CBCS SCHEME

USN 2VD17CV028 — 17CV42

Fourth Semester B.E. Degree Examination, June/July 2019
Analysis of Determinate Structures

Time: 3 hrs. Max. Marks: 100

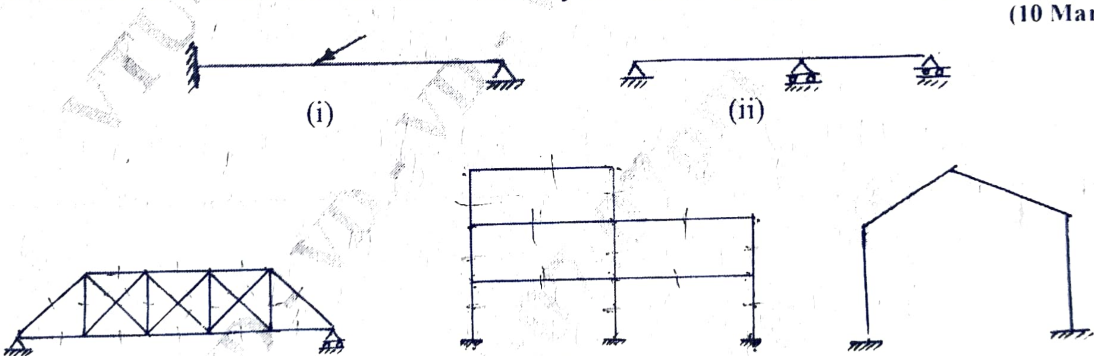
Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Assume any missing data suitably.

Module-1

1 a. Differentiate between statically determinate and indeterminate beams with an example for each. **(06 Marks)**

b. Define degree of freedom. What is the degree of freedom for a i) Fixed support ii) Hinged support. **(04 Marks)**

c. Determine static and kinematic indeterminancy for the following shown in Fig.Q.1(c). **(10 Marks)**



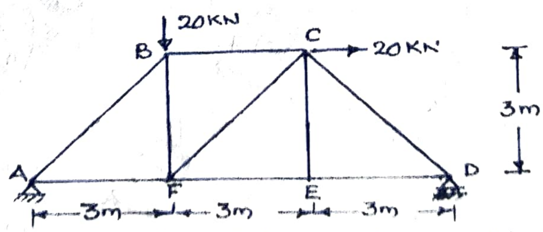
(iii) (iv) (v)

Fig.Q.1(c)

OR

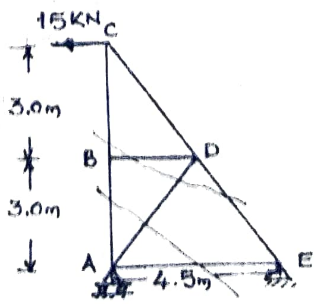
2 a. Determine the forces in all the members of the truss shown in Fig.Q.2(a) use the method of joints. **(12 Marks)**

Fig.Q.2(a)



b. Determine the forces in all the members of the truss shown in Fig.Q.2(b) by the method of section. **(08 Marks)**

Fig.Q.2(b)



Module-2

3 a. Derive moment curvature equation for deflection. **(04 Marks)**

b. Determine the slope and deflection at free end of a cantilever beam subjected to point load 'W' at free end and of span 'L' with constant EI use Maculay's method. **(08 Marks)**

c. Using Conjugate beam method Determine the maximum deflection and slopes at support for a simply supported beam subjected to udl of w/m run over a span of L m with constant EI. **(08 Marks)**

OR

4 a. Determine the slope at left support and deflection at mid span of simply supported beam subjected to the loads as shown in Fig.Q.4(a) by using Maculay's method take EI = 200MN-m^2. **(10 Marks)**

b. Determine the slope at A and deflection at mid span for the above beam shown in Fig.Q.4(b) by using moment area method EI = 200 MN-m^2. **(10 Marks)**

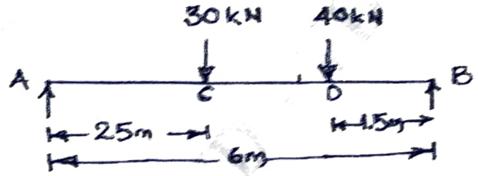


Fig.Q.4(b)

Module-3

5 a. Obtain an expression for strain energy stored in a member when it is subjected to bending moment. **(06 Marks)**

b. Find the deflection at C due to a point load acting as shown in Fig.Q.5(b) by using strain energy method. **(06 Marks)**

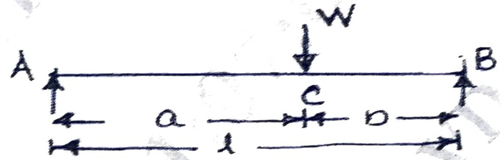
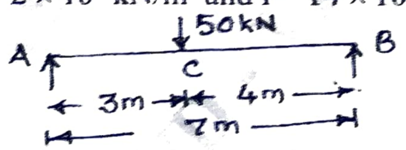


Fig.Q.5(b)

c. Find the deflection under the concentrated load for the beam shown in Fig.Q.5(c), by using Castiglino's theorem. Take E = 2 × 10^8 kN/m^2 and I = 14 × 10^{-6} m^4. **(08 Marks)**



OR

6 a. Determine the horizontal and vertical deflection at the free end of bracket shown in Fig.Q.6(a). **(10 Marks)**

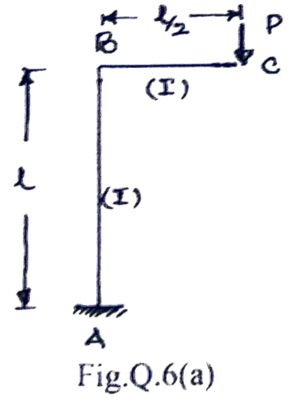


Fig.Q.6(a)

b. Determine the slope and deflection at free end of cantilever by using unit load method take $E = 2 \times 10^5$ N/mm^2 and $I = 12 \times 10^6$ mm^4 Refer Fig.Q.6(b). **(10 Marks)**

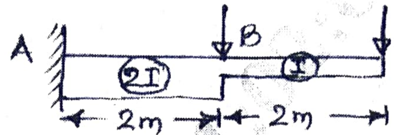


Fig.Q.6(b)

Module-4

7 a. A three hinged parabolic arch of span 20m and rise 4m carries a udl of 20kN/m run on the left half of the span find the maximum BM for the arch and also determine normal thrust and radial shear at a point 5m from left support. **(10 Marks)**

b. Show that the shape of cable is parabolic when the supports are at the same level and is subjected to udl of w force/unit length over the entire span also find the length of the cable. **(10 Marks)**

OR

8 a. A cable of span 20m and central dip 4m carries a udl of 20kN/m over the whole span. Find: i) Maximum tension in the cable ii) Minimum tension in the cable iii) Length of cable iv) Horizontal and vertical forces transmitted on to the supporting pier if the cable passed over a smooth frictionless pulley. **(10 Marks)**

b. Show that the parabolic shape is a funicular shape for a three hinged arch subjected to udl over its entire span. **(10 Marks)**

Module-5

9 a. Define influence line diagram, what are the uses of ILD? **(04 Marks)**

b. A simple girder of 20m span is traversed by a moving udl of length 6m with an intensity of 20kN/m from left to right. Find the maximum bending moment and maximum positive and negative shear force at section 4m from left support also find the absolute maximum bending moment that may occur any where in the girder. [Ref.Fig.Q.9(b)] **(16 Marks)**

