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ENGINEERING PHYSICS 18 PHY12/22

I/II SEMESTER B.E. Dec / Jan 2020

MODULE - 1

1a) Give the theory of forced vibration and obtain the expression for amplitude (8M)

Solⁿ:- consider a body of mass m executing vibrations in a damping medium acted upon by an External periodic force $F \sin(pt)$

where 'p' is the angular frequency of the External force.

If x is displacement of the body at any instant of time 't' then the damping force which acts in a direction opposite to the movement of the body is equated to the term $-r\frac{dx}{dt}$, where r is damping constant. Also the restoring force acting on the body is $-kx$, where k is force constant

∴ The net resultant restoring force acting on the body is given by.

Resultant restoring force $= -r\frac{dx}{dt} kx + F\sin(pt) \rightarrow (1)$

Resultant force on the body A/c to Newton's 2nd Law, the resultant force on the body is $-kx$, where k is force constant.

∴ The net resulatant force acting on the body is given by

Resultant restoring force $= -r\frac{dx}{dt} kx + F\sin(pt) \rightarrow (1)$

A/c to Newton 2nd Law.

Resultant force $= m\frac{d^2x}{dt^2} \rightarrow (2)$

From Equⁿ 1 & 2 we get.

$$m\frac{d^2x}{dt^2} = -r\frac{dx}{dt} - kx + F\sin(pt)$$

$$\therefore m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = F\sin(pt)$$

This is Equⁿ of forced vibration motion

on re-arranging, we get.

Dividing throught by m, we get

$$\frac{\cancel{m}}{\cancel{m}}\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + kx = F\sin(pt)$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = F\sin(pt) \rightarrow (3)$$

Let $\frac{r}{m} = 2b$ W.K.T $\omega^2 = \frac{k}{m} = \omega = \sqrt{\frac{k}{m}}$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = f\sin(pt) \rightarrow (4)$$

The solution of this differential Equation $x = a\sin(pt - \alpha)$ $\rightarrow (5)$

where a and α represent amplitude and phase of the vibrating body.

Differentiating Equⁿ 5 w.r.t. "t" we get.

$$\frac{dx}{dt} = ap\cos(pt - \alpha) \rightarrow (6)$$

Differentiating Equⁿ 6 w.r.t "t" we get

$$\frac{d^2x}{dt^2} = -ap^2\sin(pt - \alpha) \rightarrow (7)$$

∴ From Equⁿ (4), we get. Substitute Equⁿ 6 & 7 in Equⁿ 4 we get

$$-ap^2\sin(pt - \alpha) + 2bap\cos(pt - \alpha) + \omega^2 a\sin(pt - \alpha) = \frac{F}{m}\sin(pt) \rightarrow (8)$$

But $\frac{F}{m}\sin(pt) = \frac{F}{m}\sin[(pt-\alpha)+\alpha]$

Substituting in Eqn (8) we get.

$$-ap^2\sin(pt-\alpha) + 2bap\cos(pt-\alpha) + \omega^2 a\sin(pt-\alpha)$$
$$= \frac{F}{m}\sin(pt-\alpha)\cos\alpha + \frac{F}{m}\cos(pt-\alpha).\sin\alpha.$$

Equating the co-efficients of $\sin(pt-\alpha)$ and $\cos(pt-\alpha)$ on both Sides.

Separately we get

$$-ap^2 + \omega^2 a = \frac{F}{m}\cos\alpha \rightarrow (9)$$

$$2bap = \frac{F}{m}\sin\alpha \rightarrow (10)$$

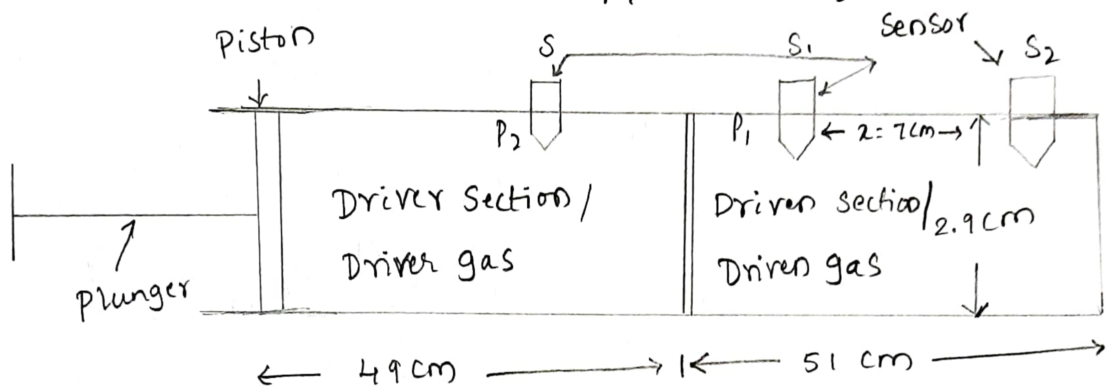
Squaring and adding Equation (9) & (10) we get.

$$[a(\omega^2-p^2)]^2 + (2bap)^2 = \left(\frac{F}{m}\right)^2(\cos^2\alpha + \sin^2\alpha)$$

$$a^2[(\omega^2-p^2)^2 + 4b^2p^2] = \left(\frac{F}{m}\right)^2 (1)$$

$$\boxed{a = \frac{\frac{F}{m}}{\sqrt{4b^2p^2 + (\omega^2-p^2)^2}}} \rightarrow (11)$$

16b) With neat diagram, Explain the Construction and working of Reddy tube. Mention four application of Shock waves.



Construction :-

① R.S.T Consists of a steel tube of length 100cm and it's diameter 2.9cm.

② A diaphragm of thickness 0.1cm divides the tube into two Compartments of length 49cm fitted with piston called Driver section filled with driver gas (Helium). The other compartment of length 51cm is called Driven Section filled with driven gas (Aargon)

③ sensor 'S' fitted to driver Section measures the ruputure Pressure P_2, temperature T_2.

④ Two sensors S_1 & S_2 separated by a distance Δx fitted to driven Section measures the pressure P_4, P_5 and temperature T_4, T_5 respectively.

Working :-

1. Driver section is filled with gas at high pressure (P_2) and driven section is filled with gas of low pressure (P_1)

2. Diaphragm is ruptured to produce shock waves by pushing the piston and the rupture pressure P_2 & temperature is measured using sensor S.

3. The time 't' taken by the shock wave to travel the distance 'x' is measured using sensors S_1 S_2 and CRO(Cathode Ray oscilloscope). The speed of the shock waves is calculated using $v = xt$.

4. Then if a is the speed of sound at laboratory ~~system~~ temperature, the mach number of the shock waves is calculated using $M = va$

5. The Mach number increases with the increase of the thickness of the diaphargm.

Applications of the Shock wave

1. Shock waves are used in the treatment of kidney stones

2. SW are used in the pensil industry for softening of pencil wood and dry painting.

3. SW are used in the extraction of sandal wood.

4. SW are used for needleless drug delivery.

c) Calculate the resonant frequency for a simple pendulum of length 1m. (4m)

Solution:- A simple pendulum set for oscillations. oscillates with a period.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1}{9.8}} = 2 \text{ seconds}$$

∴ Its frequency of oscillation is.

$$\nu = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ Hz}.$$

This is its natural frequency of oscillation. Any external periodic force with this frequency causes. resonant oscillation in the pendulum.

∴ Resonance frequency of the spring $\nu = 0.5 Hz$.

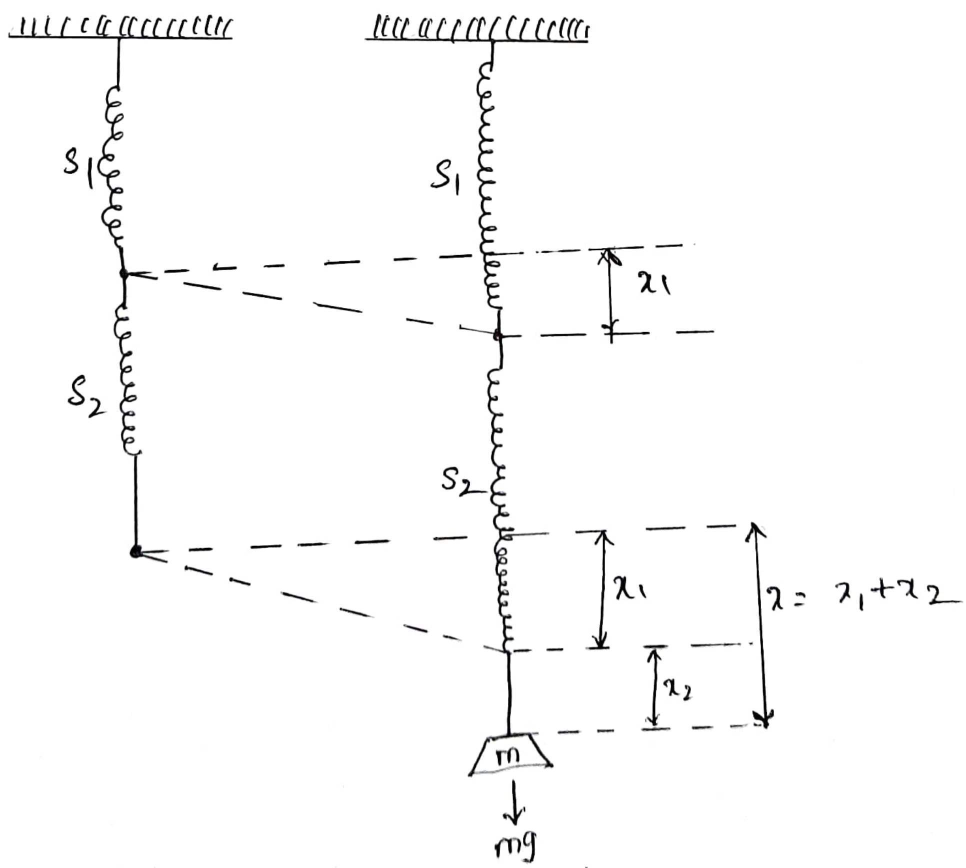
2) a) Define force constant and mention its physical significance. Derive the Expression for force constant for springs in series and parallel combination.

Solⁿ:- It is defined as the magnitude of the applied force that produces unit Extension (or Compression) in the spring while it is loaded within the Elastic limit.

Physical Significance:-

* Physically force constant is a measure of "stiffness"
* In case of springs, it represents how much force it takes to stretch the spring over a unit length.

Expression for force constant for springs in Series.



Let us consider two idealized spring S_1 & S_2 with spring constant K_1 & K_2 respectively

Here x_1 & x_2 be the Extension (elastic Limit) of the Spring.

