CBCS SCHEME

USN | | | | | | | | | | |

15ME81

Eighth Semester B.E. Degree Examination, June/July 2019
Operations Research

Time: 3 hrs. Max. Marks: 80

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. Use of statistical tables is permitted.

Module-1

1 a. List and explain briefly the phases of operations research. (06 Marks)

b. A paper manufacturing company produces two grades of papers grade 'R' and grade 'S'. Because of raw material restrictions, not more than 450 tonnes of grade R and 240 tonnes of grade S papers can be produced per week. It requires 0.2 hours to produce 1 tonne of grade R paper and 0.4 hours to produce 1 tonne of grade S paper. There are 108 production hours per week. The profit per tonne of grade R paper is Rs 400 and per tonne of grade S paper it is Rs. 500. Formulate a mathematical model to determine how many tones of grade R and grade S papers the company has to produce per week to maximize its profit. Solve graphically. (10 Marks)

OR

2 a. Discuss the limitations of operations research. (06 Marks)

b. Solve the following LPP by graphical method and indicate the solution :
Maximize $Z = 2x_1 + 3x_2$
Subject to constraints : $x_1 - 2x_2 \le 0$
$2x_1 - x_2 \ge 0$
$x_1 - x_2 \le 0$
with $x_1, x_2 \ge 0$. (10 Marks)

Module-2

3 a. What is the significance of introducing slack, surplus and artificial variables in LPP? (04 Marks)

b. Solve the following LPP by Simplex Method :
Maximize $Z = 6x_1 + 4x_2$
Subject to constraints: $-2x_1 + x_2 \le 2$
$x_1 - x_2 \le 2$
$3x_1 + 2x_2 \le 9$
with $x_1, x_2 \ge 0$. (12 Marks)

OR

4 a. Solve the following LPP by either Big–M method or two phase method :
Minimize $Z = x_1 - 2x_2 - 3x_3$
Subject to constraints : $-2x_1 + x_2 + 3x_3 = 2$
$2x_1 + 3x_2 + 4x_3 = 1$
with $x_1, x_2, x_3 \ge 0$. (08 Marks)

b. Solve the following by Dual Simplex Method :
Maximize $Z = -2x_1 - 2x_2 - 4x_3$
Subject to constrains: $2x_1 + 3x_2 + 5x_3 \ge 2$
$3x_1 + x_2 + 7x_3 \le 3$
$x_1 + 4x_2 + 6x_3 \le 5$
with $x_1, x_2, x_3 \ge 0$. (08 Marks)

Module-3

5 a. What is degeneracy in transportation problem? Discuss its consequence and how it is over come. (04 Marks)

b. Obtain the optimum solution to the following transportation problem to minimize the total transportation cost. Initial solution by Vogel's approximation method. (VAM).

Destination

		D_1	D_2	D_3	D_4	Supply
Origin	O_1	42	48	38	37	16
	O_2	40	49	52	51	15
	O_3	39	38	40	43	19
	Demand	8	9	11	16	

(12 Marks)

OR

6 a. Explain the differences between assignment problem and transportation problem. (05 Marks)

b. A small machine shop has five jobs to be assigned to five machines. The following matrix indicates the cost of assigning each of the five jobs to each of the five machines. Obtain the optimum assignment of jobs to machines, in order to minimize the total assignment cost.

Machines

		1	2	3	4	5
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

Q6(b) Cost Matrix

(11 Marks)

Module-4

7 a. Explain the Kendall and Lee's notations for representing queuing models. (04 Marks)

b. A small project consists of activities from 'A' to 'I'. The following table indicates the precedence relationship among activities and the three time estimates – optimistic, most – likely and pessimistic time for each activity in days.

Activity	Predecessor Relationship	Optimistic time 't_0'	Most likely time 't_m'	Pessimistic time 't_p'
A	–	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B, C	1	4	7
E	A	8	8	8
F	D, E	5	14	17
G	C	3	12	21
H	F, G	3	6	9
I	H	5	8	11

i) Draw the project network. Determine the expected time and variance for each activity

ii) Obtain the total expected duration of the project and critical path

iii) What is the probability of completing the project in 50 days? (12 Marks)

OR

8 a. For the following set of activities of a project, draw the network and obtain Early Start [ES], Early Finish [EF], Late Start [LS] and Late Finish [LF] for each activity. Also, indentify the critical path and project duration.

Activity	Predecessor	Duration in days
A	–	5
B	A	8
C	A	6
D	C	5
E	B, D	9

(08 Marks)

b. The mean arrival rate to a service centre is 3 per hour. The mean service time is found to be 10 minutes per service. Assuming Poisson arrival and exponential service time, find :

i) Utilization factor for the service facility
ii) Probability of two units in the system
iii) Queue length
iv) Expected waiting time in the system

(08 Marks)

Module-5

9 a. Apply the rules of dominance to reduce the game to (2 × 2) and solve the game to obtain game value and optimum strategies for both the players.

		Player B 1	2	3
Player A	1	2	–2	4
	2	–1	4	2
	3	2	1	6

(08 Marks)

b. Solve the following (2 × 4) game graphically.

