom's Taxonomy Level	Marks
Q.01	a	Given the signal $x[n] = (8-n)(u[n] - u[n-7])$, sketch the following signals: (i) $y[n] = x[4-n]$ (ii) $g[n] = x[-2n+3]$	L2	4 Marks
	b	Calculate the Energy and Power of the following signals: (i) $x(t) = e^{-0.05t}[u(t) - u(t-10)]$ (ii) $x[n] = u[n]$ (iii) $x(t) = 5\cos(200\pi t)$ (iv) $x[n] = (-2)^n[u(n+1) - u(n-2)]$	L3	10 Marks
	c	Find whether the following signals are periodic or not. If periodic, find the fundamental period. (i) $x(t) = \sin^2(400\pi t)$ (ii) $x(t) = \cos(2t) + \sin(3t)$ (iii) $x(t) = \sin(4\pi t) + \sin(5t)$	L2	6 Marks
		OR		
Q.02	a	Fig. Q2(a) shows two signals x(t) and y(t). Sketch the following signals. (i) $x(t)\,y(t-1)$ (ii) $x(t+1)\,y(t-2)$ (iii) $x(t)\,y(-1-t)$ (iv) $x(4-t)\,y(t)$ Fig. Q2(a)-(i) Fig. Q2(a)-(ii)	L3	10 Marks
	b	Evaluate the expression, $\int_1^2 t^2\,\delta(2t-3)dt + \int_{-3}^{3}\delta(3t+5)\,dt$	L2	4 Marks
	c	Given the signal, $x(t) = r(t+5) - r(t+4) - r(t-4) + r(t-5)$, sketch x(t) and its derivative.	L3	6 Marks
		Module-2		
Q. 03	a	Determine whether the following system represented by input-output relation, is (i) Stable (ii) Memoryless (iii) Causal (iv) Time-invariant and (v) Linear: $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$	L2	6 Marks

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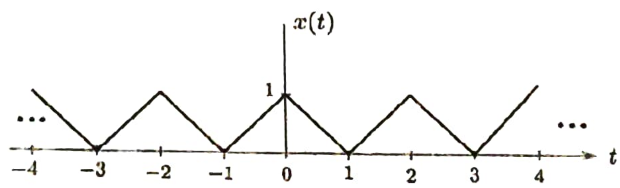
Scheme & Solution Prepared by, Prof. SURAJ KADLI.

	b	Following represents the input-output relations of various systems. Determine whether they are invertible or not: (i) $y[n] = n\,x[n]$ (ii) $y(t) = x(2t+3)$ (iii) $y[n] = \sum_{k=-\infty}^{n} x[k]$ If Invertible, represent the Inverse system.	L2	6 Marks
	c	Let $x[n] = u[n] - u[n-5]$, be the input signal applied to a Linear and Time-Invariant (LTI) discrete-time system and $h[n] = \alpha^n\,(u[n] - u[n-7])$, be the impulse response of the system. Obtain the output signal, y[n].	L3	8 Marks
		OR		
Q.04	a	Perform convolution operation on the following signals: $x_1(t) = e^{-\lvert t-2\rvert}$ and $x_2(t) = e^{-2t}u(t+4)$	L3	8 Marks
	b	Given $x[n] = \{1, 2, 3, -2\}$, here sample-value 2 appears at time-origin, and $h[n] = \delta[n+1] - 2\delta[n] + 3\delta[n-1]$. Obtain y[n] = x[n]*h[n].	L3	4 Marks
	c	Determine whether the following systems represented by input-output relations are stable and causal: (i) $y[n] = x[n]\,u[n+1]$ (ii) $y(t) = x(\sin t)$ (iii) $y[n] = \cos(x[n])$ (iv) $y(t) = t\,x(t)$	L2	8 Marks
		Module-3		
Q. 05	a	Given the impulse response, h[n], of an LTI system, obtain the condition which needs to be satisfied by h[n], for the system to be (i) causal (ii) stable.	L2	4 Marks
	b	The signal, $h(t) = u(t) - 2\,u(t-1) + u(t-2)$, represents impulse response of an LTI system. Obtain the step response of the system.	L3	6 Marks
	c	Find the complex Fourier Series coefficients X(k) for the waveform shown in the Fig.Q5(c). Sketch magnitude and phase spectra.  Fig. Q5(c)	L3	10 Marks
		OR		
Q. 06	a	State and prove the following properties of Continuous-Time Fourier Series: (i) Frequency shifting (ii) Time differentiation	L2	6 Marks
	b	For a periodic signal x(t), magnitude and phase spectral sample values are given below in the table Q6(b). They take zero values for other values of k. Determine the signal x(t), whose fundamental frequency, $\omega_0 = \pi$ rad/sec.	L3	6 Marks
	c	Determine whether the following systems represented by impulse responses are stable and causal: (i) $h[n] = u[n-1] - u[n-5]$ (ii) $h[n] = 0.5^{\lvert n\rvert}$ (iii) $h(t) = e^{-t}u(-t)$ (iv) $h(t) = u(t-1)$	L2	8 Marks

k	-2	-1	0	1	2
|X(k)|	2	1	1	1	2
θ(k)	-π/8	π/4	0	π/4	π/8

Table Q6(b)

		Module-4		
Q. 07	a	Show that for a real-valued aperiodic signal x(t), the real part of its Fourier transform is an even function of frequency and the imaginary part is an odd function of frequency.	L2	6 Marks
	b	State and prove the following properties with respect to DTFT: (i) Frequency differentiation (ii) Time-domain Convolution	L2	6 Marks
	c	Find the Fourier Transform of the following signals: (i) $x(t) = \delta(t+1) - \delta(t-1)$ (ii) $x(t) = \frac{d}{dt}[te^{-2t}\sin t\, u(t)]$	L3	8 Marks
		OR		
Q. 08	a	Find the DTFT of $x[n] = a^{\|n\|}, \|a\| < 1$. Also, sketch the magnitude and phase spectra.	L3	6 Marks
	b	State and prove the following properties with respect to continuous-time Fourier transform: (i) Time scaling (ii) Integration	L2	6 Marks
	c	The DTFT of a real signal is $X(e^{j\Omega})$. Express DTFT of each of the following signals in terms of $X(e^{j\Omega})$: (i) $x[-n]$ (ii) $x[n] * x[-n]$ (iii) $(-1)^n x[n]$ (iv) $(1 + \cos n\pi)x[n]$	L3	8 Marks
		Module-5		
Q. 09	a	What is Region of Convergence (ROC) of Z-Transform? Mention its properties.	L1	6 Marks
	b	Determine the z-transform and ROC for the following time signals. Sketch the ROC, poles and zeroes in the z-plane. (i) $x[n] = 2^n \sin\left(\frac{\pi}{4}n\right) u[-n]$ (ii) $x[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n-N])$	L3	8 Marks
	c	Find the Inverse Z-transform of $X(z) = \frac{z^4+z^2}{z^2-0.75z+0.125}$, ROC: |z| >0.5, using partial fraction method.	L3	6 Marks
		OR		
Q. 10	a	A discrete LTI system is characterized by following Difference equation: y[n] = y[n-1] +y[n-2] +x[n-1] (i) Find the system function. (ii) Indicate ROC if system is stable. (iii) Indicate ROC if system is causal. (iv) Obtain impulse response in both cases.	L3	10 Marks
	b	State and prove the following properties of Z-transform: (i) Time-reversal (ii) Multiplication by exponential function	L2	4 Marks
	c	Find the Z-transform of the following signals: (i) $x[n] = 2^n u[-n-3]$ (ii) $x[n] = \sin\left(\frac{\pi n}{8} - \frac{\pi}{4}\right) u[n-2]$	L3	6 Marks

*Bloom's Taxonomy Level: Indicate as L1, L2, L3, L4, etc. It is also desirable to indicate the COs and POs to be attained by every bit of questions.

Signals and Systems - 18EC45

Model Question Paper - 2 Scheme & Solution

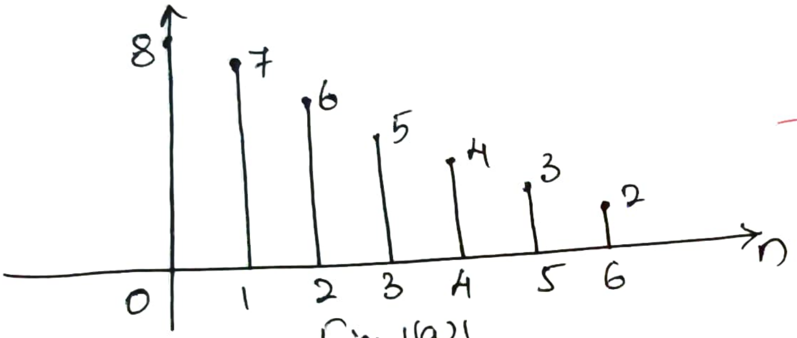
Module - 1

1.a) Given the signal, $x[n] = (8-n)(u[n] - u[n-7])$, Sketch the following signals:

[Total-4M]

i) $y[n] = x[4-n]$ ii) $g[n] = x[-2n+3]$

$\rightarrow$ $x[n] = (8-n)(u[n] - u[n-7])$.



-1M

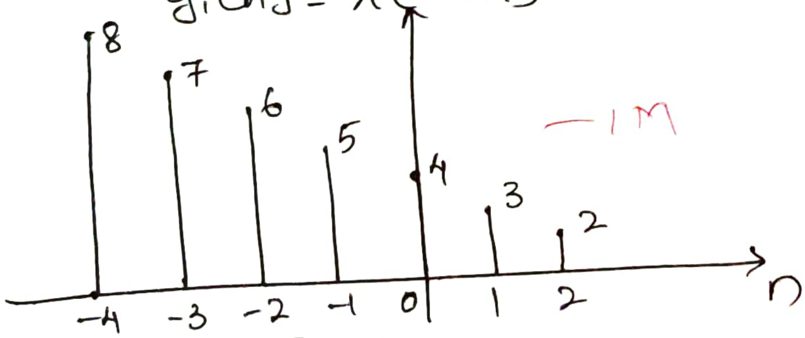
Fig 1(a)1

i) $y[n] = x[4-n]$

$y[n] = x[-n+4]$

Time-shifting

$y_1[n] = x[n+4]$

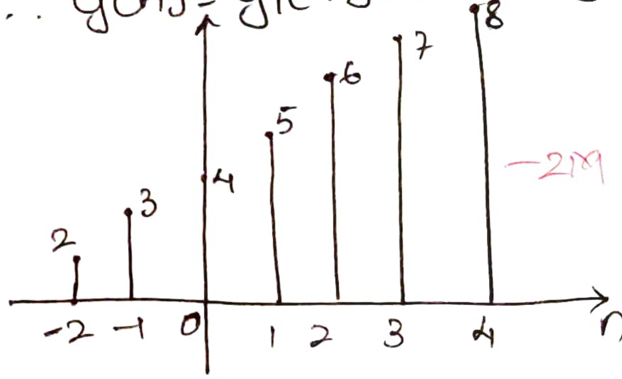


-1M

Fig 1(a)2

Time - reversal

$\therefore y[n] = y_1[-n] = x[-n+4]$



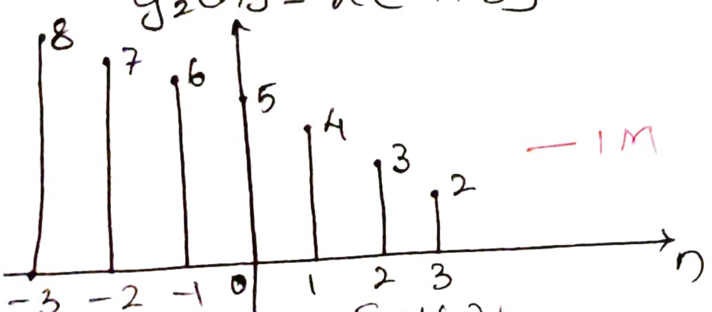
-2M

Fig 1(a)3

ii) $g[n] = x[-2n+3]$

Time-shifting

$g_2[n] = x[n+3]$

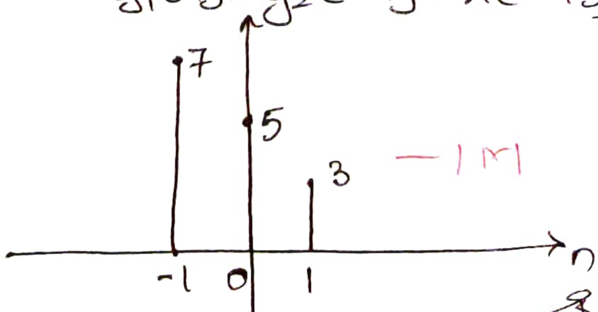


-1M

Fig 1(a)4

Time-scaling

$g_1[n] = g_2[2n] = x[2n+3]$



-1M

Fig 1(a)5

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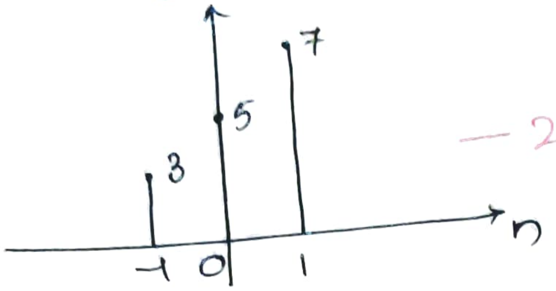
Time-reversal

$g_2[n] = g_1[-n] = x[-2n+3]$



— 2M

fig 1(a)6

1.b) Calculate the Energy and Power of the following signals:-

i) $x(t) = e^{-0.05t}[u(t) - u(t-10)]$, ii) $x[n] = u[n]$.

iii) $x(t) = 5\cos(200\pi t)$, iv) $x[n] = (-2)^n[u[n+1] - u[n-2]]$

[Total - 10M].

