GBCS SCHEME

Fourth Semester B.E. Degree Examination
Dec 2019 | Jan 2020

Kinematics of Machines

Max. Marks ÷ 100

Module-1

1)
a) Explain: (i) Kinematic Pair. (06 marks)
(ii) Types of links.
(iii) Grashaf's Criterian.

Ans:- (i) Kinematic Pair :-

When two kinematic links or elements of a machine are connected together in such a way that, their motion is either completely or successfully constrained, then these two links are said to form a kinematic pair.

[2 Marks]

Classification.

A) According to the type of relative motion between the links.
i) Sliding pair ii) Turning pair iii) Rolling pair
iv) Screw pair V) Spherical pair.

B) According to the type of contact between links.
i) Lower pair ii) Higher pair.

c) According to the mechanical arrangement
i) Self closed pair ii) Forced closed pair.

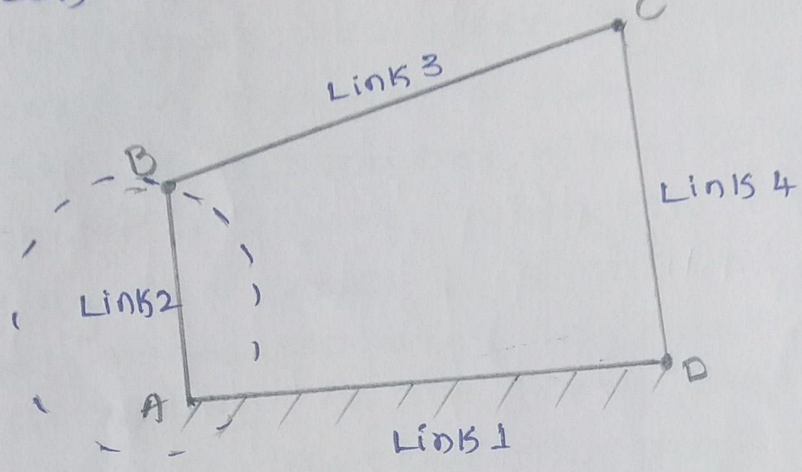
[2 Marks]

(ii) Types of Links

Kinematic link or element is a resistant body or an assembly of resistant bodies which become a part or parts of a machine connecting other parts which have motion relative to it
i) Rigid link ii) Flexible link iii) Fluid link.

iii) Grashof's Criterion

For a Planar four bar linkage, the sum of the shortest and the longest link length cannot be greater than the sum of the remaining two links lengths, if there is a continous relative motion between the two members. (2M)



$$l_2 + l_3 \leq l_1 + l_4$$

1)

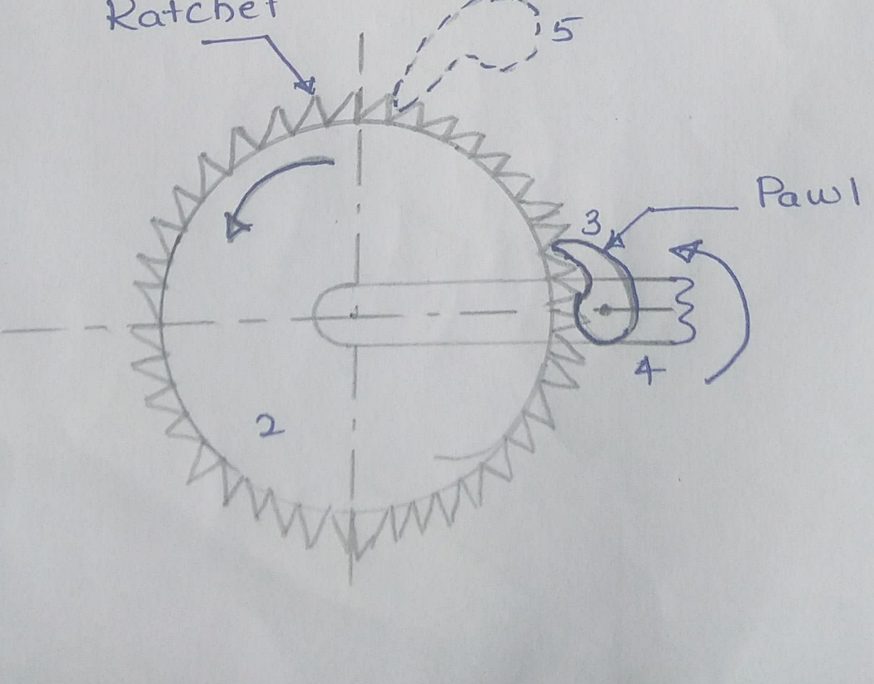
b) Explain with neat sketches: (10 Marks)

(i) Ratchet and Pawl Mechanism

(ii) Toggle mechanism.

Ratchet and Pawl Mechanism

(5M)

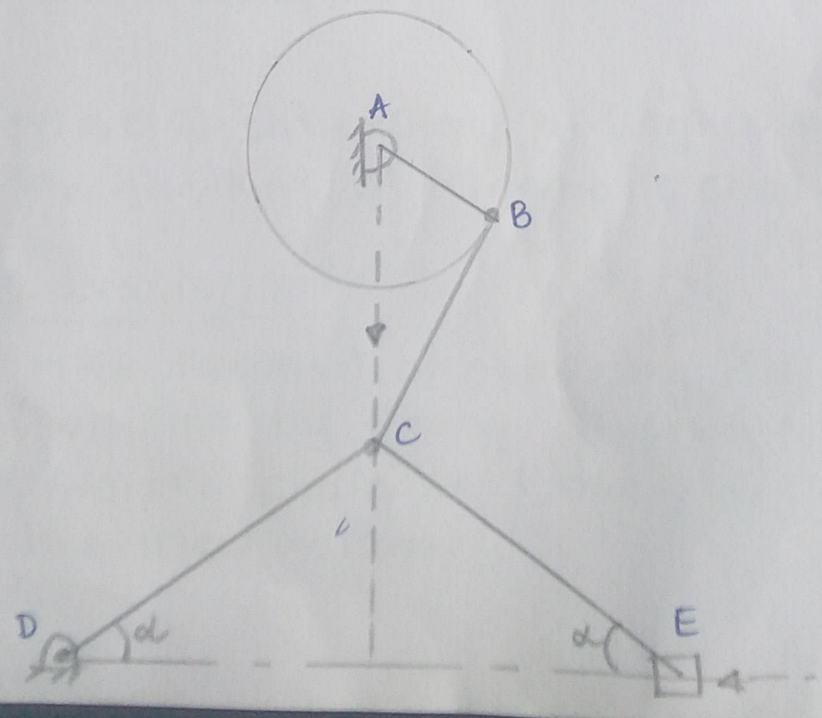


Intermittent motion means that the motion is not Continous but it is Ceased at definite intervals. There are many instances where it is necessary to Convert Continous motion in to intermittent motion.

This mechanism is used in producing intermittent rotary motion from on oscillating or reciprocating motion member. A ratchet & pawl mechanism consists of a ratchet wheel 2 & pawl 3 as shown in figure. When the lever 4 carrying the pawl 3 is raised, the ratchet wheel rotates in counter clock wise direction. As the lever carrying the pawl 3 is lowered, the pawl 3 slides over the ratchet teeth. One more pawl 5 is used to prevent the ratchet from reversing. Ratchets are used in feed mechanisms, lifting jacks, clocks, watches & Counting devices.

Toggle Mechanism

(5M)



In Slider Crank mechanism as the crank approaches one of its dead-centre position the Slider approaches Zero. The ratio of the crank movement to the Slider movement approaching infinity is proportional to the mechanical advantage. This is the principle used in toggle mechanism. A toggle mechanism is used when a large force act through a Small distance is required.

Resolving the forces at G vertically

$$F \tan\alpha = \frac{P}{2}$$

$$F = \frac{P}{2\tan\alpha}$$

Thus for a given value of P, as the links CD & CE approaches collinear position.

Toggle mechanisms are used in toggle clamps, rivetting machines, punch presses, Stone crushers, etc. The toggle principle is also used in switches, circuit breakers.

