Design of Machine Elements - II

Aug / Sept. 2020

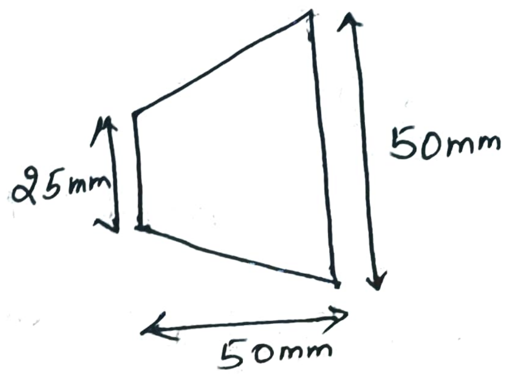
Module - 1

1.a Write a difference between straight & curved beam

Solⁿ

	Curved Beam	Straight Beam
1.	Neutral axis does not co-incide with CG axis	Neutral axis coincide with CG axis
2.	Non linear distribution of stress	Linear distrubution of stress
3.	general expression $\sigma_b = \frac{-M_b y}{Ae(R_1 - y)}$	general expression $\sigma_b = \frac{M_b}{I} \cdot y$
4.	Stress are not proportional to distance of fibres from neutral axis	Stress are proportional to distance of fibres from neutral axis

1.b The Cross-section of curved link is a symmetrical trapezium 50mm deep. The Inner width & outer width are 50mm & 25mm. Find max stress if link carries a load of 15 KN which passes through centre of link. Internal radius of link 50mm.



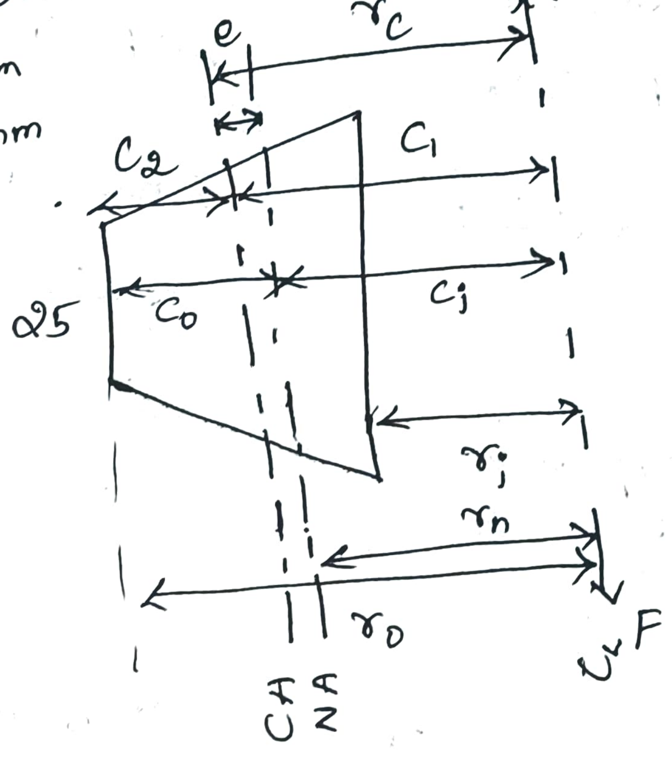
Given

$h = 50\,mm$

$F = 15\,KN$ $\quad r_i = 50\,mm$

$b_i = 50\,mm$

$b_o = 25\,mm$



Soln

$r_i = 50\,mm$

$r_o = 50 + 50 = 100\,mm$

Distance of centroidal axis from Inner fibre

$$C_1 = \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right) = \frac{50}{3}\left(\frac{50 + 2\times 25}{50 + 25}\right)$$

$$C_1 = 22.22\,mm$$

Radius of centroidal axis $r_c = r_i + c_1 = 50 + 22.2$

$\therefore \boxed{r_c = 72.2\,mm}$

Radius of neutral axis $r_n = \dfrac{\frac{1}{2} h (b_i + b_o)}{\left(\dfrac{b_i r_o - b_o r_i}{h}\right) \ln\left(\dfrac{r_o}{r_i}\right) - (b_i - b_o)}$

$$r_n = \frac{\frac{1}{2}\, 50\,(50+25)}{\left(\dfrac{50 \times 100 - 25 \times 50}{50}\right) \ln\left(\dfrac{100}{50}\right) - (50-25)}$$

$\boxed{r_n = 69.48\,mm}$

Distance of neutral axis to Centroidal axis

$e = r_c - r_n = \cancel{69.48}\ 72.2 - 69.48$

$\boxed{e = 2.74\,mm}$

Distance of neutral axis to Inner radius

$c_i = r_n - r_i = 69.48 - 50 = 19.48\,mm$

Distance of neutral axis to outer radius

$c_o = r_o - r_n = 100 - 69.48 = 30.52\,mm$

Area of C/S $= A = \frac{1}{2} (b_i + b_o)^{\times h} = \frac{1}{2} (50+25) \times 50$

$A = 1875\,mm^2$

Bending moment about C.G axis $= M_b = F \times l$

$$M_b = 15 \times 10^3 \times 72.22 = 1.08 \times 10^6 \text{ N-mm}$$

Direct stress $= \sigma_d = \dfrac{F}{A} = \dfrac{15 \times 10^3}{1875} = 8 \text{ N/mm}^2$

Bending stress at inner fibre $= \sigma_{bi} = \dfrac{M_b C_i}{A e r_i}$

$$\sigma_{bi} = \frac{1.08 \times 10^6 \times 19.48}{1875 \times 2.74 \times 50}$$

$$\sigma_{bi} = 81.9 \text{ N/mm}^2$$

Bending stress at outer fibre $= \sigma_{bo} = -\dfrac{M_b C_o}{A e r_o}$

$$\sigma_{bo} = -\frac{1.08 \times 10^6 \times 30.52}{1875 \times 2.74 \times 100}$$

$$\sigma_{bo} = -64.15 \text{ N/mm}^2$$

Combined stress at inner fibre $= \sigma_d + \sigma_{bi} = 8 + 81.9$

$$= 89.9 \text{ N/mm}^2$$

Combined stress at outer fibre $= \sigma_d + \sigma_{bo} = 8 - 64.15$

$$= -56.15 \text{ N/mm}^2$$

Max shear stress $= \tau_{max} = 0.5 \times \sigma_{max}$

$$\tau_{max} = 0.5 \times 89.9 = 44.95 \text{ N/mm}^2$$

$\therefore$ Max stress $\sigma_{max} = 89.9 \text{ N/mm}^2$

2. A tube with 50mm & 75mm as inner & outer dia is reinforced by shrinking jacket of outer dia 100mm. Compound tube is to withstand internal pressure of 35MPa. Shrinkage allowance is such that max tangential stress in each tube has same magnitude. Find shrinkage pressure & original dimensions of tube. Assume $E = 207\,kN/mm^2$