When mass m is attached to the spring at its lower end It follows Hooke's Law

So we have

$$F = -K_1 x_1$$

But In generally $F = mg$

$$\text{So } mg = -K_1 x_1$$

$$x_1 = \frac{-mg}{K_1} \rightarrow (1)$$

Similarly for $x_2 = \frac{-mg}{K_2} \rightarrow (2)$

Now let S_1 & S_2 be the Suspended in Series as shown in above fig Now attach the mass at the bottom of the spring in Series Combination Since Each of the springs S_1 & S_2 Experience the same pull by the mass m, S_1 extends by x_1 & x_2 Thus the mass m Comes down Showing a total Extension.

$$\therefore x = x_1 + x_2$$

Let the force constant for this Series combination as whole be K_s

$\therefore$ we can write

$$mg = -K_s x = -K_s(x_1 + x_2)$$

$$\text{or } \boxed{x_1 + x_2 = \frac{-mg}{K_s}} \rightarrow (3)$$

using Eqn (1) & (2), Eqn (3) can be written as

$$\frac{-mg}{K_1} - \frac{mg}{K_2} = -\frac{mg}{K_s}$$

cancel $-mg$ term on both sides we get

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_s}$$

or $$\boxed{K_s = \frac{K_1 K_2}{K_1 + K_2}}$$

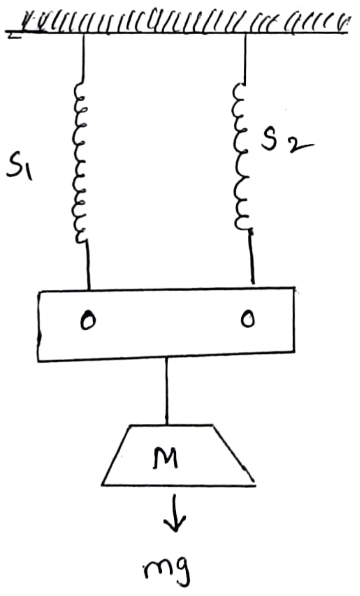
If there are number of springs in series In generally we can write

$$\frac{1}{k_s} = \sum_{i=1}^{n} \frac{1}{k_i}$$

If a mass m is attached to the bottom of such a series combination of springs & set for oscillations its period of oscillation will be

$$T = 2\pi\sqrt{\frac{m}{k_s}}$$

Equivalent force constant for springs in parallel combination.



* Consider two Idealized springs S_1 & S_2 with spring constants K_1 & K_2 respectively.

* Let x_1 and x_2 be the respective extensions that the springs S_1 & S_2 would undergo individually under the pulling action of a suspended mass m

Hence we have,

$$F_1 = mg = -k_1 x_1 \text{ or } x_1 = \frac{-mg}{k_1} \rightarrow (1)$$

and $$F_2 = mg = -k_2 x_2 \text{ or } x_2 = \frac{-mg}{k_2} \rightarrow (2)$$

Their free ends are fastened to free support to which a mass m is suspended. The free support descends a distance x due to mass m.

Let the restoring force acting on the support be F_p and the force constant for this combination be K_p

$$\therefore F_p = -K_p x \rightarrow (3)$$

The restoring force F_p is actually shared by two springs.

Let the restoring force in S_1 be F_1 & that in S_2 be F_2

$$\therefore F_p = F_1 + F_2 \Rightarrow -k_1 x_1 - k_2 x_2$$

But since both springs undergo some extension x

$$x_1 = x_2 = x$$

$$\therefore F_p = -k_1 x - k_2 x$$

$$F_p = -(k_1 + k_2) x \rightarrow (4)$$

Comparing Eqn (3) & (4) we have.

$$\boxed{K_p = k_1 + k_2}$$

In generally $K_p = \sum_{i=1}^{n} k_i = k_1 + k_2 + \dots + k_n$

For this combination of mass - spring system the period of oscillation will be

$$\boxed{T = 2\pi\sqrt{\frac{m}{K_p}}}$$

26) Define Simple harmonic motion. Derive the differential Equⁿ of motion for it using Hook's law. mention the characterstics and Examples of Simple harmonic motion (8 M)

Solⁿ:- SHM is the oscillatory motion of a body where the restoring force is proportional to the negative displacement.

Differential Equation of SHM:-

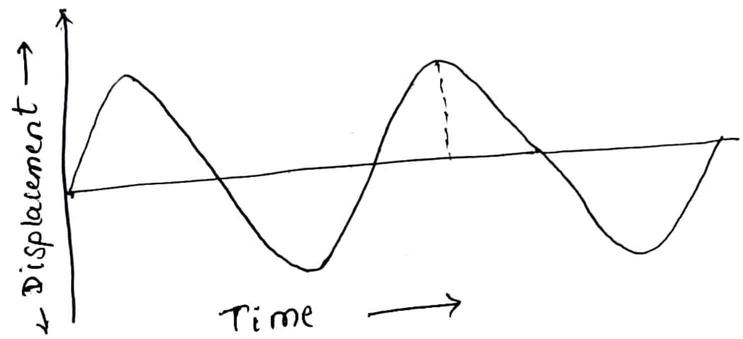


fig (1)

Let a body be initiated to an oscillatory motion after being displaced from its Equillibrium position and left free for such oscillations. the lonely force acting on the body will be the restoring force F.

We know that for a vibrating body.

$$F = -kx \rightarrow (1)$$

where x is the displacement and k is force constant

If m is the mass of the body. then as per newton's second law of motion.

$$F = m\frac{d^2x}{dt^2} \rightarrow (2)$$

From Equⁿ (1) & (2) we get.

$$m\frac{d^2x}{dt^2} = -kx.$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \rightarrow (3)$$

The above Equn represents the Equn of motion for a body Executing free vibration.

The Solution for the above Equn is given as

$$x = a\sin\omega t \quad \rightarrow (4)$$

Where a is the amplitude, ω is the angular frequency. and t is the time (period) elapsed.

Characteristics of SHM :-

All oscillatory motions need not be SHM. But for those which could be called SHM. the characterstics can be listed as follows.

1. It is a particular type of periodic motion
2. The oscillating System must have inertia which in turn means mass
3. There is a constant restoring force continuously acting on the body System.
4. The acceleration developed in the motion due to the restoring force is directly proportional to the displacement.
5. The direction of acceleration is opposite to that of the displacement $(F \propto -x)$ or $F = -kx$
6. It is represented by a sine or cosine function such as $x = a\sin\omega t$.

Examples of SHM

1. A mass suspended to a spring when pulled down and left free executed simple harmonic motion vertically.
2. A pendulum set for oscillation
3. Excited tunning fork.
4. A plucked string in a veena or a guitar
5. A shock absorber after being bumped.
6. Swings with which the children play.

2) c) The distance between the two pressure sensors in a shock tube is 150 mm. The time taken by a shock wave to travel this distance is 0.3 ms. If the velocity of sound under the same condition is $340\ ms^{-1}$. Find the mach number of the shock wave.

Sol:- Shock speed $u_s = \frac{d}{t} = \frac{150 \times 10^{-3}}{0.3 \times 10^{-3}}$

$$u_s = 500\ ms^{-1}$$

Mach no $M = \frac{u_s}{a} = \frac{500}{340} = 1.47$

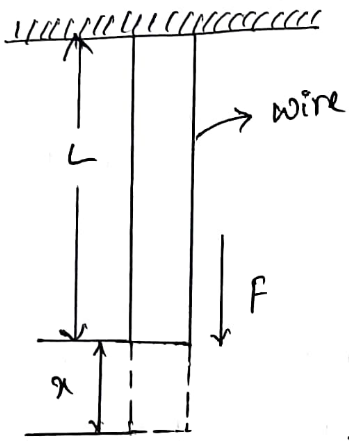
$\therefore$ Mach number of the shockwave is 1.47

Module - 2

3a) Explain longitudinal Stress, strain, volume stress & volume Strain. Discuss the effect of Stress. temperature annealing and impurities on elasticity.

Sol^n:

* **Longitudinal stress:-** It is the Streching force acting per unit area of the section of the Solid along its length



If F is the force applied normally to a cross-sectional area a then the stress is F/a

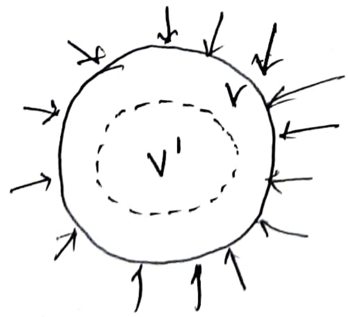
$\therefore$ Longitudinal stress $= \frac{F}{a}$

Longitudinal Strain:- If x is the change in length produced in an original length l then.

$$\frac{\text{change in length}}{\text{original length}} = \frac{x}{L}$$

$\therefore$ Linear strain $= \frac{x}{L}$

compressive stress:-



It is the uniform pressure (Force per unit area) acting normally all over the body Thus compressive stress

$$= \frac{F}{a}$$

3a * The effect of stress, temperature annealing and impurities on elasticity

① Rolling and hammering on a material generally breaks the bonding in the material which in turn increase the elastic property of the property.

② Temperature:- It is increase of the material generally decrease the elasticity of the material

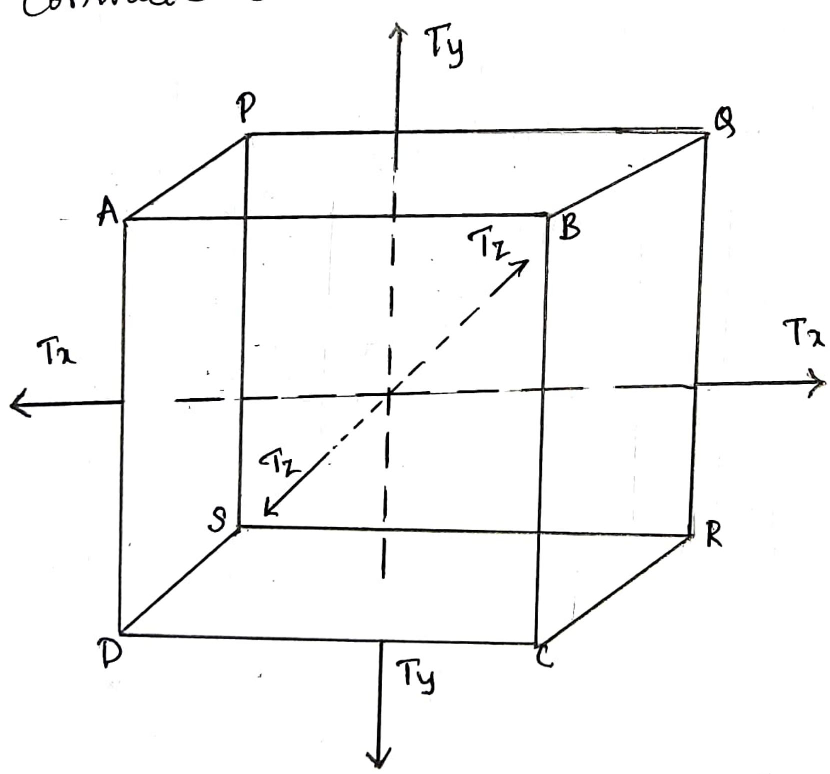
③ Impurities:- It is added to the material may increase or decrease the Elastic property of the material. The elasticity of the material increases if the elasticity of impurity is more than that of the material. The elasticity of the material decreases if the elasticity of impurity is less than that of the material.

④ Annealing is the process of heating at a particular temperature and then cooling gradually. using annealing to increase the particle size and the material hardned. which in turn reduce the elasticity of the material.

3) b) Derive the relation between bulk modulus (K) Youngs modulus (Y) and poisson's ratio (σ) what are the limiting values of poisson's ratio.?

Sol^n: Consider a unit cube ABCDPQRS of Each side '1' unit. Let T_x, T_y and T_z be the tensile stresses acting normal to parallel surfaces. $\perp^{lar}$ to x, y and z axis respectively.

