First/Second Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Physics

Time: 3 hrs. Max. Marks: 100

Note: 1. ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. *Physical constants : $h = 6.62 \times 10^{-34}$ JS ; $C = 3 \times 10^{8}$ m/s ; $K = 1.38 \times 10^{-23}$ J/K ; $N_A = 6.02 \times 10^{26}$/K mole ; $M_e = 9.1 \times 10^{-31}$ kg ; $e = 1.6 \times 10^{-19}$ C ; $g = 9.8$ m/s ; $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $\epsilon_0 = 8.852 \times 10^{-12}$ F/m.*

Module-1

1 a. Discuss the theory of forced oscillations and obtain an expression for Amplitude resonance. **(10 Marks)**

b. Define shock waves and mention the applications of shock waves. **(06 Marks)**

c. The distance between the two pressure sensors in a shock tube is 150mm. The time taken by a shock wave to travel this distance is 0.3ms. If the velocity of second is 340m/s under the same condition, find the Mach number of the shock wave. **(04 Marks)**

OR

2 a. What is Mach Number? Classify shock waves on the basis of Mach number and mention examples for each. **(06 Marks)**

b. Derive the expression for equivalent force constant for two springs in series and parallel. What is the period of its oscillations? **(10 Marks)**

c. A 20g oscillator with natural frequency 10 rad/s is vibrating in damping medium. The damping force is proportional to the velocity of the vibrator. If the damping coefficient is 0.17, how does the oscillations decays. **(04 Marks)**

Module-2

3 a. Explain stress and strain diagram. **(06 Marks)**

b. Derive an expression for couple per unit twist of a solid cylinder. **(10 Marks)**

c. A load of 2kg produces an extension of 1mm in a wire of 3m in length and 1mm in diameter. Calculate the Young's modulus of the wire. **(04 Marks)**

OR

4 a. Show that shear strain (θ) is equivalent to half of compression strain ($\theta/2$) and half of extension strain ($\theta/2$) in two mutually perpendicular directions. **(06 Marks)**

b. Derive an expression for Young's modulus (Y) using Single Cantilever method. **(10 Marks)**

c. Calculate the torque produced in a wire of length 1.5m, radius 0.0425×10^{-2}m through an angle of ($\pi/45$) radians. If the rigidity modulus of the material is $8.3 \times 10^{+10}$ N/m². **(04 Marks)**

Module-3

5 a. By using Maxwells equations develop wave equation for electric and magnetic fields in free space. **(10 Marks)**

b. Explain with neat diagram the different types of optical fibre. **(06 Marks)**

c. An optical fibre has core RI 1.5 and RI of cladding is 1.455. Calculate numerical aperture and angle of acceptance. **(04 Marks)**

OR

6 a. Obtain the expression for Numerical Aperture and angle of acceptance and hence show the condition for propagation. (08 Marks)

b. State and prove Gauss divergence theorem. (08 Marks)

c. Find attenuation in an optical fibre of length 500m when a length of power 100mw emerges out of the fiber with a power 90mw. (04 Marks)

Module-4

7 a. State Heisenberg's uncertainty principle. Show that electron do not exists inside the nucleus using it. (08 Marks)

b. With neat diagram, explain the construction and working of CO_2 laser. (08 Marks)

c. An electron is trapped in a one – dimensional potential well of infinite height and a width of 0.2nm. Calculate the energy required for ground state and its first two excited states. (04 Marks)

OR

8 a. Derive an expression for energy density in terms of Einsteins co-efficients. (10 Marks)

b. Obtain energy eigen values for a particle in a potential well of infinite height. (06 Marks)

c. The uncertainty in the measurement of time spent by Iridium – 199 nuclei in the excited state is found to be 1.4×10^{-10} sec. Estimate the uncertainty in energy in the excited state. (04 Marks)

Module-5

9 a. Explain Hall effect. Derive an expression for Hall voltage, Hall field and Hall co-efficient. (10 Marks)

b. Define Fermi factor. Explain the variation of Fermi factor with temperature. (06 Marks)

c. The intrinsic carrier concentration of Germanium is $2.4 \times 10^{19}/m^3$. Calculate its conductivity if the mobility of the electron and holes respectively are $0.39 m^2/VS$ and $0.19 m^2/V\text{-}S$. (04 Marks)

OR

10 a. Derive Clausius – Morsotti relation in a solid dielectric. (08 Marks)

b. Explain any two failures of classical free electron theory and any two merits of quantum free electron theory. (08 Marks)

c. Calculate the concentration at which donor atoms need to be added to a silicon semiconductor, so that it results in n-type semi conductivity of 2.2×10^{-4} S/m and the mobility of electron being $1.25 \times 10^{-3} m^2/VS$. (04 Marks)

KLS VDIT, HALIYAL

ENGINEERING PHYSICS 18PHY12/22

I/II SEMESTER B.E. Aug/Sept. 2020

Question Paper Solutions

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Module-I

1 a. Discuss the theory of forced oscillations and obtain an expression for amplitude resonance (10 M)

Consider a body of mass "m" executing vibrations in a damping medium acted upon by an external periodic force F.sin(pt).
where p → is the angular frequency of the external force.

If x is the displacement of the body at any instant of time "t", then the damping force which acts in a direction opposite to the movement of the body is equated to the term $-r\left(\frac{dx}{dt}\right)$,
where r → is the damping constant, and the restoring force is equated to the term $-Kx$,
where K is the force constant.

The net force acting on the body is the resultant of all the three forces.

∴ The net force acting on the body is the resultant of all the three forces.

$$\therefore \text{Resultant forces} = -r\frac{dx}{dt} - Kx + F.\sin(pt) \quad \text{--- (1)}$$

According to Newton's 2^{nd} law of motion, the resultant force on the body.

$$\text{Resultant force} = m \cdot \frac{d^2x}{dt^2} \quad \text{——} \quad (2)$$

From equn (1) & (2), we have

$$m \cdot \frac{d^2x}{dt^2} = -r\frac{dx}{dt} - Kx + F \cdot \sin(pt)$$

$$m \cdot \frac{d^2x}{dt^2} + r\frac{dx}{dt} + Kx = F \cdot \sin(pt)$$

This is the equation of motion for forced vibrations.

On re-arranging, we get

Dividing throughout by "m" we get

$$\frac{d^2x}{dt^2} + \frac{r}{m} \cdot \frac{dx}{dt} + \frac{K}{m}x = \frac{F}{m} \cdot \sin(pt). \quad \text{——} \quad (3)$$

Let $\frac{r}{m} = 2b$ & w.k.t. $\omega^2 = \frac{K}{m}$ $\quad \omega = \sqrt{\frac{K}{m}}$

$$\therefore \frac{d^2x}{dt^2} + \frac{r}{m} \cdot \frac{dx}{dt} + \omega^2 x = \frac{F}{m} \cdot \sin(pt). \quad \text{——} \quad (4)$$

The solution of this differential equation is, $x = a \cdot \sin(pt - \alpha)$ —— (5)

where a & α represent amplitude & phase of the vibrating body.

Differentiating equn (5) w.r.t. "t", we get

$$\frac{dx}{dt} = ap \cdot \cos(pt - \alpha) \quad \text{——} \quad (6)$$

Differentiating again w.r.t. 't', we get

$$\frac{d^2x}{dt^2} = -ap^2 \sin(pt-\alpha) \quad — (7)$$

Substituting in eqn (4), we get

$$-ap^2\sin(pt-\alpha) + 2bap\cos(pt-\alpha) + \omega^2 a\sin(pt-\alpha)$$
$$= \left(\frac{F}{m}\right)\cdot\sin(pt) \quad — (8)$$

But $\frac{F}{m}\cdot\sin(pt) = \frac{F}{m}\cdot\sin[(pt-\alpha)+\alpha]$

Substituting in eqn (8) & simplifying, we get

$$[-ap^2\sin(pt-\alpha) + \omega^2 a\sin(pt-\alpha) + 2bap\cos(pt-\alpha) =$$
$$= \frac{F}{m}\sin(pt-\alpha)\cdot\cos\alpha + \frac{F}{m}\cdot\cos(pt-\alpha)\sin\alpha$$

By equating the coefficients of $\cos(pt-\alpha)$ from both sides, we get

$$-ap^2 + \omega^2 a = \frac{F}{m}\cos\alpha \quad — (9)$$
$$2bap = \frac{F}{m}\cdot\sin\alpha \quad — (10)$$

Squaring and adding eqn (9) & (10), we get

$$[a(\omega^2-p^2)]^2 + (2bap)^2 = \left(\frac{F}{m}\right)^2[\cos^2\alpha + \sin^2\alpha]$$

$$a^2[(\omega^2-p^2)^2 + 4b^2p^2] = \left(\frac{F}{m}\right)^2$$

$$a^2 = \frac{(F/m)^2}{[(\omega^2-p^2)]^2 + 4b^2p^2}$$

$$a = \frac{(F/m)}{\sqrt{4b^2p^2 + (\omega^2-p^2)^2}} \quad — (11)$$

The above equation represents the amplitude of the forced vibrations.

1 b. Define shock waves and mention the applications of shock waves. (6M)

Shock waves are the waves produced due to the sudden (release) dissipation of energy.

Shock waves are the waves in which the pressure, density and temperature changes are very large.

Example: shock waves are produced during the burst of crackers, Explosion of dynamites and bombs, volcanic eruptions etc.

Applications of shock waves:

1. Shock waves (SW) are used in the treatment of kidney stones.
2. SWs are used in the pencil industry for softening of pencil wood and dry painting.
3. SWs are used in the extraction of sandal wood.
4. SWs are used in rejuvenate (activate) dried bore wells.
5. SWs are used in needleless drug delivery.
6. SWs are used to push DNA into the cell.
7. SWs are used in wood preservation.
8. SWs are used in treatment of orthopedic diseases.
9. SWs are used to heal broken bones quickly.

1 c. The distance between the two pressure sensors in a shock ~~waves~~ tubes is 150mm. The time taken by a shock wave to travel this distance is 0.3ms. If the velocity of second is 340 m/s under the same condition, find the Mach number of the shock waves. (4 M)

Given data:

Distance between the two sensors, $d = 150\,mm$

$$d = 150 \times 10^{-3}\,m$$

Time taken to travel d is, $t = 0.3\,ms$

$$t = 0.3 \times 10^{-3}\,s$$

velocity of sound, $a = 340\,m/s$.

To find:

Mach number of the shock wave, $M = ?$

solution:

Shock speed, $U_s = \frac{d}{t} = \frac{150 \times 10^{-3}}{0.3 \times 10^{-3}} =$

$$U_s = 500\,ms^{-1}$$

Mach number, $M = \frac{U_s}{a} = \frac{500}{340} = 1.47$

∴ Mach number of the shock wave is $\underline{\underline{1.47}}$

2 a. What is Mach number? Classify shock waves on the basis of Mach number and mention examples for each. (6M)

Mach number:

In aerodynamics, the speeds of bodies moving in a fluid medium are classified into different categories on the basis of Mach number.

It is defined as "the ratio of the speed of the object to the speed of the sound in the given medium".

$$\text{Mach number} = \frac{\text{object speed}}{\text{speed of sound in the medium}}$$

It is denoted by M. Thus, if v is the object speed & the speed of sound in the medium is "a", then

$$\boxed{M = \frac{v}{a}}$$

Classification of Shock waves:

Distinction between acoustic, ultrasonic, subsonic, transonic, ~~an~~ supersonic and hypersonic waves.

An acoustic wave is simply a sound wave. It moves with a speed 333 m/s in air at STP. Sound waves have frequencies between 20 Hz to 20,000 Hz.

Amplitude of acoustic wave is very small.

