USN 2VD14EE015

15EE63

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Sixth Semester B.E. Degree Examination, June/July 2019
Digital Signal Processing

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Determine DFT of sequence $x(n) = \frac{1}{3}$ for $0 \leq n \leq 2$ for $N = 4$. Plot magnitude and phase spectrum. **(08 Marks)**

b. Two length – 4 sequence are defined below :

$$x(n) = \cos\left(\frac{\pi n}{2}\right) \quad n = 0, 1, 2, 3$$

$$h(n) = 2^n \quad n = 0, 1, 2, 3$$

i) Calculate $x(n) \circledast_4 h(n)$ using circular convolution directly.

ii) Calculate $x(n) \circledast_4 h(n)$ using Linear convolution. **(08 Marks)**

OR

2 a. Compute circular convolution using DFT + IDFT for following sequence :

$$x_1(n) = \left\{\underset{\uparrow}{2}, 3, 1, 1\right\}, \quad x_2(n) = \left\{\underset{\uparrow}{1}, 3, 5, 3\right\}.$$ **(08 Marks)**

b. Find the output of the LTI system whose impulse $h(n) = \{1, 1, 1\}$ and the input signal is $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$. Using the overlap save method. Use 6-pt circular convolution. **(08 Marks)**

Module-2

3 a. What are FFT algorithms? Explain the advantages of FFT algorithms over the direct computations of DFT for a sequence $x(n)$. **(04 Marks)**

b. What are the differences and similarities between DIT and DIF –FFT algorithms? **(04 Marks)**

c. Find the 8-pt DFT of the sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$. Using DIT – FFT radix – 2 algorithm. **(08 Marks)**

OR

4 a. Find the 4-pt circular convolution of $x(n)$ and $h(n)$ given. Using radix-2 DIF – FFT algorithm. **(08 Marks)**

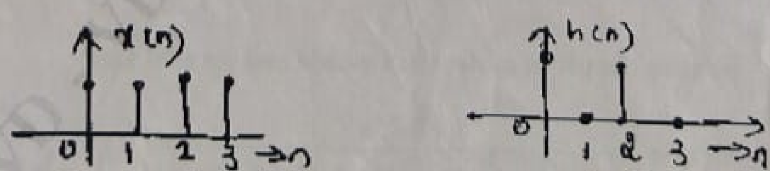


Fig.Q4(a)

b. Given $x(n) = (n + 1)$ and $N = 8$. Determine $X(K)$. Using DIF – FFT algorithm. **(08 Marks)**

Module-3

5 a. Convert the analog filter with system transfer function :

$$H(s) = \frac{(s+0.1)}{(s+0.1)^2 + 3^2}$$

into a digital IIR filter by mean of the impulse invariant method. **(06 Marks)**

b. Design a butter worth digital IIR lowpass filter using bilinear transformation by taking $T = 0.1$ sec, to satisfy the following specification :

$$0.6 \le \left|H(e^{j\omega})\right| \le 1.0; \quad \text{for } 0 \le \omega \le 0.35\pi$$
$$\left|H(e^{j\omega})\right| \le 0.1; \quad \text{for } 0.7\pi \le \omega \le \pi$$

(10 Marks)

OR

6 a. Compare analog and digital filters. **(04 marks)**

b. Determine the poles of lowpass Butterworth filter for N = 2. Sketch the location of poles on s-plane and hence determine the normalized transfer function of lowpass filter. **(08 Marks)**

c. Write difference between IIR and FIR filter. **(04 Marks)**

Module-4

7 a. Design a Chebyshev digital IIR lowpass filter using impulse invariant transformation by taking T = 1 sec to satisfy the following specifications;

$$0.9 \le \left|H(e^{j\omega})\right| \le 1.0; \quad \text{for } 0 \le \omega \le 0.25\pi$$
$$\left|H(e^{j\omega})\right| \le 0.24; \quad \text{for } 0.5\pi \le \omega \le \pi$$

Draw direct form – I and II structure of the filter. **(12 Marks)**

b. Write the relation between analog and digital frequency in Billnear transformation. **(04 Marks)**

OR

8 a. Obtain the direct form – I, direct form II realization of the LTI system governed by the relation.

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64y}y(n-3) + x(n) + 3x(n-1) + 2x(n-2).$$

(08 Marks)

b. Realize the given system in cascade and parallel form :

$$H(z) = \frac{1+0.25z^{-1}}{(1-2z^{-1}+0.25z^{-2})(1-3z^{-1}+0.25z^{-2})}.$$

(08 Marks)

Module-5

9 a. The frequency response of a filter is described by : $H(\omega) = j\omega$, $-\pi \le \omega \le \pi$. Design the filter using a rectangular window. Take N = 7. **(08 Marks)**

b. Design a lowpass digital filter to be used in A/D – H(z) – D/A structure that will have – 3dB cutoff at 30π rad/sec and attenuation factor of 5dB at 45π rad/sec. The filter is required to have a linear phase and the system will use sampling frequency of 100 samples/sec. **(08 Marks)**

OR

10 a. Deduce the equation for the following frequency spectrum for rectangular window sequence defined by :

$$w_r(n) = \begin{cases} 1, & \frac{-(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0, & \text{otherwise} \end{cases}$$ **(06 Marks)**

b. A lowpass filter has the desired frequency response :

$$H_d(\omega) = \begin{cases} e^{-j\omega 3}, & 0 < \omega < \pi/2 \\ 0, & \text{otherwise} \end{cases}$$

Determine h(n) based on frequency sampling method. Take K = 7. **(06 Marks)**

c. Realize the linear phase FIR filter having the following impulse response :

$$h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4).$$ **(04 Marks)**

* * * * *

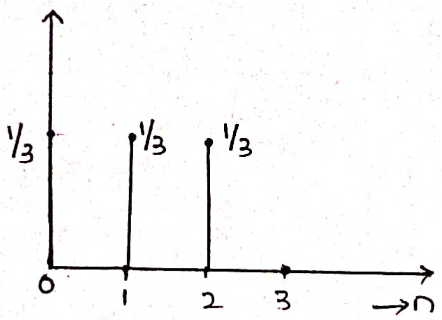
SIXTH SEMESTER B.E DEGREE EXAMINATION JUNE/JULY 2019

DIGITAL SIGNAL PROCESSING

MODULE - I

1a) Determine DFT of sequence $x(n) = 1/3$ for $0 \le n \le 2$ for $N=4$. Plot magnitude and phase spectrum (8M)

$\Rightarrow$ Given $x(n) = 1/3$ for $0 \le n \le 2$, $N=4$



$\therefore x(n) = \{1/3, 1/3, 1/3, 0\}$

Using linear transformation,

$$[X(k)] = [W_N^{kn}][x(n)]$$

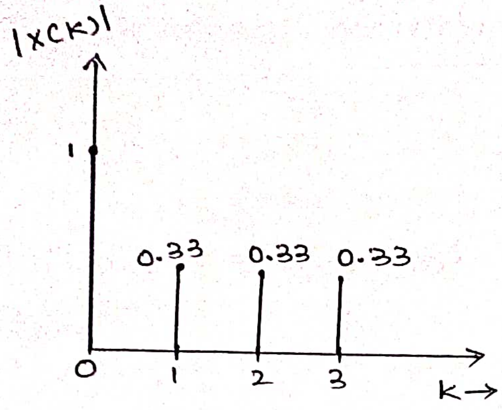
$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

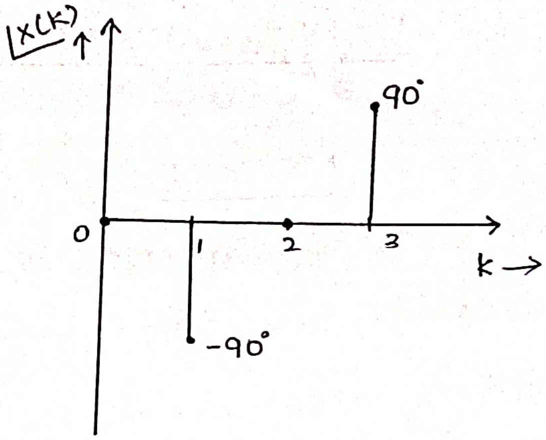
$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/3 + 1/3 + 1/3 + 0 \\ 1/3 - j/3 - 1/3 + 0 \\ 1/3 - 1/3 + 1/3 + 0 \\ 1/3 + j/3 - 1/3 + 0 \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 \angle 0^\circ \\ 0.33 \angle -90^\circ \\ 0.33 \angle 0^\circ \\ 0.33 \angle 90^\circ \end{bmatrix}$$

$\therefore X(k) = \{1\angle 0^\circ, 0.33\angle -90^\circ, 0.33\angle 0^\circ, 0.33\angle 90^\circ\}$

Magnitude and Phase Specturm:-





1b) Two length-4 sequence are defined below:

$x(n) = \cos\left(\frac{\pi n}{2}\right)$, $n = 0,1,2,3$

$h(n) = 2^n$, $n = 0,1,2,3$

i) Calculate $x(n) \circledast_4 h(n)$ using circular convolution directly

ii) Calculate $x(n) \circledast_4 h(n)$ using linear convolution. (8M)

⇒ Given $x(n) = \cos\left(\frac{\pi n}{2}\right)$ for $n = 0,1,2,3$

i.e $x(0) = \cos\left(\frac{\pi \times 0}{2}\right)$; $n=0$ ⇒ $x(0) = 1$

$x(1) = \cos\left(\frac{\pi \times 1}{2}\right) = \cos(\pi/2) = 0$; $n=1$ ⇒ $x(1) = 0$

$x(2) = \cos\left(\frac{\pi \times 2}{2}\right) = \cos(\pi) = -1$; $n=2$ ⇒ $x(2) = -1$

$x(3) = \cos\left(\frac{\pi \times 3}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$; $n=3$ ⇒ $x(3) = 0$.

$\therefore x(n) = \{1, 0, -1, 0\}$

& $h(n) = 2^n$ for $n = 0,1,2,3$.

$h(0) = 2^0 = 1$; $n=0$

$h(1) = 2^1 = 2$; $n=1$

$h(2) = 2^2 = 4$; $n=2$

$h(3) = 2^3 = 8$; $n=3$

$\therefore h(n) = \{1, 2, 4, 8\}$

i) Using circular convolution directly.

