Modified (CMM)

CBCS Scheme

USN | | | | | | | | | |

15EC61

Sixth Semester B.E. Degree Examination, June/July 2018

Digital Communication

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define Hilbert transform. List the properties of the Hilbert transform. **(04 Marks)**

b. Obtain the canonical representation of band pass signals. **(06 Marks)**

c. What is line coding? For the binary stream 011010 sketch the following line codes:

i) Unipolar NRZ
ii) Polar NRZ
iii) Unipolar RZ
iv) Bipolar RZ
v) Manchester **(06 Marks)**

OR

2 a. Define pre-envelope of a real valued signal. Given a band pass signal s(t), sketch the amplitude spectra of signal s(t), pre-envelope $s_+(t)$ and complex envelope $\tilde{s}(t)$. **(04 Marks)**

b. Derive the expression for the complex low pass representation of band pass systems. **(08 Marks)**

c. Write a note on HDBN signaling. **(04 Marks)**

Module-2

3 a. Explain the geometric representation of signals. Show that energy of the signal is equal to the squared length of the vector representing it. **(08 Marks)**

b. Derive the expressions for mean and variance of the correlator outputs. Also show that the correlator outputs are statistically independent. **(08 Marks)**

OR

4 a. Explain the Gram-Schmidt orthogonalization procedure. **(06 Marks)**

b. Obtain the maximum likelihood decision rule for the signal detection problem. **(10 Marks)**

Module-3

5 a. Explain the signal space representation for binary phase shift keying modulation. Also derive the expression for the probability of error for the binary phase shift keying. **(10 Marks)**

b. With a neat block diagram, explain the generation and coherent detection of QPSK signals. **(06 Marks)**

OR

6 a. With a neat block diagram, explain the non-coherent detection of binary frequency shift keying technique. **(04 Marks)**

b. Derive an expression for probability of error of binary frequency shift keying technique. Also draw the block diagrams of BFSK transmitter and coherent receiver. **(10 Marks)**

c. For the binary sequence given by 10010011, illustrate the operation of DPSK. **(02 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

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Module-4

7 a. With a neat block diagram of digital PAM system obtain the expression for inter symbol interference (ISI). **(06 Marks)**

b. State and prove Nyquist condition for zero ISI. **(06 Marks)**

c. For the binary data sequence $\{d_n\}$ given by 11101001. Determine the precoded sequence, transmitted sequence, received sequence and the decoded sequence. **(04 Marks)**

OR

8 a. Explain the design of band limited signals with controlled ISI. **(10 Marks)**

b. What is a zero forcing equalizer? With a neat block diagram explain the operation of linear transversal filter. **(06 Marks)**

Module-5

9 a. Explain the model of a spread spectrum digital communication system. **(06 Marks)**

b. Explain the generation and demodulation of direct sequence spread spectrum signals with necessary equation and block diagram. **(07 Marks)**

c. Write a note on low detectability signal transmission as an application of direct sequence spread spectrum. **(03 Marks)**

OR

10 a. With a neat block diagram, explain the frequency hopped spread spectrum. **(07 Marks)**

b. Explain the effect of despreading on a Narrow band interference in direct sequence spread spectrum systems. A direct sequence spread spectrum signal is designed to have the power ratio P_R/P_N at the intended receiver is 10^{-2}. If the desired $E_b/N_o = 10$ for acceptable performance, determine the minimum value of processing gain. **(06 Marks)**

c. Write a note on code division multiple access as an application of direct sequence spread spectrum. **(03 Marks)**

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June / July 2018

Subject : Digital Communication (15EC61)

Note: Answer any FIVE full questions, choosing one full question from each module.

Module I

I. a) Define Hilbert transform. List the properties of the Hilbert transform. (4 marks)

Ans: Definition : $\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau$ (01)

properties : (i) $g(t)$ & $\hat{g}(t)$ have same magnitude spectrum (ii) Hibert transform of $\hat{g}(t)$ is $g(-t)$ and (iii) $g(t)$ and $\hat{g}(t)$ are orthogonal (03)

1. b. Obtain the canonical representation of band pass signals. (6 marks)

<u>Canonical Representation of Band Pass Signals</u>

By definition of pre-envelope

$s_+(t) = s(t) + j\hat{s}(t)$ — eqn (I)

where $s(t)$ – band pass signal

$\hat{s}(t)$ – HT of $s(t)$

$s_+(t)$ – pre envelope of $s(t)$

From equn (I) it is clear that the real part of the pre-envelope $s_+(t)$ is equal to the original band pass signal $s(t)$.

The complex envelope of band pass signal s(t) is

$$S_{+}(t) = \hat{s}(t)e^{j2\pi f_c t} \quad \text{— equ}^n \text{ (II)} \qquad (01)$$

where

$\hat{s}(t)$ - is the complex envelope of the band pass signal s(t)

Similarly complex envelope of band pass signal s(t)

Similarly complex envelope of band pass signal s(t) can be written as -

$$s(t) = Re\left[\hat{s}(t)e^{j2\pi f c t}\right] \quad \text{— equ}^n \text{ (III)}$$

As $\hat{s}(t)$ is a complex - valued quantity, we emphasize this property by expressing it in Cartesian form

$$\hat{s}(t) = s_I(t) + j s_Q(t) \quad \text{— equ}^n \text{ (IV)}$$

where

$s_I(t)$ & $s_Q(t)$ are both real valued low pass function, there low pass property is inherited from the complex envelope $\hat{s}(t)$. (01)

Using equ^n (IV) in equ^n (III) we can express s(t) in the canonical form or Standard form as

$$s(t) = s_I(t)\cos(2\pi f c t) - s_Q(t)\sin(2\pi f c t) \text{ - equ}^n \text{ (V)}$$

where

$s_I(t)$ - in-pass component of the band pass signal s(t)

$s_Q(t)$ - Quadrature - phase component of s(t)

According to equ^n (II), the complex envelope may be pictured as a time-varying phase positioned at the origin of (S_I, S_Q) plane & is shown in Fig (1), Fig (2), Fig (3)

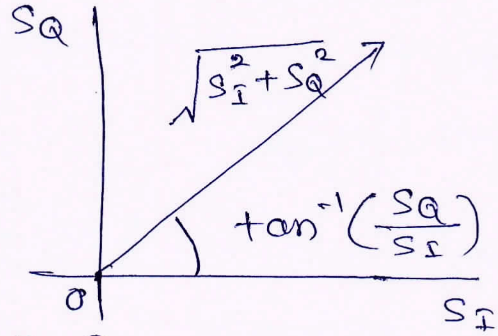


Fig (1) Complex envelope (before mod^n)

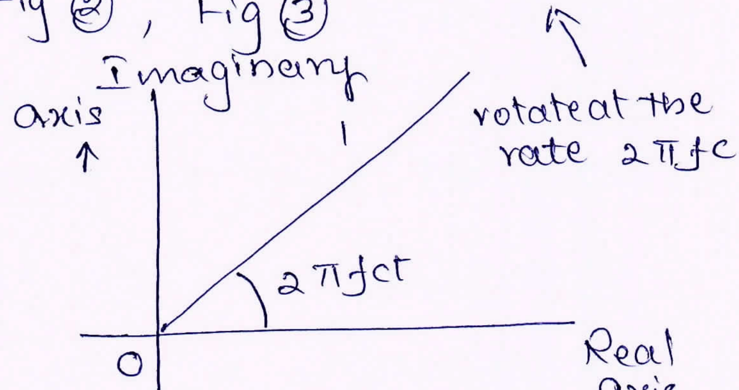


Fig (2) Exponential Signal

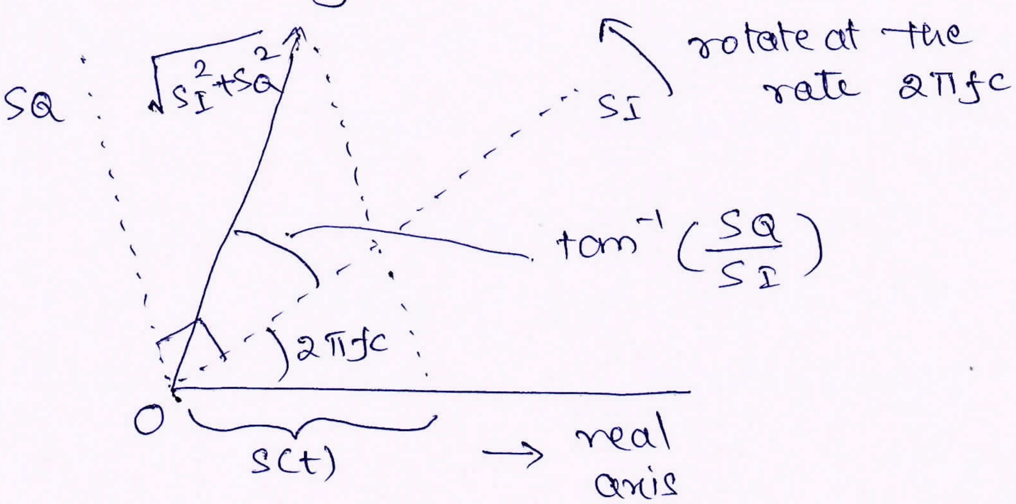


Fig (3) Complex envelope after modul^n

Fig (1), (2) & (3) Illustrating an interpretation of the complex envelope $\hat{s}(t)$ & its multiplication by $e^{j2\pi f_c t}$

→ with time t varying continuously the end of the phase moves about in the plane as shown in fig (1)

Fig (2) shows the phase representation of the complex exponential $e^{j2\pi f_c t}$

In the definition given in equ^n (III) complex envelope $\hat{s}(t)$ is multiplied by the complex exponential $e^{j2\pi f_c t}$.

The angles of these two phasors shown in fig (1) & fig (2) add and their lengths multiply and the result is shown in fig (3)

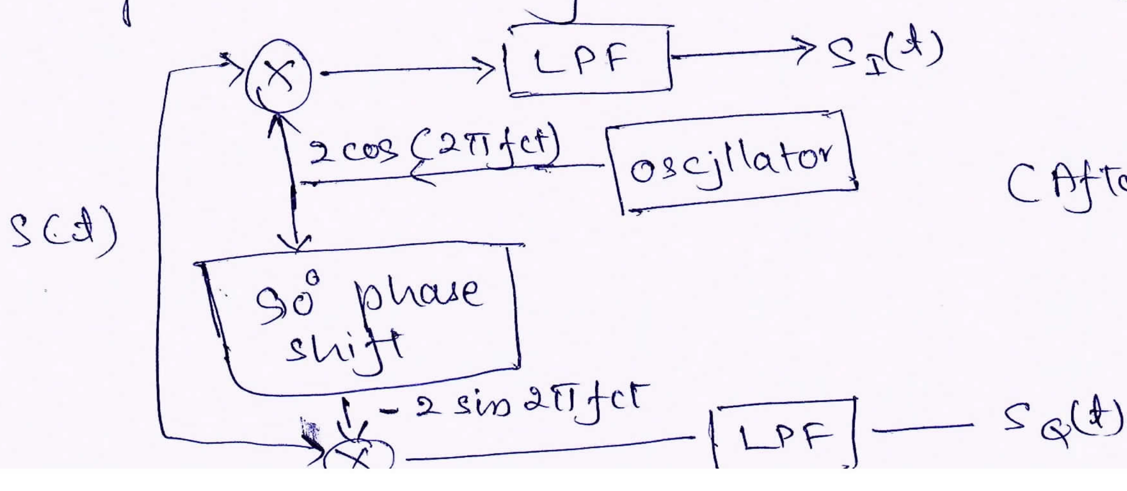
As shown in fig (3) (s_I, s_Q) phase rotating with an angular velocity $2\pi f_c$ radians/sec

Similarly the complex envelope $\hat{s}(t)$ moves in the (s_I, s_Q) plane, while at the same time the plane rotates about the origin.

The original band pass signal $s(t)$ is the projection of this time - varing phasor on a fixed line representing the real axis, as shown in fig (3)

Since both $s_I(t)$ & $s_Q(t)$ are low pass signals limited to the band $-W \leq f \leq W$

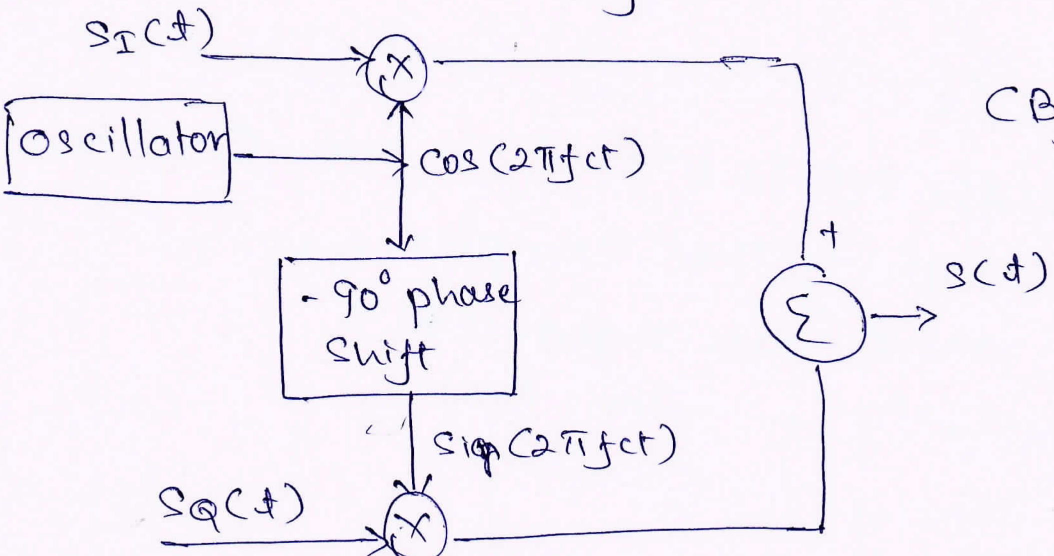
They may be extracted from the band pass signal $s(t)$ using scheme shown in fig (4) below



(After modulation)

(01)

Similarly reconstruction of $s(t)$ from its in-phase & quadrature-phase component is done using scheme shown in fig (5)



(Before modulation)

(01)

Fig (5) scheme for reconstructing $s(t)$ from $s_I(t)$ & $s_Q(t)$ (Synthesizer)

The above two schemes given in fig (4) & fig (5) are basic to the study of linear modulation schemes, be they of analog modulation or digital modulation.