Q1)

c) Define a) Inversion b) Degree of freedom c) Mechanism d) Kinematic chain. (4 Marks)

i) <u>Inversion</u>

The method of obtaining different mechanisms by fixing different links in a kinematic chain is known as inversion of the mechanism. (1M)

ii) Degree of Freedom

Degree of freedom of a pair is defined as the number of independent relative motions, both translational & rotational a pair can have. (1M)

iii) Mechanism (1M)

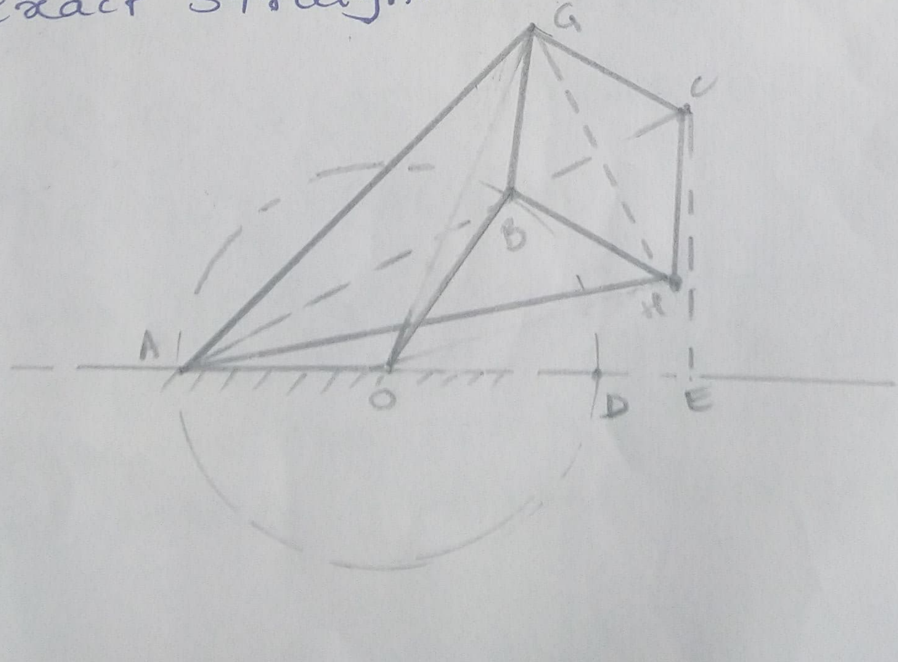
When one of the links of a kinematic chain is fixed, then the chain is called as mechanism.

iv) Kinematic Chain

When kinematic pairs are connected in such a manner that, the last link is joined to the first link to transmit completely or successfully constrained motion, then it is called as kinematic chain. (1M)

Q2)

a) Explain the construction and working of Peaucillier's mechanism with a neat sketch. Prove that it generates an exact straight line.



(5M)

→ It is an exact straight line motion mechanism. It consists of eight links, link AO is fixed and all pairs are turning pairs.

→ The pin at B is constrained to move along a circular path with O as a centre.

$BG = GC = CH = HB$,

$AG = AH$ & $OA = OB$

As link OB rotates about O, pin at C describes exact straight line.

(AB × AC remains constant)

Proof.

join GH to bisect BC at F

Consider ΔAFG

$$AG^2 = AF^2 + FG^2 \quad \text{———} (1)$$

(5 M)

Consider ΔGFC

$$GC^2 = FG^2 + CF^2 \quad \text{———} (2)$$

Substracting eqt'n (2) from Eqt'n (1)

$$AG^2 - GC^2 = (AF^2 + FG^2) - (FG^2 + CF^2)$$

$$AG^2 - GC^2 = AF^2 - CF^2 = (AF + CF)(AF - CF)$$

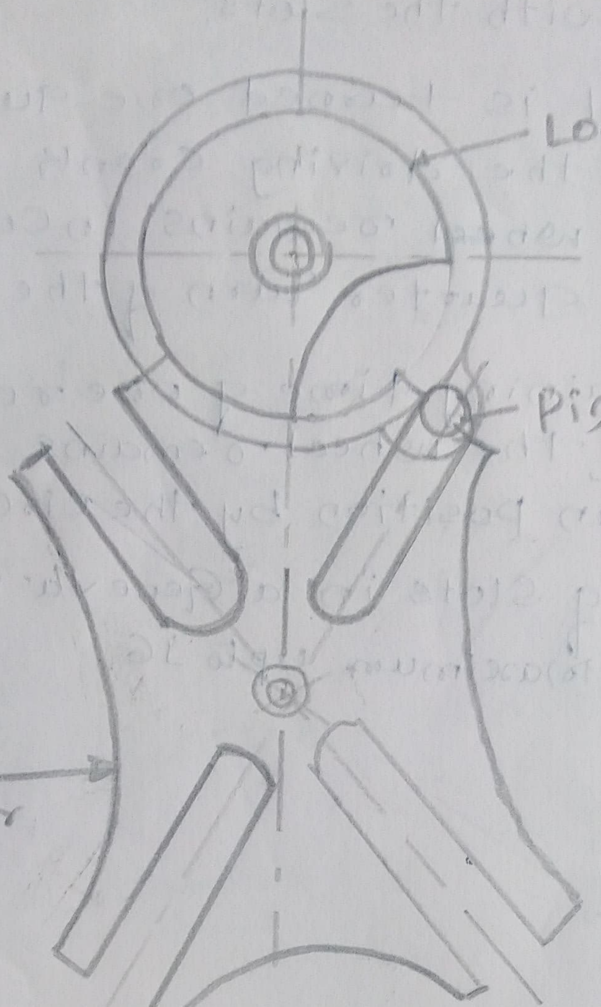
$$AG^2 - GC^2 = (AF + CF)(AF - BF)$$

$$AG^2 - GC^2 = (AC)(AB)$$

links AG & GC are constants, hence the product of AC & AB is constant. It means, the point C traces a straight path which is perpendicular to AE.

Q2)

b) With neat Sketch, explain Geneva wheel Mechanism.



Sketch = 4 m

→ Geneva Mechanism is a Cam like Mechanism which provides intermittant rotary motion

→ It is used for indexing of a work table in both low & high Speed machinery.

→ It is also used in motion - picture projection to provide intermittant advance of the film.

→ In this Mechanism, driving Crank Carries pin P which engages in a Slot of the Geneva wheel.

→ It is important to note that the centre line of the Slot & Crank are mutually ⊥ler at engagement & disengagement.

6M

- Usually Crank rotates at a uniform angular velocity and carries a roller to engage with the Slots.
- Geneva wheel is turned one quarter revolution of the driving crank it means the pin P of wheel remains in contact during one quarter turn of the wheel
- During remaining time of one revolution of the crank, the wheel remains at rest i.e, locked in position by the circular arcs.
- The number of slots in a Geneva wheel may be 3 or maximum up to 16.

Module-2

Q 3) The Crank of a Slider Crank Mechanism is 480 mm long & rotates at 20 rad/sec in the counter clockwise direction. It has a connecting rod of 1600 mm long. Determine the following when the crank is at 60° from IDC. Determine.

(i) velocity of slider

(ii) Angular velocity of connecting rod

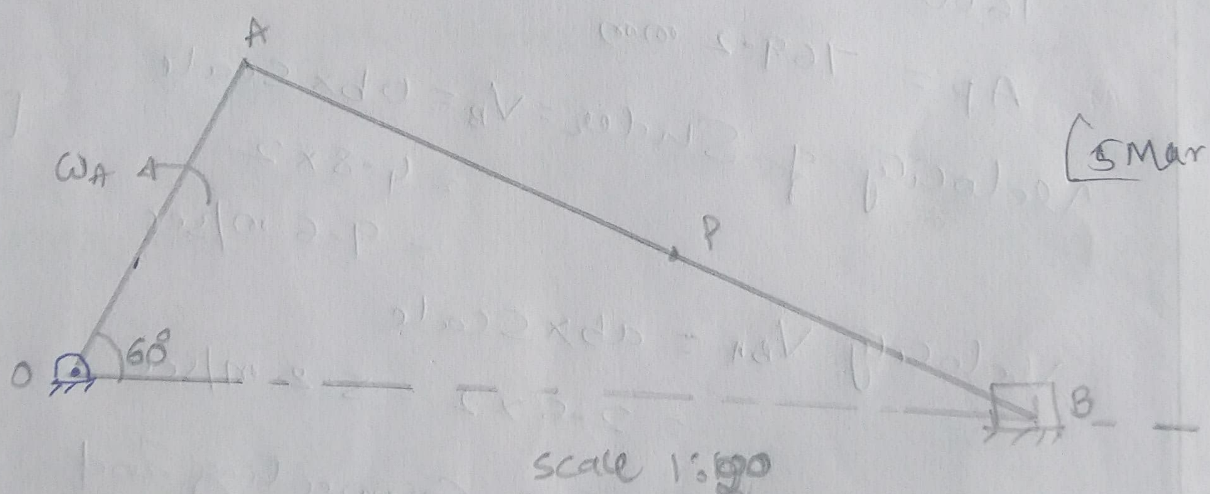
(iii) position & velocity of a point 'P' on the connecting rod and having least absolute velocity.