→ i) $x(t) = e^{-0.05t}[u(t) - u(t-10)]$.

Total Energy.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{10} |e^{-0.05t}|^2 dt = \int_0^{10} e^{-0.1t} dt$$ — 1M

$$E = \frac{1}{-0.1}[e^{-0.1t}]_0^{10} = 6.33 < \infty$$ — 1M

$P = 0.$

Because of total energy is finite, $x(t)$ is a energy signal.

ii) $x[n] = u[n]$.

Energy, $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} 1^2 = \infty$ — 1M

Average power, $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$

$$P = \lim_{N\to\infty} \frac{1}{2N+1}(N+1) = \frac{1}{2} < \infty$$ — 1M

$\therefore x[n]$ is a power signal.

22/07

iii> $x(t) = 5\cos(200\pi t)$

$\omega_0 = 200\pi$, $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\cancel{200}\pi_{100}} = \frac{1}{100} = 0.01$ — 1M

$P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{0.01}\int_0^{0.01} |5\cos(200\pi t)|^2 dt$

$P = 1250\int_0^{0.01} dt + 1250\underbrace{\int_0^{0.01}\cos(400\pi t)\,dt}_{\text{This integral will be zero.}} = 1250[t]_0^{0.01}$ — 1M

$P = 1250[0.01 - 0] = 12.5$

$P = 12.5\,W$, $E = \infty$ — 2M

It is a power signal.

iv> $x[n] = (-2)^n[u[n+1] - u[n-2]]$.

$x[n]$ is a aperiodic signal, hence let us calculate its energy.

$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-1}^{1} |-2^n|^2 = \sum_{n=-1}^{1} |-2^2|^n = \sum_{n=-1}^{1} 4^n$ — 1M

$E = \frac{4^{-1} - 4^2}{1-4} = 5.25 < \infty$ — 1M

$\sum_{n=k}^{l} \beta^n = \frac{\beta^k - \beta^{l+1}}{1-\beta}, \beta \neq 1$

$x[n]$ is a Energy signal.

1.c) Find whether the following signal are periodic or not. If periodic, find the fundamental period.

i) $x(t) = \sin^2(400\pi t)$ ii) $x(t) = \cos(2t) + \sin(3t)$

iii) $x(t) = \sin(4\pi t) + \sin(5t)$ [Total- 6M]

$\longrightarrow$ i) $x(t) = \sin^2(400\pi t)$.

$$x(t) = \frac{1 - \cos(800\pi t)}{2} \quad \text{— 1M} \qquad \because \sin^2(\theta) = \frac{1-\cos 2\theta}{2}$$

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos(800\pi t)$$

Here, $\frac{1}{2}$ is the DC shift $\therefore$ signal $\frac{1}{2}\cos(800\pi t)$ is periodic — 1M

and, $\omega_0 = 800\pi$, $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{800\pi} = \frac{1}{400} = 0.0025.$

ii) $x(t) = \cos(2t) + \sin(3t)$

$x_1(t) = \cos 2t$ & $x_2(t) = \sin 3t$

$\omega_1 = 2$ $\omega_2 = 3$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2}$ $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3}$ — 1M

$T_1 = \pi$ $T_2 = \frac{2\pi}{3}$

$\therefore \frac{T_1}{T_2} = \frac{3}{2}$, which is rational $\therefore$ signal is Periodic.

$T = LCM(T_1, T_2) = 2\pi.$ — 1M

iii> $x(t) = \sin(4\pi t) + \sin(5t)$

$x_1(t) = \sin(4\pi t)$ $\qquad\qquad$ $x_2(t) = \sin(5t)$

$\omega_1 = 4\pi$ $\qquad\qquad$ $\omega_2 = 5$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2}$ $\qquad\qquad$ $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{5}$ — 1M

$\frac{T_1}{T_2} = \frac{5}{4\pi}$, which is a irrational number.

— 1M

$\therefore$ $x(t)$ is aperiodic signal.

"OR"

2.a> Fig. 2(a) shows two signals $x(t)$ & $y(t)$ sketch the following signals.

i> $x(t)y(t-1)$ $\qquad\qquad$ ii> $x(t+1)y(t-2)$

iii> $x(t)y(-1-t)$ $\qquad\qquad$ iv> $x(4-t)y(t)$

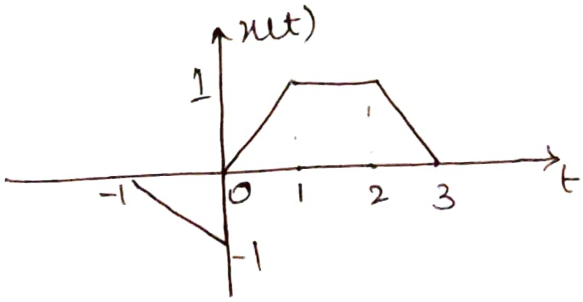
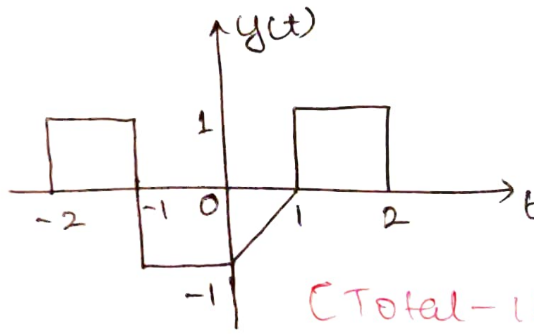


Fig 2(a1)



[Total – 10M]

Fig 2(a2).

→ i> $x(t)y(t-1)$

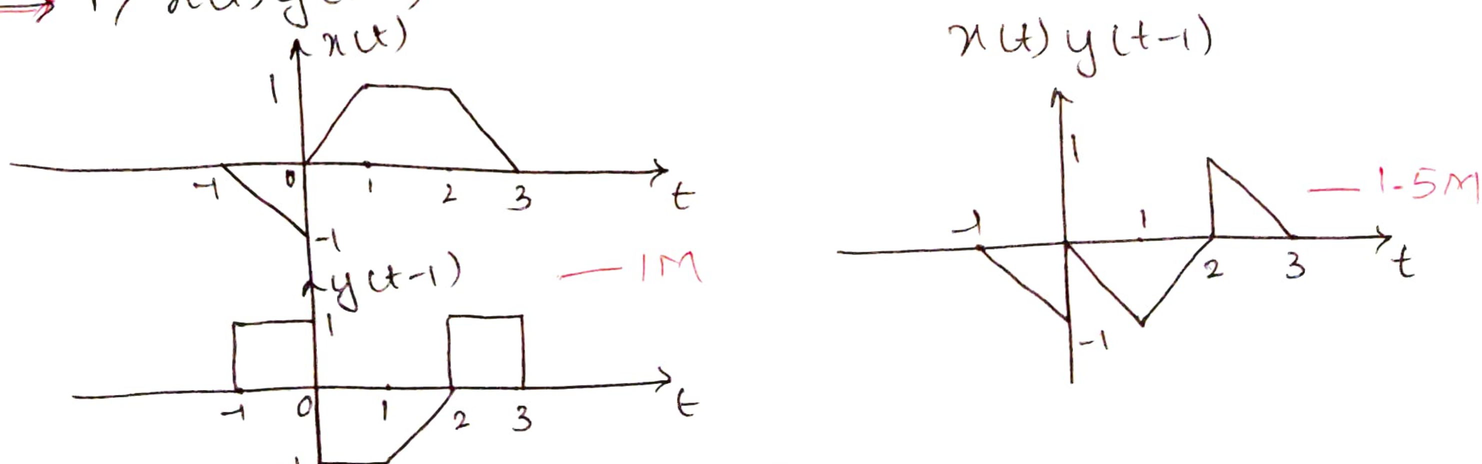


Fig 2(a3)

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ii) $x(t+1)\,y(t-2)$

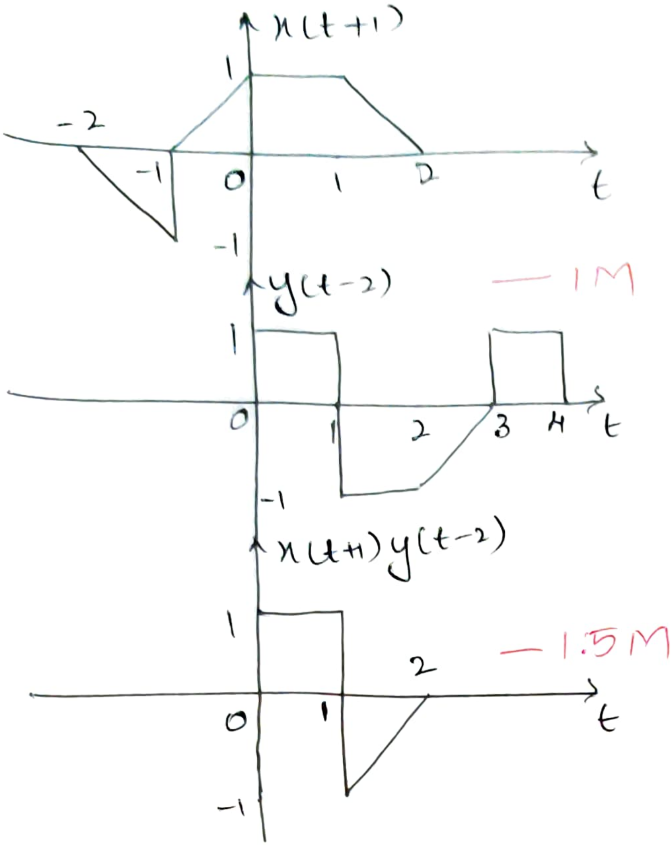


Fig 2(a4)

iii) $x(t)\,y(-1-t)$

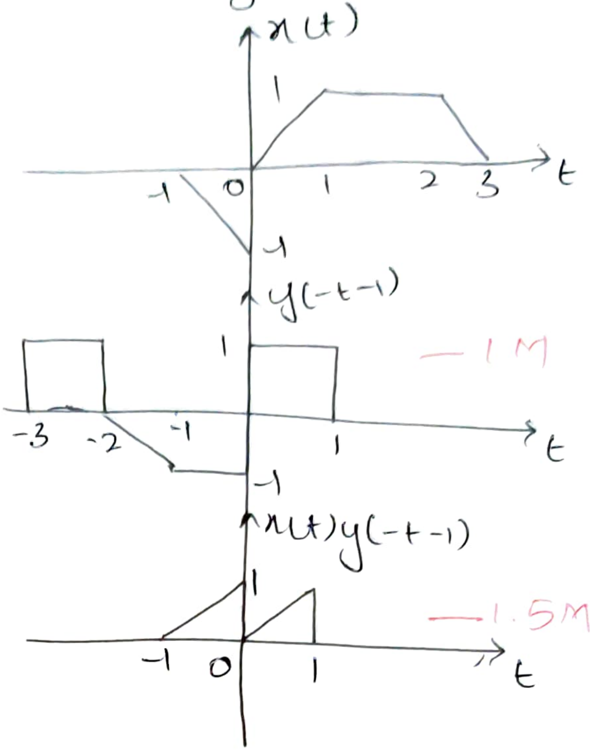


Fig 2(a5)

iv) $x(4-t)\,y(t)$

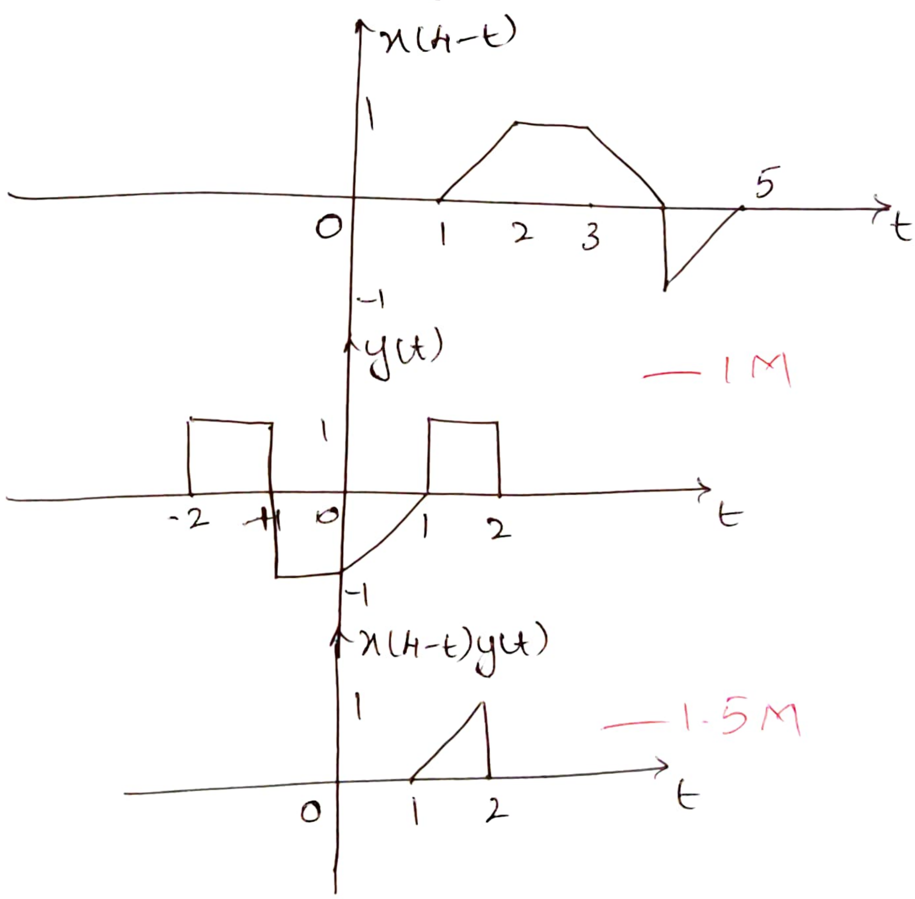


Fig 2(a6)

2. b) Evaluate the expression, $\int_{1}^{2} t^2 \delta(2t-3)\,dt + \int_{-3}^{3} \delta(3t+5)\,dt$. [Total - 4M]

$\longrightarrow$ $\int_{1}^{2} t^2 \delta(2t-3)\,dt + \int_{-3}^{3} \delta(3t+5)\,dt$.