The side AB is elongated due to T_x and contracted due to T_y and T_z. The side AD is elongated due to T_y and contracted due to T_z & T_x



The side AP is elongated due to T_z and contracted due to T_x & T_y

If α is the elongation strain co-efficient and β is contraction strain co-efficient, then the new lengths of the sides.

$AB = 1 + \alpha T_x - \beta T_y - \beta T_z$ along the x-direction.
$AD = 1 + \alpha T_y - \beta T_z - \beta T_x$ along the y-direction
$AP = 1 + \alpha T_z - \beta T_x - \beta T_y$ along the z-direction

$\therefore$ New cube volume $= AB.AD\ AP.$

$= (1 + \alpha T_x - \beta T_y - \beta T_z)(1 + \alpha T_y - \beta T_z - \beta T_x)(1 + \alpha T_z - \beta T_x - \beta T_y)$

$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z)$ (neglecting higher order terms)

If $T_x = T_y = T_z = T$

New volume $= 1 + 3T(\alpha - 2\beta)$

change in volume $\Delta V = 1 + 3T(\alpha - 2\beta) - 1$

$\Delta V = 3T(\alpha - 2\beta)$

Bulk strain $= \dfrac{\text{Change in volume}}{\text{initial volume}} = \dfrac{\Delta V}{V} = \dfrac{3T(\alpha - 2\beta)}{1} = 3T(\alpha - 2\beta)$

Bulk Modulus $= \dfrac{\text{Bulk Stress}}{\text{Bulk Strain}} = \dfrac{T}{3T(\alpha - 2\beta)} = \dfrac{1}{3(\alpha - 2\beta)}$

$$K = \frac{1}{3(\alpha - 2\beta)} = \frac{1}{3\alpha\left(1 - 2\frac{\beta}{\alpha}\right)} = \frac{Y}{3(1 - 2\sigma)}$$

where $Y = \dfrac{1}{\alpha}$ & $\sigma = \dfrac{\beta}{\alpha}$

$$\boxed{K = \frac{Y}{3(1 - 2\sigma)}}$$

3 c) Calculate the Extension produced in a wire of length 2m and radius 0.013×10^{-2} m due to a force of 14.7 N applied along its length. Given Young's modulus of the material of the wire

$Y = 2.1 \times 10^{11}\ N/m^2$

Soln:- we have.

$$Y = \frac{FL}{ax}$$

where a is the area of cross-section of the wire

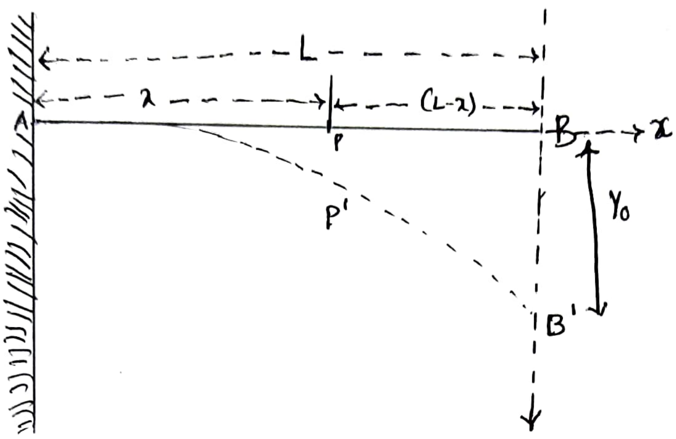
$$\therefore \text{Extension produced, } x = \frac{FL}{aY} = \frac{FL}{\pi R^2 Y} \quad (\text{since } a = \pi R^2),$$

$$x = \frac{14.7 \times 2}{\pi \times (0.013 \times 10^{-2})^2 \times 2.1 \times 10^{11}}$$

$$x = 2.6 \times 10^{-3}$$

$\therefore$ Extension produced $= 2.6 \times 10^{-3}$ m.

4 a) Describe a Single Cantilever and derive the Expression for Young's modulus of the material of rectangular beam.



Cantilever is a weight less beam whose length is very large Compared to its thickness. which is fixed at end and the other end is free.

Consider a Cantilever of length 'L' fixed at one end 'A' and a Load 'W' is attached to the other end 'B'

Let the neutral axis AB gets deflected to AB'

consider a section P (close to the free end) at a distance 'x' from the fixed end which will be at a distance of (L-x) from B.

Let 'P' be its position after the beams is bent.

Bending moment = Force X Perpendicular distance.

So that, $PB \simeq PB' = (L-x)$

Then bending moment produced at $P = W(L-x)$ and

Restoring couple acting upon $P = \frac{Y}{R} I_g$.

At Equillibrium, Restoring couple = Bending moment

$$\frac{Y}{R} I_g = W(L-x) \rightarrow (1)$$

$$\frac{1}{R} = \frac{W(L-x)}{Y I_g} \rightarrow (2)$$

But If 'y' is the depression of the point P then it can be shown that,

$$\frac{1}{R} = \frac{d^2y}{dx^2} \rightarrow (3)$$

where R is the radius of the circle to which the bent beam becomes a part. Comparing Eqn (2) and Eqn (3)

$$\frac{d^2y}{dx^2} = \frac{W(L-x)}{Y I_g}$$

or

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{W(L-x)}{Y I_g}$$

$$d\left(\frac{dy}{dx}\right) = \frac{W(L-x)\,dx}{Y I_g}$$

Integrating on both sides, we get

$$\frac{dy}{dx} = \frac{W}{Y I_g}\left(Lx - \frac{x^2}{2}\right) + C_1 \quad --- (4)$$

Where C_1 is the constant of integration. But (dy/dx) is the slope of the tangent drawn to the beam at a distance x from the fixed end.

When $x=0$, it refers to the tangent drawn at M where it is horizontal.

Hence $(dy/dx) = 0$ at $x = 0$,

Introducing this condition in Equ^n (3) we get $C_1 = 0$

Therefore Equ^n (3) now becomes.

$$\frac{dy}{dx} = \frac{W}{YI_g}\left(Lx - \frac{x^2}{2}\right)$$

$$dy = \frac{W}{YI_g}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_2 \longrightarrow (4)$$

where C_2 is the constant of integration and y is the depression produced at a known distance from the fixed end Therefore when $x = 0$, it refers to the depression at M where there is obviously no depression.

Hence $y = 0$ at $x = 0$. Introducing this conditions in Equ^n (4)

we get $0 = C_2$

Substituting this for C_2 in Equ^n (4), we get.

$$y = \frac{W}{YI_g}\left[\frac{Lx^2}{2} - \frac{x^3}{6}\right]$$

At the loaded end $y = y_0$ and $x = L$

$$y_0 = \frac{W}{YI_g}\left[\frac{L^3}{2} - \frac{L^3}{6}\right] \rightarrow (5)$$

Depression produced at the loaded end is

$$y_0 = \frac{WL^3}{3YI_g} \rightarrow (6)$$

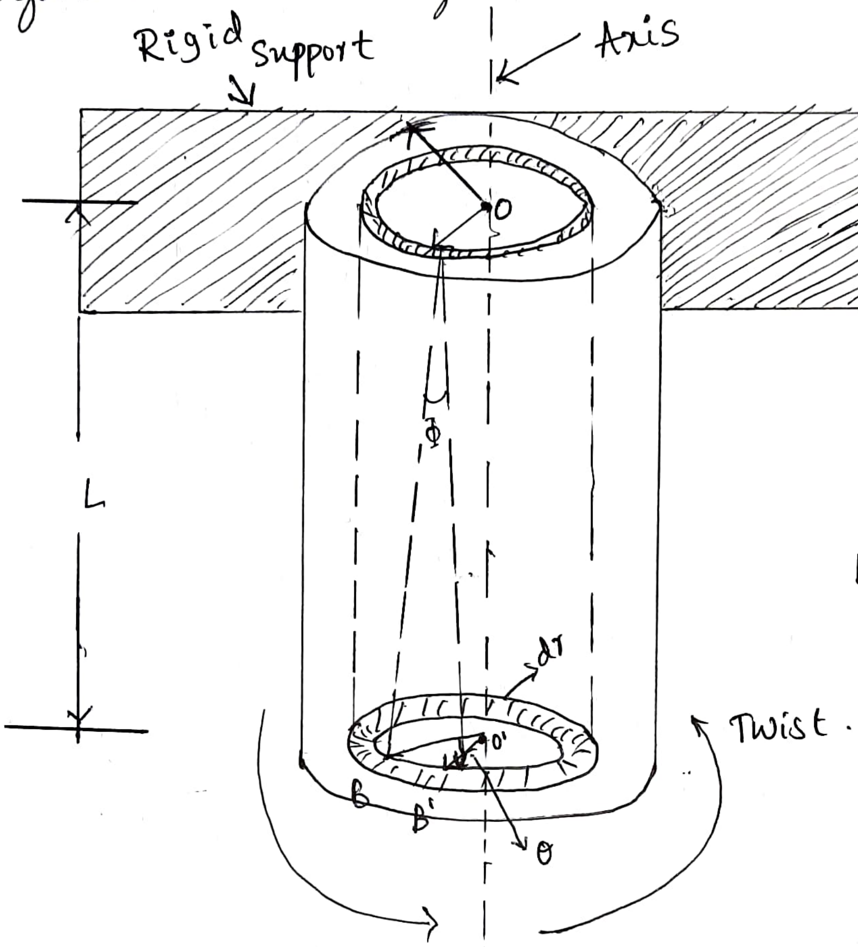
If the beam is having rectangular cross section. with breadth b and thickness d then I_g is given as

$$I_g = \frac{bd^3}{12}$$

Substituting in Equ^n (6) we get

$$Y = \frac{WL^3}{3y_0} \times \frac{12}{bd^3} \qquad \boxed{Y = \frac{4WL^3}{y_0 bd^3}}$$

4(b) Derive an Expression for couple per unit twist for a solid cylinder with a diagram (8M)



$\angle BXB' = \phi$

$\angle BOB' = \theta$

$BO' = B'O' = r$

Torsion of a cylinder

consider a long cylindrical rod of length L and radius R rigidly fixed at its upper end.

Let OO' be its axis.

We can imagine the cylindrical rod to made of thin concentric hollow cylindrical layers each of its thickness dr.

If the rod is now twisted at its lower end. then the concentric layers slide one over the other This movement will be zero at the fixed end. and it gradually increases along the downward direction.

Let us consider one concentric circular layer of radius 'r' and thickness dr.

