CBCS SCHEME

USN | | | | | | | | | | 18CIV14/24

First/Second Semester B.E. Degree Examination, June/July 2019
Elements of Civil Engineering and Mechanics

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Assume missing data suitably.

Module-1

1 a. Briefly explain the role of civil engineers in the infrastructure development of the country. **(06 Marks)**

b. Explain briefly the scope of civil engineering in (i) structural engineering (ii) geotechnical engineering. **(08 Marks)**

c. A 200 N vertical force is applied to the end of the lever which is attached to a shaft as shown in Fig.Q1(c). Determine: (i) Moment of force about 'O' (ii) Horizontal force applied at 'A' which creates same moment about 'O' (iii) Minimum force which creates the same moment about 'O'.

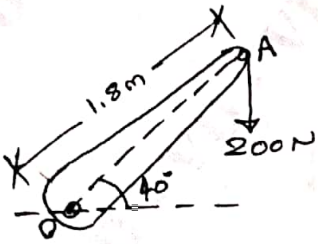

Fig.Q1(c) **(06 Marks)**

OR

2 a. Determine the resultant of the force system acting on a body as shown in the Fig.Q2(a) with respect to point 'O'.

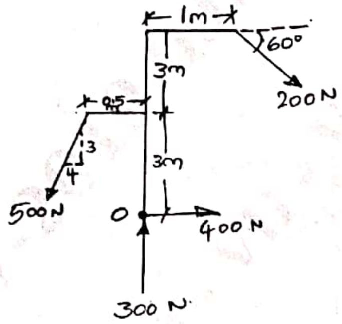

Fig.Q2(a) **(08 Marks)**

b. State and prove Varignon's theorem of moments. **(06 Marks)**

c. 2 kN force is the resultant of a system of forces acting along positive y-axis as shown in Fig.Q2(c). Determine the value of F and θ.

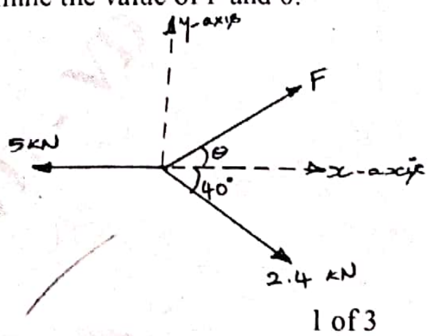

Fig.Q2(c) **(06 Marks)**

Module-2

3 a. What is meant by equilibrium? State the conditions of static equilibrium for both coplanar concurrent and non-concurrent force system. (05 Marks)

b. State and prove Lami's theorem. (05 Marks)

c. Determine the force 'P' required to cause the motion of the blocks to impend. Take the weight of A as 90 N and weight of B as 50 N. Take coefficient of friction for all contact surfaces as 0.30 as shown in Fig.Q3(c) and consider the pulley being frictionless.

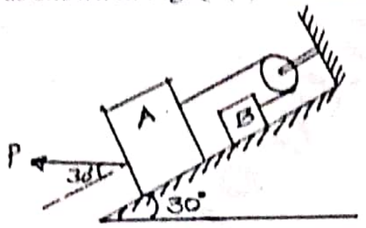

Fig.Q3(c) (10 Marks)

OR

4 a. Briefly explain: (i) Angle of friction (ii) Cone of friction. (04 Marks)

b. Calculate the tension in the strings. Also calculate 'θ' in Fig.Q4(b).

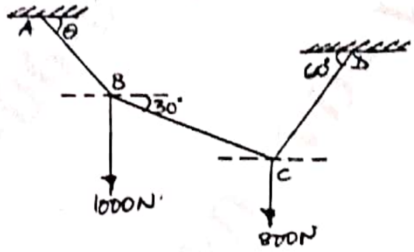

Fig.Q4(b) (10 Marks)

c. Prove that angle of repose is equal to angle of friction. (06 Marks)

Module-3

5 a. What are the different types of beams? How do you differentiate them? (06 Marks)

b. List the steps followed in the analysis of truss by method of joints. (06 Marks)

c. Find the length 'X' so that the reactions at both the supports are equal for the beam as shown in Fig.Q5(c).

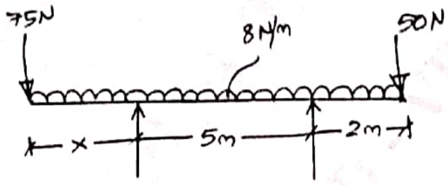

Fig.Q5(c) (08 Marks)

OR

6 a. List the assumptions made in the analysis of trusses. (04 Marks)

b. What are the different types of supports and mark their reaction lines. (06 Marks)

c. Analyze the frame and tabulate the member forces for the frame shown in Fig.Q6(c).

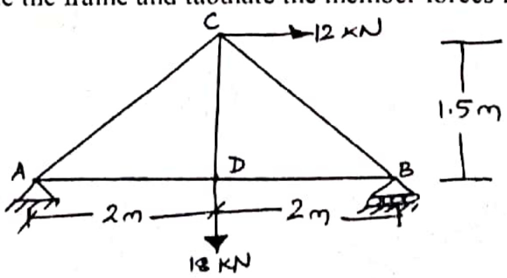

Fig.Q6(c) (10 Marks)

Module-4

7 a. Derive an expression for the centroid of a right angles triangle. **(06 Marks)**

b. State and prove perpendicular axis theorem. **(04 Marks)**

c. Determine the polar radius of gyration for the built up section as shown in Fig.Q7(c).

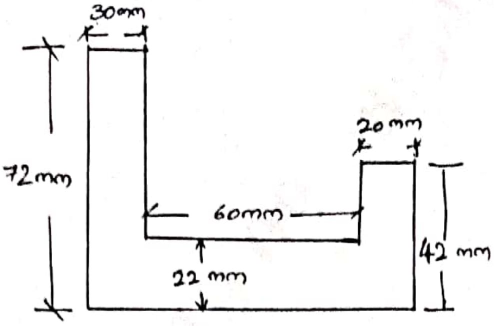

Fig.Q7(c) **(10 Marks)**

OR

8 a. Determine the moment of inertia of a semicircle with respect to its diameter line and hence determine the moment of inertia with respect to its centroidal axis parallel to the diameter line. Also write the expression for moment of inertia with respect to a line perpendicular to the diameter passing through the centroid. **(12 Marks)**

b. Determine the position of the controid for the Fig.Q8(b).