We know that,
$\delta(at) = \frac{1}{|a|}\delta(t)$

$\therefore$ time shifting for $\delta(2t-3)$

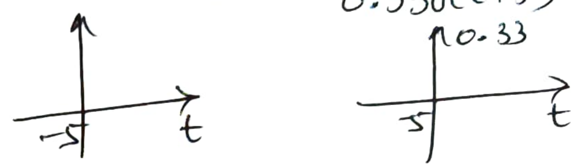
$\delta(t-3)$ $\quad \therefore \delta(2t-3) = 0.5\delta(t-3)$

— 1M

Impulse in first integral has strength of 0.5 and placed at $t=3$, which is out of limits therefore first integral value is 0. — 1M

Impulse in second integral has strength of 0.33 and is placed at $t=-5$, which is also out of second integral limits, and hence it is also zero. — 1M

for $\delta(3t+5)$ $\quad \delta(3t+5) = 0.33\delta(t+5)$

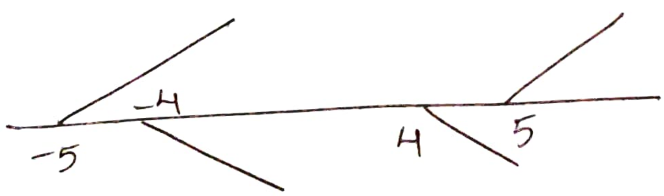


$\therefore \int_{1}^{2} t^2 \delta(2t+3)\,dt + \int_{-3}^{3} \delta(3t+5)\,dt$ — 1M

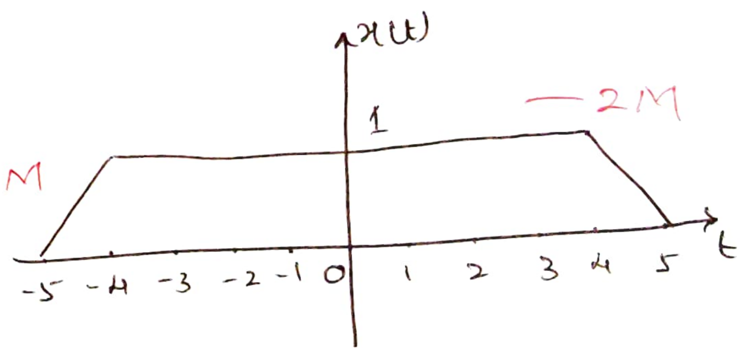
$0 + 0 = \underline{\underline{0}}$

2. c) Given the signal, $x(t) = r(t+5) - r(t+4) - r(t-4) + r(t-5)$, sketch $x(t)$ and its derivative. [Total - 6M]

$\longrightarrow$ $x(t) = r(t+5) - r(t+4) - r(t-4) + r(t-5)$

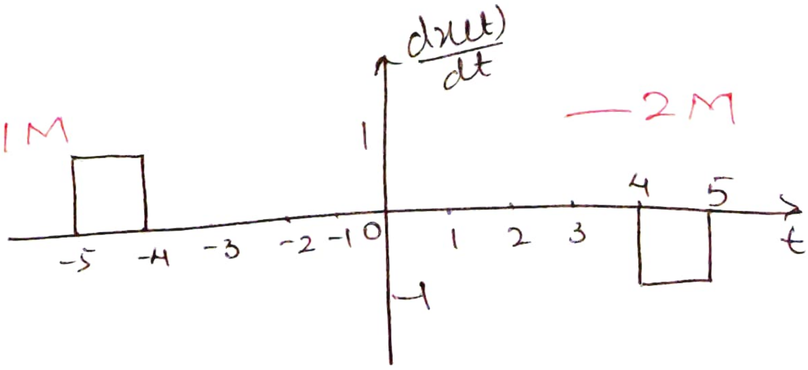


$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t+5, & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$ — 1M



— 2M

$$\frac{dx(t)}{dt} = \begin{cases} -1, & 4 \le t \le 5 \\ 1, & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$ — 1M



— 2M

22/02

Module - 2

3.a) Determine whether the following system represented by input-output relation, is i) Stable ii) Memoryless, iii) Causal, iv) Time-invariant and v) Linear;

[Total - 6M]

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

→ $y(t) = \int_{-\infty} x(\tau)d\tau$

i) Stability :-

Consider a bounded input $x(t) = u(t)$

$$\therefore y(t) = \int_{-\infty}^{t} u(\tau)d\tau = \int_{0}^{t} d\tau = \tau\Big]_{0}^{t} = t$$

— 2M

$y(t) \longrightarrow \infty$ as $t \to \infty$ $\therefore$ output is unbounded

Hence, system is unstable.

ii) Memoryless :-

— 1M

Output depends on past inputs $(-\infty)$ $\therefore$ system posses memory.

iii) Causality :-

Output depends on present & past inputs only, hence it is a causal system.

— 1M

iv) Time-invariance :-

$$T\{x(t-t_0)\} = \int_{-\infty}^{t} x(\tau - t_0)d\tau$$

— 1M

$m = \tau - t_0$, $dm = d\tau$.

$$T\{x(t-t_0)\} = \int_{-\infty}^{t-t_0} x(m)dm = y(t-t_0)$$

$\therefore$ System is time invariant.

v) Linearity :-

$$T\{a_1x_1(t) + a_2x_2(t)\} = \int_{-\infty}^{t} [a_1x_1(\tau) + a_2x_2(\tau)]d\tau$$

— 1M

$$= a_1\int_{-\infty}^{t} x_1(\tau)d\tau + a_2\int_{-\infty}^{t} x_2(\tau)d\tau = a_1y_1(t) + a_2y_2(t)$$

Hence, it is a linear system.

3.b) Following represents the input-output relations of various systems. Determine whether they are invertible or not:

i) $y[n]=n\,x[n]$ ii) $y(t)=x(2t+3)$ iii) $y[n]=\sum_{k=-\infty}^{n} x[k]$.

If invertible, represent the inverse system. [Total - 6M]

→ i) $y[n]=n\,x[n]$.

$y[n]=n\,x[n]$ is a non-invertible system, because it produces the same output if $x[n]$ is odd function of n. — 1M

Example:

consider, $x[n]=2\sin(2n)$

for, $n=2$, $y[n]=y[2]=2\sin(2n)$

for, $n=-2$, $y_1[n]=y_1[-2]=-2\sin(-2n)=2\sin(2n)$ — 1M

Two different inputs produce same output ∴ system is Non-invertible.

ii) $y(t)=x(2t+3)$

Inverse of time shifting operation is $x(t-3)$ &

Inverse of time scaling operation is $x\left(\frac{t}{2}\right)$

— 1M

$y(t)=x(2t+3)$ is an invertible system & inverse system is $y(t)=x\left(\frac{t}{2}-3\right)$ but time scaling operation has to be performed first. — 1M

iii) $y[n]=\sum_{k=-\infty}^{n} x[k]$

$y[n]=x[n]+x[n-1]+x[n-2]+\ldots$

$y[n+1]=x[n+1]+x[n]+x[n-1]+\ldots$

$=x[n+1]+y[n]$. — 1M

∴ $y[n]=x[n]+y[n-1]$

or

$x[n]=y[n]-y[n-1]$

Input $x[n]$ can be obtained back & inverse system is,

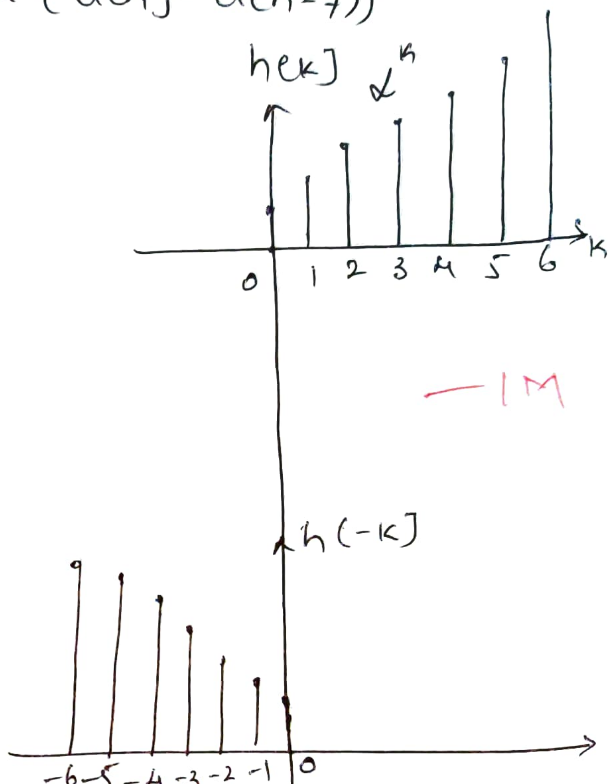
$w[n]=y[n]-y[n-1]$. — 1M

3.c) Let $x[n] = u[n] - u[n-5]$, be the input signal applied to a Linear and Time-invariant (LTI) discrete-time system and $h[n] = \alpha^n (u[n] - u[n-7]$, be the impulse response of the system. Obtain the output signal, $y[n]$. [Total- 8M]

→ $x[n] = u[n] - u[n-5]$ & $h[n] = \alpha^n (u[n] - u[n-7])$

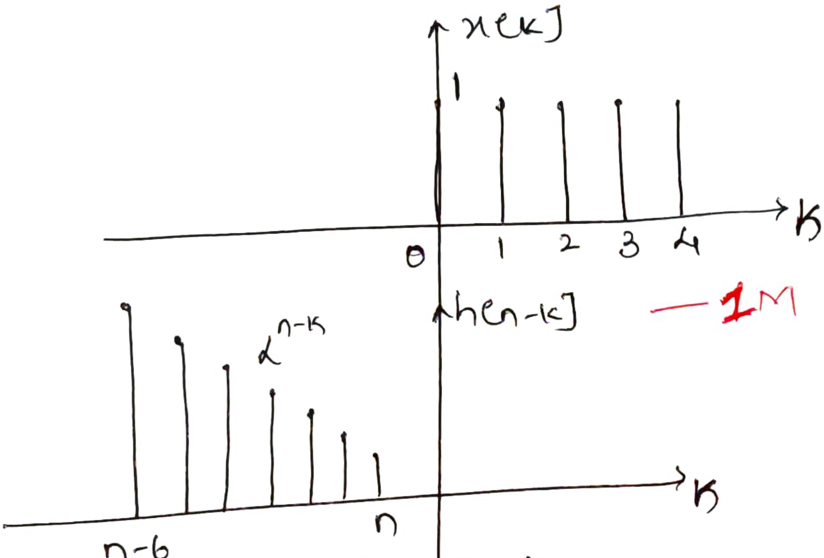
let us consider for $\alpha > 1$.

$$\therefore y[n] = \sum_{k=-\infty}^{\infty} x(k) h[n-k].$$



— 1M

Fig 3(c1)



— 1M

Fig 3(c2)

case 1) $n < 0$, $w_n[k] = x[n] h[n-k]$

$w_n[k] = 0$ — 1M

$$\therefore y[n] = \sum_{k=-\infty}^{\infty} w_n[k] = 0, \quad n < 0$$

case 2) $n \geq 0$ & $n \leq 4$ $(0 \leq n \leq 4)$

$$\because \sum_{n=0}^{M-1} \beta^n = \frac{1-\beta^M}{1-\beta}$$

$w_n[k] = \alpha^{n-k}$, $0 \leq k \leq n$

$$y[n] = \sum_{k=0}^{n} \alpha^{n-k} = \alpha^n \sum_{k=0}^{n} (\alpha^{-1})^k = \alpha^n \left(\frac{1-(\alpha^{-1})^{n+1}}{1-(\alpha^{-1})} \right).$$

$$\therefore y[n] = \frac{\alpha^n - \alpha^{-1}}{1-\alpha^{-1}} = \frac{\alpha^{n+1}-1}{\alpha} \cdot \frac{\alpha}{\alpha-1} = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad 0 \leq n < 4.$$ — 1M