A point X on the top remains fixed and a point like B at it bottom shifts to B' $\angle BXB' = \phi$ is the angle of shear since ϕ is also small. we have $BB' = L\phi$ Also if $\angle BOB' = \theta$, Then arc length $BB' = r\theta$

$$L\phi = r\theta \quad \text{or} \quad \phi = \frac{r\theta}{L} \rightarrow (1)$$

Now the cross-sectional area of the layer under consideration is $2\pi r\, dr$. if F is the shearing force then the shearing stress T is given by

$$T = \frac{\text{Force}}{\text{area}} = \frac{F}{2\pi r dr}$$

$$\therefore \text{Shearing force } F = T(2\pi r dr) \rightarrow (2)$$

If ϕ is the angle through which the layers is sheared then the rigidity Modulus.

$$\eta = \frac{\text{Shearing Stress}}{\text{Shearing Strain}} = \frac{T}{\phi}$$

$$T = \eta\phi = \frac{\eta r\theta}{L} \quad \{\text{from Equ (1)}\}$$

After substituting for T, Equn (2) becomes.

$$F = \frac{\eta r\theta}{L}(2\pi r dr) = \left(\frac{2\pi\eta\theta}{L} r^2 dr\right) r$$

$$F = \frac{2\pi\eta\theta r^3 dr}{L}.$$

This is regarding only one layer of the cylinder

$\therefore$ Twisting couple acting on the entire cylinder

$$= \int_{r=0}^{r=R} \frac{2\pi\eta\theta}{L} r^3 dr$$

$$= \frac{2\pi\eta\theta}{L}\left[\frac{r^4}{4}\right]_0^R = \frac{\pi\eta R^4\theta}{2L}$$

Couple per unit twist C is given by.

$$C = \frac{\text{Total twisting couple.}}{\text{angle of twist}}$$

$$C = \frac{\pi\eta R^4\theta/2L}{\theta}$$

$$\boxed{C = \left(\frac{\pi\eta R^4}{2L}\right)} \rightarrow (3).$$

The above Eqn (3) represents couple per unit twist.

4.c) Calculate the angular twist of a wire of length 0.3 m and radius 0.2×10^{3} m when a torque of 5×10^{4} Nm is applied (Rigidity modulus of the material is 8×10^{10} N/m^2)

Soln: The torque acting on the wire is given by

$$\tau = C\theta$$

where $C = \frac{\pi n R^4}{2L}$, the couple per unit twist of the wire

$$\tau = \frac{\pi n R^4}{2L}\theta$$

$$\theta = \frac{2L}{\pi n R^4} = \frac{2\times5\times10^{-4}\times0.3}{3.1\times8\times10^{10}\times(0.2\times10^{-3})^4}$$

$$\theta = 0.75 \text{ radian.}$$

$\therefore$ angular twist produced $= 0.75$ radian.

Module - 3

5a) Explain divergence and curl. Derive Gauss divergence Theorem.

Soln:- Divergence ($\nabla \cdot \vec{A}$) :-

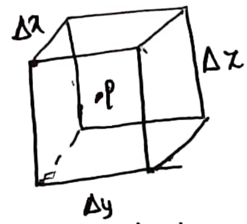
The divergence of a vector field $\vec{A}$ at a given point P means, it is outward flux per unit volume as the volume shrinks to zero about P.

Considering an elementary volume ΔV around a point P in the given space, the divergence at P can be represented as

$$\text{div } \vec{A} = \lim_{\Delta V \to 0} \frac{\text{outward flux of } \vec{A}}{\Delta V}$$

Mathematically we can rewrite as

$$\text{div } \vec{A} = \lim_{\Delta V \to 0} \frac{\oint_S \vec{A} \cdot \vec{ds}}{\Delta V} \rightarrow (1)$$



Now, by considering a rectangular parallelopiped around the given point P (Fig 1) as the elementary volume ΔV and working the total outward flux from all its six faces, it is possible to show that

$$\lim_{\Delta V \to 0} \frac{\oint_S \vec{A} \cdot ds}{\Delta V} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \Bigg|_{at\, P}.$$

Since as per Equn (1) the left side is divergence. we can write

$$\text{divergence of } \vec{A} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)$$

$$\vec{A} = \nabla \cdot \vec{A}.$$

Curl $\nabla \times \vec{A}$:-

The curl of a vector field $\vec{A}$ at a given point P means, it is the maximum circulation of $\vec{A}$ per unit area as the area shrinks to zero about P. curl $\vec{A}$ is represented as a vector whose direction is normal to the area around P when the area is oriented to make the circulation maximum.

It can be represented as

$$\text{Curl } \vec{A} = \lim_{\Delta s \to 0} \frac{\text{Max. circulation around } P}{\Delta S}$$

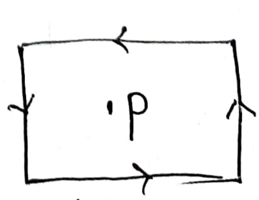
Mathematically we can write

$$\text{curl } \vec{A} = \left(\lim_{\Delta S \to 0} \frac{\oint \vec{A} \cdot dl}{\Delta S}\right) \hat{a}_n$$

where, the elementary area ΔS is bounded by the curve $L = \oint dl$, and $\hat{a}_n$ is the unit vector normal to ΔS.

Now by considering a rectangular elementary area across the point P as ΔS (fig 14) and working the closed line integral about the 4 sides of the boundary line it is possible to show that.

$$\left[\lim_{\Delta S \to 0} \frac{\oint A \cdot dl}{\Delta S}\right) \hat{a}_n = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$



Rectangular contour Around P.

where the right side is third order determinant.
Since the left side is curl of $\vec{A}$

$$\text{curl of } \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

But here, the right side corresponds to cross product $\nabla \times \vec{A}$.

∴ we can write curl of $\vec{A} = \nabla \times \vec{A}$. which is the common expression we are going to make use of henceforth

5 b) Define V-number and fractional index change. With a neat diagram. Explain the different types of optical fibre

· Mode refers to the number of paths for the light rays to propagate in the fiber.

The number of modes supported by the optical fiber can be determined by a parameter called V-number.

The V-number is given by

$$V = \frac{\pi d}{\lambda}\left(\sqrt{n_1^2 - n_2^2}\right) = \frac{\pi d}{\lambda}(NA)$$

where d is the diameter of the core

λ is the wavelength of the light propagation in the fiber

n_1 & n_2 → refractive index of core & cladding respectively

$$\text{number of modes} = \frac{V^2}{2}$$

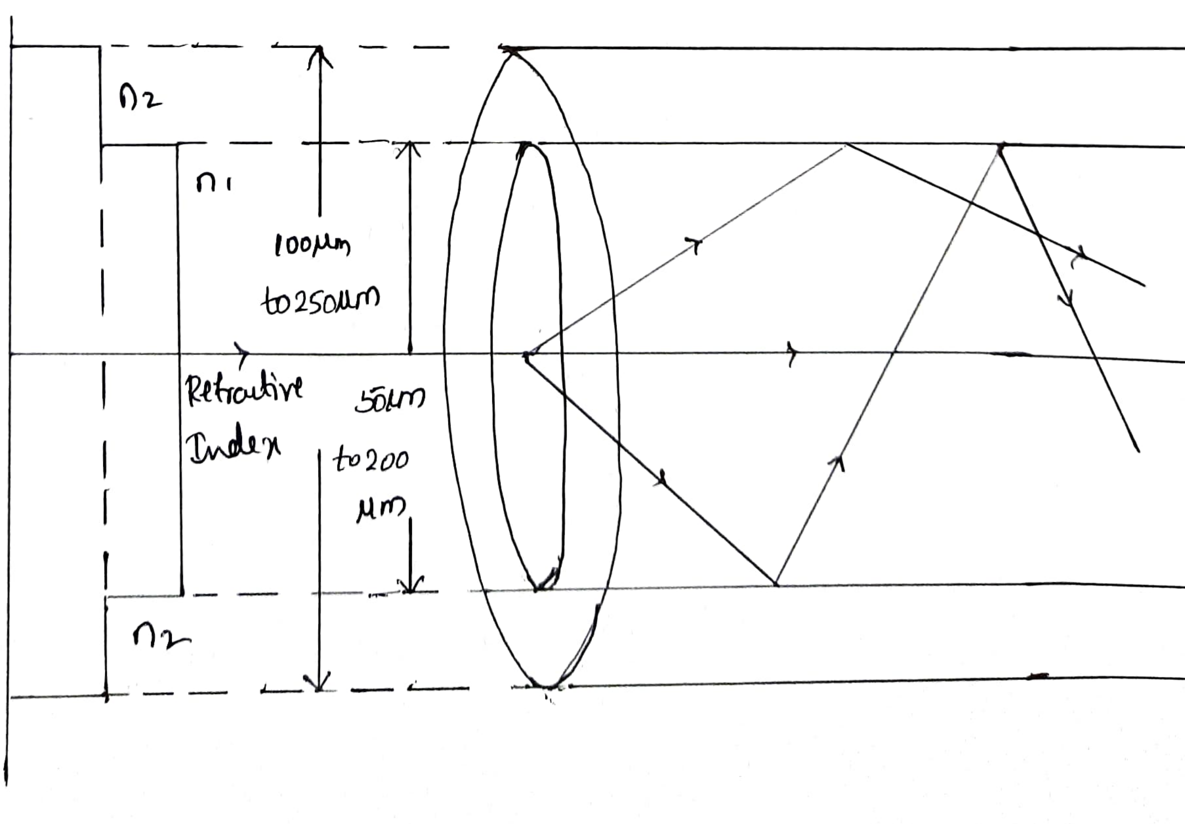
Types of optical fibers:-

optical fibers are classified into three types based on the refractive index of core and cladding and number of modes of propagation in the fiber.

① Step Index Single mode fiber :-

It consists of a core made of glass of uniform refractive Index n_1. The diameter of the core is about 10 µm.

* The core is surrounded by a cladding made of glass of uniform refractive index n_2
* The refractive Index of cladding is slightly lower than the refractive Index of core.
* The External diameter of the cladding is 60 to 70 µm
* Since the core diameter is very small therefore it can guide a single mode as shown in the below fig
* Lasers can be used as the source of light. S.M. fibers are the best guides the loss of light energy is very less They are used for long distance communications.

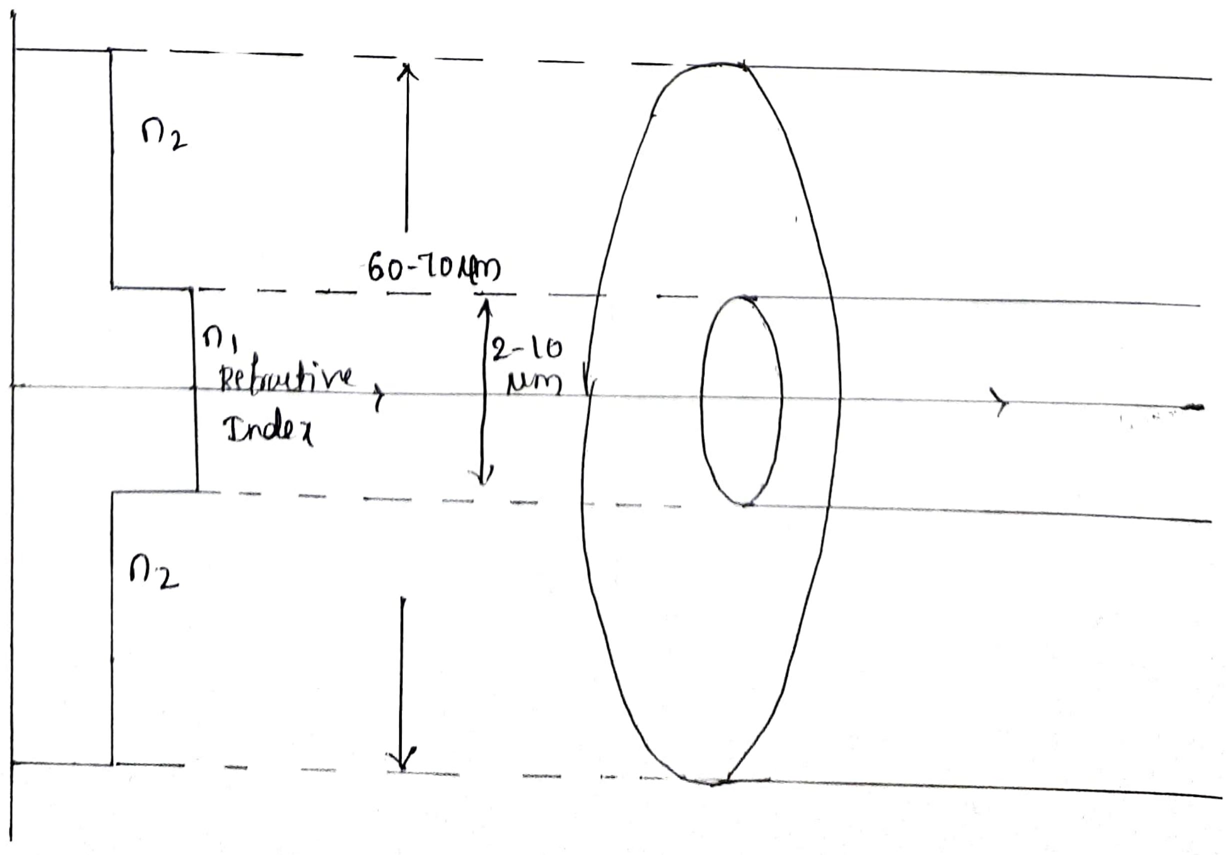


Step index fibers which have core diameters in the range of 2 µm to 10 µm are known as Single mode step index fibers.