Given

$d_i = 50\,mm$ $d_c = 75\,mm$ $d_o = 100\,mm$ $P_i = 35\,MPa$

$E = 206\,kN/mm^2$

Solution

Tangential stress distribution due to internal fluid pressure P_i

$$\sigma_t = \frac{P_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{4r^2}\right) \quad \text{when } P_o = 0.$$

Tangential stress at outer surface of outer cylinder due to internal pressure P_i

$$\sigma_{too} = \frac{2P_i d_i^2}{d_o^2 - d_i^2} = \frac{2 \times 35 \times 50^2}{100^2 - 50^2}$$

$$\sigma_{too} = 23.3\ N/mm^2$$

$$\sigma_{toi} = \sigma_{tio} = \frac{P_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_c^2}\right)$$

$$= \frac{35 \times 50^2}{100^2 - 50^2}\left(1 + \frac{100^2}{75^2}\right)$$

$$= 32.4 \text{ N/mm}^2$$

$$\sigma_{tii} = \frac{P_i d_i^2}{d_o^2 - d_i^2}\left(1 + \frac{d_o^2}{d_i^2}\right)$$

$$= \frac{35 \times 50^2}{100^2 - 50^2}\left(1 + \frac{100^2}{50^2}\right) = 58.3 \text{ N/mm}^2$$

Tangential stress distribution due to shrink fit

$$\sigma_{t-oo} = \frac{2 P_c d_c^2}{d_o^2 - d_c^2} = \frac{2 \times P_c \times 75^2}{100^2 - 75^2} = 2.57 P_c$$

$$\sigma_{t-oi} = P_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) = P_c\left(\frac{100^2 + 75^2}{100^2 - 75^2}\right)$$

$$= 3.57 P_c = 3.57 \times 3.6 = 12.9 \text{ N/mm}^2$$

$$\sigma_{t-oi} = $$

$$\sigma_{t-io} = -P_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right) = -P_c\left(\frac{\cancel{100}75^2 + 50^2}{75^2 - 50^2}\right)$$

$$= -2.6 P_c = -2.6 \times 3.61 = -9.4 \text{ N/mm}^2$$

$$\sigma_{t-io} = $$

$$\sigma_{tii} = -P_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right) = -P_c$$

$$\sigma_{t-ii} = \frac{-2P_c d_c^2}{d_c^2 - d_i^2} = \frac{-2P_c \times 75^2}{75^2 - 50^2}$$

$$\sigma_{tii} = -3.6P_c = -3.6 \times 3.61 = -13.01 N/mm^2$$

Shrinkage Pressure

Equating stress at inner fibre of tube & Jacket

$$\sigma_{tii} + \sigma'_{tii} = \sigma'_{toi} + \sigma_{toi}$$

$$58.33 - 3.6P_c = 32.41 + 3.57P_c$$

$$\therefore P_c = 3.6 N/mm^2$$

original dimensions [Assume $\nu = 0.29$]

Change in dia of inner member

$$\Delta d_i = \frac{-P_c d_c}{E}\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \nu\right)$$

$$\Delta d_i = -\frac{3.6 \times 75}{206 \times 10^3}\left(\frac{75^2 + 50^2}{75^2 - 50^2} - 0.29\right)$$

$$\Delta d_i = -3.03 \times 10^{-3} mm$$

Change in dia of outer member

$$\Delta d_o = \frac{P_c d_c}{E}\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \nu\right)$$

$$\Delta d_o = \frac{3.6 \times 75}{206 \times 10^3}\left(\frac{100^2 + 75^2}{100^2 - 75^2} + 0.29\right)$$

$\Delta d_o = 5.08 \times 10^{-3}$ mm

Outside dia of Tube $= d_s = d_c + \Delta d_i = 75 + 3.03 \times 10^{-3}$

$d_s = 75.00303$ mm

outside dia Jacket $= d_h = d_e - \Delta d_o = 75 - 5.08 \times 10^{-3}$

$d_h = 74.994$ mm.

Total Interference $= \delta = \Delta d_i + \Delta d_o$

$\delta = 3.038 \times 10^{-3} + 5.08 \times 10^{-3}$

$\delta = 8.123 \times 10^{-3}$ mm

Resultant tangential stress distribution

Tangential stress due to internal pressure & shrink fit at outside dia of outer member

$= \sigma'_{too} - \sigma_{too} = 23.33 + 9.29 = 32.62 \, N/mm^2$

Internal Pressure + Shrink fit at inside dia of outer member $= 32.41 + 12.91 = 45.3 \, N/mm^2$

Internal Pressure + Shrink fit at outside dia of Inner member $= 32.41 - 9.4 = 23.01 \, N/mm^2$

Internal Pressure + Shrink fit at inside dia of inner member $= 58.33 - 13.01 = 45.32 \, N/mm^2$

Total result & tangential stress at matting surface $= 23.01 + 45.32 = 68.3 \, N/mm^2$

Module - 2

3.a Explain effects of slip, Creep & Centrifugal tension in flat belt drive

* **Slip**

When frictional grip becomes insufficent there is possibilty of motion of driver without carrying belt with it, this is termed as slip in belts

* **Creep**

Due to continuous mmt of belt over the pulleys, belts are subjected to continuous expansion & contraction, due to which there is relative moment of belts over pulleys. This is termed as creep

* **Centrifugal Tension**

Due to rotation of pulley there will be Centrifugal force which acts away from pulley. Tensions created because of this Centrifugal force both on tight & slack side is known as Centrifugal tension.

b) Specify the details of V-belt to drive for a 10KW, 1160 rpm induction motor operating a fan at 400rpm. Centre distance between pulley is 1m & $\alpha = 34^0$.