Multiplication of the low pass in-phase component $s_I(t)$ by $\cos(2\pi f_c t)$ and

Multiplication of the low pass quadrature-phase component $s_Q(t)$ by $\sin(2\pi f_c t)$

represent linear forms of modulation.

Provided f_c (carrier frequency) is larger than the low-pass bandwidth W, then the resulting band pass signal $s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$ is referred to as a passband signal waveform.

Correspondingly, the mapping form $s_I(t)$ & $s_Q(t)$ combined into $s(t)$ is known as pass band modulation.

Polar Representation of Band pass Signals.

wkt $s(t) = s_I(t) + j\,s_Q(t)$ is the Cartesian form of defining the complex envelope $\hat{s}(t)$ of the band-pass signal $s(t)$.

Alternatively in polar form as

$$\tilde{s}(t) = a(t)\, e^{[j\phi(t)]} \quad \text{———} \quad \text{equ}^n \; \text{VI}$$

where

$a(t)$ is natural envelope or simply the envelope ~~since~~ ~~both~~ of the brand pass signal $s(t)$ and is a real-valued low-pass function.

Based on polar representation of complex envelope in equ^n VI, the original band pass signal $s(t)$ is itself defined by

$$\tilde{s}(t) = a(t)\cos[2\pi f_c t + \phi(t)) \quad \text{———} \quad \text{equ}^n \; \text{VII}$$

Relationship between Cartesian & polar Representation of band pass signal

The envelope and phase $\phi(t)$ of a brand pass signal $s(t)$ are representively related to $s_I(t)$ & $s_Q(t)$ by relations.

$$|s_+(t)| = |\tilde{s}(t)| = a(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$$

and

$$\angle \tilde{s}(t) = \phi(t) = \tan^{-1}\left(\frac{s_Q(t)}{s_I(t)}\right)$$

conversely we may write

In phase component $\rightarrow$ $S_I(t) = a(t) \cos[\phi(t)]$

Quadrature component ~~phase~~ $\rightarrow$ $S_Q(t) = a(t) \sin[\phi(t)]$

Thus both the in-phase & quadrature component of a band pass signal contain amplitude & phase information. (02)

Q I (c) What is line coding? For the binary stream 011010 sketch the following line codes:

(i) Unipolar NRZ

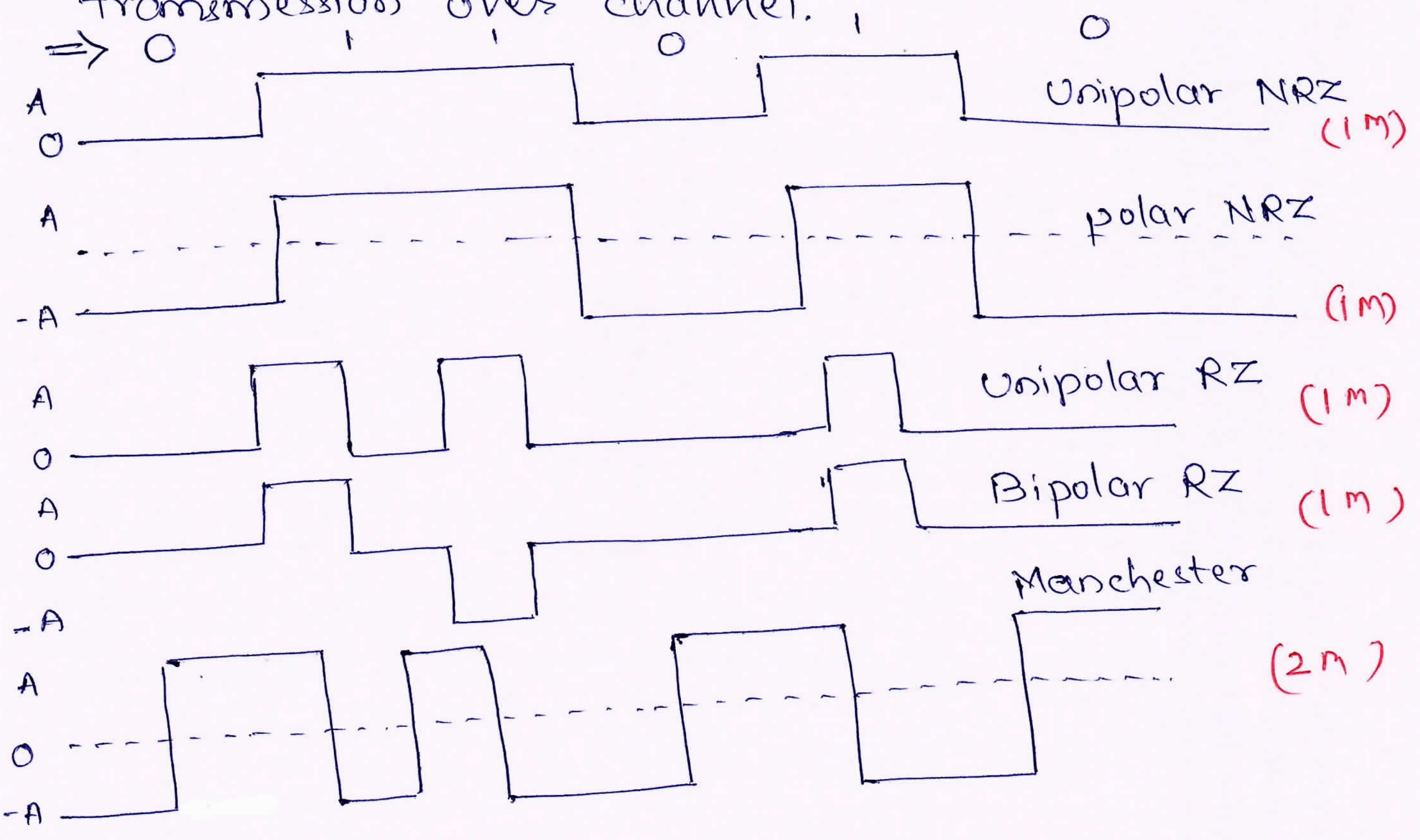
(ii) Polar NRZ

(iii) Unipolar RZ

(iv) Bipolar RZ

(v) Manchester

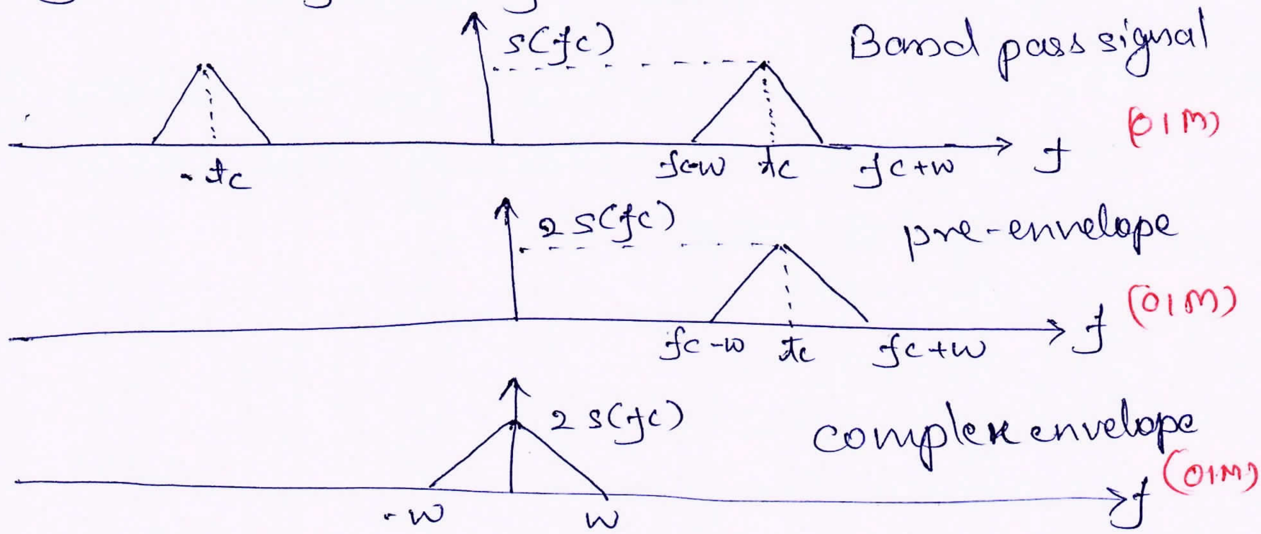
Ans: Line coding: Conversion of source encoder output to electrical pulses for purpose of transmission over channel. (06 marks)



OR

2\. a. Define pre-envelope of a real valued signal. Given a band pass signal s(t), sketch the amplitude spectra of signal s(t), pre-enevelope $s_+(t)$ and complex envelope $\tilde{s}(t)$. (4 marks)

Ans: Definition : $g_+(t) = g(t) + j\hat{g}(t)$ (01M)



2b. Derive the expression for the complex low pass representation of band pass systems. (8 marks)

Ans: Assumption 1

Let s(t) is a narrow band signal & S(f) is its F.T

We assume that the spectrum of s(t) is limited to frequencies within ± wHz of the carrier frequency fc. 01M

We also assume that $w < f_c$.

Let the signal s(t) be applied to a LTI band pass system with impulse response h(t) and frequency response H(f).

Assumptions:

We assume spectrum of $h(t)$ is ltd. within $\pm B$ Hz of the carrier f_c.

(01 M)

We assume that frequency response of the system is ltd to frequencies within $\pm B$ of the carrier frequency f_c.

$$B < f_c$$

⇒ The system bandwidth $2B$ is usually narrower than or equal to the input signal bandwidth $2W$.

⇒ Expressing $h(t)$ in canonical form as (01 M)

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t) \quad \text{--- Equ}^n \text{ I}$$

Correspondingly, we define the complex impulse response of the band pass system as

$$\hat{h}(t) = h_I(t) + j h_Q(t) \quad \text{--- Equ}^n \text{ II}$$

We may express impulse response $h(t)$ in terms of its corresponding complex envelope $\hat{h}(t)$ as

$$h(t) = Re\left[\hat{h}(t) e^{(j2\pi f_c t)}\right] \quad \text{--- Equ}^n \text{ III}$$

Note that

$h_I(t)$, $h_Q(t)$ & $\hat{h}(t)$ are all low pass functions ltd to the frequency band $-B \leq f \leq B$

Equ^n II tells us that we can determine $\hat{h}(t)$ in terms of the in-phase and quadrature-phase components of the band pass impulse response $h(t)$.

Alternatively,

we may determine complex impulse response $\hat{h}(t)$ from $H(f)$ band pass frequency response in the following way.

Use Equ^n III to write (02m)

$$2h(t) = \hat{h}(t)e^{(j2\pi f_c t)} + \hat{h}^*(t)e^{(-j2\pi f_c t)} \quad \text{— Equ}^n \text{ IV}$$

$\hat{h}(t) = \hat{h}^*(t)$ — Complex conjugation

[Addition of complex signal & its complex conjugate results in, 2 times addition of real part & cancellation of imaginary part]

Apply FT to both sides of Equ^n IV we get

$$2H(f) = \hat{H}(f-f_c) + \hat{H}^*(-f-f_c)$$

└ Equ^n V

Equ^n V satisfies the requirement that $H^*(f) = H(-f)$ for a real valued impulse response $h(t)$

Since $\hat{H}(f)$ represents a low pass frequency response ltd to $|f| \leq B$ with $B < f_c$

we conclude from Equ^n V that

$$\hat{H}(f-f_c) = 2H(f) \qquad f > 0 \quad \text{— Equ}^n \text{ VI}$$

The above equ^n VI states that

For a specified band pass frequency response $H(f)$, we may determine the corresponding complex low-freq responses $\hat{H}(f)$ by taking the part of $H(f)$ defined for positive frequencies shifting it to the origin and scaling it by factor 2.

Having determined $\hat{H}(f)$ we decompose it as

$$\hat{H}(f) = \hat{H}_2(f) + j\hat{H}_Q(f) \quad \text{——} \quad \text{Equ}^n \text{ VII}$$

where

$$\hat{H}_I(f) = \frac{1}{2}\left[\hat{H}(f) + \hat{H}^*(-f)\right] \text{——} \quad \text{Equ}^n \text{ VIII}$$

$$H_Q(f) = \frac{1}{2}\left[\hat{H}(f) - j\hat{H}^*(-f)\right] \text{——} \quad \text{Equ}^n \text{ IX} \quad (02M)$$

Finally to determine $\hat{h}(t)$ of the band pass sys we take IFT of $\hat{H}(f)$ as (01 M)

$$\hat{h}(f) = \int_{-\infty}^{\infty} \hat{H}(f) e^{(j2\pi ft)} df$$

2c. Write a note on HDBN signaling.