(20 Marks)

Solution

$$V_A = \omega_A \times OA = 20 \times 0.48 = 9.6 \text{ m/sec}$$

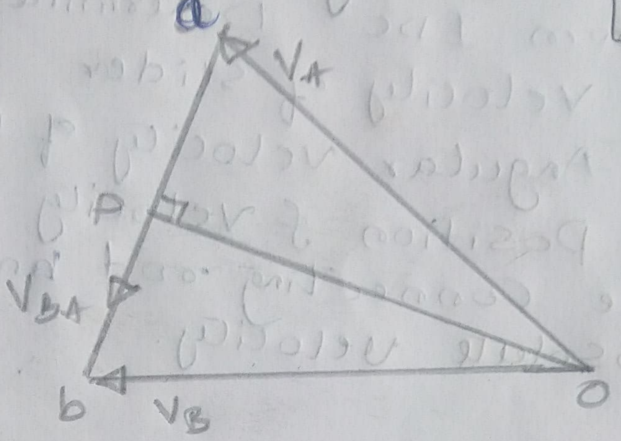
Space diagram



(5 Marks)

Velocity diagram*

[10 Marks]



$$\frac{AP}{AB} = \frac{ap}{ab}$$

$$\frac{AP}{1600} = \frac{12.5}{26}$$

$$AP = 769.2 \text{ mm}$$

Velocity of Slider $= V_B = ob \times \text{scale}$ [5 Marks]

$$= 4.8 \times 2$$

$$= 9.6 \text{ m/sec}$$

Velocity $V_{BA} = ab \times \text{Scale}$

$$= 2.6 \times 2 = 5.2 \text{ m/sec}$$

∴ Angular velocity of Connecting Rod

$$\omega_{BA} = \frac{V_{B/A}}{BA} = \frac{5.2}{1.6} = 3.25 \frac{\text{rad}}{\text{sec}}$$

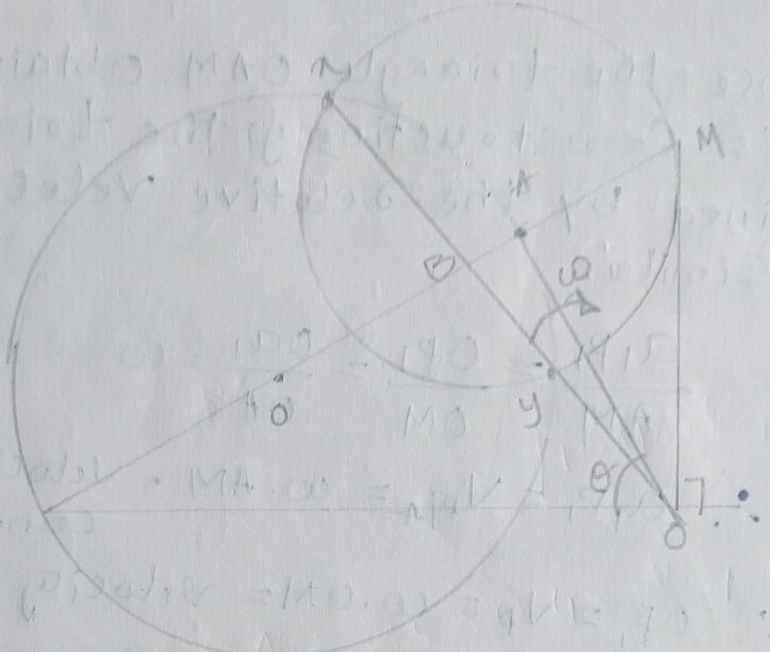
Velocity of point P $= V_P = op \times \text{Scale}$

$$= 4.6 \times 2$$

$$\boxed{V_P = 9.2 \text{ m/sec}}$$

Q4)

a) Explain Klein's Construction for Slider Crank Mechanism (10 marks)



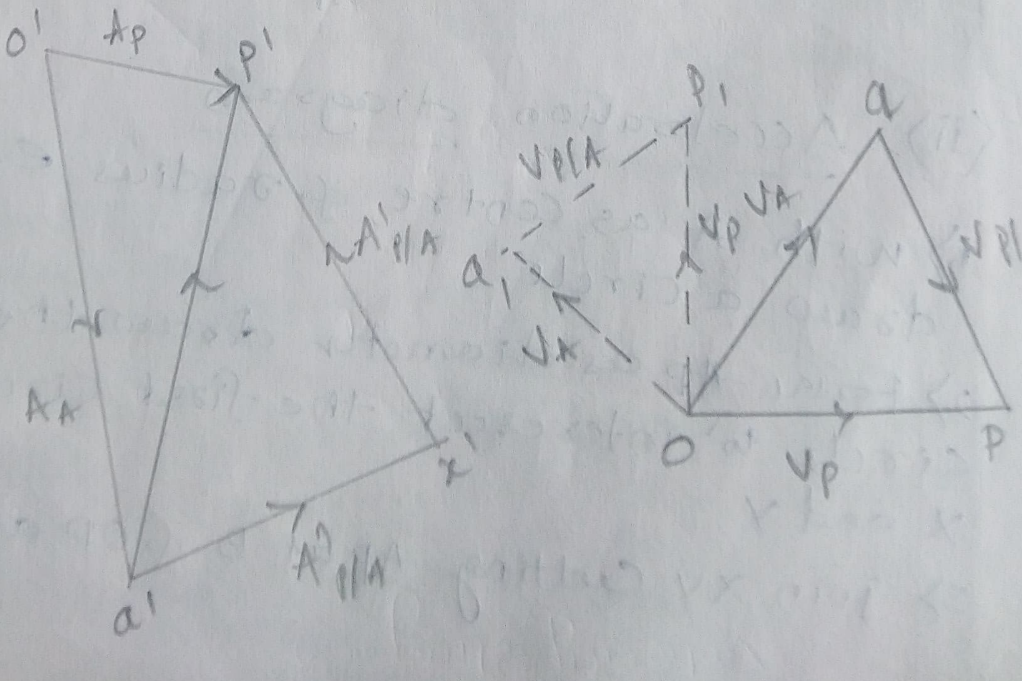
Velocity diagram

1) Draw the Slider Crank Mechanism OAP of the given position of Crank OA as shown in figure.

2) Draw a $\perp^{lar}$ from O to meet the extension of PA at M.

3) The triangle OAM is known as Klein's velocity diagram.

[5 Marks]



4) The velocity diagram obtained by Relative method is shown in fig which is rotated by 90° as triangle oap. & indicated by dotted line as oa_1p_1

5) Since the triangle OAM obtained by Klein's construction & the triangle oa_1p_1 obtained by the relative velocity method are similar.

$$\frac{a_1p_1}{AM} = \frac{op_1}{OM} = \frac{oa_1}{OA} = \omega$$

$\therefore a_1p_1 = V_{P/A} = \omega \cdot AM$ = velocity of connecting rod

$\therefore op_1 = V_P = \omega \cdot OM$ = velocity of piston

$oa_1 = V_A = \omega \cdot OA$ = velocity of crank

6) Also

$V_{P/A} = \omega_{PA} \times AP$

$\omega_{P/A} \times AP = \omega \times AM$

$\therefore$ Angular velocity of connecting rod

$$\omega_{CR} = \omega_{AP} = \frac{V_{P/A}}{AP} = \omega \cdot \frac{AM}{AP}$$

[5 Marks]

(ii) <u>Acceleration diagram</u>

1) With A as centre, & radius equal to AM draw a circle.

2) take AP as diameter draw the second circle to intersect the first circle at points X and Y

3) Join XY cutting AP at B & OP at C.

4) join OABC which forms a quadrilateral & known as Klein's acceleration diagram.