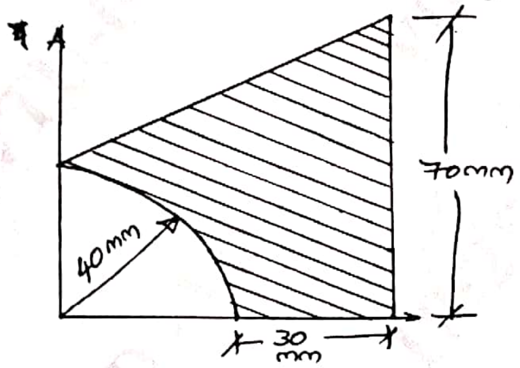

Fig.Q8(b) **(08 Marks)**

Module-5

9 a. Define displacement, distance travelled, velocity and acceleration. Mention their respective S.I units. **(04 Marks)**

b. Acceleration of a particle is given by $a = -2\ m/s^2$. If $v = 8$ m/s and $x = 0$ when $t = 0$. Determine: (i) velocity (ii) total distance travelled when $t = 6$ sec. **(08 Marks)**

c. State D'Alembert's principle and mention its application in plane motion. **(08 Marks)**

OR

10 a. Derive the equations of motion. **(08 Marks)**

b. What is superelevation? Why is it necessary? **(04 Marks)**

c. An aircraft moving horizontally at a speed of 300 km/hr at an elevation of 2.5 km targets a point on the ground and releases a bomb. If the bomb has to hit the target, determine the horizontal distance at which the aircraft should release the bomb. Also calculate the velocity and direction with which the bomb will hit the target. **(08 Marks)**

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Elements of Civil Engineering and Mechanics

(18CIV14/24)

June - July 2019.

1.

a. The role of civil engineer lies in planning the task meticulously and carrying out the designed work systematically to achieve the most optimal and efficient output that help the common people to lead a satisfactory life. They also are responsi-ble for the regular maintainence of the works carried out by them.

Civil engineering is much more than erecting skyscrapers or bridges. The civil engineers must have a thorough understanding of interaction among the various units of construction among the various structural elements, between the structure and complex environment. Since constructing a large building or public works project can involve elaborate planning, civil engineers can be outstanding project managers. They sometimes thousands of workers. They also develop advanced computerisation and planning policies. In addition many civil engineers are also involved with preserving, preventing protecting and restoring the environment.

In total a civil engineer has to perform the role of planner, a builder, an architect, a management expert and also an arbitrator.

b. i) Structural Engineering -

* The satisfactory performance of a structure requires the knowledge of materials behaviour and selection of appropriate material for use, proportioning/designing the different components of structure, estimating the stresses developed in different component of a structure and back checking the design.

* This field includes subjects like engineering mechanics strength of materials, structural analysis and design of structures. It also requires the knowledge of different tools to carry out the analysis and design of structural components. Such as matrix method of analysis, finite different techniques, finite element method of analysis etc.

ii) Geotechnical engineering - This subject encompasses

* a detailed study of rocks - types, properties, strength and deformation characteristics and their determination

* different types of foundation, their relative merits and limitations, suitability and design aspects.

* design and analysis of various earth structures such as embankments, dams, retaining walls etc.

* site investigations, sub soil explorations

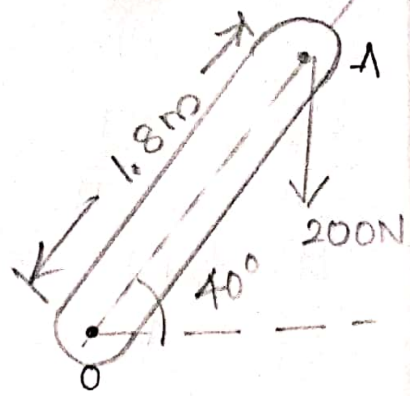
* Ground improvement techniques.

c.

i) Moment of force about O

$M_O = 200 \times 1.8\cos 40^\circ$

$\boxed{M_O = 275.7\ \text{Nm}}$



ii) Let F be the horizontal force at A.

$275.7 = F \times 1.8\sin 40^\circ$.

$\boxed{F = 238.35\ \text{Nm}}$

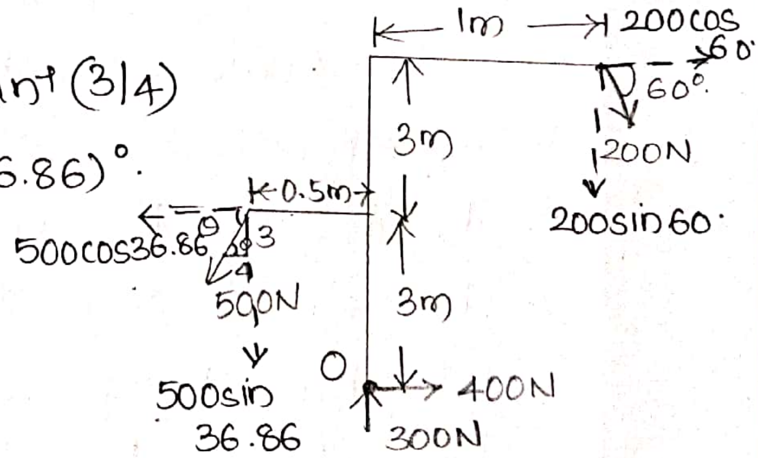
iii) The smallest force P at A is perpendicular to OA. Hence it makes angle 50° to the horizontal. The perpendicular distance will be 1.8m.

$275.7 = P \times 1.8$

$\Rightarrow \boxed{P = 153.16\ \text{N}}$

2.

a.



$\tan\theta = \frac{3}{4}$ $\quad \therefore \theta = \tan^{-1}(3/4)$

$= (36.86)^\circ$.

$\Sigma H = 200\cos 60 + 400 - 500\cos 36.86$

$\Sigma H = 99.94\ \text{N}$

$\Sigma V = -200\sin 60 - 500\sin 36.86 + 300$

$\Sigma V = 173.27\text{N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(99.94)^2 + (173.27)^2}$

$\boxed{R = 200\text{N}}$

$\Sigma M_O = -500\cos 36.86 \times 3 - 500\sin 36.86 \times 0.5 + 200\sin 60 \times 1 + 200\cos 60 \times 6$

$= -576.91\ \text{Nm}$

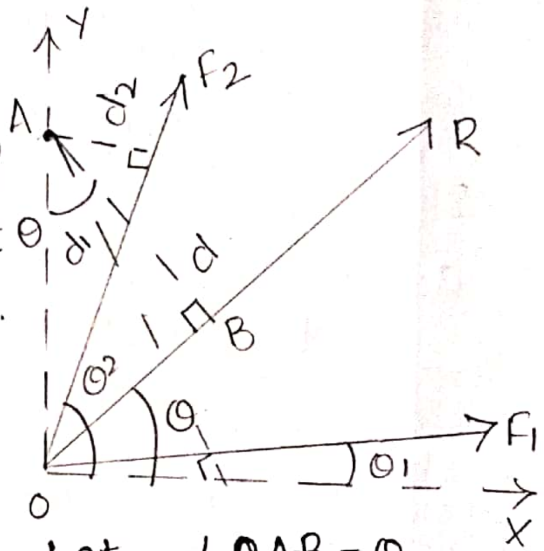
$$x = \frac{\Sigma M_o}{\Sigma V} = \frac{-576.91}{173.27} = 3.32 m$$

$$y = \frac{\Sigma M_o}{\Sigma H} = \frac{-576.91}{99.94} = 5.77 m$$

$$d = \frac{\Sigma M_o}{R} = \frac{-576.91}{200} = 2.88 m.$$

b. Varignon's theorem of moment -

Statement - " the algebraic sum of moments due to all forces acting on any object about any point is equal to the moment of their resultant about the same point".



