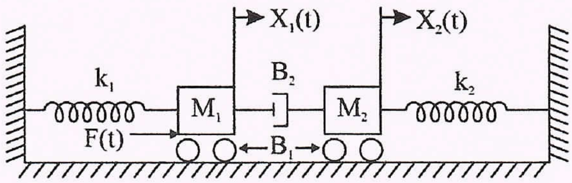
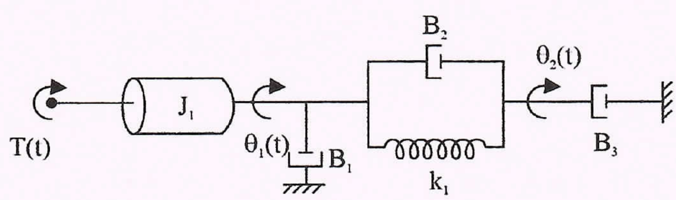
Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN | | | | | | | | | |

Fourth Semester B.E. Degree Examination
Subject CONTROL SYSTEMS

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02.
03.

		Module -1	*Bloom's Taxonomy Level	Marks
Q.01	a	Compare closed loop and open loop control systems. Give one example for each.	L1 CO1	06 M
	b	What are the components of a closed loop control system?	L1 CO1	04 M
	c	Find the Force Voltage analogous electrical network for the given Translational Mechanical system shown in Fig 1(c).  FIG 1(C)	L1, L2 CO1	10 M
		OR		
Q.02	a	What are the effects of negative feedback in control systems?	L1 CO1	05 M
	b	What are the requirements of a good control system?	L1 CO1	05 M
	c	Find the transfer function $\theta_2(s) / T(s)$ and $\theta_1(s) / T(s)$ for the Rotational Mechanical system shown in Fig 2 c.  FIG 2 (C)	L1, L2 CO1	10 M

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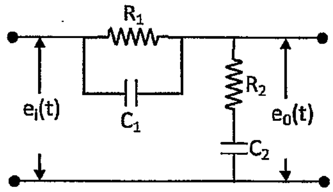
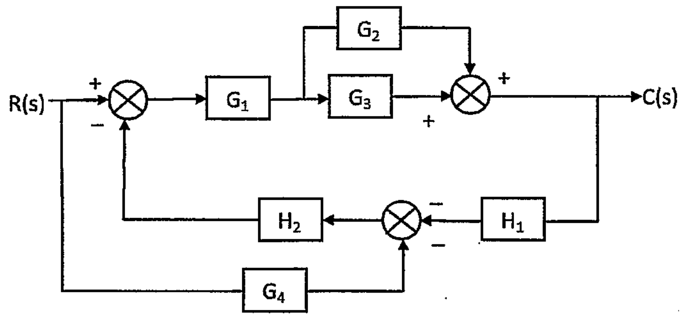
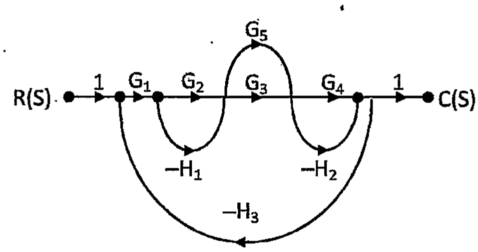
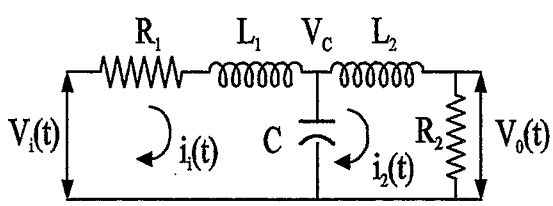
		Module-2		
Q. 03	a	Derive the transfer Function for the lead lag network shown in Fig 3 a. when $R_1=R_2=1\ M\Omega$, $C_1 = C_2 = 1\ \mu F$.  FIG 3 (a)	L1 CO2	06 M
	b	Write any four rules of reducing Block diagram.	L1 CO2	04 M
	c	Find the transfer function by reducing the block diagram shown in fig 3(c).  FIG 3 (c)	L1, L2 CO2	10 M
		OR		
Q.04	a	Define Mason's gain formula in Signal flow Graph.	L1 CO2	04 M
	b	Find Transfer function by block diagram reduction for the signal flow graph shown in Fig 4 b.  FIG 4 (b)	L1, L2 CO2	08 M
	c	Find the transfer function by Mason's Gain formula for the Signal flow Graph shown in Fig 4 b.	L1,L2 CO2	08 M
		Module-3		
Q. 05	a	List the standard test inputs used in control system with their Laplace transform.	L1 CO3	04 M
	b	Derive the step input response of a first order system.	L1, L2 CO3	08 M
	c	A unity negative feedback control system with $G(s) = \frac{100}{s^2(s+4)(s+12)}$ (i) What is the type of the system? (ii) Find static error coefficients. (iii) Find steady state error if the input is **r(t) = $2t^2 + 5t + 10$.**	L1,L2 CO3	08 M
		OR		

Q. 06	a	Starting from the output equation C(t) derive expressions for: (i) Rise time (t_r) (ii) Peak overshoot (M_p) of an under damped second order system subjected to unit step input.	L1, L2 CO3	08 M
	b	For servomechanism system $G(s)=\frac{K_1}{s^2}$ and $H(s)=1+K_2s$. Determine the value of K_1, K_2 so that peak overshoot is 0.25 and peak time is 2 seconds for a unit step input.	L3 CO3	08 M
	c	With a neat block diagram explain PI and PD controllers.	L1 CO3	04 M
		Module-4		
Q. 07	a	Explain Routh – Hurwitz criterion for stability of the system and what are its limitations.	L1 CO4	04 M
	b	Find the range of K so that system with characteristic equation as: $S^4 + 22 S^3 + 10 S^2 + S + K = 0$ is stable. Also find frequency of oscillation at marginal value of K.	L1,L2 CO4	06 M
	c	Plot root locus for $s^3 + 6s^2 + 8s + K = 0$	L2, CO4	10 M
		OR		
Q. 08	a	For a closed loop control system, $G(s)=\frac{100}{s(s+8)}$ $H(s)=1$, determine resonant peak and resonant frequency.	L3 CO4	04 M
	b	State any three rules of plotting root locus.	L1 CO4	06M
	c	Sketch the Bode plot for open loop transfer function. $G(s)H(s)=\frac{K}{s(1+0.2s)(1+0.05s)}$ Determine the value of K for gain margin (GM) of 6 dB.	L2,L3 CO5	10 M
		Module-5		
Q. 09	a	State mapping theorem and explain Nyquist stability criterion.	L1 CO5	04 M
	b	Draw polar plot for $G(s)H(s)=\frac{10}{s(s+1)(s+2)}$ Find GM and PM, also comment on stability.	L1 CO5	10 M
	c	Explain lead lag compensating networks.	L1,CO5	06 M
		OR		
Q. 10	a	What are the advantages of State Space analysis?	L1,CO5	04 M
	b	Obtain the state equations for the electrical network shown in fig 10 b.  FIG 10 (b)	L2, L3 CO5	08 M
	c	Compute the STM for the system given system matrix $A=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ using Laplace approach technique.	L2,L3 CO5	08 M

*Bloom's Taxonomy Level: Indicate as L1, L2, L3, L4, etc. It is also desirable to indicate the COs and POs to be attained by every bit of questions.

Model Question paper-1 (18EC43)

Module-1

Q1a. Compare closed loop and open loop control systems. Give one example for each. — 06M —

Open loop	Closed loop
1) Feedback Element is absent	1) Feedback Element is present
2) Simple and Economical	2) Complex and costlier
3) Consume less power	3) Consume more power

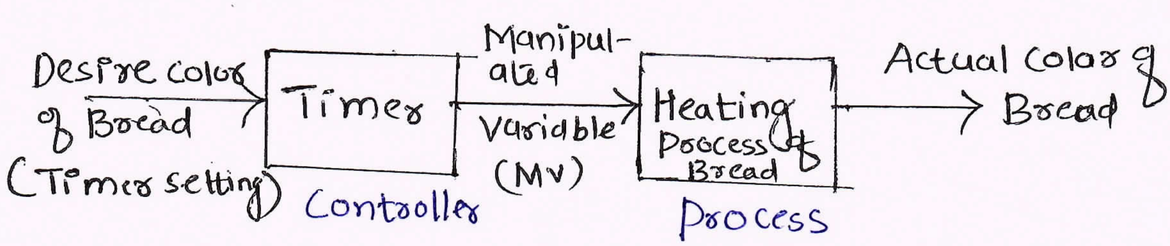
— 02M —

Example of open loop control system

Bread Toaster

Objective: To toast the Bread with desire color.

Block Diagram

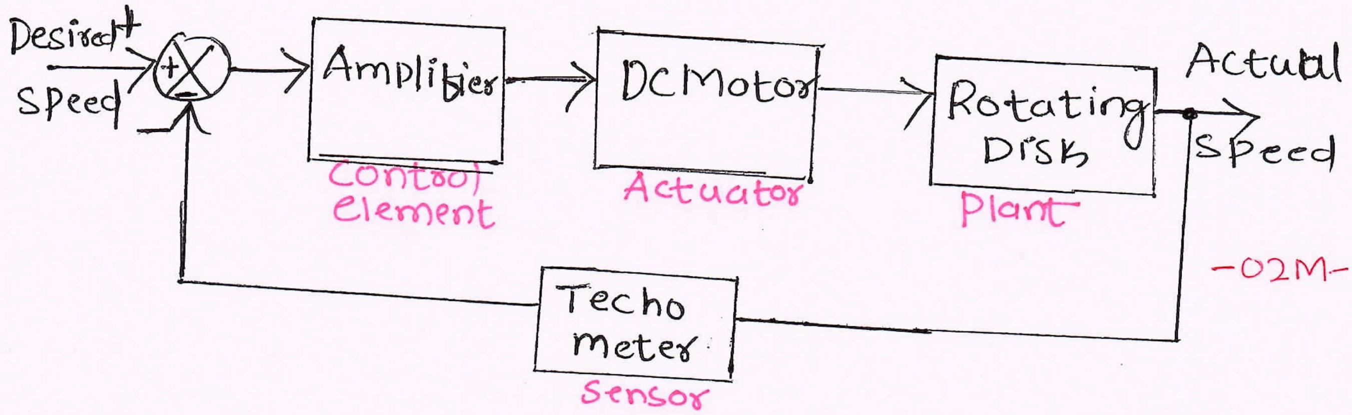


— 02M —

Example of closed loop control system

Rotating Disk Speed Control

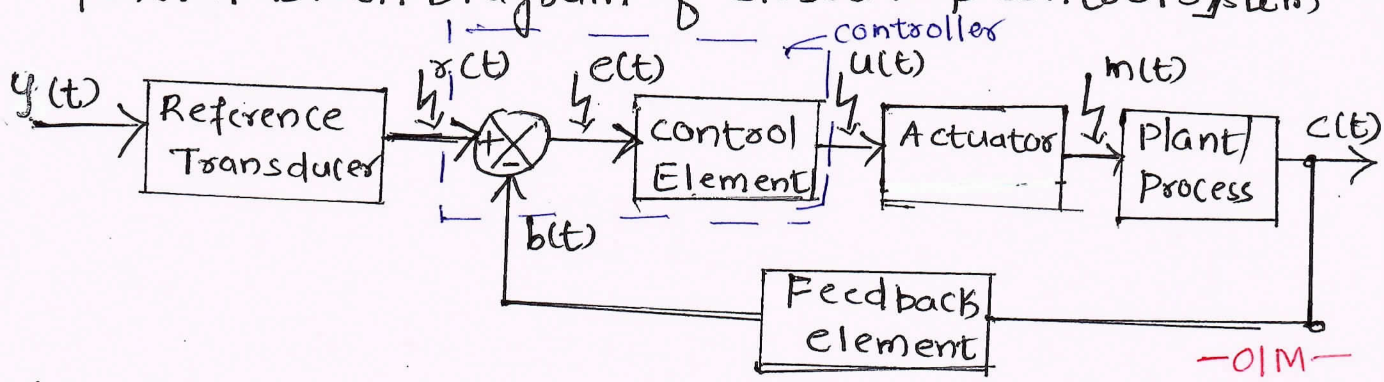
Objective: To control the speed of rotating disk