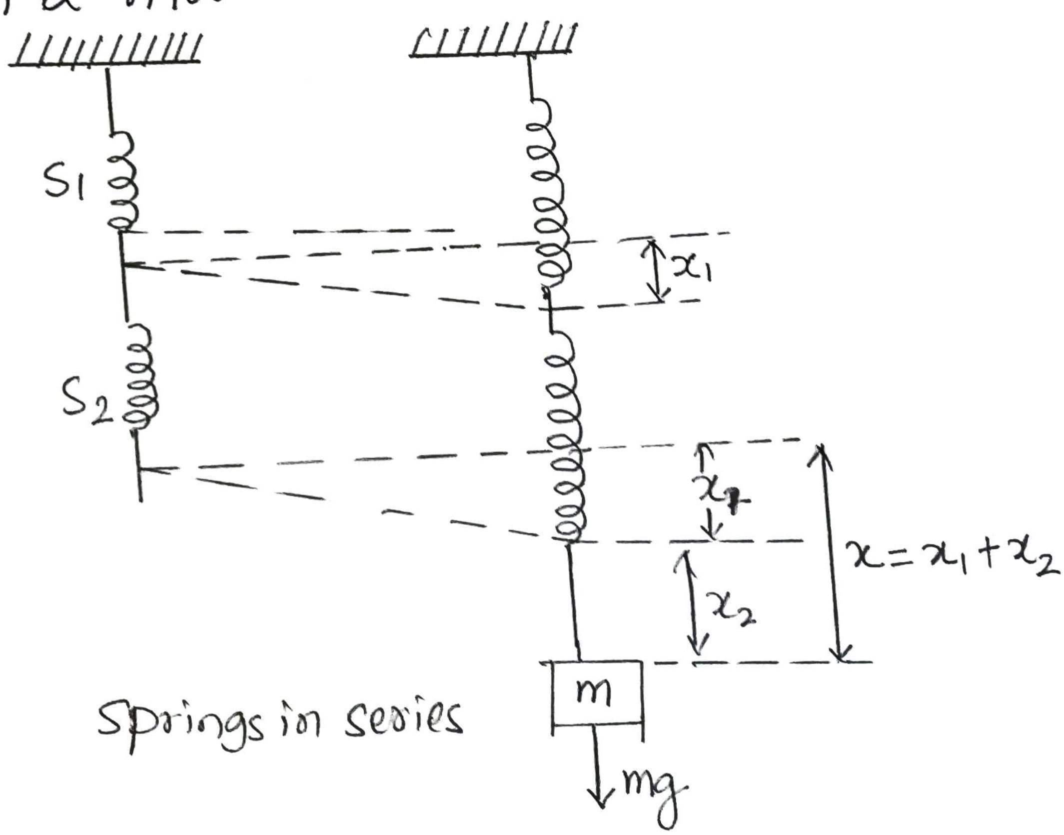
Shock wave classifications:

Name	Mach No. Range
Subsonic	$M < 1$
Transonic	$0.8 < M < 1.2$
Supersonic	$M > 1$
Hypersonic	$M > 5$

2 b. Derive the expression for equivalent force constant for two springs in series and parallel. What is the period of its oscillations. (10M)

Equivalent force constant for springs in series combination:

Consider two identical springs S_1 & S_2 with spring constants K_1 and K_2 resp. x_1 be the extensions (within elastic limit) in S_1 when a mass "m" is attached at its lower end.



Springs in series

Using Hooke's law, we have

$$F = -K_1 x_1$$

But $F = mg$ Hence $mg = -K_1 x_1$

$$\boxed{x_1 = -\frac{mg}{K_1}} \quad \text{— (1)}$$

Similarly, Let x_2 be the extension (within elastic limit) in S_2 when the same mass 'm' is attached to it.

In similarity to equn (1), we have

$$F = K_2 x_2$$

But $F = mg$ Hence $mg = -K_2 x_2$

$$\boxed{x_2 = -\frac{mg}{K_2}} \quad \text{— (2)}$$

Now, let S_1 & S_2 be suspended in series as shown in figure.

Let the load "m" be suspended now at the bottom of this series combination.

Since each of the springs S_1 & S_2 experience the same pull by the mass m, S_1 extends by x_1 and S_2 by x_2.

Thus the mass "m" comes down showing a total extension,

$$\boxed{x = x_1 + x_2} \quad \text{— (3)}$$

Let the force constant for this series combination as a whole be K_s

∴ We can write,

$$mg = -K_s x$$

$$mg = -K_s (x_1 + x_2)$$

$$x_1 + x_2 = -\frac{mg}{K_s} \quad \text{— (4)}$$

Using equⁿ 1, 2 & 4, can be written as

$$-\frac{mg}{K_1} - \frac{mg}{K_2} = -\frac{mg}{K_s}$$

Removing the common factor $-mg$ & rearranging, we have.

$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\boxed{K_s = \frac{K_1 K_2}{K_1 + K_2}}$$

If there are "n" number of springs are connected in series, then

$$\frac{1}{K_s} = \sum_{i=1}^{n} \frac{1}{K_i}$$

If a mass "m" is attached to the bottom of such a series combination of springs & set for oscillations, its period of oscillations will be

$$\boxed{T = 2\pi\sqrt{\frac{m}{K_s}}}$$

Equivalent force constant for springs in parallel combination:

Consider two identical springs S_1 & S_2 with spring constants K_1 and K_2 resp.

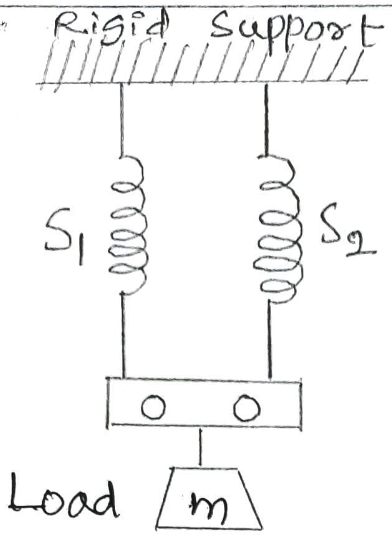
Let x_1 & x_2 be the respective extensions that the springs S_1 & S_2 would undergo individually under the pulling action of a suspended mass "m".

Hence we have,

$$mg = -K_1 x_1 \quad \text{or} \quad x_1 = -\frac{mg}{K_1} \quad \text{--- (1)}$$

and

$$mg = -K_2 x_2 \quad \text{or} \quad x_2 = -\frac{mg}{K_2} \quad \text{--- (2)}$$



Let the two springs be suspended from a rigid support parallel to each other as shown in the figure.

Their free ends are fastened to a free support to which a mass "m" is suspended. The free support descends a distance 'x' due to the mass "m".

Let the ~~supporting~~ restoring force acting on the support be F_p and the force constant for this combination be K_p.

$$\therefore \boxed{F_p = -K_p x} \longrightarrow (3)$$

The restoring force F_p is actually shared by the two springs.

Let the restoring forces in S_1 be F_1 & that in S_2 be F_2

$$\therefore F_p = F_1 + F_2 = -K_1 x_1 - K_2 x_2$$

But, Both springs undergo same extension x $\quad x_1 = x_2 = x$ (Using equ^n (1) & (2))

$$F_p = -K_1 x_1 - K_2 x$$

$$F_p = -(K_1 + K_2) x \longrightarrow (4)$$

Comparing equ^n (3) & (4), we have

$$\boxed{K_p = K_1 + K_2} \longrightarrow (5)$$

K_p is the equivalent force constant for the parallel combination.

If there are "n" number of springs connected in parallel, then,

$$K_p = K_1 + K_2 + \cdots + K_n.$$

For this combination of mass-spring system, the period of oscillation will be $T = 2\pi\sqrt{\frac{m}{K_p}}$

1 c. A 20 gm oscillator with natural angular frequency 10 rad/second is vibrating in damping medium. The damping force is proportional to the velocity of the vibrator. If the damping coefficient is 0.17, how does the oscillators decay?

Given data:

Mass of the oscillator, $m = 20\ gm = 0.2\ kg$

Natural angular frequency, $\omega = 10$ rad/second.

Damping force $\propto \frac{dx}{dt}$

damping force $= \gamma \left(\frac{dx}{dt}\right)$

where $\gamma \to$ is the damping constant coefficient.

Damping coefficient, $\gamma = 0.17$ Kg/s.

To find: To identify the case under which the decay of oscillations takes place.

Solution:

The equation of motion for damped oscillations is,

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + Kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\cdot\frac{dx}{dt} + \frac{K}{m}x = 0$$

Let $\frac{\gamma}{m} = 2b$ —— (1)

& $\frac{K}{m} = \omega^2$ —— (2)

where ω, is natural angular frequency.

W.K.T, the nature of damping is decided by comparing the value of b^2 with that of ω^2.

Now, $b = \frac{\gamma}{2m}$ (From eqn (1))

$b = \frac{0.17}{2 \times 0.02} = 4.25$ $\boxed{b^2 = 18.06}$

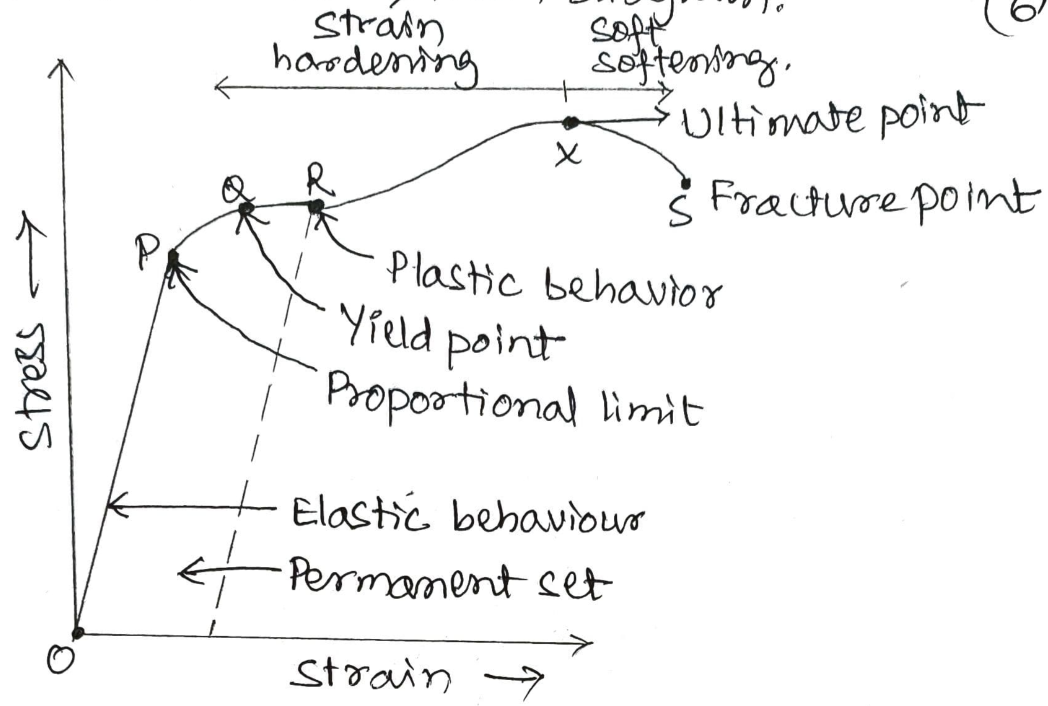
From data, $\omega = 10$ rad/s $\omega^2 = 100$

comparing values of b^2 & ω^2, we find that

$$b^2 < \omega^2$$

$\therefore$ It is the case of underdamping.