5) Since the quadrilateral OABC obtained by Kelein's construction of the quadrilateral shown in fig.

$$\frac{o'a'}{OA} = \frac{a'x'}{AB} = \frac{x'p'}{BC} = \frac{p'o'}{CO} = \omega^2$$

$o'a' = \omega^2 \times OA$ = Acceleration of crank OA

$a'x' = \omega^2 \times AB = A^n_{A/B}$ = Normal acceleration of connecting rod

$x'p' = \omega^2 \times BC = A^t_{P/A}$ = Tangential acceleration of connecting rod.

$p'o' = \omega^2 \times OC = A_P$ = Acceleration of piston

$$A^t_{P/A} = \alpha_{PA} \times AP$$

$$\omega^2 \times BC = \alpha_{PA} \times AP$$

$$\therefore \alpha_{PA} = \omega^2 \times \frac{BC}{AP}$$ = Angular Acceleration of connecting rod.

Q4)

b) Define instantaneous Centre & State, explain the types of instaneous centres. [10 Marks]

Definition :-

(i) An instant centre is a point on one body about which some other body is rotating either permanently or at the instant

and

(ii) An instant centre is a point common to two bodies having the same linear velocity in both magnitude & direction.

Number of Instantaneous Centres

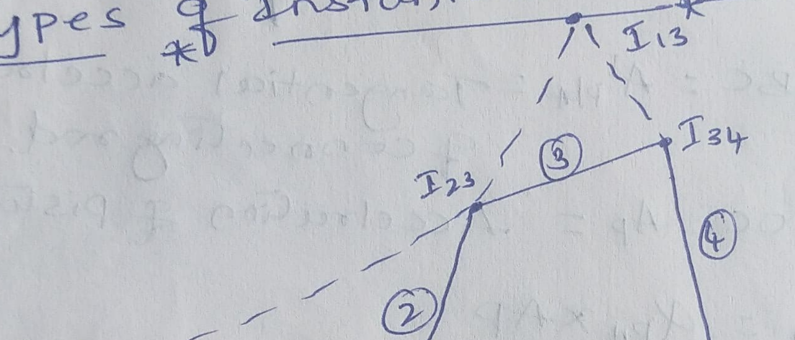
The number of instantaneous Centres in a Constrained Kinematic chain is equal to the number of possible combination of two links i.e. number of Combinations of 'n' links taken two at a time.

∴ Number of instantaneous Centres [4 Marks]

$$N = \frac{n(n-1)}{2}$$

where n = number of links or bodies.

Types of Instantaneous Centres [6 Marks]



1> Fixed Instantaneous Centres

From fig I_{12} & I_{14} are fixed instantaneous centres bcz they do not change their positions for all Configurations of the mechanism. These Centres are located at pin-joints of links 1 & 2 (I_{12}) & pin joints of links 1 & 4 (I_{14})

2> Permanent Instantaneous Centres

From fig I_{23} & I_{34} are permanent instantaneous centres bcz they move when the mechanism moves but the joints are of Permanent nature. These Centres are located at pin-joints of links 2 & 3 (I_{23}) & pin joints of links 3 & 4 (I_{34}).

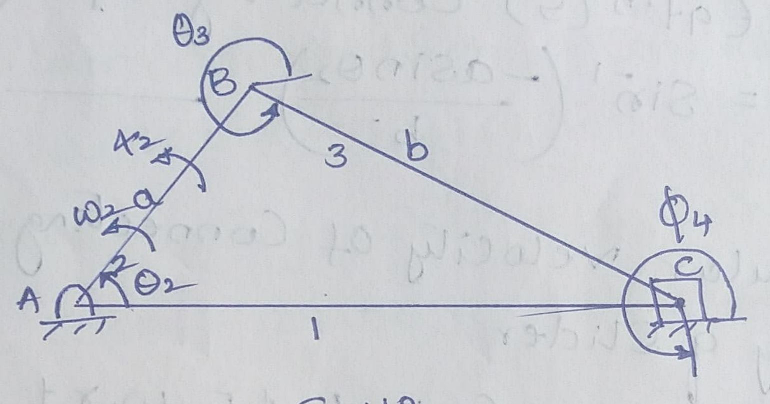
3) Neither fixed nor permanent instantaneous centres: From fig I_{13} & I_{24} are neither fixed nor permanent instantaneous Centres, bcz they vary with the configuration of the mechanism.

Q 5)

a) Using Complex algebra derive expression for velocity & acceleration of the Piston & angular acceleration of connecting rod for a reciprocating engine mechanism. Use these expressions to find the above, if the crank length is 50mm, connecting rod is 200mm long, crank angle is 30°. The crank rotates at a constant speed of 3000 rpm.

[20 Marks]

Ans:-



loop Closure Eqt'n

$-\bar{d}+\bar{a}+\bar{b}=0$

i.e $\bar{d}=\bar{a}+\bar{b}$ ——— (1)

Expressing the vectors in exponential or Complex number in polar form

$\bar{d} = de^{i\theta_1} = d(\cos\theta_1 + i\sin\theta_1) = d \quad (\theta_1 = 0)$

$\bar{a} = ae^{i\theta_2} = a(\cos\theta_2 + i\sin\theta_2)$

$\bar{b} = be^{i\theta_3} = b(\cos\theta_3 + i\sin\theta_3)$

Eqt'n (1) Can be written as

$d = ae^{i\theta_2} + be^{i\theta_3}$ ——— (2)

i.e $d = a(\cos\theta_2 + i\sin\theta_2) + b(\cos\theta_3 + i\sin\theta_3)$ —— (3)

(i) Connecting rod angle, θ_3

The real part of equation 3 or Displacement along x-axis is

$$d = a\cos\theta_2 + b\cos\theta_3$$

$$b\cos\theta_3 = d - a\cos\theta_2 \quad \text{—— (4)}$$

Imaginary part of Eqt'n (3)

$$0 = a\sin\theta_2 + b\sin\theta_3$$

$$b\sin\theta_3 = -a\sin\theta_2 \quad \text{—— (5)}$$

From Eqt'n (5) Connecting Rod angle

$$\theta_3 = \sin^{-1}\left(\frac{-a\sin\theta_2}{b}\right) \quad \text{—— (6)}$$

(ii) Angular velocity of Connecting Rod & velocity of Slider

Differentiating Eqt'n 4 & 5 w.r.t time

$$-b\omega_3\sin\theta_3 = V_c + a\sin\theta_2\,\omega_2$$

$$-a\omega_2\sin\theta_2 - b\omega_3\sin\theta_3 - V_c = 0 \quad \text{—— (8)}$$

$$b\cos\theta_3\,\omega_3 = -a\cos\theta_2\,\omega_2$$

$$a\cos\theta_2\,\omega_2 + b\cos\theta_3\,\omega_3 = 0 \quad \text{—— (9)}$$

Multiply Eqt'n 8 by $\cos\theta_3$ & Eqt'n (9) by $\sin\theta_3$ & add.

i.e

$$-a\omega_2\sin\theta_2\cos\theta_3 - b\omega_3\sin\theta_3\cos\theta_3 + V_c\cos\theta_3$$

$$+ a\cos\theta_2\,\omega_2\sin\theta_3 + b\cos\theta_3\,\omega_3\sin\theta_3 = 0$$

$$-a\omega_2\sin\theta_2\cos\theta_3 + a\omega_2\cos\theta_2\sin\theta_3 - V_c\cos\theta_3 = 0$$

$$a\omega_2(\sin\theta_3\cos\theta_2 - \cos\theta_3\sin\theta_2) - V_c\cos\theta_3 = 0$$

$$a\omega_2\sin(\theta_3 - \theta_2) - V_c\cos\theta_3 = 0$$

Velocity of Slider

$$V_c = \frac{a\omega_2\sin(\theta_3 - \theta_2)}{\cos\theta_3} \quad \text{——— (10)}$$

$$\omega_3 = -\frac{a\omega_2\cos\theta_2}{b\cos\theta_3}$$

Angular accelration of Connecting Rod and accelration of piston

Differentiating Eqt'n 8 & 9 w.r.t. time

$$-a\omega_2^2\cos\theta_2 - a\sin\theta_2\alpha_2 - b\omega_3^2\cos\theta_3 - b\sin\theta_3\alpha_3 - A_c = 0 \quad \text{——— (12)}$$

$$-a\omega_2^2\sin\theta_2 + a\cos\theta_2\alpha_2 - b\omega_3^2\sin\theta_3 + b\cos\theta_3\alpha_3 = 0 \quad \text{——— (13)}$$