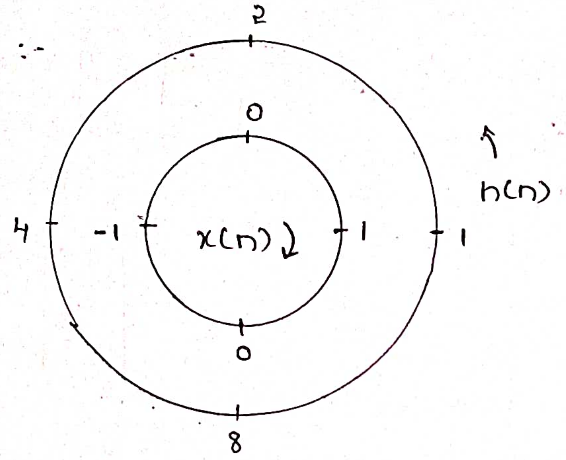
$$y(n) = x(n) \circledast_N h(n)$$

$$y(n) = x(n) \circledast_4 h(n)$$

$$x(n) = \{1, 0, -1, 0\}$$

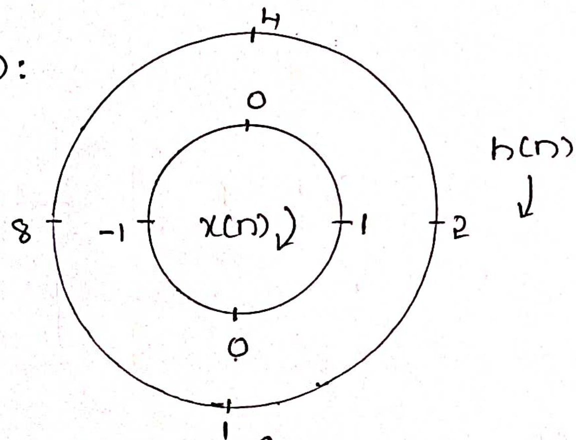
$$h(n) = \{1, 2, 4, 8\}$$

a) $y(0)$:-



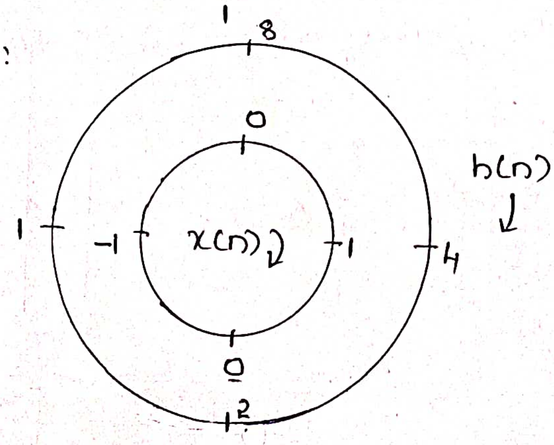
$$y(0) = 1(1) + 0(8) + (-1)(4) + 0(2)$$
$$= 1 + 0 - 4 + 0$$
$$y(0) = -3$$

b) $y(1)$:



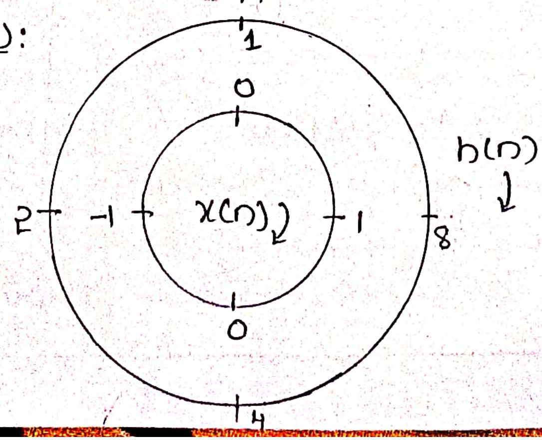
$$y(1) = 1(2) + 0(-1) + (-1)(8) + 0(4)$$
$$= 2 + 0 - 8 + 0$$
$$y(1) = -6$$

c) $y(2)$:



$$y(2) = 1(4) + 0(2) + (-1)(1) + 0(8)$$
$$= 4 + 0 - 1 + 0$$
$$y(2) = 3$$

d) $y(3)$:



$$y(3) = 1(8) + 0(4) + (-1)(2) + 0(1)$$
$$= 8 + 0 - 2 + 0$$
$$y(3) = 6$$

$\therefore y(n) = \{-3, -6, 3, 6\}$

ii) calculate $x(n) \circledast_4 h(n)$ using linear convolution.

$x(n) = \{1, 0, -1, 0\}$

$h(n) = \{1, 2, 4, 8\}$

$x(n)\rightarrow$ / $\downarrow h(n)$	1	0	-1	0
1	1	0	-1	0
2	2	0	-2	0
4	4	0	-4	0
8	8	0	-8	0

$\therefore y(n) = \{1, 2, (4-1), (8-2), -4, -8, 0\}$

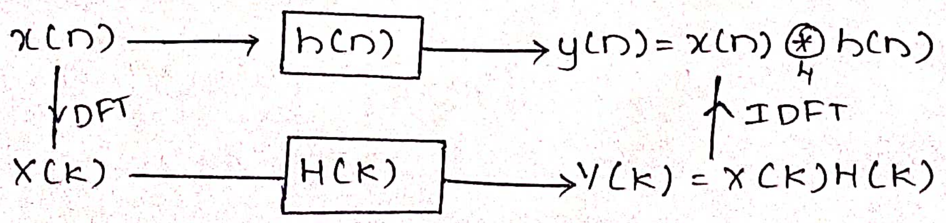
$\therefore y(n) = \{1, 2, 3, 6, -4, -8, 0\}$

2a) Compute circular convolution using DFT + IDFT for following sequence:

$x_1(n) = \{\underset{\uparrow}{2}, 3, 1, 1\}$ $\quad x_2(n) = \{\underset{\uparrow}{1}, 3, 5, 3\}$ (8M)

=> Given :- $x_1(n) = \{\underset{\uparrow}{2}, 3, 1, 1\} = x(n)$

$x_2(n) = \{\underset{\uparrow}{1}, 3, 5, 3\} = h(n)$



i) Finding DFT of $x(n)$:-

$[X(k)] = [W_N^{kn}][x(n)]$ ⇒ linear form.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+3+1+1 \\ 2-j3-1+j \\ 2-3+1-1 \\ 2+j3-1-j \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 7 \\ 1-j2 \\ -1 \\ 1+j2 \end{bmatrix}$$

$\therefore X(k) = \{7, 1-j2, -1, 1+j2\}$

ii) Finding DFT of $h(n)$

From Linear form :- $[H(k)] = [W_N^{kn}][h(n)]$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \end{bmatrix}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+3+5+3 = 12 \\ 1-j3-5+3j = -4 \\ 1-3+5-3 = 0 \\ 1+3j-5-3j = -4 \end{bmatrix}$$

$H(k) = \{12, -4, 0, -4\}$

iii) Finding Y(k) :-

$Y(k) = X(k) H(k)$

$$Y(k) = \begin{bmatrix} 7 & 1-j2 & -1 & 1+j2 \end{bmatrix} \times \begin{bmatrix} 12 & -4 & 0 & -4 \end{bmatrix}$$

$$Y(k) = \begin{bmatrix} 84 & -4+j8 & 0 & -4-j8 \end{bmatrix}$$

iv) <u>Taking</u> <u>IDFT of Y(k)</u>

$$y(n) = \frac{1}{N} [W_N^{-kn}][Y(k)]$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^{-1} & W_4^{-2} & W_4^{-3} \\ W_4^0 & W_4^{-2} & W_4^{-4} & W_4^{-6} \\ W_4^0 & W_4^{-3} & W_4^{-6} & W_4^{-9} \end{bmatrix} \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 84 \\ -4+j8 \\ 0 \\ -4-j8 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 84-4+j8+0-4-j8 = 76 \\ 84-4j-8+0+j4-8 = 68 \\ 84+4-j8+0+4+j8 = 92 \\ 84+j4+8+0-j4+8 = 100 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 76 \\ 68 \\ 92 \\ 100 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \\ 23 \\ 25 \end{bmatrix}$$

$\therefore y(n) = \{19, 17, 23, 25\}$

2b) Find the output of the LTI system whose impulse h(n) = {1, 1, 1} and input signal is x(n) = {3, −1, 0, 1, 3, 2, 0, 1, 2, 1}. Using overlap save method. Use 6-pt circular convolution (8M)

⇒ Given :- $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$

$h(n) = \{1, 1, 1\}$

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) \circledast h(n)$$

$L = 10$ $\quad$ $M = 3$ $\quad$ $L+M-1 = 10+3-1 = 12$.

$\therefore x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1, 0, 0\}$

For $N = 6$ Given :- 6 point circular convolution.

Step 1 :- Finding the blocks.

$x_1(n) = \{\underbrace{0.\ 0}_{(m-1)\text{zero's}}\ 3\ -1\ 0\ 1\}$

$x_2(n) = \{0.1\ 3\ 2\ 0\ 1\}$

$x_3(n) = \{0\ 1\ 2\ 1\ 0\ 0\}$

Step 2 : Block circular convolution.

$y_1(n) = x_1(n) \underset{6}{\circledast} h(n)$ $\qquad$ $y_3(n) = x_3(n) \underset{6}{\circledast} h(n)$.

$y_2(n) = x_2(n) \underset{6}{\circledast} h(n)$

Finding $y_1(n) = x_1(n) \circledast_6 h(n)$

$x_1(n) = \{0 \quad 0 \quad 3 \quad -1 \quad 0 \quad 1\}$

$h(n) = \{1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\}$

$$
\begin{array}{cccc}
h(n) & x_1(n) & & y_1(n) \\
\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} &
\begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 0 \\ 1 \end{bmatrix} & = &
\begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 2 \\ 0 \end{bmatrix}
\end{array}
$$

Finding $y_2(n) = x_2(n) \circledast_6 h(n)$

$x_2(n) = \{0 \quad 1 \quad 3 \quad 2 \quad 0 \quad 1\}$

$h(n) = \{1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\}$

$$
\begin{array}{cccc}
h(n) & x_2(n) & & y_2(n) \\
\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} &
\begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \\ 0 \\ 1 \end{bmatrix} & = &
\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 5 \\ 3 \end{bmatrix}
\end{array}
$$

Finding $y_3(n) = x_3(n) \circledast_6 h(n)$

$x_3(n) = \{0 \quad 1 \quad 2 \quad 1 \quad 0 \quad 0\}$

$h(n) = \{1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\}$

$$\overset{h(n)}{\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}} \overset{x_3(n)}{\begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}} = \overset{y_3(n)}{\begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}}$$

Step 3: Combining $y_1(n)$, $y_2(n)$ & $y_3(n)$.

$y_1(n) = \underbrace{1 \quad 1}_{(m-1)\text{discard}} \quad 3 \quad 2 \quad 2 \quad 0$

$y_2(n) = \qquad\qquad \underbrace{1 \quad 2}_{(m-1)\text{discard}} \quad 4 \quad 6 \quad 5 \quad 3$

$y_3(n) = \qquad\qquad\qquad\qquad \underbrace{0 \quad 1}_{(m-1)\text{discard}} \quad 3 \quad 4 \quad 3 \quad 1$

$y(n) = \{3 \quad 2 \quad 2 \quad 0 \quad 4 \quad 6 \quad 5 \quad 3 \quad 3 \quad 4 \quad 3 \quad 1\}$

$\therefore y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\} = 12$.

$\because L+M-1 = 10+3-1 = 12$.