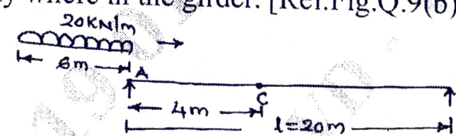


Fig.Q.9(b)

OR

10 a. Draw the unit load influence line diagrams for the reactions at supports of a simply supported beam. **(04 Marks)**

b. A simply supported beam shown in Fig.Q.10(b) is subjected a set of four concentrated loads which move from left to right. Determine: i) Maximum bending moment and shear force at a section of 6m from left support ii) Absolute maximum shear force and absolute maximum bending moment. Use influence line principle. **(16 Marks)**

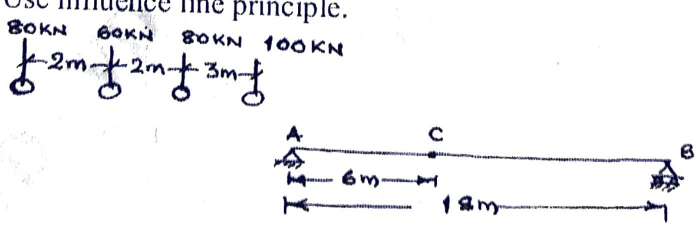


Fig.Q.10(b)

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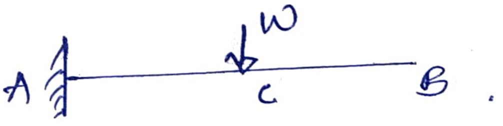
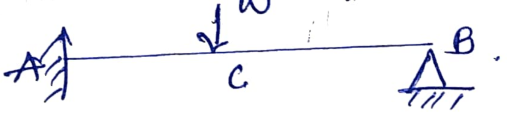
Solution for ADS (18CV42) June/July - 2019.

Sub: Analysis of Determinate Structures.

Q1. (a) Difference between statically determinate & indeterminate beams with examples.

Sl no	Statically Determinate	Statically Indeterminate
1.	The determination of internal forces do not require the C/s area & M.I of beam.	Here during analysis the C/s area and M.I are required as first step.
2.	Settlement of support / lack of fit will not affect internal forces and moments	Settlement of support / lack of fit are important factor in determining of internal force / moment.
3.	Equation of Equilibrium are sufficient to find the internal force / moments. Ex: Cantilever beam.  total 3 unknown.	Additional compatibility condition of displacement are required along with Equation of Equilibrium to find internal forces / moments. Ex: propped cantilever beam  total 5 unknown.

(b) Degree of freedom:-

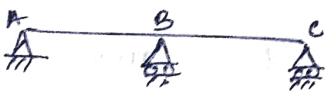
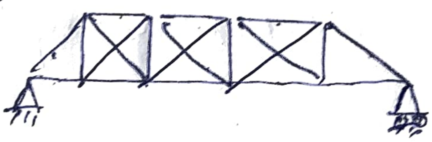
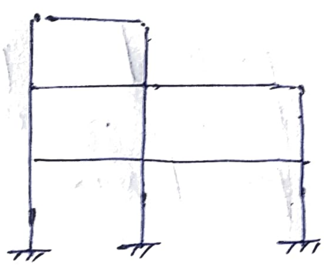
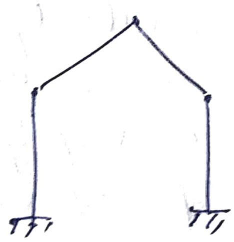
the no of independent displacements that reflects the structural displacement configuration.

Q. (b) (i)

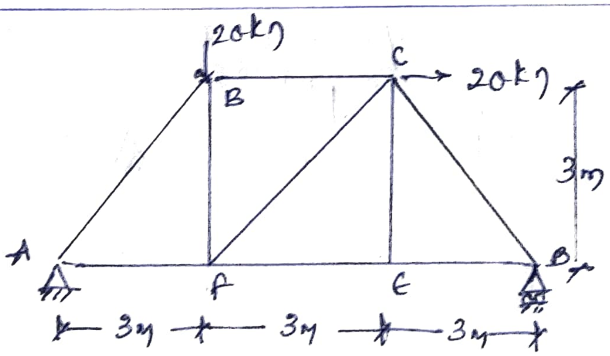
 Fixed support DOF 0.

 Hinged support DOF 1.

(c)

	Static	Kinematic
	$D = 3m + r - 3j$	
(i) 	$3\times1 + 5 - 3\times2 = 2$	@A = 0 @B = 1 $D_k = 01$.
(ii) 	$3\times2 + 4 - 3\times3 = 1$	$D_k = 3$.
(iii) 	$20 + 3 - 2\times10 = 3$. $(m + r - 2j)$	$D_k = 20 - 3 = 17$
(iv) 	$3\times13 + 9 - 3\times11 = 15$	$D_k = 11\times3 = 33$ $- 09$ 24
(v) 	$3\times4 + 6 - 3\times5 = 3$.	$D_k = 10 - 6 = 4$

2(a)

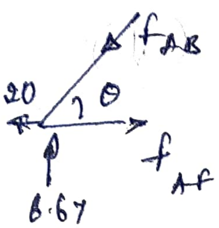


$\Sigma f_y = 0$. $R_A + R_B = 20kN$. $\Sigma f_x = 0$ $H_A = 20kN$ (←)

$\Sigma M_A = 0$. $20\times3 + 20\times3 - R_B\times9$ $\therefore R_B = 13.33$ kN ↑

$R_A = 6.667$ kN ↑

2(a)



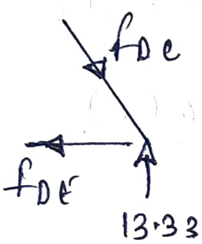
$\tan\theta = 4/3 \quad \therefore \theta = 50^\circ.$

$\Sigma f_y = 0 \quad f_{AB} = \dfrac{6.67}{\sin\theta} = 9.43 \ (C).$

$\Sigma f_x = 0 \quad f_{AF} = 20 + f_{AB}\cdot\cos\theta = 26.67 \ (T)$

Joint A.

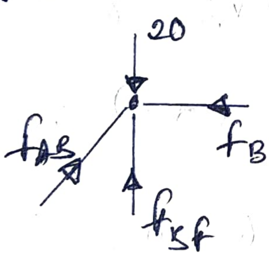
Joint D.



$\Sigma f_y = 0 \quad f_{DC}\cdot\sin\theta = 13.33 \quad \therefore f_{DC} = 18.85 \ (C)$

$\Sigma f_x = 0 \quad f_{DE} - f_{DC}\cdot\cos\theta = 0. \quad \therefore f_{DE} = 13.33 \ (T)$

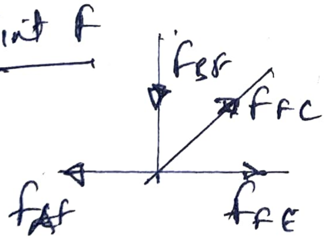
Joint B



$\Sigma f_x = 0. \quad f_{BC} = f_{AB}\cdot\cos\theta \quad \therefore f_{BC} = 6.67 \ (C)$

$\Sigma f_y = 0 \quad 20 - f_{BF} - f_{AB}\cdot\sin\theta \quad \therefore f_{BF} = 13.33 \ (C)$