		Player B 1	2	3
Player A	1	1	3	12
	2	8	6	02

(08 Marks)

OR

10 a. There are seven jobs to be processed on a single machine. The following table indicates the jobs and corresponding processing time in hours. Obtain the optimum sequence of jobs by Shortest Processing Time [SPT] rule that minimizes the mean flow time. Also obtain average in process inventory. (06 Marks)

Jobs (j)	A	B	C	D	E	F	G
Processing time (t_j) in hr	8	3	5	4	3	9	6

b. There are six jobs to be processed on three machines A, B and C in the order CAB. The following table indicates the processing time in hours for the six jobs on the three machines. Obtain optimum sequence of jobs that minimizes the total elapsed time for completing all the jobs on the three machines. Also indicate the idle time of each machine.

Jobs	1	2	3	4	5	6
Processing time in hours on M/C A	4	6	7	4	5	3
Processing time in hours on M/C B	8	10	7	8	11	8
Processing time in hours on M/C C	5	6	2	3	4	9

(10 Marks)

Sub: OPERATION RESEARCH.
(15ME81)

: **SOLUTIONS**:

Q.1. (a) List & Explain the phases of OR.

As OR can be defined as the scientific method to solve any problem where no. of deterministic & non deterministic factors are involved. Therefore the solution should have proper approach or phases which result into proper solution.

Following are the phases of OR.

1. Formulating the Problem.
2. Construction of Mathematical model.
3. Deriving a solution.
4. Testing the model.
5. Establishing the control. over the solution.
6. Implementation.

1. Formulation of Problem:

This step. no. of variables associated with the problem are identified. These variables are further distinguished as. constraint variables & non-constraint variables.

2. Construction of Mathematical Model.

In this step, set of equations relating the variables are constructed. The inter relationship like, linearity, or non-linearity among the variables are considered during the construction of the equations.

3. Deriving the Solution:

The set of equations are ~~do~~ Solved by either exact or numerical or logical method to get initial solution to the problem.

4. Testing the model.

The mathematical model is solved by substituting the initial solution, and is examined for the optimality of the ~~so~~ problem.

5. Establishing the control:

In this step, the solution to the variables are re-examined for the required optimality by applying excact or numerical methods.

6. Implementation:

At this step. a detailed solution is derived and released for the further action of the process.

Q2 (b) Given.

Raw Matl. restrictions.

For Matl. R - 450 T.

Matl. S - 240 T.

Production time

For R - 0.2 Hours/T.

S - 0.4 Hours/T.

Max. prod. hours available = 108 hours/Week

Considering the profit,

For R = Rs 400/T.

S - Rs 500/T.

No. of paper R type in tonne/week = ?

No. of paper S type in " " = ?

Let x_1 be the R type of papers in tonne per week & x_2 be the not S type of papers in tonne per week.

(a) Mathematical Model.

Considering the profit.

$Z = 400x_1 + 500x_2$

Considering Prod. time availability

$0.2x_1 + 0.4x_2 \leq 108.$

Considering raw matl. availiblity

$x_1 \leq 450.$

$x_2 \leq 240.$

∴ The mathematical model for the manufacturing of papers. are

Objective function.