MODULE - II

3 a) What are FFT algorithms? Explain the advantages of FFT algorithms over derect computations of DFT for a sequence $x(n)$ (4M)

=> FFT [Fast Fourier Transform] is an algorithm that computes the discrete Fourier transform [DFT] of a sequence or its inverse (IDFT). Fourier analysis converts a signal from its original domain to frequency domain and vice versa. The DFT is obtained by decomposing a sequence of values into components of different frequencies. so FFT rapidly

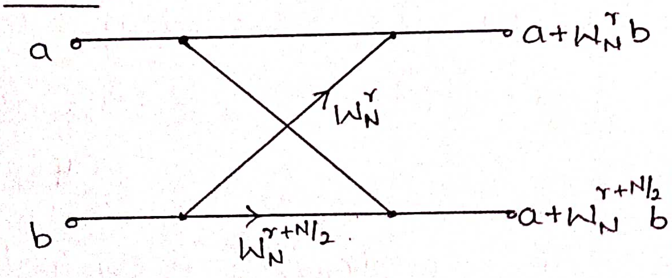
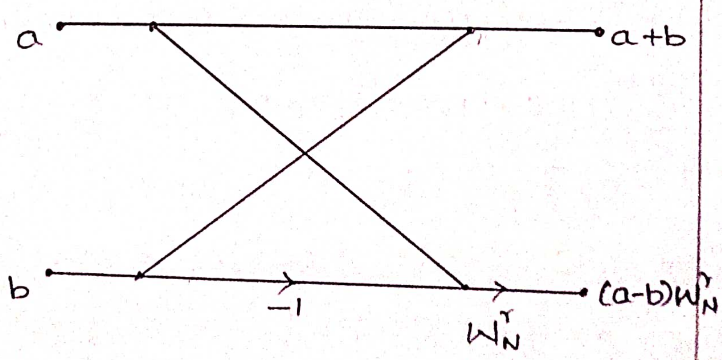
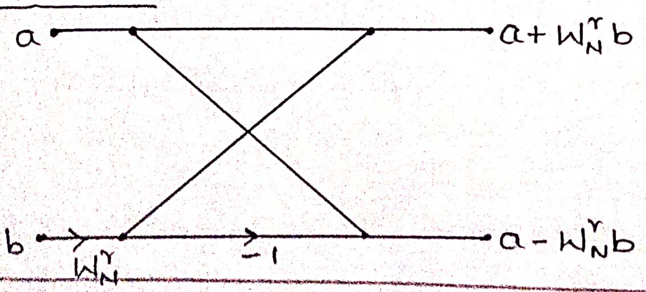
Computes such transformation by factorizing the DFT matrix into a product of sparse factors. Hence it reduces the complexity of computing the DFT. It makes use of the symmetry and periodicity properties of twiddle factor to reduce DFT computation.

<u>Advantages of FFT algorithm over direct computation of DFT</u>

- The computation complexity of FFT algorithm greatly reduced compare to direct computation
- FFT algorithm can be used to find DFT as well as IDFT very effectively.
- FFT helps in converting the time domain to frequency domain.

3b) What are the differences and similarities between DIT and DIF-FFT algorithms? (4M).

⇒

DIT	DIF
In this algorithm time index 'n' is decimated into even & odd in various level.	In this algorithm frequency index 'k' is decimated into even & odd at various level.
Butterfly Structure:	Butterfly Structure:
<u>Basic</u>:- 	<u>Modified</u>:- 
<u>Modified</u>:- 	

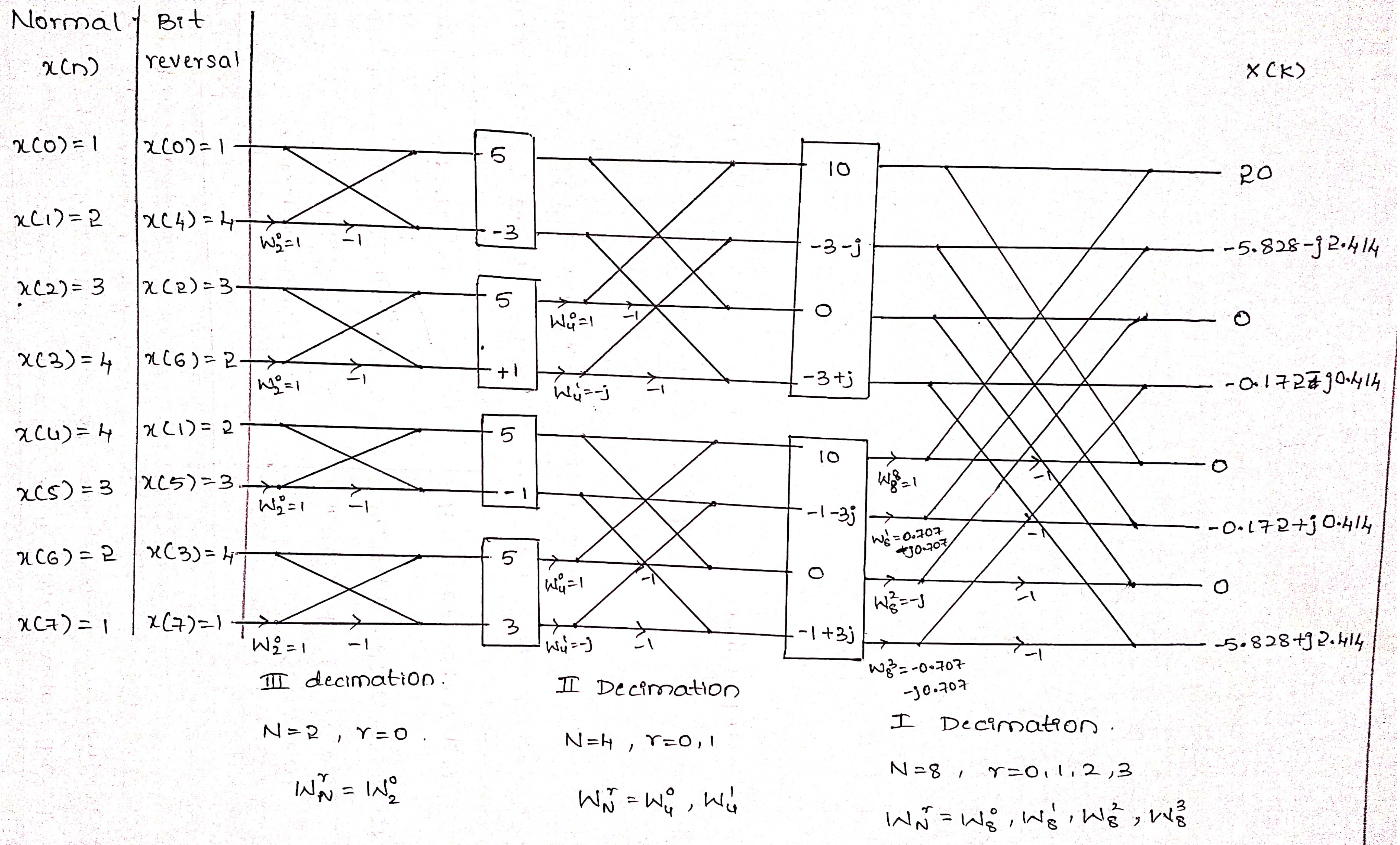
DIT	DIF
*> In this algorithm the input sequence has to be bit reversal order and output sequence will be normal.	*> In this algorithm the input sequence is normal and output sequence is bit reverse.
Similarities:- *> Memory requirement is same. *> In DIT computation for modified structure. Multiplication $= \frac{N}{2} \log_2 N$ Addition $= N \log_2 N$	*> Memory requirement is same. *> In DIF computation for modified structure. Multiplication $= \frac{N}{2} \log_2 N$ Addition $= N \log_2 N$

3c) Find the 8 point DFT of the sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ Using DIT-FFT radix-2 algorithm (8M).

=> Given:-

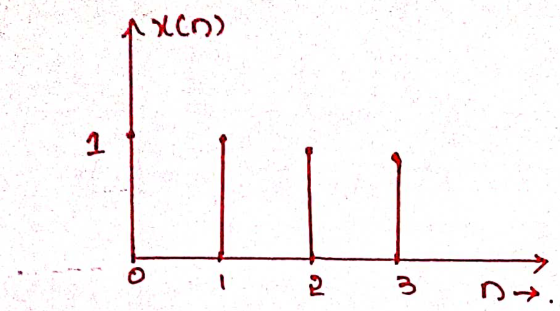
$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

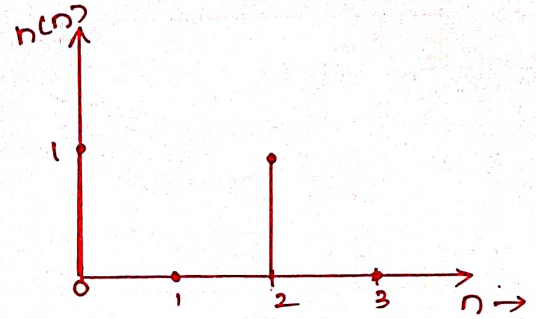
To solve using DIF-FFT radix-2 algorithm.



$X(k) = \{20, -5.828-j2.414, 0, -0.172-j0.414, 0, -0.172+j0.414, 0, -5.828+j2.414\}$

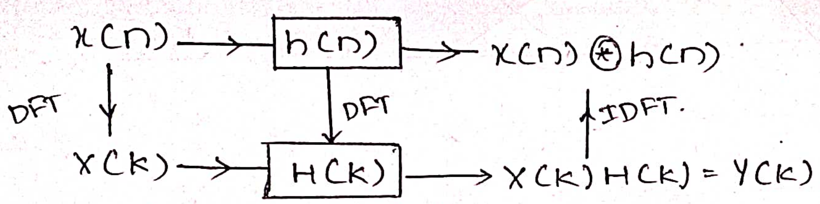
4a) Find the 4-pt circular convolution of $x(n)$ and $h(n)$ given using radix-2 DIF-FFT algorithm.



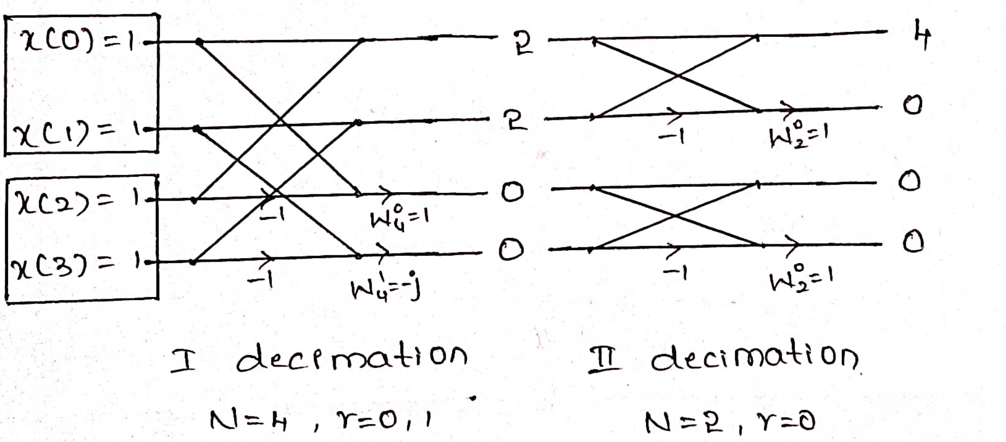


(8M)

Soln :- Given : $x(n) = \{1, 1, 1, 1\}$, $h(n) = \{1, 0, 1, 0\}$



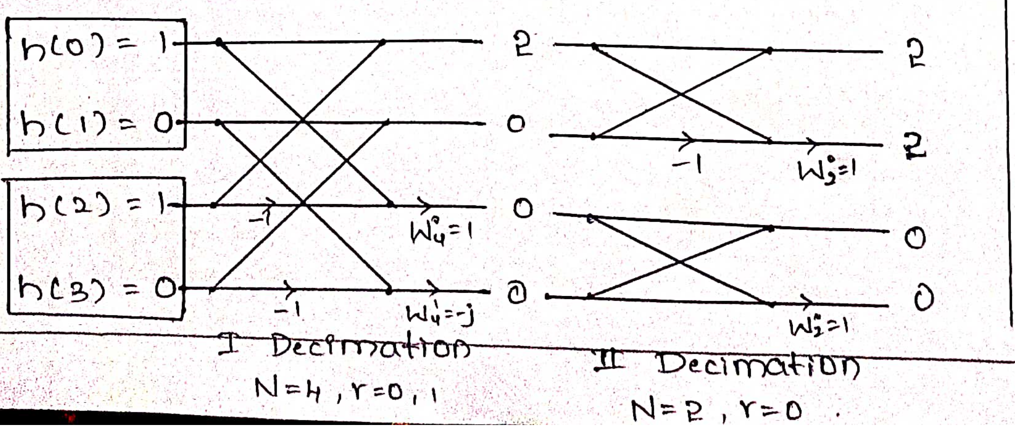
i) Finding DFT X(k) by DIF-FFT Radix-2 Algorithm.