* These fibers essentially transmit only the axial mode due to very small core diameter as shown in fig (1).
* The Thickness of the core is typically 25 µm to 30 µm. These fibers are used for long distance communication as intermodal dispersion is eliminated.

Graded index multimode fibers:-

In graded index fibers, the refractive index is maximum at the axis of the core, decreases gradually upto the cladding and then remains constant throughout the cladding as shown in fig (1). The rays travelling at an angle to the axis travel in curved paths due to gradually decreasing refractive index.



Although these rays travel longer distances compared to the axial ray, they travel in region of small refractive index and hence travel faster. This reduces the intermodal dispersion. The core diameter is typically about 20μm to 100μm with cladding thickness of about 25μm. They are commonly used for medium distance communications.

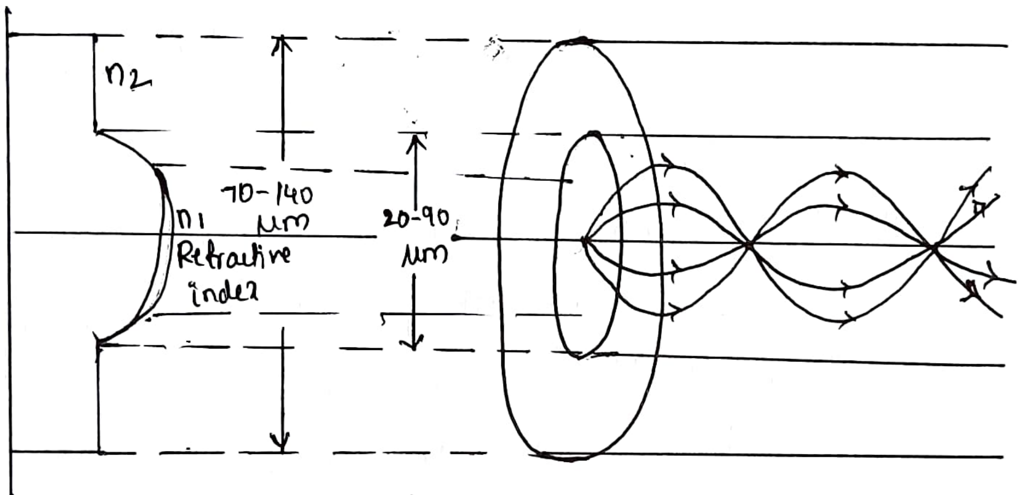


fig (3)

5. c) Find the divergence of the vector field $\vec{A}$ given by

$$A = 6x^2\hat{a}_x + 3xy^2\hat{a}_y + xyz^3\hat{a}_z$$

at a point $p(1,3,6)$.

Solⁿ: The given vector field is

$$\vec{A} = 6x^2\hat{a}_x + 3xy^2\hat{a}_y + xyz^3\hat{a}_z \rightarrow (1)$$

writing the general Expression for $\vec{A}$ we have

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z \rightarrow (2)$$

comparing Eqnⁿ (1) & (2), we have

$$A_x = 6x^2 \quad A_y = 3xy^2, A_z = xyz^3$$

The respective partial derivatives are

$$\frac{\partial A_x}{\partial x} = 12x \quad \frac{\partial A_y}{\partial y} = 6xy, \frac{\partial A_z}{\partial z} = 3xyz^2$$

The divergence of the vector field is given as

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Substituting the values for the respective vector components.

we have $\nabla \cdot \vec{A} = 12x + 6xy + 3xyz^2$

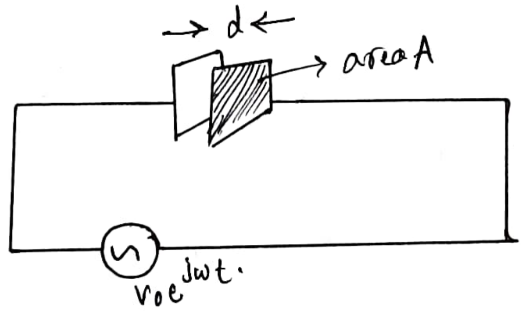
At the point p as per data $x = 1$, $y = 3$ & $z = 6$

$\therefore \ \mathrm{div}\,\vec{A} = 12(1) + 6(1)(3) + 3(1)(3)(6^2)$

$\mathrm{div}\,\vec{A} = 354.$

6(a) Derive the expression for displacement current. mention 4 maxwell's Equations in differential form for time varying fields.

Sol^n: consider a parallel plate capacitor. Connected across an ac Source. Let the area of each of the plate.



(i) Conduction current:-

Let the potential be $V = V_0 e^{j\omega t}$.

we have $D = \epsilon E$.

For a parallel plate Capacitor $E = \frac{V}{d}$

$$D = \frac{\epsilon V}{d} = \frac{\epsilon}{d} V_0 e^{j\omega t}$$

The displacement current density is given by $\left(\frac{\partial D}{\partial t}\right)$

if I_D is the displacement current, then $\left(\frac{\partial D}{\partial t}\right) = \frac{I_D}{A}$

$$\therefore \ I_D = \left(\frac{\partial D}{\partial t}\right) A = \frac{\partial}{\partial t}\left(\frac{\epsilon}{d} V_0 e^{j\omega t}\right) A$$

displacement current $I_D = \frac{j\omega \epsilon A}{d} V_0 e^{j\omega t} \longrightarrow (6)$

The name displacement current was introduced by maxwell for resons that are not relevant to the modern view of electromagnetic field in vacum.

A displacement current in vacum is not a current in the sense that it describes any motion of charges. But it is a current in the sense that it possesses the essential property of electric currents. (It has associated with itself a magontic field). Thus we can define the displacement current as follows.

Displacement current is the correction factor in Maxwell's Equation that appears in time-varying condition but doesn't describe any movement of charges though it has an associated magnetic field.

Maxwell's Equations:

Now we list the 4 Maxwell's Equations in differential form for time-varying fields and in conditions in vaccum as follows.

Time-varying fields.

(Differential form or point form)

1) From Gauss's law in electro statics $\nabla \cdot \vec{D} = \rho_v$

2) From faradays law $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

3) From Gauss's law for magtic fields $\nabla \cdot \vec{B} = 0$

4) From Ampere's law $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

6b) derive an Expression for numerical aperature in an optical fiber and state the condition for propagation.

Soln:- Consider n_1 and n_2 be the refractive indices of core and cladding respectively.

Let N_0 be the refractive Index of Surrounding medium (air) when a ray of light Ao incident on the core at 'O' It refracts along OB we have applying Snell's law

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\therefore \sin\theta_0 = \frac{n_1 \sin\theta_1}{n_0} \rightarrow (1)$$

At the point B on the interface the refracted ray grazes along BC

Angle of incidence $= 90^\circ - \theta_1$

Angle of refraction $= 90^\circ$

Again by Snell's law

$$n_1 [\sin(90-\theta_1)] = n_2 \sin 90^\circ$$

$$n_1 \sin(90-\theta_1) = n_2$$

$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = \frac{n_2}{n_1} \rightarrow (2)$$

from Eqn (1)

$$\Rightarrow \sin\theta_0 = \frac{n_1 \sin\theta_1}{n_0}$$

$$= \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_1}$$

$$\Rightarrow \sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_c^2}{n_2^2}}$$

$$= \frac{\cancel{n_1}}{n_0 \cancel{n_1}}\sqrt{n_1^2-n_2^2}$$ $\Big|$ $\because n_0 = 1$ for air or vaccum

$$\therefore \sin\theta_0 = \sqrt{n_1^2-n_2^2}$$

$$NA = \sin\theta_0 = \sqrt{n_1^2-n_2^2}$$

This is the Expression for numerical aperature.

The condition for propagation is angle of incidence θ should be less than acceptance angle θ_0

i.e $\theta_1 < \theta_0$

or $\sin\theta_1 < \sin\theta_0$

$\sin\theta_1 < \sqrt{n_1^2-n_2^2}$

Thus the condition for propagation is

$$\boxed{\sin\theta_1 < N.A}$$

Fractional change index (Δ):-

It is the ratio of refractive index difference between core and cladding to the refractive index of core of an optical fiber

$$\Delta = \frac{n_1-n_2}{n_1}$$

Relation between NA and Δ

$$\Delta = \frac{n_1-n_2}{n_1} \quad = \quad n_1-n_2 = n_1\Delta \rightarrow (1)$$

we have $NA = \sqrt{n_1^2 - n_2^2}$

$NA = \sqrt{(n_1+n_2)(n_1-n_2)}$ = $n_1 \simeq n_2$

$NA = \sqrt{2n_1 n_1 \Delta}$ = $n_1\sqrt{2\Delta}$

$$\boxed{NA = n_1\sqrt{2\Delta}}$$

6 c) Find the attenuation in an optical fiber of length 500 m. when a light signal of power 100mW emerges out of the fiber with a power 90 mW

Sol: The attenuation is given by.

$P_L = 10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)$ in dB

$P_{in} = 100mW$ $P_{out} = 90mW$

$P_L = 10 \log_{10}\left(\frac{100}{90}\right)$

$$\boxed{P_L = 1.04 \text{ dB}}$$

Module - 4

7) a) State and Explain Heisenberg's uncertainity principle. Show that the electron cannot exist inside the nucleus.

Solⁿ :- The Heisenberg's uncertainity principle States that it is impossible to determine both the exact position and exact momentum of a particle at the same time.