Ans. The HDB scheme is an ITU (formerly CCITT) standard. In this scheme the problem of non transparency in bipolar signaling is eliminated by adding pulse when the number of consecutive 0s exceeds N. Such a modified coding is designated as high-density bipolar coding (HDBN), where N can take on any value 1, 2, 3, ... The most important of the HDB codes is HDB3 format, which has been adopted as an international standard. — (1 M)

The basic idea of the HDBN code is that when a run of N+1 zeros occurs, this group of zeros is replaced by one of the special N+1 binary digit sequences. To increase the timing content of the signal, the sequences are chosen to include some binary 1s. The 1s is included deliberately violate the bipolar rule for easy

identification of the substituted sequences.
In HDB3 coding, for example, the special sequences used are OOOV and BOOV where B=1 that —(1M) conforms to the bipolar rule and V=1 that violates the bipolar rule. The choice of sequence OOOV or BOOV is made in such a way that consecutive V pulses alternate signs to avoid dc wander (1M) and to maintain the dc null in the PSD. This requires that the sequence BOOV be used when there are an even number of 1s following the last special sequence and the sequence OOOV be used when there are an odd number of 1s following the last sequence.

Module - 2

3.a. Explain the geometric representation of signals. Show that energy of the signal is equal to the squared length of the vector representing it.

Ans: **Geometric Representation of Signals.**

→ The essence of geometric representation of signals is to express any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \leq M$.

→ That is to say, given a set of real-valued energy signals, $s_1(t)$, $s_2(t)$ ..... $s_M(t)$ each of duration T seconds we write

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t) \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \ldots M \end{cases} \quad \text{— Equn (I)}$$

where the coefficients of expansion are defined by

$$s_{ij} = \int_{t=0}^{t=T} s_i(t)\,\phi_j(t)\,.dt \quad \begin{cases} i = 1,2 \ldots M \\ j = 1,2, \ldots N \end{cases} \quad \text{— Equ}^n \; 2$$

The real valued basis functions $\phi_1(t)$, $\phi_2(t) \ldots \phi_N(t)$

Form an orthonormal set by which we mean

$$\int_0^T \phi_i(t)\,\phi_j(t)\,.dt = \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

Equ^n 3

where

δ_{ij} = Kronecker delta (02M)

Thus first condition of equ^n 3 states that

1. Inner product with itself results in 1 i.e. unit energy

Second condition

2. Inner product with other functions results in Zero

OR

we say that

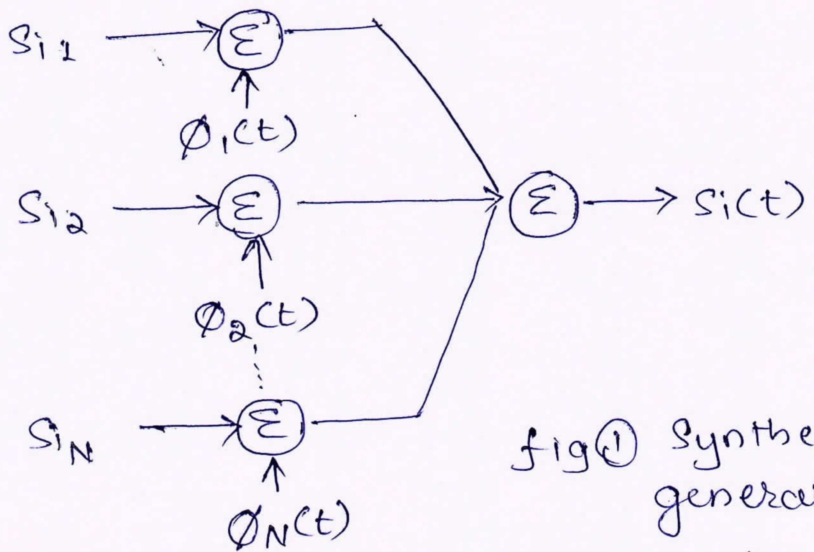
basic function $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ are orthogonal w.r.t. each other over the interval $0 \leq t \leq T$.

→ Let set of coefficients $\{s_{ij}\}_{j=1}^{j=N}$ may be viewed as an N-dimentional signal vector s_i

s_i bears one to one relationship with the Txed Signal $s_i(t)$

→ Geometric Representation of message vectors

→ Given the N elements of the vector s_i operation as input, we may use scheme shown in fig ① below to generate the signals $s_i(t)$ which follow directly from Equⁿ ①



(1M)

fig ① Synthesizer for generating the signal $s_i(t)$

Conversely, given the signals $s_i(t)$, $i = 1, 2, \ldots M$ operating as input we may use the scheme shown in fig ② to calculate the coefficients $s_{i1}, s_{i2}, \ldots s_{iN}$ which follows directly from equⁿ ②.

This second scheme consists of a bank of N product integrators or correlators with a common input and with each one of them supplied with its own basis function.

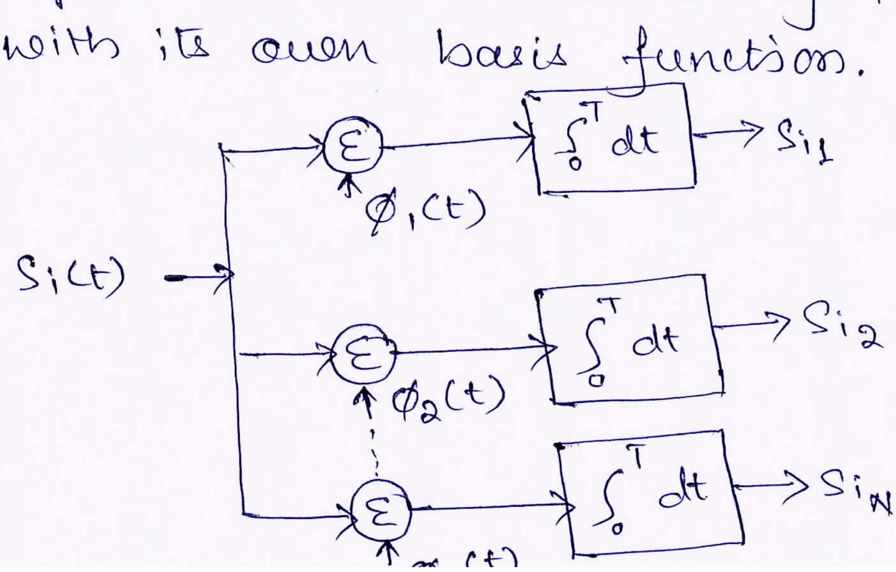


fig ② Analyzer for reconstructing the signal vector $\{s_i\}$

(1M)

In an N-dimentional Euclidean Space, we may define lengths of vectors.

$$\|s_i\|^2 = s_i^T s_i$$ - inner product or dot product of s_i with itself.

$$= \sum_{j=1}^{j=N} s_{ij}^2 \qquad i = 1, 2, \ldots M \quad \text{— (1M)}$$

Relationship between energy content of a signal and its representation as a vector.

$$E_i = \int_0^T s_i^2(t)\,dt \qquad i = 1, 2, \ldots M$$

(1M)

WKT $s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t)$ $\quad \left\{ \begin{array}{l} 0 \le t \le T \\ i = 1, 2, \ldots M \end{array} \right.$

$$\therefore E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right] dt$$

Interchanging the order of summation & integration,

$$E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}\, s_{ik} \int_0^T \phi_j(t)\,\phi_k(t)\,dt \quad \text{— (1M)}$$

Since by definition, the $\phi_j(t)$ form an orthonormal set in accordance with the two conditions

$$\int_0^T \phi_i(t)\phi_j(t)\,dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \ne j \end{cases}$$

$$\therefore E_i = \sum_{j=1}^{N} s_{ij}^2 \quad \text{— (1M)}$$

$$= \|s_i\|^2$$

3 b. Derive the expressions for mean & variance of the correlator outputs. Also show that the correlator outputs are statistically independent.

Ans. → Here we wish to develop a statistical characterization of the set of N correlator outputs.

→ Let $X(t)$ — denote the stochastic process and let $x(t)$ is sample function of stochastic process $X(t)$.

→ Correspondingly,

Let X_j - denote the random variable whose sample value is represented by the correlator outputs x_j, $j = 1, 2, \ldots N$

→ Accordingly to AWGN model, the stochastic process $X(t)$ is a Gaussian process. (1m)

→ It follows, therefore that X_j is a Gaussian random variable for all j in accordance with property 1.

[If a Gaussian process $X(t)$ is applied to a stable linear filter, then the stochastic process $Y(t)$ developed at the output of the filter is also Gaussian]

→ Hence X_j is characterized completely by its mean & variance.

Let W_j - denote the random variable represented by the sample value w_j produced by the jth Correlator in response to white Gaussian noise component $w(t)$.

→ The random variable W_j has zero mean because the channel noise process $W(t)$ (1m) represented by $w(t)$ in the AWGN model has zero mean by definition.

→ As a result, mean of X_j dependents only on s_{ij} as shown by

$$\mu_{xj} = E[x_j]$$
$$= E[s_{ij} + w_j]$$
$$= s_{ij} + E[w_j] \quad \text{--- Equ}^n \ (1)$$
$$= s_{ij} \quad \text{--- (1 M)}$$

→ To find the variance of x_j we start with definition.

$$\sigma^2_{xj} = \mathrm{Var}[x_j]$$
$$= E[(x_j - s_{ij})^2] \quad [\because x_j = s_{ij} + w_j]$$
$$= E[w^2_j] \quad \text{--- Equ}^n \ (2) \ (1M)$$

→ Here x_j & w_j replaced by x_j & w_j

→ WKT random variable w_j is defined by (1M)

$$w_j = \int_0^T w(t)\, \phi_j(t).dt \quad \text{--- Equ}^n \ (3)$$

Use Equ^n (3) in Equ^n (2) then

$$\sigma^2_{xj} = E \int_0^T w(t)\, \phi_j(t).dt \int_0^T w(u)\, \phi_j(u).du$$

$$= E\left[\int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, w(t)\, w(u).dt.du\right] \text{... Equ}^n \ (4)$$

For the linear operations we can Interchange the order of summation & expectation

then $$\sigma^2_{xj} = \int_0^T \int_0^T \phi_j(t) \phi_j(u)\, E[w(t) . w(u)].dt.du$$

$$= \int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, R_w(t,u).dt.du \text{ ---... Equ}^n \ (5)$$

(1 M)

where

$R_w(t,u)$ = Auto correlation function of the noise process $w(t)$

→ Since this noise is stationary $R_w(t,u)$ depends only on the time difference $t-u$

→ WKT $w(t)$ is white Gaussian noise with PSD = $\frac{N_0}{2}$

therefore we may express

$$R_w(t,u) = \left(\frac{N_0}{2}\right)\delta(t-u) \quad ---- \quad \text{Eq}^n (6)$$

Put Eq^n (6) in Eq^n (5) and then using shifting property of the delta fun $\delta(t)$, we get

$$\sigma^2_{xj} = \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t)\phi_j(u).\delta(t-u).dt.du \quad -(1m)$$

$$= \frac{N_0}{2}\int_0^T \phi_j^2(t).dt$$

→ Since $\phi_j(t)$ have unit energy, by definition, the expression for noise variance σ^2_{xj} reduce to (1m)

$$\sigma_{xj} = \frac{N_0}{2} \quad \text{for all } j \quad ----- \quad \text{Eq}^n (7)$$

→ This important result shows that all the correlator outputs denoted by x_j with $j=1,2...N$ have a variance equal to the PSD $= \frac{N_0}{2}$ of the noise process $w(t)$

Moreover, since the basic functions $\phi_j(t)$ from an orthonormal set x_j and x_k are mutually uncorrelated as shown by

$$Cov[x_j x_k] = E[(x_j - \mu_{xj})(x_k - \mu_{xk})] \quad (2m)$$

$$= E[(x_j - s_{ij})(x_k - s_{ik})]$$

$$= E[w_j w_k]$$

$$= E\left[\int_0^T w(t)\phi_j(t).dt . \int_0^T w(u).\phi_k(u).du\right]$$

$$= \int_0^T\int_0^T \phi_j(t)\phi_k(u) R_w(t,u)\, dt.du \quad --- \text{Eq} (8)$$

$$= \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t).\phi_k(u)\delta(t-u).dt.du$$

$$= \frac{N_0}{2}\int_0^T \phi_j(t)\phi_k(u).dt$$

$$Cov[x_j x_k] = 0 \quad \text{if } j \neq k$$

Since x_j are Gaussian random variables, equ^n (8) implies that they are also statistically in accordance with property 4 of a Gaussian process.

$\boxed{\text{OR}}$

4.a. Explain the Gram-Schmidt orthogonalization procedure.

Ans: → Suppose we are given a signal set

$\{s_1(t), s_2(t) \ldots\ldots s_m(t)\}$

→ Find the orthogonal basis functions for this signal set

$\{\phi_1(t), \phi_2(t) \ldots\ldots \phi_N(t)\}$ $N \leq M$

Step 1 Construct the first basis function

→ Complete $E_1 = \int_{t=-\infty}^{t=\infty} s_1^2(t) \cdot dt$

→ The first basis function is $\phi(t) = \frac{s_1(t)}{\sqrt{E_1}}$

$\Rightarrow s_1(t) = \sqrt{E_1}\,\phi(t) = s_{11}\phi_1(t)$

$$s_{11} = \int_{t=-\infty}^{\infty} s_1(t)\,\phi_1(t) \cdot dt = \sqrt{E_1} \quad - (1M)$$

Step 2 Construct the second basis function

→ Complete the correlation between signal 2 and basis function 1

$$s_{21} = \int_{t=-\infty}^{\infty} s_2(t)\,\phi_1(t) \cdot dt \quad - (1M)$$

→ Subtract off the correlation portion

$$g_2(t) = s_2(t) - s_{21}(t)\,\phi_1(t) \quad - (1M)$$

$\Rightarrow g_2(t)$ is orthogonal to $\phi_1(t)$

→ Complete the energy in the remaining portion

$$E_{g_2} = \int_{t=-\infty}^{\infty} [g_2(t)]^2 . dt$$

→ $$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}}$$

⇒ $$S_{22} = \int_{t=-\infty}^{\infty} s_2(t)\,\phi_2(t) = \sqrt{E_{g_2}}$$ — (1m)

Step 3 Construct the successive basis functions

→ For signal $S_k(t)$, complete $s_{ki} = \int_{t=-\infty}^{\infty} s_k(t)\,\phi_j(t).dt$

→ Define $g_k(t) = s_k(t) - \sum_{i=1}^{i=k-1} s_{ki}\,\phi_i(t)$

→ Energy of $g_k(t)$

$$E_{gk} = \int_{t=-\infty}^{\infty} [g_k(t)]^2 . dt$$

→ Therefore k^{th} basis function

$$\phi_k(t) = \frac{g_k(t)}{\sqrt{E_{gk}}}$$

→ In general

$$s_{kk} = \int_{t=-\infty}^{\infty} s_k(t)\,\phi_k(t).dt = \sqrt{E_{gk}}$$ (2m)

4b. Obtain the maximum likelihood decision rule for the signal detection problem.