Bit reversal
$X(0) = 4$
$X(2) = 0$
$X(1) = 0$
$X(3) = 0$

$\therefore X(k) = \{4, 0, 0, 0\}$

ii) Finding DFT H(k) by DIF-FFT Radix-2 Algorithm :-



Bit Reversal
$H(0) = 2$
$H(2) = 2$
$H(1) = 0$
$H(3) = 0$

$H(k) = \{2, 0, 2, 0\}$

iii) <u>Finding</u> <u>$Y(k) = X(k)H(k)$</u>

$Y(k) = \{4, 0, 0, 0\}\{2, 0, 2, 0\}$

$Y(k) = \{8, 0, 0, 0\}$

iv) <u>Finding</u> <u>IDFT of</u> <u>Y(k) to</u> <u>find y(n)</u>

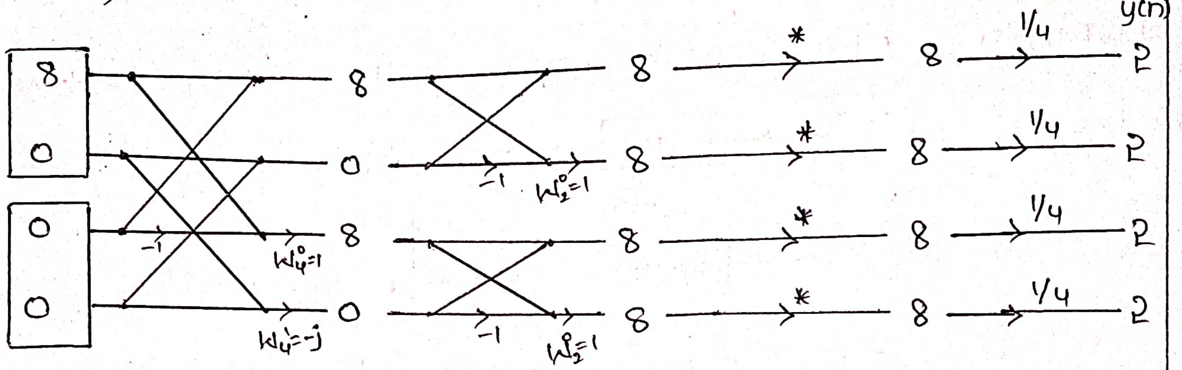
$Y(k)$ $\quad$ $Y^*(k)$

$Y(0) = 8$

$Y(1) = 0$

$Y(2) = 0$

$Y(3) = 0$



y(n)	Bit reversal y(n)
2	$y(0) = 2$
2	$y(2) = 2$
2	$y(1) = 2$
2	$y(3) = 2$

$\therefore y(n) = x(n) \circledast h(n) = \{2, 2, 2, 2\}$ //

4b) Given $x(n) = (n+1)$ and $N = 8$. Determine $X(k)$. Using DIF - FFT Algorithm (8M)

$\Rightarrow$ Given $x(n) = (n+1)$, $N = 8$.

$\therefore x(0) = (0+1) = 1$

$x(1) = 1+1 = 2$

$x(2) = 2+1 = 3$

$x(3) = 3+1 = 4$

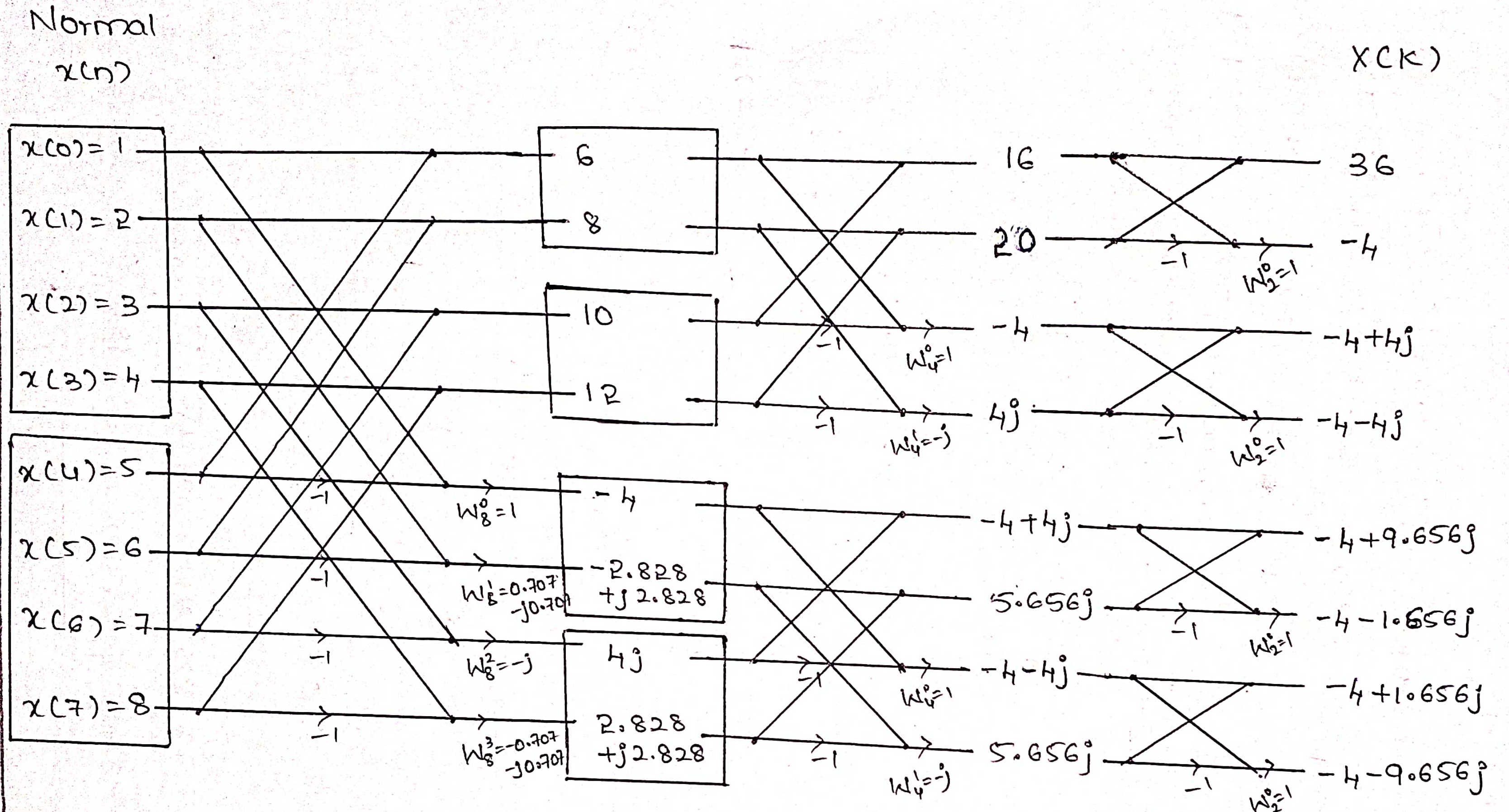
$x(4) = 4+1 = 5$

$x(5) = 5+1 = 6$

$x(6) = 6+1 = 7$

$x(7) = 7+1 = 8$

$\therefore x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$



Bit reversal
X(K)

X(0) = 36

X(4) = -4

X(2) = -4+4j

X(6) = -4-4j

X(1) = -4+9.656j

X(5) = -4-1.656j

X(3) = -4+1.656j

X(7) = -4-9.656j

I Decimation

N=8, r=0,1,2,3

II Decimation

N=4, r=0,1

III Decimat^n

N=2, r=0

$\therefore X(K) = \{36, -4+9.656j, -4+4j, -4+1.656j, -4, -4-1.656j, -4-4j, -4-9.656j\}$

MODULE - 03

5a) Convert the analog filter with system transfer function $H(s) = \frac{(s+0.1)}{(s+0.1)^2+3^2}$ into a digital IIR filter by mean of the impulse invariant method. (6M)

⇒ Given $H(s) = \frac{(s+0.1)}{(s+0.1)^2+3^2}$.

WKT $\frac{(s+a)}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$

comparing the above formula with $H(s)$ we get.

$a = 0.1 \quad ; b = 3$.

$$H(z) = \frac{1-e^{-0.1T}(\cos 3T)z^{-1}}{1-2e^{-0.1T}(\cos 3T)z^{-1}+e^{-2(0.1)T}z^{-2}}$$

$$H(z) = \frac{1-e^{-0.1T}(\cos 3T)z^{-1}}{1-2e^{-0.1T}(\cos 3T)z^{-1}+e^{-0.2T}z^{-2}} \quad //$$

5b) Design a butter worth digital IIR low pass filter using bilinear transformation by taking T=0.1sec. to satisfy the following specification:

$0.6 \leq |H(e^{j\omega})| \leq 1.0$; for $0 \leq \omega \leq 0.35\pi$

$|H(e^{j\omega})| \leq 0.1$; for $0.7\pi \leq \omega \leq \pi$. (10M)

⇒ Given :- $T = 0.1$ sec , $0.6 \leq |H(e^{j\omega})| \leq 1$; for $0 \leq \omega \leq 0.35\pi$.

$|H(e^{j\omega})| \leq 0.1$; for $0.7\pi \leq \omega \leq \pi$.

Given :- $A_p = 0.6$ for $\omega_p \leq 0.35\pi$.

$A_s = 0.1$ for $\omega_s \geq 0.7\pi$.

i) To obtain specification of equivalent analog filter for billinear transformation:-

$\Omega_p = \tan\frac{\omega_p}{2} = \tan\frac{0.35\pi}{2} = 0.6128$

$\Omega_s = \tan\frac{\omega_s}{2} = \tan\frac{0.7\pi}{2} = 1.9626$.

Hence specification of equivalent analog filter are.