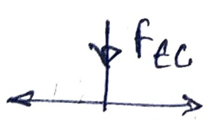
Joint F



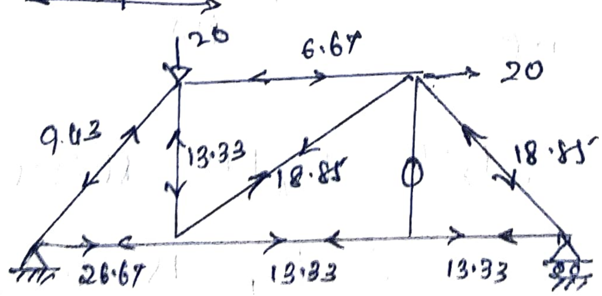
$\Sigma f_y = 0 \quad f_{FC}\cdot\sin\theta - f_{BF} \quad \therefore f_{FC} = 18.85 \ (T)$

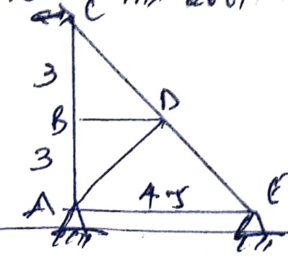
$\Sigma f_x = 0 \quad f_{FE} = f_{AF} + f_{FC}\cdot\cos\theta \quad \therefore f_{AF} = 13.3 \ (T)$

Joint E



$\Sigma f_y = 0 \quad f_{EC} = 0.$





2(b)

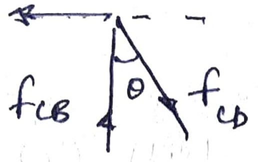
$\Sigma f_y = 0 \quad R_A + R_E = 0.$

$\Sigma f_x = 0 \quad H_E = 15 \ (\rightarrow)$

$\Sigma M_A = 0. \quad 15 \times 6 + R_E \times 4.5 = 0 \qquad \therefore R_E = -20\ (\downarrow)$

$\therefore R_A = \uparrow (20)\ kN.$

Joint C (Taking section along CB & CD)

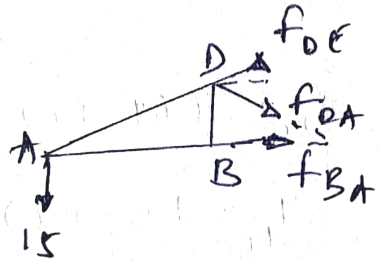


$\Sigma f_x = 0. \quad f_{CD} \sin\theta - 15 = 0 \qquad f_{CD} = \frac{15}{\sin\theta}.$

$(\theta = \tan^{-1} 4.5/6) \qquad f_{CD} = 25 (T)$

$\Sigma f_y = 0. \quad f_{CB} - f_{CD} \cos\theta = 0 \qquad f_{CB} = 20\ (C).$

(Taking section along DE, DA & BA)



$\Sigma M_D = 0. \quad f_{BA} \times 2.25 + 15 \times 3 = 0$

$\therefore f_{BA} = -20\ (\leftarrow)\ C$

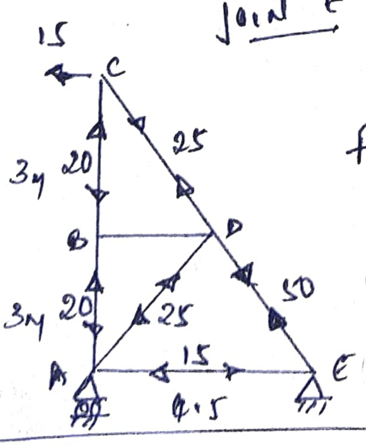
$\Sigma M_A = 0. \quad f_{DE} \sin\theta \times 3 + 15 \times 6 = 0.$

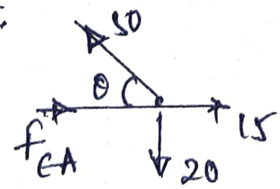
$\therefore f_{DE} = \frac{90.}{(3 \times \sin\theta)} = -50 (kN) (T)$

$\Sigma f_y = 0. \quad -15 - f_{DA} \sin\theta + f_{DE} \sin\theta = 0.$

$(-f_{DA} + f_{DE}) = \frac{15}{\sin\theta} = 25 \quad \therefore f_{DA} = 25 (T)$

Joint E

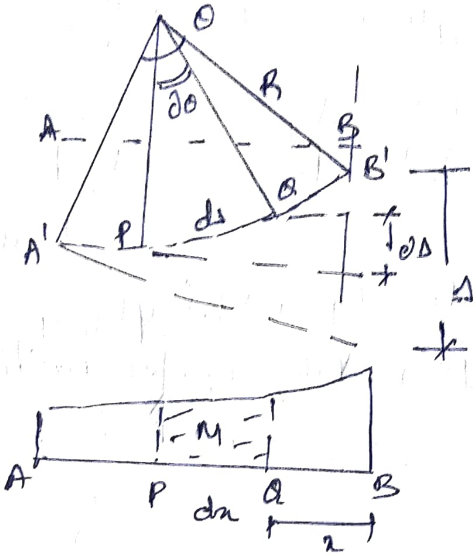




$\Sigma f_x = 0 \quad f_{EA} + 15 - 50 (\cos 53°13')$

$\therefore f_{EA} = 15\ (C)$

3 (a)



from diagram $d\Delta = x \cdot d\theta$. —(1)

But from fig $R = \frac{ds}{d\theta}$

Theory of bending $\frac{M}{I} = \frac{E}{R}$

$$\boxed{\frac{1}{R} = \frac{M}{EI}}$$

But $ds = dx$.

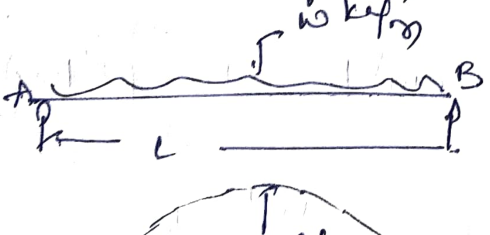
$\therefore d\theta = \frac{ds}{R} = \frac{M}{EI} \cdot ds = \frac{M}{EI} \cdot dx$

$\therefore d\Delta = x \cdot \frac{M}{EI} \cdot dx$

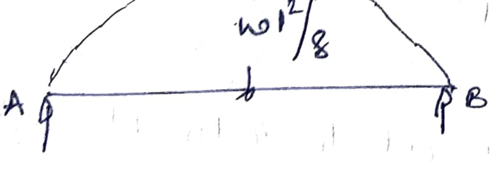
for overall beam AB $\int_0^{\Delta} d\Delta = \int_A^B x \cdot M/EI \cdot dx$

$t_{B/A} = \int_A^B \frac{M \cdot x}{EI} \cdot dx \Rightarrow t_{B/A} =$ area of $\frac{M}{EI}$ dia $\Big|_A^B \times \bar{x}_B$

(b) Calculate Max deflection & slope for S/s beam by CB method.



$R_{A_1} + R_{B_1} = w \cdot l/2$.



Reaction of conjugate beam

$R_A = R_B = \frac{2}{3}\left(\frac{l}{2}\right) \cdot \left(\frac{wl^2}{8EI}\right)$

ie $\theta_A = \theta_B = \frac{wl^3}{24EI}$

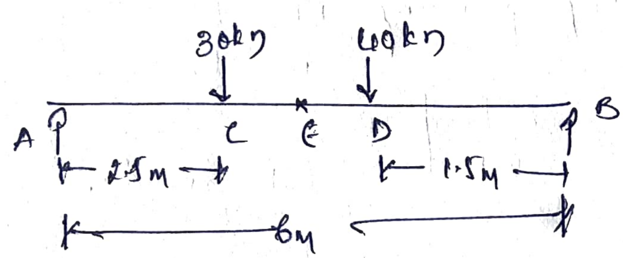
Hence deflection @ mid-span

$\Delta_c = R_A \times l/2 - \frac{wl^3}{24EI}\left(\frac{3}{8} \times l/2\right)$.