The product of uncertainity in these quantities is always greater than or equal to $\frac{h}{4\pi}$

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

The uncertainity principle in y and z-directions can be written as

$$\Delta y \, \Delta p_y \geq \frac{h}{4\pi}$$

$$\Delta z \, \Delta p_z \geq \frac{h}{4\pi}$$

The uncertainty principle can also be written for the pairs of variable energy-time and angular displacement -angular momentum as

$$\Delta E \, \Delta t \geq \frac{h}{4\pi} \quad , \quad \Delta \theta \, \Delta L \geq \frac{h}{4\pi}$$

(*) Show that the electron cannot exist inside the nucleus.

If an electron is confined to the nucleus which has a radius of the order of 10^{-14} m. The maximum uncertainity in position of electron will be the order of the radius.

$$\therefore \quad (\Delta x)_{max} = 10^{-14} \, m$$

By ~~a~~ Heisenberg uncertainity principle.

$$(\Delta x)_{max} (\Delta P)_{min} = \frac{h}{4\pi}$$

$$\therefore (\Delta P)_{min} = \frac{h}{4\pi (\Delta x)_{max}}$$

$$= \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$(\Delta P)_{min} = 5.276 \times 10^{-21} \text{ kg m/s}.$$

The momentum of electron has to be atleast comparable in magnitude to this uncertainity

$$\therefore P_{min} \simeq (\Delta P)_{min} = 5.276 \times 10^{-21} \text{ kgm/s}.$$

The Equation for energy from theory of relativity is

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$\text{Here } m_0^2c^4 << p^2c^2$$

$$\therefore E = PC$$

$$E_{min} = 5.276 \times 10^{-21} \text{ kg.m/s}.$$

The Equation for energy from theory of relativity is

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$\text{Here } m_0^2c^4 << p^2c^2$$

$$\therefore E = PC$$

$$E_{min} = 5.276 \times 10^{-21} \times 3 \times 10^{8}$$

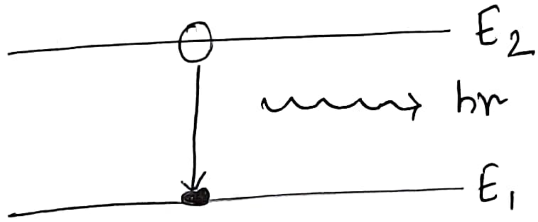
$$E_{min} = 1.583 \times 10^{-12} J = 9.9 \text{ Mev}$$

Thus for an electron to Exist in the nucleus its energy must be atleast 9.9 Mev.

Experimentally it has been observed during betadecay that the β-particles (electrons emitted by the nucleus) never have energy Exceeding about 4 Mev. As the minimum energy of an electron in a nucleus is 9.9 Mev. The electron can never exist in a nucleus. In fact. the β-particles are electrons which are produced in the nucleus due to decay of neutrons.

7.6) Define spontaneous emission and Stimulated emission. Explain the construction and working Semiconductor laser.

Soln:- ① Spontaneous emission:- In Spontaneous emission. the atoms or molecules in the higher energy state E_2 eventually return to the ground state by emitting their excess energy spontaneously



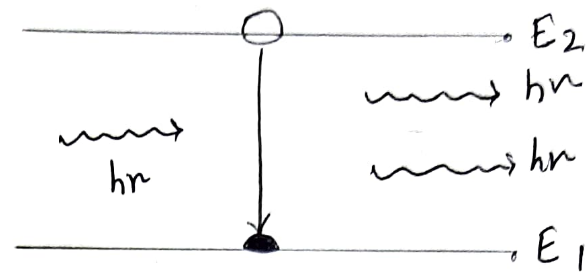
In the above emission there is no required any external agency. So it is independent of external radiation. The rate of spontaneous emission is directly proportional to the population of the energy level E_2

i.e $R_{21}(SP) \propto N_2$

$$R_{21}(SP) = A_{21} N_2 \rightarrow (2)$$

where A_{21} is the probability per unit time that the atoms will spontaneously fall to the ground state and N_2 be the atoms per unit volume.

② Stimulated emission:- In stimulated emission. a photons having energy E equal to the difference in a energy between two level E_2 and E_1. Stimulates an atom in the higher state. to make a transistion to the lower state with the creation of second photon.



The rate of stimulated emission (ST) is given as

$$R_{21}(ST) \propto N_2 E_\nu$$

$$R_{21}(ST) = B_{21} N_2 E_\nu \rightarrow (3)$$

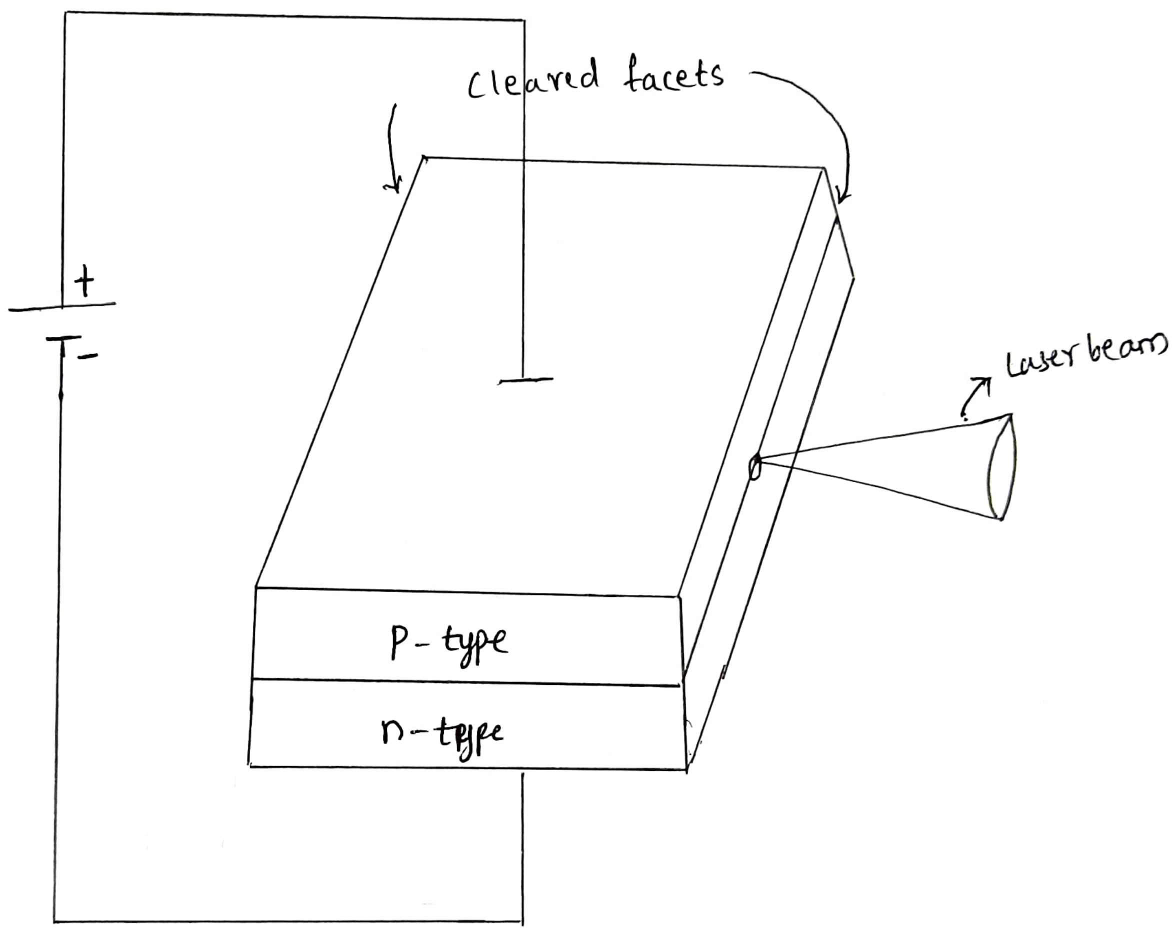
Where B_{21} is the probability per unit time that the atoms undergo transistion from higher energy state to lower state.

Under condition of thermal Equillibrium. The population of energy level obey Boltzman's distribution function.

Construction of Semiconductor laser:

* Gallium Arsenide laser is a Semiconductor laser. The Gallium Arsenide laser diode is a single crystal of GaAs.
* It consists of heavily doped n & p sections.
  The n-section is formed by doping with tellurium
  p-section is formed by doping with zinc.
* The doping concentration is very high and is of the order of 10^{17} to 10^{19} dopant/cm^3
* The overall size of the diode is very small and each of its sides of the order of 1mm
* The p-n Junction layer has width varying from 1μm to 100μm

* Two opposite faces which are $\perp^{lar}$ to the plane of the junction are polished and made parallel to each other
* These parallel faces constitute the resonant cavity and laser is obtained through these faces as shown in below fig.



working :-
* As the P and N types are heavily doped the fermi level (E_F) in N-type lies in the conduction band.
* In P-type lies in the valence band.
* The fermi level is uniform throught the unbiased diode.
* when the junction is forward biased the energy levels Shifts as shown in ~~below fig.~~ above fig
* The width of depletion region decreases due to injection of electrons and holes.

* At low forward currents, the electron-hole recombination causes spontaneous emission of radiation and diode acts as LED.
* When current is increased and reaches a threshould value population inversion is achieved in the depletion region due to large concentration of electrons in conduction band and hole (i.e valancies) in valence band.
* The narrow region where population inversion is achieved becomes the active region. where lasing action takes place
* The forward bias applied to the junction is thus the pumping mechanism which produces population inversion.
* The photons travelling in the junction along the resonant cavity stimulate recombination of electron-hole pairs due to which intensity of coherent light builds up along the axis of the cavity.
* The Semiconductor lasers have low power consumption are compact and highly efficient But the laser output is less monochromatic and more divergent compared to other lasers.

7c) A particle of mass 0.5 Mev/c^2 has kinetic energy 100eV. Find its de broglie wavelength where c is the velocity of light

Solⁿ:-

mass = 0.5 Mev/c^2 K.E = 100 eV = $100 \times 1.60 \times 10^{-19}$ J

$\lambda = ?$

we have mass of the particle $m = 0.5\ \text{Mev}/c^2 = 0.5 \times 10^6\ \text{eV}/c^2$

$$m = (0.5 \times 10^6) \times (1.602 \times 10^{-19})\ J/c^2$$

$$m = \frac{0.5 \times 10^6 \times 1.602 \times 10^{-19}}{(3 \times 10^8)^2}$$

$$m = 8.9 \times 10^{-31}\ kg$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 8.9 \times 10^{-31} \times 100 \times 1.602 \times 10^{-19}}}$$

$$\boxed{\lambda = 1.24 \times 10^{-10}\ m}$$

8.a) Assuming the time independent Schrodinger wave Equation discuss the Solution for a particle in one dimensional potential well of infinite height. Hence the Obtain the normalized wave function.

Soln:- The general differential Equation of a wave travelling in x-direction with velocity 'u' having wave function ψ is given by

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 \psi}{\partial t^2} \rightarrow (1)$$

The general Solution of Equation (1) is of the form

$$\psi = \psi_0 e^{i(kx-\omega t)} \rightarrow (2)$$

where ψ_0 is a constant.