$A_p = 0.6$, $\Omega_p = 0.613$

$A_s = 0.1$, $\Omega_s = 1.963$.

ii) To obtain order of filter:-

$$N = \frac{1}{2}\frac{\log\left[\left(\frac{1}{A_s^2}-1\right)\left(\frac{1}{A_p^2}-1\right)\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{1}{2}\left[\frac{\log\left[\left(\frac{1}{0.1^2}-1\right)\left(\frac{1}{0.6^2}-1\right)\right]}{\log\left(\frac{1.963}{0.613}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{\log(176)}{\log(3.2022)}\right] = \frac{1}{2}[4.44]$$

$N = 2.22 \approx 2$

To obtain Ω_c.

$$\Omega_c = \frac{1}{2}\left[\frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{\frac{1}{2N}}} + \frac{\Omega_s}{\left(\frac{1}{A_s^2}-1\right)^{\frac{1}{2N}}}\right] = \frac{1}{2}\left[\frac{0.613}{\left(\frac{1}{0.6^2}-1\right)^{\frac{1}{2\times2}}} + \frac{1.963}{\left(\frac{1}{0.1^2}-1\right)^{\frac{1}{2\times2}}}\right]$$

$\Omega_c = 0.576$

To obtain poles of $H_a(s)$

$P_k = \pm\Omega_c\, e^{j(N+2k+1)\pi/2N}$; $k = 0, 1, \ldots\ldots N-1$

For $N=2$, $P_k = \pm\Omega_c e^{j(2+2k+1)\frac{\pi}{2\times2}}$.

$P_k = \pm\Omega_c\, e^{j(3+2k)\pi/4}$

$P_k = \pm 0.576\, e^{j(3+2k)\frac{\pi}{4}}$; $k = 0, 1$.

For $k=0$; $P_0 = \pm 0.576\, e^{j(3+2(0))\frac{\pi}{4}} = \pm 0.576\, e^{j\frac{3\pi}{4}}$

$P_0 = +\left(0.576\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right)$, $-\left(0.576\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right)$

$P_0 = (-0.407 - j0.407)$, $(0.407 + j0.407)$

For $k=1$; $P_1 = \pm 0.576\, e^{j(3+2(1))\pi/4} = \pm 0.576\, e^{\frac{5\pi}{4}j}$.

$P_1 = +\left(0.576 \cos\frac{5\pi}{4} - j\,0.576 \sin\frac{5\pi}{4}\right),\ -\left(0.576 \cos\frac{5\pi}{4} - j\,0.576 \sin\frac{5\pi}{4}\right)$

$P_1 = (-0.407 + j0.407),\ (0.407 - j0.407)$.

$\therefore\ s_1 = -0.407 - j0.407$ & $s_1^* = -0.407 + j0.407$.

To obtain system function $H_a(s)$.

For $N=2$,

$$H_a(s) = \frac{\Omega_c^2}{(s-s_1)(s-s_1^*)}$$

$$H_a(s) = \frac{0.576^2}{(s+0.407+j0.407)(s+0.407-j0.407)}$$

$$= \frac{0.576^2}{(s+0.407)^2-(j0.407)^2} = \frac{0.576^2}{(s+0.407)^2+(0.407)^2}$$

$$= \frac{0.3317}{s^2+0.1656+0.8148+0.1656}.$$

$$H_a(s) = \frac{0.3317}{s^2+0.814s+0.3312}$$

To obtain $H(z)$ by billinear transformation.

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

$$= \frac{0.3317}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.814\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.3312} = \frac{0.3317(1+z^{-1})^2}{\frac{(1-z^{-1})^2+0.814(1-z^{-1})(1+z^{-1})+0.3312}{(1+z^{-1})^2}}$$

$$= \frac{0.3317(1+z^{-2}+2z^{-1})}{1+z^{-2}-2z^{-1}+0.814(1-z^{-2})+0.3312(1+z^{-2}+2z^{-1})}$$

$$H(z) = \frac{0.3317(1+z^{-2}+2z^{-1})}{2.1452-1.3376z^{-1}+0.5172z^{-2}}.$$

$$\therefore\ \boxed{H(z) = \frac{0.3317z^{-2}+0.6634z^{-1}+0.3317}{0.5172z^{-2}-1.3376z^{-1}+2.1476}}$$

6a) Compare analog and digital Filter:- (4M)

Analog Filter	Digital Filter.
+> It processes continuous time signal using passive elements (R,L,C) called as passive filter. or using active elements (opamp) called as active filter.	+> It processes digital signal through numerical computation using digital circuits [FPGA or DSA].
+> Types of Analog Filters:- 1> LPF - Low pass filters 2> HPF - High pass filters 3> BPF - Band pass filters...	+> Types of Digital Filters:- 1> IIR - Infinite impulse response Filters. 2> FIR - Finite impulse response filters.
+> Infinite frequency range because FT is non periodic.	+> Finite frequency range i.e ($-\pi$ to π) because DTFT is periodic in 2π.
+> Mathematically, it is defined by transfer function in 's' domain 'H(s)'.	+> Mathematically, it is defined by transfer function in 'z' domain 'H(z)'.
+> Frequency response characteristic is defined by $H(j\Omega)$ where Ω is Analog frequency (rad/sec)	+> Frequency response characteristic is defined by $H(e^{j\omega})$ where ω is digital frequency (rad)
+> For stable analog filter, all poles lie on left half of 's' plane	+> For stable digital filter all poles lie within the unit circle in 'z' plane.
+> Design of Analog filter, means for given frequency response char. $H_d(j\omega)$, finding H(s).	+> For given frequency response char. $H_d(e^{j\omega})$ design of digital filter means finding 'H(z)'

6b) Determine the poles of lowpass butterworth filter for N=2. Sketch the location of poles on s-plane and hence determine the normalized transfer function of lowpass filter. (8M).

=> Design of Low pass butterworth Filter for N=2.

Low pass filter can be designed using the butterworth polynomial

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

, N = Order.

Ω_c = -3dB cuttoff frequency of the filter.

To find equation for 'N':

Take log on both sides.

$$20\log|H(j\Omega)| = -\log\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right] = k.$$

at $\Omega = \Omega_p$, $k = k_p$

& at $\Omega = \Omega_s$, $k = k_s$.

we get.

$$k_p = -10\log\left[1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right]$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-k_p/10} - 1 \longrightarrow (1)$$

Similary we get.

$$\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{-k_s/10} - 1 \longrightarrow (2)$$

Divide equation (1) by (2).

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{(10^{-k_p/10}-1)}{(10^{-k_s/10}-1)}$$

Taking log on both side.

$$2N\log\left(\frac{\Omega_p}{\Omega_c}\right) = \log\left[\frac{(10^{-k_p/10}-1)}{(10^{-k_s/10}-1)}\right]$$

$$N = \frac{1}{2}\left[\frac{\log\left[\frac{10^{-kp/10}-1}{10^{-ks/10}-1}\right]}{\log\left(\frac{\Omega_p}{\Omega_c}\right)}\right]$$

To find equation for Ω_c.

From equation ① we have :

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-kp/10} - 1$$

$$\frac{\Omega_p}{\Omega_c} = \left(10^{-kp/10}-1\right)^{1/2N}.$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{-kp/10}-1\right)^{1/2N}}$$

To find the transfer function H(s)

Normalised transfer function is determined by setting $\Omega_c = 1$.

WKT, the poles of Transfer function lie on a circle of radius $\Omega_c = 1$. The total number of poles will be equal to N.

Then angle between the poles is $\theta = \frac{360}{2N}$.

The location of 2N pole around a circle is made as per the following rule.

For N = odd.

The first pole will lie along +ve real axis of s plane.

For N = Even.

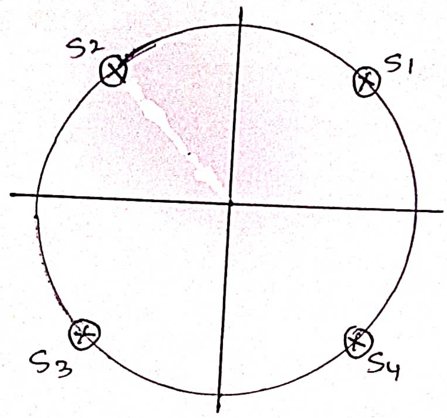
The first pole lies at angle $\theta/2$ from the +ve real axis.

Now to find for N=2 :-

Total poles $= 2N = 4$.

Angle between poles $\quad \theta = \frac{360^\circ}{2N} = \frac{360^\circ}{4} = 90^\circ$.

First pole will lie at an angle $\frac{\theta}{2} = 45^\circ$ from the real axis.



s-plane

For stable filter, we have to consider poles lying on LHS.

$S_2 = -0.707 + j0.707$.

$S_3 = -0.707 - j0.707$.

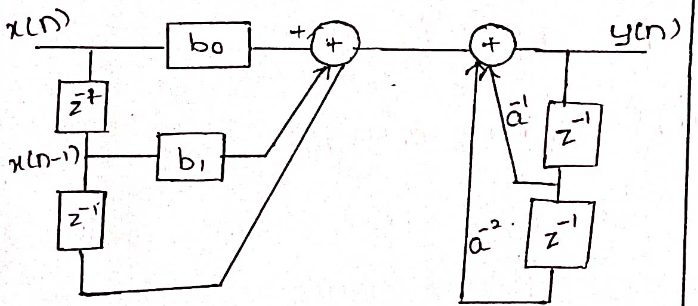
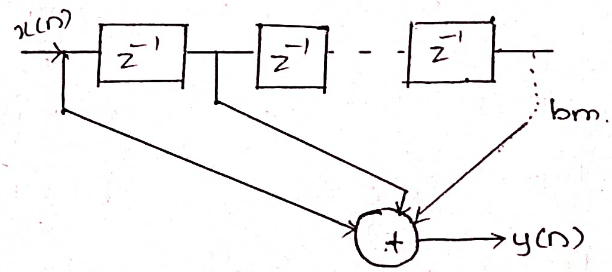
$\therefore$ Transfer function $\quad H(s) = \frac{1}{(s - s_2)(s - s_3)}$.

$$H(s) = \frac{1}{(s + 0.707 - j0.707)(s + 0.707 + j0.707)}$$

$$H(s) \quad \frac{1}{s^2 + \sqrt{2}s + 1}$$

Here $H(s) = s^2 + \sqrt{2}s + 1$ is called normalized butterworth polynomial for $N = 2$ //.

6c) Write difference between IIR and FIR Filter (4M)

IIR	FIR
+> It is an acronym for Infinite Impulse response.	+> It is an acronym for Finite Impulse Response.
+> It is defined by differential equation $y(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=1}^{N} a_k y(n-1)$	+> It is defined by differential equation $y(n) = \sum_{k=0}^{M} b_k x(n-k)$.
+> Since present output depends upon past values of output. It has a feedback Hence this filter is called recursive type of filter.	+> As there is no feedback so it is called as non-recursive type of filter.
+> Block Diagram 	+> Block diagram. 
+> Transfer function is given by $H(z) = \dfrac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$. So this type of filter has zeros and poles. For stable system the poles should be made to lie within the unit circle.	+> Transfer function is given by $H(z) = \sum_{k=0}^{M} b_k z^{-k}$. This type of filter has only zeros. Poles lies at z=0 in z plane. so it is always stable
+> computational efficiency is comparatively more	+> computational efficiency is less.
+> Memory requirement is less	+> Memory requirement is more
+> controllability is difficult	+> Controllability is easy.