-02M-

Q1b. What are the components of a closed loop control system. -04M-

General Block Diagram of Closed loop control system



-01M-

* Plant : The system which is to be controlled or regulated is called plant /process

* Controller : The component of control system that generates control signal to reduce the deviation of the actual value from the desired value to almost zero or lowest possible value.

* Actuator : An actuator is device that converts energy to motion -02M

* Feedback element : Feedback element senses the output & feed to comparator

* Reference transducer : Converts the (command) input signal in to suitable form for controller

* e(t) : error signal * u(t) = control signal * m(t) = manipulated variable, * c(t) = controlled variable * b(t) = feedback signal -01M-

Q1c. Find the Force Voltage analogous electric network for the given Translational Mechanical System shown in Fig 1(c) -10M-

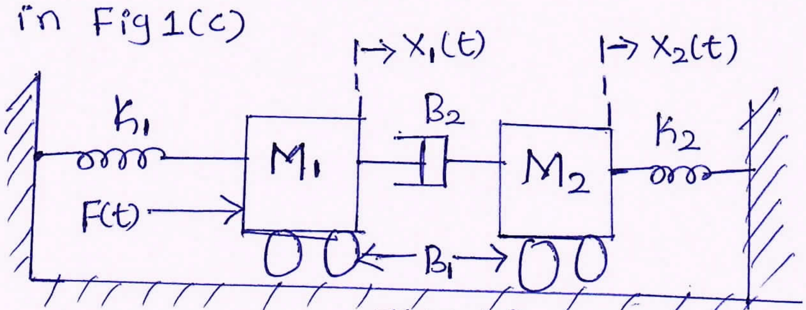
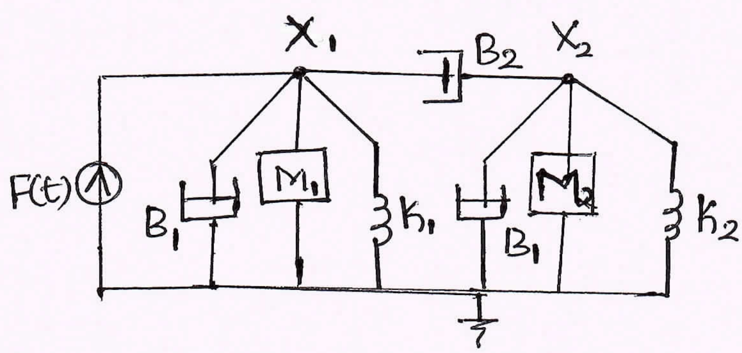


Fig 1(c)

Step 1 Nodal Diagram



-02M-

Step 2 Netwon's Law at x_1

$$F(t) = M_1\ddot{x}_1 + K_1x_1 + B_1\dot{x}_1 + B_2(\dot{x}_1 - \dot{x}_2) \quad -①$$

Netwon's Law at x_2

$$M_2\ddot{x}_2 + K_2x_2 + B_1\dot{x}_2 + B_2(\dot{x}_2 - \dot{x}_1) = 0 \quad -②$$

-03M-

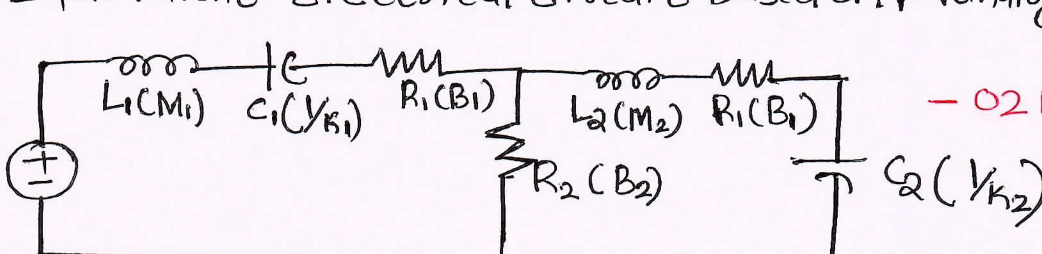
Step 3 Equivalent Electrical ckt D.E eqn based on F-V analogy

$$V = L_1\ddot{q}_1 + \frac{1}{C_1}q_1 + R_1\dot{q}_1 + R_2(\dot{q}_1 - \dot{q}_2) \quad -③$$

$$L_2\ddot{q}_2 + \frac{1}{C_2}q_2 + R_1\dot{q}_2 + R_2(\dot{q}_2 - \dot{q}_1) = 0 \quad -④$$

-03M-

Step 4 Equivalent electrical circuit based on F-V analogy



-02M-

-OR-

Q2a. What are the effects of Negative feedback -05M.

① The feedback reduces the effect of the disturba[nce]
with presence of feedback; the system output due to noise acting alone is

$$Y = \left(\frac{G_2}{1+G_1G_2H}\right)n$$ — noise signal

without feedback

$$y = G_2 n$$

-01M-

② The overall gain of the system reduces due to feedback.

$$y = \left(\frac{G}{1+GH}\right)r$$ — with feedback.

-01M-

③ The feedback reduces the sensitivity of the control systems to the parameter varation

$$S_G^M = \frac{\partial M}{\partial G} \cdot \frac{G}{M} = \frac{1}{GH+1}$$

-02M-

If GH is +ve mag of sensitivity can be made small by increasing GH.

④ The feedback increases the stability of system, by adding feed path with gain 'F' for unstable system. The overall system can be made stable by properly selecting the outer loop feedback gain 'F'

-01M-

Q2b. What are requirements of a good Control system. -05M

(E)

① **Accuracy**: A Good control system must be highly accurate -01M

② **Sensitivity**: A Good control system should very insensitivity to parameter variations but Sensitive to the input commands. -01M-

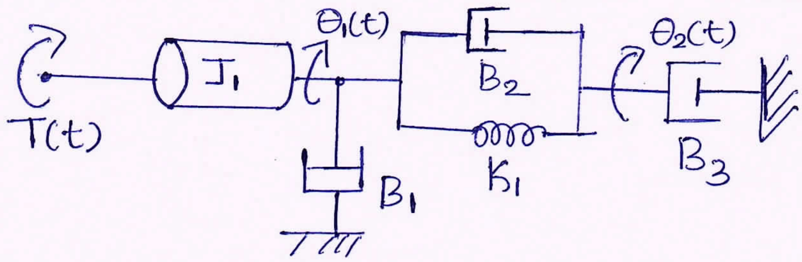
③ **Stability**: A Good control system should be stable Over the variation of parameter in system -01M-

④ **Bandwidth**: A Good Control system should give Satisfactory output over the range of input frequency -01M-

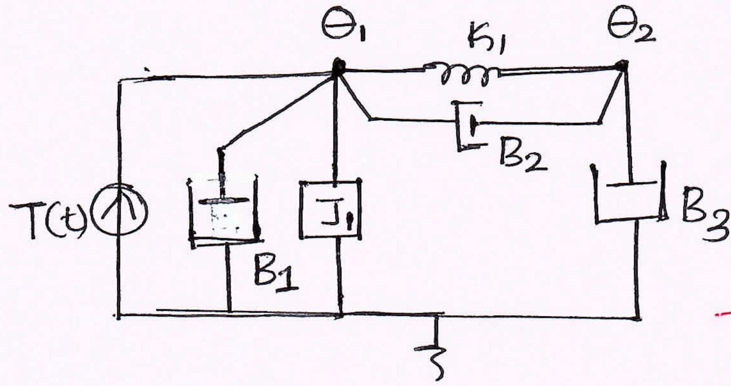
⑤ **Oscillations**: A Good Control system should Exhibit suitable damping. -01M-

Q2c. Find the transfer function $\Theta_2(s)/T(s)$ and $\Theta_1(s)/T(s)$ for the Rotational Mechanical System shown in Fig2(c)

—10M—



Step1 Nodal Diagram



—02M—

Step2 Newton's Law at θ_1

$$T(t) = J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_1(\theta_1-\theta_2) + B_2(\dot{\theta}_1-\dot{\theta}_2) \quad -①-$$

Newton's Law at θ_2

$$K_1(\theta_2-\theta_1) + B_2(\dot{\theta}_2-\dot{\theta}_1) + B_3\dot{\theta}_2 = 0 \quad -②-$$

—02M—

Step3 let us take Laplace Transform of Eqn ① & Eqn ②

$$T(s) = J_1 s^2\Theta_1(s) + B_1 s\Theta_1(s) + K_1(\Theta_1(s)-\Theta_2(s)) + B_2 s(\Theta_1(s)-\Theta_2(s)) \quad -③$$

—02M—

$$0 = K_1(\Theta_2(s)-\Theta_1(s)) + B_2 s(\Theta_2(s)-\Theta_1(s)) + B_3 s\Theta_2(s) \quad -④$$

Step4 Rearranging Eq ③ & solving for $\Theta_1(s)$ from Eqn ④

$$T(s) = \Theta_1(s)[J_1s^2 + s(B_1+B_2)+K_1] - \Theta_2(s)[B_2s+K_1] \quad -③a$$

$$\theta_2(s)\left[K_1 + s(B_2+B_3)\right] - \theta_1(s)\left[K_1 + B_2 s\right] = 0$$

$$\theta_1(s) = \frac{\left[K_1 + s(B_2+B_3)\right]\theta_2(s)}{K_1 + B_2 s} \quad -⑤$$

lly

$$\theta_2(s) = \frac{\left[K_1 + B_2 s\right]\theta_1(s)}{K_1 + s(B_2+B_3)} \quad -⑥$$