Multiply Eqt'n (12) by $\cos\theta_3$ & Eqt'n (13) by $\sin\theta_3$ & add

$$-a\omega_2^2\cos\theta_2\cos\theta_3 - a\sin\theta_2\alpha_2\cos\theta_3 - b\omega_3^2\cos^2\theta_3$$
$$- b\sin\theta_3\alpha_3\cos\theta_3 - A_c\cos\theta_3 - a\omega_2^2\sin\theta_2\sin\theta_3$$
$$+ a\cos\theta_2\alpha_2\sin\theta_3 - b\omega_3^2\sin^2\theta_3 + b\cos\theta_3\alpha_3\sin\theta_3 = 0$$

i.e $-a\omega_2^2(\sin\theta_3\sin\theta_2 + \cos\theta_3\cos\theta_2) + a\alpha_2(\sin\theta_3\cos\theta_2 - \cos\theta_3\sin\theta_2) - b\omega_3^2(\sin^2\theta_3 + \cos^2\theta_3) - A_c\cos\theta_3 = 0$

Acceleration of piston

$$A_c = \frac{a\alpha_2 \sin(\theta_3-\theta_2) - a\omega_2^2 \cos(\theta_3-\theta_2) - b\omega_3^2}{\cos\theta_3} \quad \text{——— (14)}$$

$$\alpha_3 = \frac{a\omega_2 \sin\theta_2 - a\cos\theta_2\,\alpha_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3} \quad \text{——— (15)}$$

(b) Data

$a = 50mm = 0.05m$

$b = 200mm = 0.2m$

$\theta_2 = 30^\circ$

$N_2 = 3000\ rpm$

Sol'n

Angular velocity of crank

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi\, 3000}{60} = 314.16\ \frac{rad}{sec}$$

$\alpha_2 = 0$

(i) Connecting rod angle

$$\theta_3 = \sin^{-1}\left(-\frac{a\sin\theta_2}{b}\right)$$

$$\theta_3 = \sin^{-1}\left(-\frac{0.05\times\sin 30}{0.2}\right)$$

$\theta_3 = 352.82^\circ$

(ii) Angular velocity of Connecting Rod

$$\omega_3 = -\frac{a\omega_2 \cos\theta_2}{b\cos\theta_3} = -\frac{0.05\times 314.16\times\cos 30}{0.2\times\cos 352.82}$$

$\omega_3 = 68.56\ \frac{Rad}{sec}$ (cw)

$$\omega_3 = -\frac{300 \times \sin(60 - 286.194)}{360 \times \sin(13.806 - 286.194)} \times (-10)$$

$$\omega_3 = 6.02 \ \frac{rad}{sec}$$

Angular velocity of CD

$$\omega_4 = \frac{q \sin(\theta_2 - \theta_3)}{s . \sin(\theta_3 - \theta_4)} \times \omega_2$$

$$\omega_4 = \frac{300 \sin(60 - 13.80)}{360 \sin((13.806 - 286.194)} \times (-10)$$

$$\omega_4 = -6.02 \ \frac{rad}{sec}$$

Angular Accelration of link BC, α_3

$$= \frac{\omega_3}{\omega_2}\alpha_2 - \frac{q . \omega_2^2 \cos(\theta_2 - \theta_4) + r\omega_3^2 \cos(\theta_3 - \theta_4) + s\omega_4^2}{r . \sin(\theta_3 - \theta_4)}$$

$$= \frac{6.02}{(-10)}(-30) - \frac{300 \times (-10)^2 \cos(60 - 286.194) + 360 \times 6.02^2 \cos(13.806 - 286.194) + 360(-6.02)^2}{360 \times \sin(13.806 - 286.194)}$$

$$= 38.01 \ \frac{rad}{sec^2}$$

Angular accelration of link CD

$$\alpha_4 = \frac{\omega_4}{\omega_2}\alpha_2 + \frac{q\omega_2^2 \cos(\theta_2 - \theta_3) + r\omega_3^2 + s\omega_4^2 \cos(\theta_3 - \theta_4)}{s . \sin(\theta_3 - \theta_4)}$$

$$= \frac{(-6.02)}{(-10)} \times (-30) + \frac{300(-10)^2 \cos(60 - 13.80) + 360 \times 6.02^2 + 360 \times (-6.02)^2 \cos((13.806 - 286.19)}{360 \sin(13.806 - 286.194)}$$

$$= 77.24 \ \frac{rad}{sec^2}$$

Let BD = t & now

$$\frac{\sin\theta_2}{t} = \frac{\sin\alpha}{q}$$

$$\frac{\sin 60}{519.615} = \frac{\sin\alpha}{300}$$

$$\boxed{\alpha = 30^\circ}$$

Δ^{le} BCD, applying cosine rule

$$CD = s = \sqrt{BD^2 + BC^2 - 2BD \cdot BC \cos\phi}$$

$$= \sqrt{t^2 + r^2 - 2t \cdot r\cos\phi}$$

$$360 = \sqrt{519.615^2 + 360^2 - 2 \times 519.61 \times 360 \cos\phi}$$

$$\boxed{\phi = 43.80^\circ}$$

$$\theta_3 = \phi - \alpha = 43.80 - 30$$

$$\boxed{\theta_3 = 13.806^\circ}$$

Also $\frac{\sin\beta}{r} = \frac{\sin\phi}{s}$

$$\frac{\sin\beta}{360} = \frac{\sin 43.80}{360}$$

$$\boxed{\beta = 43.806^\circ}$$

$$\alpha + \beta = 30 + 43.80 = 73.806^\circ$$

$$\theta_4 = 360 - (\alpha + \beta)$$

$$\theta_4 = 360 - 73.80$$

$$\boxed{\theta_4 = 286.194^\circ}$$

Angular velocity of link AB

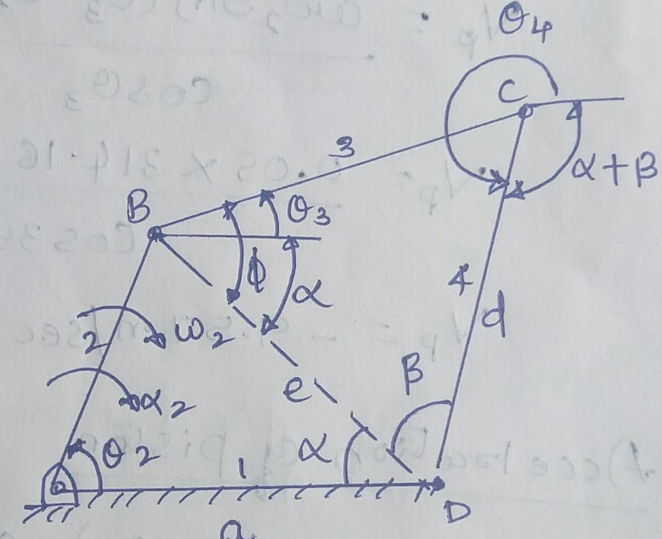
$$\omega_2 = -10 \text{ rad/sec}$$

Angular velocity of link BC

$$\omega_3 = -\frac{q\sin(\theta_2 - \theta_4)}{r\sin(\theta_3 - \theta_4)} \times \omega_2$$

Q6. In a 4-bar mechanism ABCD, link AB = 300 mm, BC = 360 mm, CD = 360 mm & the fixed link AD is 600 mm. The angle BAD = 60°. The link AB has an angular velocity of 10 rad/sec and angular acceleration of 30 rad/sec² both CW. Determine the angular velocity & angular acceleration of link BC & CD by using Complex algebra method. (20 marks)

Solution



$AD = p = 600$ mm $CD = s = 360$ mm

$AB = q = 300$ mm $\theta_2 = 60°$

$BC = r = 360$ mm $\omega_2 = -10 \frac{rad}{sec}$

$\alpha_2 = -30 \frac{rad}{sec^2}$

Δ^{le} ABD, applying cosine rule

$$BD = \sqrt{AB^2 + AD^2 - 2AB \cdot AD \cos BAD}$$

$$= \sqrt{q^2 + p^2 - 2qp\cos 60}$$

$$= \sqrt{300^2 + 600^2 - 2 \times 300 \times 600 \times \cos 60}$$

$BD = 519.615$ mm [5M]