MODULE - 4

7a) Design a chebyshev digital filter IIR low pass filter using bilinear transformation to satisfy the following specifications;

$$0.9 \le |H(e^{j\omega})| \le 1.0 \; ; \quad \text{for } 0 \le \omega \le 0.25\pi$$

$$|H(e^{j\omega})| \le 0.24 \; ; \quad \text{for } 0.5\pi \le \omega \le \pi .$$

(12M)

=> i) Given :- $A_p = 0.9$, $\omega_p = 0.25\pi$

$A_s = 0.24$, $\omega_s = 0.5\pi$.

ii) Prewraping:-

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2} = \tan \frac{\omega}{2} \quad \text{assuming } \frac{2}{T} = 1$$

$$\Omega_p = \tan \frac{\omega_p}{2} = \tan \frac{0.25\pi}{2} = 0.414 \text{ rad/sec.}$$

$$\Omega_s = \tan \frac{\omega_s}{2} = \tan \frac{0.5\pi}{2} = 1 \text{ rad/sec.}$$

Thus specification of equivalent analog filter

$A_p = 0.9$, $\Omega_p = 0.414$ rad/sec.

$A_s = 0.24$, $\Omega_s = 1$ rad/sec.

iii) Order of chebyshev filter :-

Let us first calculate $\varepsilon = \sqrt{\frac{1}{A_p^2} - 1} = \sqrt{\frac{1}{0.9^2} - 1} = 0.484$

& $\delta = \sqrt{\frac{1}{A_s^2} - 1} = \sqrt{\frac{1}{0.24^2} - 1} = 4.045$

The order of chebyshev is given by

$$N = \frac{\cosh^{-1}(\delta/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} = \frac{\cosh^{-1}\left(\frac{4.045}{0.484}\right)}{\cosh^{-1}\left(\frac{1}{0.414}\right)} = 1.84 \approx 2 .$$

$$H_a(s) = \frac{K}{(s-0.242)^2 + (0.361)^2} = \frac{K}{[s^2 - 0.484s + 0.0686] + 0.130}$$

$$H_a(s) = \frac{K}{(s^2 - 0.484s + 0.1868)}$$

$\therefore b_0 = 0.1868$.

$$K = \frac{b_0}{\sqrt{1+\epsilon^2}} = \frac{0.1868}{\sqrt{1+0.484}} = 0.1533.$$

$$\therefore H_a(s) = \frac{0.1533}{(s^2 - 0.484s + 0.1868)}$$

vi) H(z) using billinear transformation:-

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.1533}{(s^2 - 0.484s + 0.1868)}\Bigg|_{s=\frac{1-z^{-1}}{1+z^{-1}}} \qquad \text{Since } \frac{2}{T} = 1$$

$$= \frac{0.1533}{\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 - 0.484\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.1868}$$

$$= \frac{0.1533(1+z^{-1})^2}{[1-z^{-1}]^2 - 0.484(1-z^{-1})(1+z^{-1}) + 0.1868(1+z^{-1})^2}$$

$$= \frac{0.1533(1+z^{-1})^2}{1-2z^{-1}+z^{-2} - 0.487(1-z^{-2}) + 0.1868(1+z^{-2}+2z^{-1})}$$

$$H(z) = \frac{0.1533z^{-2} + 0.3066z^{-1} + 0.0235}{1.6738z^{-2} - 1.626z^{-1} + 0.6998} \quad //$$

iv) Poles of $H_a(s)$.

Let us calculate the values of $\mu = \frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon} = \frac{1+\sqrt{1+0.484^2}}{0.484}$

$\mu = 4.36$.

$$a = \Omega_p \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 0.414\left[\frac{4.36^{\frac{1}{2}} - 4.36^{-\frac{1}{2}}}{2}\right] = 0.33$$

$$b = \Omega_p = \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.414\left[\frac{4.36^{1/2} + 4.36^{-1/2}}{2}\right] = 0.53$$

$$\phi_k = \frac{(2k+N+1)\pi}{2N} \quad ; \; k = 0,1,2 \ldots N-1$$

For $N=2$ $\quad \phi_k = \frac{(2k+2+1)}{2\times 2} \quad ; \; k=0,1.$

$$\therefore \quad \phi_k = \frac{(2k+3)}{4} \quad ; \; k = 0,1$$

k	$\phi_0 = \frac{2k+3}{4}$	$\sigma_k = a\cos\phi_k$	$\Omega_k = b\sin\phi_k$	$P_k = \sigma_k + j\Omega_k$
0	$\phi_0 = 2(0) + \frac{3}{4}$ $\phi_0 = 0.75$.	$\sigma_0 = 0.33\cos\left(\frac{3}{4}\right)$ $\sigma_0 = 0.242$.	$\Omega_0 = 0.53\sin\left(\frac{3}{4}\right)$ $\Omega_0 = 0.361$	$P_0 = 0.242 + j0.361$
1	$\phi_1 = 2(1) + \frac{3}{4}$ $\phi_1 = 2.75$.	$\sigma_1 = 0.33\cos(2.75)$ $\sigma_1 = -0.305$	$\Omega_1 = 0.53\sin(2.75)$ $\Omega_1 = 0.202$.	$P_1 = -0.305 + j0.202$.

$S_1 = P_0 = 0.242 + j0.361 \quad , \; S_1^* = 0.242 - j0.361$

v) System equation/function $H_a(s)$

$$H_a(s) = \frac{K}{(s-s_1)(s-s_1^*)} = \frac{K}{(s-0.242-j0.361)(s-0.242+j0.361)}$$

7b) Write the relation between analog and digital frequency in Bilinear transformation. (4M).

⇒ Analog frequency is denoted by (Ω) & digital frequency is denoted by (ω). Analog frequency is converted to digital a signal by sampling.

$$\cos(\Omega t+\phi) \xrightarrow[t=nTs]{\text{Sampling}} \cos(\Omega nT+\phi)$$

$$\cos(\omega n+\phi)$$

$$\omega=\Omega T_s \qquad \Rightarrow \quad T_s=\frac{1}{f_s}$$

$$\omega=\frac{\Omega}{f_s}$$

In analog frequency Ω unit is rad/sec. but in digital frequency ω is rad. Analog frequency are transformed using Laplace transform while digital frequency are transformed using z transform. The bilinear transform is mathematical relationship which converts the transfer function of a particular filter from laplace domain to z-domain and viceversa.

8a) Obtain direct form-I and direct form II realization of the LTI system governed by the relation.

$$y(n)=-\frac{3}{8}y(n-1)+\frac{3}{32}y(n-2)+\frac{1}{64}y(n-3)+x(n)+3x(n-1)$$

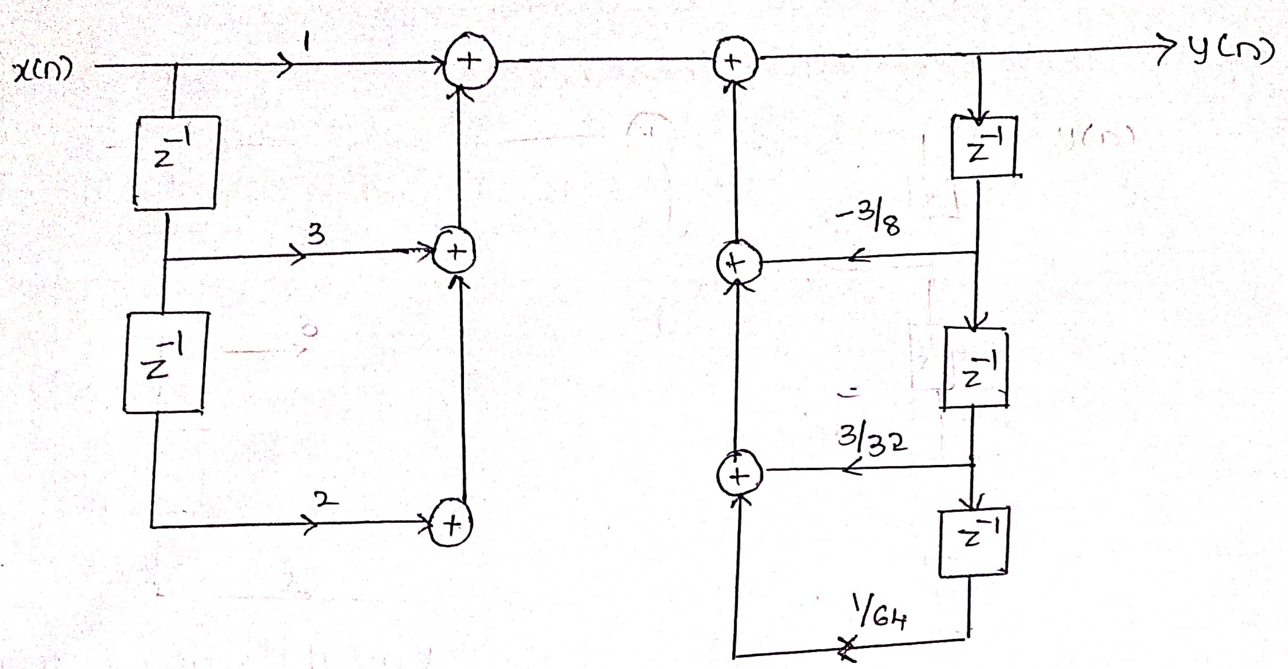
$$+2x(n-2)$$ (8M).

⇒ Direct form I :-

From given equation.