$R_A = \frac{wl^3}{24EI}$

$$\Rightarrow \frac{wl^3}{24EI}\left[\frac{5}{8} \times \frac{1}{2}\right] \rightarrow \boxed{\frac{5}{384}\frac{wl^4}{EI}}$$

Q4 (b)

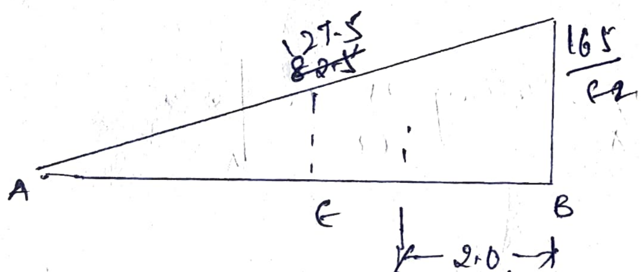


Calculate slope @ A & defl @ mid-span, take $EI = 200 \ MNm^2$

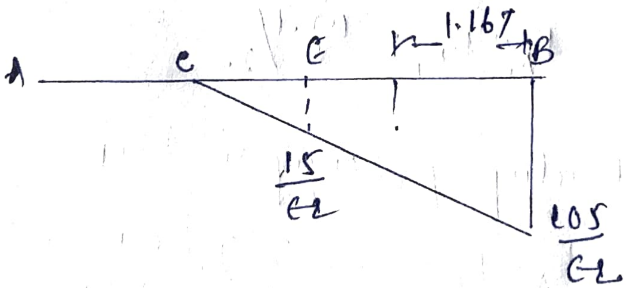
$\Sigma F_y = 0 \quad V_A + V_B = 30 + 40 = 70 \ kN.$

$\Sigma M_A = 0. \quad 30 \times 2.5 + 40 \times 4.5 - V_B \times 6 \quad \therefore V_B = 42.5 \ kN.$

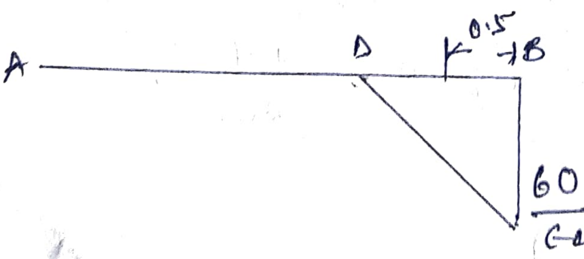
$\therefore V_A = 70 - 42.5 = 27.5 \ kN.$



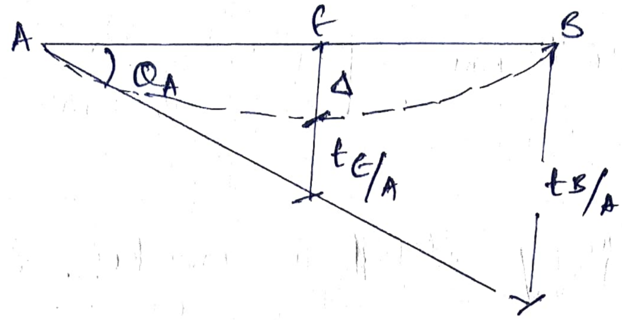
BM due to reaction A.



BM due to 30 kN pt load.



BM due to 40 kN pt load.



The vertical deviation $t_{B/A}$ is obtained by taking all the moment about B.

$$t_{B/A} = \left(\tfrac{1}{2} \times 6 \times \frac{165}{EI}\right) \times 2.0 - \left(\tfrac{1}{2} \times 3.5 \times \frac{105}{EI}\right) \times (1.167) - \left(\tfrac{1}{2} \times 1.5 \times \frac{60}{EI}\right) \cdot (0.5)$$

$$t_{B/A} = \frac{990}{EI} - \frac{214.5}{EI} - \frac{22.5}{EI} = 753/EI$$

$$\theta_A = \frac{t_{B/A}}{L_{AB}} = \frac{753/EI}{6.0} = 125.5/EI$$

To calculate deflection @ midspan, taking all moments about C point.

$$t_{C/A} = \left(\tfrac{1}{2} \times 3.0 \times \frac{82.5}{EI}\right) \times (1.0) - \left(\tfrac{1}{2} \times 0.5 \times \frac{15}{EI}\right)(0.167)$$

$$t_{C/A} = \frac{123.75}{EI} - \frac{0.63}{EI} = 123.12/EI$$

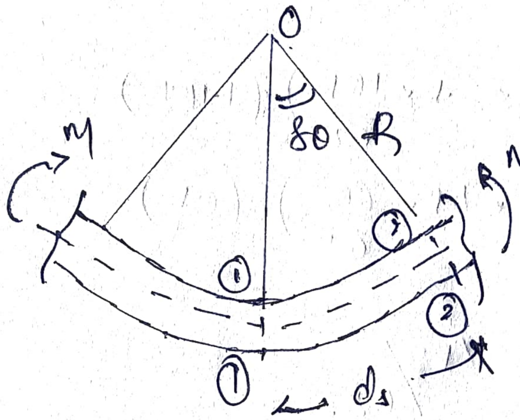
The deflection Δ_C is obtained from

$$\Delta_C = \theta_A \times AC - t_{C/A}$$

$$= \frac{125.5}{EI} \times 3.0 - 123.12/EI = \frac{253.38}{EI}$$

$\Delta_E = \frac{253.38}{200} = 1.267$ mm

(5) Expression for strain Energy stored in member subjected
(a) to bending moment.



AB initially a straight beam, due to application of bending moment two normal sections in the st beam are deformed as shown in figure. let δs be curved distance δθ be the subtended angle

strain energy due to external loads can be expressed as

$U = \frac{1}{2}\sum W.\Delta$

consider a small element

$\delta U_b = \frac{1}{2} M.\delta\theta$

Since R is radius of curvature

$R\, d\theta = ds$

$\therefore d\theta = ds/R$

$\delta U_b = \frac{1}{2} M.\frac{\delta s}{R}$

from theory of bending, curvature can be expressed as

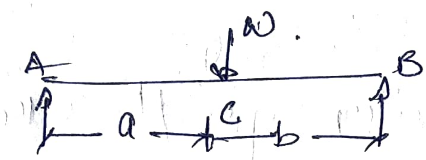
$1/R = \frac{M}{EI}$ $\therefore \delta U_b = \frac{1}{2} M.\frac{M}{EI}.\delta s$

$\therefore \delta U_b = \frac{1}{2}\frac{M^2}{EI}.\delta s$

On integration, we obtain total strain energy due to bending

$$U_b = \int \frac{M^2 \cdot ds}{2EI}$$

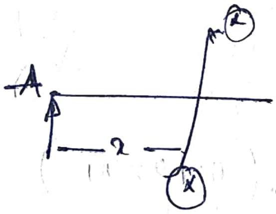
5(b)



find deflection @ 'C' pt.