Ans: For an AWGN Channel, a sample fun of the received random process $x(t)$ is given by

$$x(t) = S_i(t) + w(t) \qquad 0 \leq t \leq T, \quad i = 1, 2 \ldots M \quad \ldots \text{Equ}^n (1)$$

where $w(t)$ - sample fun of white Gaussian noise process $w(t)$, with zero mean and PSD = $N_0/2$ — (1m)

The set of outputs of the correlations constitutes a vector x

$$x = S_i + W \qquad i = 1, 2, \dots M \quad \text{----- equn (2)}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}$ $\quad S_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \\ \vdots \\ s_{iN} \end{bmatrix}$ $\quad W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$ ← (1m)

→ Vectors x & ~~W~~ W are sample values of the random vectors X and W respectively.

$$P_c = P(m_i \text{ sent} \mid x \text{ received}) \text{ —— Equ}^n \text{ (3)}$$

← (1m)

→ The average probability of symbol error P_e is given by

$$P_e = 1 - P_c \text{ (Probability of correct decision)}$$

$$P_e = 1 - P(m_i \text{ Sent} \mid x \text{ received}) \text{ —— Equ}^n \text{ (4)}$$

← (1m)

The estimate of Rx^{ed} symbol

$\hat{m} = m_i$ if, $P(m_i \text{ sent} \mid x \text{ received}) \geq P(m_k \text{ sent} \mid x \text{ received})$

for all $k = 1, 2, \dots M \quad k \neq i$ —— Equ^n (5) – (1m)

→ The decision rule given by Equ^n (5) is called max. a posteriori probability.

→ The likelihood function $f_X(x \mid m_i)$ is the conditional PDF of the received vector X given that m_i was transmitted.

→ The decision rule is to choose estimate

$\hat{m} = m_i$ if ← (1m)

$$f_X(x \mid m_k) \text{ is maximum for } k = i \text{ —— Equ}^n \text{ (6)}$$

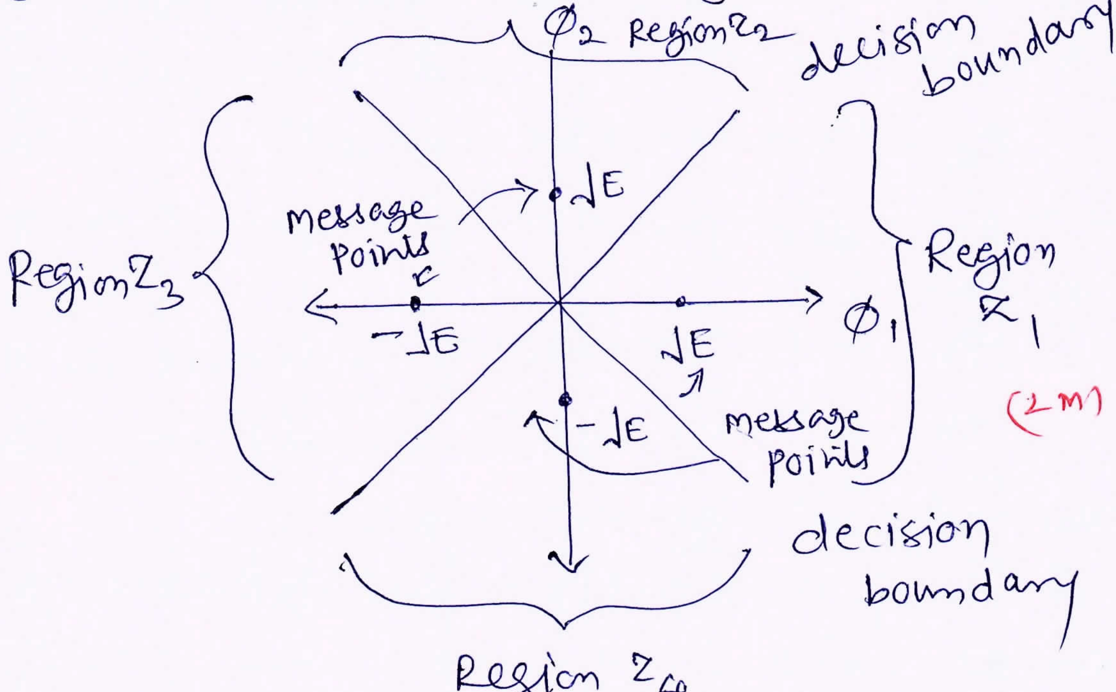
set $\hat{m} = m_i$ if $L(m_k)$ is maximum for $k = i$

Observation vector x lies in region Z_i if $L(m_k)$ is maximum for $k = i$ — (1m)

Observation vector x lies in region Z_i if $\sum_{j=1}^{N}(x_j - s_{ij})^2$ is minimum for $k = i$ —

Observation vector x lies in region Z_i if $\|x - s_k\|$ is minimum for $k = i$

Observation vector x lies in region Z_i if $\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2}E_k$ is maximum for $k = i$ —(1M)



(2M)

Figure illustrates partitioning of the observation space into decision regions for the case when $N = 2$, $M = 4$

Module 3

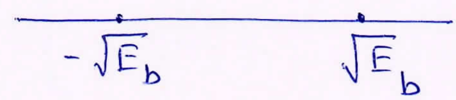
5.a. Explain the signal space representation for binary phase shift keying modulation. Also derive the expression for the probability of error for the binary phase shift keying. (10M)

Ans: Signal space representation of BPSK

Basic function : $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

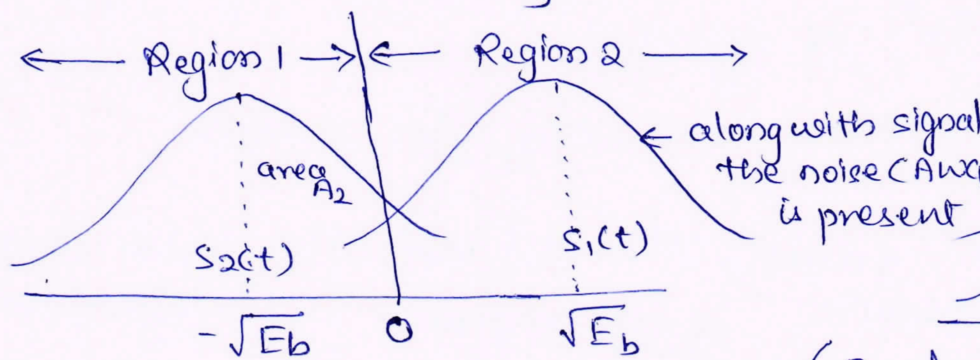
$S_1(t) = \sqrt{E_b}\,\phi(t)$ $\quad$ $S_2(t) = -\sqrt{E_b}\,\phi_1(t)$ — (1 M)

$S_{11} = \sqrt{E_b}$ $\quad$ $S_{21} = -\sqrt{E_b}$



BPSK – Probability of Error Calculation

Decision boundary



AWGN of Zero mean
PSD = $\frac{N_0}{2}$
Stationary

— (1 M)

(Message pt 2) $\quad$ (Message Pt 1

Read as probability of $T_x$ed as '0' but $R_x$ed as 1

average probability of error = $P_{e1} + P_{e0}$

The observation vector x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^T x(t)\,\phi_1(t).dt$$ — (1 M)

The error is of two types

(1) $P_e(0/1)$ i.e. transmitted ($T_x$ed) as '1' but received as '0'

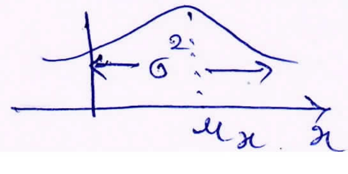
(2) $P_e(1/0)$ i.e. transmitted as '0' but received as '1'

we assume that in

→ most of the cases we have the areas A_1 and A_2 are same.

→ How we assume Gaussian noise

Any Gaussian fn is given by



mean value
variance σ^2
random variable x

$$f(x) = \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-4x)^2}{2\sigma^2}\right] dx \quad \text{—(1m)}$$

$$Pe(1|0) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty \exp\left[\frac{-(x_1-4)^2}{2\sigma^2}\right] dx_1 \quad \text{—(1m)}$$

μ = mean value (expected value) = $-\sqrt{E_b}$ for the Tx^n of symbol '0'

σ^2 = variance = $N_0/2$ for AWGN —(1m)

value = Th = 0 [indicates lower limit in integration]
of AWGN of zero shifted to

Mean value ($+\sqrt{E_b}$ or $\sqrt{E_b}$ derivates because of noise

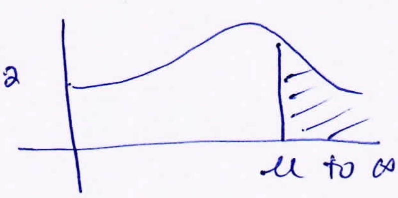
$$\therefore Pe_0 = Pe(1|0) = \frac{1}{\sqrt{2\pi(\frac{N_0}{2})^2}} \int_0^\infty \exp\left[-\frac{[x_1-(-\sqrt{E_b})]^2}{2\times(\frac{N_0}{2})^2}\right] dx_1$$

$$Pe_0 = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1+\sqrt{E_b})^2}{[\sqrt{N_0}]^2 = N_0}\right] dx_1 \quad \text{—(1m)}$$

It is not in standard for. The two standard forms are

(i) erfc (ii) Φ form

$$\therefore Pe_0 = Z = \frac{x_1+\sqrt{E_b}}{\sqrt{N_0}} \quad \text{—(1m)} \quad dx_1 = \sqrt{N_0} : dz$$



$$\therefore Pe_0 = \frac{1}{\sqrt{\pi}} \int_0^\infty \exp[(-z)^2]\, dx$$

$$\left(\frac{1}{2}\,\mathrm{erfc}(u) = \frac{1}{\sqrt{\pi}} \int_u^\infty e^{(-z)^2} dz\right)$$

$$Z = \sqrt{E_b/N_0}$$

$$u = \sqrt{E_b/N_0}$$

$$\therefore Pe_0 = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/N_0}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\text{Energy of the signal}}{\text{Noise parer}}}\right) \quad \text{—(1m)}$$

Similarly $P_e(0|1) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{0} \exp\left[-\frac{(x_1-\mu)^2}{2\sigma^2}\right] dx_2$

$P_e(0|1) = \frac{1}{2} \operatorname{erfc}\sqrt{E_b/N_0}$

∴ Total probability of error $P_e = P_e(1|0)\,P_e(0) + P_e(0|1)\,P_e(1)$ — (1m)

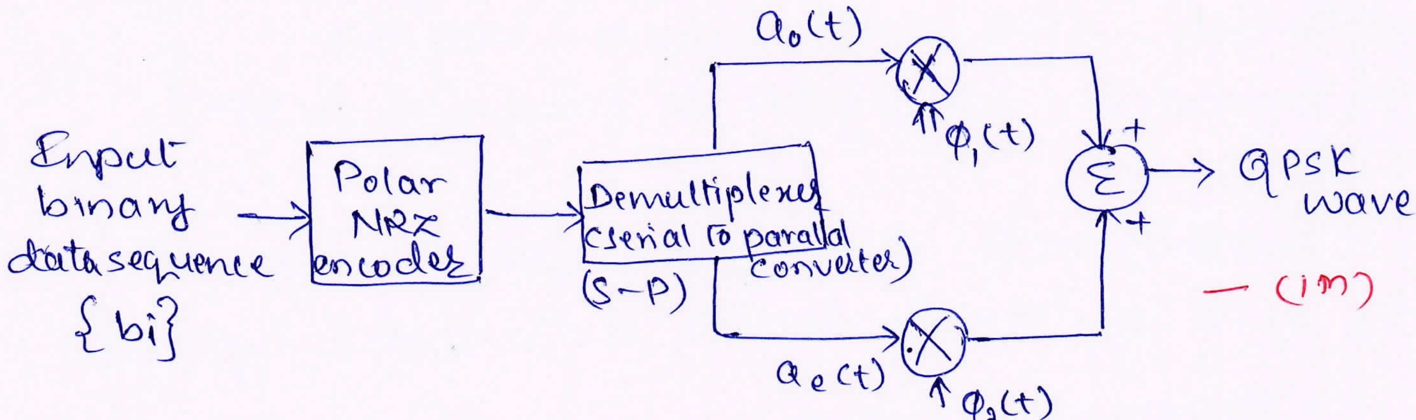
Assuming probability of 1's & 0's are equal

$P_e = \frac{1}{2}\left[P_e(1|0) + P_e(0|1)\right]$

$P_e = \frac{1}{2} \operatorname{erfc}\sqrt{\frac{E_b}{N_0}}$ — (1m)

5 b. With a neat block diagram, explain the generation and coherent detection of QPSK signals. (6m)

Ans: Coherent QPSK Transmitter



— (1m)

fig ① coherent QPSK Tx

Operation –

→ $\{b_i\}$ is the input binary sequence with a bit interval of T_b seconds.