Given

$P = 10KW$ $\quad n_1 = 1160$ rpm $\quad n_2 = 400$ rpm

$C = 1m = 1000mm$ $\quad \alpha = 34^0$

Solution

As given $P = 10KW$ c/s of belt selected is "C" as it is in power range of 10-70KW

$\therefore d_e = 300$ mm $\quad K_d$ for $\frac{n_1}{n_2} = 2.9 = 1.14$

$\therefore d_e = d \times K_d \Rightarrow d = \frac{d_e}{K_d} = \frac{300}{1.14}$

$\therefore$ Pitch dia of smaller Pulley $d = d_1 = 263$ mm

$\therefore n_1 d_1 = n_2 d_2$

$1160 \times 263 = 400 \times d_2$

$\therefore d_2 = 762$ mm

Velocity

$V = \frac{\pi d_1 n_1}{60 \times 10^3} = \frac{\pi \times 263 \times 1160}{60 \times 10^3} = 15.9$ m/s

Power Capacity

$$KW = \left(2.01\, V^{-0.09} - \frac{194.8}{de} - 3.18\times10^{-4} V^2\right) \times 0.7355\, V$$

$$KW = \left(2.01\times15.9^{-0.09} - \frac{194.8}{300} - 3.18\times10^{-4}\times15.9^2\right)\times 0.7355\times15.9$$

$$KW = 9.8\, KW$$

Number of V-belts

$$n' = \frac{P\times K_S}{(KW)\, K_L\, K_a}$$

Assume 10-16 hrs/day $K_S = 1.2$

Pitch length

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$L = 2\times1000 + \frac{\pi}{2}(263+762) + \frac{(762-263)^2}{4\times1000}$$

$$L = 3672.3\, mm$$

$\therefore$ From Table 14.21 for std Length 4013 mm

$K_L = 1.02$

Angle of Contact

$$\theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right) = 2\cos^{-1}\left(\frac{762-263}{2\times1000}\right)$$

$$\theta = 151.1^\circ$$

$\therefore$ From Table 14.22 for $\theta = 151.1$ $K_a = 0.93$

$$n' = \frac{10 \times 1.12}{9.8 \times 1.02 \times 0.93}$$

$$n' = 1.29 \simeq 2 \text{ belts}$$

Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left(\frac{L}{4} - \frac{\pi(D+d)}{8}\right)^2 - \left(\frac{D-d}{8}\right)^2}$$

$$C = \frac{4013}{4} - \frac{\pi(762+263)}{8} + \sqrt{\left\{\frac{4013}{4} - \frac{\pi(762+263)}{8}\right\}^2 - \left(\frac{762-263}{8}\right)^2}$$

$$C = 21.28 \text{ mm}$$

Specification of belt

$W = 22\text{mm}$ $T = 14 \text{ mm}$

4.a One helical spring is nested inside another; dimensions are as tabulated. Both spring has same free length & carry load of 2500N.

	outer spring	Inner spring
No of coils	6	10
Wire dia	12.5	9
Mean coil dia	100	70

Determine (i) Max load carried by spring
(ii) Total deflection of each spring
(iii) Max stress in 2 springs

Take $G = 83 GN/m^2$

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$P = 2500N$ $\quad G = 83 \times 10^3 N/mm^2$ $\quad N_i = 10$ $\quad N_o = 6$

$D_i = 70mm$ $\quad D_o = 100mm$

$d_i = 9mm$ $\quad d_o = 12.5mm$

As both springs have same deflection

$\delta_i = \delta_o$

$$\therefore \frac{8 P_i D_i^3 N_i}{G d_i^4} = \frac{8 P_o D_o^3 N_o}{G d_o^4}$$

$$\frac{P_i \times 70^3 \times 10}{9^4} = \frac{P_o \times 100^3 \times 6}{12.5^4}$$

$$522.78 P_i = 245.76 P_o$$

$$\therefore P_i = 0.47 P_o \quad — (1)$$

Also $P_i + P_o = 2500 \quad — (2)$

By (1) & (2) $\quad 0.47 P_o + P_o = 2500$

$P_o = 1700.56 N$

$P_i = 0.47 \times 1700.56 = 799.26N$

$\therefore$ Max load $= P_o = 1700.56N$

(ii) Max deflection

$$\delta_i = \delta_o = \frac{8 P_o D_o^3 N_o}{G d_o^4} = \frac{8 \times 1700.56 \times 100^3 \times 6}{83 \times 10^3 \times 12.5^4}$$

$$\delta_i = \delta_o = 40.28 \text{ mm}$$

(iii) Max stress generated

$$C = \frac{D_o}{d_o} = \frac{100}{12.5} = 8$$

$$C = \frac{D_i}{d_i} = \frac{70}{9} = 7.77$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

Outer spring

$$\tau_o = K\left(\frac{8 P_o C}{\pi d_o^2}\right) = 1.184\left(\frac{8 \times 1700.56 \times 8}{\pi \times 12.5^2}\right)$$

$$\tau_o = 262.5 \text{ N/mm}^2$$

Inner spring

$$\tau_i = K\left(\frac{8 \times P_i C}{\pi \times d_i^2}\right) = 1.189\left(\frac{8 \times 799.26 \times 7.77}{\pi \times 9^2}\right)$$

$$\tau_i = 232.13 \text{ N/mm}^2$$

b) A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05m apart & central band is 85mm wide. The central load is to be 5.4KN with a permissible stress of 280N/mm², Determine thickness & width of steel spring leaves. The ratio of total depth to width of spring is 3. Also determine deflection of spring. $E = 0.26 \times 10^6$ MPa

Given

$n = 12 \quad n_f = 2 \quad n_g = 10 \quad L = 1050\,mm$

$l_b = 85\,mm \quad 2F = W = 5.4 \times 10^3\,N \quad \sigma_f = 280\,MPa$

$\frac{b'}{h} = 3 \quad E = 0.26 \times 10^6\,N/mm^2$

Permissible stress

$$\sigma_f = \frac{18Fl}{b'h^2(2i_g + 3i_f)}$$

$$l = \frac{L - l_b}{2} = \frac{1050 - 85}{2} = 482.5\,mm$$

$$F = \frac{5.4 \times 10^3}{2} = 2.7 \times 10^3\,N \qquad b' = 3h$$

$$\therefore 280 = \frac{18 \times 2.7 \times 10^3 \times 482.5}{3h \times h^2 (2 \times 10 + 3 \times 2)}$$