Case 3) $n \geq 4$ and $n-6 \leq 0$, $(4 \leq n \leq 6)$

$$w_n[k] = \alpha^{n-k}, \quad 0 \leq k \leq 4$$

$$y[n] = \sum_{k=0}^{4} \alpha^{n-k} = \alpha^n \sum_{k=0}^{4} (\alpha^{-1})^k = \alpha^n \left(\frac{1-(\alpha^{-1})^5}{1-\alpha^{-1}} \right) = \frac{\alpha^n - \alpha^{n-5}}{1-\alpha^{-1}}$$

$$y[n] = \frac{\alpha^n - \alpha^{n-5}}{1} \cdot \frac{\alpha}{\alpha-1} = \frac{\alpha^{n+1} - \alpha^{n-4}}{\alpha-1} = \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}, \quad 4 \leq n \leq 6$$

— 1M

Case 4) $n > 6$ and $n-6 \leq 4$ $(6 < n \leq 10)$

$$w_n[k] = \alpha^{n-k}, \quad n-6 \leq k \leq 4$$

$$y[n] = \sum_{k=n-6}^{4} \alpha^{n-k} = \frac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}, \quad 6 < n \leq 10.$$

— 1M

Case 5) $n-6 > 4$ or $n > 10$

$$w_n[k] = 0, \quad y[n] = 0, \quad n > 10$$

$$y[n] = \begin{cases} 0, & n < 0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha}, & 0 \leq n < 4 \\ \dfrac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}, & 4 \leq n \leq 6 \\ \dfrac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}, & 6 < n \leq 10 \\ 0, & n > 10 \end{cases}$$

— 1M

$y[n]$ for $\alpha = 2$.

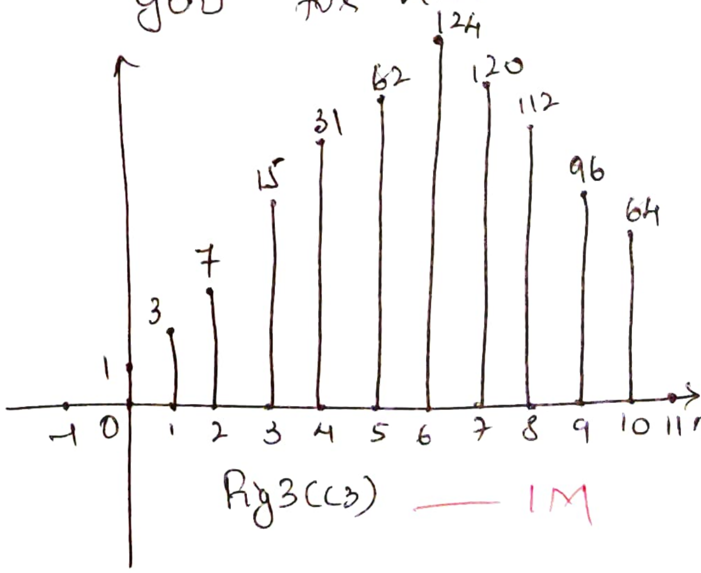


Fig3(c3) — 1M

"OR"

4.a) Perform convolution operation on the following signals:

$x_1(t) = e^{-|t-2|}$ and $x_2(t) = e^{-2t}u(t+4)$ [Total – 8M]

$\rightarrow$

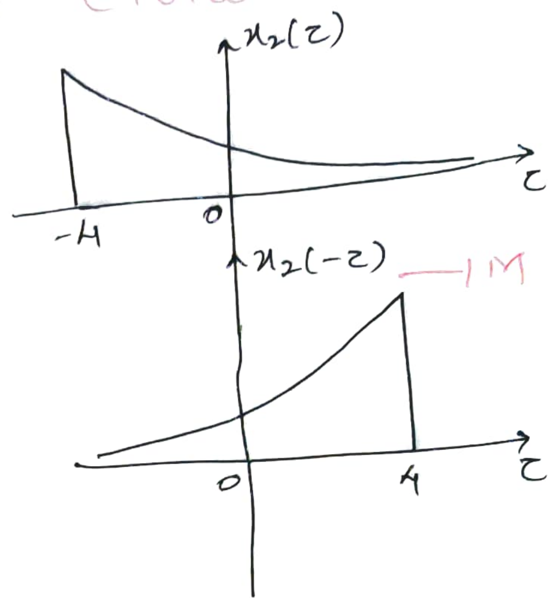
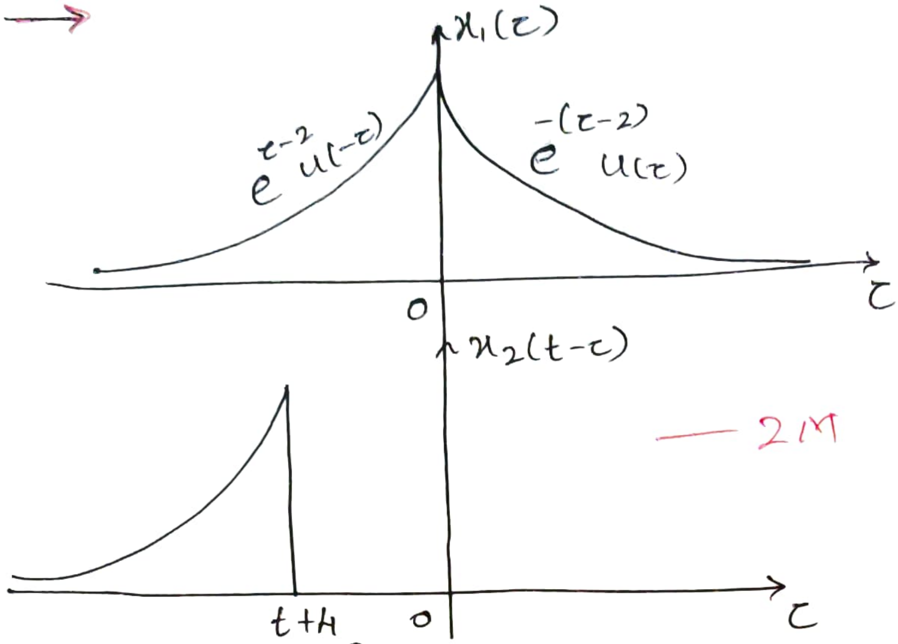


Fig 4(a1)



— 2M

Fig 4(a2)

case 1) for, $t+4<0$ or $t<-4$

$$w_t(\tau) = e^{\tau-2} \cdot e^{-2(t-\tau)} = e^{-2-2t} \cdot e^{3\tau}, \quad \text{for } -\infty \le \tau \le t+4$$

$$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\,d\tau = \int_{-\infty}^{t+4} e^{-2-2t} \cdot e^{3\tau}\,d\tau = e^{-2-2t}\int_{-\infty}^{t+4} e^{3\tau}\,d\tau \quad \text{— 1M}$$

$$y(t) = \frac{e^{-2-2t}}{3}\left[e^{3\tau}\right]_{-\infty}^{t+4} = \frac{e^{-2-2t} \cdot e^{3t+12}}{3}$$

$$\therefore y(t) = \frac{e^{t+10}}{3}, \quad t<-4 \quad \text{— 1M}$$

case 2) for, $t+4>0$, $t>-4$

$$y(t) = \int_{-\infty}^{0} e^{\tau-2} \cdot e^{-2t+2\tau}\,d\tau + \int_{0}^{t+4} e^{-\tau+2} \cdot e^{-2t+2\tau}\,d\tau.$$

$$y(t) = \int_{-\infty}^{0} e^{-2-2t}\, e^{3\tau}\,d\tau + \int_{0}^{t+4} e^{2-2t}\, e^{\tau}\,d\tau. \quad \text{— 1M}$$

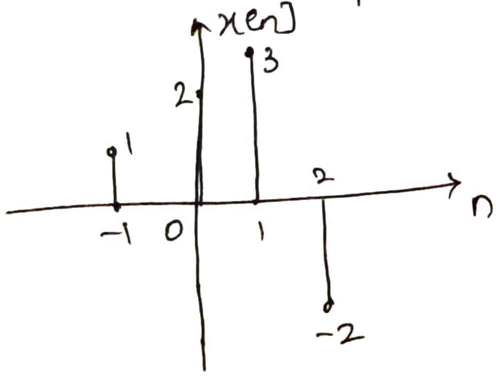
$$y(t) = \frac{e^{-2-2t}}{3}\left[e^{3\tau}\right]_{-\infty}^{0} + e^{2-2t}\left[e^{\tau}\right]_{0}^{t+4}$$

$$y(t) = \frac{e^{-2-2t}}{3} + e^{-t+6} - e^{2-2t}, \quad t > -4 \quad \text{— 1M}$$

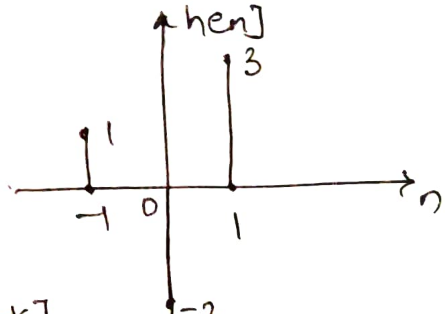
$$\therefore y(t) = \begin{cases} \dfrac{e^{t+10}}{3}, & t < -4 \\[2ex] \dfrac{e^{-2-2t}}{3} + e^{-t+6} - e^{2-2t}, & t > -4. \end{cases} \quad \text{— 1M.}$$

4.b) Given, $x[n] = [1, 2, 3, -2]$, here sample-value 2 appears at time-origin, and $h[n] = \delta[n+1] - 2\delta[n] + 3\delta[n-1]$.
Obtain $y[n] = x[n] * h[n]$. [Total-4M]

→ $x[n] = [1, \underset{\uparrow}{2}, 3, -2]$ $\quad$ $h[n] = \delta[n+1] - 2\delta[n] + 3\delta[n-1]$

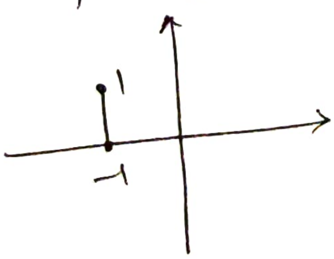


— 1M

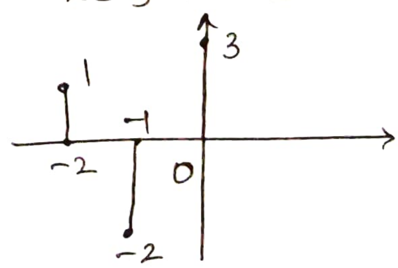


$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k].$$

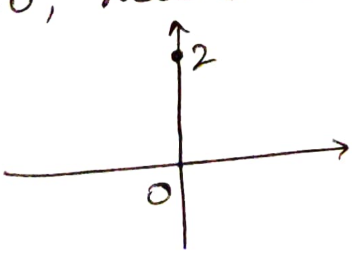
$k=-1$, $x[-1]\,\delta[n+1]$



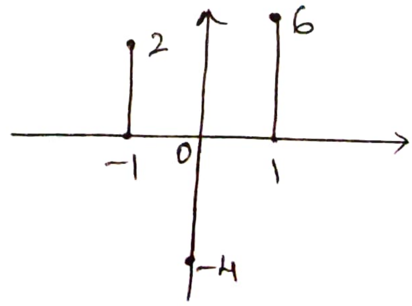
$x[-1]\,h[n+1]$



$k=0$, $x[0]\,\delta[n]$



$x[0]\,h[n]$



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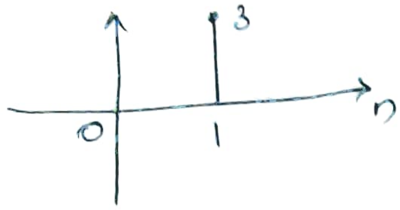
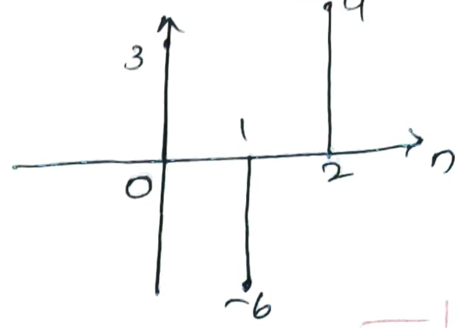
$K=1,\ x[1]\delta[n-1]$