Step 5 Sub^n of eq^n ⑤ in (3a)

$$T(s) = \frac{\left[J_1 s^2 + s(B_1+B_2)+K_1\right]\left[K_1 + s(B_2+B_3)\right]\theta_2(s)}{K_1 + B_2 s} - \theta_2(s)\left[B_2 s + K_1\right]$$

$$T(s) = \frac{\left[J_1 s^2 + s(B_1+B_2)+K_1\right]\left[K_1 + s(B_2+B_3)\right]\theta_2(s)}{K_1 + B_2 s} - \theta_2(s)\left[B_2 s + K_1\right]$$

$$T(s) = \theta_2(s)\left[\frac{\left[J_1 s^2 + s(B_1+B_2)+K_1\right]\left[K_1 + s(B_2+B_3)\right] - \left[B_2 s + K_1\right]^2}{K_1 + B_2 s}\right]$$

– 02M –

$$\frac{\theta_2(s)}{T(s)} = \frac{K_1 + B_2 s}{\left(J_1 s^2 + s(B_1+B_2)+K_1\right)\left(K_1 + s(B_2+B_3)\right) - \left(B_2 s + K_1\right)^2}$$

Step 6 Sub^n of eq^n ⑥ in (3a) – 01M –

$$T(s) = \left(J_1 s^2 + s(B_1+B_2)+K_1\right)\theta_1(s) - \frac{\left(K_1 + B_2 s\right)^2\theta_1(s)}{K_1 + s(B_2+B_3)}$$

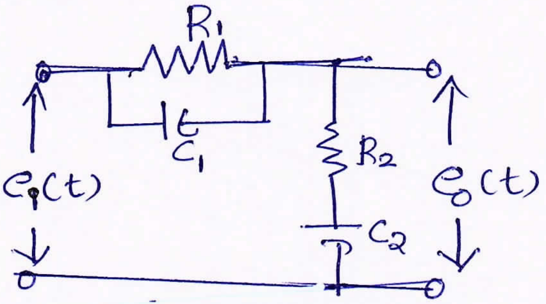
$$\frac{\theta_1(s)}{T(s)} = \frac{K_1 + s(B_2+B_3)}{\left(J_1 s^2 + s(B_1+B_2)+K_1\right)\left(K_1 + s(B_2+B_3)\right) - \left(K_1 + B_2 s\right)^2}$$

– 01M –

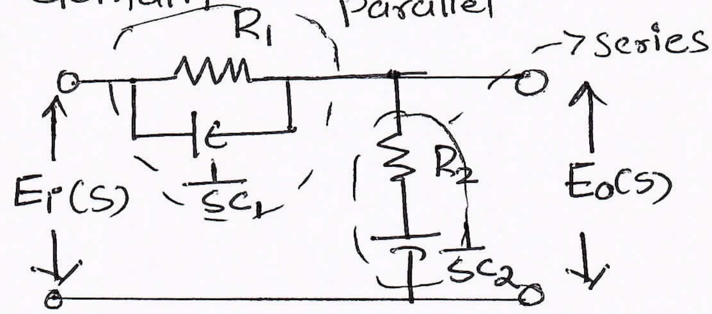
Module-2

Q3a. Derive the transfer function for the lead lag network shown in Fig 3a. When $R_1 = R_2 = 1M\Omega$, $C_1 = C_2 = 1\mu F$

—06M—



Step1 Transform the Given ckt in Laplace domain



—01M—

Step2 Combining Parallel and series elements

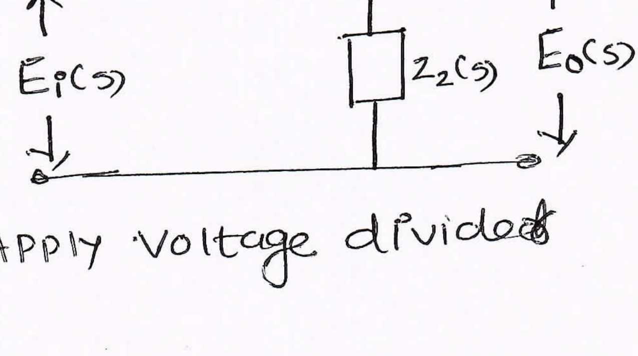
$$Z_1(S) = \frac{R_1 \times \frac{1}{SC_1}}{R_1 + \frac{1}{SC_1}}$$

$$Z_1(S) = \frac{R_1}{SC_1R_1 + 1}$$

$$Z_2(S) = R_2 + \frac{1}{SC_2}$$

$$Z_2(S) = \frac{SC_2R_2 + 1}{SC_2}$$

step3 Equivalent ckt after series and parallel combn



—02M—

Step4 Apply voltage divider

$$E_o(s) = \frac{E_i(s)\left[\frac{sC_2R_2+1}{sC_2}\right]}{\frac{R_1}{sC_1R_1+1} + \frac{sC_2R_2+1}{sC_2}} \qquad E_o(s) = \frac{E_i(s)\,Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$E_o(s) = \frac{E_i(s)\,(sC_2R_2+1)}{\frac{sC_2R_1}{sC_1R_1+1} + sC_2R_2+1} = \frac{E_i(s)(sC_2R_2+1)(sC_1R_1+1)}{sC_2R_1+(sC_2R_2+1)(sC_1R_1+1)}$$

-02M-

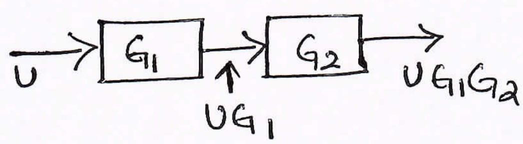
$C_1R_1 = 1$, $C_2R_2 = 1$, $R_1C_2 = 1$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{(s+1)^2}{(s+1)^2+s}}$$

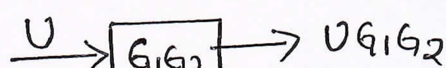
-01M-

Q3b. Write any four rules of Reducing Block diagram -04M-

① Combining Blocks in cascade



Original Diagram

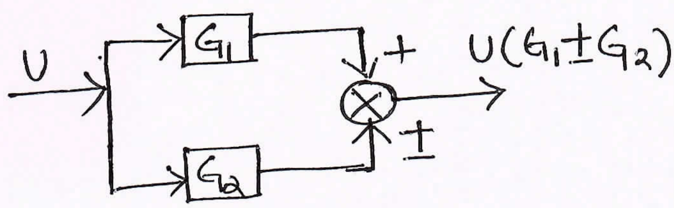


Equivalent Diagram

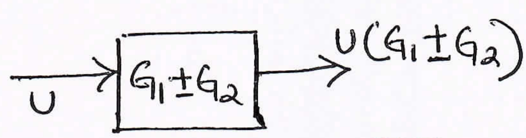
-01M-

② Combining Blocks in parallel

Original diagram



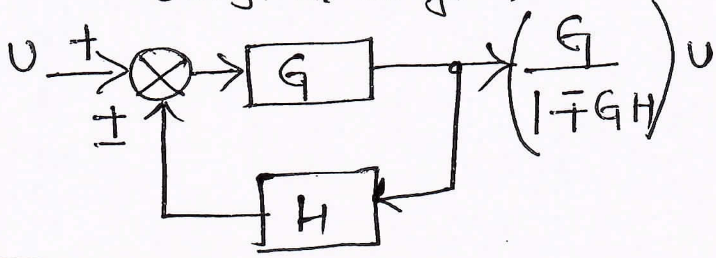
Equivalent diagram



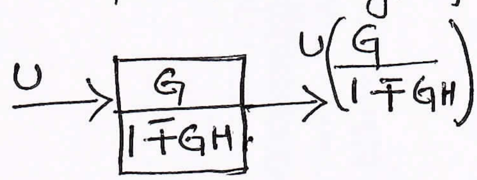
-01M-

③ Eliminating feedback loop

Original diagram



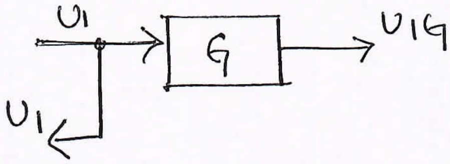
Equivalent Diagram



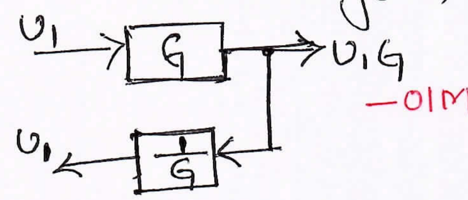
-01M-

(4) Moving Take off Point after Block

Original diagram



Equivalent diagram



—01M

Q3C. Find the transfer function by reducing the blockdiagram shown in fig 3(c) -10M-

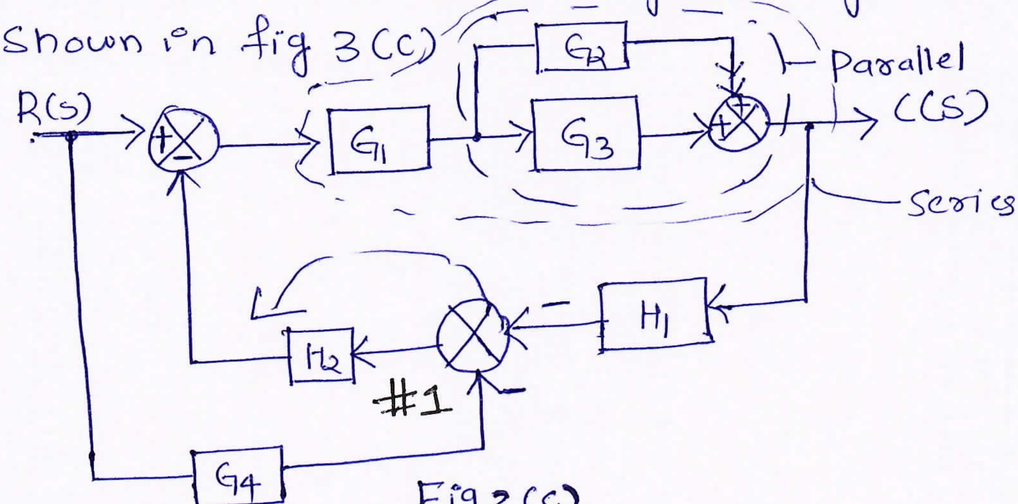
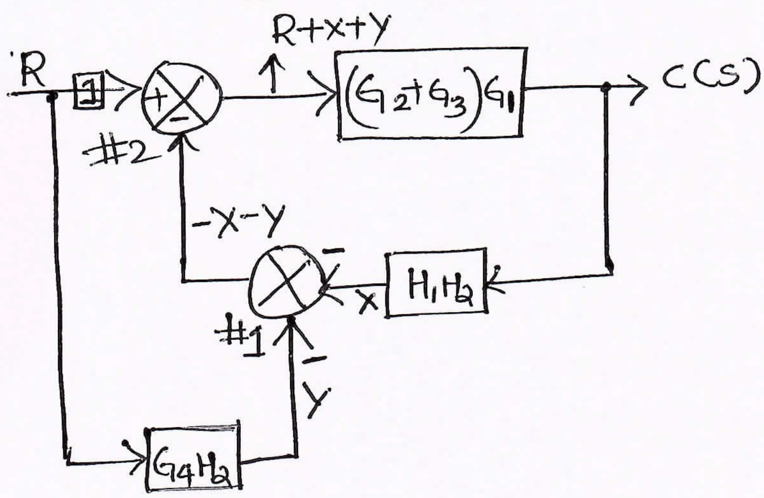