$h^3 = 1073.7$

$\therefore h = 10.23\,mm$

$$h = \frac{b'}{3} = \frac{10.23}{3} = 3.4\ mm$$

$$b' = 3h = 3 \times 10.23 = 30.7\ mm$$

Max deflection

$$y = \frac{12 F L^3}{b' h^3 E (2 i_g + 3 i_f)}$$

$$y = \frac{12 \times 2.7 \times 10^3 \times 482.5^3}{30.7 \times 10.23^2 \times 0.26 \times 10^6 (2 \times 10 + 3 \times 2)}$$

$$y = 167.5\ mm$$

Module - 3

5. A pair of spur gear with 20^0 full depth transmits 20KW at 1500rpm the pinion. Speed reduction ratio is 4. Allowable static stress of pinion is $220 N/mm^2$ & gear is $193.2 N/mm^2$. Check for dynamic & wear load.

Given

$\alpha = 20^0$ $\quad$ $P = 20KW$ $\quad$ $n_1 = 1500 rpm$ $\quad$ $i = 4$

$\sigma d_1 = 220 N/mm^2$ $\quad$ $\sigma d_2 = 193.2 N/mm^2$

Soln

$$\frac{n_1}{n_2} = 4 \quad \therefore n_2 = \frac{n_1}{4} = \frac{1500}{4} = 375\ rpm$$

Determine weaker member

Assume $Z_1 = 20$ $\therefore Z_2 = i \times Z_1 = 4 \times 20 = 80$

As $\alpha = 20^\circ$

$$y = 0.154 - \frac{0.912}{Z}$$

$$\therefore y_1 = 0.154 - \frac{0.912}{20} = 0.1084$$

$$y_2 = 0.154 - \frac{0.912}{80} = 0.1426$$

$$\sigma_{d_1} y_1 = 220 \times 0.1084 = 23.84 \ N/mm^2$$

$$\sigma_{d_2} y_2 = 193.2 \times 0.1426 = 27.55 \ N/mm^2$$

$\therefore$ Pinion is weaker member

Determining module

$$F_{t_1} = \frac{1000 \times P \times C_s}{V}$$

$\cancel{F_{t_1} =}$

$$V = \frac{\pi d_1 n_1}{60 \times 10^3} = \frac{\pi \times d_1 \times 1500}{60 \times 10^3}$$

$$V = \frac{\pi \times m Z_1 \times 1500}{60 \times 10^3}$$

To determine min of teeth on Pinion we have

$$Z_1^2 + 2 Z_1 Z_2 = \frac{4 K_1 (Z_2 + K_1)}{\sin^2 \alpha}$$

$$Z_1^2 + 2 Z_1 \times i Z_1 = \frac{4 K_1 (i \times Z_1 + K_1)}{\sin^2 \alpha}$$

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Now $K_1 = 1$ for $\alpha = 20$

$i = 4$

$\therefore z_1^2 + 8z_1^2 = \dfrac{4(4z_1 + 1)}{\sin^2 20}$

$0.26z_1^2 - 4z_1 - 1 = 0$

$\therefore z_1 = 15.63 = 16$

$V = \dfrac{\pi \times m \times 16 \times 1500}{60 \times 10^3} = 1.25m$

$F_{t_1} = \dfrac{1000 \times 20 \times 1.5}{1.25m} = \dfrac{23.87 \times 10^3}{m}$ — (1)

By Lewis tangential Eqn

$F_{t_1} = \sigma d_1 C_v b y_1 \pi m$

$F_{t_1} = 220 \times C_v \times 10m \times 0.097 \times \pi \times m$

As $b = 10m$ $y_1 = 0.154 - \dfrac{0.912}{16} = 0.097$

$F_{t_1} = 670.41\, m^2 C_v$ — (2)

By (1) & (2)

$\dfrac{23.87 \times 10^3}{m} = 670.41\, m^2 C_v$

$m^3 C_v = 35.60$

By itteration Assume $m = 4mm$

$\therefore V = 1.25 \times 4 = 5\, m/s$

$C_v = \dfrac{3.05}{3.05 + V} = \dfrac{3.05}{3.05 + 5} = 0.378$

(19)

$4^3 \times 0.378 = 35.60$

$24.24 < 35.60$

$\therefore$ Assume $m = 5\,mm$

$v = 1.25 \times 5 = 6.25\,m/s$

$C_v = \dfrac{3.05}{3.05 + 6.25} = 0.328$

$5^3 \times 0.328 = 35.60$

$40.99 > 35.60$

$\therefore$ Module selected is $m = 5\,mm$

Design Check

Allowable stress $= \sigma d_1 \times C_v = 220 \times 0.328 = 72.15\,N/mm^2$

Induced stress $= \sigma d_1 C_v = \dfrac{F_{t_1}}{\pi \times m \times b \times y_1} = \dfrac{23.87 \times 10^3/5}{\pi \times 5 \times 50 \times 0.097}$

$(\sigma d_1 C_v)_{ind} = 62.66\,N/mm^2$

As $(\sigma d_1 C_v)_{all} > (\sigma d_1 C_v)_{ind}$

Design is safe

$\therefore$ $m = 5\,mm$ $b = 50\,mm$ $Z_1 = 16$ $Z_2 = 64$

$F_{t_1} = 4774\,N/mm^2$

Dynamic Load

$$Fd_1 = F_{t_1} + \frac{K_3 v\,(Cb + F_{t_1})}{K_3 v + \sqrt{Cb + F_{t_1}}}$$

Determining value of C

For $v = 6.25\,m/s$ $\quad e = 0.057$

For steel & steel combination $\alpha = 20^0$

For $e = 0.057$ $\quad C = 686.7$

$$Fd_1 = 4774 + \frac{20.67 \times 6.25\,(686.7 \times 50 + 4774)}{20.67 \times 6.25 + \sqrt{686.7 \times 50 + 4774}}$$

$$Fd_1 = 20.23 \times 10^3\,N$$

Wear Load

$$F_w = d_1 b Q K \geq Fd_1$$

$$\left[\; Q = \frac{2Z_2}{Z_1 + Z_2} = \frac{2 \times 64}{16 + 64} \qquad Q = 1.6 \;\right]$$

$$K \geq \frac{Fd_1}{d_1 b Q}$$

$$K \geq \frac{20.23 \times 10^3}{5 \times 16 \times 50 \times 1.6}$$

$$K \geq 3.16$$

$\therefore$ Pinion $= 450\,BHN$

gear $= 450\,BHN$

67) Design a pair of bevel gears at acute angle to transmit 40KW at 1200rpm of pinion with velocity ratio of 6. Assume C-45 steel for both pinion & gear having allowable stress of 233.4 N/mm² BHN 200. Consider $Z_1 = 25$ $\alpha = 14.5$ $\theta = 45$ & continuous service of medium shocks