Consider two forces F_1 and F_2 making angles θ_1 & θ_2 resply with x axis. Let their resultant R make angle θ with x axis. The perpendicular distances of F_1, F_2 and R from points A be d_1, d_2 and d resply. Let $\angle OAB = \theta$.

We have to prove,

$$Rd = F_1 d_1 + F_2 d_2 \quad \text{--- (1)}$$

Consider $R \times d = R \times OA \cos\theta$

$= OA \times R\cos\theta$

$R \times d = OA \times R_x$ --- (2)

Similarly for F_1 and F_2 we can write,

$F_1 \times d_1 = OA \times F_{1x}$ and $F_2 d_2 = OA\, F_{2x}$

Adding,

$F_1 d_1 + F_2 d_2 = OA\, F_{1x} + OA\, F_{2x}$

$= OA(F_{1x} + F_{2x})$

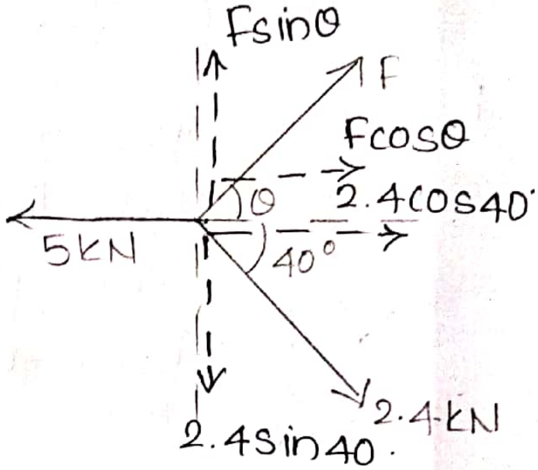
$F_1d_1 + F_2d_2 = OA\,R_x$ — ③

Comparing equation no ② and ③ we get,

$F_1d_1 + F_2d_2 = R \times d$

Hence the theorem is proved.

c.



$R = 2\,KN$.

R acting along y axis,

$\Sigma H = 0$.

$\Sigma V = \sqrt{0 + (\Sigma V)^2} = R$

$\Rightarrow R = \Sigma V$.

$\Sigma H = 0$

$\Rightarrow 2.4\cos 40 + F\cos\theta - 5 = 0$.

$\Rightarrow F\cos\theta = 3.16$ — ①

$\Sigma V = R$

$\Rightarrow F\sin\theta - 2.4\sin 40 = 2$.

$\Rightarrow F\sin\theta = 3.54$ — ②

Dividing eqn ② by eqn ①,

$\dfrac{\cancel{F}\sin\theta}{\cancel{F}\cos\theta} = \dfrac{3.54}{3.16} \Rightarrow \tan\theta = 1.12$

$\therefore \boxed{\theta = (48.24)^\circ}$

∴ From eqn ①, $F\cos 48.24 = 3.16$.

$\Rightarrow \boxed{F = 4.74\,KN}$

3.

a. Equilibrium is a condition in which a body remains constant or it will not undergo any translatory / rotatory motion even after application of load.

The conditions of static equilibrium for

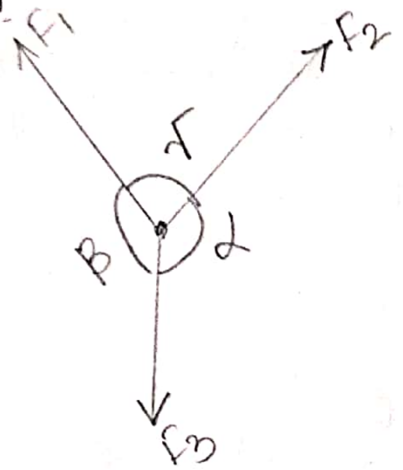
① Concurrent Force system-

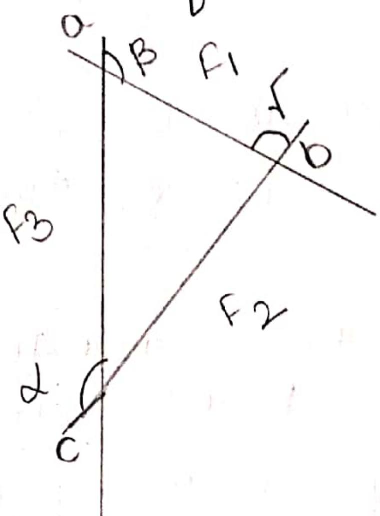
$\Sigma H = 0$, $\Sigma V = 0$.

② Non concurrent force system-

$\Sigma H = 0$ $\Sigma V = 0$ $\Sigma M = 0$.

b. Lami's theorem - It states that 'a body in equili-brium under the action of three concurrent force system, each force is proportional to sine of angle between other two forces'.





Consider a body under the action of F_1, F_2 & F_3. Let the angle made by the forces be α, β, γ. Construct a triangle ABC such that $ab = F_1$ $bc = F_2$ and $ac = F_3$ in proportion to magnitude and direction.

Applying sine rule to Δabc.

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(180-\gamma)}$$

$$\Rightarrow \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Hence the theorem is proved.

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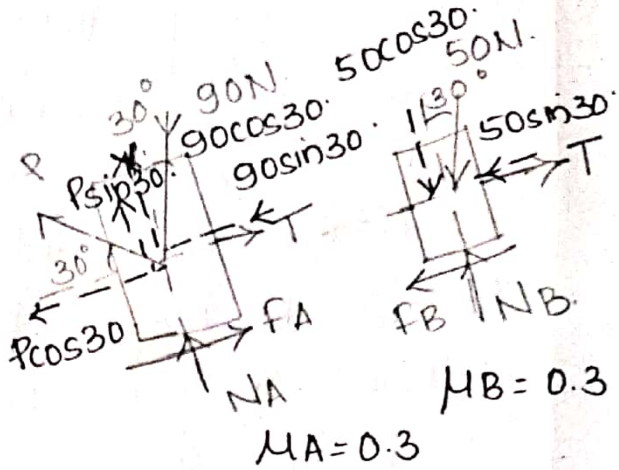
c. Consider FBD of block B.