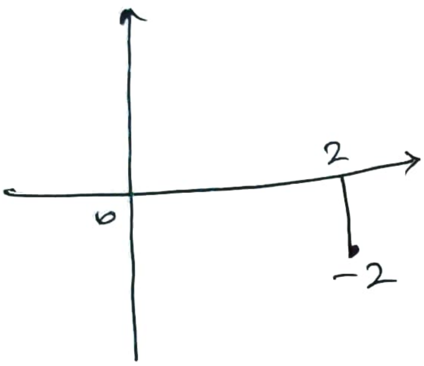


$x[1]h[n-1]$

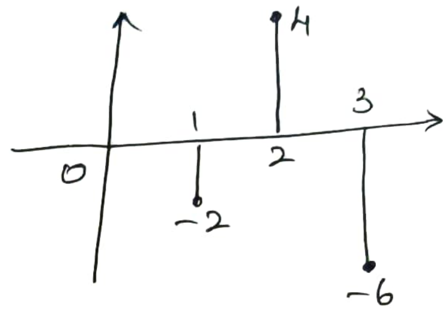


— 1M

$K=2,\ x[2]\delta[n-2]$



$x[2]h[n-2]$



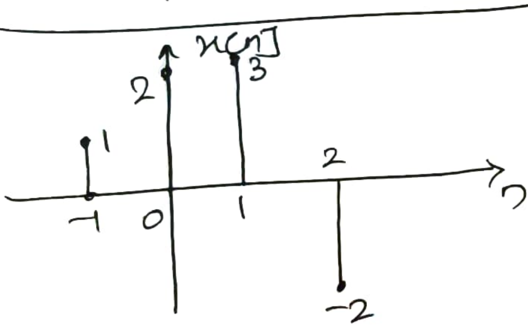


Fig 4(b1)

— 1M

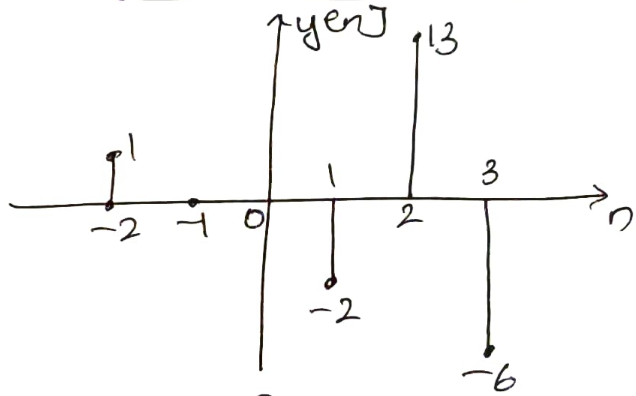


Fig 4(b2)

$\therefore\ y[n] = [1, 0, \underset{\uparrow}{2}, -2, 13, -6]$ — 1M

4.c) Determine whether the following systems represented by input-output relations are stable and causal:-

i) $y[n] = x[n]\,u[n+1]$ ii) $y(t) = x(\sin t)$

iii) $y[n] = \cos(x[n])$ iv) $y(t) = t\,x(t)$

[Total-8M]

→ i) $y[n] = x[n]\,u[n+1]$

a) let us consider input $x[n]$ is bounded ($|x[n]| \le M_x < \infty$)

$\therefore |y[n]| = |x[n]|\,|u[n+1]|$ $\qquad$ $|u[n+1]|$ is bounded

$|y[n]| = M_x\,|u[n+1]| < \infty$ $\qquad \therefore$

$\therefore$ Bounded input produces bounded output

$\therefore$ system is stable. — 1M

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b) Output depends on present input only ∴ system is causal.
— 1M

ii) $y(t) = x(\sin t)$

a) $|\sin t|$ is bounded ∴ bounded input will produce bounded output.

∴ system is stable. — 1M

b) Here, present instant output depends on past values of input
∴ system is causal. — 1M

iii) $y[n] = \cos(x[n])$

a) For bounded value of $x[n]$ cosine function has bounded value, hence $y[n]$ has bounded value.
∴ system is stable. — 1M

b) Here, n^{th} sample of output depends on n^{th} sample of input. ∴ system is causal. — 1M

iv) $y(t) = t\, x(t)$

a) As, $t \to \infty$, output will be unbounded, even if input is bounded. Hence the system is unstable. — 1M

b) t^{th} output depends on t^{th} input, hence the system is causal. — 1M

Module - 3

5.a) Given the impulse response, $h[n]$, of an LTI system, obtain the condition which needs to be satisfied by $h[n]$, for the system to be i) causal, ii) stable. [Total - 4M]

→ i) Causal LTI System:-

Output of a causal LTI system depends only on past and present values of input. Therefore, we require that $h[k]=0$ for $k<0$. — 1M

Hence, for a discrete-time causal LTI system,

$$h[k]=0 \quad \text{for} \quad k<0, \quad \text{and}$$

— 1M

for a continuous-time causal LTI system,

$$h(\tau)=0 \quad \text{for} \quad \tau<0.$$

ii) Stable LTI System:-

Bounded input, $|x[n]| \leq M_x < \infty$, should produce a bounded output, $|y[n]| \leq M_y < \infty$.

$$\therefore |y[n]| = |h[n] * x[n]|$$

$$= \left| \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \right|$$ — 1M

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]\, x[n-k]| \qquad \because |a+b| \leq |a|+|b|$$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]|\,|x[n-k]| \qquad \because |ab| = |a|\,|b|.$$

and, $|x[n-k]| \leq M_x$.

$$|y[n]| \leq M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

Now, for output to be bounded impulse response should be absolutely summable for DTS & absolutely integrable.

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \qquad \text{and,} \quad \int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty.$$ — 1M

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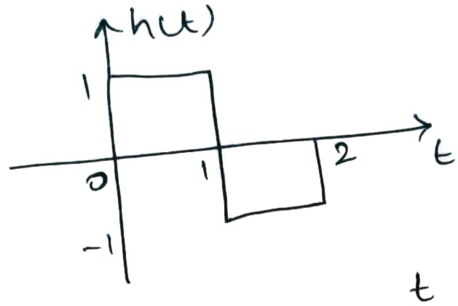
5.b) The signal, $h(t) = u(t) - 2u(t-1) + u(t-2)$, represents impulse response of an LTI system. Obtain the step response of the system.

[Total - 6M]

$\longrightarrow$ $h(t) = u(t) - 2u(t-1) + u(t-2)$



— 2M

Step response, $s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$. — 1M

$t < 0, \quad s(t) = 0$

$0 \le t \le 1, \quad s(t) = \int_0^1 1\, dt = t\Big]_0^1 = 1$ — 2M

$t \geq 2, \quad s(t) = \int_0^1 dt - \int_1^2 dt = 0$

$$s(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \le t \le 1 \\ 0, & t \geq 2 \end{cases}$$ — 1M.

5. c) Find the complex Fourier Series coefficients $X(k)$ for the waveform shown in the Fig. Q5(c). Sketch magnitude and phase spectra.

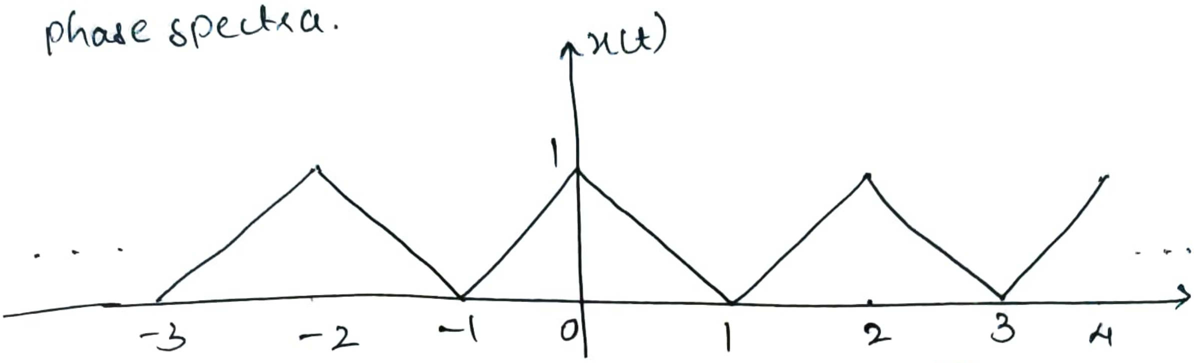


Fig. Q5(c).

[Total - 10M]

$$x(t) = \begin{cases} t+1, & -1 \le t \le 0 \\ 1-t, & 0 \le t \le 1 \end{cases}$$ — 1M

$T = 2, \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ — 1M

$$X(k) = \frac{1}{T}\int_{\langle T \rangle} x(t)\, e^{jk\omega_0 t}\, dt$$ — 1M

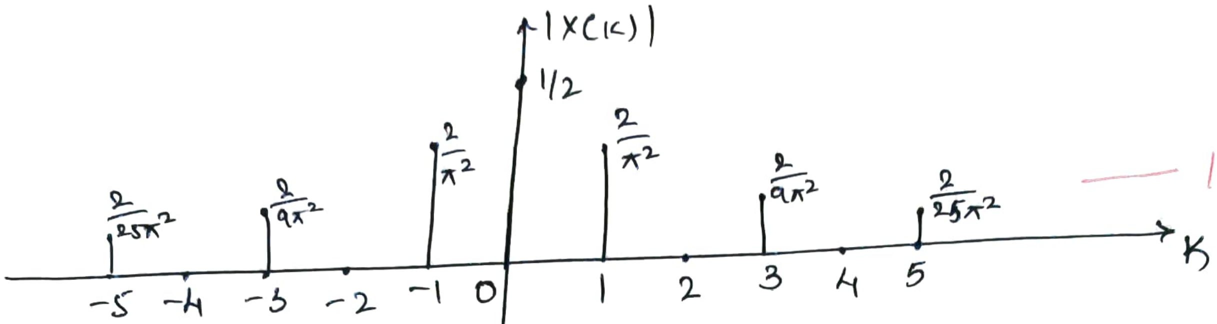
$$X(k) = \frac{1}{2}\left[\int_{-1}^{0}(t+1)\,e^{jk\pi t}\,dt + \int_{0}^{1}(1-t)\,e^{jk\pi t}\,dt\right]$$

$$\therefore \quad X(k) = \frac{1}{\pi^2 k^2}\left[1-(-1)^k\right] \quad ; \quad k \ne 0$$ — 2M

To find $X(k)$ for $k=0$,

$$X(0) = \frac{1}{2}\left[\int_{-1}^{0}(1+t)\,dt + \int_{0}^{1}(1-t)\,dt\right] = \frac{1}{2}$$ — 1M

$$\therefore X(k) = \begin{cases} \frac{1}{2}, & k=0 \\ 0, & k = \pm 2, \pm 4, \pm 6, \pm 8, \ldots \\ \frac{2}{\pi^2 k^2}, & k = \pm 1, \pm 3, \pm 5, \pm 7, \ldots \end{cases}$$ — 2M



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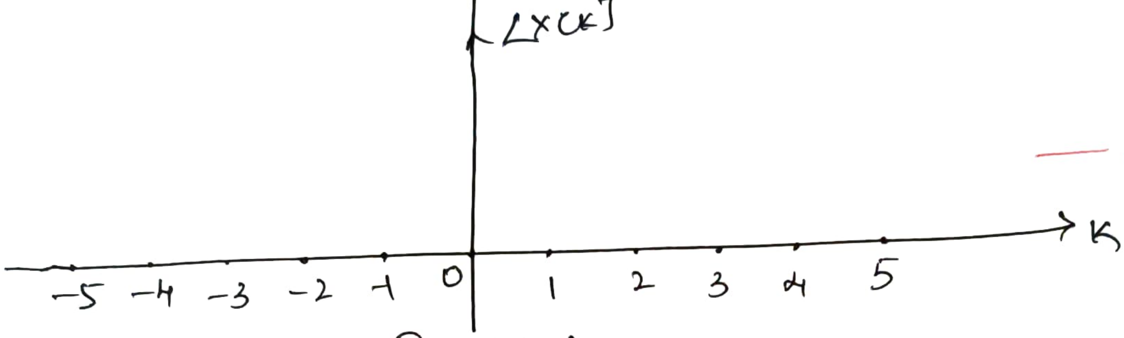


— 1M

Fig 5 (ii)

Phase spectrum, $\angle X[k] = 0$, because the signal $x(t)$ is real & even.

"OR"

6. a) State and prove the following properties of continuous-time Fourier series: i) Frequency shifting, ii) Time differentiation.

[Total – 6M]

→ i) Frequency shifting:-

If, $x(t) \xleftrightarrow{FS} X(k)$ — 1M

then, $z(t) = e^{jk_0\omega_0 t} x(t) \xleftrightarrow{FS} Z(k) = X(k-k_0)$.

from Fourier series coefficients,

$$Z(k) = \frac{1}{T}\int_0^T z(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T e^{jk_0\omega_0 t} x(t)\, e^{-jk\omega_0 t}\, dt \quad \text{— 1M}$$

$$= \frac{1}{T}\int_0^T x(t)\, e^{-j(k-k_0)\omega_0 t}\, dt.$$

$$\therefore \boxed{Z(k) = X(k-k_0)} \quad \text{— 1M}$$

ii) Time-differentiation:-

If, $x(t) \xleftrightarrow{FS} X(k)$ — 1 M

then, $\frac{dx(t)}{dt} \xleftrightarrow{FS} jk\omega_0 X(k)$.