3 a. Explain stress and strain diagram. (6M)



Consider a material subjected to continuo-ouisly load/stress. If we plot a graph of stress along x-axis and strain along x-axis, we get stress strain graph OPQRS as shown in figure.

From O to P stress varies directly as strain. When load is removed the material regains the its original dimension. The point P is called elastic limitup to which Hooke's law is obeyed. If the stress is increased beyond P, the strain increases and realesed Q. Now the material is partly elastic and partly plastic. If stress is removed, the material returns to the original state along BS. The original region OS is the residual strain acquired by the material called permanent set.

The strain increases Q to R without further increase of stress. The region BC is in irregular shape. Q is called upper yield point and the stress is called yield stress.

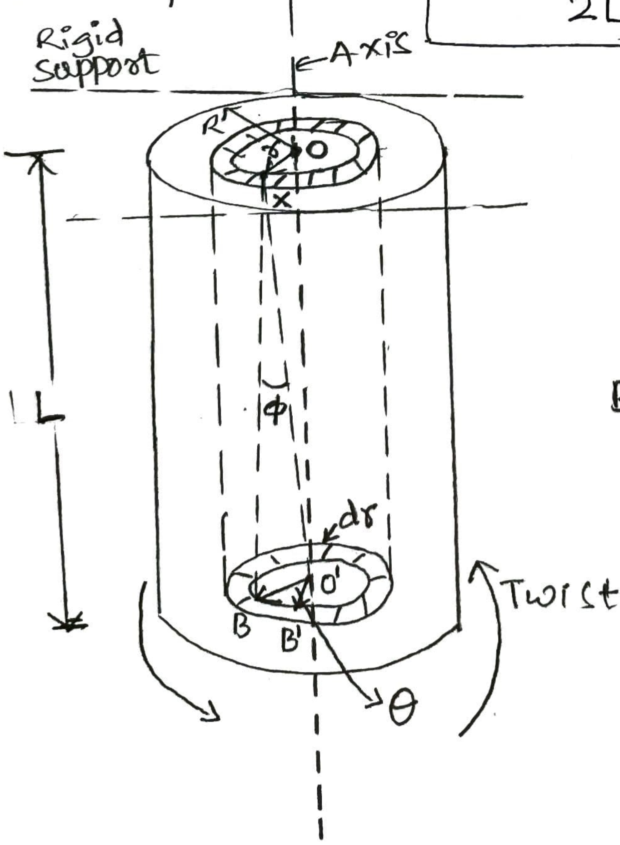
The sudden increase in strain gets stopped at C called Lower yield point.

If stress is gradually increased beyond C, the strain increases and follow the path QR. Region QR is called plastic region. In this region the thickness of the material decreases without change in volume. The point S is the maximum stress the material can with stand and the corresponding deforming force is called ultimate/tensile strength.

In the region SE strain increases without increase of stress. In this region neck is formed in the material (wire). Due to the neck the material breaks even though the stress is decreased. The stress corresponding to the point T is called breaking stress.

The area under the curve OPQRSTSO gives work done per unit volume.

3 b. Derive an expression for couple per twist of a solid cylinder. $\boxed{C = \dfrac{n\pi R^4}{2L}}$



$\angle BXB' = \phi$

$\angle BOB' = \theta$

$BO' = B'O' = r$

Consider a long cylindrical rod of length L and radius R, rigidly fixed at its upper end.

Let OO' be its axis

We can imagine the cylindrical rod to be made of thin concentric, hollow cylindrical layers each of thickness dr.

If the rod is now twisted at its lower end, then the concentric layers slide one over the other. This movement will be zero at the fixed end, and it gradually increases along the downward direction.

Let us consider one concentric circular layer of radius r and thickness dr.

A point X on the top remains fixed and, a point like B at its bottom shifts to B'.

$\angle BXB' = \phi$ is the angle of shear.

Since ϕ is also small, we have

$BB' = L\phi$.

Also, if $\angle BO'B' = \theta$, then arc length $BB' = r\theta$

$\angle BXB' = \phi$, angle of shear

$\angle BOB' = \theta$, angle of twist

$$L\phi = r\theta$$

$$\boxed{\phi = \frac{r\theta}{L}} \quad \text{——} (1)$$

Now, the cross sectional area of the layer under consideration is $2\pi r dr$.

If F is the shearing force, then the shearing stress T is given by

$$T = \frac{\text{Force}}{\text{Area}} = \frac{F}{2\pi r dr}$$

$\therefore$ Shearing force $F = T(2\pi r . dr)$ —— (2)

If ϕ is the angle through which the layer is sheared then the rigidity modulus(η),

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{T}{\phi}$$

$$\therefore \; T = \eta\phi = \frac{\eta r \theta}{L} \quad \text{[From equn (1)]}$$

After substituting for T, Equn (2) becomes.

$$F = \frac{\eta r \theta}{L} \cdot (2\pi \cdot r \cdot dr) = \frac{2\pi\eta\theta}{L} r^2 \cdot dr$$

Moment of force = F x distance

$$\therefore \text{Moment of the force about } OO' = \left(\frac{2\pi\eta\theta}{L} r^2 \cdot dr\right) r$$

(distance = r)

$$= \frac{2\pi\eta\theta}{L} \cdot r^3 dr$$

This is regarding only one layer of the cylinder.

$\therefore$ Twisting couple acting on the entire cylinder,

$$= \int_{r=0}^{r=R} \frac{2\pi\eta\theta}{L} r^3 \cdot dr$$

$$= \frac{2\pi\eta\theta}{L} \left[\frac{r^4}{4}\right]_0^R$$

$$\text{Twisting couple} = \frac{\pi\eta R^4 \theta}{2L}$$

Couple per twist C, is given by,

$$C = \frac{\text{Total twisting couple}}{\text{angle of twist}}$$

$$= \frac{\pi\eta R^4\theta / 2L}{\theta}$$

$$\boxed{C = \left(\frac{\pi\eta R^4}{2L}\right)}$$

3 c.
a. A load of 2 kg produces an extension of 1 mm in a wire of 3 m in length and 1 mm in diameter. Calculate the young's modulus of the wire

Given data:

Load $m = 2\,kg$ $\quad \Delta l = 1\,mm = 1\times10^{-3}\,m$

$L = 3\,m$

$d = 1\,mm \quad r = \frac{d}{2} = \frac{1\,mm}{2} = 0.5\,mm = 0.5\times10^{-3}\,m$

$A = \pi r^2 \quad A = \pi\times(0.5\times10^{-3})^2 = 0.7855\times10^{-6}\,m^2$

$F = mg = 2\times9.8 = 19.6\,N.$

Young's modulus $= \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/L}$

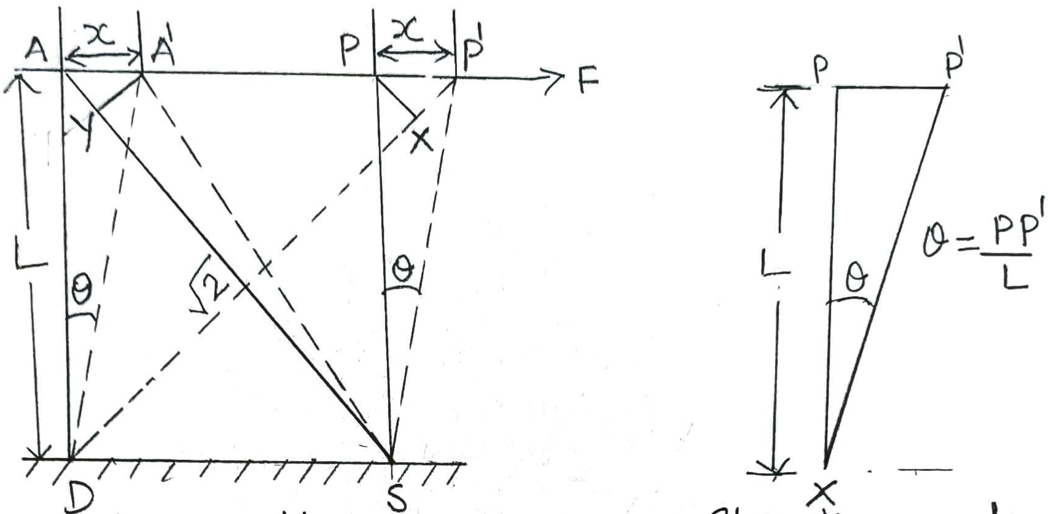
$$Y = \frac{F.L}{A.\Delta L}$$

$$= \frac{19.6\times3}{0.7855\times10^{-3}\times10^{-6}} = \frac{19.6\times3}{0.7855\times10^{-9}}$$

$$= 74.856\times10^{+9}$$

$$\boxed{Y = 7.4856\times10^{10}\,N/m}$$

4 a. Show that shear strain (θ) is equivalent to half of compression strain ($\theta/2$) and half of extension strain ($\frac{\theta}{2}$) in two mutually perpendicular directions.



Deformation

Shearing angle.

Consider a cube whose lower surface is fixed to a rigid support.

Let APSD be one of its faces with the side DS along the fixed support. When a deforming force is applied to its upper face along AP, it causes relative displacements at different parts of the cube so that, A moves to A' and P to P'.

Let "θ" be the angle of shear which is very small in magnitude. Also one can notice that the diagonal AS of the cube is now shrunk to a length A'S and that PD is stretched to a length P'D, the two taking place at right angles to each other.

If PX is drawn $\perp^{lar}$ to P'D and A'Y to AS,

then, DP = DX and A'S = YS.

So, it could be approximated that P'X is the extension in an original length PD & AY is the contraction in an original length AS.

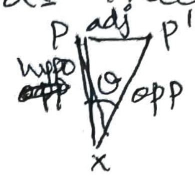
$\therefore$ Elongation strain $= \dfrac{P'X}{PD}$ —— (1)

and compression strain $= \dfrac{AY}{AS}$ —— (2)

If L is the length of each side of the ~~side~~ cube, then,

$AS = PD = \sqrt{2}\,L$ (3) (By Pythagoras' theorem) — (3)

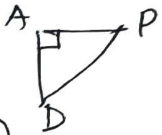
Now, $\cos\theta = \dfrac{opp}{adj} = \dfrac{P'X}{PP'}$



$P'X = PP' \cos \angle PP'X$

$= PP' \cos \angle AP'D$

In the isoscele's right angled triangle APD, $\angle APD = 45^\circ$



Since θ is very small, $\angle AP'D = \angle APD = 45^\circ$

$\left\{ \cos\theta = \dfrac{opp}{adj} \right\}$ $\therefore P'X = PP' \cdot \cos 45^\circ = \dfrac{PP'}{\sqrt{2}}$ — (4)

Substituting from equ^n (3) and (4),

Equ^n (1) becomes,

Elongation strain $= \dfrac{P'X}{PD} = \dfrac{PP'/\sqrt{2}}{\sqrt{2}L} = \dfrac{PP'}{2L}$

Elongation strain $= \dfrac{PP'}{2L} = \dfrac{\theta}{2}$ — (5)

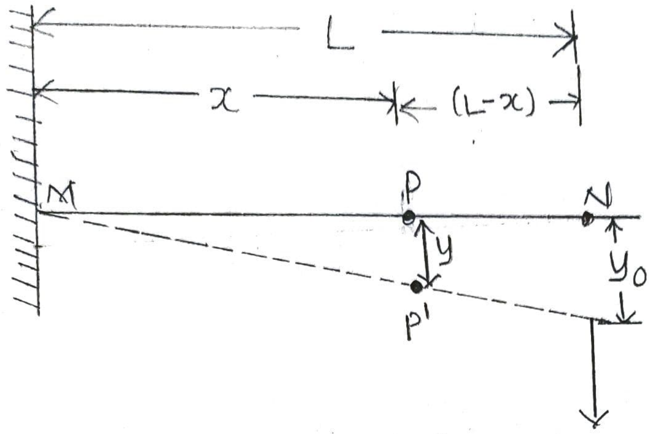
Similarly,

Compression strain observed along $AS = \dfrac{\theta}{2}$ — (6)

Adding equ^n (5) & (6)

Elongation strain + compression strain $= \dfrac{\theta}{2} + \dfrac{\theta}{2} = \theta$, the shearing strain.