(iii) Angular acceleration of connecting rod

$$\alpha_3 = \frac{a\omega_2^2 \sin\theta_2 - a\cos\theta_2\alpha_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3}$$

$$\alpha_3 = \frac{0.05 \times 314.16^2 \times \sin 30 - 0 + 0.2 \times (-68.56)^2 \sin 352.8}{0.2 \times \cos 352.82}$$

$$\alpha_3 = 11842.43 \ \frac{rad}{sec^2} \ (ccw) \quad [5M]$$

(iv) Velocity of piston

$$V_p = \frac{a\omega_2 \sin(\theta_3 - \theta_2)}{\cos\theta_3}$$

$$V_p = \frac{0.05 \times 314.16 \times \sin(352.82 - 30)}{\cos 352.82}$$

$$V_p = -9.57 \ m/sec \quad [5M]$$

(v) Acceleration of piston

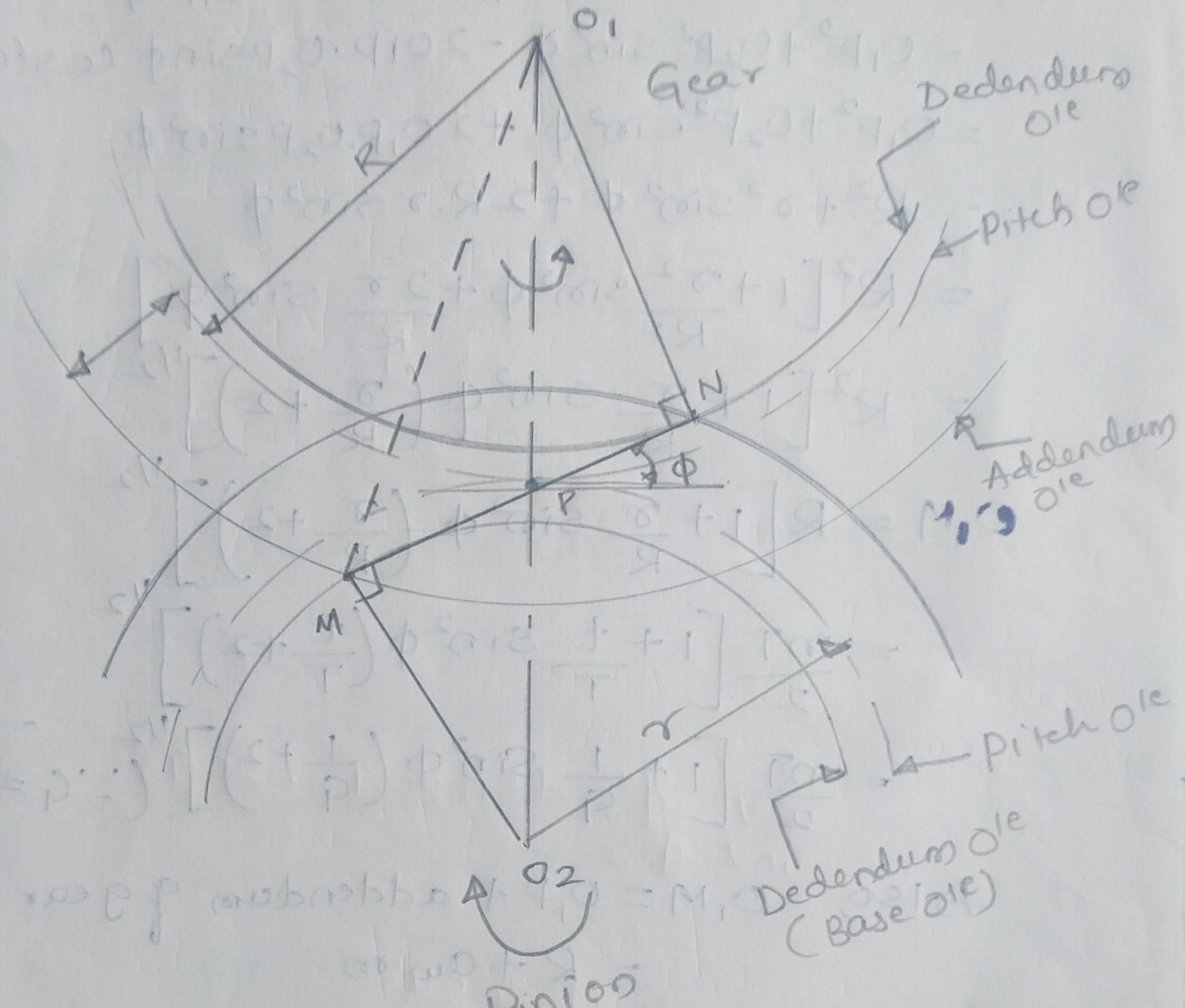
$$A_P = \frac{a\alpha_2 \sin(\theta_3 - \theta_2) - a\omega_2^2 \cos(\theta_3 - \theta_2) - b\omega_3^2}{\cos\theta_3}$$

$$= \frac{0 - 0.05 \times 314.16^2 \times \cos(352.82 - 30) - 0.2 \times (-68.56)^2}{\cos 352.82}$$

$$A_p = -4910.4 \ \frac{m}{sec^2} \quad [5 \text{ Marks}]$$

Module - 4

Q7) a) Derive an expression for minimum number of teeth on pinion to avoid interference. [10 Marks]



Let ϕ = pressure angle

R = Pitch circle radius of gear $= \frac{mT}{2}$

r = pitch circle radius of pinion $= \frac{mt}{2}$

T = Number of teeth on gear.

t = Number of teeth on pinion

m = module

a_w = Addendum Constant of gear

a_p = Addendum Constant of pinion

$a_w \cdot m$ = Addendum of gear

$a_p \cdot m$ = Addendum of pinion

G = Gear ratio $= \frac{T}{t}$

join O_1 to M and from Δ^{le} O_1MP applying cosine rule.

$$O_1M^2 = O_1P^2 + PM^2 - 2O_1P \cdot PM \cos O_1PM$$
$$= O_1P^2 + O_2P^2 \sin^2\phi - 2O_1P \cdot O_2P\sin\phi \cos(90+\phi)$$
$$= O_1P^2 + O_2P^2 \sin^2\phi + 2O_1P \cdot O_2P \sin^2\phi$$
$$= R^2 + r^2\sin^2\phi + 2R \cdot r\sin^2\phi$$
$$= R^2\left[1 + \frac{r^2}{R^2}\sin^2\phi + \frac{2r}{R}\sin^2\phi\right]$$
$$= R^2\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R}+2\right)\right]^{1/2}$$
$$O_1M = R\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R}+2\right)\right]^{1/2}$$
$$= \frac{mT}{2}\left[1 + \frac{t}{T}\sin^2\phi\left(\frac{t}{T}+2\right)\right]^{1/2}$$
$$= \frac{mT}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G}+2\right)\right]^{1/2} \quad \left(\because G = \frac{T}{t}\right) \quad \text{--- (1)}$$

Also, $O_1M = O_1P$ + addendum of gear

$$= R + a_w \cdot m$$
$$= \frac{mT}{2} + a_w \cdot m \quad \text{--- (2)}$$

Equating (1) & (2)

$$\frac{mT}{2} + a_w m = \frac{mT}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G}+2\right)\right]^{1/2}$$
$$a_w = \frac{T}{2}\left[\left\{1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G}+2\right)\right\}^{1/2} - 1\right]$$
$$T = \frac{2a_w}{\left\{1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G}+2\right)\right\}^{1/2} - 1}$$

= Number of teeth on gear to avoid interferences

Number of teeth on pinion to avoid interference $= t = \frac{T}{G}$.