$\max(Z) = 400x_1 + 500x_2$

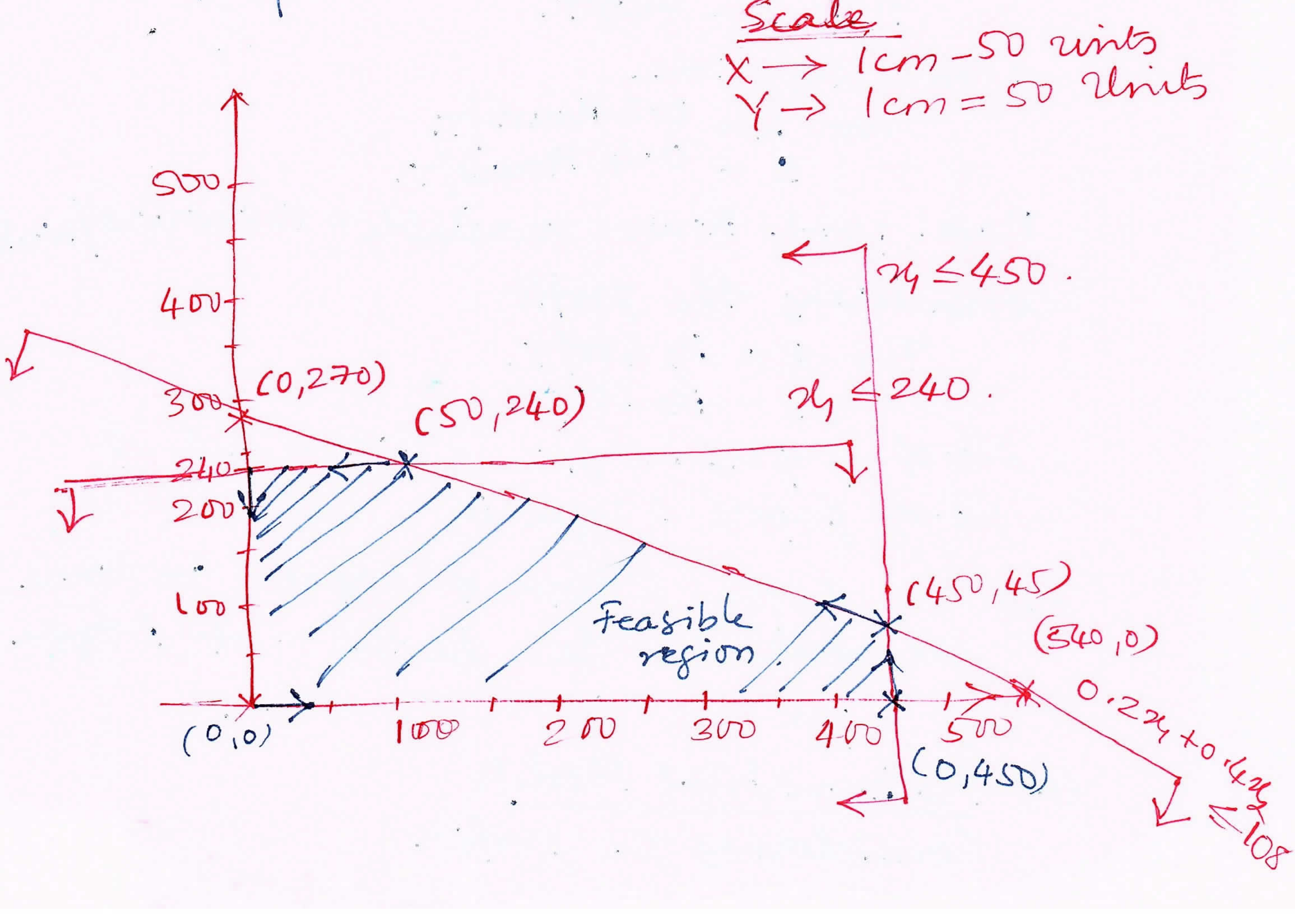
Subjected to,

$x_1 \leq 450$

$x_2 \leq 240,$

& $x_1, x_2 \geq 0.$

Graphical Solution to the Problem.



Considering the constraint-equations.

(1) $0.2x_1 + 0.4x_2 = 108$.

when $x_1 = 0$. $x_2 = 270$. $\Rightarrow (0, 270)$

$x_2 = 0$. $x_1 = 540$. $(540, 0)$.

(2) $x_1 = 450$.

(3) $x_2 = 240$.

From the Graph,

The boundries of feasible region are,

$(0,0)$, $(450,0)$ $(450,45)$, $(50,240)$ & $(0,240)$.

$\therefore Z_1 = 0$.

$Z_2 = 400 \times 450 + 500 \times 0 = 1,80,000/-$

$Z_3 = 400 \times 450 + 500 \times 45 = 2,02,500/-$

$Z_4 = 400 \times 50 + 500 \times 240 = 1,40,000/-$

$Z_5 = 400 \times 0 + 500 \times 240 = 1,20,000/-$

$\therefore$ Max $(Z) = 2,02,500/-$

$x_1 = 450$, $x_2 = 45$.

$\therefore$ Solution to the Problem:

R type of paper = 450 tonne/week

S type of paper = 45 tonne/week

Profit = 2,02,500/-

Q2(a) Limitations of Operation Research.

1. The mathematical models will not consider qualitative factors.
2. Mathematical models can be applied to specific problems. and are not universal.

3. OR will fail is establishing relations with variables considering wide factors.
4. Difficult to elaborate the steps.
5. Numerical steps may not convince the decision makers.
6. Difficulty in relativeg men & machine issues.

Q 2(b)) Graphical Solution to LPP.

$\max(Z) = 2x_1 + 3x_2$.

Constraints

$x_1 - 2x_2 \leq 0$.

$2x_1 - x_2 \geq 0$

$x_1 - x_2 \leq 0$

$x_1, x_2 \geq 0$.

(1) As Formulation is already available, let us find out extreme co-ordinate, for the constraint equations.

(1) $x_1 - 2x_2 = 0$.

when $x_1 = 0$, $x_2 = 0.5$. $\Rightarrow (0, 0.5)$.

$x_2 = 0$. $x_1 = 0$. $\Rightarrow (0, 0)$.

$x_1 = 1$, $x_2 = 0.5$ $\Rightarrow (1, 0.5)$

$x_2 = 1$ $x_1 = 2$ $\Rightarrow (2, 1)$.

(2) $2x_1 - x_2 = 0$.