→ Demultiplexer is serial to parallel converter. Thus using Demux, a binary sequence is divided into an odd indexed sequence $a_0(t)$ and an even indexed sequence $a_e(t)$. (2m)

→ If $\{b_i\} = \{b_0, b_1, b_2, b_3, b_4, b_5, b_6 \ldots\}$ then even seq $\{a_e(t)\} = \{b_0, b_2, b_4, b_6, b_8, \ldots\}$ and odd seq $\{a_e(t)\} = \{b_1, b_3, b_5, b_7 \ldots\}$

→ For the sequences, $\{a_o(t)\}$ and $\{a_e(t)\}$, the bit interval $T = 2T_b$ seconds

→ It can be regarded that $\{a_o(t)\}$ and $\{a_e(t)\}$ are two sequences with bit interval $2T_b$, twice that of sequence $\{b_i\}$

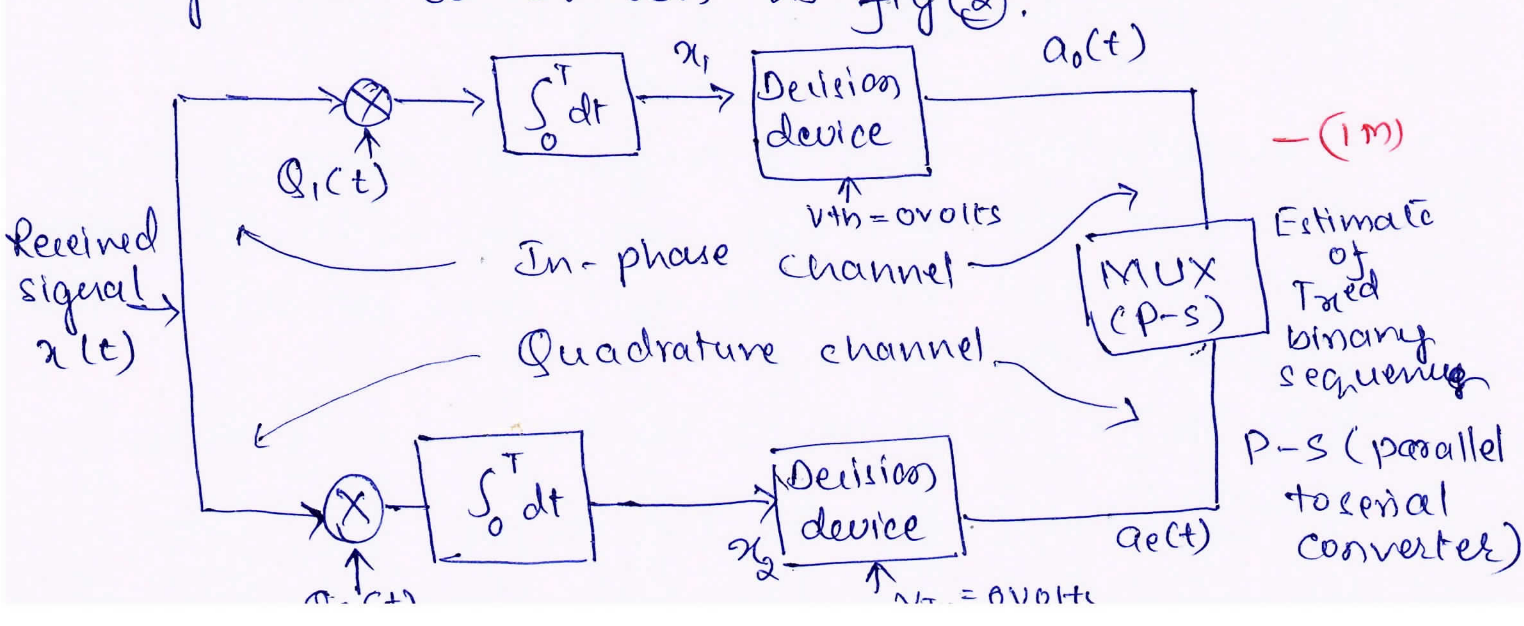
→ These even indexed and odd indexed sequences Phase modulate two carrier ($\cos 2\pi f_c t$ & $\sin 2\pi f_c t$) with same frequency but that are in phase quadrature.

→ Since bandwidth is inversely related to symbol interval it can concluded that, for a given binary data rate, QPSK signal occupies half the transmission bandwidth as compared to BPSK signal.

In other words for a given BW, the data rate of QPSK systems is double that of BPSK systems.

Coherent Detection of QPSK signal.

→ The scheme of coherent detection for a QPSK signal is as shown in fig ②.



– (1M)

Operation:

→ It consists of a pair of correlators with locally generated pair of coherent referance signals OR (orthonormal basis functions) $\phi_1(t)$ & $\phi_2(t)$ which means that R_x is synchronised with T_x.
Let x_1 & x_2 are the correlator outputs in response to the received signal $x(t)$

→ A decision is made by comparing x_1 & x_2 with a threshold of zero volts ($V_{Th} = 0$ volts)

If inphase channel output $x_1 > 0$ a decision is made in favour of symbol 1, otherwise it is in favour of symbol 0 (2M)

Likewise,

→ If quadrature channel output $x_2 > 0$, a decision is made in favour of symbol 1, otherwise it is favour of symbol 0.

→ The binary sequence from inphase and quadrature channels are multiplexed to get an estimate of original binary sequence.

→ The coherent detection scheme is optimum & thus ensures minimum probability of error.

OR

6. a. With a neat diagram, explain the non-coherent detection of binary frequency shift keying techniques. (4M)

Ans: WKT in BFSK the transmitted signal is defined as

$$S_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$$

where T_b - bit duration

$f_i = \frac{n_i}{T_b}$ (carrier frequency equals one of two possible values f_1 & f_2;

to ensure that the signals representing these two frequencies are orthogonal

$\boxed{n_i = \text{is an integer}}$ — (1m)

→ The transmission of frequency f_1 represent symbol 1 & transmission of frequency f_2

→ Represents symbol 0

→ For the non coherent detection of this frequency modulated signal, the receiver consists of a pair of matched filters followed by envelope detectors and is shown in fig ③

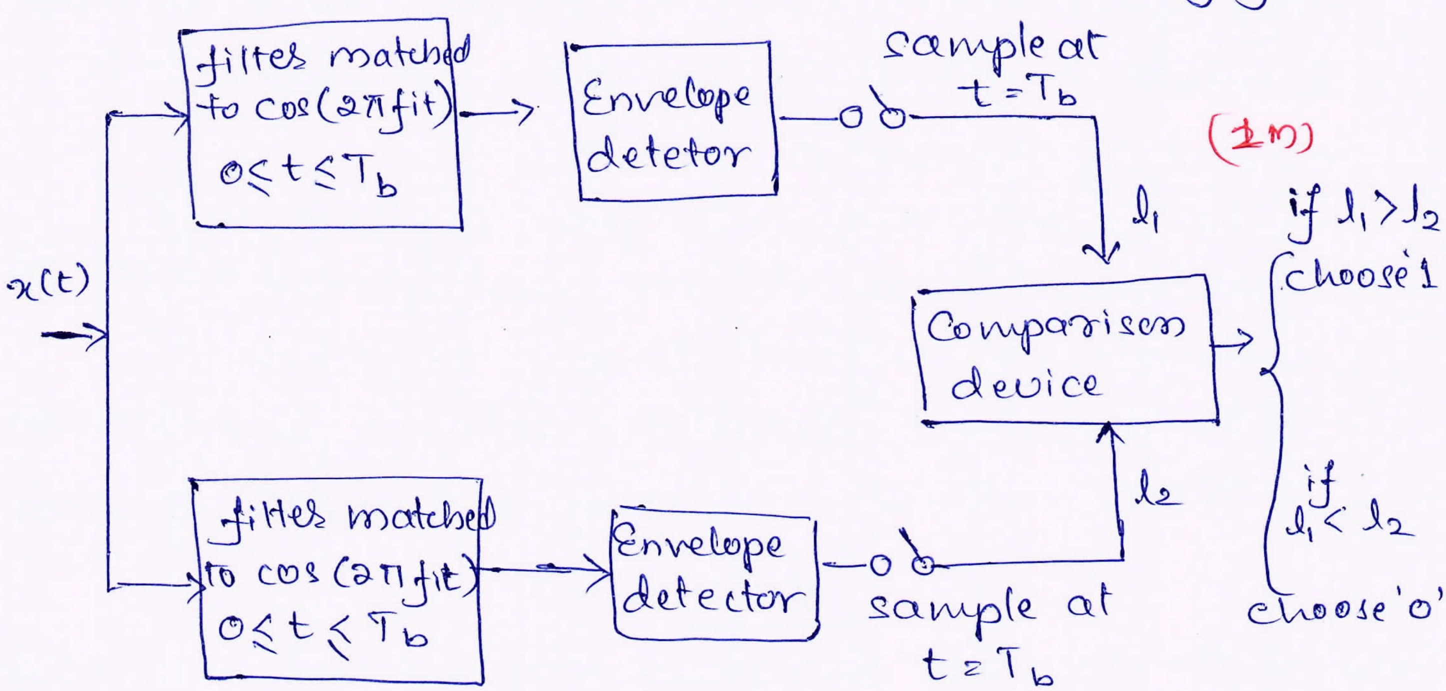


fig ③ Non coherent Rx for detection of BFSK

→ The filter in the upper path of the Rx is matched to $\cos(2\pi f_1 t)$ and filter in the lower path matched to $\cos(2\pi f_2 t)$ for the signaling interval $0 \le t \le T_b$.

→ The resulting envelope detector outputs are sampled at $t = T_b$ and their values are compared.

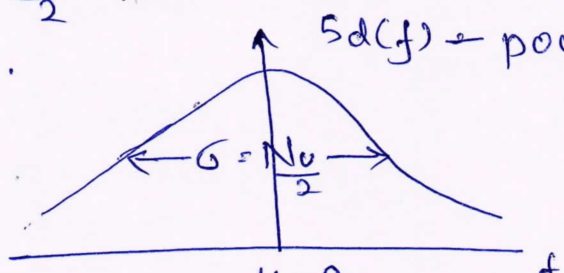
→ Here envelope detector outputs are shown as l_1 & l_2.

→ The Rx decides in favor of symbol 1 if $l_1 > l_2$ and Rx decides in favour of symbol 0 if $l_1 < l_2$.

→ If $l_1 = l_2$ the Rx simply guesses randomly in favor of symbol 1 or 0. (2M)

6 b. Derive an expression for probability of error of binary frequency shift keying technique. Also draw the block diagrams of BFSK transmitter and coherent receiver. (10M)

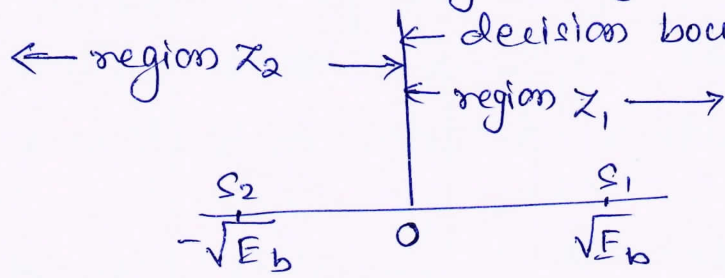
Ans: It starts with assumption that the noise which adds with original signal is of type AWGN i.e. at the receiver input we are receiving random signal $x(t) = s(t) + w(t)$. Here this $w(t)$ we are assuming as Additive White Gaussian Noise (AWGN), which has mean = 0 and PSD = $\frac{N_0}{2}$ wide sense stationary has following nature.



→ Thus alongwith Gaussian noise we receive signals $s(t)$ i.e. $x_1(t) = s_1(t) + w_1(t)$ and → (1M)

$x_2(t) = s_2(t) + w_2(t)$

→ WKT signal space diagram for coherent BFSK



– (1 M)

→ The receiver decides in favour of symbol '1' if the received signal point represented by the observation vector x falls inside region Z_1

This occurs when $x_1 > x_2$

→ On the otherhand, we have $x_1 < x_2$, the received signal point falls inside region Z_2 and the receiver decides in favor of symbol 0.

→ Let us assume here Y as a new Gaussian random variable whose sample value y is equal to the difference between x_1 & x_2

i.e. $y = x_1 - x_2$

→ The conditional mean value of the random variable Y depends on which binary symbol was transmitted i.e.

when symbol 1 was transmitted/sent then the conditional mean value f random variable Y is given by

$$E[Y|1] = E[X_1|1] - E[X_2|1]$$
$$= E[\sqrt{E_b}] - E[0]$$
$$= \sqrt{E_b}$$

[**Note**: $E[X_2|1] = 0$ because ϕ_1 is orthogonal to ϕ_2]

(1 M)

Similarly when symbol 0 is sent then conditional mean value of Y is → $E[Y|0] = E[X_1|0] - E[X_2|0]$

$$= E[0] - E[\sqrt{E_b}]$$
$$= -\sqrt{E_b}$$

→ The variance of the random variable Y is independent of which symbol was sent.