Differentiating Equation (2) partially w.r.t 't' twice we get

$$\frac{\partial \psi}{\partial t} = (-i\omega)\psi_0 e^{i(kx-\omega t)}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi_0 e^{i(kx-\omega t)}$$

$$= -\omega^2 \psi_0 e^{i(kx-\omega t)}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi.$$

Substituting in Eqn (1)

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{\omega^2}{u^2}\psi. \rightarrow (3)$$

$$\omega = 2\pi\nu = \frac{2\pi u}{\lambda}$$

$$\frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2}$$

By De Broglie hypothesis $\lambda = \frac{h}{p}$

$$\frac{\omega^2}{u^2} = \frac{4\pi p^2}{h^2} \rightarrow (4)$$

The total energy 'E' is a sum of the K.E $\frac{1}{2}mv^2$ and potential energy V.

$$E = \frac{1}{2}mv^2 + V$$

$$= \frac{1}{2}\frac{m^2v^2}{m} + V$$

$$E = \frac{p^2}{2m} + V \quad \text{as } P = mv$$

$$p^2 = 2m(E-V)$$

Substitutin in Eqn (4)

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m(E-V)}{h^2}$$

Substituting in Eqn (3)

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{8\pi^2 m(E-V)\psi}{h^2}.$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-V)\psi}{h^2} = 0.$$

The above Eqn is used when the potential energy is constant in time but varies in space. Hence the wave function will also depend only on space co-ordinates. The partial derivative of ψ with respect to x can therefore be replaced by the total derivative.

$$\therefore \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \rightarrow (5)$$

Eqn (5) which is used for potential energy distributions that remains constant with time and only vary with x and hence gives solution for ψ as function of x only is knows as shroedinger time independent wave Equation.

b) Derive the Expression for energy density interms of Einstein's co-efficient.

Sol^n: Consider a system of atoms havings a ground state energy E_1 and Excited state energy E_2, With number densities of atoms in these states N_1 and N_2 respectively. If photons of frequency

$$\nu = \frac{E_2 - E_1}{h}$$

are incident on the system of atoms there will be induced absorption. The rate of absorption of photons will be proportional to the number density N_1 of the atoms in ground state and the energy density E_ν in the frequency range ν to $\nu + d\nu$ in incident radiation

$\therefore$ Rate of absorption $\propto N_1 E_\nu$

$\therefore$ Rate of absorption $= B_{12} N_1 E_\nu$

where B_{12} is a constant known as Einstien's co-efficient of induced absorption.

Atoms in Excited state E_2 can come down to ground state either through spontaneous emission or through stimulated emission of radiation. In the case of spontaneous emission The rate of transition of atoms from E_2 to E_1 does not depend on the energy density in the incident radition. and is proportional only to the number density of atoms in the Excited state i.e

Rate of spontaneous emission $\propto N_2$

$\therefore$ Rate of spontaneous emission $= A_{21} N_2$

where A_{21} is a constant known as Einstien's co-efficient of spontaneous emission.

In the case of stimulated emission, a photon of frequency $\nu = \frac{E_2 - E_1}{h}$ is required to stimulate the atoms. Hence the rate of stimulated emission is proportional to the energy density E_γ and the number density N_2 of the atoms in the excited energy state E_2 i.e,

Rate of stimulated emission $\propto N_2 E_\gamma$

Rate of stimulated emission $= B_{21} N_2 E_\gamma$

where B_{21} is ~~the~~ a constant known as Einstein's co-efficient of stimulated emission.

In a state of thermal equilibrium, the rate of transition of atoms from E_1 to E_2 must equal the total rate of transition from E_2 to E_1

$\therefore$ Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission.

$$B_{12} N_1 E_\gamma = A_{21} N_2 + B_{21} N_2 E_\nu$$

Dividing by N_1, we get.

$$B_{12} E_\gamma = A_{21} \frac{N_2}{N_1} + B_{21} \frac{N_2}{N_1} E_\gamma$$

$$\therefore E_\gamma \left[B_{12} - B_{21} \frac{N_2}{N_1} \right] = A_{21} \frac{N_2}{N_1}$$

$$E_\gamma = \frac{A_{21} \left[\frac{N_2}{N_1} \right]}{B_{12} \left(\frac{N_1}{N_2} \right) - B_{21}}$$

$$E_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{B_{12}}{B_{21}} \right) \left(\frac{N_1}{N_2} \right) - 1} \right]$$

From Maxwell - Boltzman distribution.

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/KT} = e^{-h\nu/KT}$$

$$\therefore \frac{N_1}{N_2} = e^{h\nu/KT}$$

$$\therefore E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}}{B_{21}}\right)e^{\frac{h\nu}{KT}} - 1}\right] \rightarrow (1)$$

Comparing with the energy density from planck's law.

$$\therefore E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/KT}-1}\right] \rightarrow (2)$$

we get $\frac{B_{12}}{B_{21}} = 1$

$$B_{12} = B_{21} \rightarrow (3)$$

and $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \rightarrow (4)$

Eqn (3) indicated that the probability of induced absorption is same as the probability of stimulated emission.

from Eqn (4).

$$\frac{A_{21}}{B_{21}} \propto \nu^3.$$

For large ν, $A_{21} >> B_{21}$, As $\nu = \frac{E_2-E_1}{h}$ for large energy difference betwen the ground state and Excited state, the probability of spontaneous emission is much larger than the probability of stimulated emission.

8.c) The ratio of population of two energy levels is 1.059×10^{-30}. Find the wavelength of light emitted by spontaneous emissions at 330K.

Sol^n:

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT} = e^{-\frac{hc}{\lambda KT}}$$

$$\therefore \log_e\left(\frac{N_2}{N_1}\right) = \frac{-hc}{\lambda KT}$$

$$\frac{N_2}{N_1} = 1.059 \times 10^{-30}$$

$h = 6.625 \times 10^{-34}$ J s $c = 3 \times 10^8$ m/s $K = 1.38 \times 10^{-23}$ J/s $T = 330$K

$$\log_e(1.059 \times 10^{-30}) = \frac{-6.626 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.38 \times 10^{-23} \times 330}$$

$$\lambda = 6.324 \times 10^{-7} \text{ m}$$

$$\boxed{\lambda = 6324 \, A^\circ}$$

9a) Give the assumptions of quantum free electron theory. Discuss two success of quantum free electron theory.

Sol^n:- Assumptions:-

① The valence electrons are free to move inside the metal.

② The electrons are confined to the metal by potential barrier at the boundaries. The potential is constant inside the metal.

③ The electrostatic forces of attraction between the free electrons and the ion cores are negligible

④ The electrostatic forces of repulsion amongst the free electrons are negligible.

⑤ The energies of electrons are quantized and the distribution of electrons in the allowed descrete energy levels is acording to pauli's exclusion principle which prohibits more than one electron in a single quantum state.

Success of quantum free electron theory (merits):

① Temperature dependence of electrical conductivity:-

The experimentally observed fact that the electrical conductivity σ has a dependence on $(1/T)$, but not on $(1/\sqrt{T})$ can be Explained as follows.

As per quantum free electron theory: the electrical conductivity for metals is given by

$$\sigma = \frac{ne^2\lambda}{m^* V_F} \rightarrow ①$$

Also as per the theory E_F and therefore V_F are essentially independent* of temperature. But λ is dependent on temperature. The nature of dependence of λ on T can be analyzed as follows.

If r is the amplitude of vibrations, then the ions can be considered to present effectively a circular cross-section of area πr^2 that blocks the path of the electron waves irrespective of the direction of approach. Since vibrations of larger area of cross-section cause more deviation in the periodicity of the lattice, the electron waves are scattered more efficiently. Increased scattering of waves results in a reduction in the value of mean free path of the electrons.

$$\therefore \quad \lambda \propto \frac{1}{\pi r^2} \quad \rightarrow \quad (1)$$

Now considering the facts that,

(a) the energy of a vibrating body is proportional to the square of the amplitude

(b) the energy of ions is due to thermal energy and

(c) the thermal energy is proportional to the temperature

(T) we can write.

$$r^2 \propto T$$

$$\therefore \quad \lambda \propto \frac{1}{T} \quad \rightarrow \quad (2)$$

comparing Eqn (1) and the proportionality (ii) we have

$$\sigma \propto \frac{1}{T}$$

Thus the depedence of σ on T is correctly explained by the quantum free electron theory.

2) Electrical conductivity and electron concentration:-

It was not possible to understand why metals such as Aluminium and Gallium, which have three free electrons/atom have lower electrical conductivity that metals such as copper and silver which poses only one free electron/atom. The explanation for the same could be given as follows

As per quantum free electron theory, we have the equation for electrical conductivity as

$$\sigma = \frac{ne^2}{m^*}\left(\frac{\lambda}{V_F}\right)$$

From the above equation it is clear that, the value of σ depends on n, the ratio (λ/V_F) and m^*

Further the value of m^* for Aluminium is 1.08 times that for Copper. Because of the inverse dependence of σ on m^* this also serves as a contributing factor for the higher value of σ for Copper

9(b) What are polar and non-polar dielectrics? Explain types of polarization.

Soln:- There are two types of dielectrics namely:-

(1) Non polar dielectrics are the dielectrics in which the centres of gravity of positive and negative charges of the molecules coincide with each other.

Ex:- Symmetric molecules like (mono atomic) He, Ne, Ar, Xe etc and H_2, N_2, Cl_2 etc

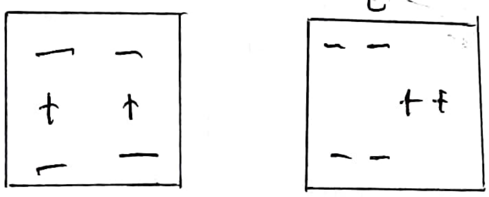
2) Polar dielectrics are the dielectrics in which the centres of gravity of positive & negative charges of the molecules do not co-incide with each other

Ex: NaCl, HC

There are four types of dielectric polarization namely,

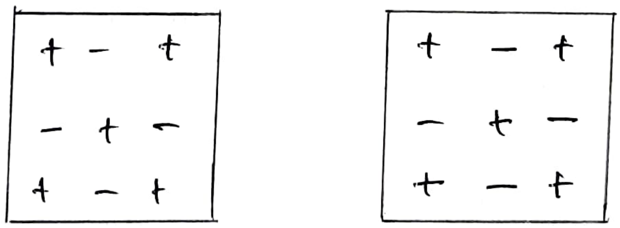
(1) Electronic polarization ($P_e = N\alpha_e E$) is the polarization in dielectrics due to displacement of the positive and negative charges under the influence of External electric field

$E \rightarrow$



E = 0 E > 0

2) Ionic polarization ($P_i = N\alpha_i E$) is the polarization in dielectrics having ionic bonds like NaCl. under External electric field displacement of ions takes place resulting in polarization



E = 0 $\rightarrow$ E > 0

3) Orientation polarization ($P_o = N\alpha_o E$) is the polarization in dielectrics having molecules of permanent dipole moment. under External electric field randomly oriented dipoles are aligned along the field resulting in polarization

E = 0

$\rightarrow$ E > 0

9.c) Calculate the probability of an electron occupying an energy level 0.02eV above the fermi level at 200K and 400K in a material.

Solⁿ :-

$E - E_F = 0.02eV = 0.02 \times 1.6 \times 10^{-19} J.$

(i) $f(E)$ at $200K = ?$

(ii) $f(E)$ at $400K = ?$

Solⁿ :-

(i) Evaluation of $f(E)$ at 200K:

we have

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$$

$$= \frac{1}{e^{\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}} + 1} = \frac{1}{e^{1.1594} + 1}$$

$$= \frac{1}{3.188 + 1} = 0.24$$

(ii) Evaluation of $f(E)$ at 400K.