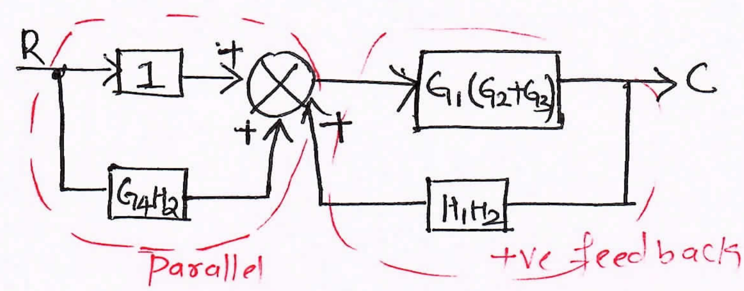
Fig 3(c)

Step1 combining G_2 & G_3 block in parallel (G_2+G_3), Parallel Combination is combined in Series with G_1 $(G_2+G_3)G_1$, shifting summer #1 in forward direction. -04M-



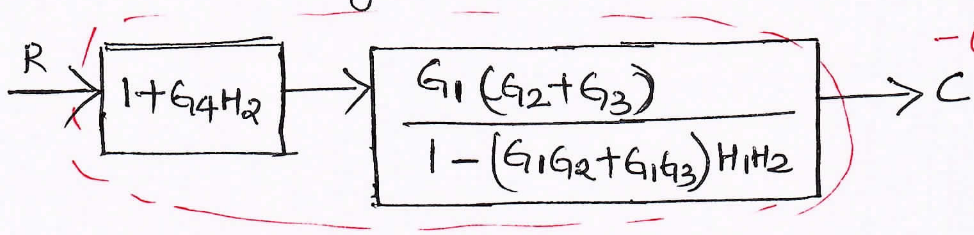
Step2 Combining summer #1 and #2

Step2 ---



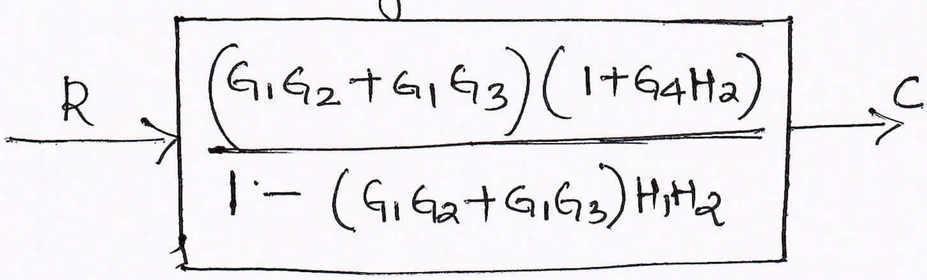
−02M

Step3 Combining parallel block G_4H_2 with Gain of 1 & eliminating +ve feedback



−02M−

Step4 Combining Blocks in Series



−02M−

−OR−

Q4a. Define Mason's gain formula in Signal flow graph. −04M−

Mason's gain formula for the determination of the Overall system gain (Transfer function) is given by

$$T = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K$$

T = Overall gain of the system (Transfer function)

−02M−

Δ : Determinant of the graph

$\Delta = 1 -$ (sum of individual loop gain) + (Sum of gain Products of all Combⁿ of two non-touching loops) − (Sum of gain Products of all Combⁿ of three non-touching loops) + - - -

N : No' of forward path in the Signal flowgraph

−02M−

Δ_k : The value of delta for the part of the graph not touching the k^{th} forward path

P_k : Path gain of k^{th} forward path

Q4b. Find Transfer function by Block diagram reduction for the signal flow graph shown in Fig 4(b)

–08M–

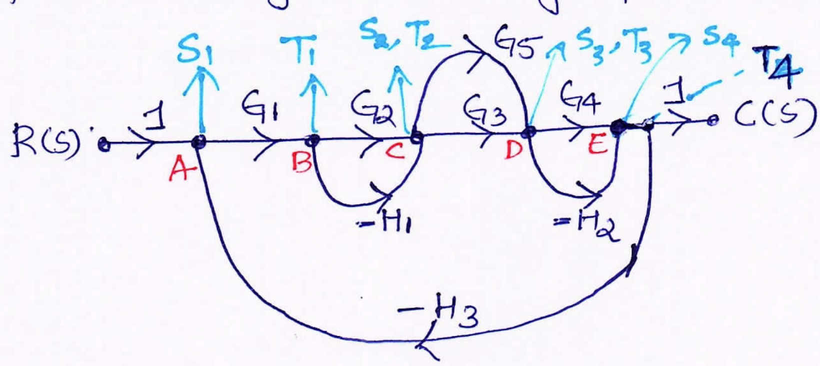
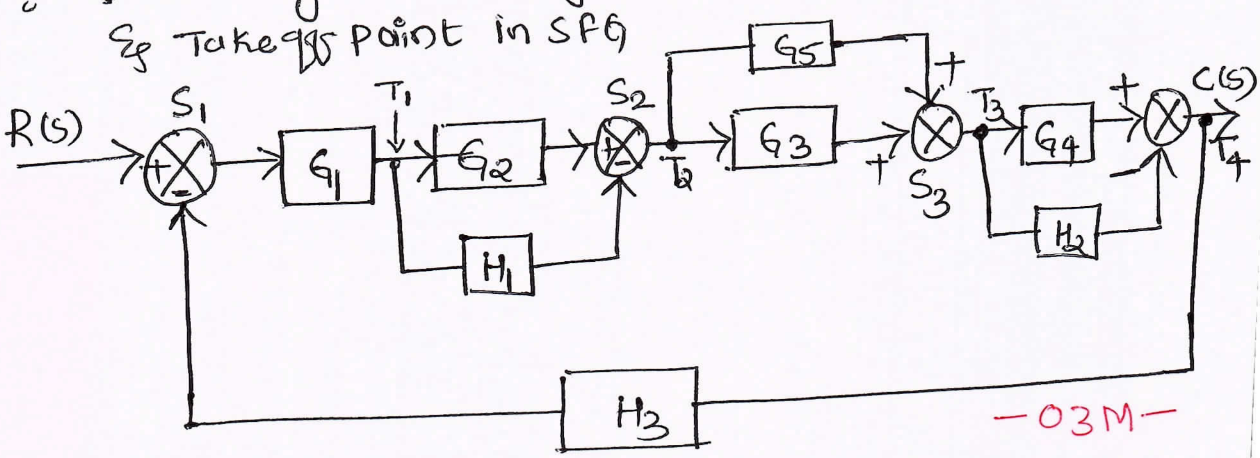


Fig 4(b)

Step1 : Identifying summer & Takeoff point in SFG

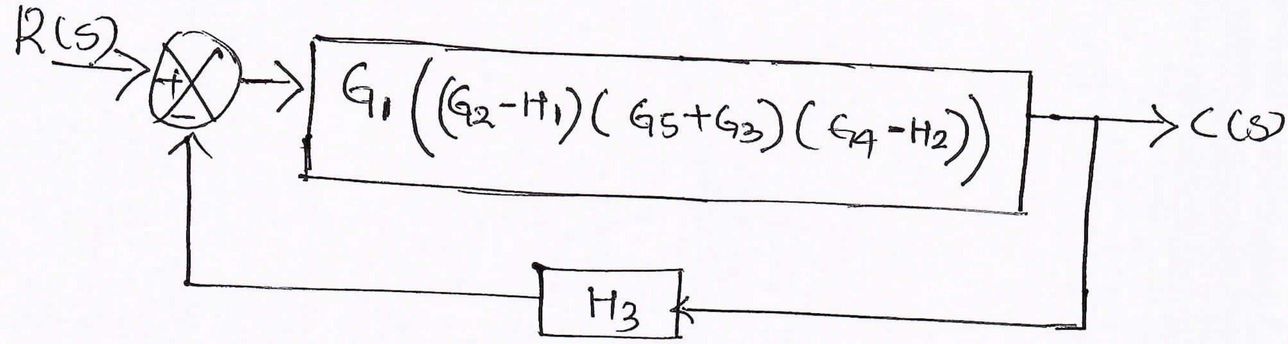
Step2 : Drawing Block diagram with identified summer & Takeoff point in SFG



–03M–

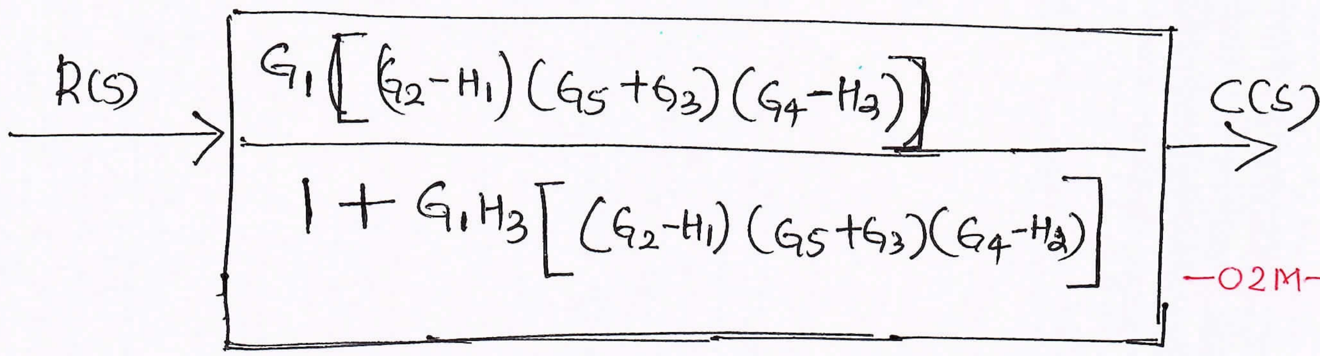
Step3 : Reducing Block diagram, (G_2, H_1), (G_5, G_3), (G_4, H_2) are all in parallel $(G_2 - H_1)$, $(G_5 + G_3)$, $(G_4 - H_2)$, this whole parallel combn is series with each & this whole combn is series with G_1.