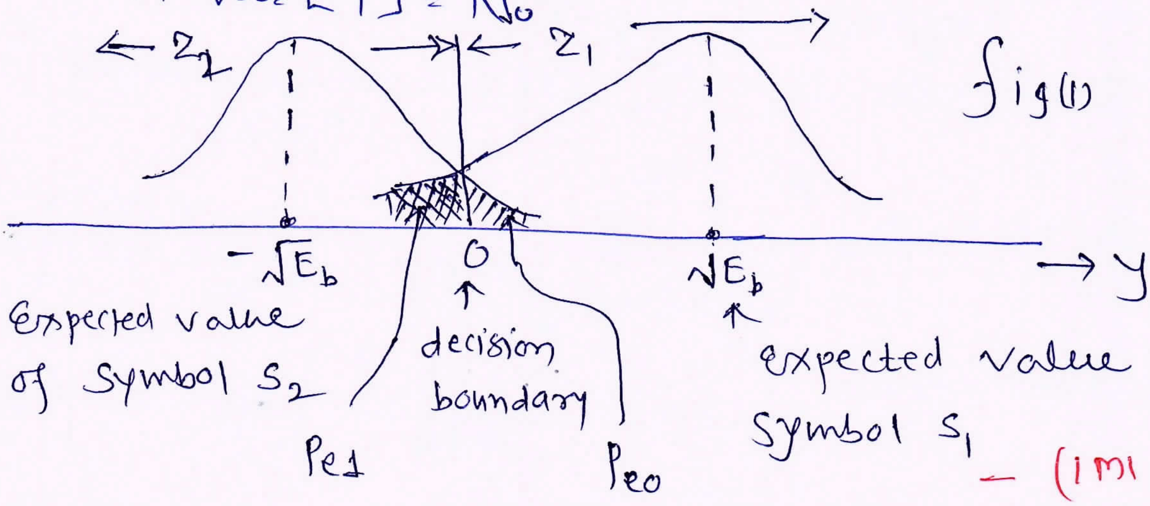
→ Since the random variables X_1 & X_2 are statistically independent, each has variance $(\sigma^2) = \frac{N_0}{2}$

→ It follows that $var[X_1 \pm X_2] = Var[X_1] \pm var[X_2]$

$$var[Y] = Var[X_1] + var[X_2]$$ (1 M)

$$= \frac{N_0}{2} + \frac{N_0}{2}$$

$$\therefore var[Y] = N_0$$



fig(1) — (1 M)

→ The fig(1) shows probability distribution of random variable Y, where $E[Y] = E[X_1] - E[X_2]$

→ The observation vector x has two elements x_1 & x_2 that are defined by respectively

$$x_1 = \int_{t=0}^{t=T_b} x(t)\,\phi_1(t).dt$$ and

$$x_2 = \int_{t=0}^{t=T_b} x(t)\,\phi_2(t).dt$$ — (1 M)

→ where $x(t)$ is the received signal, whose form depends on which symbol (either 1 or 0) was transmitted.

→ we know that the Gaussian function is written as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_x)}{2\sigma^2}}$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int exp\left[-\frac{(x-\mu_x)^2}{2\sigma^2}\right] dx \quad -(1M)$$

where x is a Gaussian random variable

→ But in BFSK we have consider Y as Gaussian random variable, therefore the above equation is written as with substitution.

replace X by Y

Substitute $\mu_x = -\sqrt{E_b}$ (if we are calculating P_{e0})

$\sigma^2 = N_0$

$$\therefore P_e(1|0) = P_{e0} = \frac{1}{\sqrt{2\pi(N_0)}} \int_0^\infty exp\left[-\frac{(Y-(-\sqrt{E_b}))^2}{2N_0}\right] dy$$

$$P_{e0} = \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty exp\left[-\frac{(Y+NE_b)^2}{2N_0}\right] dy \quad \text{—— Eq}^n \text{ (I)}$$

$$\frac{1}{2} erfc(u) = \frac{1}{\sqrt{\pi}} \int_u^\infty exp(-z)^2 dz \quad \text{—— Eq}^n \text{ (II)}$$

Substitute

$$Z = \frac{Y+\sqrt{E_b}}{\sqrt{2N_0}} \quad \text{in Eq}^n \text{ (I) with}$$

change in limits of Y as (2M)

for $Y = 0 \longrightarrow Z = \frac{\sqrt{E_b}}{\sqrt{2N_0}}$

$Y = \infty \longrightarrow Z = \infty$, also $dy = \sqrt{2N_0} \cdot dz$

Eq^n (I) becomes

$$P_{e0} = \frac{1}{\sqrt{2\pi N_0}} \int_{Z=\sqrt{\frac{E_b}{2N_0}}}^{Z=\infty} Exp(-z)^2 (\sqrt{2N_0}\, dz)$$

$$\therefore P_{e0} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_0}}^{z=\infty} e^{(-z)^2} \, dz \quad \text{———} \quad \text{Equ}^n \text{ (III)}$$

where $u = \sqrt{E_b/2N_0}$

$\therefore$ Equ^n (III) is in required standard form

$\Rightarrow$ Compare Equ^n (II) and (III)

$$u = \sqrt{E_b/2N_0}$$

$$\therefore \boxed{P_{e0} \text{ or } P_e(1|0) = \frac{1}{2} \, erfc \sqrt{\frac{E_b}{2N_0}}}$$

This is in — (1M)

$\Rightarrow$ Similarly the probability of error when symbol 1 is sent but received as 0 can be calculated with following changes

Here u_x = mean value = $+\sqrt{E_b}$

$$\sigma^2 = N_0$$

$$\rightarrow P_{e1} \text{ or } P_e(0|1) = \frac{1}{\sqrt{2\pi N_0}} \int_0^{\infty} \exp\left[-\frac{(y-(\sqrt{E_b}))^2}{2N_0}\right] . dy$$

$$P_{e1} = \frac{1}{\sqrt{2\pi N_0}} \int_0^{\infty} \exp\left[-\frac{(y-\sqrt{E_b})^2}{2N_0}\right] . dy$$

put $z = \frac{y-\sqrt{E_b}}{\sqrt{2N_0}}$ then $dy = \sqrt{2N_0} \, . \, dy$

and for $y=0 \rightarrow z = -\sqrt{E_b/2N_0}$ and $y=\infty \rightarrow z=\infty$

$$\therefore P_{e1} = \frac{1}{\sqrt{2\pi N_0}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e(-z)^2 . \sqrt{2N_0} \, . \, dz$$

Comparing with equ^n (II) we get

$$P_{e1} = \frac{1}{2} \, erfc \sqrt{\frac{E_b}{2N_0}}$$

→ Total probability $= \frac{1}{2}[P_{e0} + P_{e1}]$ or $\begin{bmatrix} [P(0)\,P_{e0} + P(1)\,P_{e1}] \\ P(0) = P(1) = \frac{1}{2} \end{bmatrix}$

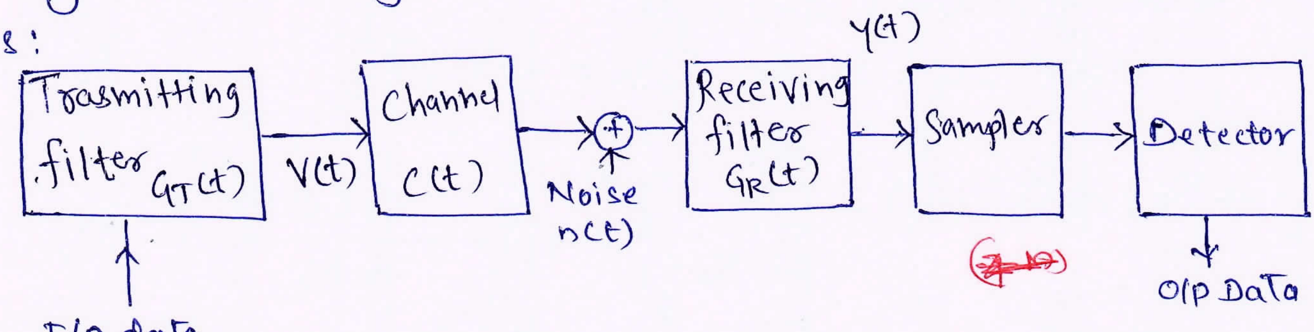
$$\therefore P_e = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{2N_0}}\right] \quad \text{— (1m)}$$

Thus comparing the probability of error of BFSK with BPSK for coherent detection, BFSK give more probability of error as compared to BPSK.

Module 4

7. a. With a neat block diagram of digital PAM system obtain the expression for inter symbol interference (ISI)

Ans:



$$v(t) = \sum_{n=-\infty}^{\infty} a_n\, g_T(t - nT) \quad \text{— (1m)}$$

$$y(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + n(t) \quad \text{— (2m)}$$

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n x(mT - nT) + w(mT) \quad \text{(2m)}$$

$$y_m = x_0 a_m + \sum_{n \neq m} a_n x_{n-m} + w_m \quad \text{— (1m)}$$

The second term is ISI

Block diagram explaination (2m) + Fig -(2m) — (4m)

7 (b) State ~~and~~ ~~prove~~ Nyquist condition for zero ISI

Ans: Statement: For zero ISI a necessary & sufficient condition for $x(t)$ to satisfy $x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$ — (1M)

is that its Fourier transform $X(f)$ must satisfy

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$ — (2M)

7 (c) For the binary data sequence $\{d_n\}$ given by 1110 1001. Determine the precoded sequence, transmitted sequence, received sequence and the decoded sequence.

Ans:

Data sequence d_n		1	1	1	0	1	0	0	1
Precoded sequence P_n	* 0	1	0	1	1	0	0	0	1 — (1M)
Transmitted sequence a_n	−1	1	−1	1	1	−1	−1	−1	1 — (1M)
Received sequence b_n		0	0	0	2	0	−2	−2	0 — (1M)
Decoded sequence		1	1	1	0	1	0	0	1 — (1M)

* Student may choose $P_{-1} = 1$ instead of 0.

8. a. Explain the design of band limited signals with controlled ISI.

Ans: To achieve a symbol rate of 2W symbols/seconds, zero ISI can be relaxed by allowing one additional non-zero value in the samples $\{x(nT)\}$. Since the introduced ISI is controlled, it can be taken into account at the receiver.

Duobinary signal pulse — $x(nT) = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$

$$X(f) = \begin{cases} \frac{1}{W} e^{-j\pi f/2W} \cos \frac{\pi f}{2W} & |f| < W \\ 0 & \text{otherwise} \end{cases}$$ — (2M)

$x(t) = \text{sinc}(2wt) + \text{sin}\,c(2wt-1)$ — (1m)

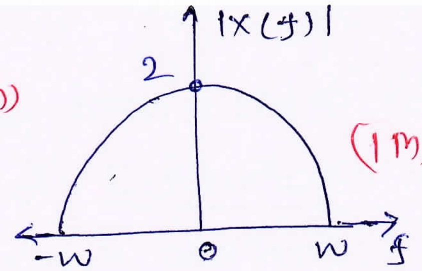
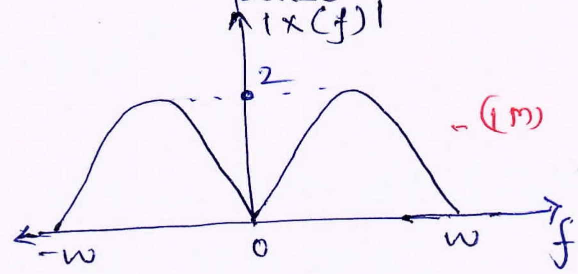


fig Duobinary Signal pulse

Modified Duobinary signal pulse

$$x(nT) = \begin{cases} 1 & n=-1 \\ -1 & n=+1 \\ 0 & \text{otherwise} \end{cases}$$ — 1m)



$x(t) = \text{sinc}(t+T)/T - \text{sinc}(t-T)/T$ — (1m)

$$x(f) = \begin{cases} \frac{j}{w} \sin\frac{\pi f}{w} & |f| \le w \\ & |f| > w \end{cases}$$ — (1m)

Generally pulses of the form

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2w}\right) \frac{\sin 2\pi w(t - n/2w)}{2\pi w(t-n/2w)}$$

$$X(f) = \begin{cases} \frac{1}{2w} \sum_{n=-\infty}^{\infty} x(n/2w) e^{-jn\pi f/w} & |f| \le w \\ 0 & |f| > w \end{cases}$$

are called partial response signals when ISI is purposefully introduced. (2m)

8 b. What is a zero forcing equalizer? With a neat block diagram explain the operation of linear transversal filter.

Ans: <u>Definition of Zero forcing equilizer:</u>

we know that inverse channel filter (equilizer) completely eliminates ISI caused by channel. Since it forces the ISI to be zero at the sampling times $t = nT$, the equalizer is called a zero forcing equalizer.