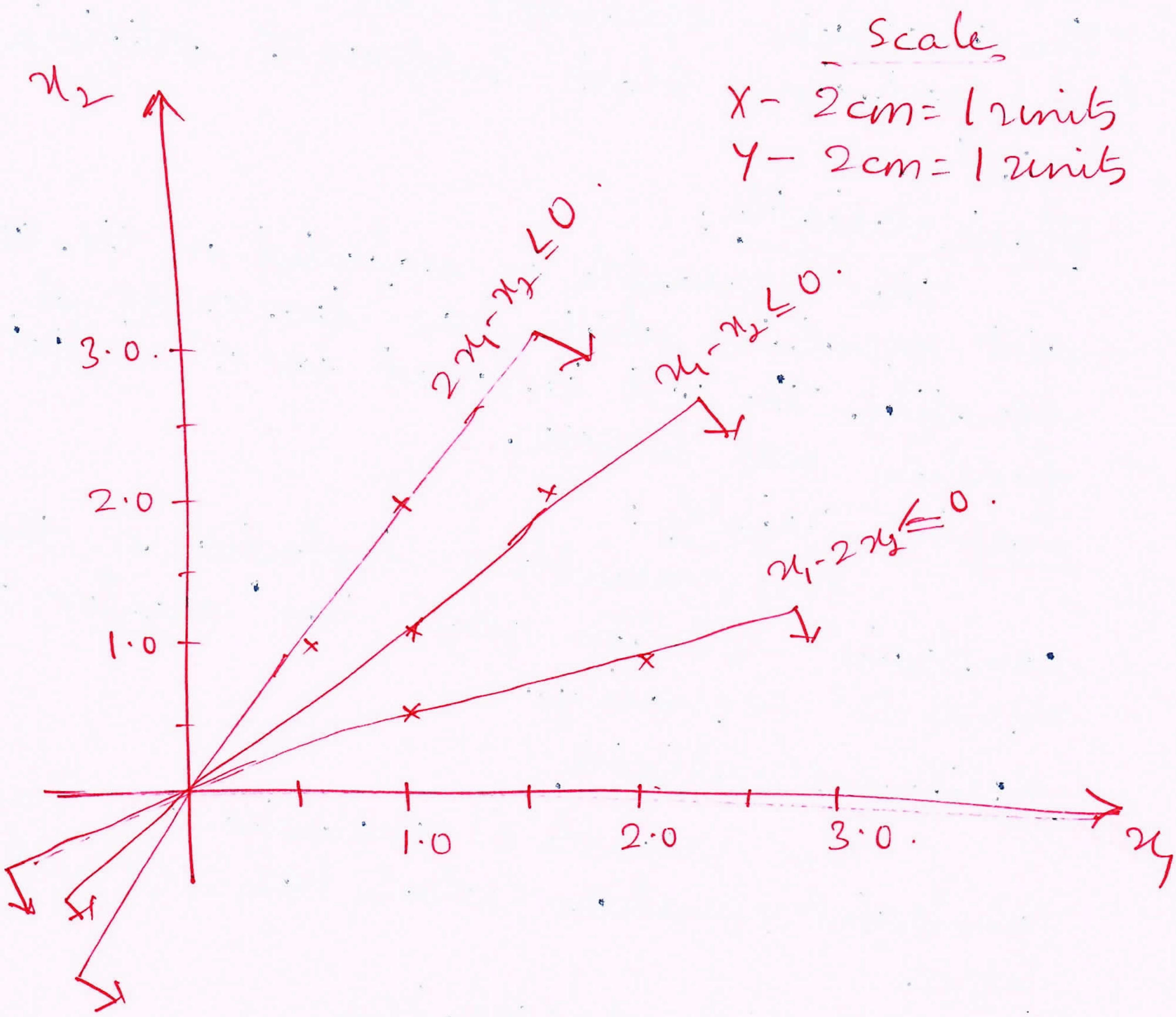
when $x_1 = 1$, $x_2 = 2$. $\Rightarrow (1, 2)$

$x_2 = 1$ $x_1 = 0.5$ $\Rightarrow (0.5, 1)$.

(3) $x_1 - x_2 = 0$. $\Rightarrow x_1 = x_2$

$x_1 = 1$, $x_2 = 1$. $\Rightarrow (1, 1)$

$x_2 = 2$, $x_1 = 2$ $\Rightarrow (2, 2)$.



From above Graph, it is clear that, feasibility region cannot be bound by the constraint equations.

$\therefore$ The problem is unbound solution.

Q.3 (a). Simplex method:

This method is used to solve linear programming problems with 'n' no. of variables. During the formulation, we get constraint equations which will have inequality.

The solution to objective function can be achieved only if these constraint equations are converted into equality

The constraint equations can be made equal by introducing slack, surplus & artificial variables.

<u>Slack variable</u>:

This variable is included in the constr-aint equation which has "less or equal to" equality. The cost involved in the objective function will be zero.

<u>Surplus Variable</u>:

This variable is included in the constraint equation which has "greater or equal to" equality.

<u>Artificial variable</u>

This variable is included in the constraint equation which has equality sign.

<u>Significance of Variables</u>

1. These variables place very important role in getting optimal solution as the cost involved is zero.
2. These variables are taken as basic variables which will be further elimi-nated from the solution.
3. The allocations to these variables will not alter the basic theme of the solution.

Q 3 (b).

$max(Z) = 6x_1 + 4x_2$

Subjected to,

$-2x_1 + x_2 \leq 2$

$x_1 - x_2 \leq 2$

$3x_1 + 2x_2 \leq 0$

$x_1, x_2 \geq 0$..

Solution:

There are three constraint equations, let us introduce slack variables.

$\therefore$

$-2x_1 + x_2 + S_1 = 2$

$x_1 - x_2 + S_2 = 2$

$3x_1 + 2x_2 + S_3 = 0$.

$\therefore$ objective function is

$max(Z) = 6x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$.

Step 2. Initial basic feasible solution.

	C_j	6	4	0	0	0	b.	b_j
Br.	C_B	x_1	x_2	S_1	S_2	S_3		
S_1	0	-2	1	1	0	0	2	-1
S_2	0	1	-1	0	1	0	2	2
S_3	0	(3)	2	0	0	1	0	0 ← leaving variable
Z_f		0	0	0	0	0		
$C_j - Z_f$		6	4	0	0	0		
		↑ Entering Variable						

Step 3:

→ Basic Element = (3)

Factor Ratio = 3.