4 b. Derive an expression for Young's modulus (Y) using Single cantilever method.



Consider a uniform beam of length L fixed at M. Let a load W act on ~~a~~ the load at N.
As a result, the beam bends as shown (Fig 1).

Consider a point P on the free beam at a distance x from the fixed end which will be at a distance of (L-x) from N.
Let P' be its position after the beam is bent.

∴ Bending moment = Force × perpendicular distance

$$= W(L-x)$$

But bending moment of a beam is given by

$$B.M. = \frac{Y}{R} I_g \quad \text{--- (1)}$$

$$\frac{Y}{R} I_g = W(L-x)$$

$$\frac{1}{R} = \frac{W(L-x)}{Y I_g} \quad \text{--- (2)}$$

But, if y is the depression of the point P then it can be shown that

$$\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}} \text{ appox} \quad \text{--- (3)}$$

$$\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}} \quad \text{--- (3)}$$

where, R is the radius of the circle to which the bent beam becomes a part.

Comparing, eqn (2) & (3), we have

$$\frac{d^2y}{dx^2} = \frac{W(L-x)}{Y\,I_g}.$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{W(L-x)}{Y\,I_g}.$$

$$d\left(\frac{dy}{dx}\right) = \frac{W(L-x)}{Y\,I_g}.dx = \frac{W}{Y\,I_g}(L.dx - x.dx)$$

Integrating on both sides, we get

$$\frac{dy}{dx} = \frac{W}{Y.I_g}\left[Lx - \frac{x^2}{2}\right] + C_1 \quad \text{--- (4)}$$

where $C_1 \rightarrow$ is the constant of integration.

But $\left(\frac{dy}{dx}\right)$ is the slope of the tangent drawn to the bent beam at a distance x from the fixed end.

When $x=0$, it refers to the tangent drawn at M, where it is horizontal.

Hence $\left(\frac{dy}{dx}\right) = 0$ at $\boxed{x=0}$

Introducing this condition in eqn (4), we get

$$\boxed{C_1 = 0}$$

Therefore, eqn (4) now becomes

$$\frac{dy}{dx} = \frac{W}{Y\,I_g}\left[Lx - \frac{x^2}{2}\right]$$

$$dy = \frac{W}{Y\,I_g}\left[Lx - \frac{x^2}{2}\right]dx$$

$$dy = \frac{kl}{Y I_g}\left[Lx - \frac{x^2}{2}\right]dx$$

Integrating on both sides, we get

$$y = \frac{kl}{Y \cdot I_g}\left[\frac{Lx^2}{2} - \frac{x^3}{6}\right] + C_2. \quad \text{--- (5)}$$

where, C_2 is the constant of integration and y is the depression produced at a known distance from the fixed end.

Therefore, when $\boxed{x=0}$, it refers to the depression at M, where, there is obviously no depression.

Hence, $y=0$ at $x=0$

Introducing this condition in equ^n (5), we get

$$\boxed{C_2 = 0}$$

Substituting this for C_2 in equ^n (5), we get

$$y = \frac{kl}{Y \cdot I_g}\left[\frac{Lx^2}{2} - \frac{x^3}{6}\right]$$

At the loaded end, $y = y_0$ and $\boxed{x = L}$

$$y_0 = \frac{kl}{Y \cdot I_g}\left[\frac{L^3}{2} - \frac{L^3}{6}\right]$$

Depression produced at the loaded end is,

$$\boxed{y_0 = \frac{klL^3}{3 Y I_g}} \quad \text{--- (6)}$$

∴ The Young's modulus of the material of the cantilever

$$\boxed{Y = \frac{klL^3}{3 y_0 I_g}} \quad \text{--- (7)}.$$

4 c. Calculate the torque produced in a wire of length 1.5m, radius 0.0425×10^{-2}m through an angle of $\left(\frac{\pi}{45}\right)$ radians. If the rigidity modulus of the material is $8.3 \times 10^{+10}$ N/m². (4M)

Given data: Length of the wire, $L = 1.5$m

Radius, $R = 0.0425 \times 10^{-2}$m.

Angle of twist $= \theta = \left(\frac{\pi}{45}\right)$ radians.

Rigidity modulus of the material, $\eta = 8.3 \times 10^{10}$ N/m²

To find:

Torque required to produce the twist $\tau = ?$

Solution:

We know, couple per unit twist is given by

$$C = \frac{\pi \eta R^4}{2L} = \frac{\pi \cdot 8.3 \times 10^{10} \times (0.0425 \times 10^{-2})^4}{2 \times 1.5}$$

$$C = 2.8357 \times 10^{-3}$$

But, torque required

$$\tau = C\theta$$

$$\therefore \quad \tau = 2.8357 \times 10^{-3} \times \frac{\pi}{45}$$

$$\boxed{\tau = 1.98 \times 10^{-4} \text{ N m}}$$

$\therefore$ Torque required to produce the twist $=$

$\tau = 1.98 \times 10^{-4}$ N-m.

5 a- By using Maxwell's equations develop wave equation for electric and magnetic fields in free space. (10M)

Let us consider the two curl equations of Maxwell.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

But, we know that $D = \varepsilon E$ and $B = \mu H$

$$\therefore \quad \nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (1)}$$

and $$\nabla \times \vec{H} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (2)}$$

Let us obtain the wave equation in terms of the electric field, for which we must obtain an equation relating the spatial co-ordinates of $\vec{E}$ to its time co-ordinate. This is done by eliminating $\vec{H}$ between the two equations in the following way.

Taking curl for both sides of equ (2), we have,

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left[\nabla \times \vec{H}\right] \quad \text{--- (3)}$$

As per vector analysis,

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 E$$

$$= \nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2 \vec{E} \quad \text{--- (4)}$$

[Since, as per Maxwell's equations.

$$\nabla \cdot \vec{D} = \rho \quad \text{or} \quad \nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$$
]

∴ From equ^n (3) & equ^n (4), we have,

$$\nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left[\nabla\times\vec{E}\right]$$

Using equ^n (1) in the right side, we have.

$$\nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left[\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

or $$\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\partial\vec{J}}{\partial t} + \nabla\left(\frac{\rho_v}{\varepsilon}\right) \quad \text{—— (5)}$$

The left side of the above equation is in the characteristic form of a wave equation. The solution of such an equation represents a propagating wave. The right side represents the sources which are responsible for the wave field. i.e. the charges & current. Hence equ^n (5) represents the wave equation in $\vec{E}$ for a medium with constant μ & ε. i.e. a homogeneous and isotropic medium.

If we consider free space, i.e. space where there are no charges or currents, then

$\rho_v = 0$ and $J = 0$

Equ^n (5) becomes.

$$\boxed{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{—— (6)}$$

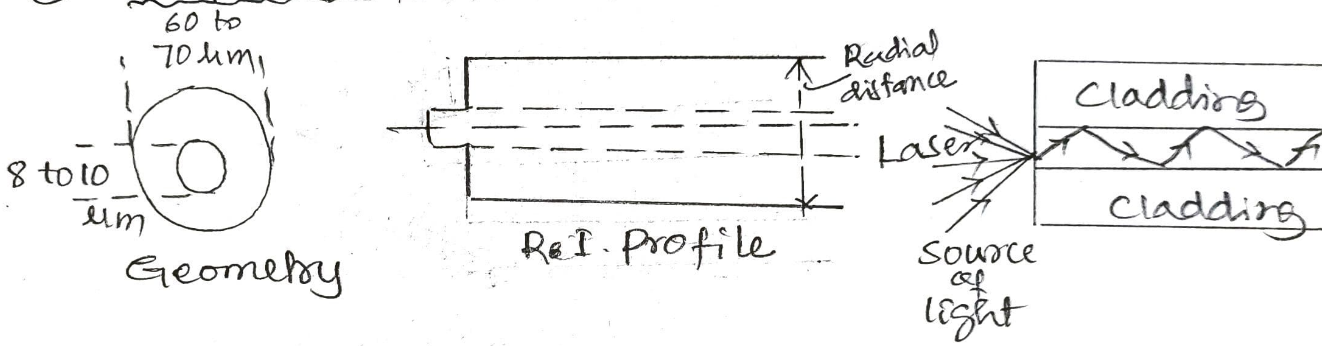
5 b. Explain with neat diagram the different types of optical fibers. (6M)

The optical fiber are classified into 3 types. namely

1. Step index single mode fiber
2. Step index multi " "
3. Graded Index " " "

This classification is done depending on the refractive index profile. and the number of modes that the fiber can guide.

① Step index single mode fiber:



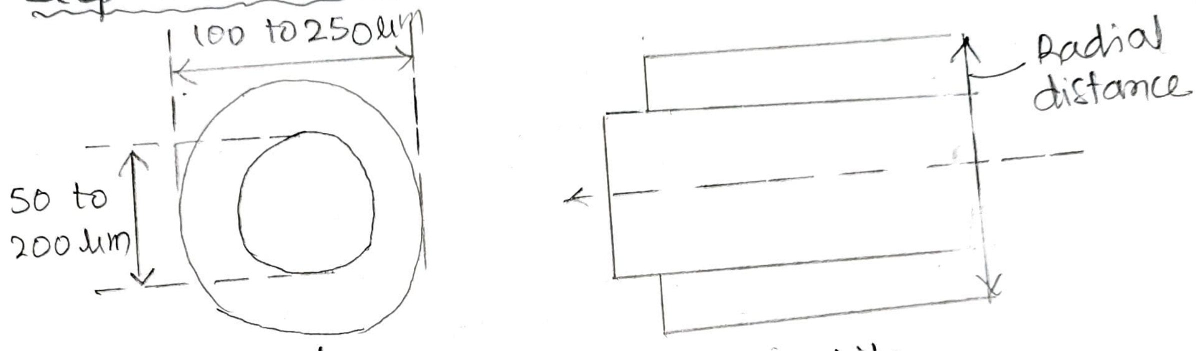
A single mode fiber has a core material of uniform R.I. value. Similarly cladding also has a material of uniform index but of lesser value. This results in a sudden increase in the value of R.I. from cladding to core. Thus its R.I. profile takes the shape of ~~the~~ a step. The ~~value~~ diameter value of the core is about 8 to 10 μm and external of cladding is 60 to 70 μm.

Because of its narrow core, it can guide just a single mode as shown in fig. Hence it is called single mode fiber.

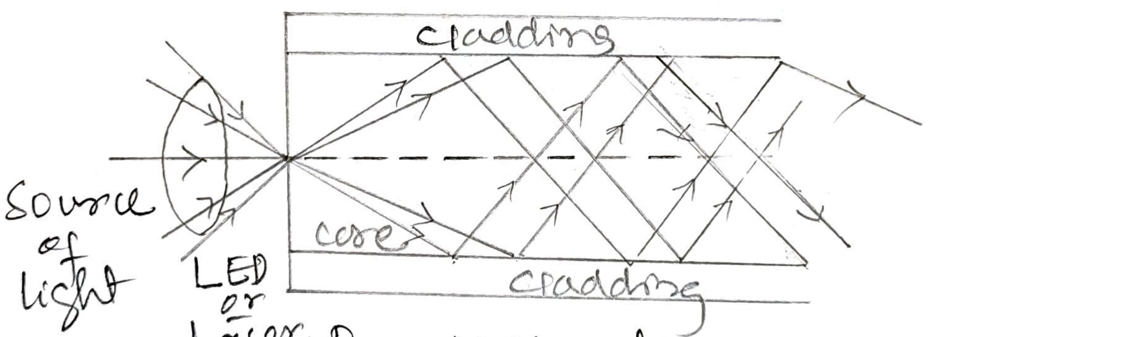
Applications:

Single mode fibers are used in long distance communication because they have higher bandwidth.
Ex: Submarine cable system.