$\Sigma H = 0$

$\Rightarrow T - 50\sin 30 - f_B = 0$ — (1)

$\Sigma V = 0$

$\Rightarrow N_B - 50\sin 30 = 0$

$\Rightarrow \boxed{N_B = 25N}$

But $\frac{f_B}{N_B} = \mu_B = 0.3$

$\Rightarrow f_B = 0.3N_B = 0.3 \times 25$

$\therefore \boxed{f_B = 7.5N}$

$\therefore$ from eqn (1), $T = 50\sin 30 + f_B = 50\sin 30 + 7.5$

$\therefore \boxed{T = 32.5N}$

Consider FBD of block A,

$\Sigma H = 0$

$\Rightarrow T - 90\sin 30 \cancel{+} Pcos 30 + F_A = 0$.

$\Rightarrow 32.5 - 90\sin 30 - P\cos 30 + F_A = 0$ — (2)

$\Sigma V = 0$

$\Rightarrow N_A - 90\cos 30 + P\sin 30 = 0$.

$\Rightarrow N_A = 77.9 + P\sin 30$ — (3)

But $\frac{F_A}{N_A} = \mu_A = 0.3$

$\Rightarrow F_A = 0.3\, N_A = 0.3(77.9 - P\sin 30)$

$F_A = 23.38 - 0.15P$.

Sub F_A in equation (2),

$32.5 - 90\sin 30 - P\cos 30 + (23.38 - 0.15P) = 0$.

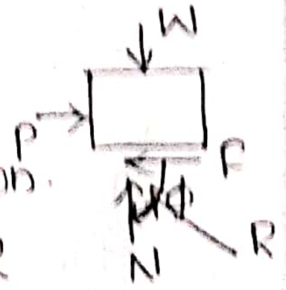
$\Rightarrow -12.5 - P\cos 30 + 23.38 - 0.15P = 0$

$\Rightarrow -1.01P + 10.88 = 0$

$\Rightarrow \boxed{P = 10.77N}$

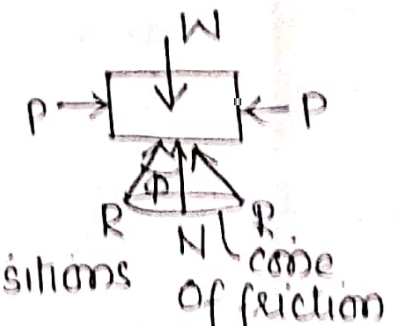
4.

a. i) <u>Angle of friction</u> – The normal reaction N and the frictional force F can be combined to give a resultant R called the resultant reaction as shown. The angle made by the resultant R with the normal reaction N is called as angle of friction ϕ.

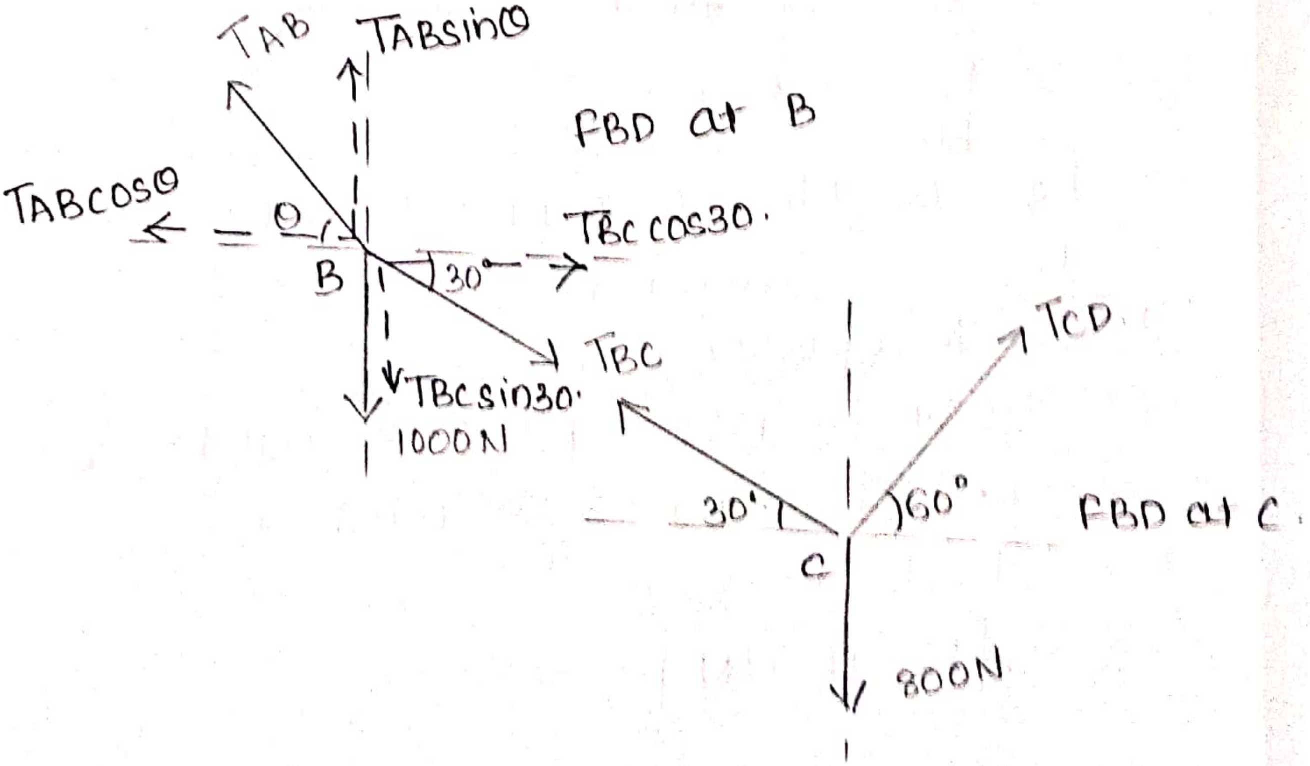


$$\tan\phi = \frac{Fr}{N}.$$

ii) <u>Cone of friction</u> – If the direction of applied force P is changed, keeping its angle with the surface of contact same throughout, the resultant R will take different positions in space, but making the same angle ϕ with the normal reaction N. In such case, R lies on the surface of a cone known as cone of friction.



b.