Given

$P = 40KW$. $n_1 = 1200$ rpm $i = 6$ $\sigma d_1 = \sigma d_2 = 233.4 N/mm^2$

$Z_1 = 25$ $\alpha = 14\frac{1}{2}$ $\theta = 45^0$

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$$\frac{n_1}{n_2} = \frac{Z_2}{Z_1} = \frac{d_2}{d_1} = 6$$

$$n_2 = \frac{n_1}{6} = \frac{1200}{6} = 200 \text{ rpm}$$

$$Z_2 = 6 \times Z_1 = 6 \times 25 = 150$$

For acute bevel gears

$$\tan \delta_1 = \frac{\sin\theta}{i + \cos\theta} = \frac{\sin 45}{6 + \cos 45} = 0.0105$$

$$\delta_1 = \tan^{-1}(0.0105) = \cancel{0.604}\ 6.018$$

$$\tan \delta_2 = \frac{\sin\theta}{\frac{1}{i} + \cos\theta} = \frac{\sin 45}{\frac{1}{6} + \cos 45} = 0.809$$

$$\delta_2 = \tan^{-1}(0.809) = 38.98$$

$$Ze_1 = \frac{Z_1}{\cos\delta_1} = \frac{25}{\cos 6.018} = 25.14$$

$$Ze_2 = \frac{Z_2}{\cos\delta_2} = \frac{150}{\cos 38.98} = 192.95$$

Determine Weaker member

$$y_1 = 0.124 - \frac{0.684}{Ze_1} = 0.124 - \frac{0.684}{25.14}$$

$$y_1 = 0.096$$

$$y_2 = 0.124 - \frac{0.684}{Ze_2} = 0.124 - \frac{0.684}{192.95}$$

$$y_2 = 0.120$$

$$\sigma d_1 y_1 = 233.4 \times 0.096 = 22.40 \, N/mm^2$$

$$\sigma d_2 y_2 = 233.4 \times 0.120 = 28.11 \, N/mm^2$$

$$\sigma d_1 y_1 < \sigma d_2 y_2$$

$\therefore$ Pinion is weaker member

Determining module

$$Ft_1 = \frac{1000 \times P \times C_s}{v} = \frac{1000 \times 40 \times 1.5}{\frac{\pi d_1 \times 1200}{60 \times 10^3}}$$

$$Ft_1 = \frac{38.19 \times 10^3}{m} \quad - (1)$$

$\therefore\ m = 5\,mm$

Design check

Allowable stress $= \sigma_{d_1} C_v = 233.4 \times 0.437 = 102\ N/mm^2$

Induced stress $= \sigma_{d_1} C_v = \dfrac{F_{t_1}}{\left(\frac{L-b}{L}\right) \pi b y_1 m}$

$$\sigma_{d_1} C_v = \frac{7638}{0.666 \times \pi \times 126.7 \times 0.096 \times 5}$$

$$\sigma_{d_1} C_v = 60\ N/mm^2$$

$\therefore$ Design is safe

$m = 5\,mm$ $\quad b = 126.7\,mm$ $\quad d_1 = m \times z_1$

$d_1 = 5 \times 25 = 125\,mm$ $\quad d_2 = 5 \times 150 = 750\,mm$

$L = 380\,mm$

Dynamic Load

$$F_{d_1} = F_{t_1} + \frac{K_3 V (Cb + F_{t_1})}{K_3 V + \sqrt{Cb + F_{t_1}}}$$

Finding value of C for $V = 7.85\,m/s$

$e = 0.05$ $\quad C = 551.65$

$$F_{d_1} = 7638 + \frac{20.67 \times 7.85 (551.65 \times 126.7 + 7638)}{20.67 \times 7.85 + \sqrt{551.6 \times 126.7 + 7638}}$$

$$F_{d_1} = 36.18 \times 10^3\,N$$

$$F_{t_1} = \sigma_{d_1} C_v b \pi y_1 m \left(\frac{L-b}{L}\right)$$

$$L = \frac{1}{2}\sqrt{d_1^2+d_2^2} = \frac{1}{2} m \sqrt{Z_1^2+Z_2^2}$$

$$L = \frac{m}{2}\sqrt{25^2+150^2} = 76.03\,m$$

$$b \leq \frac{L}{3} \leq \frac{76.03\,m}{3} = 25.34m$$

$$F_{t_1} = 233.4 \times C_v \times 25.34m \times \pi \times 0.096 \times m \left(\frac{76.03m - 25.34m}{76.03m}\right)$$

$$F_{t_1} = 1189.11 m^2 C_v \quad - (2)$$

By (1) & (2)

$$\frac{38.19\times10^3}{m} = 1189.11 m^2 C_v$$

$$m^3 C_v = 32.11$$

Assume $m = 4mm$

$$\therefore V = 1.57m = 1.57\times4 = 6.28\,m/s$$

$$C_v = \frac{6.1}{6.1+V} = \frac{6.1}{6.1+6.28} = 0.492$$

$$4^3\times0.492 = 32.11$$

$$31.52 < 32.11$$

$\therefore$ Assume $m = 5mm$

$$V = 1.57\times5 = 7.85\,m/s$$

$$C_v = \frac{6.1}{6.1+7.85} = 0.437$$

$$5^3\times0.437 = 32.11$$

$$54.65 > 32.11$$

Wear load

$$F_w = \frac{d_1 b Q_e k}{\cos\delta_1} \geq Fd_1 \qquad Q_e = \frac{2Z_{e2}}{Z_{e1}+Z_{e2}}$$

$$K \geq \frac{Fd_1 \times \cos\delta_1}{d_1 b Q_e} \qquad Q_e = \frac{2\times 192.9}{25.14+192.9}$$

$$K \geq \frac{36.18\times10^3 \times \cos 6.08}{125 \times 126.7 \times 1.76} \qquad Q_e = 1.76$$