From the synthesis equation of Fourier series,

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

differentiating on both sides,

$$\frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} X(k) jk\omega_0 e^{jk\omega_0 t}$$ — 1 M

$$\frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} [X(k) jk\omega_0] e^{jk\omega_0 t}$$

$$\therefore \boxed{\frac{dx(t)}{dt} \xleftrightarrow{FS} jk\omega_0 X(k).}$$ — 1 M

6.b) For a periodic signal $x(t)$, magnitude and phase spectral sample values are given below in the table Q6(b). They take zero values for other values of k. Determine the signal $x(t)$, whose fundamental frequency, $\omega_0 = \pi$ rad/sec.

K	-2	-1	0	1	2
$\lvert X(k)\rvert$	2	1	1	1	2
$\theta(k)$	$-\pi/8$	$-\pi/4$	0	$\pi/4$	$\pi/8$

Table Q6(b). [Total - 6M]

→ The magnitude and phase of a Fourier coefficients is represent as,

$$X(k) = |X(k)| e^{j\angle X(k)}$$ — 1 M

$\therefore$ from the given table we can write.

$X(-2) = 2e^{-j\frac{\pi}{8}}$, $X(-1) = e^{-j\frac{\pi}{4}}$, $X(0) = 1$, $X(1) = e^{j\frac{\pi}{4}}$, $X(2) = 2e^{j\frac{\pi}{8}}$ — 1M

Complex Exponential Fourier Series is given by,

$$x(t) = \sum_{K=-\infty}^{\infty} X(K) e^{jK\omega_0 t}$$

for, $K = -2$ to 2.

$$x(t) = X(-2)e^{-j2\omega_0 t} + X(-1)e^{-j\omega_0 t} + X(0) + X(1)e^{j\omega_0 t} + X(2)e^{j2\omega_0 t}$$ — 1M

Using Fourier series values for $\omega_0 = \pi$ rad/sec

$$x(t) = 2e^{-j\frac{\pi}{8}} e^{-j2\pi t} + e^{-j\frac{\pi}{4}} e^{-j\pi t} + 1 + e^{j\frac{\pi}{4}} e^{j\pi t} + 2e^{j\frac{\pi}{8}} e^{j2\pi t}$$ — 1M

$$\therefore x(t) = 2\left[e^{j(2\pi t+\frac{\pi}{8})} + e^{-j(2\pi t+\frac{\pi}{8})}\right] + \left[e^{j(\pi t+\frac{\pi}{4})} + e^{-j(\pi t+\frac{\pi}{4})}\right] + 1.$$ — 1M

wkt, $e^{jx} + e^{-jx} = 2\cos x$.

$$\therefore \boxed{x(t) = 4\cos\left(2\pi t + \frac{\pi}{8}\right) + 2\cos\left(\pi t + \frac{\pi}{4}\right) + 1}$$ — 1M

6.c) Determine whether the following systems represented by impulse responses are stable and causal:

i) $h[n] = u[n-1] - u[n-5]$

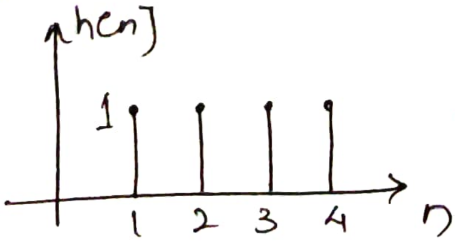
ii) $h[n] = 0.5^{|n|}$

iii) $h(t) = e^{-t}u(-t)$

iv) $h(t) = u(t-1)$.

[Total - 8M]

→ i) $h[n] = u[n-1] - u[n-5]$



— 1M

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a) $S = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=1}^{4} 1 = 4 < \infty$

impulse response is absolutely summable $\therefore$ the system is stable.

— 1M

b) The system is causal, since $h[n]$ is zero for $n<0$.

ii) $h[n] = 0.5^{|n|}$, $h[n] = (0.5)^{-n} u[-n-1] + (0.5)^{n} u[n]$.

a) System is non causal, because $h[n] \neq 0$ for $n<0$. — 1M

b) $S = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{-1} (0.5)^{-n} + \sum_{n=0}^{\infty} (0.5)^{n}$

for, $n = -m$ in first summation.

$$S = \sum_{m=\infty}^{1} (0.5)^{m} + \sum_{n=0}^{\infty} (0.5)^{n} = \sum_{m=1}^{\infty} (0.5)^{m} + \sum_{n=0}^{\infty} (0.5)^{n}$$

$$S = \frac{0.5}{1-0.5} + \frac{1}{1-0.5} = 1+2 = 3 < \infty$$

— 1M

impulse response is absolutely summable hence system is stable.

iii) $h(t) = e^{-t} u(-t)$.

a) System is non causal, because $h(t) \neq 0$ for $t<0$. — 1M

b) $S = \int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{0} e^{-t} dt = -e^{-t}\Big]_{-\infty}^{0} = -[1-e^{\infty}]$

System is unstable because impulse response is not ~~absolut~~ absolutely integrable — 1M

iv) $h(t) = u(t-1)$

a) System is causal, because $h(t) = 0$ for $t<0$. — 1M

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b) $S = \int_{-\infty}^{\infty} |h(t)| dt = \int_{1}^{\infty} dt = t\Big]_1^{\infty} = \infty$

— 1M

System is unstable.

Module-4

7.a) Show that for a real-valued aperiodic signal $x(t)$, the real part of its Fourier transform is an even function of frequency and the imaginary part is an odd function of frequency.

[Total-6M]

→ Symmetry property of Fourier Transform:-

Consider a real signal $x(t)$ and let, $X(j\omega)$ be its Fourier transform expressed as, $FT\{x(t)\} = A(j\omega) + jB(j\omega)$ and $x(t) = x_e(t) + x_o(t)$, where $x_e(t)$ & $x_o(t)$ are the even and odd components of $x(t)$ respectively.

Then,

$$x_e(t) \xleftrightarrow{FT} A(j\omega) \quad \text{— 1M}$$

$$x_o(t) \xleftrightarrow{FT} jB(j\omega).$$

we know that,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], \quad \text{and}$$

— 1M

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Given, $x(t) \xleftrightarrow{FT} X(j\omega) = A(j\omega) + jB(j\omega)$ — 1M

Since, $x(t)$ is real, we have

$$x(-t) \xleftrightarrow{FT} X(-j\omega) = X^*(j\omega) = A(j\omega) - jB(j\omega)$$

— 1M

Thus, we find that

$$x_e(t) \xleftrightarrow{FT} \frac{1}{2}[X(j\omega) + X^*(j\omega)]$$

— 1M

$$= A(j\omega) \quad , \quad \text{and.}$$

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$$x_o(t) \xleftrightarrow{FT} \frac{1}{2}[X(j\omega) - X^*(j\omega)]$$

$$= jB(j\omega) \quad \text{— 1M}$$

Thus, we have shown that the Fourier transform of a real even signal is a real function of frequency, and that of a real odd signal is an imaginary function of frequency.

7.b) State and prove the following properties with respect to DTFT: i) Frequency differentiation, ii) Time-domain convolution. [Total - 6M]

→ i) Frequency differentiation:

If, $x[n] \xleftrightarrow{DTFT} X(\Omega)$ — 1M

then, $-jn x[n] \xleftrightarrow{DTFT} \dfrac{dX(\Omega)}{d\Omega}$

from DTFT,

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{j\Omega n}$$

differentiating both sides with respect to Ω, we get

$$\frac{dX(\Omega)}{d\Omega} = \sum_{n=-\infty}^{\infty} x[n] (-jn) e^{j\Omega n}$$

— 1M

$$= \sum_{n=-\infty}^{\infty} [x[n](-jn)] e^{j\Omega n}$$

comparing with standard DTFT.

$$\boxed{-jn\, x[n] \xleftrightarrow{DTFT} \frac{dX(\Omega)}{d\Omega}.} \quad \text{— 1M}$$

ii) Time-domain convolution:-

If, $x[n] \xleftrightarrow{DTFT} X(\Omega)$ — 1M

and, $y[n] \xleftrightarrow{DTFT} Y(\Omega)$

then, $z[n] = x[n] * y[n] \xleftrightarrow{DTFT} Z(\Omega) = X(\Omega) . Y(\Omega)$.

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} z[n]\, e^{-j\Omega n}$$

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} [x[n] * y[n]]\, e^{-j\Omega n}$$

using convolution sum expression,

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x[k]\, y[n-k]\right] e^{-j\Omega n} \quad \text{— 1M}$$

put, $m = n-k$ & interchanging the order of summation.

$$Z(\Omega) = \sum_{k=-\infty}^{\infty} x[k] \sum_{m=-\infty}^{\infty} y[m]\, e^{-j\Omega(m+k)}$$

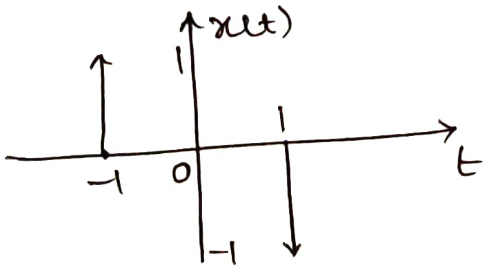
$$= \sum_{k=-\infty}^{\infty} x[k]\, e^{-j\Omega k} \sum_{m=-\infty}^{\infty} y[m]\, e^{-j\Omega m}$$

$$\boxed{\therefore Z(\Omega) = X(\Omega) \cdot Y(\Omega)} \quad \text{— 1M}$$

7. c) Find the Fourier transform of the following signals:
i) $x(t) = \delta(t+1) - \delta(t-1)$ ii) $x(t) = \frac{d}{dt}[t e^{-2t} \sin t\, u(t)]$

[Total - 8M]

→ i) $x(t) = \delta(t+1) - \delta(t-1)$



— 1M

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} [\delta(t+1) - \delta(t-1)]\, e^{-j\omega t}\, dt$$

$$= e^{-j\omega t}\Big|_{t=-1} - e^{-j\omega t}\Big|_{t=1} \quad \text{— 1M}$$

$$X(j\omega) = e^{j\omega} - e^{-j\omega}$$

multiplying & dividing by $2j$

$$X(j\omega) = 2j \, \frac{e^{j\omega} - e^{-j\omega}}{2j} \quad \text{— 1 M}$$

$$\therefore \boxed{X(j\omega) = 2j \sin(\omega)} \quad \text{— 1 M}$$

ii) $$x(t) = \frac{d}{dt}\left[t e^{-2t} \sin t \, u(t)\right]$$

$$= \frac{d}{dt}\left[t e^{-2t} u(t) \left(\frac{e^{jt} - e^{-jt}}{2j}\right)\right] \quad \text{— 1 M}$$

Since, $$t e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{(a + j\omega)^2}$$

we get, $$t e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{(2 + j\omega)^2} \quad \text{— 1 M}$$

using frequency shift property.

also, $$e^{jt} s(t) \xleftrightarrow{FT} S(\omega - 1)$$

Hence, $$e^{jt} t e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{[2 + j(\omega - 1)]^2}$$

— 1 M

and, $$e^{-jt} t e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{[2 + j(\omega + 1)]^2}$$

Applying linearity property,

$$\frac{1}{2j} e^{jt} t e^{-2t} u(t) - \frac{1}{2j} e^{-jt} t e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2j}\left[\frac{1}{[2 + j(\omega - 1)]^2} - \frac{1}{[2 + j(\omega + 1)]^2}\right]$$

Applying the time differentiation property,

$$\frac{d\,s(t)}{dt} \xleftrightarrow{FT} j\omega \, S(\omega)$$

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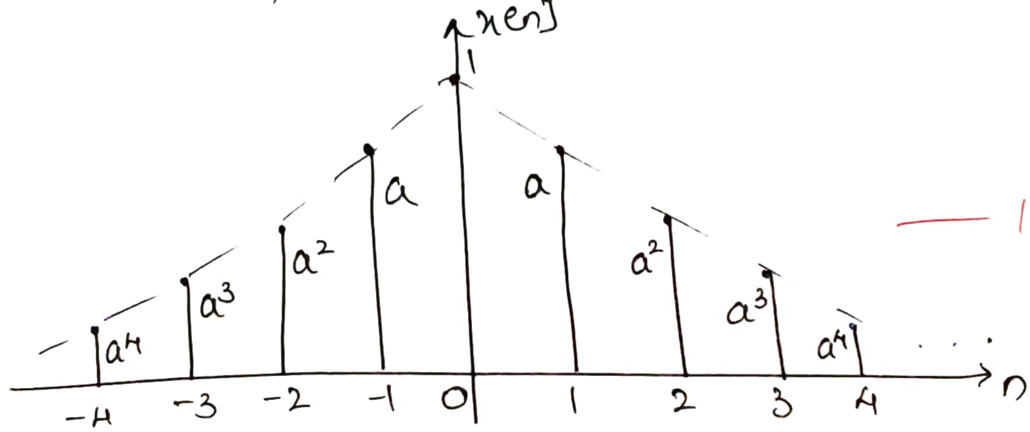
$$X(j\omega) = j\omega \, \frac{1}{2j}\left[\frac{1}{[2+j(\omega-1)]^2} - \frac{1}{[2+j(\omega+1)]^2}\right]$$

$$\therefore \boxed{X(j\omega) = \frac{\omega}{2}\left[\frac{1}{[2+j(\omega-1)]^2} - \frac{1}{[2+j(\omega+1)]^2}\right]} \quad \text{— 1M}$$