The output of the receiving filter is given by

$$Y(t) = \sum_{k=-\infty}^{\infty} \mu A_k P(t - kT_b)$$ — (2m)

<u>Operation of Linear Transversal filter:</u>

We know that in real channels, the ISI is limited to a finite number of

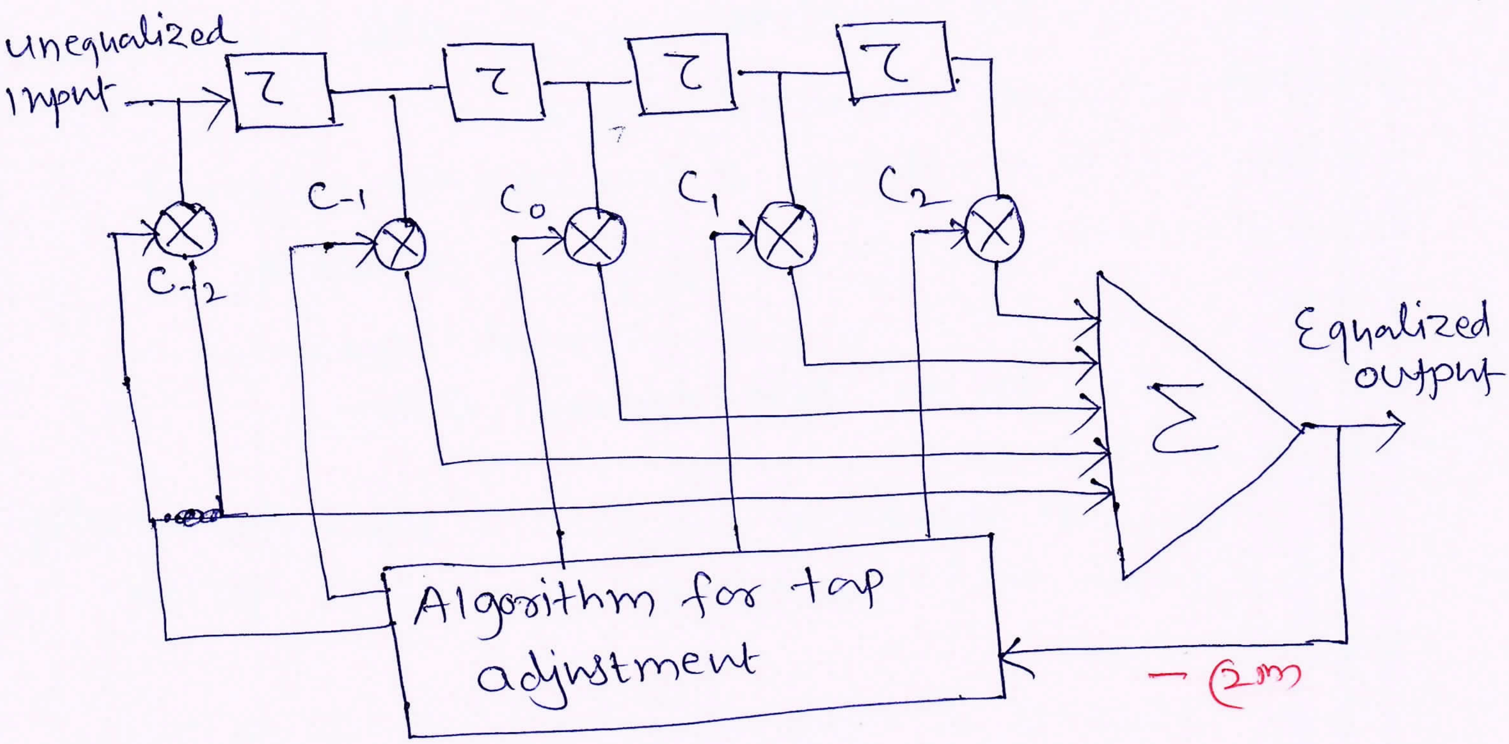


fig: Linear Transversal Filter

→ We know that in real channels, the ISI is limited to a finite no. of samples L samples.

→ As a result, in practice for example, the channel equalizer is approximated by a finite duration impulse response (FIR) filter or transversal filter, with adjustable tap coefficients $\{c_n\}$

→ The time delay τ between adjcent taps may be selected as large a T, the symbol interval in which case the FIR equalizer is called a symbol spaced equalizer.

→ In this case the input to the equalizer is the sampled sequence given by

$$y_m = x_0 a_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} a_n x_{m-n} + w_m$$

→ However, we note that when $\frac{1}{T} < 2W$, frequencies in the received signal that are above the folding frequency $\frac{1}{T}$ are aliased into frequencies below $\frac{1}{T}$

→ In this case the equalizer compensates for the aliased channel-distorted signal.

→ When the time delay τ between the adjecent taps is such that $\frac{1}{\tau} \geq 2W \geq \frac{1}{T}$, no aliasing occurs, hence the inverse channel equalizer compensates for the true channel distortion

→ Since $\tau < T$,

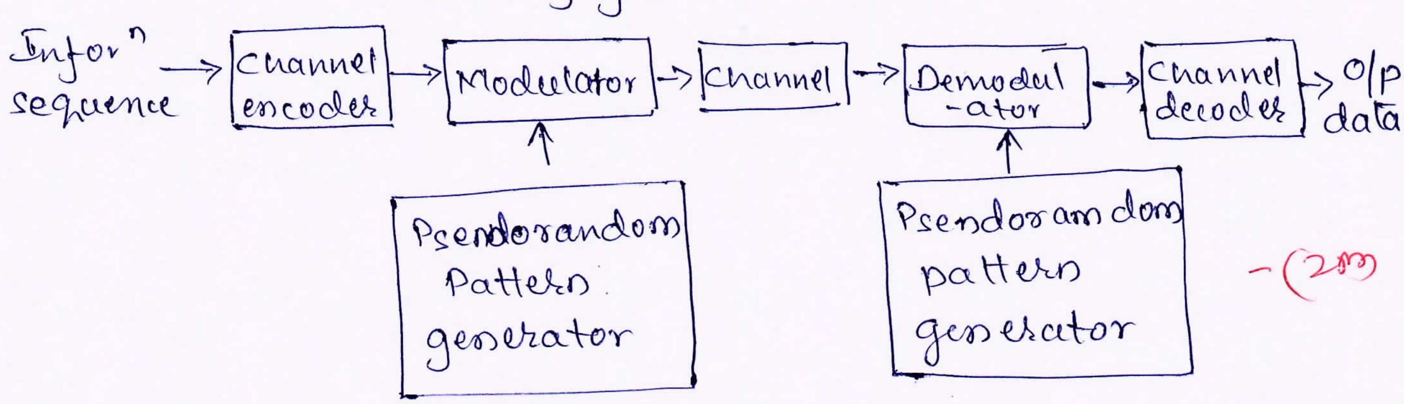
Channel equalizer is said to have fractionally spaced taps, and it is called a fractionally spaced equalizer.

→ In practice τ is often selected as $\tau = \frac{T}{2}$, in this case, the sampling rate at the output of the filter $G_R(f)$ is $\frac{2}{T}$. — (2m)

Module 5

9. a. Explain the model of a spread spectrum digital communication system.

Ans: The basic elements of a SS DC system are illustrated in fig ①



-(2m)

Fig ① model of a spread-spectrum (ss) Digital Communication (DC) system

→ The channel encoder and decoder, modulator & demodulator are the basic elements of a conventional DC sys.

→ In addition, a SS sys uses two identical pseudorandom sequence generators are at the Tx end and other at the Rx end.

→ These two generators produce a pseudorandom binary valued sequence, which is used to spread the transmitted signal at the modulator and to despread the received signal at the demodulator.

→ Time synchronization of the PN sequence generated the Rx with the PN sequence contained in the Rxed signal is required to properly despread the Rxed SS signal.

→ In practical sys

synchronization is established prior to the Txn of inforn

→ The synchronization is achieved by Txing a fixed PN bit bit pattern, which is designed so that Rx will detect it with high probability in the presence of interference.

→ once the time synchronization of PN sequence generators at Txing end and Rxing end is achieved, then the information is transmitted.

→ In the data mode, the comm. sys. usually tracks the timing of the incomming Rxed signal and keeps the PN sequence generator in synchronism.

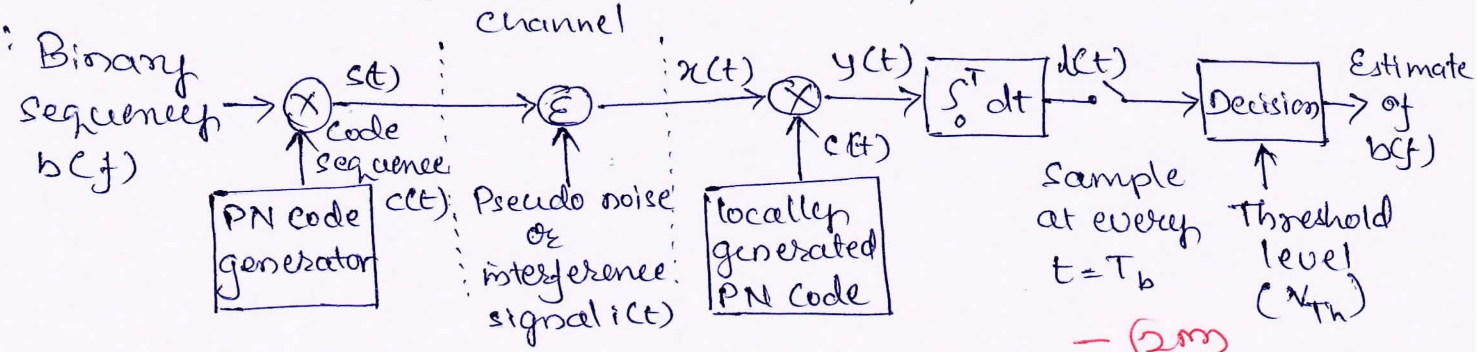
→ Interference is introduced in the Txn of the SS signal through the channels.

→ The interferance may be categorized as either ~~wi~~ wideband OR narrowband relative to the BW of the inforn- bearing signal either continuous or discontinuous (pulsed) in time. — (4m)

~~q.b. What is a zero forcing~~

q. b. Explain the generation and demodulation of direct sequence spread spectrum signals with necessary equation and block diagram.

Ans:



— (2m)

Fig Baseband DSSS system.

→ The input binary sequence b(t) is the baseband signal in NRZ polar format in which symbol 1 is represented by +1 volt and symbol 0 by −1 volt, with each bit occupying a time of T_b seconds.

→ c(t) is a wideband code sequence which has noise like properties.

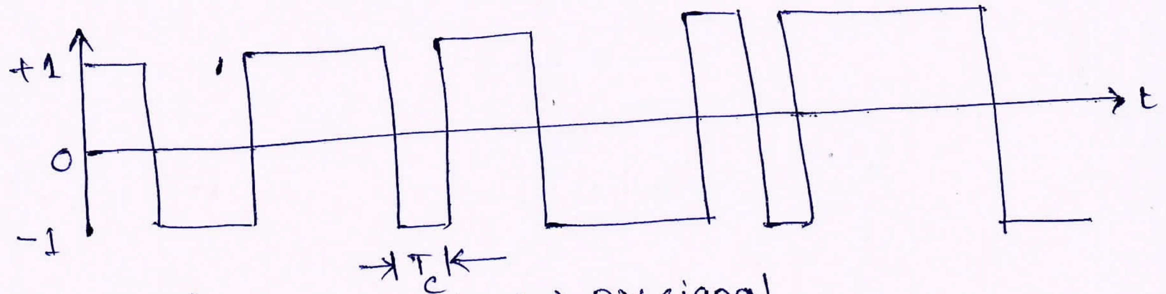
This sequency c(t), also called spreading sequence is generated by a feedback shift register.

→ Like b(t), c(t) is also expressed in NRZ polar format so that it has two levels. +1 volt and −1 volt. (1m)

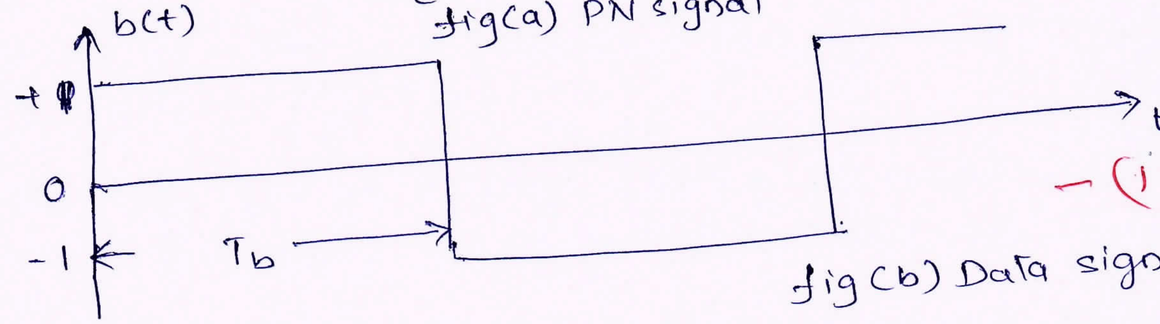
→ It is important to note that, the duration of each bit (T_c) in c(t) is the period of the clock pulse that is applied to the feedback shift register used for generating c(t).