$\Sigma F_y = 0 \quad V_A + V_B = W.$

$\Sigma M_A = 0 \quad W \cdot a - R_B \times l \quad \therefore V_B = Wa/l \;\&\; V_A = \frac{Wb}{L}$



Consider length AC. Section @ 'x' from A.

$$U_{AC} = \int_0^a \frac{M^2 \cdot dx}{2EI} =$$

$$U_{AC} = \int_0^a \frac{1}{2EI} \left(\frac{Wbx}{l} \right)^2 \cdot dx$$

$M = R_A \cdot x = \frac{Wb}{L} \cdot x$

$$= \frac{1}{2EI} \frac{W^2 \cdot b^2}{l^2} \int_0^a x^2 \cdot dx \Rightarrow \frac{W^2 b^2}{2l^2 EI} \left[\frac{x^3}{3} \right]_0^a$$

$$\therefore U_{AC} = \frac{W^2 b^2 a^3}{6 l^2 \cdot EI}$$

IIIly from member BC $\quad U_{BC} = \frac{W^2 a^2 b^3}{6L^2 \cdot EI}$

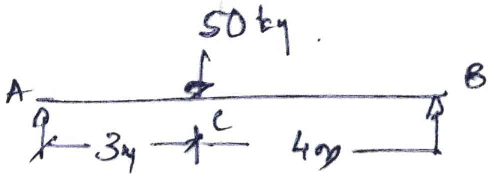
Total strain energy = $U = U_{AC} + U_{BC}$

$$= \frac{W^2 a^2 b^3}{6l^2 EI} + \frac{W^2 a^3 b^2}{6l^2 EI}$$

$$U = \frac{W^2 a^2 b^2}{6EIL}$$

deflection $\Delta = \frac{2U}{P} = \frac{2 \times W^2 a^2 b^2}{W \, 6EIL} = \frac{W a^2 b^2}{3EIL}$

5(c)

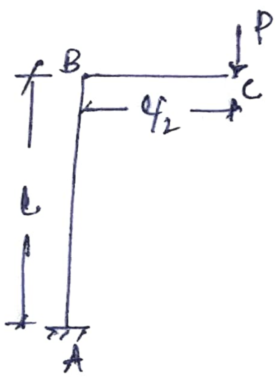


find deflection @ 'C' pt, Take $E = 2\times10^8\ kN/m^2$ & $I = 16\times10^{-6}\ m^4$

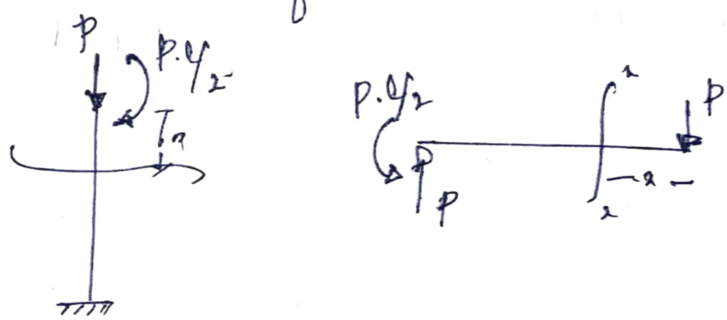
$$\Delta = \frac{W a^2 b^2}{3EIl}$$

$$= \frac{50 \times 3^2 \times 4^2}{3 \times 7 \times 2\times10^8 \times 16\times10^{-6}} = 122\ mm\ (0.122\ m)$$

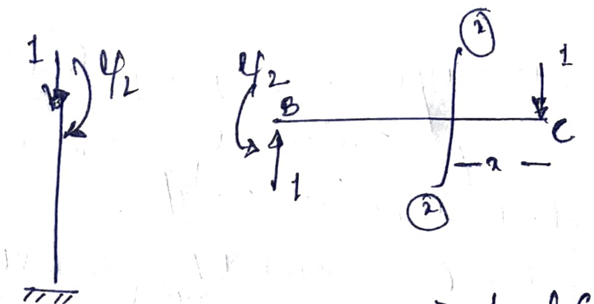
6 (a)



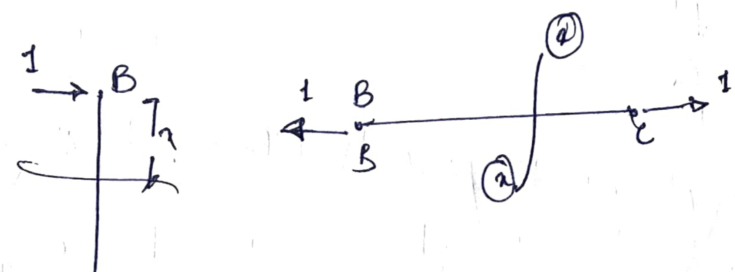
Determine horizontal & vertical deflection at free end of bent frame.



FBD due to applied load



FBD for applied load (V).



FBD for applied load (H)

Member	Origin	limit	M	m_1	m_2
BC	C	$0-l/2$	$-Px$	$-x$	0
AB	B	$0-l$	$-P\cdot l/2$	$-l/2$	$-x$

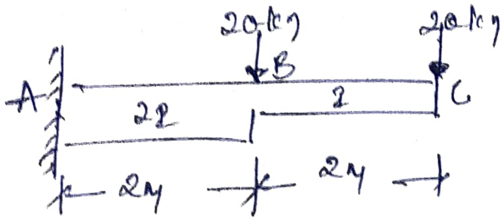
$$\Delta_V = \int \frac{M\cdot m_1\cdot dx}{EI} = \int_0^{l/2} (-Px)(-x)\cdot\frac{dx}{EI} + \int_0^{l} (-Pl/2)(-l/2)\cdot\frac{dx}{EI}$$

$$\Delta_{VC} = \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^{l/2} + \frac{Pl^2}{4EI}\left[x\right]_0^{l} = \frac{Pl^3}{24EI} + \frac{Pl^3}{4EI} = \frac{7Pl^3}{24EI}$$

$$\Delta_{HC} = \int \frac{M\cdot m_2\cdot dx}{EI} = \int_0^{l/2} -Px\cdot 0\cdot\frac{dx}{EI} + \int_0^{l} -Pl/2\cdot(-x)\cdot dx/EI$$

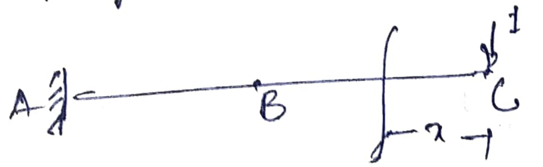
$$\Delta_{HC} = \frac{Pl}{2EI}\left[x^2/2\right]_0^{l} \Rightarrow \frac{Pl^3}{4EI}$$

6(B)

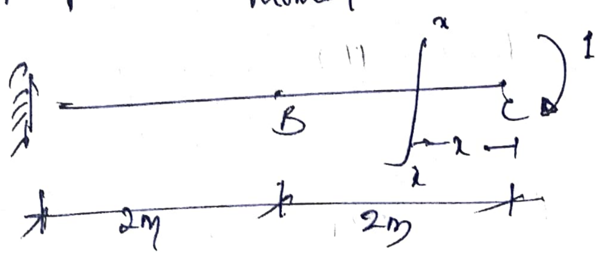


determine slope & deflection @ free end of cantilever beam using unit load method.
Take $E = 2\times10^5 \ N/mm^2$, $I = 12\times10^6 \ mm^4$.

apply unit load @ pt 'c' in vertical direction.



apply unit moment @ pt 'c' to find rotation.