Q7
b) A 2.5 mm module, 20° pinion with 36 teeth drives a gear with 60 teeth. If the centre distance is increased by 0.65 m Calculate.
(i) The radii of the operating pitch circle
(ii) The operating pressure angle.
(iii) Backlash produced.
[10 Marks]

Sol^n. Given:

$\phi = 20^\circ$ $m = 2.5\,mm$
$t = 36$ $T = 60$
$\Delta_c = 0.65$

(i) Standard pitch circle radius of pinion

$$r = \frac{mt}{2} = \frac{2.5 \times 36}{2}$$

$r = 45\,mm$

∴ Standard pitch circle radius of gear.

$$R = \frac{mT}{2} = \frac{2.5 \times 60}{2}$$

$R = 75\,mm$

∴ Standard centre distance $c = r + R$

$c = 45 + 75$
$c = 120\,mm$

Operating centre distance

$c' = c + \Delta_c = 120 + 0.65$

$c' = 120.65\,mm$

We know

$$\frac{r}{r'} = \frac{c}{c'}$$

$$\frac{45}{r'} = \frac{120}{120.65}$$

Operating pitch circle radius of pinion $r' = 45.24\,mm$

Operating pitch cle radius of gear

$R' = c' - r' = 120.65 - 45.24$

$R' = 75.41 \text{ mm}$

(ii) Since, $c \cos\phi = c' \cos\phi'$

~~[illegible]~~

$120 \cos 20 = 120.65 \cos\phi'$

$\cos\phi' = 0.934$

Operating pressure angle $\phi' = 20.93^\circ$

Now, $\text{inv.}\,\phi = \tan\phi - \phi$

$= \tan 20 - 20 \times \frac{\pi}{180}$

$\text{inv.}\,\phi = 0.0149$

$\text{inv.}\,\phi' = \tan\phi' - \phi'$

$\text{inv.}\,\phi' = \tan 20.93^\circ - 20.93^\circ$

$\text{inv.}\,\phi' = -20.54$

Back lash $B = 2c'\left[\text{inv}\,\phi' - \text{inv}\,\phi\right]$

$= 2\left[120.65\right]\left[-20.54 - 0.0149\right)$

Q 8) An epicyclic gear train, the internal wheels A, B & the compound wheel C & D rotate independently about the axis "O". The wheel E & F rotate on pin fixed to the arm G. E gears with A and C, & F gears with B & D. All the wheels have same pitch & the number of teeth on E & F are 18, C = 28 D = 26

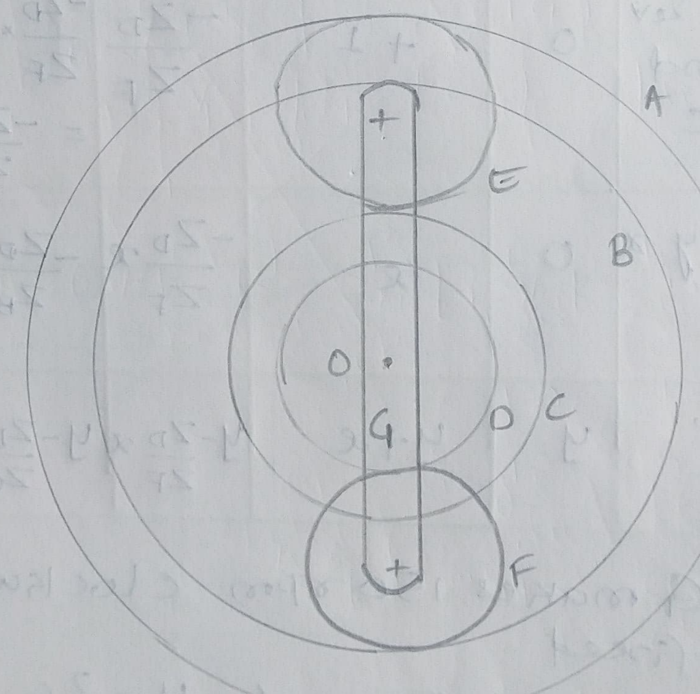
(i) Sketch arrangement.

(ii) Find the number of teeth on A & B

(iii) If arm G makes 150 rpm CW & A is fixed find speed of B. [20 Marks]

Sol^n (i)

[Diagram 5M]



Given

$Z_F = 18 \quad Z_E = 18 \quad Z_C = 28 \quad Z_D = 26$

$n_L = 150$ rpm (CW)

(ii) Number of teeth on A and B

$$r_A = r_C + 2r_E$$

$$Z_A = Z_C + 2Z_E$$

$$Z_A = 28 + 2(18)$$

$$Z_A = 64$$

Now, $r_B = r_D + 2r_F$

$$Z_B = Z_D + 2Z_F \qquad \text{[5 Marks]}$$

$$Z_B = 26 + 2 \times 18$$

$Z_B = 62$ = Number of teeth on gear B.

Condition of Motion	Arm G	Compound gear D & C	Gear F	Gear B	Gear E	Gear A
Fix the arm G & give +1 rev to compound gear D & C	0	+1	$-\frac{Z_D}{Z_F}$	$-\frac{Z_D}{Z_F} \times \frac{Z_F}{Z_B}$ $= -\frac{Z_D}{Z_B}$	$-\frac{Z_C}{Z_E}$	$-\frac{Z_C}{Z_E} \times \frac{Z_E}{Z_A}$ $= -\frac{Z_C}{Z_A}$
Mutiply by x	0	x	$-\frac{Z_D}{Z_F}x$	$-\frac{Z_D}{Z_B}x$	$-\frac{Z_C}{Z_E}x$	$-\frac{Z_C}{Z_A}x$
Add 'y'	y	$y+x$	$y-\frac{Z_D}{Z_F}x$	$y-\frac{Z_D}{Z_B}x$	$y-\frac{Z_C}{Z_E}x$	$y-\frac{Z_C}{Z_A}x$

(iii) Arm G makes 150 rpm clockwise & wheel A is fixed

i.e $y = 150$ & $y - \frac{Z_C}{Z_A}x = 0$

$$150 - \frac{28}{64}x = 0$$

$$x = 342.86$$

[10 Marks]

Speed of B $= y - \frac{Z_D}{Z_B}\cdot x$

$$= 150 - \frac{26}{62}(342.86)$$

Speed of gear B = 6.22 rpm (CW)

Q.No 9

Module -5

Construct the Profile of a Cam to Suit the following Specification.

Cam shaft Diameter = 40mm

Least radius of CAM = 25mm

Diameter of roller = 25mm

Angle of lift = 120°

Angle of fall = 150°

Lift of the follower = 40mm

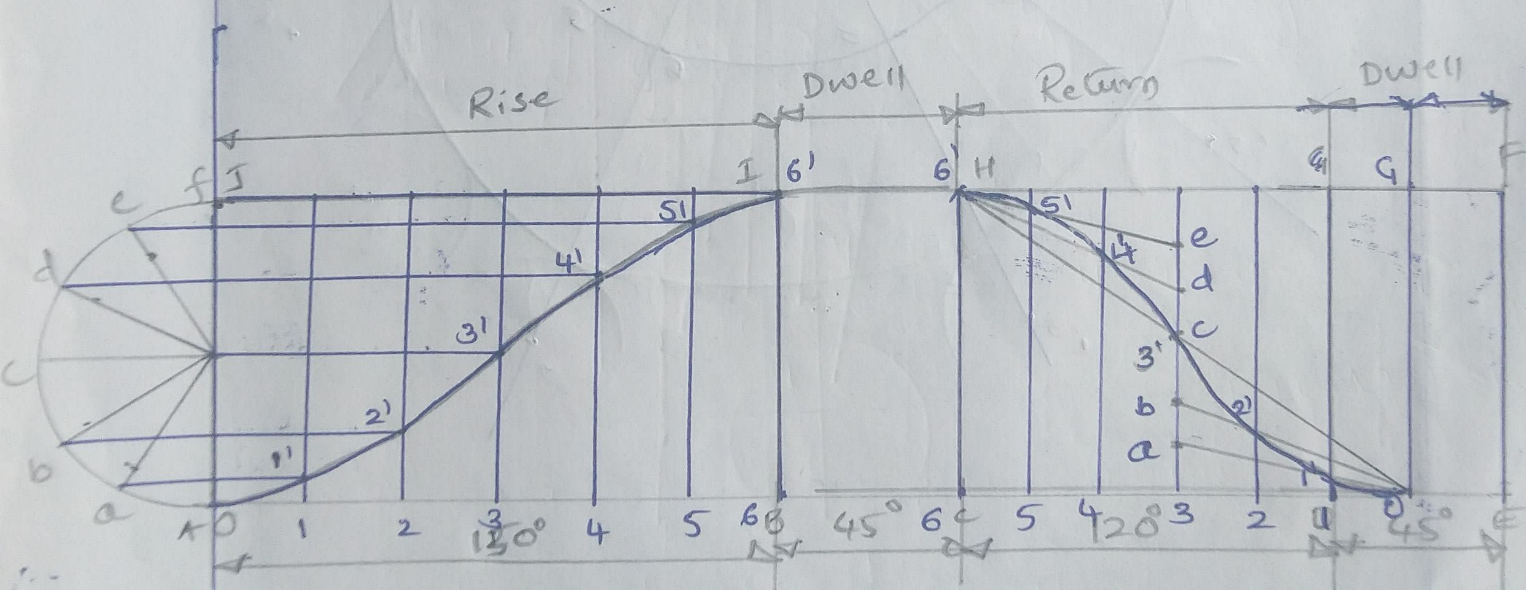
Number of Pauses are two of equal interval between motions. During the lift the motion is SHM. During the fall the motion is UARM. the speed of the Cam shaft is uniform. the line of the Stroke is Center of the Cam.