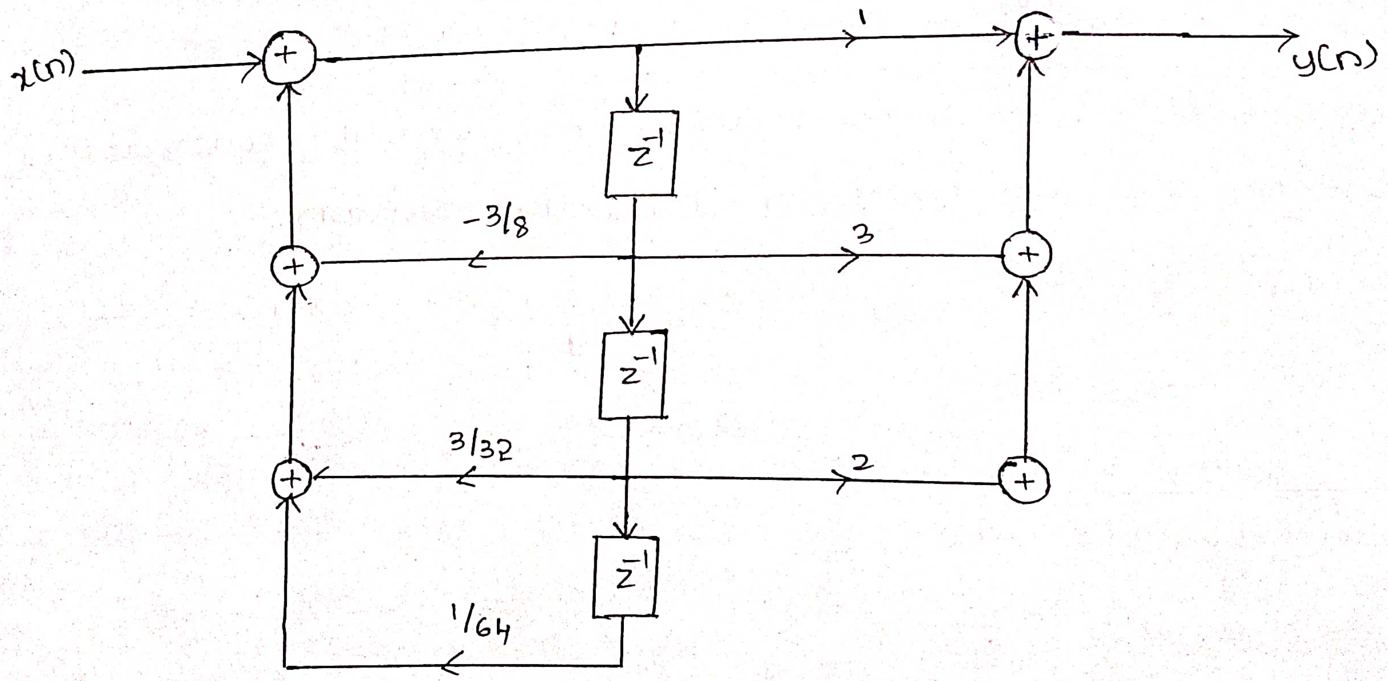
$$y(n)=\frac{1}{4}y(n-1)+\frac{3}{32}y(n-2)+\frac{1}{64}y(n-3)+x(n)+3x(n-1)$$

$$+2x(n-2).$$



Direct Form II :-

$$y(n) = -\frac{3}{8} y(n-1) + \frac{3}{32} y(n-2) + \frac{1}{64} y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$$



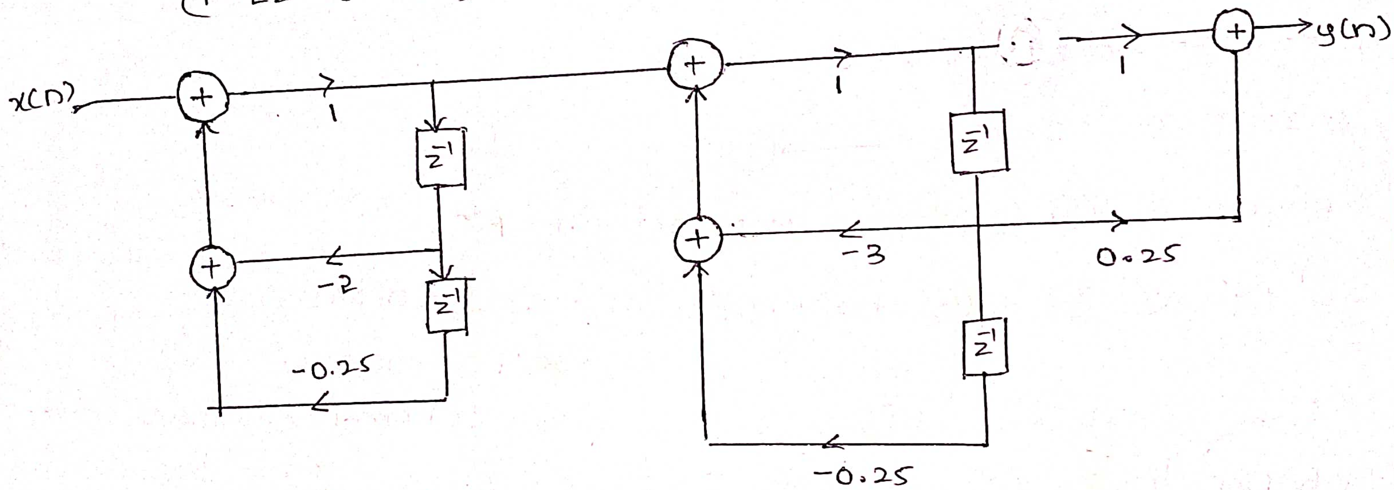
8b) Realise the given system in cascade and parallel form:- $H(z) = \dfrac{1+0.25z^{-1}}{(1-2z^{-1}+0.25z^{-2})(1-3z^{-1}+0.25z^{-2})}$ (8M)

Cascade:-

$\Rightarrow$ Given:- $H(z) = \dfrac{1+0.25z^{-1}}{(1-2z^{-1}+0.25z^{-2})(1-3z^{-1}+0.25z^{-2})}$

$H(z) = H_1(z).H_2(z)$.

$\therefore H_1(z) = \dfrac{1}{(1-2z^{-1}+0.25z^{-2})}$ & $H_2(z) = \dfrac{1+0.25z^{-2}}{1-3z^{-1}+0.25z^{-2}}$.



Parallel form:-

$$H(z) = \frac{1+0.25z^{-1}}{(1-2z^{-1}+0.25z^{-2})(1-3z^{-1}+0.25z^{-2})}$$

$$H(z) = \frac{z^4(1+0.25z^{-1})}{z^2(1-2z^{-1}+0.25z^{-2})(1-3z^{-1}+0.25z^{-2})z^2}$$

$$\frac{H(z)}{z} = \frac{z^3+0.25z^2}{(z^2-2z+0.25)(z^2-3z+0.25)}$$

$$\frac{z^3+0.25z^2}{(z^2-2z+0.25)(z^2-3z+0.25)} = \frac{Az+B}{(z^2-2z+0.25)} + \frac{Cz+D}{(z^2-3z+0.25)}$$

$$z^3+0.25z^2 = (Az+B)(z^2-3z+0.25) + (Cz+D)(z^2-2z+0.25).$$

$$z^3+0.25z^2 = Az^3-3Az^2+0.25Az+Bz^2-3Bz+0.25B+Cz^3-2Cz^2+0.25Cz+Dz^2-2Dz+0.25D.$$

$$z^3+0.25z^2 = (A+C)z^3 + (-3A+B-2C+D)z^2 + (0.25A-3B+0.25C-2D)z + (0.25B+0.25D)$$

Comparing coefficients on both sides.

$A+C=1$

$-3A+B-2C+D=0.25$

$0.25A-3B+0.25C-2D=0$

$0.25B+0.25D=0.$

Solving above eqn we get $A=1-C.$

$B+C+D=3.25 \rightarrow (1)$

$-3B+0C-2D=-0.25 \rightarrow (2)$

$0.25B+0.25D=0 \rightarrow (3)$

Solving eqn (1), (2), (3) we get.

$B=0.25$, $C=3.25$, $D=-0.25$; $A=1-C=1-3.25=-2.25$.

$$\frac{H(z)}{z} = \frac{-2.25z+0.25}{z^2-2z+0.25} + \frac{3.25z-0.25}{z^2-3z+0.25}.$$

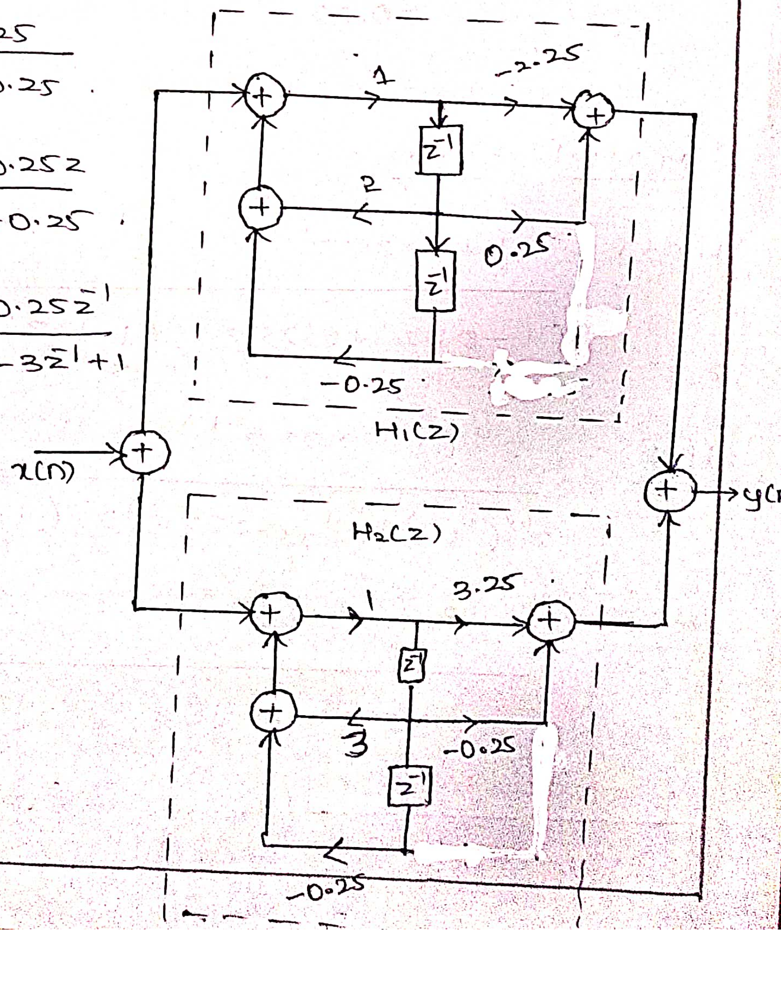
$$H(z) = \frac{-2.25z^2+0.25z}{z^2-2z+0.25} + \frac{3.25z^2-0.25z}{z^2-3z+0.25}.$$

$$H(z) = \frac{-2.25+0.25z^{-1}}{0.25z^{-2}-2z^{-1}+1} + \frac{3.25-0.25z^{-1}}{0.25z^{-2}-3z^{-1}+1}$$

$\Rightarrow H(z) = H_1(z) + H_2(z)$

$$\therefore H_1(z) = \frac{0.25z^{-1}-2.25}{0.25z^{-2}-2z^{-1}+1}$$

$$H_2(z) = \frac{-0.25z^{-1}+3.25}{0.25z^{-2}-3z^{-1}+1}$$



MODULE -05.

9a) The frequency response of a filter is described by:

$H(\omega) = j\omega$, $-\pi \le \omega \le \pi$. Design the filter using rectangular window. Take $N=7$. (8M)

⇒ The desired impulse response is given as,

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} j\omega\, e^{j\omega n} d\omega.$$

$$= \begin{cases} \frac{\cos n\pi}{n} & n \neq 0 \\ 0 & \text{for } n = 0 \end{cases}$$

To have linear phase, we will shift h(n) to right. This shift is equal to $\frac{N-1}{2} = \frac{7-1}{2} = 3$ samples i.e

$$h(n) = \begin{cases} \cos\pi\,(n-3) & \text{for } n \neq 3 \\ 0 & \text{for } n = 3 \end{cases}$$

Values of h(n) can be calculated as follows:

$$h(0) = \frac{\cos(-3\pi)}{-3} = \frac{1}{3}$$

$$h(1) = \frac{\cos(-2\pi)}{-2} = -1/2.$$

$$h(2) = \frac{\cos(-\pi)}{-1} = 1$$

$h(3) = 0.$

$$h(4) = \frac{\cos(\pi)}{1} = -1$$

$$h(5) = \frac{\cos(2\pi)}{2} = \frac{1}{2}.$$

$$h(6) = \frac{\cos(3\pi)}{3} = -\frac{1}{3}.$$

$\therefore h(n) = \{ 1/3, -1/2, 1, 0, -1, 1/2, -1/3 \}$

9b) Design a low pass digital filter to be used in A/D – H(z) – D/A structure that will have −3dB cutoff at 30π rad/sec and attenuation factor of 50dB at 45π rad/sec. The filter is required to have a liner phase and system will use sampling frequency of 100 sample/sec. (8M)

⇒ i) Given:-

3 dB cuttoff frequency of 30π rad/sec i.e.

$$\Omega_c = 30\pi \text{ rad/sec}$$

sampling frequency $F_{SF} = 100$ Hz.