Applying Lami's theorem to FBD at C.

$$\frac{T_{BC}}{\sin(90+60)} = \frac{T_{CD}}{\sin(90+30)} = \frac{800}{\sin(180-30-60)}$$

$$\Rightarrow \frac{T_{BC}}{\sin 150} = \frac{800}{\sin 90^\circ} \quad \Rightarrow \frac{T_{CD}}{\sin 120} = \frac{800}{\sin 90^\circ}$$

$$\Rightarrow T_{BC} = \frac{800 \times \sin 150}{\sin 90^\circ} \quad \Rightarrow T_{CD} = \frac{800 \times \sin 120}{\sin 90^\circ}$$

$$\therefore \boxed{T_{BC} = 400\text{N}} \quad \Rightarrow \boxed{T_{CD} = 692.82\text{N}}$$

Consider FBD. of B.

$\Sigma H = 0$

$\Rightarrow 400\cos 30 - T_{AB}\cos\theta = 0.$

$\Rightarrow \quad T_{AB}\cos\theta = 346.41$ —(1)

$\Sigma V = 0$

$\Rightarrow -400\sin 30 + 1000 + T_{AB}\sin\theta = 0$

$\Rightarrow \quad T_{AB}\sin\theta = 1200$ —(2)

Dividing eqn (2) by eqn (1)

$$\Rightarrow \frac{\cancel{T_{AB}}\sin\theta}{\cancel{T_{AB}}\cos\theta} = \frac{1200}{346.41} \quad \Rightarrow \tan\theta = 3.46$$

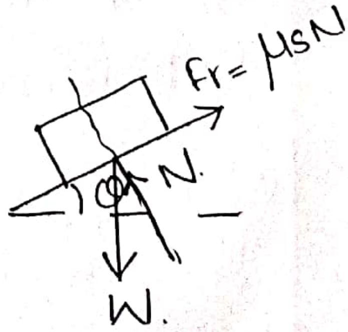
$$\boxed{\therefore \theta = 73.89^\circ}$$

$\therefore$ From eqn (1),

$$T_{AB} = \frac{346.41}{\cos 73.89}$$

$$\therefore \boxed{T_{AB} = 1248.4\text{N}}$$

c. Consider FBD of an object kept on an inclined plane at angle θ equal to angle of repose as shown.



$\Sigma V = 0$ $\qquad\qquad\qquad\qquad$ $\Sigma H = 0$

$\Rightarrow N - W\cos\theta = 0$ $\qquad\qquad$ $\Rightarrow \mu_s N - W\sin\theta = 0$

$N = W\cos\theta$ $\qquad\qquad\qquad\qquad$ $\mu_s W\cos\theta = W\sin\theta$

$\Rightarrow \mu_s = \tan\theta$

But $\mu_s = \tan\phi_s$.

$\tan\phi_s = \tan\theta$.

$\Rightarrow \boxed{\phi_s = \theta}$

5.

a. The different types of beams are –

① Cantilever beam – Here one end is fixed and other end is free.

② Fixed beam – Here both the ends of the beam are fixed ie on fixed supports.

③ One end hinged and other on roller beam – In this beam, one end is on hinged support and other is on roller support.

④ Both ends fixed beam – In this beam, both the supports are fixed.

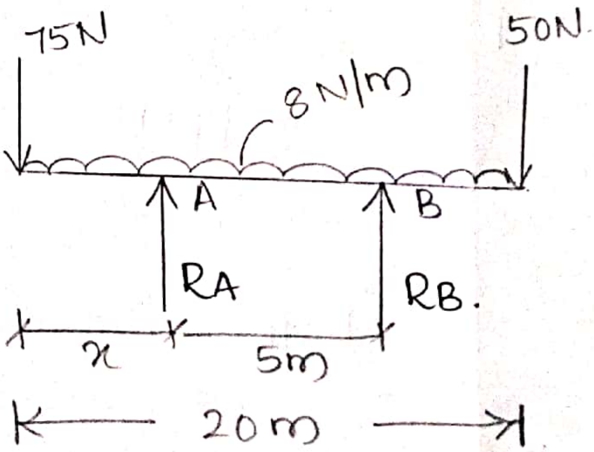
⑤ Continuous beam – In this type of beam, the beam is supported on intermediate supports continuously.

⑥ Propped cantilever beam – In this beam, one end is fixed and other on roller beam.

b. Following are the steps followed for the analysis of truss by method of joints (6)

1. Determine the inclination of all the inclined members.
2. Look for a joint where there are only two unknowns. If such a joint is not available, determine the reactions at support and then the unknowns at one of the support may reduce to only two.
3. On the diagram of truss, mark the arrow on the members near the joints analysed to indicate the forces on the joint. At the other end, mark the arrows in the reverse direction.
4. Look for the next joint for analysis where there are only two unknown forces.
5. Repeat steps 3 and 4 till all the forces in all the members are determined.
6. Determine the nature of forces in all members and tabulate the results.

c.



$\Sigma V = 0.$

$\Rightarrow -75 - 8\times 20 - 50 + V_A + V_B = 0$

$\Rightarrow V_A + V_B = 285$

$\Rightarrow V_A = V_B$ (Given)

$\Rightarrow V_A = V_B$

$\boxed{V_A = 142.5 \text{ kN} = V_B}$

$\Sigma M_A = 0.$

$\Rightarrow -75 \times x - 8 \times 20 \times (10-x) + V_B \times 5 + 50 \times (20-x) = 0.$

$\Rightarrow -75x - 1600 + 160x + 142.5 \times 5 + 1000 - 50x = 0$

$\Rightarrow -35x = 112.5$

$\Rightarrow x = 3.21 m$

$\therefore \boxed{x = 3.21 m}$ for the assumed beam figure.

6.

a. The assumptions made in the analysis of truss are -

① The ends of the members are pin connected (hinged)

② The loads act only on the joints

③ Self weight of the members are negligible.

④ Members are either having uniform cross section throughout or if they have varying cross section the centroid is located among the same longitudinal line.

b.

Supports

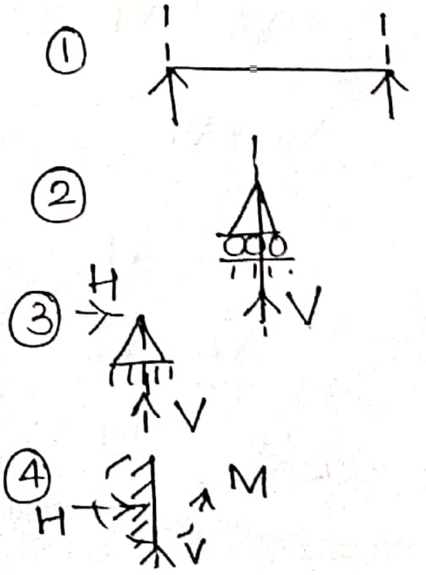
① Simple

② Roller

③ Hinged

④ Fixed

Reaction line

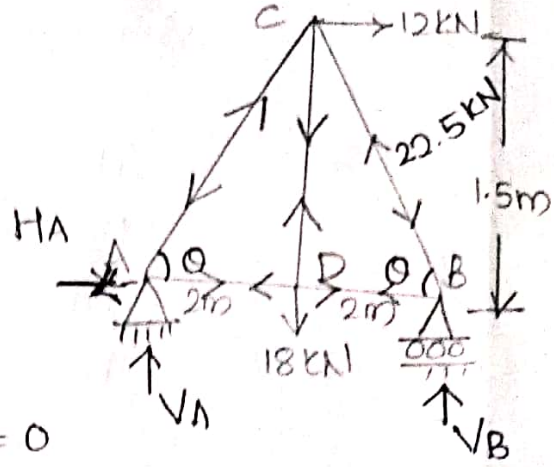


C. ⑦

① $\tan\theta = \frac{1.5}{2}$

$\therefore \theta = \tan^{-1}\left(\frac{1.5}{2}\right)$

$\theta = 36.86^\circ$.