–03M–



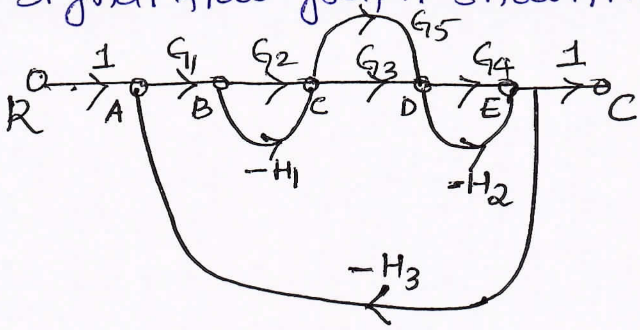
—02M—

Step4 Eliminating −ve feedback



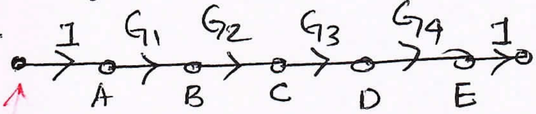
—02M—

Q4C Find the Transfer function by Mason's Gain formula for the signal flow graph shown in Fig (4b) —08M—



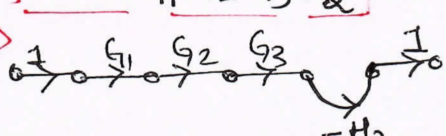
Step1 No' of forward path N=8

Step2 ↓ [Individual forward path with its gain]



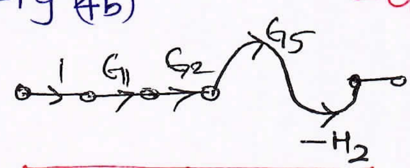
$P_1 = G_1G_2G_3G_4$

$P_2 = -G_1G_2G_3H_2$

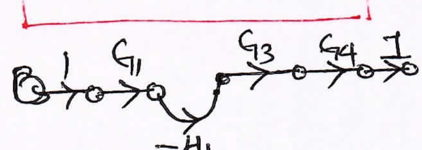


—03M—

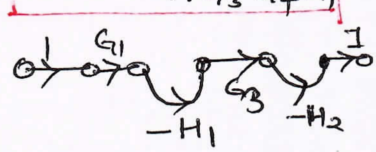
$P_3 = G_1G_2G_5G_4$



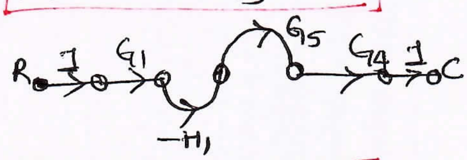
$P_4 = -G_1G_2G_5H_2$



$P_5 = -G_1G_3G_4H_1$



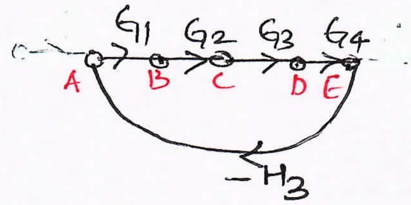
$P_6 = G_1G_3H_1H_2$



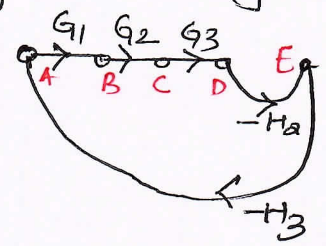
$P_7 = -G_1G_5G_4H_1$

$P_8 = G_1G_5H_1H_2$

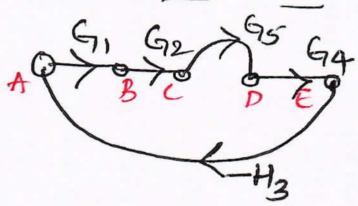
Step3 Individual loops along with its gain



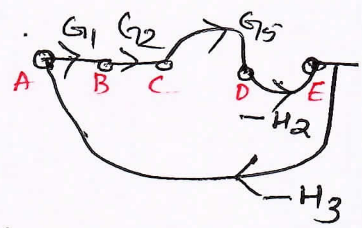
$L_1 = -G_1G_2G_3G_4H_3$



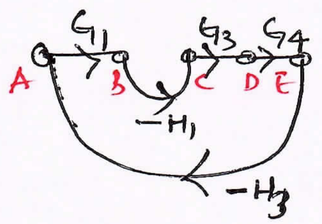
$L_2 = +G_1G_2G_3H_2H_3$



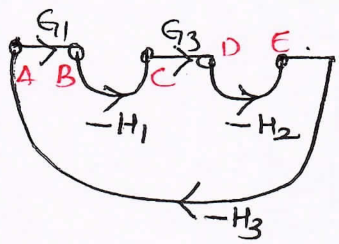
$L_3 = -G_1G_2G_4G_5H_3$



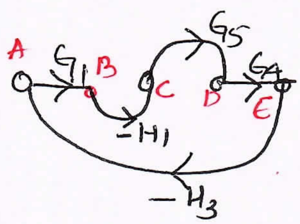
$L_4 : G_1G_2G_5H_2H_3$



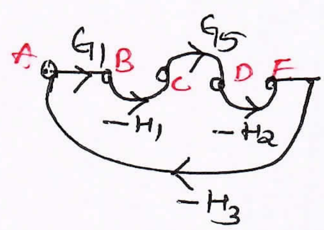
$L_5 = G_1G_3G_4H_1H_3$



$L_6 = -G_1G_3H_1H_2H_3$



$L_7 = G_1G_4G_5H_1H_3$



— 03M —

$L_8 = -G_1G_5H_1H_2H_3$

Step4 : Non touching loops : All loop have common Nodes A, B, C, D, E. ∴ There is no combⁿ of Non-touching loop — NiL

Step5 : $\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8]$

Step6 : Δ_k : All forward path and all loops have common node ∴ $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = \Delta_7 = \Delta_8 = 1$

— 01M —

Step 7 $T.F = \frac{1}{\Delta}\sum_{K=1}^{8} P_K \Delta_K$ | $\Delta_K = 1$ | K varies from 1-8

$$= \frac{P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8}{\Delta}$$

$$T.F = \frac{G_1G_2G_3G_4 - G_1G_2G_3H_2 + G_1G_2G_5G_4 - G_1G_2G_5H_2 - G_1G_3G_4H_1 + G_1G_3H_1H_2 - G_1G_5G_4H_1 + G_1G_5H_1H_2}{\Delta}$$

— 01M —

$$\Delta = 1 + G_1G_2G_3G_4H_3 - G_1G_2G_3H_2H_3 + G_1G_2G_4G_5H_3 - G_1G_2G_5H_2H_3 - G_1G_3G_4H_1H_3 + G_1G_3H_1H_2H_3 - G_1G_4G_5H_1H_3 + G_1G_5H_1H_2H_3$$

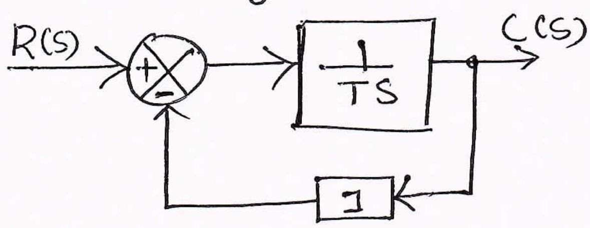
— Module-3 —

Q5a. List the standard test inputs used in control system with their Laplace transform. — 04M —

Test signals	Laplace transform of test signal
1) Unit Impulse $\delta(t)$	1 — 01M —
2) Unit step $u(t)$	$\frac{1}{s}$ — 01M —
3) Unit Ramp $r(t) = t$	$\frac{1}{s^2}$ — 01M —
4) Unit Parabolic $x(t) = \frac{t^2}{2}$	$\frac{1}{s^3}$ — 01M —

Q5b. Derive the step input response of a First order System. —08M—

Standard Block Diagram of a first order System is given by



* where 'T' is called time constant of System

—01M—

Step1 Transfer function of above Block Diagram

$$\frac{C(S)}{R(S)} = \frac{1}{Ts+1}$$

Step2 Solving for o/p from T.F

$$\underset{(o/p)}{C(S)} = \underset{(T.F)}{\frac{1}{Ts+1}} \cdot \underset{(i/p)}{R(S)}$$

Step3 for unit step resp $R(S) = \frac{1}{S}$, sub^n of R(S) in above o/p eq^n

$$C(S) = \frac{1}{Ts+1} \cdot \frac{1}{S}$$ —03M—

Step4 Apply Paratial fraction Expansion for above o/p eq^n

$$C(S) = \frac{K_1}{Ts+1} + \frac{K_2}{S}$$

$K_1 = \frac{1}{S}\Big|_{S=-\frac{1}{T}}$

$\boxed{K_1 = -T}$

$K_2 = \frac{1}{Ts+1}\Big|_{S=0} = 1$

$\boxed{K_2 = 1}$ —03M—

$$C(S) = \frac{-T}{Ts+1} + \frac{1}{S}$$

$$\therefore C(S) = \frac{1}{S} - \frac{1}{S+1/T}$$

Step5 Inverse Laplace of above o/p eq^n

$$\boxed{C(t) = \left(1 - e^{-t/T}\right)u(t)}$$ —01M—

Q5C. A unity negative feedback control system with $G(s) = \frac{100}{s^2(s+4)(s+12)}$

(i) What is the type of system

(ii) Find static error coefficient

(iii) find steady state error if the i/p is $r(t) = 2t^2 + 5t + 10$

— 08M —

Step 1 (i) Type of system is no. of poles at origin from above Given G(s), the no. of poles at origin is '2' ∴ Type of system is '2'

— 01M —

Step 2 (ii) Static error coefficients: 'Kp' position error constant $K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{100}{s^2(s+4)(s+12)}$

$\boxed{K_p = \infty}$

'Kv' velocity error constant

$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{s\,100}{s^2(s+4)(s+12)} = \infty$

$\boxed{K_v = \infty}$

'Ka' acceleration error constant

$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} \frac{s^2\,100}{s^2(s+4)(s+12)} = \frac{100}{4 \times 12}$

$\boxed{K_a = \frac{25}{12}}$

— 03M —

Step 3 (iii) error for sum of three input is sum of individual error $e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$ | $A_1 = 10$ | $A_3 = 4$

$e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a}$ | $A_2 = 5$ |

$e_{ss} = \frac{10}{1+\infty} + \frac{5}{\infty} + \frac{4}{\frac{25}{12}}$

$e_{ss} = \frac{48}{25} = 1.92$

$\boxed{e_{ss} = 1.92}$

— 04M —

-OR-

Q6a. Starting from the output equation c(t) derive expression for (i) Rise time (t_r) (ii) Peak overshoot (M_p) of under damped second order system subjected to unit step input. — 08M —

(i)