$K \geq 1.29$

BHN of Pinion = 350

BHN of gear = 350

Module -4

7 Design a worm gear to transmit 5 kW at 1200 rpm. Speed ratio to be 25 & Centre distance 250 mm. Worm wheel is made of phosphor bronze with $\sigma d_2 = 82.4\ N/mm^2$ & hardness 100 BHN & worm made of C-45 steel with $\sigma d_1 = 233.4\ N/mm^2$ & 200 BHN. Load factor 1.25 & $\alpha = 14.5^\circ$

given

$P = 5\ kW$ $\quad n_2 = 1200\ rpm$ $\quad i = 25$ $\quad a = 250\ mm$

$\sigma d_2 = 82.4\ N/mm^2$ $\quad \sigma d_1 = 233.4\ N/mm^2$ $\quad K = 1.25$

$\alpha = 14.5^\circ$

Dimensions of worm & worm gear

$$i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1}$$

$$\frac{n_1}{n_2} = 25$$

$$\frac{1200}{n_2} = 25 \Rightarrow n_2 = 48 \text{ rpm}$$

Pitch dia of worm

$$d_1 = \frac{a^{0.875}}{\cancel{1}.466} = \frac{250^{0.875}}{\cancel{1}.466} = 85.52 \text{ mm}$$

$$d_1 = 86 \text{ mm}$$

$$a = \frac{d_1 + d_2}{2} \Rightarrow 250 = \frac{86 + d_2}{2}$$

$$d_2 = 500 - 86 = 414 \text{ mm}$$

Also $d_1 \approx 3 P_c \approx 3\pi m$

$$m \approx \frac{d_1}{3\pi} \approx \frac{86}{3\pi} \approx 9.12$$

$$\therefore m = 9 \text{ mm}$$

$$i = \frac{\pi d_2}{P_z} = \frac{\cancel{\pi} d_2}{\cancel{\pi} m Z_1} = \frac{d_2}{m Z_1}$$

$$\therefore d_2 = i \times m \times Z_1 = 9 \times 25 \times Z_1$$

$$d_2 = 225 Z_1$$

Z_1	1	2	3
d_2	225	450	675

(27)

As 450 mm is closest to 414 mm

$\therefore$ take $d_2 = 450$ mm

$\therefore d_1 = 2a - d_2 = 2 \times 250 - 450 = 50$ mm

Lead angle $\gamma = \tan^{-1}\left(\frac{m Z_1}{d_1}\right) = \tan^{-1}\left(\frac{9 \times 2}{50}\right)$

$\gamma = 19.8^\circ$

Now $Z_1 = 2$ $\quad Z_2 = i \times Z_1 = 25 \times 2 = 50$

Axial module $m = 9$ mm

Normal module $m_n = m \cos\gamma = 9 \times \cos 19.8 = 8.46$ mm

Dimensions of worm

Pitch diameter $= d_1 = 50$ mm

Face length $= L_{w_1} = (14.14 + 0.063 Z_1)\, m$

$L_w = 128.4$ mm

Depth of tooth $= h = 2.16\, m = 2.16 \times 9 = 19.44$ mm

Addendum $= h_a = m = 9$ mm

outside dia of worm $= d_{o_1} = d_1 + 2m = 50 + 2 \times 9 = 68$ mm

Dimensions of worm wheel

Outside dia $= d_{o_2} = d_2 + 3.1854 \times m = 478.66$ mm

Face width $= b = 7.48m + 6.35 = 73.67$ mm

Throat dia $= d_t = d_2 + 2m = 468$ mm

Radius of gear $= r_b = 2.72m + 14 = 38.94$ mm

Input Power Capacity

$F_{t_2} = \sigma_{d_2} C_v b \pi y_2 m$

$V = \frac{\pi d_2 n_2}{60 \times 10^3} = \frac{\pi \times 450 \times 48}{60 \times 10^3} = 1.13 \, m/s$

$C_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 1.13} = 0.84$

$y_2 = 0.124 - \frac{0.684}{Z_2} = 0.124 - \frac{0.684}{50} = 0.110$

$F_{t_2} = 82.4 \times 0.84 \times 73.67 \times \pi \times 0.110 \times 9$

$F_{t_2} = 15.9 \times 10^3 \, N$

Dynamic Load $= F_{d_2} = \frac{F_{t_2}}{C_V} = \frac{15.9 \times 10^3}{0.84} = 18.93 \, KN$

Wear Load $= F_W = d_2 b K = 450 \times 73.63 \times 1.25$

$F_W = 41.4 \, KN$

Effeciency

$\eta = \frac{\tan\gamma (1 - \mu \tan\gamma)}{\tan\gamma + \mu}$

$\mu = \frac{0.0422}{V_r^{0.28}} = \frac{0.0422}{3.34^{0.28}} = 0.03$

$V_r = \frac{\pi d_1 n_1}{60 \times 10^3 \times \cos\gamma} = \frac{\pi \times 50 \times 1200}{60 \times 10^3 \times \cos 19.8} = 3.34 \, m/sec$

$\eta = \frac{\tan 19.8 (1 - 0.03 \tan 19.8)}{\tan 19.8 + 0.03} = 91.31\%$

Heat balance

Heat generated $= Q = \frac{\mu F_n V_r}{\cos\gamma}$

$$F_n = \frac{F_{t_2}}{\cos\alpha} = \frac{15.9\times10^3}{\cos 14.5} = 16.42\,KN$$

$$Q = \frac{0.03\times16.42\times10^3\times3.34}{\cos 19.8} = 1.74\,KW$$

Heat dissipated $= Q = \frac{0.407}{10^3}(A_g + A_w)(t_2 - t_1)$

$$A_g = \frac{\pi}{4}\times d_2^2 = \frac{\pi}{4}\times450^2 = 159.04\times10^3\,mm^2$$