Member	Origin	Limit	M	m_1	m_2	I
BC	C	0-2	$-20x$	$-x$	-1	I
AB	C	2-4	$-20x-20(x-2)$	$-x$	-1	2I

$$\theta_c = \int \frac{M \cdot m_2 \cdot dx}{EI} = \int_0^2 \frac{-20x \times (-1) \cdot dx}{EI} + \int_2^4 \frac{[-20x - 20(x-2)] \times (-1) \cdot dx}{2EI}$$

$$= \frac{20}{EI}\left[\frac{x^2}{2}\right]_0^2 + \int_2^4 \frac{(-40x+40)(-1) \cdot dx}{2EI}$$

$$\theta_c = \frac{10 \times 4}{EI} + \frac{40}{2EI}\left[\frac{x^2}{2}\right]_2^4 - \frac{40}{2EI}\left[x\right]_2^4$$

$$\theta_c = \frac{40}{EI} + \frac{10}{EI}\cdot(4^2-2^2) - \frac{20}{EI}\cdot(4-2) \Rightarrow \frac{120}{EI}.$$

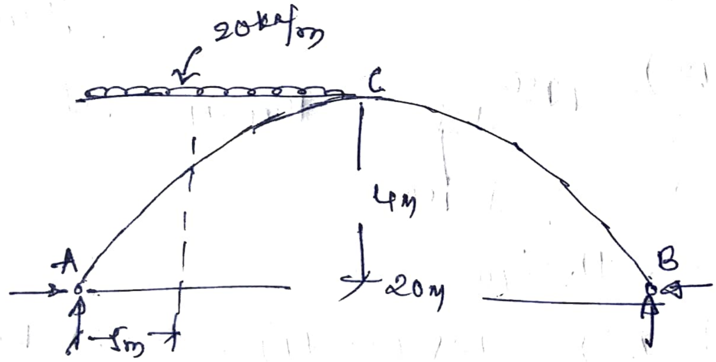
$$\Delta_c = \int m \cdot m_1 \cdot \frac{dx}{EI} = \int_0^2 (-20x)(-x)\frac{dx}{EI} + \int_2^4 \left(-20x - 20(x-2)\right) - x \cdot \frac{dx}{2EI}$$

$$\Delta_c = \frac{20}{EI}\left[\frac{x^3}{3}\right]_0^2 + \int_2^4 (-40x + 40)\times(-x)\cdot\frac{dx}{2EI}$$

$$\Delta_c = \frac{160}{3EI} + \frac{20}{EI}\left[\frac{x^3}{3}\right]_2^4 - \frac{20}{EI}\left[\frac{x^2}{2}\right]_2^4$$

$$\Delta_c = \frac{160}{3EI} + \frac{20}{3EI}(4^3 - 2^3) - \frac{10}{EI}(4^2 - 2^2) = \frac{306.67}{EI}$$

$\therefore \Delta_c = 0.128\ m$ $\theta_c = 0.05$ Radian



$\Sigma V = 0.$ $V_A + V_B = 20 \times 10 = 200$ kN.

$\Sigma M_B = 0.$ $V_A \times 20 - 20 \times 10 \times (15) = 0$ $V_A = 150$ kN.

$V_B = 50$ kN.

$\Sigma M_c = 0.$ $V_A \times 10 - H_A \times 4 - 20 \times 10 \times 10/2 = 0.$

$$H_A = \frac{150 \times 10 - 1000}{4} = 125\ kN.$$

@ $x = 5$ m. $y = \frac{4h}{l^2}(x)(l - x)$

$$y = \frac{4(4)}{20^2} x(20-x) \Rightarrow y = 0.8x - 0.04x^2.$$

$$\frac{dy}{dx} = \tan\theta = 0.8 - 0.04(2x) =$$

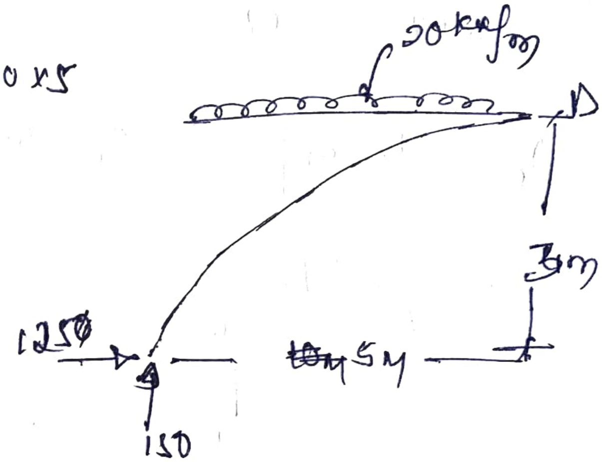
$$y_D = 0.8(5) - 0.04(5)^2 = 3m.$$

$$\tan\theta = 0.8 - 0.08(5) = 0.4 \quad \therefore \theta = 21^\circ 48' 50'' \ (21.80)$$

$$\sin\theta = 0.371 \quad \cos\theta = 0.928$$

Vertical shear $V = 150 - 20 \times 5$

$= 50$ kN.



$NT = V.\sin\theta + H.\cos\theta$

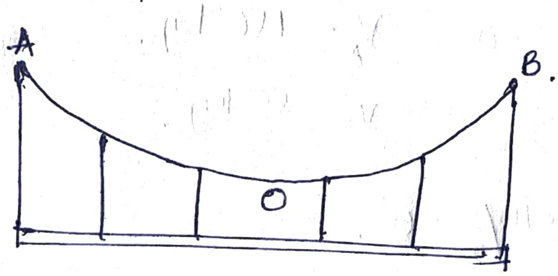
$= 50.(0.371) + 125(0.928)$

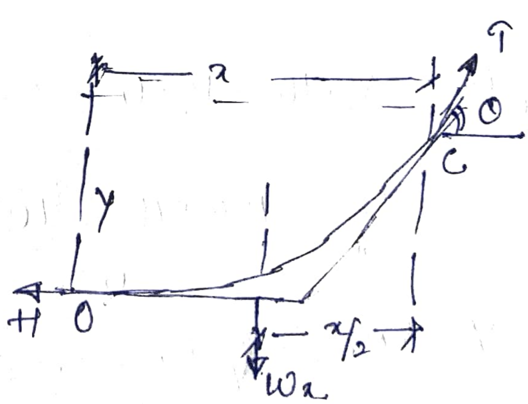
$NT = 134.55$ kN

Radial shear $Rs = V.\cos\theta - H.\sin\theta$

$= 50(0.928) - 125(0.371) = 0.025$ kN.

7(b) Equilibrium of cable subjected to UDL





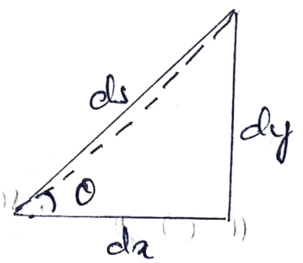
$\Sigma M_0 = 0$. $\quad Hy - w.x(x/2) = 0$. $\quad y = \frac{wx^2}{2H}$

Consider "O" as Origin if cable is parabola where $x = l/2$ & $y = d$

$\therefore d = \frac{w}{2H}(l/2)^2$

$H = \frac{wl^2}{8d}$

<u>Length of cable.</u>



$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} . dx$

$ds = \sqrt{1 + \left(\frac{8dx}{l^2}\right)^2} . dx$

$\therefore S = 2 . \int_0^{l/2} ds$

$S = 2 \int_0^{l/2} \left[1 + \frac{1}{2}\left(\frac{8xd}{l^2}\right)^2\right] . dx$

$S = 2 \int_0^{l/2} \left(1 + \frac{32x^2.d^2}{l^4}\right) . dx \qquad \left[\therefore S = l + \frac{8d^2}{3l}.\right]$

Max slope @ support $\quad y = kx^2$.