Step 3. Iteration towards optimal solution.

	C_j	6	4	0	0	0	
B.V.	C_B	x_1	x_2	S_1	S_2	S_3	b
S_1	0	-3	$1/3$	1	0	$-1/3$	2
S_2	0	0	$-5/3$	0	1	$-1/3$	2
x_1	6	1	$2/3$	0	0	$1/3$	0
Z_j		6	4	0	0	2	
$C_j - Z_j$		0	0	0	0	-2	

Since all values of $(C_j - Z_j)$ are 0 & negative the basic feasible solution itself is optimal.

i.e. $x_1 = 0$, $S_1 = 2$, $S_3 = 2$.

$\therefore \max(Z) = 6 \times 0 + 4 \cdot (0) + 0 \cdot 2 + 0 \times 2$.
$= 0$.

$\therefore$ optimal value of $Z = 0$.

Q4. Given,

$$\min(Z) = x_1 - 2x_2 - 3x_3$$

Subjected to,

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

Solution,

Adding artificial variables.

$$-2x_1 + x_2 + 3x_3 + A_1 = 2$$
$$2x_1 + 3x_2 + 4x_3 + A_2 = 1$$

$$\min(Z) = x_1 - 2x_2 - 3x_3 + MA_1 + MA_2 =$$

	C_j	1	-2	-3	M	M		
C_B	B.V.	x_1	x_2	x_3	A_1	A_2	b_i	b_j
M	A_1	-2	1	3	1	0	2	-1
M	A_2	(2)	3	4	0	1	1	$1/2 \leftarrow$
Z_j		0 ↑	4M	7M	M	M		

Step 2: Iteration towards optimality

C_B	B.V.	x_1	x_2	x_3	A_1	A_2	b_j
M	A_1	0	4	7	1	1	3
1	x_1	1	$3/2$	2	0	$1/2$	$1/2$
	Z_j	1	$(4M+\frac{3}{2})$	$(7M+2)$	M ↑	$(M-1/2)$	

The problem should be proceeded to eliminate A_1 from the basic variable.

4 (b). Dual Simplex Method:

$\max(Z) = -2x_1 - 2x_2 - 4x_3$

Subjected to,

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

Solution

Since given ob. function is max. Keeping as it is,

$\max(Z) = -2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3$

Converting $\geq$ constraint to $\leq$ by multiplying with -ve sign and introducing slack variable we get

$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$

$3x_1 + x_2 + 7x_3 + S_2 = 3$

$x_1 + 4x_2 + 6x_3 + S_3 = 5$

$x_1, x_2, x_3 \geq 0, \quad S_1 S_2 S_3 \geq 0.$

Step 2. Construction of matrix:

	C_j	-2	-2	-4	0	0	0		
C_B	BV	x_1	x_2	x_3	S_1	S_2	S_3	b_i	
0	S_1	-2	(-3)	-5	1	0	0	-2	$\leftarrow 2/3$
0	S_2	3	1	7	0	1	0	3	3
0	S_3	1	4	6	0	0	1	5	$5/4$
$Z_f = C_B \cdot x_i =$		0	0	0	0	0	0		
$(C_j - Z_f)$		-2	-2	-4	0	0	0		
			$\uparrow$						

Step 3.

		-2	-2	-4	0	0	0	
C_B	BV	x_1	x_2	x_3	S_1	S_2	S_3	b
-2	x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$
0	S_2	$1/3$	0	$16/3$	$4/3$	1	0	$7/3$
0	S_3	$-5/3$	0	$-2/3$	$+4/3$	0	1	$7/3$
	Z_f	$-4/3$	-2	$-10/3$	$2/3$	0	0	
$(C_j - Z_f)$		$-2/3$	0	$-2/3$	$-2/3$	0	0	

Since b_i values are +ve
all $(C_j - Z_f)$ values are -ve
the above solution is optimal.

$\therefore$ optimal solution.

$x_2 = 2/3, \quad x_1 = 0, \quad x_3 = 0,$

$S_1 = 0, \quad S_2 = 7/3 \quad S_3 = 7/3.$

$\max(Z_f) = -2(0) - 2(2/3) + (-4)0 + 0$

$= \underline{\underline{-4/3}}.$

Q5 (a). <u>Degeneracy:</u>

Every problem in operation research are checked for the optimality.

In transportation model, after finding basic feasible solution by any one method, optimality test is applied.

To get optimal solution by MODI method, the no. of allocated cells should be equal to $(m+n-1)$. where m = no. of rows & n = no. of columns.

If No. of allocated cells are not equal to $(m+n-1)$. then the problem leads to degeneracy.

<u>Solution to degeneracy:</u>

<u>step 1</u>. An infinitesmally small unit 'ϵ' is allocated to a cell of min. of unallocated cell.

<u>Step 2</u>: In allocating 'ϵ' units it is checked that the cell should be independent, ie it should not form a closed loop in the matrix.

<u>Step 3</u>: If tie exist between the cost one need to check the independent status of the cell.

Following the above steps one can solve the degeneracy in transportation model.

Q 5.
(b).