Step 1 The output expression for under damped second order system is given by

$$c(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$$ — 01M —

According to defn of rise time @ $t = t_r$ $c(t) = 1$

$$\therefore \quad c(t_r) = 1 = 1 - \frac{e^{-\zeta \omega_n t_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_r + \theta)$$

$$\sin(\omega_d t_r + \theta) = 0$$

$\omega_d t_r + \theta = \sin^{-1}(0) = n\pi$; $n = 0, 1, 2 \ldots$ for first instant $n = 1$

$$\omega_d t_r + \theta = \pi$$

Step 2 Solving for t_r from above eqn

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n \sqrt{1-\zeta^2}}$$ where $\theta = \cos^{-1}\zeta$ — 02M —

(ii) According to defn of Peak overshoot

$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$, for unit step i/p $c(\infty) = 1$ — 01M —

$$\therefore \quad M_p = c(t_p) - 1$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\zeta \omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \theta)$$

$$M_p = c(t_p) - 1 = \frac{-e^{-\zeta \omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \theta)$$

. Sub^n of t_p expression $t_p = \frac{\pi}{\omega_d}$ in M_p

$$M_p = \frac{-e^{-\zeta\omega_n \frac{\pi}{\omega_d}}}{\sqrt{1-\zeta^2}} \cdot \sin\left(\omega_d \cdot \frac{\pi}{\omega_d} + \theta\right)$$ —02M—

$$= \frac{-e^{-\zeta\omega_n \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \cdot (-\sin\theta)$$

but $\sin\theta = \sqrt{1-\zeta^2}$

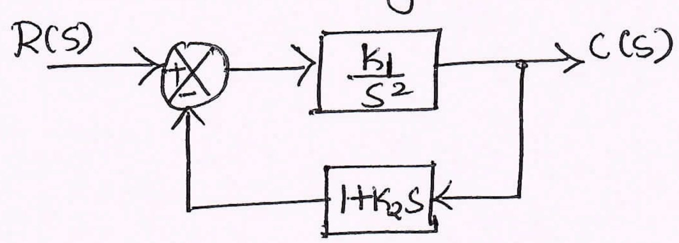
$$M_p = \frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \cdot \sin\theta$$

$$\boxed{M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

—02M—

Q6b. For servomechanism system with $G(s) = \frac{K_1}{s^2}$ and $H(s) = 1 + K_2 s$. Determine the value of K_1, K_2 so that peak overshoot is 0.25 and peak time is '2' seconds for a unit step input. —08M—

Step1 The Block Diagram for Given system is



$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + K_1(1+K_2 s)} = \frac{K_1}{s^2 + K_1K_2 s + K_1}$$ —02M—

Step2 standard 2nd order T.F $= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$

Comparing standard T.F with obtained T.F

$\omega_n^2 = K_1$ —①; $2\zeta\omega_n = K_1K_2$ —②—

Step3 Given $M_p = 0.25$ (25%) we can solve for ζ,

$$\zeta = \frac{-\ln(0.25)}{\sqrt{\pi^2 + \ln^2(0.25)}}$$

$$\boxed{\zeta = 0.403}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2 \qquad \omega_n = \frac{\pi}{2\sqrt{1-\zeta^2}}$$

$$\boxed{\omega_n = 1.716 \text{ rad/s}}$$ — 04M —

Step 4: Subⁿ of ζ & ω_n in Eqⁿ ① & ② & solving for K_1 and K_2

$K_1 = \omega_n^2 = (1.716)^2 = 2.947$

$$\boxed{K_1 = 2.947}$$

$2\zeta\omega_n = K_1 K_2$

$K_2 = \frac{2 \times 0.403 \times 1.7}{2.947}$

$$\boxed{K_2 = 0.469}$$ — 01M —

$\therefore$ $\boxed{K_1 = 2.947, \quad K_2 = 0.469}$ — 01M —

Q6.c With a neat Block diagram explain PI and PD Controller — 04M —

Proportional-Integral Controller (PI Controller):

The Controller with PI control action, the relationship between the output of controller u(t) and the actuating error signal e(t) is given by

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt$$

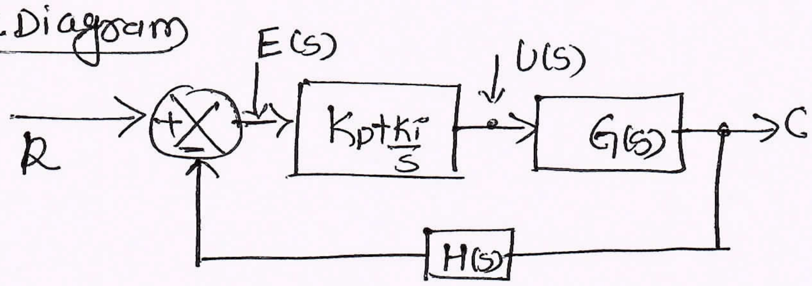
Laplace Transform of above Eqⁿ

$$U(s) = K_p E(s) + \frac{K_i}{s} E(s)$$

$$\boxed{\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s}}$$

PI Controller Block is represented by Block with gain $K_p + \frac{K_i}{s}$

Block Diagram



— 02M —

Proportional-Derivative controller (PD controller)

The controller with PD control action the relation between the output of controller u(t) and the actuating error signal e(t) is given by

$$u(t) = K_p e(t) + K_D \frac{de(t)}{dt}$$

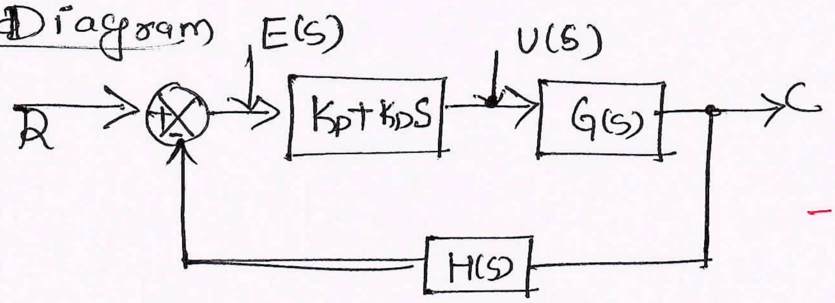
Laplace Transform of above eqn

$$U(S) = K_D E(S) + K_D S E(S)$$

$$\frac{U(S)}{E(S)} = K_D + K_D S$$

PD controller Block is represented by Block with gain $K_p + K_D S$

Block Diagram



— 02M —

— Module-4 —

Q7a Explain Routh-Hurwitz criterion for stability of the systems and what are its limitations. — 04M —

The Routh-Hurwitz criterion is necessary & sufficient condition for the stability of LTI systems

* The method requires two steps
  (i) Generating Routh array
  (ii) Interpreting Routh array for location of poles in the s-plane

— 02M —

* The Routh-Hurwitz criterion states that the number of roots of CE eqn with postive real parts is equal to the number of changes in sign of the first column of the Routh array

— 01M —

* Limitations

1) Applicable only to linear system
2) It doesnot provide exactlocation ((co-ordinates) in LHP or RHP
3) It is valid only for real coefficients of the C.E Equation.

—01M—

Q7b. Find the range of K so that system with C.E as: $s^4 + 22s^3 + 10s^2 + s + K = 0$ is stable. Also find frequency of oscillation at marginal value of K.

—06M—

Step1: Constructing (Generating) Routh table

s^4	1	10	K
s^3	22	1	0
s^2	$\frac{219}{22}$	K	0
s^1	$\frac{219-484K}{219}$	0	
s^0	K	0	

$$\frac{\frac{219}{22} - 22K}{\frac{219}{22}} = \frac{219-484K}{219}$$

—02M—

Step2: For system to stable all the elements of first column shoulde Postive

$K>0$ & $\frac{219-484K}{219} > 0$; $K < \frac{219}{484}$, $\boxed{K<0.45}$

∴ range of 'K' for stable System is $\boxed{0<K<0.45}$

Step3: For marginal stable system, RH table require Row of zero & A(s) ∴ from s^1 row

$$\frac{219-484K}{219} = 0 \Rightarrow K = \frac{219}{484} = 0.45$$

$\boxed{K = 0.45 = K_{mar}}$

—02M—

Step4 $A(s) = \frac{219}{22}s^2 + K = 0$ | $K = 0.45$

$A(s) = \frac{219}{22}s^2 + 0.45 = 0$

$\therefore \quad s^2 = \frac{-0.45 \times 22}{219}$

$s^2 = -0.045$

$\therefore$ $\boxed{\omega = 0.213 \text{ rad/s}}$ (frequency of oscillation)

$\boxed{s = \pm j\,0.213}$

$s = \pm j\omega$

$\boxed{K_{mar} = 0.45}$ (Marginal value of K)

— 02M —

Q7c. Plot root locus for $s^3+6s^2+8s+K=0$

— 10M —

Step1: From Give C.E we need find open loop Transfer function $G(s)H(s)$.

Step2: Rewritting C.E as

$(s^3+6s^2+8s)+K=0$

dividing above eqn by s^3+6s^2+8s

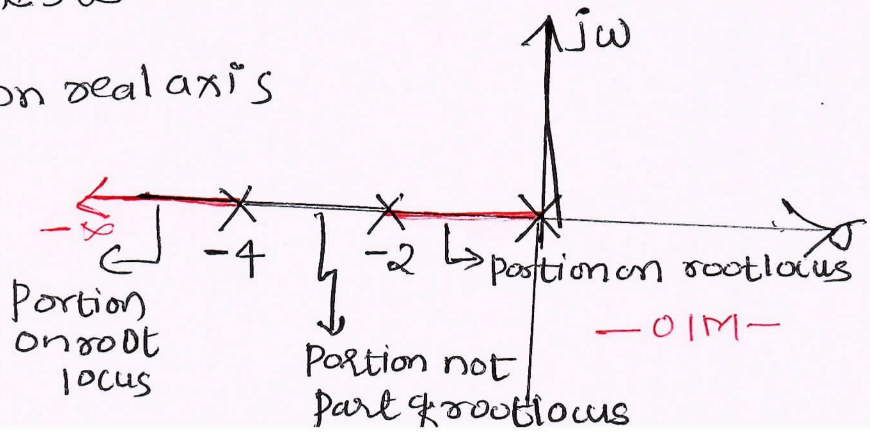
$\boxed{1+\frac{K}{s^3+6s^2+8s}=0}$ | $1+G(s)H(s)=0$

$\therefore \quad G(s)H(s) = \frac{K}{s^3+6s^2+8s} = \frac{K}{s(s^2+6s+8)} = \frac{K}{s(s+4)(s+2)}$