$$A_w = L_w \times d_1 = 128.4\times50 = 6420\,mm^2$$

Assume $t_2 = 65^\circ C$ $\quad t_1 = 27^\circ C$

$$Q = \frac{0.407}{10^3}\times(159.04\times10^3 + 6420)(65-27)$$

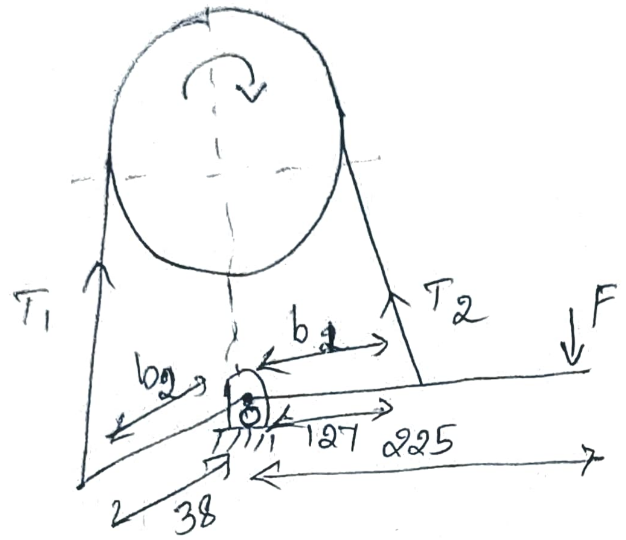
$$Q = 2.55\,KW$$

No artifical cooling is required as Heat dissipated is higher than Heat generated.

8) a A differential band brake operates on drum of dia 600mm. Band is 3.2 x 100mm $\mu = 0.22$ $\theta = 300°$

(i) Find force required at end of operating lever, if band subjected to 55 N/mm²

(ii) Find Torque applied to brake drum shaft



Given

$D = 600mm$ $R = 300mm$ $\mu = 0.22$ $\theta = 300°$

$h = 3.2mm$ $W = 100mm$ $\sigma_d = 55 N/mm^2$

Force

$T_1 = \sigma_d W h = 55 \times 100 \times 3.2 = 17600N$

$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.22 \times 300 \times \frac{\pi}{180}} = 3.164$

$T_2 = \frac{T_1}{e^{\mu\theta}} = \frac{17600}{3.164} = 5562.1N$

Taking moments about O

$F \times a + T_1 \times b_2 = T_2 \times b_1$

$F \times 225 + 17600 \times 38 = 5562.1 \times 127$

$F = 167.1 N$

Taking mmt about O for ccw rotation

$$F \times a + T_2 b_2 = T_1 b_1$$

$$F \times 225 + 5562.1 \times 38 = 17600 \times 127$$

$$F = 8994.8\,N$$

$\therefore$ Least Force $= F = 167.1\,N$

Braking Torque

$$M_t = (T_1 - T_2)\,R = (17600 - 5562.1) \times 300$$

$$M_t = 3611370\ N\text{-}mm$$

86) A Cone Clutch of semi cone angle 12° has to transmit 10KW at 750rpm. The width of face is $\frac{1}{4}$ Dm. Pressure acting is 0.85 bar, assuming uniform wear criteria & $\mu = 0.2$. Calculate dimension & axial force to engage/disengage the clutch.

Given

$\alpha = 12°$ $\quad P = 10KN$ $\quad n = 750rpm$ $\quad b = \frac{1}{4} D_m$

$P = 0.85 \text{ bar} = 0.085 N/mm^2$ $\quad \mu = 0.2$

Sol^n

Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 10000 \times \frac{10}{750}$$

$$M_t = 127333.33 \text{ N-mm}$$

Axial force — 13.10(C)

$$F_a = \pi D_m P b \sin\alpha$$

$$F_a = \pi D_m \, 0.085 \times \frac{D_m}{4} \sin 12$$

$$F_a = 0.0138 D_m^2$$

Dimension

$$M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha} \quad — \quad 13.10(d)$$

$$M_t = 127333.33 = \frac{1}{2} \times \frac{0.2 \times 0.0138 D_m^2 \times D_m}{\sin 12}$$

$$\therefore D_m = 267.2\,mm \simeq 268\,mm$$

$$\text{Face width} = b = \frac{D_m}{4} = \frac{268}{4} = 67$$

$$\text{Inner dia} = D_1 = D_m - b\sin\alpha$$

$$D_1 = 268 - 67\sin 12$$

$$D_1 = 254\,mm$$

$$\text{outer Dia} = D_2 = D_m + b\sin\alpha$$

$$= D_2 = 268 + 67\sin 12$$

$$D_2 = 282\,mm$$

$$\text{Axial } \overset{\text{load}}{\cancel{\text{Force}}} = F_a = 0.0138 \times D_m^2 = 0.0138 \times 268^2$$

$$F_a = 996.9\,N$$

Axial force required for Engagement / Disengagement

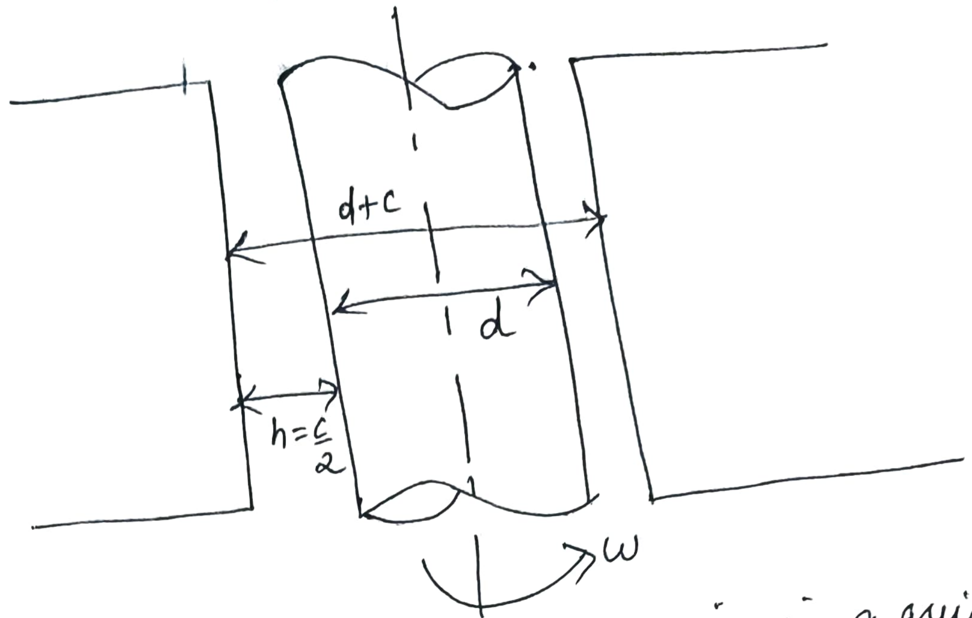
$$F_a' = F_n(\sin\alpha + \mu\cos\alpha) - 13.10(g) \quad \left[\because F_n = \frac{F_a}{\sin\alpha} \quad 13.10(a)\right]$$

$$F_a' = \frac{F_a}{\sin\alpha}(\sin\alpha + \mu\cos\alpha)$$

$$F_a' = \frac{996.9}{\sin 12}(\sin 12 + 0.2\cos 12)$$

$$F_a' = 1934.9\,N$$

9.a Derive Petroff's equation of lightly loaded bearing.