	D_1	D_2	D_3	D_4	Supply
O_1	42	48	38	37	16
O_2	40	49	52	51	15
O_3	39	38	40	43	19
Demand	8	9	11	16.	50 / 44.

Soln.

Step1. the given problem is unbalanced transportation problem.

Demand > Supply.

Let us introduce Dummy destination with demand of 6 units.

	D_1	D_2	D_3	D_4	Dm.	
O_1	42	48	38	37	0	16
O_2	40	49	52	51	0	15
O_3	39	38	40	43	0	19
	8	9	11	16	6	50 / 50.

Step 2. Basic feaseable solution. by VAM. method.

42	48	38	37 [16]	0	16/0	(37) (1) (1) (1)
40 [8]	49.	52 [1]	51	0 [6]	15/8/1/0.	(40) ← (9) (11) ← (1)
39	38 [9]	40 [10]	43	0	19/10/0	(38) (1) (1) (3)
8/0	9/0.	11/1/0	16/0	6/0		
(1)	(10)	(2)	(6)	(0)		
(1)	(10)	(2)	(6)			
(1)	↑	(2)	(6)			
		(2)	(6)			
			↑			

Step.3. Optimality test.

No. of Allocated Cells = 6.

Condition $= (m+n-1)$ $\qquad m = 3$

$= (3+5-1) = 7.$ $\qquad n = 5$

$\therefore (m+n-1) > 6.$

$\therefore$ Problem leads to degeneracy.

Step 4. Solution to Degeneracy.

42	48	38	37 [16]	0 [ε]	16
40 [8]	49	52 [1]	51	0 [6]	15
39	38 [9]	40 [10]	43	0	19.
8	9	11	16	6.	

Let us allocate ε units to (1,5) cell.

steps. optimality test by MODI method.

					u_i
42	48		37 [16]	0 [ε]	$u_1 = 0.$
40 [8]		52 [1]		0 [6]	$u_2 = 0.$
	38 [9]	40 [10]			$u_3 = \cancel{92}\ -12$
v_j: v_1 (+40)	v_2 +54.	v_3 (+52)	v_4 +37	$v_5 = 0.$	

Constraint eqn are.

$u_1+v_4 = 37,\ u_1+v_5 = 0;\ u_2+v_1 = 40,\ u_2+v_3 = 52.$

$u_2+v_5 = 0.\ u_3+v_2 = 38.\ u_3+v_3 = 40.$

Let $u_1 = 0.$

$0+v_4 = 37 \Rightarrow v_4 = -37, \quad 0+v_5 = 0 \quad v_5 = 0.$

$u_2+0 = 0 \Rightarrow u_2 = 0, \quad u_2+v_1 = 40. \quad v_1 = 40$

$0+v_3 = 52 \quad v_3 = -52, \quad -52+u_3 = 40 \quad u_3 = 92$

$92+v_2 = 38 \quad v_2 = 54.$

Step 5 Cell Evaluation Variables.

42 [2]	48 [-2]	38 [-14] δ	37 [16]	0 [ε] -δ	0
40 [8]	49 [-1]	52 [1]	51 [14]	0 [6] +δ.	0
39 [11]	38 [9]	40 [10]	43 [18]	0.	-12
40	50	52	37	0	

The cell (1,3) has max. negative variable.
Let 'δ' be the new unit alloted to (1,3) cell.
The min. units that can be allotted to (1,3) cell is ε.

Step 6. 2nd b.s.f.c.

42 [16]	48 [12]	38 [ε]	37 [16]	0 [14]	$u_1 = 0$.
40 [8]	49 [-1] δ	52 [1] -δ	51 [0]	0. [6]	$u_2 = 14$.
39 [11]	38 [9] -δ	40 [10] +δ	43 [4]	0 [12]	$u_3 = 2$
$v_1 = 26$.	$v_2 = 36$.	$v_3 = 38$	$v_4 = 37$	$v_5 = -14$.	

$\boxed{\delta = 1.}$

Step 7: 3rd b.s.f.c.

42 [15]	48 [12]	38 [ε]	37 [16]	0 [+13]	$u_1 = 0$.
40 [8]	49 [1]	52 [1]	51 [1]	0 [6]	$u_2 = 13$
39 [10]	38 [8]	40 [11]	43 [2]	0 [11]	$u_3 = 2$
v_1 27	v_2 36.	v_3 (38)	v_4 37.	$v_5 = 13$	

From above table it is clear that the above solution is optimal.

$\therefore$ min(z) = $38\times0+37\times16+40\times8$
$+49\times1+0\times6+38\times8$
$+40\times11$.
$= ₹\ 1705/-$

Q 6 (a) Difference betn. Assignment problem and Transportation problem.

(1). In transportation model, min cost are Considered where as in Assignment model, each cost are modified by comparing with other cost.

(2). Assignment problems with degeneracy will also have optimal solution.

(3) Assignment model gives solution to a transportation problem with linking destination & origin.

(4) optimality can be acheived quickly in Assignment problem.

Q 6 (b). Assignment model is removed from the syllabus. for 2017 scheme.

Q 7. (a)

Kendall & Lee Notations.

According to Kendall & Lee, any Queueing model can be represented as follows.