$\Sigma H = 0$

$\Rightarrow -H_A + 12 = 0$.

$\Rightarrow H_A = +12\,KN$

$\Sigma V = 0$

$\Rightarrow V_A + V_B = 18$

$\Sigma M_A = 0$.

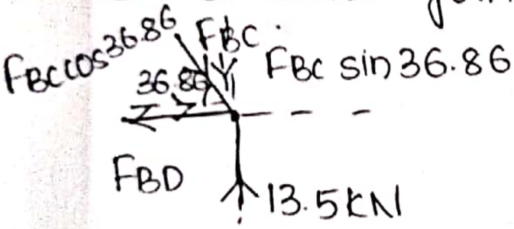
$\Rightarrow 12 \times 1.5 + 18 \times 2 - V_B \times 4 = 0$.

$\rightarrow \boxed{V_B = 13.5\,KN}$

$\therefore V_A + V_B = 18$

$\Rightarrow V_A = 18 - 13.5 = 4.5\,KN$.

② Consider joint B.



$\Sigma H = 0$

$\Rightarrow -F_{BD} + F_{BC}\cos 36.86 = 0$

$\Sigma V = 0$.

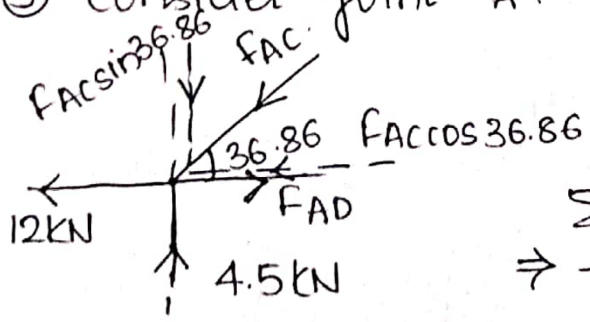
$\Rightarrow 13.5 - F_{BC}\sin 36.86 = 0$

$\Rightarrow \boxed{F_{BC} = 22.5\,KN}$

$\Rightarrow F_{BD} = 22.5\cos 36.86$

$\boxed{F_{BD} = 18\,KN}$

③ Consider joint A.



$\Sigma V = 0$.

$\Rightarrow 4.5 - F_{AC}\sin 36.86 = 0$

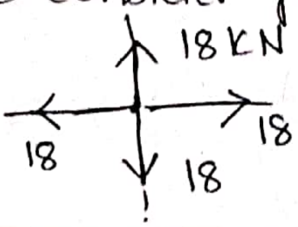
$\Rightarrow \boxed{F_{AC} = 7.5\,KN}$

$\Sigma H = 0$

$\Rightarrow -12 - 7.5\cos 36.86 + F_{AD} = 0$

$\boxed{\therefore F_{AD} = 18\,KN.}$

④ Consider joint D.

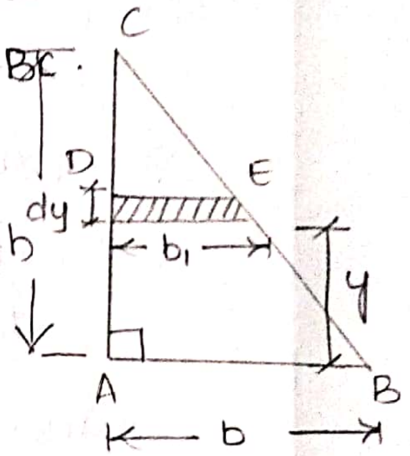


$\boxed{F_{DC} = 18\,KN}$

Sl No	Member	Forces (kN)	Nature of force
1	AC	7.5	Compression
2	AD	18	Tension
3	DC	18	Tension
4	DB	18	Tension
5	BC	22.5	Compression

7.

a. Centroid of a right angle triangle –



Consider a right angle triangle ABC of base b and height h.

Consider an elemental strip at a distance y from base of thickness dy and base b_1.

we have $\bar{x} = \dfrac{\int y \, dA}{A}$.

Area $dA = b_1 \times dy$.

From similar triangles ΔCDE and ΔCAB,

$$\frac{b_1}{b} = \frac{h-y}{h}. \quad \therefore b_1 = b\left(\frac{h-y}{h}\right) = b\left(1-\frac{y}{h}\right).$$

$$\therefore \quad dA = b\left(1-\frac{y}{h}\right) dy.$$

$$\therefore \quad \bar{x} = \frac{\int_0^h y \, b\left(1-\frac{y}{h}\right) dy}{\frac{1}{2} \times b \times h}$$

$$= \frac{\cancel{b}\int_0^h \left(y-\frac{y^2}{h}\right) dy}{\frac{1}{2} \times \cancel{b} \times h} = \frac{\left[\frac{y^2}{2}-\frac{y^3}{3h}\right]_0^h}{\frac{1}{2} \times h} = \frac{\left[\frac{h^2}{2}-\frac{h^2}{3}\right]}{1/2 \times h}$$

$$= \frac{\frac{3h^2-2h^2}{6}}{\frac{1}{2} \times h} = \frac{h^2}{6} \times \frac{2}{h} = \frac{h}{3} \qquad \boxed{\therefore \bar{x} = h/3}$$

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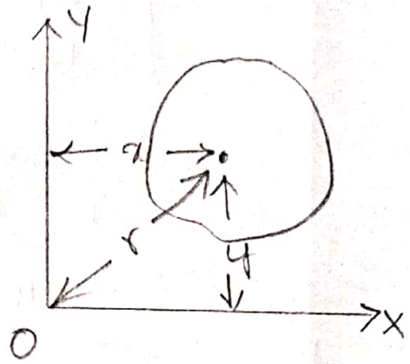
b. **Perpendicular axis theorem** – It states that
"The moment of inertia about a point / axis perpendicular through the plane is equal to the moment of inertia about two mutually perpendicular axis passing through the same point".

ie $I_{zz} = I_{xx} + I_{yy}$.