"OR"

8.a) Find the DTFT of $x[n] = a^{|n|}$, $|a|<1$. Also, sketch the magnitude and phase spectra. [Total − 6M]

→ $x[n] = a^{|n|}$, $|a|<1$



— 1M

$$x[n] = a^{|n|}$$

$$x[n] = a^{-n}u[-n-1] + a^{n}u[n]$$

$$\therefore X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} \quad \text{— 1M}$$

$$X(\Omega) = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\Omega n}$$

Put, $n=-m$ in first summation.

$$X(\Omega) = \sum_{m=\infty}^{1} a^{m} e^{j\Omega m} + \sum_{n=0}^{\infty} a^{n} e^{-j\Omega n}$$

— 1M

$$X(\Omega) = \sum_{m=1}^{\infty} (a e^{j\Omega})^{m} + \sum_{n=0}^{\infty} (a e^{-j\Omega})^{n}$$

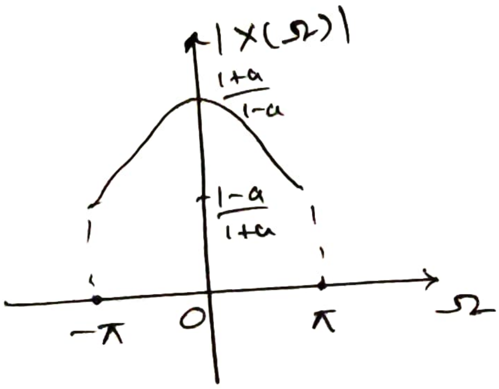
we know that,

$$\sum_{n=0}^{\infty} \beta^n = \frac{1}{1-\beta}, \quad |\beta|<1 \quad \text{and,} \quad \sum_{n=k}^{\infty} \beta^n = \frac{\beta^k}{1-\beta}, \quad |\beta|<1$$

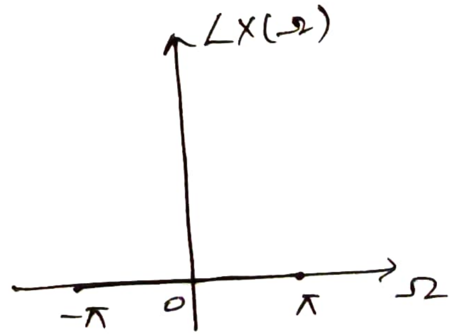
$$\therefore X(\Omega) = \frac{a e^{j\Omega}}{1-a e^{j\Omega}} + \frac{1}{1-a e^{-j\Omega}}$$

$$\boxed{X(\Omega) = \frac{1-a^2}{1+a^2-2a\cos\Omega}, \quad |a|<1}$$ — 1M

Ω	$X(\Omega)$	$\lvert X(\Omega)\rvert$	$\angle X(\Omega)$
$-\pi$	$\frac{1-a}{1+a}$	$\frac{1-a}{1+a}$	0
$-\frac{\pi}{2}$	$\frac{1-a^2}{1+a^2}$	$\frac{1-a^2}{1+a^2}$	0
0	$\frac{1+a}{1-a}$	$\frac{1+a}{1-a}$	0 — 1M
$\frac{\pi}{2}$	$\frac{1-a^2}{1+a^2}$	$\frac{1-a^2}{1+a^2}$	0
π	$\frac{1-a}{1+a}$	$\frac{1-a}{1+a}$	0



— 1M



8.b) State and prove the following properties with respect to continuous-time Fourier transform:-

i) Time-scaling ii) Integration. [Total-6M]

→ i) Time-Scaling:-

If, $$x(t) \xleftrightarrow{FT} X(j\omega)$$ — 1M

then, $$y(t) = x(at) \xleftrightarrow{FT} Y(j\omega) = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right).$$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(at) e^{-j\omega t} dt$$

let, $a > 0$, put, $at = \lambda$, $\therefore dt = \frac{d\lambda}{a}$

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega \frac{\lambda}{a}} \frac{d\lambda}{a}$$ — 1M

$$Y(j\omega) = \frac{1}{a} \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega \frac{\lambda}{a}} d\lambda$$

$$\boxed{Y(j\omega) = \frac{1}{a} X\left(\frac{j\omega}{a}\right)}$$ — 1M

The scaling property is its own dual such that a scaling by 'a' in time-domain results in inverse scaling by $\frac{1}{a}$ and amplitude scaling by $\frac{1}{|a|}$ in the frequency domain. The multiplier $\frac{1}{|a|}$ ensures that the scaled signal in time-domain and the scaled spectrum in frequency-domain possess the same energy.

ii) Integration:-

If, $$x(t) \xleftrightarrow{FT} X(j\omega)$$ — 1M

then, $$\int_{-\infty}^{t} x(\tau) d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(j\omega) + \pi X(0) \delta(j\omega)$$

we can write, $$\int_{-\infty}^{t} x(\tau) d\tau = x(t) * u(t).$$

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Hence, $\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FT} X(j\omega)U(j\omega)$

we know that,

$$u(t) \xleftrightarrow{FT} U(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FT} X(j\omega)\left[\pi\delta(j\omega) + \frac{1}{j\omega}\right] \text{ — 1M}$$

$$\xleftrightarrow{FT} \pi X(j\omega)\delta(j\omega) + \frac{1}{j\omega}X(j\omega)$$

$$\xleftrightarrow{FT} \underline{\pi X(0)\delta(j\omega) + \frac{1}{j\omega}X(j\omega)}$$

note that, $X(j\omega)\delta(j\omega) = X(0)\delta(j\omega)$ — 1M

8.c) The DTFT of a real signal is $X(e^{j\Omega})$. Express DTFT of each of the following signals in terms of $X(e^{j\Omega})$:

i) $x[-n]$

ii) $x[n] * x[-n]$

iii) $(-1)^n x[n]$

iv) $(1+\cos n\pi)x[n]$.

[Total-8M]

→ i) $x[-n]$.

$y[n] = x[-n]$

$\rightarrow Y(\Omega) = X(-\Omega)$ or $Y(e^{j\Omega}) = X(-e^{j\Omega})$. — 2M

ii) $x[n] * x[-n]$

$g[n] = x[n] * x[-n]$

$\therefore G(e^{j\Omega}) = X(e^{j\Omega}).X(-e^{j\Omega}) = |X(e^{j\Omega})|^2$ — 2M

iii) $(-1)^n x[n]$

$s[n] = (-1)^n x[n]$

$= e^{j\pi n} x[n]$

$S(e^{j\Omega}) = X(e^{j\Omega} - \pi)$ — 2M

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iv) $[1+\cos n\pi]x[n]$

$$z[n] = [1+\cos n\pi]x[n]$$

$$z[n] = [1+(-1)^n]x[n] \quad \text{— 1M}$$

$$z[n] = x[n] + e^{j\pi n}x[n]$$

$$\therefore \quad Z(e^{j\Omega}) = X(e^{j\Omega}) + X(e^{j\Omega} - \pi). \quad \text{— 1M}$$

Module - 5

9.a) What is Region of convergence (RoC) of z-transform? Mention its properties. [Total-6M]

→ **Region of Convergence:-**

RoC is the region (values of z), where z-transform converges. — 1M

Properties of RoC:-

1) The RoC cannot contain any poles.

2) If $x[n]$ is a finite causal sequence, then the RoC is the entire z-plane except at $z=0$. — 1M

3) If $x[n]$ is a finite non-causal sequence, then the RoC is the entire z-plane except at $z=\infty$.

4) If $x[n]$ is a finite double-sided sequence, then the RoC is the entire z-plane except at $z=0$ & $z=\infty$. — 1M

5) If $x[n]$ is a causal infinite length sequence, then the RoC is of the form.

$|z| > r_{max}$, r_{max} = largest magnitude of any poles of $X(z)$. — 1M

6) If $x[n]$ is a non-causal infinite length sequence, then the RoC is of the form.

$|z| < r_{min}$, r_{min} = smallest magnitude of any poles of $X(z)$.

7) If $x[n]$ is a two-sided sequence of infinite duration, then the RoC is of the form, $r_1 < |z| < r_2$. — 1M

8) The RoC of an LTI stable system contains the unit circle in the Z-plane. — 1M

9) The RoC must be a connected region (circle).

9.b) Determine the Z-transform and RoC for the following time signals. Sketch the RoC, poles and zeros in the Z-plane.

i) $x[n] = 2^n \sin\left(\frac{\pi}{4}n\right) u[-n]$ ii) $x[n] = \left(\frac{1}{4}\right)^n [u[n] - u[n-N]]$.

[Total-8M].

→ i) $x[n] = 2^n \sin\left(\frac{\pi}{4}n\right) u[-n]$

$$x[n] = 2^n \left[\frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{2j}\right] u[-n]$$

$$x[n] = \frac{1}{2j}\left[(2e^{j\frac{\pi}{4}})^n u[-n] - (2e^{-j\frac{\pi}{4}})^n u[-n]\right] \quad \text{— 1M}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(z) = \frac{1}{2j}\left[\sum_{n=-\infty}^{0} (2e^{j\frac{\pi}{4}})^n z^{-n} - \sum_{n=-\infty}^{0} (2e^{-j\frac{\pi}{4}})^n z^{-n}\right]$$

$$X(z) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} (0.5e^{-j\frac{\pi}{4}} z)^n - \sum_{n=0}^{\infty} (0.5e^{j\frac{\pi}{4}} z)^n\right] \quad \text{— ①}$$

$$X(z) = \frac{1}{2j}\left[\frac{1}{1-0.5e^{-j\frac{\pi}{4}} z} - \frac{1}{1-0.5e^{j\frac{\pi}{4}} z}\right]$$

$$X(z) = \frac{1}{2j}\left[\frac{\cancel{1} - 0.5e^{j\frac{\pi}{4}} z - \cancel{1} + 0.5e^{-j\frac{\pi}{4}} z}{(1-0.5e^{-j\frac{\pi}{4}} z)(1-0.5e^{j\frac{\pi}{4}} z)}\right]$$

$$X(z) = \frac{1}{2j}\left[\frac{0.5z(e^{-j\frac{\pi}{4}} - e^{j\frac{\pi}{4}})}{(1-0.5e^{-j\frac{\pi}{4}} z)(1-0.5e^{j\frac{\pi}{4}} z)}\right] \quad \text{— 1M}$$

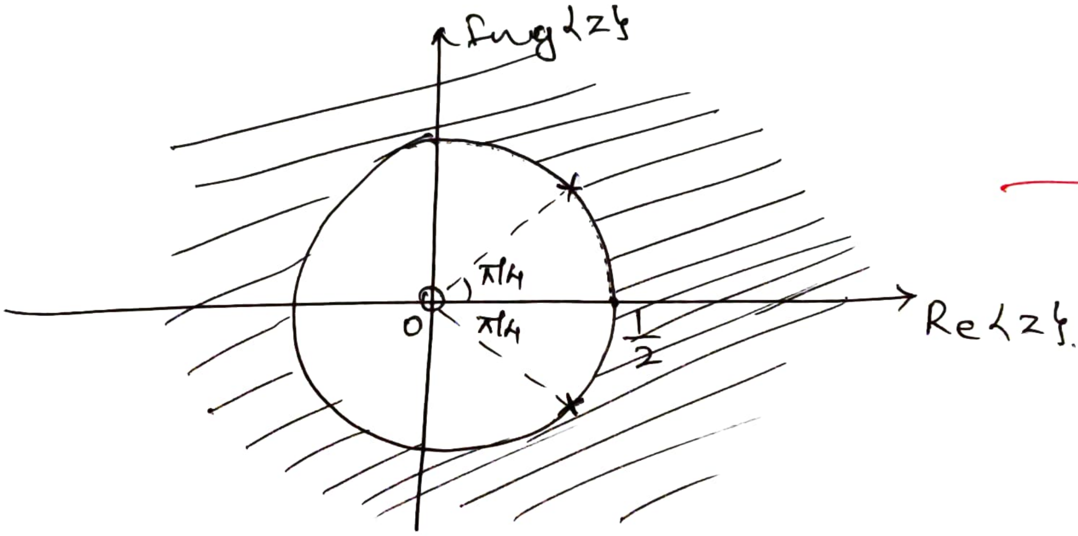
$$X(z) = -\frac{(e^{j\frac{\pi}{4}} - e^{-j\frac{\pi}{4}})}{2j}\left[\frac{0.5z}{(1-0.5e^{-j\frac{\pi}{4}}z)(1-0.5e^{j\frac{\pi}{4}}z)}\right]$$

$$\boxed{X(z) = \left[\frac{\frac{1}{2\sqrt{2}}z}{(1-0.5e^{-j\frac{\pi}{4}}z)(1-0.5e^{j\frac{\pi}{4}}z)}\right]}$$ — 1M

In eq^n ① both the sums should converge.

$$|0.5e^{-j\frac{\pi}{4}}z| < 1 \text{ and } |0.5e^{j\frac{\pi}{4}}z| < 1$$

i.e., $|z| > \frac{1}{2}$



— 1M

ii) $x[n] = \left(\frac{1}{4}\right)^n [u[n] - u[n-N]]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (0.25)^n [u[n] - u[n-N]] z^{-n}$$ — 1M