(a/b/c) : (d:e:f).

where a = Arrival distribution theory
b : Departure or service distribution.

c: No. of parallel service channels.

d: Service Discipline.

e: Max. customers allowed in the system.

f: Calling source or population

Example:

$$(M:M:1):(FCFC/\infty/\infty).$$

M = arrival rate follows Poisson's distribution

M = Service at poisson's distribution

1 : One service channel.

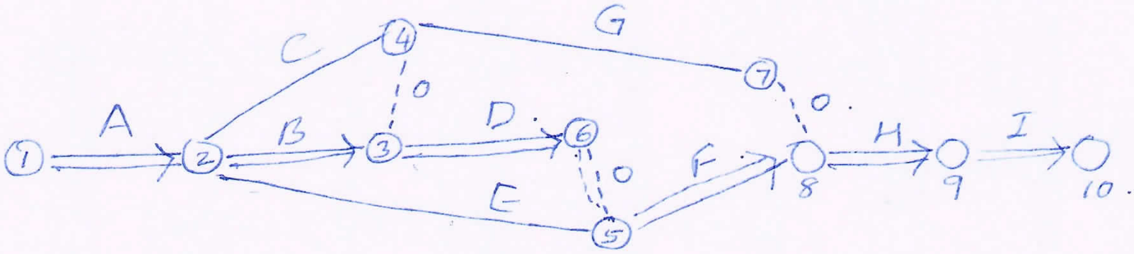
FCFS = First come first service discipline

∞ = Max customers & queue length are infinity.

Q 7(b).

Activity	Predecessor	opt. time	Most likely time	Pessimistic time
A	—	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B.C.	1	4	7
E	A.	8	8	8
F	D, E.	5	14	17
G	C.	3	12	21
H	F, G.	3	6	9
I	H	5	8	11.

(a) Construction of Network.



(b) Calculation of CPM, Average time & Varience.

Activity	t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$	$\sigma = \left(\frac{t_p-t_o}{6}\right)^2$
A	2	5	8	5	1.0
B	6	9	12	9	1.0
C	6	7	8	~~8~~ 7.0	0.11
D	1	4	7	4	1.0
E	8	8	8	8	0
F	5	14	17	13	4.0
G	3	12	21	10	9.0
H	3	6	9	5	1.0
I	5	8	11	8	1.0

Earliest / latest time.

Activity	t_e	Early Start	Early Finish	Latest start	Latest finish	Float LS-EF
A	5	0	5	0	5	0
B	9	5	14	5	14	0
C	7	5	12	7	14	2
D	4	14	18	14	18	0
E	8	5	13	10	18	5
F	13	18	31	18	31	0
G	10	12	22	21	31	11
H	5	31	36	31	36	0
I	8	36	44	36	44	0

(c) Mean standard Deviation of CPM.

ie $\sigma = \Sigma(\sigma_{ij})^2$

$= \sqrt{1^2+1^2+1^2+4^2+1^2+1^2} = \sqrt{21}$.

$= 4.58$.

Probability of completing the project before 50 days. Considering Normal distribution Curve

$$Z = \frac{(50-44)}{4.58} = 1.31.$$

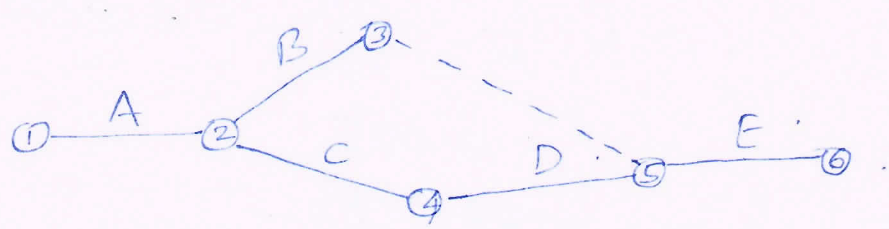
Probability $= 0.9049$.

ie 90.49%

Q 8 (a).

Activity	Predecessor	Duration
A	—	5
B	A	8
C	A.	6.
D	C	5
E	B, D.	9.

Network Construction



Time Calculations

Activity	Duration	Early Start	Early Finish	Latest Start	Latest Finish	Float
A	5	0	5	5	5	0
B	8	5	13	8.	16	3
C	6	5	11	5	11	0
D	5	11	16.	11	16.	0
E	9	16	25.	16.	25	0.

Critical Activities are

A – C – D – E.

Total project Duration = 25 days.

Q 8(b).

Given,

Mean arrival rate $= \lambda = 3/\text{hour}$

Mean service time $= \mu = 10$ min.

Arrival = Poisson distribution.

Service = Exponential.

(a) Utilization factor.

(b) Probability of 2 units.

(c) Queue length =

(d) Expected waiting time.

(a) Utilization factor

$$\rho = \frac{\lambda}{\mu} = \frac{3/60}{10} =$$

$$= 0.005.$$

(c) Queue length $\frac{\mu}{\mu - \lambda}$.

$$= \frac{10}{10 - 0.05}.$$

$$= 1.005 \text{ units.}$$

(d) Expected waiting time

$$= \frac{1}{(\mu-\lambda)} = \frac{1}{(10-0.05)} \doteq 0.1 \text{ min},$$

(b) Probability of 2 units

$$= \frac{\lambda}{\mu} \cdot e^{-(\mu-\lambda) \cdot n}.$$

$$= \frac{0.05}{10} \cdot e^{-(10-0.05)\times 2}.$$

$$\simeq \underline{\underline{0}}.$$

Q 9 (a) Given,

Player B.