Stopband attenuation of 50 dB at 45π rad/sec i.e

$A_s = 50$ dB for $\Omega_s = 45\pi$ rad/sec.

ii) To obtain specifications of equivalent digital filter:

WKT $\omega = 2\pi f$, & $f = \frac{F}{F_{SF}}$ hence we can write.

$$\omega = 2\pi \frac{F}{F_{SF}}.$$

Let $\Omega_c = \Omega_1 = 30\pi$ rad/sec & $\Omega_s = \Omega_2 = 45\pi$ rad/sec.

Hence $\omega_1 = \frac{\Omega_1}{F_{SF}} = \frac{30\pi}{100} = 0.3\pi$ rad/sample.

& $\omega_2 = \frac{\Omega_2}{F_{SF}} = \frac{45\pi}{100} = 0.45\pi$ rad/sample

Thus we have specification of equivalent digital filter as

3 dB attenuatn at $\omega_1 = 0.3\pi$ rad/sample.

$A_s = 50$ dB at $\omega_2 = 0.45\pi$ rad/sample.

iii) To select type of window:-

Here minimum stop band attenuation is 50dB. WKT hamming window provides −53 dB of stopband attenuation. Hence hamming window is used to get required attenuation of 50 dB.

iv> <u>To determine order of the filter :-</u>

For Hamming window, the width of the main lobe of window is $\frac{8\pi}{M}$, Hence $k\left(\frac{2\pi}{M}\right) = \frac{8\pi}{M} \Rightarrow k = 4$.

The order of filter is given by $N = k\left(\frac{2\pi}{\omega_2 - \omega_1}\right)$

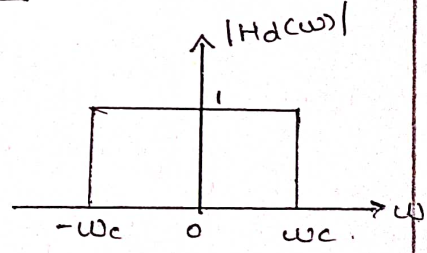
$$N = 4\left[\frac{2\pi}{0.45\pi - 0.3\pi}\right] = 53.33 .$$

Hence we design FIR filter for odd length. Hence we select next order integer $N = 55$.

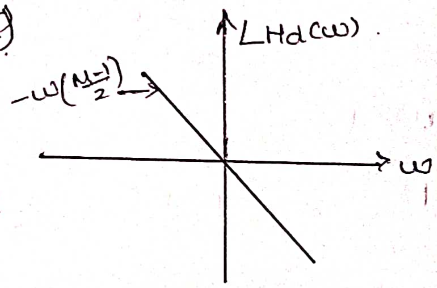
$\therefore$ Length of filter, $M = 55$.

v> <u>To obtain desired unit sample response $h_d(n)$:</u>

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 & \text{elsewere.} \end{cases}$$



This is ideal lowpass filter having cutoff frequency ω_c & linear phase response of $-\omega\left(\frac{M-1}{2}\right)$



The desired unit sample can be obtaind by taking inverse fourier transform of $H_d(\omega)$ i.e

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}\,d\omega .$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\left(\frac{M-1}{2}\right)} e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega\left(n-\frac{M-1}{2}\right)}\,d\omega . \rightarrow (1)$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega\left(n-\frac{M-1}{2}\right)}}{j\left(n-\frac{M-1}{2}\right)}\right]_{-\omega_c}^{\omega_c} = \frac{1}{2\pi}\left\{\frac{e^{j\omega_c\left(n-\frac{M-1}{2}\right)} - e^{-j\omega_c\left(n-\frac{M-1}{2}\right)}}{j\left(n-\frac{M-1}{2}\right)}\right\}.$$

$$= \frac{\sin\left[\omega_c\left(n-\frac{M-1}{2}\right)\right]}{\pi\left(n-\frac{M-1}{2}\right)} \quad \text{for } n \ne \frac{M-1}{2}.$$

when $n = \frac{M-1}{2}$, eq^n (1) becomes

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} d\omega = \frac{1}{2\pi}\left[\omega\right]_{-\omega_c}^{\omega_c} = \frac{\omega_c}{\pi}.$$

Thus we obtained $h_d(n)$ as.

$$h_d(n) = \begin{cases} \dfrac{\sin\left[\omega_c\left(n-\frac{M-1}{2}\right)\right]}{\pi\left(n-\frac{M-1}{2}\right)} & \text{for } n \neq \frac{M-1}{2} \\ \dfrac{\omega_c}{\pi} & \text{for } n = \frac{M-1}{2}. \end{cases}$$

vi) To obtain $h(n)$ by windowing:-

For hamming window :, $w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$ for $0 \le n \le M-1$

$h(n)$ is given as $h(n) = h_d(n)\,w(n)$.

Putting respective expressions in above equation

$$h(n) = \begin{cases} \dfrac{\sin\left[\omega_c\left(n-\frac{M-1}{2}\right)\right]}{\pi\left(n-\frac{M-1}{2}\right)} \cdot \left[0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)\right] & \text{for } n \neq \frac{M-1}{2} \\ \dfrac{\omega_c}{\pi}\left[0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)\right] & \text{for } n = \frac{M-1}{2}. \end{cases}$$

we have.

$M = 55$, hence $\frac{M-1}{2} = 27$.

$|||^{rly}$ $\omega_1 = \omega_c = 0.3\pi$ rad/sample.

$$h(n) = \begin{cases} \dfrac{\sin\left[0.3\pi(n-27)\right]}{\pi(n-27)} \cdot \left[0.54 - 0.46\cos\left(\frac{\pi n}{27}\right)\right] & \text{for } n \neq 27. \\ 0.3\left[0.54 - 0.46\cos\left(\frac{\pi n}{27}\right)\right] & \text{for } n = 27. \end{cases}$$

$\therefore$ The values of unit sample response can be obtained from above equation to get required linear phase FIR filter.

a) Deduce the equation for the following frequency spectrum or rectangular window sequence defined by.

$$W_r(n) = \begin{cases} 1, & -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}. \\ 0, & \text{Otherwise}. \end{cases}$$

(6M)

⇒ DTFT of window function is given by

$$W(\omega) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} w(n) e^{-j\omega n} = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{-j\omega n}.$$

Here let us use $\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1} - a^{N_2+1}}{1-a}.$

$$W(\omega) = \frac{(e^{-j\omega})^{-\frac{N-1}{2}} - (e^{-j\omega})^{\frac{N-1}{2}+1}}{1 - e^{-j\omega}}.$$

By rearranging above equation,

$$W(\omega) = \frac{e^{j\frac{\omega N}{2}} e^{j\frac{\omega}{2}} - e^{-j\frac{\omega N}{2}} e^{-j\frac{\omega}{2}}}{e^{j\frac{\omega}{2}} e^{-j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} e^{-j\frac{\omega}{2}}} = \frac{e^{j\frac{\omega N}{2}} - e^{-j\frac{\omega N}{2}}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} = \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

Frequency spectrum will be, $|W(\omega)| = \frac{\left|\sin\left(\frac{\omega N}{2}\right)\right|}{\left|\sin\left(\frac{\omega}{2}\right)\right|}$ //.

10b) A lowpass filter has the desired frequency response:

$$H_d(\omega) = \begin{cases} e^{-j\omega 3}, & 0 < \omega < \pi/2. \\ 0, & \frac{\pi}{2} \le \omega \le \pi. \end{cases}$$

Determine h(n) based on frequency sampling method. Take k=7

(6M)

⇒ Given:- $H_d(\omega) = \begin{cases} e^{-j\omega 3}, & \text{for } 0 \le \omega \le \pi/2. \\ 0, & \text{for } \frac{\pi}{2} \le \omega \le \pi. \end{cases}$

i) To sample $H_d(e^{j\omega})$.

To sample $H_d(e^{j\omega})$, put $\omega = \frac{2\pi k}{N}$, $k = 0, 1, \ldots N-1$.

For $N = 7$, $\omega = \frac{2\pi k}{N}$,

$\therefore H_d(e^{j\omega})$ becomes,

$$H(k) = \begin{cases} e^{-j\frac{2\pi k}{7}\cdot 3} & \text{for } 0 \le \frac{2\pi k}{7} \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le \frac{2\pi k}{7} \le \pi \end{cases}$$

$$H(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le k \le \frac{7}{4} \\ 0 & \text{for } \frac{7}{4} \le k \le \frac{7}{2} \end{cases}$$

The range of 'k' in above equation can be written in nearest integer as follows.

$$H(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le k \le 2 \\ 0 & \text{for } 2 < k \le 4 \end{cases}$$

ii) To obtain $h(n)$

The expression for $h(n)$ is

$$h(n) = \frac{1}{M}\left\{ H(0) + 2\sum_{k=1}^{P} Re\left[H(k)e^{j2\pi kn/M}\right]\right\}$$

Here $H(0) = 1$ for $k = 0$, $M = N = 7$

$$P = \frac{M-1}{2} = \frac{7-1}{2} = 3.$$

Hence above equation becomes,

$$h(n) = \frac{1}{7}\left\{ 1 + 2\sum_{k=1}^{2} Re\left[e^{-j\frac{6\pi k}{7}} \cdot e^{j2\pi kn/7}\right]\right\}$$

$$= \frac{1}{7}\left\{ 1 + 2\sum_{k=1}^{2} Re\left[e^{-j2\pi k(3-n)/7}\right]\right\}$$

Here $Re[e^{j\theta}]$ is $\cos\theta$.

Hence $h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2}\cos\left[\frac{2\pi k(3-n)}{7}\right]\right\}$, $n = 0, 1, 2 \ldots 6$.

10 c) Realize the linear phase FIR filter having the following impulse response:

$$h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4)$$ (4M)

⇒ Given :-

$$h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4).$$

It can be written as.

$$h(n) = \left\{1, \frac{1}{4}, -\frac{1}{8}, \frac{1}{4}, 1\right\}$$

Here $h(0) = 1$, $h(1) = 1/4$, $h(2) = -1/8$, $h(3) = 1/4$, $h(4) = 1$.

Here $M=5$, the impulse response is symmetric. i.e $h(n)$ satisfy the following condition

$$h(n) = h(M-1-n).$$

i.e $h(0) = h(4)$, $h(1) = h(3)$.

As 'M' is odd, we have to use linear phase structure.