Proof – Consider a body at a distance 'x' from y axis, 'y' from x axis and 'r' from the axis perpendicular to the plane of the body.



We have $r^2 = x^2 + y^2$

But
$$I_{zz} = \Sigma r^2 dA$$
$$= \Sigma (x^2+y^2) dA$$
$$= \Sigma x^2 dA + \Sigma y^2 dA$$
$$= I_{yy} + I_{xx}$$

$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}}$ Hence proved.

c.

$A_1 = 30 \times 72 = 2160\ mm^2$

$x_1 = 15\ mm$

$y_1 = 36\ mm$

$A_2 = 60 \times 22 = 1320\ mm^2$

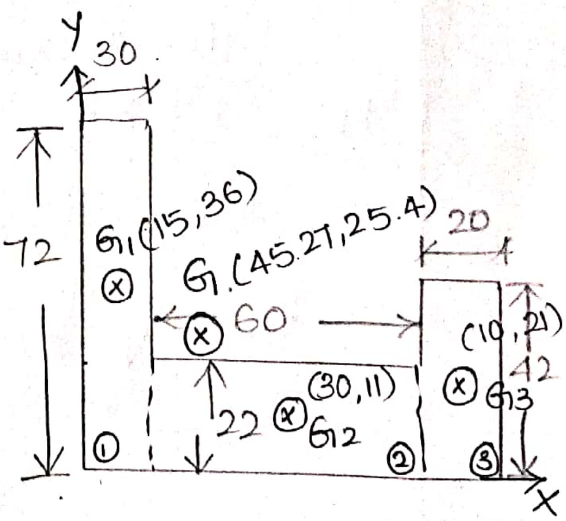
$x_2 = 60\ mm$

$y_2 = 11\ mm$

$A_3 = 20 \times 42 = 840\ mm^2$

$x_3 = 100\ mm$

$y_3 = 21\ mm$.



$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} \qquad \bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{2160 \times 15 + 1320 \times 60 + 840 \times 100}{2160 + 1320 + 840} \qquad \bar{y} = \frac{2160 \times 36 + 1320 \times 11 + 840 \times 21}{2160 + 1320 + 840}$$

$$\bar{x} = 45.27 \text{ mm}$$

$$\bar{y} = 25.44 \text{ mm}$$

$$I_{xx} = \left[\frac{30 \times 72^3}{12} + 2160 \times (45.27 - 15)^2\right] + \left[\frac{60 \times 22^3}{12} + 1320 \times (60 - 45.27)^2\right]$$

$$+ \left[\frac{20 \times 42^3}{12} + 840 \times (100 - 45.27)^2\right]$$

$$= 2.91 \times 10^6 + 392.8 \times 10^3 + 2.63 \times 10^6$$

$$\boxed{I_{xx} = 5.92 \times 10^6 \text{ mm}^4}$$

$$\therefore K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{5.92 \times 10^6}{2160 + 1320 + 840}} = 37 \text{ mm.}$$

$$I_{yy} = \left[\frac{72 \times 30^3}{12} + 2160 \times (36 - 25.42)^2\right] + \left[\frac{22 \times 60^3}{12} + 1320 \times (25.4 - 11)^2\right]$$

$$+ \frac{42 \times 20^3}{12} + 840 \times (25.4 - 21)^2\Big]$$

$$= 403.78 \times 10^3 + 669.71 \times 10^3 + 44.26 \times 10^3$$

$$I_{yy} = 1.11 \times 10^6 \text{ mm}^4$$

$$\therefore I_{zz} = I_{xx} + I_{yy} = 5.92 \times 10^6 + 1.11 \times 10^6 = 7.03 \times 10^6 \text{ mm}^4$$

$$\therefore K_{zz} = \sqrt{\frac{I_{zz}}{A}} = \sqrt{\frac{7.03 \times 10^6}{2160 + 1320 + 840}} = \underline{\underline{40.33 \text{ mm}}}$$

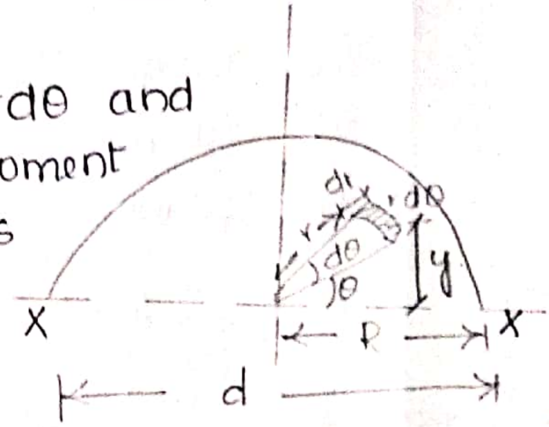
8.

a.

8.

a. Moment of inertia of semicircle wrt diametrical axis –

Consider an element of sides $r d\theta$ and dr as shown in the figure. Its moment of inertia about diametrical axis x-x is



$$= y^2 dA$$
$$= (r\sin\theta)^2 \, r \, d\theta \, dr$$
$$= r^3 \sin^2\theta \, d\theta \, dr$$

∴ Moment of inertia of semi circle about x-x is

$$I_{xx} = \int_0^R \int_0^\pi r^3 \sin^2\theta \, d\theta \, dr$$
$$= \int_0^R \int_0^\pi r^3 \left(\frac{1-\cos 2\theta}{2}\right) d\theta \, dr$$
$$= \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi dr$$
$$= \left(\frac{r^4}{\cancel{4}}\right)_0^R (\pi - 0) = \frac{\pi R^4}{8}$$

ie
$$\boxed{I_{xx} = \frac{\pi R^4}{8} = \frac{\pi d^4}{128}}$$

Moment of inertia about centroidal axis X-X =

$$y_c = \frac{4R}{3\pi} = \frac{2d}{3\pi}$$

Area $A = \frac{1}{2} \times \frac{\pi d^2}{4} = \frac{\pi d^2}{8}$.

From parallel axis theorem,

$$I_{AB} = I_{xx} + A y_c^2$$
$$\frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2$$
$$\therefore I_{xx} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi}$$
$$\boxed{I_{xx} = 0.0068 \, d^4 = 0.11 R^4}$$

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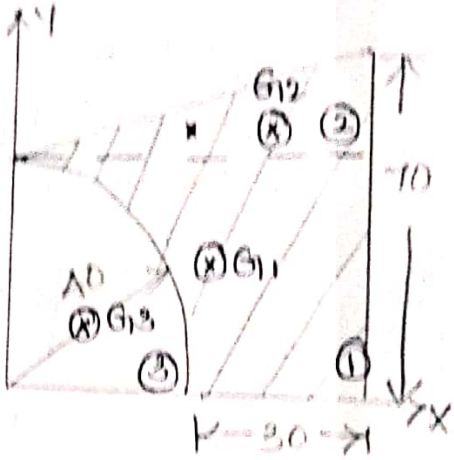
$$\boxed{I_{yy} = \frac{\pi d^4}{128}}$$

b.