(2) Step-Index Multimode Fiber:



Geometry R.I. profile

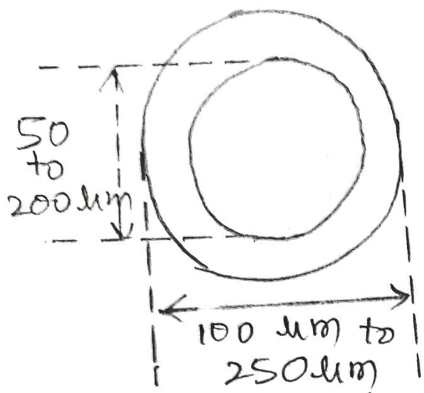


Ray propagation

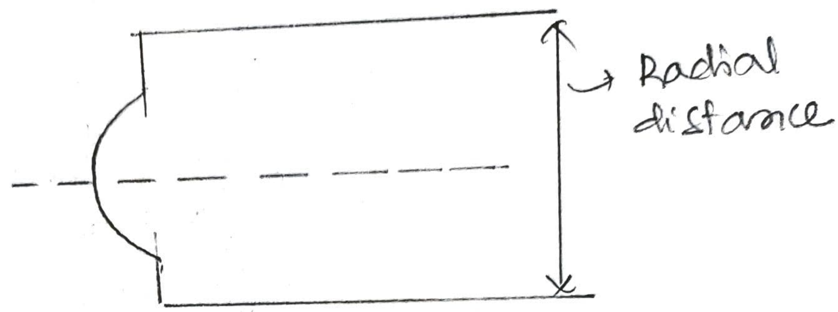
It is similar to single mode fiber, but core has large diameter. The diameter value of the core is about 50 μm to 200 μm and external diameter of cladding is 100 μm to 250 μm. But the core is comparatively larger in diameter. It can propagate large number of modes as shown in the figure. Laser or LED is used as source of light. It has an application in data links.

Applications: They are used in submarine. It is used in short distance communication due to lower bandwidth.

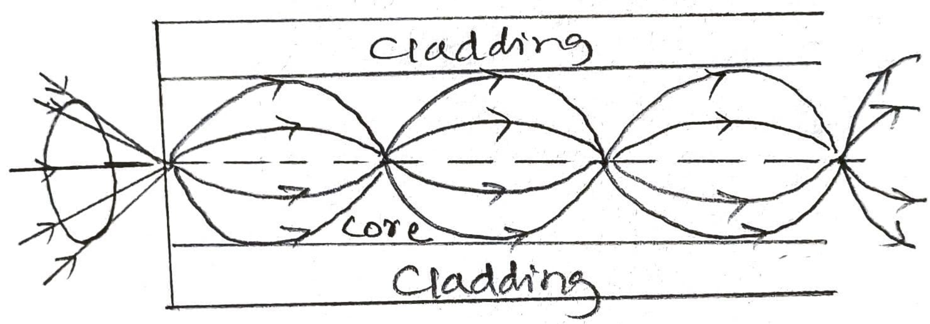
3. Graded Index Multimode Fiber: (GRIN)



Geometry



R. I. Profile



Ray propagation

It is also called GRIN. The geometry of the GRIN multimode fiber is similar to that of step index multimode fiber. Its core material has a special feature that is R.I. value decreases in the radially outwards direction from the axis and becomes equal to that of the cladding at the interface. But the R.I. of the cladding remains uniform. The refractive index profile is shown in figure.

The incident rays bends and takes a periodic path along the axis. The rays have different paths with same period. Laser or LED is used as a source of light. It is the expensive of all. It is used to telephone trunk between central offices.

Applications:

i) They are used in the telephone trunk between central offices.

(ii) It is used to short distance communication due to lower bandwidth. They transmit transformation to shorter distance.

5 c. An optical fiber has core R.I. 1.5 and R.I. of Cladding is 1.455. Calculate numerical aperture and angle of acceptance (4M)

Given Data:

R.I. of the core, $n_1 = 1.5$

R.I. of the cladding $n_2 = 1.455$ $\quad n_0 = 1$

To find:

Numerical aperture, N.A = ?

Angle of acceptance, $\theta_0 = ?$

Solution:

We have the relation for N.A. is

$$N.A = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$N.A = \sqrt{n_1^2 - n_2^2} = \sqrt{1.5^2 - 1.455^2}$$

$$\boxed{N.A = 0.24}$$

The angle of acceptance θ is related to N.A. through the equation.

$\cancel{\theta = \sin\theta}$ $\quad \sin\theta_0 = N.A.$

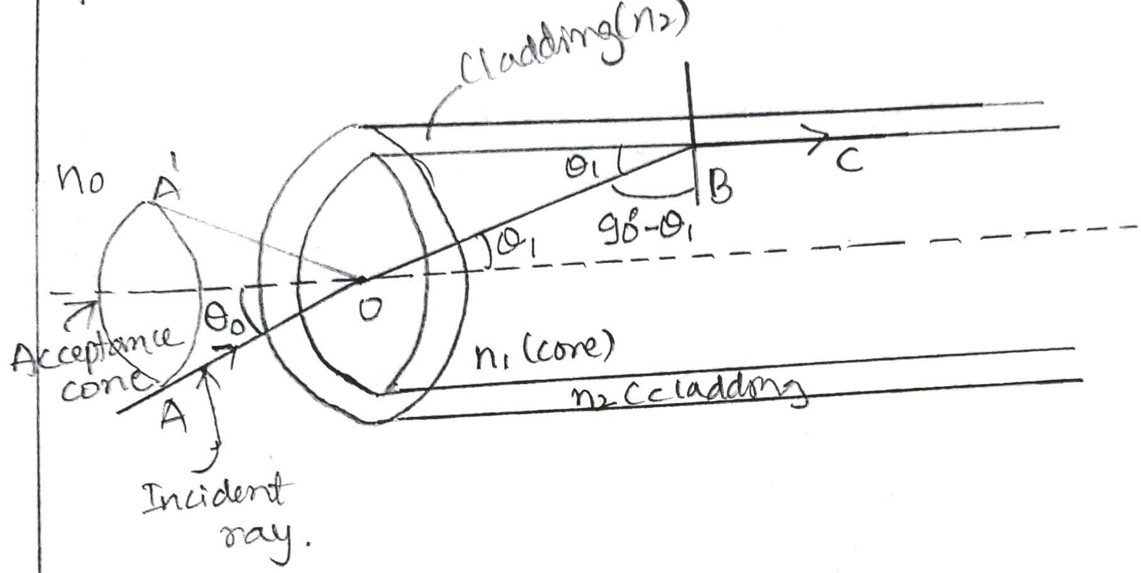
$$\theta_0 = \sin^{-1}(N.A)$$

$$= \sin^{-1}(0.244)$$

$$\boxed{\theta_0 = 14.1^\circ}$$

∴ The values of N.A. ~~a~~ = 0.244 and angle of acceptance $\theta_0 = 14.1^\circ$.

6 a. obtain the expression for Numerical aperture and angle of acceptance and hence show the condition for propagation.



Let us consider the special case of a ray which suffers critical incidence at the core cladding interface.

The ray, to begin with travels along AO, entering into the core at an angle θ_0 to the fiber axis. Then it is refracted along OB at an angle θ_1 in the core and further proceeds to fall at critical angle of incidence (equal to $90°-\theta_1$) at B on the interface between core and cladding. Since we are considering the incidence as critical angle of incidence, the ray is refracted at $90°$ to the normal drawn to the interface i.e., it grazes along BC.

Let n_0, n_1 and n_2 be the ~~interface~~ refractive indices of surrounding medium, core of the fiber, and cladding resp.

Now, for refraction at the point of entry of the ray AO into the core, we have applying the Snell's law that,

$$n_0 \cdot \sin\theta_0 = n_1 \sin\theta_1 \quad \text{—— (1)}$$

At the point B on the interface betn core & cladding.

angle of incidence $= 90^\circ - \theta_1$

angle of refraction, $\theta_2 = 90^\circ$

Again applying Snell's law, we have.

$$n_1 \sin(90^\circ - \theta_1) = n_2 \cdot \sin\theta_2$$

$$n_1 \cdot \sin(90^\circ - \theta_1) = n_2 \sin 90^\circ$$

$$n_1 \cdot \sin(90^\circ - \theta_1) = n_2 \quad \text{—— (2)}$$

$$n_1 \cdot \cos\theta_1 = n_2 \qquad \cos\theta_1 = \frac{n_2}{n_1} \quad \text{—— (2)}$$

Rewriting eqn (1), we have.

$$n_0 \cdot \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sin\theta_1 \quad \text{—— (3)}$$

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sqrt{(1 - \cos^2\theta_1)} \qquad (\because \sin\theta_1 = \sqrt{1-\cos^2\theta_1}).$$

Put the value of $\cos\theta_1$ from eqn (2), we have.

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0} \times \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

$$\boxed{\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}}$$

If the medium surrounding the fiber is air, $n_0 = 1$

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\boxed{N.A = \sqrt{n_1^2 - n_2^2}}$$

If θ_i is the angle of incidence

$\theta_i < \theta_0$

$\sin\theta_i < \sin\theta_0$

$\sin\theta_i < \sqrt{n_1^2 - n_2^2}$

This is the condition for propagation. $\sin\theta_i < N.A.$

6b. State and prove Gauss divergence theorem.

Statement:

The integral of the normal component of the flux density over any closed surface in an electric field is equal to the volume integral of the divergence of the flux throughout the space enclosed by the surface.

It is represented mathematically as,

$$\oint_S \vec{D} \cdot d\vec{s} = \int_v \nabla \cdot \vec{D} \cdot dv. \quad \text{--- (1)}$$

Proof:

Consider a gaussian surface in a region with certain charge density (Figure)

Inside the surface, consider a differential volume element ΔV.

Let ΔQ be the charge within the element

If "ρ" is the charge density, and since ρ can vary continuously in the volume, we have.

$$\rho_v = \lim_{\Delta v \to 0} \left[\frac{\Delta Q}{\Delta v}\right] = \frac{dQ}{dv} \quad \text{--- (2)}$$

$$\therefore \quad dQ = \rho_v \, dv$$

If Q is the total charge enclosed by the gaussian surface, then

$$Q = \int dQ = \int_V \rho_v \cdot dv.$$

But, we know that

$$\nabla \cdot \vec{D} = \rho \quad \text{(By maxwell's first equation).}$$

$$Q = \int_V \nabla \cdot \vec{D}\, dv \longrightarrow (3)$$

Now, by applying Gauss' law to the gaussian surface, we have

$$\oint_S \vec{D} \cdot ds = Q \longrightarrow (4)$$

$\therefore$ By equ^n (3) & (4), we have

$$\boxed{\oint_S \vec{D} \cdot d\vec{s} = \int_V \nabla \cdot \vec{D}\, dv} \longrightarrow (5)$$

This is Gauss divergence theorem or simply the divergence theorem.

6 C. Find attenuation in an optical fiber of length 500 m when a length of power 100 mW emerges out of the fiber with a power 90 mW.