Consider a vertical shaft rotating in a guide bearing.

Let d = diameter of shaft

C = Diametral Clearance

$n' = \frac{n}{60}$ = Speed of shaft in rps

L = Length of bearing

$\psi = \frac{C}{d}$ = diametral clearance ratio

η = Viscosity of oil in Pas

V = velocity $= \frac{\pi d n}{60} = \pi d n'$ m/sec

Shear stress $= \tau = \eta \frac{V}{h} = \frac{\eta \pi d n'}{h} = \frac{\eta \pi d n'}{C/2}$

$\therefore \tau = \frac{2\pi \eta d n'}{C}$

$\therefore$ Force $= F = \tau \times A$ $\qquad [A = \pi d L]$

$$F = \frac{2\pi \eta n' d}{c} \cdot \pi d L$$

$$F = \frac{2\pi^2 d^2 n' \eta L}{c} \cdot \frac{d}{2}$$

$$\therefore M_t = F \times r = F \times \frac{d}{2}$$

$$M_t = \frac{2\pi^2 d^2 n' \eta L}{c} \cdot \frac{d}{2}$$

$$M_t = \frac{\pi^2 d^2 n' \eta L}{(c/d)} = \frac{\pi^2 d^2 n' \eta L}{\psi} \quad - (1)$$

But $M_t = (\mu W) \frac{d}{2}$ $\qquad [W = P \times A = P \times L \times d]$

$$M_t = \mu (P L d) \frac{d}{2} \quad - (2)$$

By (1) & (2)

$$\mu (P L d) \frac{d}{2} = \frac{\pi^2 d^2 n' \eta L}{\psi}$$

$$\boxed{COF = \mu = 2\pi^2 \left(\frac{\eta n'}{P} \right) \frac{1}{\psi}}$$

This is Petroff's Eq^n

9 b A roller bearing has dynamic load capacity of 26 KN. Desired life for 90% of bearing is 8000 hr & speed 300 rpm. Find equivalent radial load that bearing can carry.

Given

$C = 26\ KN$ $L_{10h} = 8000\ hr$ $n = 300\ rpm$

Bearing Life

$$L_{10} = \frac{60 \times n \times L_{10h}}{10^6} = \frac{60 \times 300 \times 8000}{10^6} = 144 \text{ million rev}$$

Equivalent radial load

$$C = P (L_{10})^{0.3}$$

$$P = \frac{C}{(L_{10})^{0.3}} = \frac{26000}{(144)^{0.3}}$$

$$P = 5854.16\ N$$

10 a List the factors to be considered for selecting bearing material

* Material should have good antiweld & Antiscoring property
* High thermal conductivity
* High fatigue strength
* Good corrosion resistance
* Low cost
* Good Machinability.
* Soft enough to absorb foreign materials

10.b A ~~b~~ full bearing of 200mm dia, 200mm long supports radial load of 45KN. Journal rotates at 1200rpm & $r/c = 1000$. Viscosity of oil at its operating temperature of 80°c is 0.1766N/mm², ambient temperature is 20°c. Determine oil film thickness Co-efficient of friction, Heat generated & Heat dissipated.

Given $d = 200$ mm $L = 200$mm $W = 45$KN

$n = 1200$ rpm $n' = 1200/60 = 20$rps

$\eta = 0.1766 N/mm^2$ $t_0 = 80°c$ $t_a = 20°c$

$\frac{r}{c} = 1000 \Rightarrow C = \frac{r}{1000} = \frac{100}{1000} = 0.1$

Soln

Co - effecient of friction

$$f = \frac{\pi^2}{0.5\times10^6}\left(\frac{Zn}{P}\right)\left(\frac{r}{c}\right)$$

$$P = \frac{W}{dL} = \frac{45\times10^3}{200\times200} = 1.125\ N/mm^2$$

$$f = \frac{\pi^2}{0.5\times10^6}\left(\frac{0.1766\times20}{1.125}\right)(1000)$$

$$\boxed{f = 0.062}$$

Oil film thickness

$$S = \left(\frac{r}{c}\right)^2\left(\frac{ZN}{P}\right)\times10^{-6}$$

$$S = (1000)^2\left(\frac{0.1766\times1200}{1.125}\right)\times10^{-6}$$

$$S = 188.37\ S/min$$

$$\frac{h_0}{C} = 0.95$$

$$h_0 = 0.95\times0.1$$

$$\boxed{h_0 = 0.095mm}$$

Heat generated

$$H_g = fWV$$

$$V = \frac{\pi dn}{60\times10^3} = \frac{\pi\times200\times1200}{60\times10^3} = 12.56\ m/s$$

$$H_g = 0.062\times45\times10^3\times12.56 = 35.06\ KW$$

Heat dissipated

$H_D = K_1 A (t_B - t_A)$

$t_R = \frac{t_0 - t_A}{2} = \frac{80 - 20}{2} = 30^\circ$

$t_B = t_A + t_R = 30 + 20 = 50^\circ$

$\therefore t_B - t_A = 30^\circ C$

From Table for $t_B - t_A = 30^\circ$ value of

$K_1 (t_B - t_A) = 0.4428 \times 10^{-3}$ [well ventilated]

$A = 2 d L = 20 \times 200 \times 200$

$A = 800 \times 10^3 \ mm^2 = 0.8 \ m^2$

$H_D = 0.4428 \times 10^{-3} \times 800 \times 10^3$

$H_D = 354.24 \ J/s = 0.354 \ KW$