$A_1 = 70 \times 40 = 2800\ mm^2$

$x_1 = 35\ mm$

$y_1 = 20\ mm$



$A_2 = \frac{1}{2} \times 70 \times 30 = 1050\ mm^2$

$x_2 = 2/3 \times 70 = 46.67\ mm$

$y_2 = 1/3 \times 30 + 40 = 50\ mm$

$A_3 = -\frac{1}{4} \times \pi \times 40^2 = -1256.63\ mm^2$

$x_3 = 4 \times 40/3\pi = 16.97\ mm$

$y_3 = 4 \times 40/3\pi = 16.97\ mm.$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} \qquad \bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{2800 \times 35 + 1050 \times 46.67 - 1256.63 \times 16.97}{2800 + 1050 - 1256.63} \qquad \bar{y} = \frac{2800 \times 20 + 1050 \times 50 - 1256.67 \times 16.97}{2800 + 1050 - 1256.63}$$

$$\boxed{\bar{x} = 48.46\ mm} \qquad \boxed{\bar{y} = 33.61\ mm}$$

9.

a. ① Displacement – The shortest distance travelled between two points is called as displacement. The SI unit of displacement is metres.

② Distance travelled – The actual distance travelled between any two points is called as distance. The SI unit of distance is metres.

(3) Velocity - The rate of change of displacement is called as velocity. SI unit is metre/second.

(4) Acceleration - The rate of change of velocity is called as acceleration. SI unit is metre/second^2.

b. Given -

$a = -2\ m/s^2$

$v = 8\ m/s$.

$t = 0$, $x = 0$.

we have $v = u + at$.

$8 = u + (-2) \times 0$.

$\therefore \boxed{u = 8\ m/s.}$

$\therefore$ Velocity $= 8\ m/s$. at $t = 0$.

$S = ?$ when $t = 6$ sec.

$V = u + at$.

$\Rightarrow 8 = u + (-2) \times 6$.

$\therefore u = 12 + 8 = 20\ m/s$.

we have $v^2 - u^2 = 2as$.

$\Rightarrow 8^2 - 20^2 = 2 \times (-2) \times s$.

$\Rightarrow \boxed{S = 84\ m}$

c. D'Alembert's principle states that " the system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body".

Application of D'Alembert's principle -

(1) Mass under gravitational force.

(2) Frictionless vertical hop with a bead.

(3) Parallel axis theorem.

10.

a) 1. Derivation of $S = ut + \frac{1}{2}at^2$.

Let a body is moving with uniform acceleration a.

$$\frac{d^2s}{dt^2} = a.$$

$$d\left(\frac{ds}{dt}\right) = a \cdot dt.$$

$$\int d\frac{ds}{dt} = \int a\,dt \quad \text{or} \quad \frac{ds}{dt} = at + c_1$$

But $\frac{ds}{dt}$ = velocity at any instant.

when $t=0$, velocity $= u$.

$\therefore$ at $t=0$, $\frac{ds}{dt} = u$.

$\therefore$ $u = a \times 0 + c_1$

$\therefore$ $c_1 = u$.

$$\therefore \frac{ds}{dt} = at + u$$

Integrating above equation,

$$S = \frac{at^2}{2} + ut + c_2.$$

where c_2 is constant of integration.

when $t=0$, $S=0$.

$$\Rightarrow \quad 0 = \frac{a}{2} \times 0 + u \times 0 + c_2 = c_2.$$

$$\therefore c_2 = 0.$$

$$\therefore \boxed{S = \frac{at^2}{2} + ut}$$

2. Derivation of $V = u + at$

We have $\frac{ds}{dt} = at + u$.

$\therefore$ $\frac{ds}{dt}$ after time $t = v$.

$$\therefore \boxed{V = at + u.}$$

3. Derivation of $v^2 = u^2 + 2as$.

$$a = \frac{v\,dv}{ds}$$

$$\frac{v\,dv}{ds} = a \quad \text{or} \quad v\,dv = a\,ds.$$

Integrating,

$$\frac{v^2}{2} = as + C_3$$

when $s=0$, velocity $= u$.

$\therefore$ At $s=0$, $v=u$.

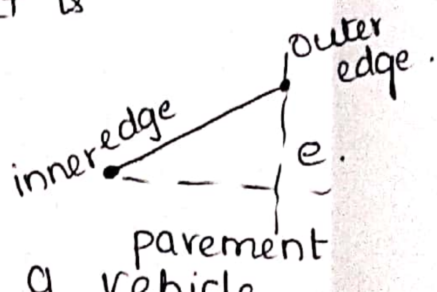
$$\therefore \frac{u^2}{2} = 0 + C_3$$

$$\therefore C_3 = \frac{u^2}{2}.$$

$$\therefore \frac{v^2}{2} = as + \frac{u^2}{2} \quad \text{or} \quad \boxed{v^2 = 2as + u^2}$$

b. Superelevation is a technique in which the outer edge of a pavement is raised comparatively with respect to the inner edge. It is denoted by 'e'.

Superelevation is provided to counteract the centrifugal force which is developed when a vehicle encounters a horizontal curve.



c.

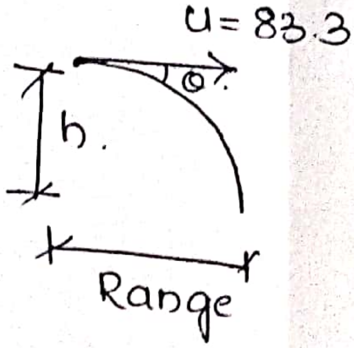
$u = 300$ km/hr

$$u = \frac{300 \times 1000}{3600}$$

$$\boxed{u = 83.3 \text{ m/s.}}$$

$h = 2.5$ km

$h = 2500$ m



We have.

$$\text{Range} = R = u \times t.$$

$$h = \frac{1}{2} \times g \times t^2.$$

$$\Rightarrow 2500 = \frac{1}{2} \times 9.81 \times t^2$$

$$\therefore t = 20.19 \text{ sec.}$$

$$\therefore R = 83.3 \times 20.19$$

$$\boxed{R = 1682.\text{ m}}$$

Direction $\theta \Rightarrow \tan\theta = \frac{2500}{1682}.$

$$\therefore \boxed{\theta = 56.06^\circ.}$$