Given data:

Length of the optical fiber $= L = 500\,m = 0.5\,Km$

Input power of the signal, $P_{in} = 100\,mW = 100 \times 10^{-3}\,W$

output power of the signal, $P_{out} = 90\,mW = 90 \times 10^{-3}\,W$

To find:

Fiber attenuation, $\alpha = ?$

Solution:

The fiber attenuation α is given by

$$\alpha = -\frac{10}{L} \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad dB/Km$$

$$\alpha = \frac{-10}{0.5} \cdot \log\left(\frac{90 \times 10^{-3}}{100 \times 10^{-3}}\right)$$

$$\boxed{\alpha = 0.915\ dB/Km}$$

$\therefore$ Fiber attenuation $\alpha = 0.915\ dB/Km$.

7 a. State Heisenberg's uncertainty principle. Show that electron do not exist inside the nucleus using it. (8M)

Statement :

Heisenberg's uncertainty principle states that " It is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to $\frac{h}{4\pi}$.

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi}$$

$\Delta x \rightarrow$ are the uncertainties in the measurement of position.

$\Delta P_x \rightarrow$ are the uncertainties in the measurement of momentum of the particle

Show that electrons donot present inside the nucleus using HUP.

Electron to be present in the nucleus, maximum uncertainty in position $\Delta x = 10^{-14} m$ (diameter

According to HUP,

The minimum uncertainty in momentum

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi}$$

$$\Delta P_x \geq \frac{h}{4\pi \cdot \Delta x}$$

$$\geq \frac{6.25 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}}$$

$$\Delta P_x \geq 5.275 \times 10^{-21} \text{ Kg.m/s.}$$
$$= P \text{ (say)}.$$

The minimum energy of the electron in the nucleus is given by

$$\therefore E \geq \frac{P^2}{2m}$$

$$\geq \frac{(5.275 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}}$$

$$E \geq 1.527 \times 10^{-11} \text{ J}$$

$$\geq \frac{1.527 \times 10^{-11}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E \geq 95.45 \times 10^{6} \text{ eV}$$

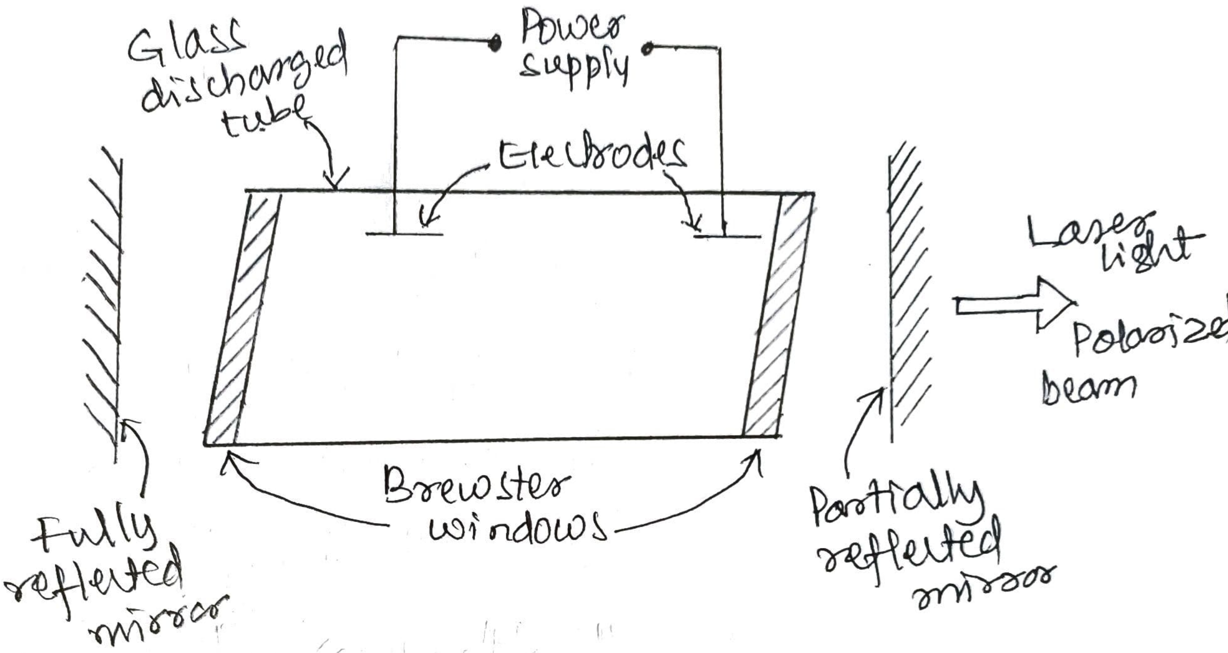
$$\geq 95.45 \text{ MeV}$$

$$\boxed{E = 95 \text{ MeV}}$$

But the maximum kinetic energy of the electrons (β-particle) emitted from the nucleus does not exceed 4 MeV, H.

Hence electrons cannot present inside the nucleus of an atom.

7 b. With neat diagram, explain the construction and working of CO_2 Laser.



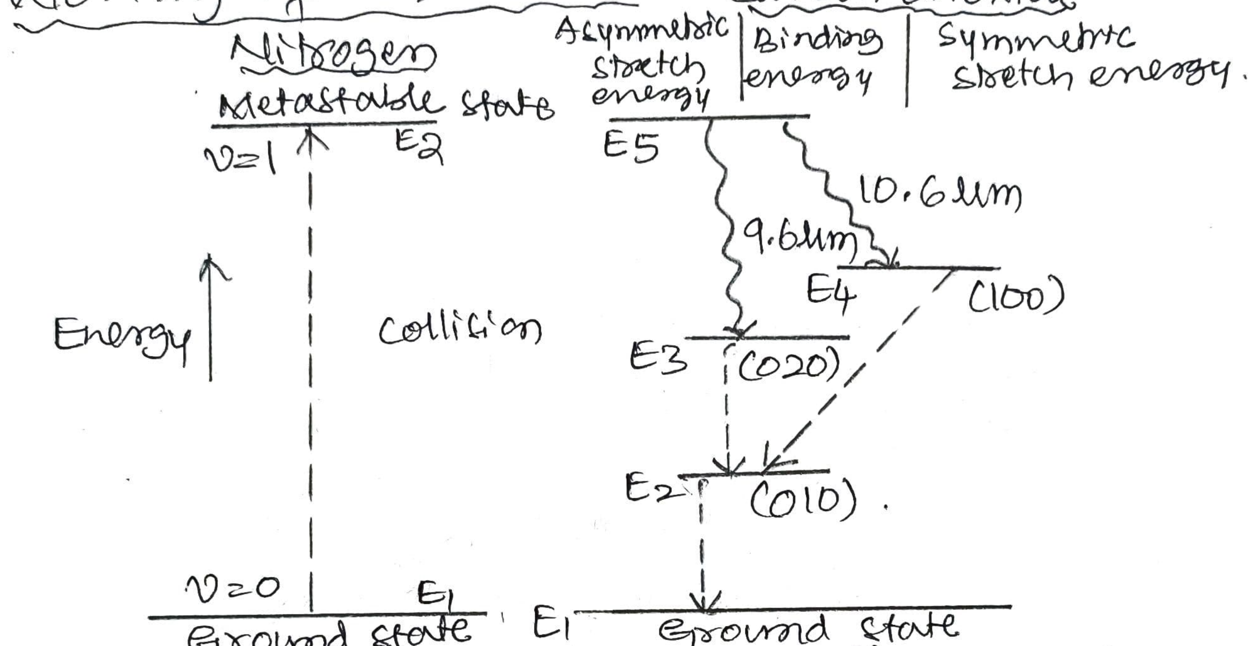
It consists of quartz tube 5m long and 2.5cm in the diameter. This discharge tube is filled with gaseous mixture of CO_2 (active medium), Helium and Nitrogen with suitable pressures.

Construction:

* The schematic diagram CO_2 Laser is as shown in the figure & this laser is invented by C.K.N Patel an Indian engineer.
* It consists of a glass discharge tube of length 5m & 2.5cm in diameter filled with a mixture of CO_2, N_2 & He in the ratio 1:2:3.
* High DC voltage can be applied to the ~~glass~~ gas between the electrodes.
* Ends of the tube is fitted with (NaCl) Brewster windows to get polarized laser beam.

* Two concave focal silicon mirrors coated with aluminium are provided at the ends of the tube which acts as a optical resonators.
* Cold water is circulated through a tube surrounding the discharge tube.

Working of CO_2 Laser:



Principle: CO_2 laser works on the principle of stimulated emission.

Working:

* CO_2 laser is a ~~four~~ five level molecular gas laser which produces continuous or pulsed laser beam.
* It works on the principle of stimulated emission between the rotational sublevels of an upper & lower vibrational levels of CO_2 molecules.
* Ionization takes place due to electric discharge when high DC voltage is applied between electrodes producing electrons.
* The accelerated $\bar{e}$s excite both N_2 & CO_2 atoms to their higher energy levels $V=1$ & C_5 from their ground states 0 & C_1 due to collision as follows:

$e_1 + N_2 \longrightarrow N_2^* + e_2$ and.

$e_1 + CO_2 \rightarrow CO_2^* + e_2$

where e_1 & e_2 are the energies of $\bar{e}$ before and after collision.

* N_2^* molecule in excited state level collide with CO_2 molecules in their ground state C_1 & excite it to metastable state C_5 by resonant energy transfer as level C_5 of CO_2 is same as level $\nu = 1$ of N_2 given by

$$N_2^* + CO_2 \longrightarrow CO_2^* + N_2$$

* ~~As~~ this process continues due to electric discharge pumping, population inversion takes place between C_5 & C_4 and C_5 & C_3.
* The transition/de-excitation takes place as follows.

$C_5 \rightarrow C_4$ producing laser 10.6 μm (IR region)

$C_5 \rightarrow C_3$ " " 9.6 μm (")

$C_4 \rightarrow C_2$

$C_3 \rightarrow C_2$ Radiation less transitions.

$C_2 \rightarrow C_1$

* Due to high thermal conductivity of He, it removes heat from mixture & de-populate the lower states C_3 & C_2 quickly.
* Laser beam is amplified by using optical resonators.
* The laser outputs is 100 KW for continuous mode & 10 KW in pulsed mode.

7 c. An electron is trapped in a 1-dimensional potential well of infinite height and a width of 0.2nm. Calculate the energy required for ground state & its first two excited state. (04M)

Given data:

width of potential well, $a = 0.2nm = 0.2\times10^{-9}m$.

To find:

Energy of ground state $E_1 = ?$ $n=1$

Energy of 1st excited state $E_2 = ?$ $n=2$

Energy of 2nd " " $E_3 = ?$ $n=3$.

$$E_n = \frac{n^2h^2}{8ma^2}$$

$h = 6.625\times10^{-34}$ J-s.

$m = 9.1\times10^{-31}$ Kg

$a = 0.2nm = 0.2\times10^{-9}m$

For ground state, $n=1$

$$E_1 = \frac{h^2}{8ma^2} =$$

$$= \frac{(6.625\times10^{-34})^2}{8\times9.1\times10^{-31}\times(0.2\times10^{-9})^2}$$

$E_1 =$ J

$E_1 =$ - eV.