$y = d$ & $x = l/2 \qquad d = k(l/2)^2 \qquad \therefore k = \frac{4d}{l^2}$

$y = \frac{4d}{l^2}x^2 \implies \frac{dy}{dx} = \frac{8dx}{l^2}$

8(a) Given data: $l = 20\,m$, dip $= 4\,m$, $w = 20\,kN/m$

$$V_A = \frac{wl}{2} = \frac{20 \times 20}{2} = 200\ kN.$$

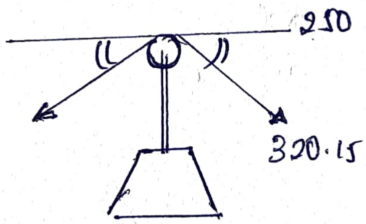
$$H_A = \frac{wl^2}{8h} = \frac{20 \times 20^2}{8 \times 4} = 250\ kN.$$

$$T_{max} = \sqrt{V_A^2 + H_A^2} = \sqrt{200^2 + 250^2} = 320.15\ kN$$

$$L = l + \frac{8h^2}{3l}$$

$$= 20 + \frac{8 \times 4^2}{3 \times 20} = 22.133\ m$$

Cable is supported by smooth pulley.



$$\theta = \cos^{-1}\left(\frac{250}{320.15}\right) = 38°39'$$

Horizontal force $= T \cdot \cos\theta$

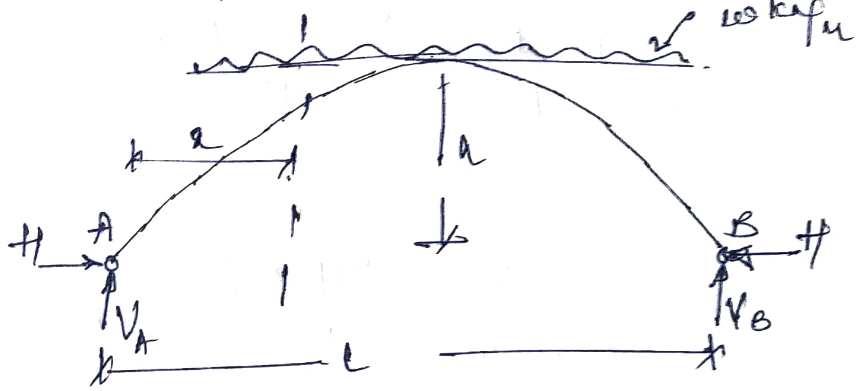
$= 320.15 \times \cos 38°39' + 250 = 500\ kN$

Vertical force $= T \cdot \sin\theta$

$= (320.15 \times \sin 38°39') \times 2$

$= 400\ kN$

8(b) prove that parabolic shape is funicular shape for 3 hinge arch. subjected to UDL over entire span.



$$y = \frac{4h}{l^2} x(l-x) = \frac{4h}{l^2}(lx - x^2)$$

$$\frac{\partial y}{\partial x} = \frac{4h}{l^2}(l - 2x)$$

$M_c = 0$. $\quad V_A \cdot l/2 - w(l/2)(l/4) - H \cdot h = 0$.

$$\frac{wl^2}{4} - \frac{wl^2}{8} = Hh. \qquad \therefore H = \frac{wl^2}{8h}$$

$$M_x = \frac{wl}{2} \cdot x - H \cdot y - w \cdot x \cdot x/2$$

$$M_x = \frac{wlx}{2} - \frac{wl^2}{8h}\left[\frac{4h}{l^2} x(l-x)\right] - w x^2/2$$

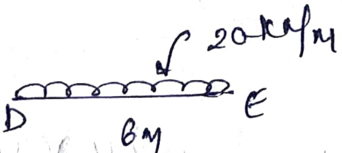
$$= \frac{wlx}{2} - \frac{wl^2}{2l^2} x(l-x) - wx^2/2$$

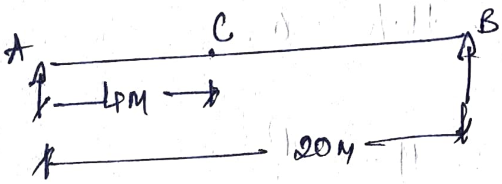
$$= \frac{wlx}{2} - \frac{wl}{2l} xl + \frac{wx^2}{2} - wx^2/2 = 0$$

9(a) **Definition**: It is a curve showing the variation of any resultant action, shear, bending moment, deflection or slope at a particular point in a structure under the influence of a unit load that rolls across the span.

(i) These diagram are used for all types of loads
(ii) Suitable for both stationary and moving loads
(iii) The shear stress reversal in panels can be assessed in trusses.
(iv) Max value of SF and BM can be obtained for the design.

(b)

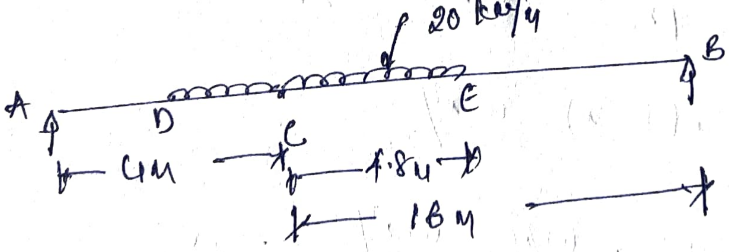




find max BM, SF (+ve & -ve) and absolute BM.

To get Max BM arrange load in this form

$$CE = \frac{DE}{AB} \times CB \Rightarrow \frac{6}{20} \times 16 = 4.8\,m$$



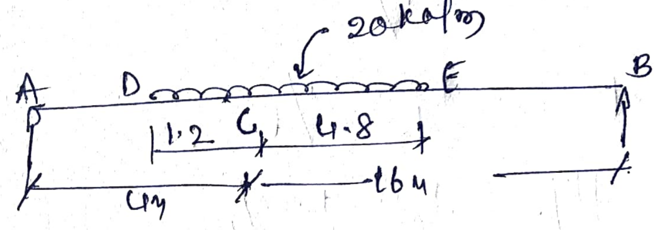
$\Sigma F_y = 0$ $\quad V_A + V_B = 20 \times 6 = 120\ kN.$

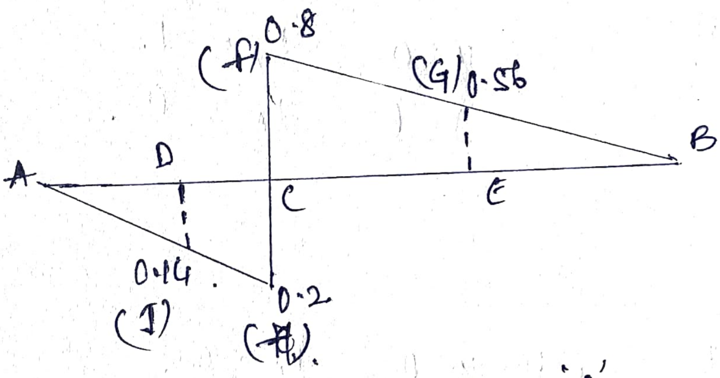
$\Sigma M_B = 0$ $\quad V_A \times 20 = 20 \times 6 \times 14.2$

$\therefore V_A = 85.2\ kg$

Max BM $\quad \Sigma M_C = 0 \quad V_A \times 4 - 20 \times (1.2)^2/2$

$M_C = 326.6\ kNm$





Max shear force @ section 'C'

V_c = [area of trapezium DCHI, CFGE) intensity of udl

$$V_c = \left[\frac{1}{2} \times 1.2 \times (0.2 + 0.44) + \frac{1}{2} \times 4.8 \times (0.8 + 0.56)\right] \times 20$$

$$= (0.204 + 3.264)\,20 = (3.468) \times 20$$

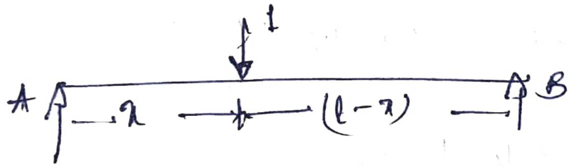
$V_c = 69.36\ kg$

Absolute Max BM $\quad M_c = \frac{wa}{4}(l - a/2)$

$$= \frac{20 \times 6}{4}(20 - 6/2)$$

$\therefore M_{c(max)} = 510\ kNm$

10 (a) ILD for Reactions



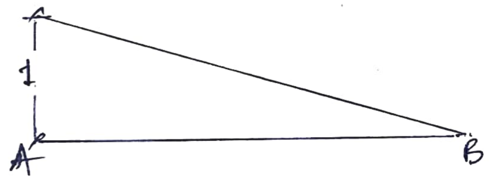
using equations of equilibrium $\Sigma F_y = 0$ $R_A + R_B = 1$.