— 01M —

Step3 $P=3,\ Z=0,\ N=P=3,\ P-Z=3$ branches approaches ∞

Step4 section on real axis



— 01M —

Step 5: Angle of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2 \ldots$$

$$= \frac{(2q+1)180^\circ}{3}$$

$$\boxed{\theta = 60^\circ, 180^\circ, 300^\circ}$$

Step 6: Centroid = $\dfrac{\sum \text{Realpart of openloop Poles} - \sum \text{Realpart of open loop Zeros}}{P-Z}$

$$\sigma = \frac{0-4-2-0}{3} = -\frac{6}{3} = -2$$

$$\boxed{\sigma = -2}$$

—01M—

Step 7: Breakaway Point

$$1+G(s)H(s) = 1+\frac{K}{s^3+6s^2+8s} = 0$$

$s^3+6s^2+8s+K=0$

$K = -s^3-6s^2-8s$

$\frac{dK}{ds} = -3s^2-12s-8 = 0$

$\therefore 3s^2+12+8=0$

$\boxed{s_1 = -0.845,}$ $s_2 = -3.154$

$s_2 = -3.154$ is not part of root locus on real axis, hence s_2 is not valid Breakaway poin

$s_1 \Rightarrow$ is part of root locus on real axis $\therefore s_1$ is valid Breakaway point

— 02M —

Step 8 Intersection with imaginary axis

C.E $s^3+6s^2+8s+K=0$

R.H Table

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	

$\frac{48-K}{6}=0$ | $K = 48 = K_{mar}$

— 02M —

$A(s) = 6s^2+K=0$

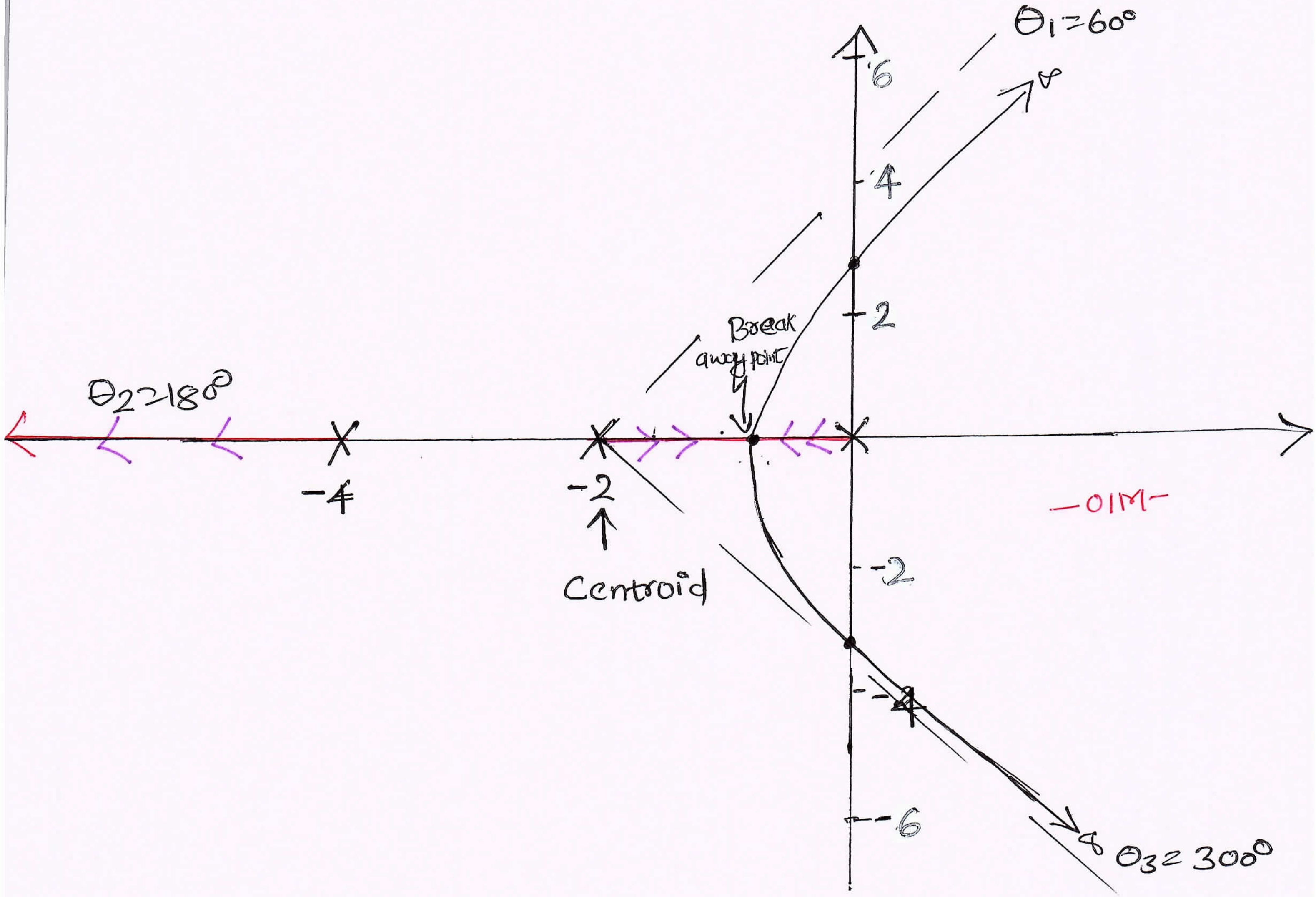
$6s^2+48=0$

$s^2=-8$ | $s = \pm j\sqrt{8}$

$\therefore \boxed{\omega = 2.82 \text{ rad/s}}$

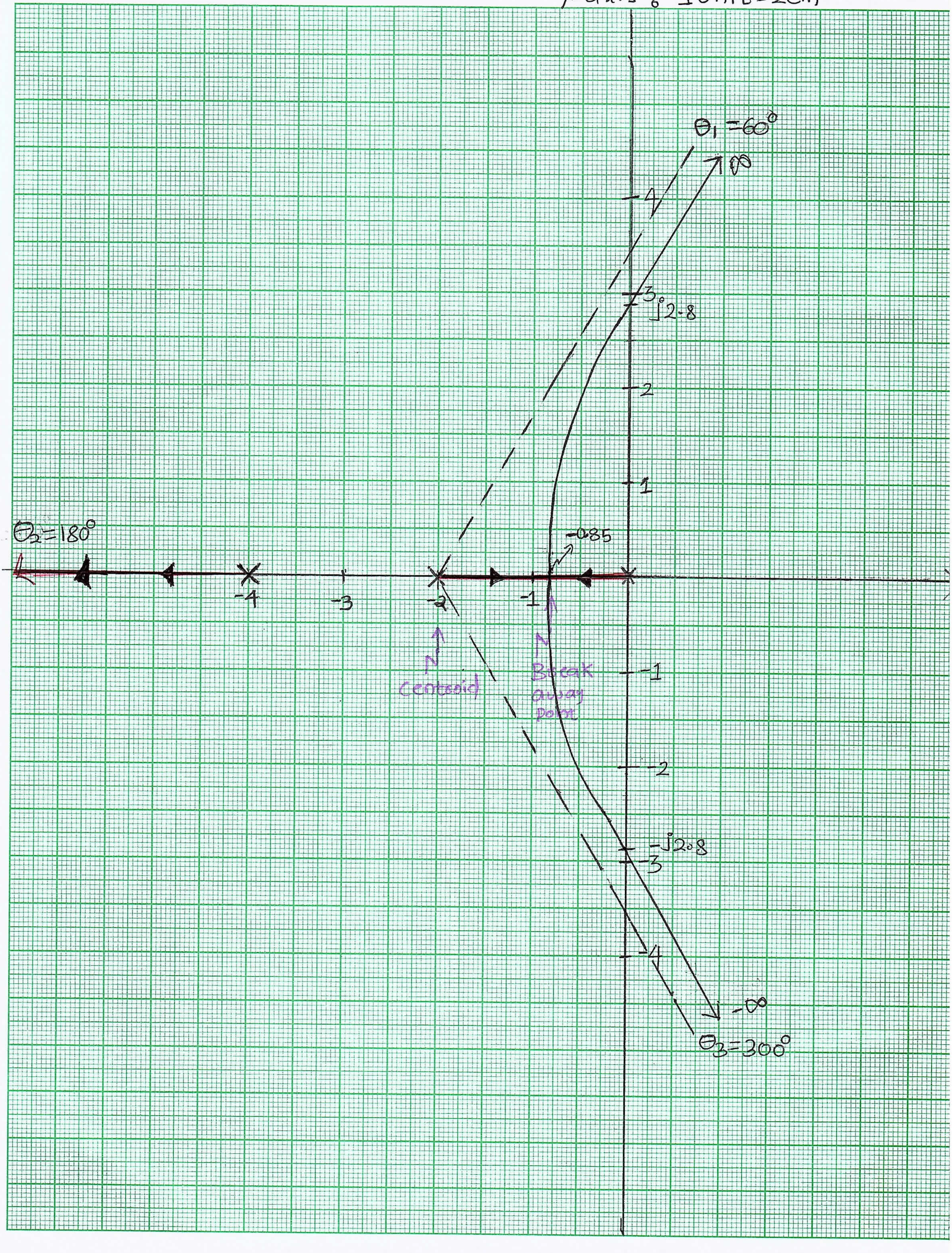
$\therefore \boxed{\omega = \sqrt{8} \ldots}$

* The complete Root locus can be plotted with above Obtained Information



Q4c -02M-

Scale: X-axis : 1unit=2cm
Y-axis : 1unit=2cm



—OR—

Q8a. For a closed loop control system, $G(s) = \frac{100}{s(s+8)}$, $H(s)=1$, determine resonant peak and resonant frequency.

—04M—

Step 1: Finding Transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{100}{s^2+8s+100}$$

Step 2: Comparing with standard 2nd order T.F

$$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n^2 = 100$; $\omega_n = 10$

$2\zeta\omega_n = 8$; $\zeta = \frac{8}{2\times 10} = 0.4$

$\omega_n = 10, \zeta = 0.4$

Step 3 Resonant peak is given by

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.36$$

—02M—

Resonant frequency is given by

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$\omega_r = 8.24$ rad/s —02M—

$M_r = 1.36$
$\omega_r = 8.24$ rad/s

Q8b. State any three rules of plotting root locus.

—06M—

① Root locus is symmetrical about the real axis

—02M—

② Segments of the real axis having an odd number of real axis open loop poles + zeros to their right are part of root locus.

—02M—

③ The $(n-m)$ root locus branches that tend to ∞, do so along straight line asymptotes making angle with real axis given by

—02M—

$$\phi_A = \frac{180°\cdot(2q+1)}{n-m} ; \quad q = 0,1,2,\dots(n-m-1)$$

Q8c

Sketch the Bode plot for open loop transfer function $G(S)H(S) = \frac{K}{S(1+0.2S)(1+0.05S)}$

Determin the value of K for gain margin (GM) of 6dB.