Energy of 1st excited state, E_2

$E_2 = 4E_1$

$= 4\times$ eV

$E_2 =$ eV

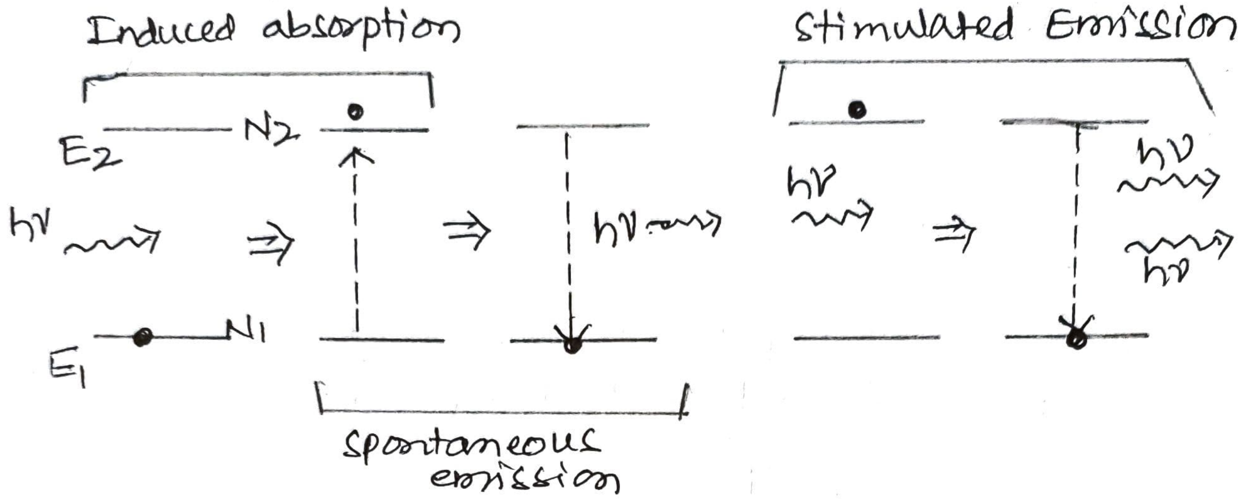
Energy of 2nd excited state, E_3

$E_3 = 9E_1$

$= 9\times$

$E_3 =$ eV

8 a. Derive an expression for energy density of radiation in terms of Einstein's coefficients.



Consider two energy states E_1 & E_2 of a system of atoms ($E_2 > E_1$). Let there be N_1 atoms with energy E_1 and N_2 atoms with energy E_2, per unit volume of the system.

N_1 & N_2 are called number density of atoms in the state 1 & 2 resp.

Let radiations with a continuous spectrum of frequencies be incident upon the system.

Let there be radiation of frequency ν, such that $\nu = \frac{(E_2 - E_1)}{h}$ & let E_ν be the energy density of radiations of frequency ν.

Then $E_\nu \cdot d\nu$ will be the energy density of radiations whose frequencies lies in the range ν and $\nu + d\nu$.

Let us now consider the absorption & the two emission process case by case.

(i) Induced absorption:

In the case of induced absorption, an atom in the level E_1 can go the level E_2 when it absorbs a radiation of frequency ν, such that

$$\nu = \left[\frac{E_2 - E_1}{h}\right] \quad \text{--- (1)}$$

The number of such absorptions per unit time per unit volume, is called rate of absorption.
The rate of absorption depends upon,
a) the no. density of lower energy state i.e. N_1 and
b) the energy density i.e. $E\nu$.

Rate of induced absorption $\propto N_1 E\nu$

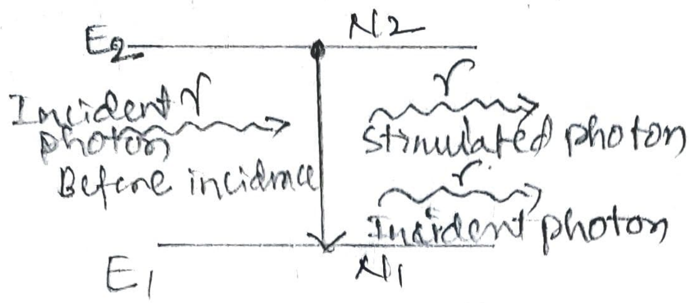
$\therefore$ Rate of induced absorption
Rate of I.A $= B_{12} N_1 E\nu$. --- (2)
where B_{12} is the constant of proportionality called Einstein's coefficients of induced absorption.

(ii) Spontaneous emission:

In case of spontaneous emission, an atom in the higher energy level E_2 undergoes transition to the lower energy level E_1 voluntarily by emitting a photon.
The number of such spontaneous emission per unit time per volume, is called rate of spontaneous emission, which is proportion to only the ~~en~~ number density in the higher energy state. i.e. N_2

$\therefore$ Rate of spontaneous emission $= A_{21} N_2$ --- (3).
Rate of spontaneous emission
where A_{21} is coeff. of spontaneous emission

(iii) Stimulated emission:



The number of stimulated emissions per unit time per unit volume is called Rate of stimulated emission.

Rate of stimulated emission $\propto N_2 E_\nu$.

Rate of stimulated emission $= B_{21} N_2 E_\nu$ —— (4)

where B_{21} is coeff. of stimulated emission.

At thermal equilibrium condition,

Rate of induced absorption = Rate of spontaneous emission + Rate of stimulated emission.

Using equn (2), (3) & (4), we have

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu \quad \text{—— (5)}$$

$$E_\nu \left[B_{12} N_1 - B_{21} N_2 \right] = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Dividing both Nr & Dr by $B_{21} N_2$, we get

$$E_\nu = \frac{A_{21} N_2 / B_{21} N_2}{B_{12} N_1 - B_{21} N_2 / B_{21} N_2}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \right] \quad \text{—— (6)}$$

According to Boltzmann's law, $\frac{N_1}{N_2} = e^{\left(\frac{E_2 - E_1}{KT}\right)} = e^{\frac{h\nu}{KT}}$ —— (7)

Equn (6) becomes.

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \cdot e^{\frac{h\nu}{KT}} - 1} \right] \quad \text{—— (7)}$$

According to Planck's law, the equation for E_ν is

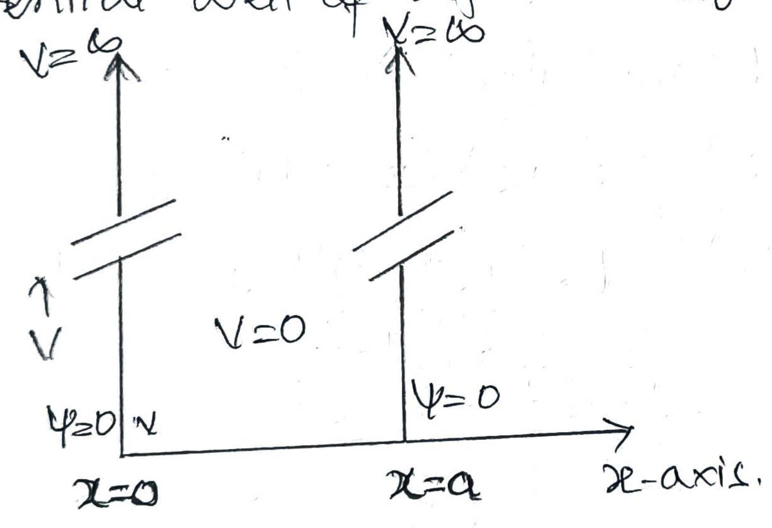
$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/KT}-1}\right] \quad \text{--- (8)}$$

Now, comparing the equ^n (7) & (8) terms by term on the basis of

$$\frac{A_{21}}{B_{21}} = \frac{8h\nu^3}{c^3} \qquad \text{and} \qquad \frac{B_{12}}{B_{21}} = 1$$

$$\boxed{B_{12} = B_{21}}$$

8 b. Obtain energy eigen value for a particle in a potential well of infinite height.



Consider a particle of mass "m" free to move in one dimension along +ve x-direction between $x=0$ to $x=a$. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state.

Such a configuration of potential in space is called infinite potential well. It is also called particle in a box.

The Schrodinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

This is Schrodinger's time independent equation for a particle.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \quad \text{—(1)} \qquad (\because V=\infty)$$

For outside, the equation hold good,
if $\psi=0$ & $|\psi|^2=0$.
That is particle cannot be found outside the well & also at the walls.
The Schrodinger's equation inside the well is,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E)\psi = 0 \quad \text{—(2)}$$

This is Eigen-value equation.
Putting $\frac{8\pi^2 m}{h^2}E = K^2$ —(3) in equ^n (2), we have

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad \text{—(4)}$$

The general solution of the quadratic equ^n (4) is of the form.
The solution of this equ^n

$$\psi = C.\cos Kx + D\sin Kx \quad \text{—(5)}.$$

where C & D are constants determined from boundary condition as follows:

$\psi(x)=0$ at $x=0$

From equ^n (5)

case (i) ~~At~~ $x=0$ at $\psi=0$.

Equ^n (5) becomes.

$0 = C \cdot \cos(0) + D \cdot \sin(0)$

$\therefore \boxed{C=0}$ — (6)

case (ii) $x=a$ at $\psi=0$

$0 = C \cdot \cos(Ka) + D \cdot \sin(Ka)$

From equ^n (6), we have $\boxed{C=0}$

$\therefore D \cdot \sin(Ka) = 0$ — (7).

$D \neq 0$ (Because the wave concept vanishes).

$\sin(Ka) = 0$

$Ka = \sin^{-1}(0)$

$Ka = n\pi$ where $n = 0, 1, 2, 3 \ldots\ldots)$

$\boxed{K = \frac{n\pi}{a}}$ — (8)

Using this equation (5), we have.

$\boxed{\psi_n = D \cdot \sin\left(\frac{n\pi}{a}\right)x}$ — (9)

which gives permitted wave functions.

8 c. The uncertainty in the measurement of time spent by Iridium - 199 nuclei in the excited state is found to be 1.4×10^{-10} sec. Estimate the uncertainty in energy in the excited state.

Given data:

Uncertainty in the life time in the excited state,

$\Delta t = 1.4 \times 10^{-10}$ sec.

To find:

The uncertainty in the energy of the nuclei in the excited state $\Delta E = ?$

Solution:

We have the equation:

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\Delta E \geq \frac{h}{4\pi \cdot \Delta t} = \frac{6.625 \times 10^{-34}}{4\pi \times 1.4 \times 10^{-10}}$$

$$= 3.77 \times 10^{-25} \text{ J}$$

$$= \frac{3.77 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\Delta E \geq 2.35 \times 10^{-6} \text{ eV}$$

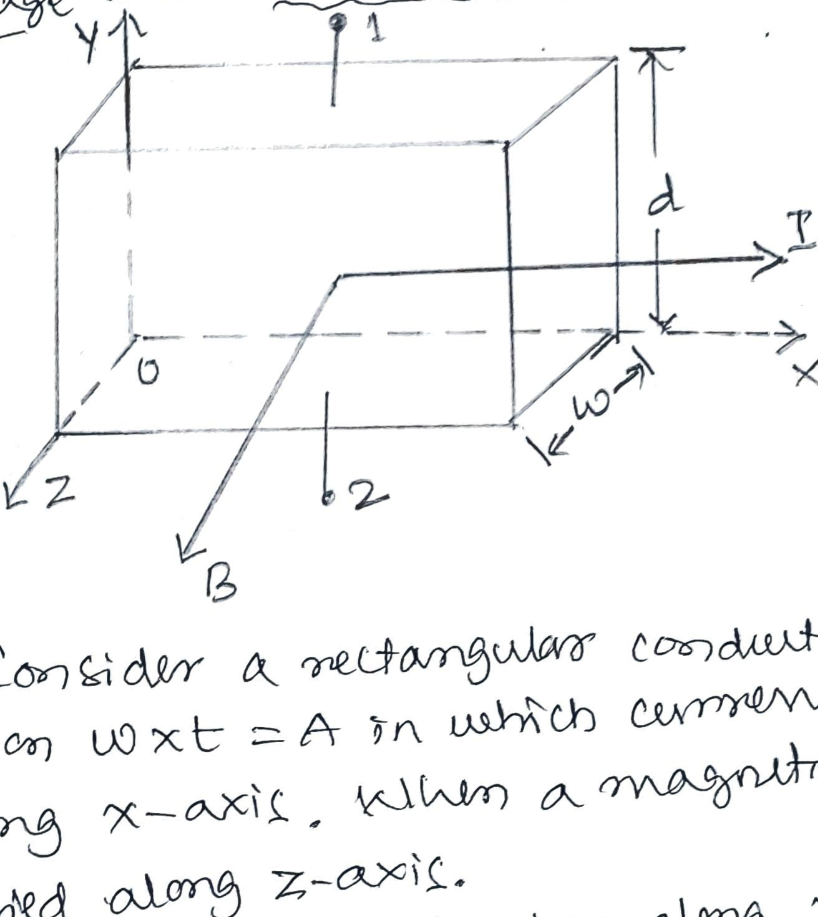
∴ The minimum uncertainty in the energy of the nuclei in the excited state is 2.35×10^{-6} eV.