→ Normally $T_c << T_b$ and hence bit rate R_c of the $c(t)$ is much greater than input bit rate $R_b = \frac{1}{T_b}$

→ Often R_c is also called Chip rate while T_c is called as chip interval.



fig(a) PN signal



— (jm)

fig (b) Data signal

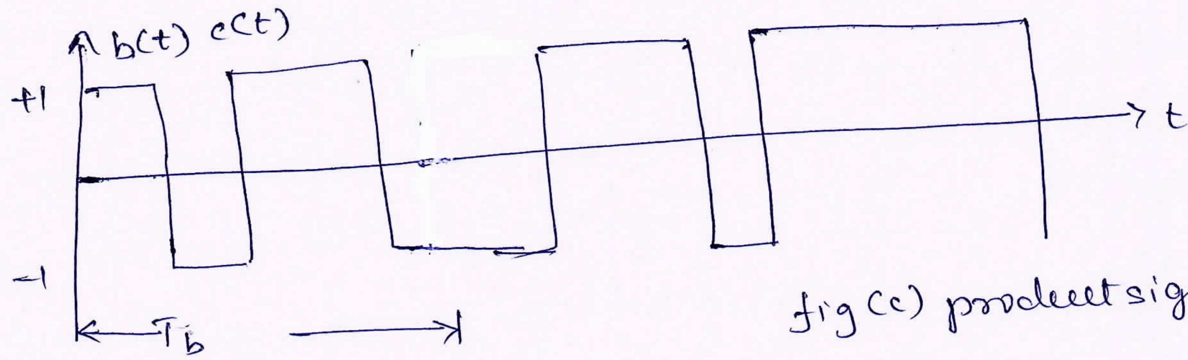
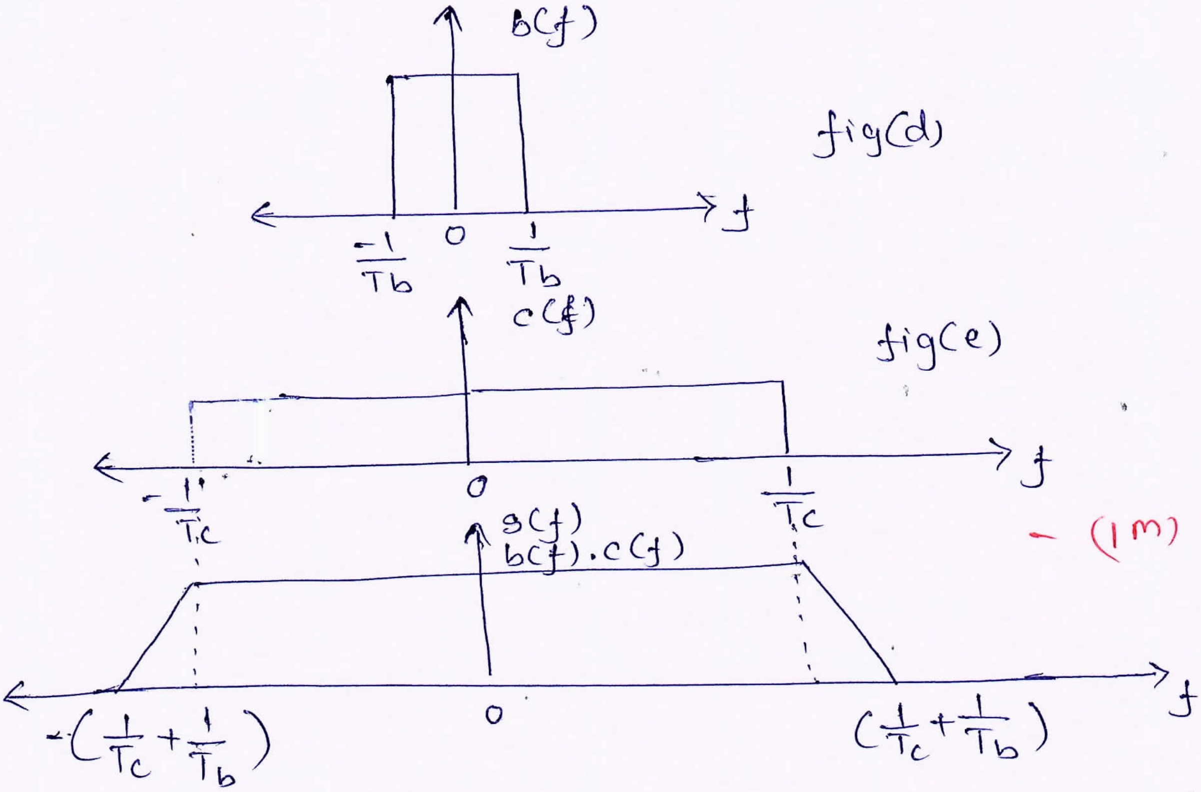


fig (c) product signal

fig② Generation of a DSSS signal.

Fig shows the basic method for accomplishing the spreading

→ This multiplication operation serves to spread the BW of the information-bearing signal (whose BW is approximately R_b Hz) into the wider BW occupied by PN generator signal $c(t)$ (whose BW is approximately $1/T_c$)



– (1m)

fig③ Convolution of power spectra of b(f) (data signal and c(f) (spreading or PN sequence)

This figure shows spectrum spreading where input binary sequence b(f) is narrowband and PN sequence c(t) is wideband, the BW of the spectrum s(f) nearly equals that of c(f)

→ Therefore PN sequence performs the role of a spreading code.

→ Let the signal Rxed at the Rx x(t).

Then

$$x(t) = s(t) + i(t) \quad \text{------} \quad \text{Equ}^n (1)$$ – (1m)

where i(t) – denotes an interference signal or a jamming signal aimed at disrupting the infor^n

→ The first step in the detection process is to multiply the Rxed signal x(t) by a locally generated PN sequence, which is the exact replica of that used at the Tx.

→ This multiplication is performed by a product modulator shown at the receiver (fig ②)

→ The output of product modulator is given by

$$r(t) = x(t) \cdot c(t)$$

$$\therefore r(t) = [s(t) + i(t)] \cdot c(t)$$

$$= [b(t) \cdot c(t) + i(t)] \cdot c(t) \qquad (1M)$$

$$= b(t)\, c^2(t) + i(t) \cdot c(t) \quad \text{—— Equ}^n \text{ (II)}$$

Since $c^2(t) = 1$ for all t

$$\therefore r(t) = b(t) + i(t) \cdot c(t) \quad \text{—— Equ}^n \text{ (III)}$$

Thus from equation (III) it is clear that the spectrum of $s(t)$ is despread, resulting in spectrum of $b(t)$

9.c. Write a note on low detectability signal transmission as an application of direct sequence spread spectrum.

Ans: → In this application the information bearing signal is Tx^{ed} at a very low power level relative to the background channel noise and thermal noise that is generated in the front end of a receiver. (1M)

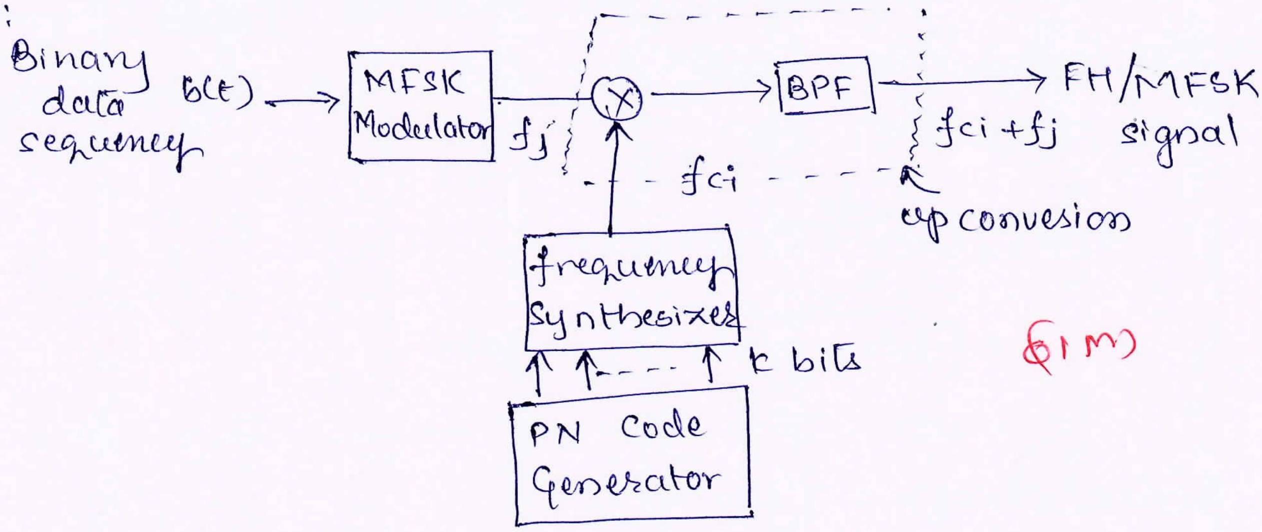
→ If the DSSS signal occupies a BW W Hz and PSD of the additive noise $= N_0$ W/Hz, then the avg noise power in the BW W is $P_N = W N_0$.

→ The avg Rx^{ed} signal power at the intended Rx is P_R. —(1M)

→ If we wish to hide the presence of the signal from other receivers (Rx^{ers}) which are close to intended Rx, the signal is Tx^{ed} at a power (low) level such that $P_R/P_N \ll 1$. — (1M)

10.a. With a neat block diagram, explain the frequency hopped spread spectrum.

Ans:



(01 M)

Fig ① FH/MFSK transmitter.

→ The first stage of an FH/MFSK transmitter is the frequency modulator and second stage is frequency mixer.

→ The incoming binary sequence is applied to a serial-to parallel converter to get blocks of k-bits.

→ Depending on k-bit binary pattern, any one of the 2^k discrete amplitude levels of M-ary PAM signal is obtained. — (01 M)

→ This M-ary PAM signal is applied to a voltage controlled oscillator (VCO)

→ For every amplitude level in the m-ary PAM signal, the VCO produces one of the 2^k descreate frequencies.

→ Thus, s(t) is an MFSK wave.

→ The output of the MFSK modulator is then mixed with the output of a frequency synthesizer.

→ The frequency synthesizer's output is one of $y = 2^k$ values, where k-equals no. of bits of the PN sequence generator output.

(01 M)

→ As a result, frequency hops over 2^k distinct values.

→ The BPF (Band pass Filter) passes the sum frequency for the transmission and rejects the difference frequency components.

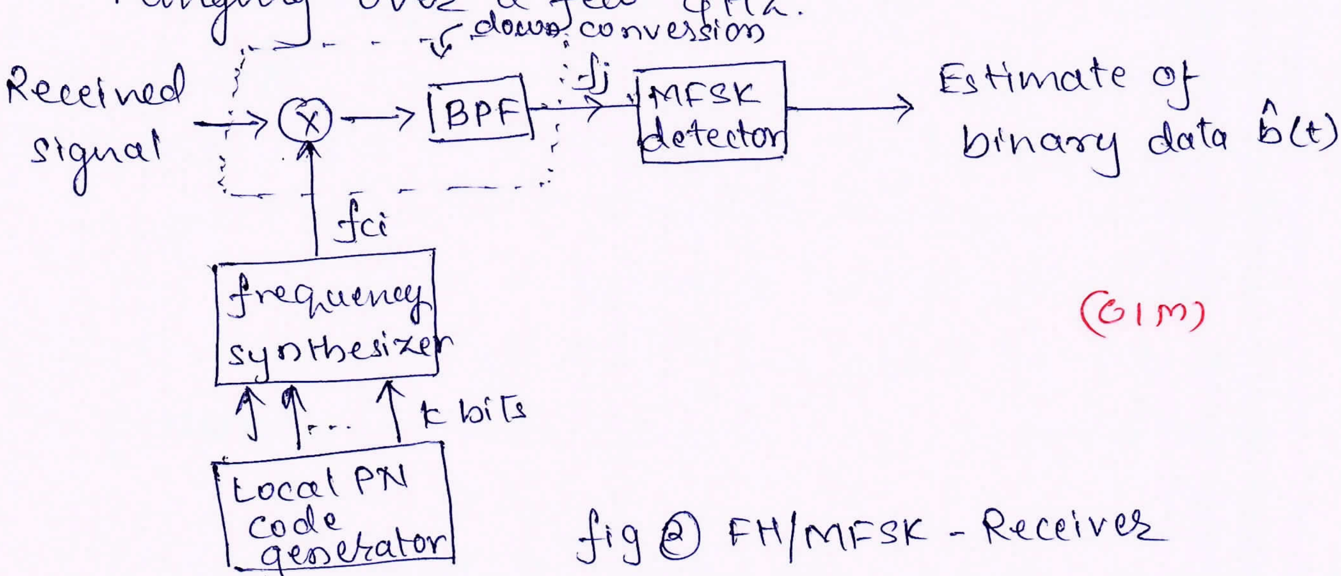
→ BPSK modulation generally gives better performance than BFSK.

→ However, it is difficult to maintain phase coherence in the synthesis of frequencies used in the hopping pattern and also in the popagation of the signal over the channel as the signal is hoped from one frequency to another over a wide BW.

– (1m)

OR

→ FHSS signal occupies much larger ~~BW~~ BW ranging over a few GHz.



(01m)

fig ② FH/MFSK - Receiver

→ In the first stage, mixing operation (down-conversion) removes the frequency hopping.

→ The mixer inputs are the received signals and the output of a local frequency synthesizer that is in synchronization with that of transmitter.

→ The output of mixer is passed through BPF which selects the different frequency component from the mixer.

→ The output of the BPF is the MFSK signal, which is demodulated using noncoherent MFSK detector. – (1M)

→ The noncoherent MFSK detector consists of a bank of M matched filters each of which is matched to one of the $M = 2^k$ MFSK tones.

→ An estimate of the original symbol transmitted is obtained by selecting the largest filter output.

→ In an FH system, an FH tone of short duration is referred to as a chip.

The chip rate, R_C, for an FH system is defined by $R_C = \max(R_h, R_s)$ – (1M)

where R_h - hop rate, R_s - symbol rate.

10 b. Explain the effect of despreading on a Narrow band interference in direct sequence spread spectrum systems. A direct sequence spread spectrum signal is designed to have the power ratio P_R/P_N at the intended receiver is 10^{-2}. If the desired $E_b/N_0 = 10$ for acceptable performance, determine the minimum value of processing gain.

Ans: Received signal $r(t) = A_c v(t) c(t) \cos 2\pi f_c t + i(t)$ (02M)

Despreading $r(t) c(t) = A_c v(t) \cos 2\pi f_c t + i(t) c(t)$

The total power in interfering signal at the output of demod is

$$P_I/(W/R_b) = P_I/(T_b/T_c) = P_I/L_c$$

∴ Power is reduced by an amount L_c. L_c is called processing gain. – (02M)

$$\frac{E_b}{N_0} = \frac{P_R T_b}{N_0} = \frac{P_R L_c T_c}{N_0} = \left(\frac{P_R}{W N_0}\right) L_c = \left(\frac{P_R}{P_N}\right) L_c \quad (2M)$$

$$\therefore 10 = 10^{-2} L_c \Rightarrow L_c = 1000$$

10. c. Write a note on code division multiple access as an application of direct sequence spread spectrum.

Ans: The enhancement in performance obtained through processing gain can enable many DS-SS signals to occupy the same channel bandwidth provided each signal has its own PN sequence. This is called CDMA. – (01M)

In demodulation, the signals from other simultaneous users of the channel appear as interference. – (01M)

A major advantage is large number of users can be accomodated if each user transmits message for a short period of time. – (01M)