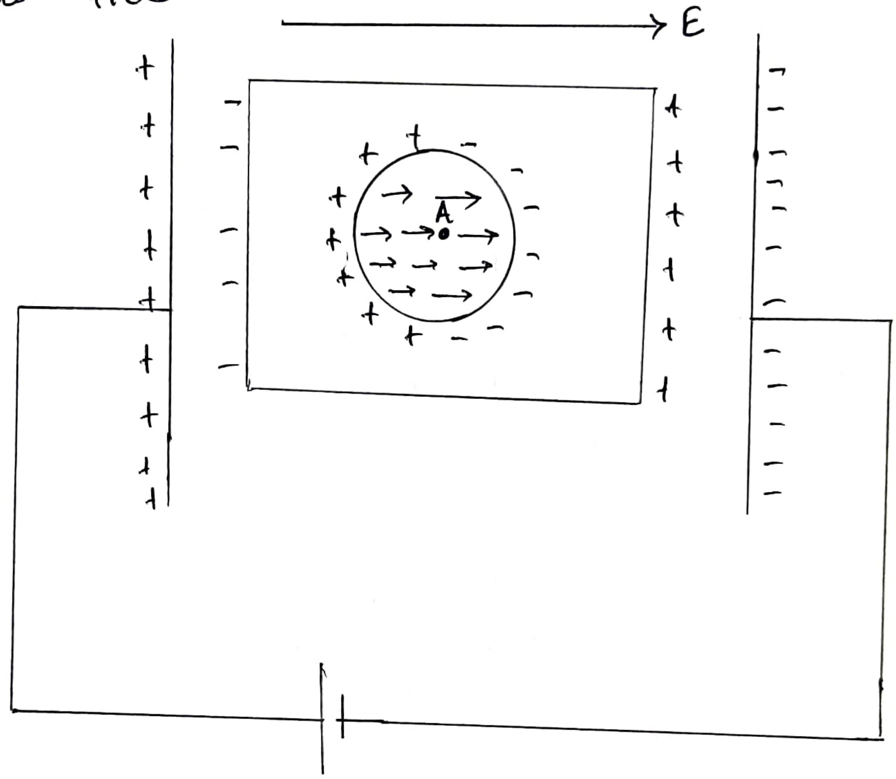
$$f(E) = \frac{1}{e^{\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 400}} + 1}$$

$$= \frac{1}{e^{0.5797} + 1} = \frac{1}{1.7855 + 1} = 0.36$$

$f(E) = 0.36$ at 400K.

10 a) Define internal field. mention the Expressions for internal field for one dimension, for three dimensional and Lorentz field for dielectrics. Derive Clausius - Mosotti Equation. (8M)

Sol^n:- The total electric field at an atom site in a dielectric material due to applied External electric field and induced various fields is called internal field.



Consider atoml dipole 'A' at the centre of an imaginary speherical cavity of large radius compared to intermolecular distance in a dielectric placed between the plates of a parallel plate capacitor.

The electric field at 'A' is called the internal field 'E_i' given by Lorentz.

$$E_i = E_1 + E_2 + E_3 + E_4 + \cdots \quad (1)$$

Where E_1 = field due to charge density on the plates of the capacitor given by

$$E_1 = E + \frac{P}{\epsilon_0} \rightarrow (2)$$

P = Polarisation, ϵ_0 = Permitivity of free space.

E_2 = field due to polarized charges induced on the plane ends of dielectric given by.

$$E_2 = \frac{-P}{\epsilon_0} \longrightarrow (3)$$

E_3 = Field due to polarized charges induced on the surface on the spherical cavity given by.

$$E_3 = \frac{P}{3\epsilon_0} \rightarrow (4)$$

E_4 = Field due to all dipoles inside the spherical cavity given by.

$$E_4 = 0 \longrightarrow (5)$$

From Equn (1) to (5) we get.

$$E_i = E + \cancel{\frac{P}{\epsilon_0}} - \cancel{\frac{P}{\epsilon_0}} + \frac{P}{3\epsilon_0} + 0$$

$$E_i = E + \frac{P}{3\epsilon_0}$$

<u>Expressions for internal fields in 1D & 3D Dielectrics and Lorentz field.</u>

Internal field in 1D dielectric material.

$$E_i = E + \frac{1.2\,\mu_{ind}}{\pi\epsilon\epsilon_0 a^3}$$

where E = applied electric field. μ_{ind} = induced dipolemoment, ϵ_0 = permittivity in free space and a = inter atomic distance.

Internal field in 3D dielectric material, $E_i = E + \frac{\gamma P}{\epsilon_0}$

where polarizability $P = \frac{\mu_{ind}}{a^3}$ and internal field constant

$$\gamma = \frac{1.2}{\pi}$$

Lorentz field $E_L = E + \frac{P}{3\epsilon_0}$ where $\gamma = \frac{1}{3}$ for cubic dielectric.

Note:- Internal field for 3D cubic material is called Lorentz field.

Internal (Local) field is the resultant field experienced by a molecule (or atom in a solid due to the external field and the fields produced by dipoles. Internal field is the sum of the applied field and the induced field.

Clausius - Mosotti Equation :-

W.K.T If P_e, P_i & P_o are the electronic, ionic and orienatation polarizations respectively. then total polarization of the dielectric material.

$$P = P_e + P_i + P_o$$

But for an elemental dielectric material P_i & P_o are Equal to zero.

$\therefore P = P_e$ also $P_e = N\alpha_e E_i$

Where α_e = electronic polarizability, E_i = Internal electric field

$P = N\alpha_e E_i$ but $E_i = (E + P3\epsilon_0)$

$$\therefore P = N\alpha_e \left[E + \frac{P}{3\epsilon_0}\right] \rightarrow (1)$$

$$\text{Also } P = \epsilon_0(\epsilon_r - 1)E \rightarrow (2)$$

From Eqn (1) & (2) we get.

$$\epsilon_0(\epsilon_r - 1)E = N\alpha_e\left[E + \frac{\epsilon_0(\epsilon_r - 1)E}{3\epsilon_0}\right]$$

on simplification $\epsilon_0(\epsilon_r - 1) = N\alpha_e\left[1 + \frac{(\epsilon_r - 1)}{3}\right]$

$$\epsilon_0(\epsilon_r - 1) = N\alpha_e\left[\frac{3 + (\epsilon_r - 1)}{3}\right]$$

$$3\epsilon_0(\epsilon_r - 1) = N\alpha_e\left[(\epsilon_r + 2)\right]$$

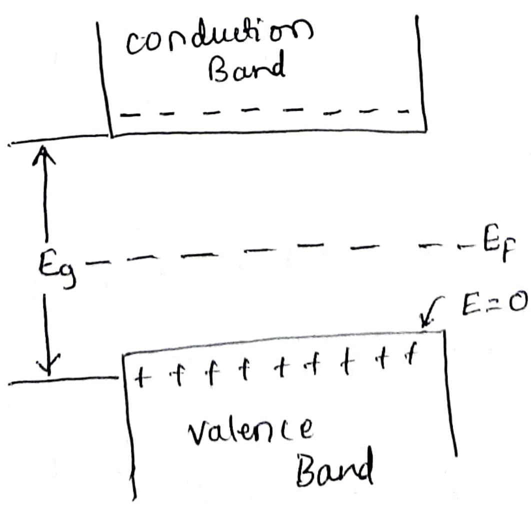
Rearranging, we can show that

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0},$$

This is the Clausius - Mosotti Equation used to obtain the value of dielectric constant of an ~~elect~~ elemental dielectric material.

6) Describe Fermi level in an intrinsic semiconductor and hence obtain the expression for Fermi energy in terms of energy gap of intrinsic semiconductor.

Sol^n:- For a semiconductor, the energy gap is much lesser than that for insulators. The energy gap is the energy difference between the bottom of the conduction band and top of the valence band. It is denoted as Eg. in fig 1. By convention, the energy at the top of the valence band is taken as zero for reference.



Fermi level in Intrinsic semiconductor.

At T = 0°K, all the energy levels in the valence band are completely filled and all the energy levels in the Conduction band are Completely filled. and all the energy But at ordinary temperatures such as room temperature, Some of the electrons at the top of the valence band are able to jump the energy gap due to the thermal Excitation. Due to this the average energy of the conduction electrons will be almost Equal to $1/2\ E_g$. Thus the fermi level lies in the mid-part of the forbidden gap for an intrinsic Semiconductor.

Expression for electron Concentration (Ne) :-

The Expⁿ for electron concentration (Ne) is given as

$$N_e = \frac{4\sqrt{2}}{h^3}\left(\pi m_e^* KT\right)^{3/2} e^{\left(\frac{E_F - E_g}{KT}\right)} \rightarrow (1)$$

where m_e^* is the effective mass of electron

K is the Boltzmann constant.

T is the temperature in absolute scale

E_F is the Fermi energy

E_g is the Energy gap

h is the planck's constant.

The Expⁿ for hole concentration (N_h) is given as

$$N_h = \frac{4\sqrt{2}}{h^3}\left(\pi m_h^* KT\right)^{3/2} e^{-E_F/KT} \rightarrow (2)$$

Relation between fermi energy & Energy Gap for an intrinsic Semiconductor.

For an intrinsic Semiconductor the number of holes / unit volume in valance band is Equal to number of electrons/unit volume in conduction band

$$\therefore \ N_e = N_h$$

$\therefore$ By Equating the right side of Eqn (1) & Eqn (2), we have

$$(m_e^*)^{3/2} e^{(E_F - E_g)/KT} = (m_h^*)^{3/2} e^{-E_F/KT},$$

$$\text{or} \quad e^{(2E_F - E_g)/K_BT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

By taking the natural logarithm on both sides, we get.

$$\frac{2E_F - E_g}{KT} = \frac{3}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$E_F = \frac{3}{4} KT \ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_g}{2} \rightarrow (3)$$

under practical considerations $m_e^* = m_h^*$ because of which the first term in the right-hand side of the above Equation goes to zero, since $\ln 1 = 0$

$$\therefore \ E_F = \frac{1}{2} E_g$$

Thus the fermi level is in the middle of the band gap for an intrinsic Semiconductor.

10. c) An elemental Solid dielectric material has polarizability 7×10^{-40} Fm^2. Assuming the internal field to Lorentz field. Calculalate the dielectric constant for the material, if the material has 3×10^{28} atoms/m^3.

Solⁿ:- Since the internal field is Lorentz field, and material is elemental Solid dielectric type. we can apply Clausius-Mossotti Eqn as

$$\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right) = \frac{N\alpha_e}{3\epsilon_0}$$

Substituting the values we have

$$\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right) = \frac{3 \times 10^{28} \times 7 \times 10^{-40}}{3 \times 8.854 \times 10^{-12}}$$

$$= 0.7906$$

$$(\epsilon_r - 1) = (\epsilon_r + 2) \times 0.7906$$

$$\epsilon_r - 1 = 0.7906\,\epsilon_r + 1.5812$$

$$\epsilon_r (1 - 0.7906) = 2.5812$$

$$\epsilon_r = \frac{2.5812}{0.2094} = 12.33$$

∴ The dielectric Constant of the material is 12.33.