[20 Marks]

Sol'n

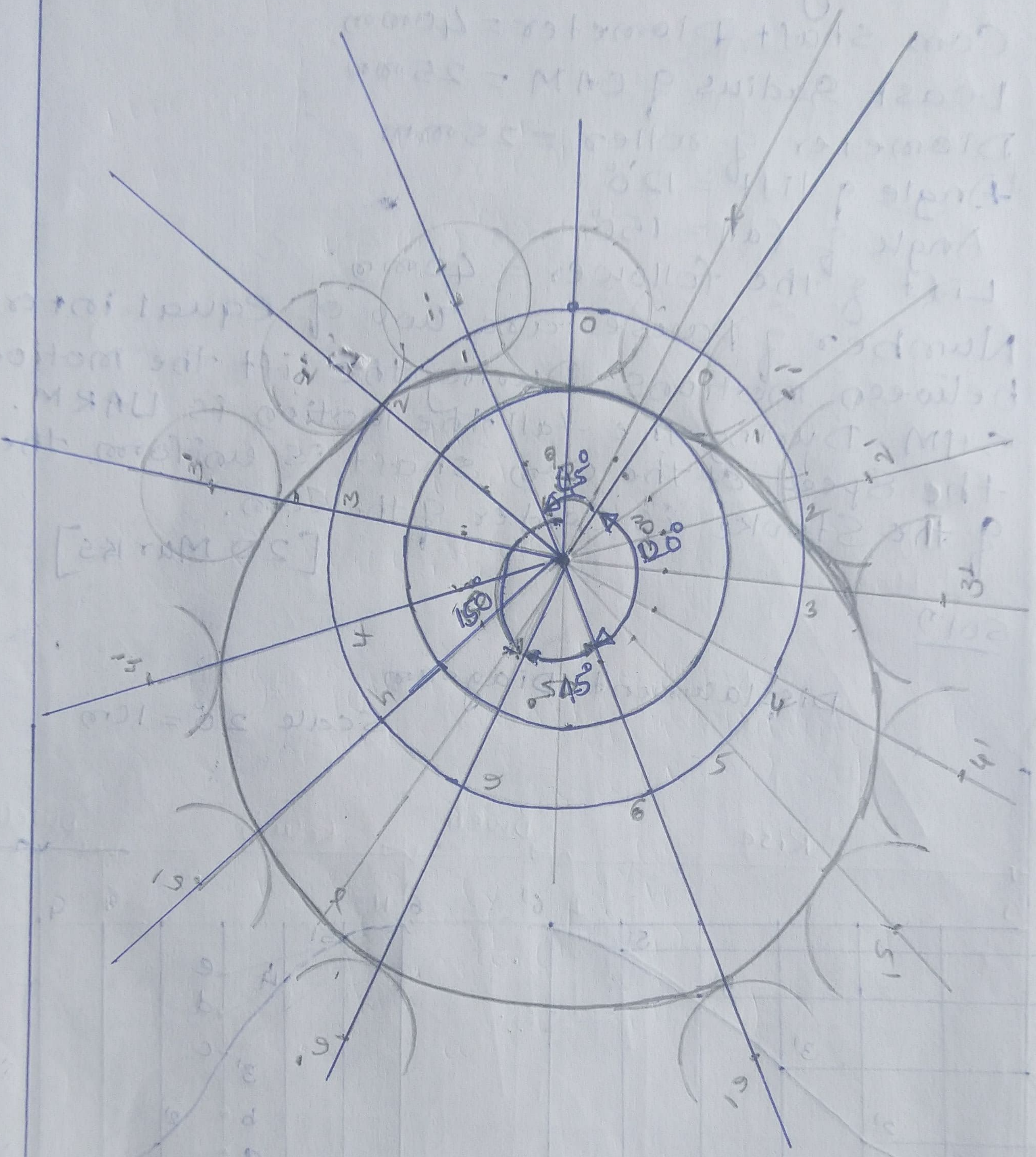
Displacement Diagram

Scale 20° = 1cm



[8 Marks]

[12 Marks]



Q10

a) Define the following terms related to cam:
(i) Lift (ii) Dwell (iii) Pressure angle (iv) Base angle
(8 marks)

Ans:-

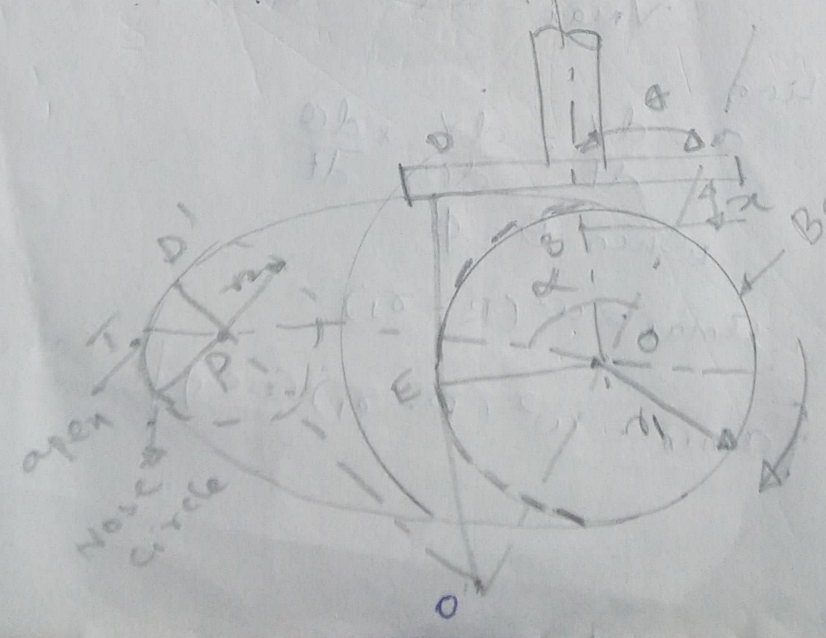
(i) Lift :- It is the maximum displacement of the follower from its lowest position to the topmost position. [2 Marks]

(ii) Dwell :- It is the period during which the follower is at rest. [2 Marks]

(iii) Pressure angle :-
It is the angle between the normal to the pitch curve & the instantaneous direction of the follower motion. [2 marks]

(iv) Base angle :-
It is the smallest circle drawn to the cam profile from the cam centre. [2 Marks]

(b) Derive an expression for displacement, velocity & acceleration for a circular arc cam operating a flat faced follower where the contact is on the circular flank.
[12 Marks]



[Diagram 3 Marks]

Let $r_1 = OB =$ Least base circle radius
$r_2 =$ Nose circle radius
$R = QD =$ Flank circle radius
$d =$ Distance between the centre of cam & nose circles.
$\alpha =$ angle of ascent
$\phi =$ Angle of contact on circular flank.

Displacement

$$x = BC = OC - OB = DE - r_1$$
$$= (QD - QE) - r_1$$
$$= (R - OQ\cos\theta) - r_1$$
$$= R - (R - r_1)\cos\theta - r_1$$
$$\boxed{x = (R - r_1)(1 - \cos\theta)}$$ [2 Marks]

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$
$$= (R - r_1)(\sin\theta)\omega$$
$$\boxed{v = \omega(R - r_1)\sin\theta}$$ [2 Marks]

From this Eqn it is evident that at the begining of the ascent, the velocity is Zero (when $\theta = 0$) & it increases with θ. It will be maximum when the follower is just shift from our flank to circular nose.

$$v_{max} = \omega(R - r_1)\sin\phi$$

Acceleration

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

when $\theta = \phi$

$$a_{max} = \omega^2(R - r_1)$$

[5 Marks]

$$\boxed{a_{min} = \omega^2(R - r_1)\cos\phi}$$