$\Sigma M_A = 0$ $1 \cdot x - R_B \times l$ $\therefore R_B = x/l$.

lly $R_A = 1 - x/l$.

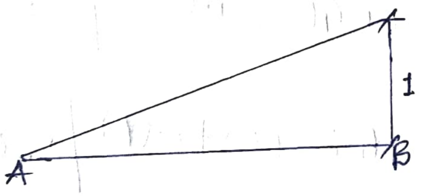
ILD for Reaction A

when $x = 0$ $R_A = 1$.
$x = l$ $R_A = 0$.

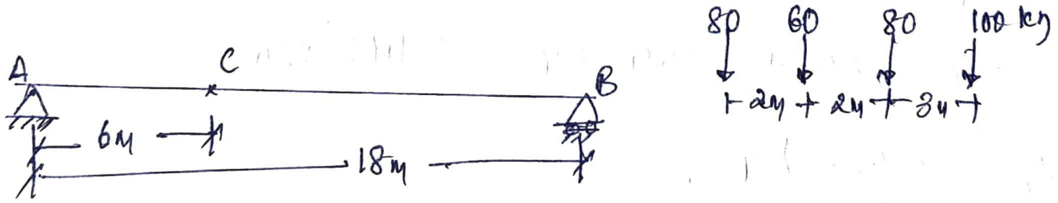


ILD for Reaction B

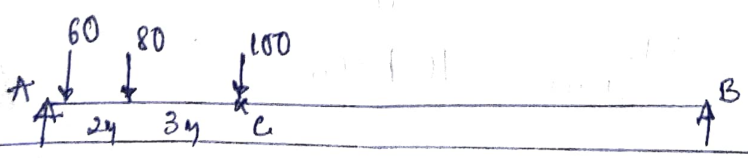
when $x = 0$ $R_B = 0$.
$x = l$ $R_B = 1$



(b)



(i) find Max BM & SF @ 6m from left support.
(ii) find absolute Max SF & absolute BM.



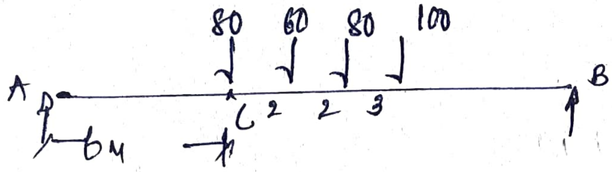
$\Sigma F_y = 0 \quad V_A + V_B = 60 + 80 + 100 = 240\ kN.$

$\Sigma M_A = 0 \quad V_B = \frac{60 \times 1 + 80 \times 3 + 100 \times 6}{18} = 60\ kN.$

$V_A = 240 - 60 = 180\ kN$

∴ Max +ve shear = 180 kN

Calculation of Max -ve shear.



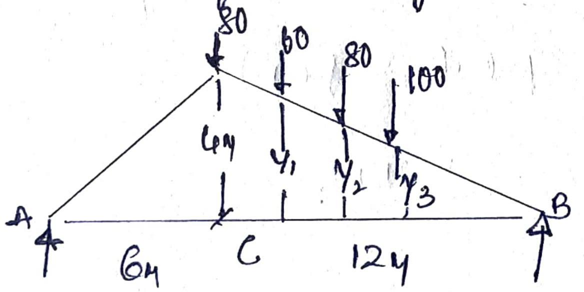
$\Sigma F_y = 0 \quad V_A + V_B = 60 + 2 \times 80 + 100 = 320\ kN.$

$\Sigma M_B = 0 \quad V_A = \frac{100 \times 5 + 80 \times 8 + 60 \times 10 + 80 \times 12}{18} = 150\ kN$

$V_B = 320 - 150 = 170\ kN.$

∴ Max -ve shear = 170 kN.

Calculation of bending moment @ 6m from left.



Using similar triangles.

$y_1 = \frac{10}{12} \times 4 = 3.33$

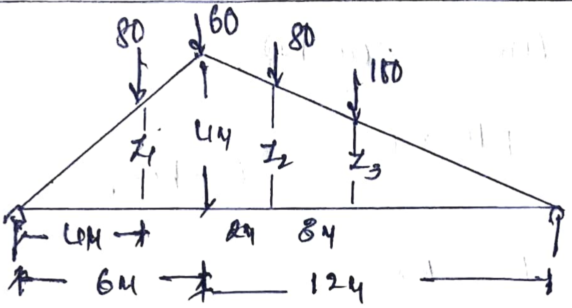
$y_2 = \frac{8}{12} \times 4 = 2.67$

$y_3 = \frac{5}{12} \times 4 = 1.67$

$\therefore \Sigma M_C = 0. \quad 80 \times 4 + 60 \times 3.33 + 80 \times 2.67 + 100 \times 1.67 =$

$M_C = 900\ kNm$

At the same time, check max BM by keeping 60 kN @ 'C' pt.



ILD for Ordinates

$Z_1 = \frac{4}{6} \times 4 = 2.67$

$Z_2 = \frac{10}{12} \times 4 = 3.33$

$Z_3 = \frac{7}{12} \times 4 = 2.33$

$\Sigma M_c = 0 \quad 80 \times 2.67 + 60 \times 4 + 80 \times 3.33 + 100 \times 2.33 =$

$M_c = 953$ kN.

Hence Max BM = 953 kN

Absolute max BM:

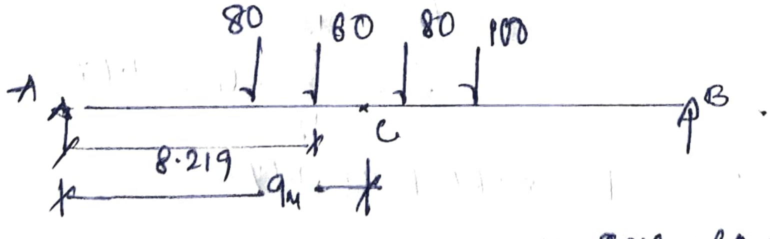
The CG of loads is obtained as

$$\bar{x} = \frac{60 \times 2 + 80 \times 4 + 100 \times 7}{80 + 60 + 80 + 100} = 3.562 \text{ m}$$

Absolute max BM occurs near centre of span is midway between CG of loads & 60 kN load. Hence 60 kN from A.

x = Half span $- \frac{1}{2}$ ($\bar{x}$ − sp bet 80 & 60)

$$= \frac{18}{2} - \frac{1}{2}(3.562 - 2) = 8.219$$



$V_A + V_B = 320$ kN. $V_B = \frac{80 \times 6.219 + 60 \times 8.219 + 80 \times 10.219}{18} = 173.89$

$V_A = 146.10$

$M_c = 146.1 \times 9 - 80 \times 2.781 - 60 \times 0.781$ = 1040 kNm