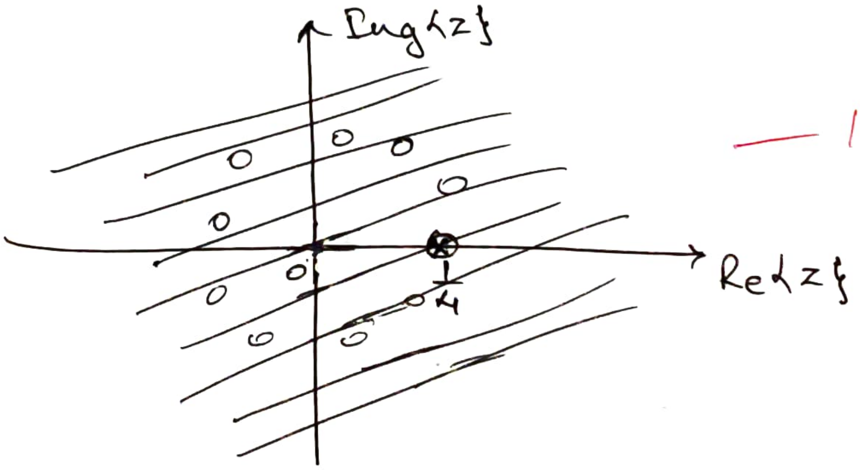
$$u[n] - u[n-N] = \begin{cases} 1, & \text{for } 0 \le n \le N-1 \\ 0, & \text{for } n \ge N \text{ \& } n \le -1 \end{cases}$$ — 1M

$$X(z)=\sum_{n=0}^{N-1}(0.25)^n z^{-n}$$

$$=\sum_{n=0}^{N-1}(0.25z^{-1})^n \qquad\qquad \sum_{n=0}^{M-1}\beta^n=\frac{1-\beta^M}{1-\beta},\ \beta\neq 1.$$

$$\boxed{X(z)=\frac{1-(0.25z^{-1})^N}{1-0.25z^{-1}}}$$ — 1M

The above eqn is convergent if $\sum_{n=0}^{N-1}|0.25z^{-1}|^n<\infty$. This summation will be finite if $|0.25z^{-1}|<\infty$. This condition states that RoC is entire z-plane except at $z=0$.



— 1M

9.c) Find the Inverse z-transform of $X(z)=\dfrac{z^4+z^2}{z^2-0.75z+0.125}$, RoC: $|z|>0.5$, using partial fraction method. [Total-6M]

→ $$X(z)=\frac{z^4+z^2}{z^2-0.75z+0.125},\quad \text{RoC}: |z|>0.5$$

$$\frac{X(z)}{z}=\frac{z^3+z}{z^2-0.75z+0.125}$$ — 1M

$\frac{X(z)}{z}$ is not a proper function, therefore

$$
\begin{array}{r|l}
 & z+0.75 \\
\hline
z^2-0.75z+0.125 & \cancel{z^3}+z \\
 & \cancel{z^3}-0.75z^2+0.125z \\
 & {\scriptstyle (-)\quad (+)\qquad\quad (-)} \\
\hline
 & \cancel{0.75z^2}+0.875z \\
 & \cancel{0.75z^2}-0.562z+0.093 \\
 & {\scriptstyle (-)\qquad\quad (+)\qquad\quad (-)} \\
\hline
 & 1.43z-0.093
\end{array}
$$

— 1M

$$\therefore \frac{X(z)}{z} = z+0.75+\frac{1.43z-0.093}{z^3-0.75z+0.125}$$

$$\frac{X(z)}{z} = z+0.75+\frac{1.43z-0.093}{(z-0.5)(z-0.25)}$$ — 1M

$$\frac{X(z)}{z} = z+0.75+\frac{A}{(z-0.5)}+\frac{B}{(z-0.25)}$$

$$A = \left.\frac{1.43z-0.093}{z-0.25}\right|_{z=0.5} = 2.488 \simeq 2.5$$

— 1M

$$B = \left.\frac{1.43z-0.093}{z-0.5}\right|_{z=0.25} = -1.058$$

$$\therefore \frac{X(z)}{z} = z+0.75+\frac{2.5}{(z-0.5)}+\frac{1.058}{(z-0.25)}$$ — 1M

$$X(z) = z^2+0.75z+\frac{2.5}{1-0.5z^{-1}}+\frac{1.058}{1-0.25z^{-1}}$$

taking inverse z-transform.

$$\boxed{x[n] = \delta[n+2]+0.75\,\delta[n+1]+2.5(0.5)^n u[n]+1.058(0.25)^n u[n]}$$

— 1M

"OR"

10.a) A discrete LTI system is characterized by following difference equation: $y[n] = y[n-1] + y[n-2] + x[n-1]$

i) Find the system function

ii) Indicate RoC if system is stable

iii) Indicate RoC if system is causal

iv) Obtain impulse response in both cases.

[Total - 10M]

→ i) System Function:-

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

taking z-transform,

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$$

$$Y(z)[1 - z^{-1} - z^{-2}] = z^{-1}X(z) \quad \text{— 1M}$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z}{z^2 - z - 1}$$

$$\boxed{H(z) = \frac{z}{(z-1.61)(z+0.618)}} \quad \text{— 1M}$$

Poles are at 1.61 & −0.61

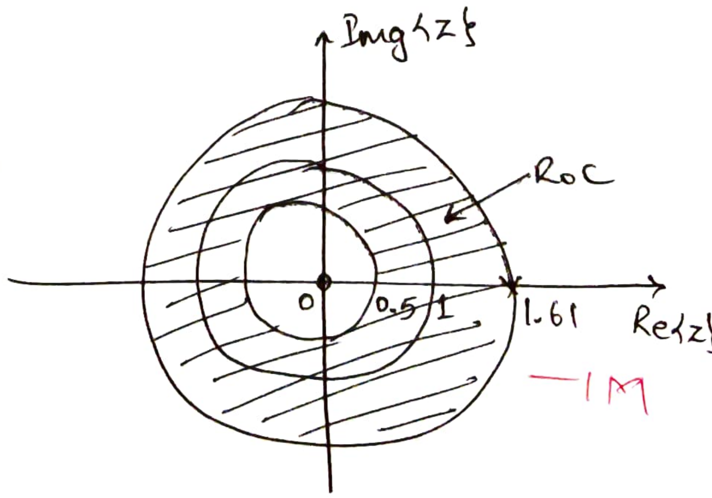
zeros are at 0.

ii) RoC of stable system:-

RoC of stable system includes unit circle.

RoC: $-0.61 < |z| < 1.61$

— 1M

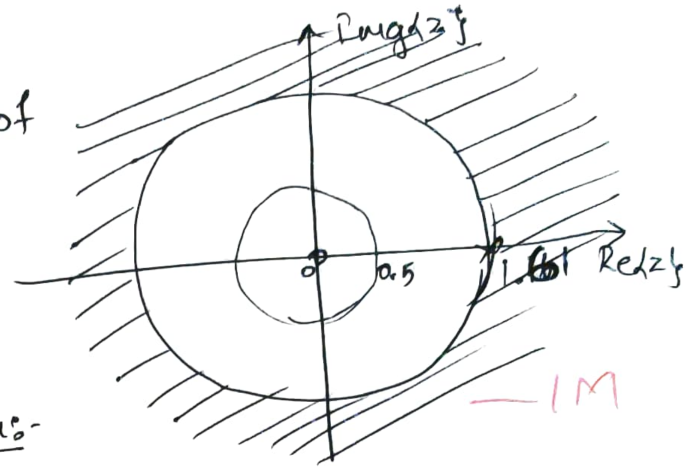


— 1M

iii) RoC of causal system:

For causal system, RoC is outside of circle of finite radius.

RoC: $|z| > 1.61$ — 1M



— 1M

iv) Impulse response of stable system:-

$$H(z) = \frac{z}{(z-1.61)(z+0.61)}, \quad RoC: -0.61 < |z| < 1.61.$$

$$\frac{H(z)}{z} = \frac{1}{(z-1.61)(z+0.61)} = \frac{A}{(z-1.61)} + \frac{B}{(z+0.61)} \quad \text{— 1M}$$

$$A = \left.\frac{1}{(z+0.61)}\right|_{z=1.61} = 0.45, \quad B = \left.\frac{1}{z-1.61}\right|_{z=-0.61} = -0.45$$

$$\frac{H(z)}{z} = \frac{0.45}{(z-1.61)} - \frac{0.45}{(z+0.61)} \quad \text{— 1M}$$

$$H(z) = \frac{0.45}{1-1.61z^{-1}} - \frac{0.45}{1+0.61z^{-1}} \quad \text{with } |z| < 1.61 \text{ \& } |z| > -0.618$$

— 1M

Here RoC is $|z| < 1.61$, hence time domain sequene, corresponding to pole 1.61 will be non causal. And $|z| > -0.61$, hence time domain sequence corresponding to pole -0.61 will be causal. Taking inverse z-transform of H(z) with this RoC,

$$\boxed{h[n] = 0.45\left[-(1.61)^n u[-n-1] - (-0.618)^n u[n]\right].}$$

— 1M

This is unit sample response of a stable system.

10.b) State and prove the following properties of Z-transform:

i) Time-reversal ii) Multiplication by exponential function.

[Total - 4M]

→ i) Time-reversal:-

If, $Z\{x[n]\} = X(z)$, $z \in R_x$.

then, $Z\{x[-n]\} = X(1/z)$, $z \in 1/R_x$.

let us consider,

$$Z\{x[-n]\} = \sum_{n=-\infty}^{\infty} x[-n] z^{-n}$$

Put, $-n = l$

$$Z\{x[-n]\} = \sum_{l=\infty}^{-\infty} x[l] z^{l} \quad \text{— 1M}$$

$$= \sum_{l=-\infty}^{\infty} x[l] (z^{-1})^{-l}$$

$$Z\{x[-n]\} = X(1/z) \; ; \; z^{-1} \in R_x \text{ or } z \in 1/R_x$$

— 1M

ii) Multiplication by exponential function:-

If, $Z\{x[n]\} = X(z)$; $R_{x^-} < |z| < R_{x^+}$

— 1M

$$Z\{a^n x[n]\} = X(z)\Big|_{z \to \frac{z}{a}} \; ; \; |a| R_{x^-} < |z| < |a| R_{x^+}$$

$$Z\{a^n x[n]\} = \sum_{n=-\infty}^{\infty} a^n x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left(\frac{z}{a}\right)^{-n}$$

$$Z\{a^n x[n]\} = X\left(\frac{z}{a}\right) \quad \text{— 1M}$$

$$R_{x^-} < \left|\frac{z}{a}\right| < R_{x^+} = |a| R_{x^-} < |z| < |a| R_{x^+}$$

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10.c) Find the z-transform of the following signals:-

i) $x[n] = 2^n u[-n-3]$ ii) $x[n] = \sin(\frac{\pi}{8}n - \frac{\pi}{4}) u[n-2]$

[Total - 6M].

→ i) $x[n] = 2^n u[-n-3]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u[-n-3] z^{-n} = \sum_{n=-\infty}^{-3} (2z^{-1})^n = \sum_{n=3}^{\infty} (2^{-1}z)^n \quad - 1M$$

$$X(z) = \frac{(2^{-1}z)^3}{1-2^{-1}z}$$

$$\boxed{X(z) = \frac{-0.25z^3}{z-2}} \quad - 1M \qquad |2^{-1}z| < 1 \text{ or } |z| < 2. \quad - 1M$$

ii) $x[n] = \sin(\frac{\pi}{8}n - \frac{\pi}{4}) u[n-2]$.

$$x[n] = \left[\sin(\tfrac{\pi}{8}n)\cos(\tfrac{\pi}{4}) - \cos(\tfrac{\pi}{8}n)\sin(\tfrac{\pi}{4})\right] u[n-2] \quad - 1M$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(z) = \frac{1}{\sqrt{2}} \sum_{n=2}^{\infty} \sin(\tfrac{\pi}{8}n) z^{-n} - \frac{1}{\sqrt{2}} \sum_{n=2}^{\infty} \cos(\tfrac{\pi}{8}n) z^{-n}$$

$$X(z) = \frac{1}{\sqrt{2}} \left[\sum_{n=0}^{\infty} \sin(\tfrac{\pi}{8}n) z^{-n} - \sum_{n=0}^{\infty} \cos(\tfrac{\pi}{8}n) z^{-n}\right] - [0 + 0.38z^{-1}] - [1 + 0.92z^{-1}] \quad - 1M$$

$$X(z) = \frac{1}{\sqrt{2}} \left[\frac{1 - z^{-1}\sin(\frac{\pi}{8})}{1 - 2z^{-1}\cos(\frac{\pi}{8}) + z^{-2}} - \frac{1 - z^{-1}\cos(\frac{\pi}{8})}{1 - 2z^{-1}\cos(\frac{\pi}{8}) + z^{-2}}\right] - 1 - 1.3z^{-1}$$

$$\boxed{X(z) = \frac{1}{\sqrt{2}} \left[\frac{0.54z^{-1}}{1 - 2z^{-1}\cos(\frac{\pi}{8}) + z^{-2}}\right] - 1 - 1.3z^{-1}} \quad - 1M.$$

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