		1	2	3
player A	1	2	−2	4
	2	−1	4	2
	3	2	1	6.

By method of Dominance.
strategy 3 for player B can be eliminated.

B.

		1	2
A	1	2	−2
	2	−1	4
	3	2	1.

stategy 3 for player A can be eliminated.

B.

		1	2
player A	1	2	−2
	2	−1	4

∴ Solution to the game.

		min.
+2	−2	−2
−1	4	−1. (max. min).
2	4	

max. (min max)

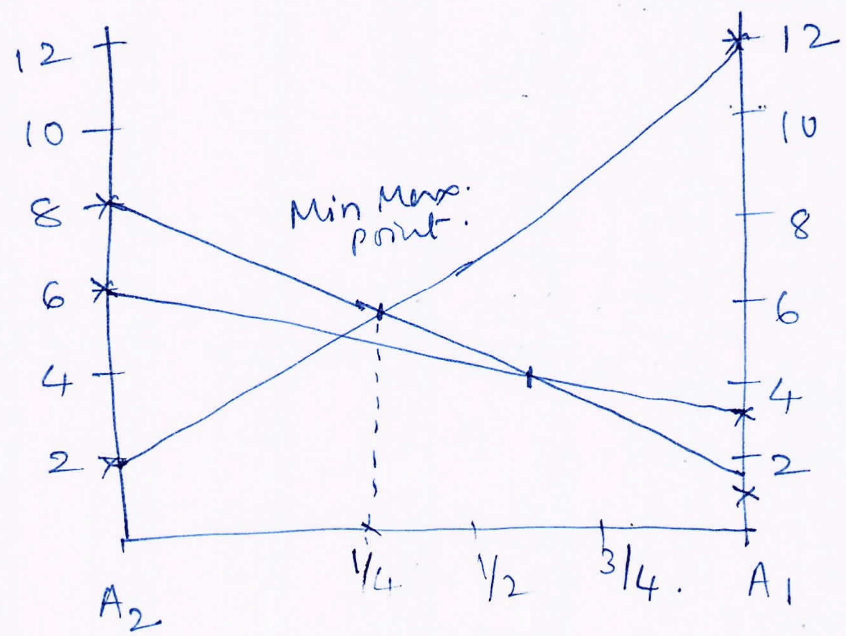
No saddle point will exist.
Therefore the method of dominance will not result any Game Value.

Q 9 (b). <u>Graphical Method.</u>

B.

		1	2	3
A.	1	1	3	12
	2	8	6	2

Let us solve the problem for the max. profit to B.
Let A_1 & A_2 are two strategies.



From the graph the game reduces to.

player B

player A.	1	12	1	1/4
	8	2	3.	3/4
	3	1		
	3/4	1/4.		

Optimal strategy

B. $(\frac{1}{4}, 0\ 3/4, 0)$

A $(\frac{3}{4}, 1/4\ 0\ 0)$.

Value of the game

$$V = \frac{12\times1+2\times3}{(1+3)} = \frac{18}{4}$$

10 (a) n jobs with one machine.

Jobs.	A	B	C	D	E	F	G
time (hrs)	8	3	5	4	3	9	6.

By. shortest processing time rule

The sequence of operation is

B – E – D – C – G – A – F

Jobs	B.	E	D	C	G	A	F
time	3	3	4	5	6	8	9
start time	0	3	6	10	15	21	29
end time	3	6	10	15	21	29	38
Inventory	7	6	5	4	3	2	1

Mean flow time $= \frac{3+6+10+15+21+29+38}{\cancel{6}\ 7.}$

$= 17.43.$ ~~jobs~~ minute.

(b) No. of jobs in inventory,

0-3, 7 jobs.,

$$\therefore \text{Av. in-process Inventory} = \frac{3\times7+3\times6+5\times4+6\times3+8\times2+9\times1}{3+3+4+5+6+8+9}$$

$$= 2.68 \text{ jobs.}$$

10(b).

Given.

order of Sequencing CAB.

Jobs		1	2	3	4	5	6.
time (hrs)	A	4	6	7	4	5	3
	B	8	10	7	8	11	8
	C	5	6	2	3	4	9

Let G be the arbitary. m/c which is Sum of C&A.

H be the sum of A&B.

Jobs		1	2	3	4	5	6.
M/c	G	9	12	9	7	9	12
	H	12	16	14	12	16	11

Sequencing of Machines.

4	1	3	5	2.	6

The time calculation are shown in the table

Sequence	Machine C In	Machine C Out	Machine A In	Machine A Out	Machine B in	Machine B out	Ideal time C	Ideal time A	
4	0	~~4~~3	~~4~~3	~~9~~7	~~9~~7	15	—	3	[illegible]
1	3	6	7	11	15	23	—	—	—
3	6	8	~~8~~11	18	23	30	—	—	—
5	8	9	18	23	30	41	—	—	—
2	9	15	23	29	41	51	—	—	—
6	15	24	29	27	51	59.	25	—	—

Ideal time for M/c C = 25. hours.

For M/c A = 3 hours

For M/c B = 7 hours

Total elapsed time = 59 hours.