9 a. Explain Hall effect. Derive an expression for Hall voltage. Hall field and Hall co-efficients.

Hall Effect:

When magnetic field is applied $\perp^{lar}$ to the direction of the current in a conductor, a potential difference develops along a direction $\perp^{lar}$ to both current and magnetic field. This effect is known as Hall effect. The potential difference developed is known as Hall voltage.

Hall Voltage and Hall coefficient:



Consider a rectangular conductor of cross section $w \times t = A$ in which current "I" flows along x-axis. When a magnetic field "B" is applied along z-axis.

Hall voltage V_H develops along y-axis between the terminals 1 & 2.

If E_H be the Hall ~~effect~~ electric field and 'v' be the drift velocity (of charges).
Then under equilibrium conditions,
Force on charges due to Hall ~~effect~~ electric field
= Force on charges due to magnetic field

$$q E_H = q v B.$$

$\therefore E_H = vB$ also $E_H = \frac{V_H}{d}$.

$\therefore \frac{V_H}{d} = vB$ or $V_H = vBd$ —(1).

Also w.r.t $I = nqAv$ or $v = \frac{I}{nqA}$ —(2).

$\therefore$ From eqn (1) & (2), we get

$$V_H = \frac{IBd}{nqA}$$

But $A = w \times d$.

$\therefore V_H = \frac{IBd}{nq \cdot w \cdot d} \Rightarrow \boxed{V_H = \frac{IB}{nqw}}$ —(3)

The quantity $\frac{I}{nq}$ is the coefficient of charge density & is called the Hall coefficient "R_H".

$$\boxed{R_H = \frac{I}{nq}} \text{ —(4)}$$

From eqn (3) & (4), we get

$V_H = R_H \cdot \frac{IBd}{A}$ $\quad \boxed{R_H = V_H \cdot \frac{A}{IBd}}$ —(5)

R_H & charge density can be determined as V_H, B, d and A are all measurable quantities.

9 b. Define Fermi factor. Explain the variation of Fermi factor with temperature.

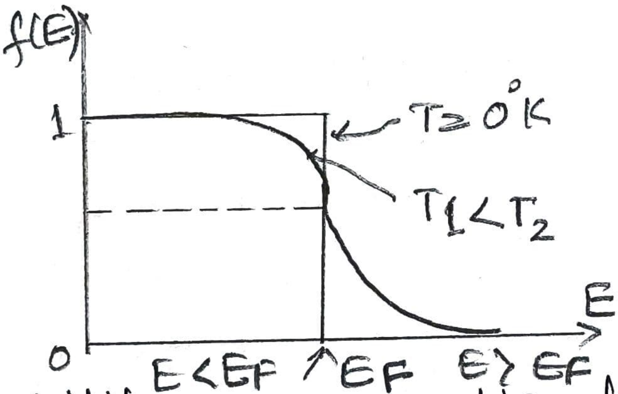
Fermi factor:

Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium.

Dependence of Fermi factor on temperature & effect on occupancy of Energy levels:

The dependence of fermi factor on temp. and the effect on occupancy of energy levels is as shown in fig.

Let us consider the different cases of distribution as follows.



(i) **Probability of occupation for $E < E_F$ at $T = 0°K$.**

When T=0 and $E < E_F$, we have for the probability.

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1} = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

$\boxed{f(E) = 1}$ for $E < E_F$

Hence, $f(E)=1$ means the energy level is certainly occupied, and $E < E_F$ applies to all the energy levels below E_F.

∴ At T=0, all the energy levels below the Fermi level are occupied.

(ii) Probability of occupation for $E > E_F$ at $T = 0°K$:

When $T = 0°K$ and $E > E_F$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1} = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0.$$

$$\boxed{f(E) = 0} \text{ for } E > E_F$$

∴ $T = 0°K$, all the energy levels above Fermi levels are unoccupied.

(iii) Probability of occupation at ordinary temp. $(T > 0°K)$ $(E = E_F)$

At ordinary temp, though the value of probability remains 1 for $E \ll E_F$, it starts decreasing from 1. as the values of E becomes closure to E_F

The value of $f(E)$ becomes $1/2$ at $E = E_F$

For $E = E_F$

$$e^{(E-E_F)/KT} = e^{0} = 1$$

$$\therefore f(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1} = \frac{1}{1+1} = \frac{1}{2}.$$

Further, for $E > E_F$, the probability value falls off to zero rapidly.

9c. The intrinsic carrier concentration of Germanium is $2.4 \times 10^{19}/m^3$. Calculate its conductivity, if the mobility of the electron & holes resp. are $0.39\, m^2/V\text{-}s$ and $0.19\, m^2/V\text{-}s$.

Given data:

Intrinsic carrier concentration for germanium $= n_i = 2.4 \times 10^{19}/m^3$

Electron mobility, $\mu_e = 0.39\, m^2 V^{-1} s^{-1}$

Hole mobility, $\mu_h = 0.19\, m^2 V^{-1} s^{-1}$

To find:

Resistivity of the sample, $\rho = ?$

Solution:

We have the relation for conductivity σ_i as,

$$\sigma_i = n_i e(\mu_e + \mu_h)$$

But, Resistivity $\rho_i = \frac{1}{\sigma_i}$

$$\rho_i = \frac{1}{n_i e(\mu_e + \mu_h)}$$

$$= \frac{1}{(2.4 \times 10^{19})(1.6 \times 10^{-19})(0.39 + 0.19)}$$

$$\rho_i = 0.449\ \Omega\text{-}m$$

$\therefore$ Resistivity of germanium is $0.449\, m$.

10 a.

Clausius Mossotti Equation:

Consider an elemental solid dielectric material of dielectric constant ε_r.

If N is the number of atoms/unit volume of the material.

$\mu \rightarrow$ is the atomic dipole moment, then we have.

Dipole moment/unit volume $= N\mu$ —— (1)

Here, the field experienced by the atoms is the internal field E_i.

Hence, if α_e is the electronic polarizability of the atoms, we can write the equation for μ as.

$$\boxed{\mu = \alpha_e E_i} \quad \text{—— (2)}$$

∴ Equn (1) becomes.

dipole moment/unit volume $= N \alpha_e E_i$ —— (3)

In equn (3), its left side is same as polarization P.

$$\boxed{P = N \alpha_e E_i} \quad \text{—— (4)}$$

$$E_i = \frac{P}{N \alpha_e} \quad \text{—— (5)}$$

But, we have the relation for P as,

$$P = \varepsilon_0 (\varepsilon_r - 1) E,$$

where E is the applied field.

$$\therefore \quad E = \frac{P}{\varepsilon_0(\varepsilon_r - 1)} \quad \text{——} (6)$$

Also we have the equation for internal field as,

$$E_i = E + \gamma \frac{P}{\varepsilon_0} \quad \text{——} (7)$$

Substituting for E_i and E from eqn (5) & (6) in eqn (7), we have,

$$\frac{P}{N\alpha_e} = \frac{P}{\varepsilon_0(\varepsilon_r - 1)} + \gamma \frac{P}{\varepsilon_0}$$

or $$\frac{1}{N\alpha_e} = \frac{1}{\varepsilon_0(\varepsilon_r - 1)} + \frac{\gamma}{\varepsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\varepsilon_0}\left[\frac{1}{(\varepsilon_r - 1)} + \gamma\right]$$

Considering the internal field in the material to be Lorentz field, we have $\gamma = \frac{1}{3}$. Substituting the same in the above eqn, we get

$$\frac{1}{N\alpha_e} = \frac{1}{\varepsilon_0}\left[\frac{1}{(\varepsilon_r - 1)} + \frac{1}{3}\right] = \frac{1}{\varepsilon_0}\left[\frac{3 + \varepsilon_r - 1}{3(\varepsilon_r - 1)}\right] = \frac{1}{\varepsilon_0}\left[\frac{\varepsilon_r + 2}{3(\varepsilon_r - 1)}\right]$$

By rearranging the above, we have

$$\boxed{\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}} \quad \text{——} (8)$$

This is called Clausius Mossotti equn & it holds good for crystals of high degree of symmetry. The equn provides a relation between ε_r, the dielectric constant which is a macroscopically measurable quantity & α_e be the electronic polarizability which is an atomic property i.e. microscopic.

10 b. Explain any two failures of Classical free electrons theory and any two merits of quantum free electron theory.

Failures of Classical Free Electron theory:

1. The Classical free electron theory predicts that the resistivity is directly proportional to square root of the absolute temperature. However, experimental evidence shows that the resistivity is directly proportional to the absolute temp.

2. According to classical theory, the free electron gas should contribute to the molar specific heat at constant volume of metal an amount equal to

$$\boxed{C_V = \frac{3}{2} R}$$

Experiment shows that the contribution of free electron to the molar specific heat at constant volume of any metal is much smaller ($C_V = 10^{-4} RT$). Also the contribution of free electron of free electron is found to be temperature dependent which is not predicted by the classical free electron theory.

Merits of Quantum free electron theory:

1. According to classical theory, which follows Maxwell's Boltzmann statistics, all the free electrons ~~gtta~~ gain energy, So it leads to much ~~energy~~ larger predicted quantities than that is actually observed. But according to quantum mechanics only one present of the free electrons can absorb the energy. So the resulting specific heat and paramagnetic succeptability values are in much better agreement with experimental values.

2. According to quantum free electron theory, both experimental and theoretical values of Lorentz number are in good agreement with each other.

3. The concept of electrical conductivity and thermal condultivity are explained by this theory.

4. The concept of photoelectric effect, compton effect and blackbody radiation are explained by this theory.

10 c. Calculate the concentration at which donor atoms need to be added to a silicon semiconductor, so that it results in n-type semi-conductivity of 2.2×10^{-4} S/m and the mobility of electron being 1.25×10^{-3} $m^2/V\text{-}s$.

Given data:

conductivity, $\sigma_e = 2.2 \times 10^{-4}$ S/m or $\mho$/m.

Mobility of electrons, $\mu_e = 1.25 \times 10^{-3}\ m^2 V^{-1} s^{-1}$

To find:

Concentration of donor impurity, $N_e = ?$

Solution:

The conductivity σ_e is given by

$$\sigma_e = N_e e \mu_e$$

$$\therefore N_e = \frac{\sigma_e}{e \mu_e}$$

$$= \frac{2.2 \times 10^{-4}}{(1.602 \times 10^{-19})(1.25 \times 10^{-3})}$$

$$\boxed{N_e = 1.1 \times 10^{16}\ m^{-3}}$$

$\therefore$ The concentration of donor impurity is $N_e = 1.1 \times 10^{16}\ m^{-3}$.

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