—10M—

Step1 : Given open loop T.F is in Time constant form

$$G(S)H(S) = \frac{K}{S(1+0.2S)(1+0.05S)}$$

Step2: Identifying different factors

① $\frac{1}{S}$, Pole at origin, st line of slope -20dB/decade Passing through intersection point of $\omega=1$ & 0dB

② $\frac{1}{1+0.2S}$, simple pole, $T_1 = 0.2$, $\omega_{c_1} = \frac{1}{T_2} = 5$
st line of slope -20dB/decade for $\omega > 5$

③ $\frac{1}{1+0.05S}$, simple pole, $T_2 = 0.05$, $\omega_{c_2} = 20$
St line of slope -20dB/decade for $\omega > 20$

—03M—

Range of ω	Resultant slope in dB/dec
$0 < \omega < 5$	-20
$5 \leq \omega < 20$	-20-20 = -40
$20 \leq \omega < \infty$	-40-20 = -60

—01M—

Step3 Phase angle table

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+0.2j\omega)(1+0.05j\omega)}$$

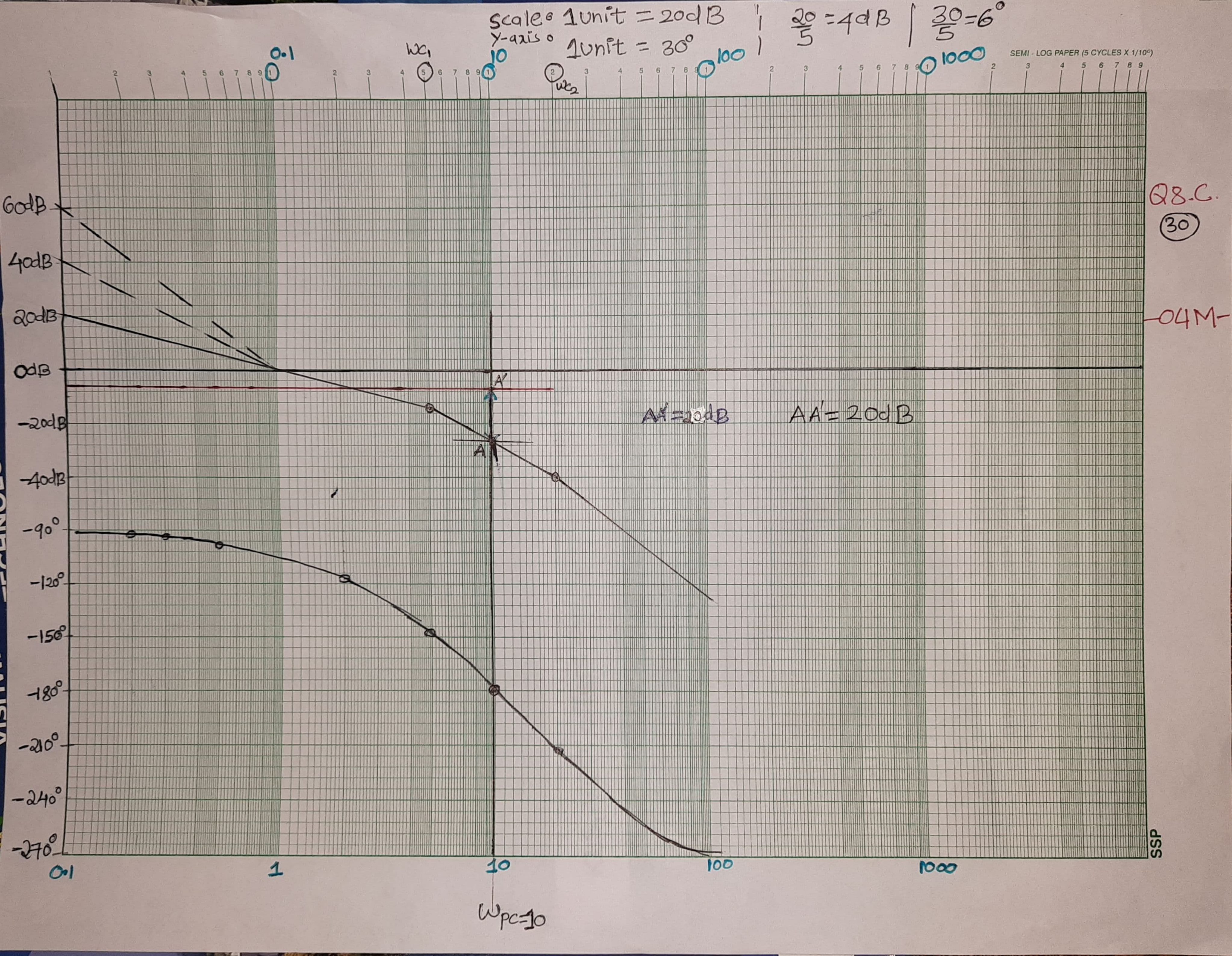
ω	$\frac{1}{j\omega}$	$-\tan^{-1} 0.2\omega$	$-\tan^{-1} 0.05\omega$	ϕ_R
0.2	$-90°$	-2.29	$-0.57°$	$-92.86°$
0.3	$-90°$	$-3.43°$	$-0.859°$	-94.29
0.5	$-90°$	$-5.71°$	$-1.432°$	$-97.142°$
2	$-90°$	$-21.8°$	$-5.71°$	$-117.51°$
5	$-90°$	$-45°$	$-14.0°$	$-149°$
7	$-90°$	-54.46	$-19.29°$	$-163.7°$
10	$-90°$	-63.43	-26.56	$-180°$
20	$-90°$	$-75.9°$	$-45°$	$-211°$
∞	$-90°$	$-90°$	$-90°$	$-270°$

— 02M —

* From Bode plot graph, we observe that amount of shift required is $AA' = 20\,dB$

$\therefore \quad 20\log_{10} K = 20 \;;\; \log_{10} K = 1$

$K = 10^1 \;;\; \boxed{K = 10}$ — 01M —



Scale: 1unit = 20dB | 20/5 = 4dB | 30/5 = 6°
Y-axis: 1unit = 30°
SEMI - LOG PAPER (5 CYCLES X 1/10°)
Q8.C.
30
-04M-
60dB
40dB
20dB
0dB
-20dB
-40dB
-90°
-120°
-150°
-180°
-210°
-240°
-270°
0.1
1
10
100
1000
ω_{c1}
ω_{c2}
A
A'
AA' = 20dB
AA' = 20dB
ω_{pc} = 10
SSP

Q99. State Mapping Theorem and explain Nyquist Stability Criterion. (3)

—04M—

Mapping Theorem

Let F(s) be a ratio of two polynomials in 's'. let 'p' be the number of poles and Z be the number of zeros of F(s) that lies inside some closed contour in the s-plane, with multiplicity of poles and zeros accounted for. Let this contour be such that it does not pass through any poles or zeros of F(s). This closed contour in the s-plane is then mapped into F(s) plane as closed curve. The total number N of clockwise encirclements of the origin of the F(s) plane, as a representative point 's' traces out the entire contour in the clockwise direction, is equal to Z−P, $\boxed{N = Z - P}$

—02M—

Nyquist stability criterion

If the contour Γ_{GH} of the open loop transfer function G(s)H(s) corresponding to the Nyquist contour in the s-plane encircles the point $(-1+j0)$ in the counter clockwise direction as many time as the number of right half s-plane poles of G(s)H(s), the closed loop system is stable.

* The Mapping of the Nyquist contour into contour Γ_{GH} is carried out as follow

① The Mapping of the Imaginary axis is carried out by sub^n of $s=j\omega$ in G(s) H(s) $[G(j\omega)\,H(j\omega)]$

② In physical systems $(m \leq n)$, $\lim_{\substack{s=Re^{j\theta} \\ R\to\infty}} G(s)H(s) = \text{real constant}$

Thus infinite arc of Nyquist contour maps into a point on the real axis

—02 M—

Q9b. Draw polar plot for $G(s)H(s) = \frac{10}{s(s+1)(s+2)}$ Find GM and PM, also comment on stability. —10M—

Step1 Given open loop Transfer function

$$G(s)H(s) = \frac{10}{s(s+1)(s+2)}$$

$$\therefore G(j\omega)H(j\omega) = \frac{10}{j\omega(j\omega+1)(j\omega+2)}$$

$$M = \frac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}, \quad \phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Step2 for Polar plot ω varies from 0 to ∞

$\omega = 0$	$M = \infty$	$\phi = -90^\circ$	$-270-(-90^\circ) = 180^\circ$
$\omega = \infty$	$M = 0$	$\phi = -270^\circ$	clockwise

—02M—

Step3 Let us rationalize $G(j\omega)H(j\omega)$.

$$G(j\omega)H(j\omega) = \frac{(-j\omega)(1-j\omega)(2-j\omega)\times 10}{j\omega(j\omega+1)(j\omega+2)(-j\omega)(-j\omega+1)(-j\omega+2)}$$

$$= \frac{(-j\omega)(-j\omega+1)(-j\omega+2)\times 10}{\omega^2(1+\omega^2)(\omega^2+4)}$$

$$= \frac{10(-j\omega)(-\omega^2-3j\omega+2)}{\omega^2(\omega^2+1)(\omega^2+4)} = \frac{-30\omega^2 - 10j\omega(-\omega^2+2)}{\omega^2(\omega^2+1)(\omega^2+4)}$$

$$G(j\omega)H(j\omega) = \frac{-30\cancel{\omega^2}}{\cancel{\omega^2}(\omega^2+1)(\omega^2+4)} - \frac{j\omega(20-10\omega^2)}{\omega^2(\omega^2+1)(\omega^2+4)}$$ —02M—

Step4 at Intersection on real axis Img part is zero. Equating Img =0, we get corresponding ω, with this ω in real part we get Point of intersection

$$\frac{-j\omega(20-10\omega^2)}{\omega^2(\omega^2+1)(\omega^2+4)} = 0$$

$10\omega^2 = 20$

$\omega^2 = 2$

$\omega = \sqrt{2} = 1.414$

$\omega = \omega_{pc} = 1.414$ rad/s

$$Q\Big|_{\omega=\omega_{pc}} = \frac{-30}{(2+1)(2+4)} = \frac{-30}{3\times6} = -1.66$$

$$\boxed{Q = -1.66}$$

—02M—

Step5 Magnitude and Phase table

ω	M	φ	$G_R(j\omega)$	$G_I(j\omega)$
0.5	8.67	-130.6°	-5.6	-6.5
0.6	6.8	-137.6°	-5.02	-4.58
0.7	5.5	-144.2°	-4.4	-3.2
0.8	4.5	-150°	-3.89	-2.25
1	3.16	-161.5°	-2.496	-1.002
2	0.79	-198.43°	-0.749	0.249

—01M—

Step6 $GM = 20\log\frac{1}{|Q|} = 20\log\frac{1}{1.66} = -4.40\,dB$

PM from graph ⇒ $\phi_{gc} = -192°$

$PM = 180 + \phi_{gc} = -12°$

Both GM and PM are negative system is Unstable

—01M—

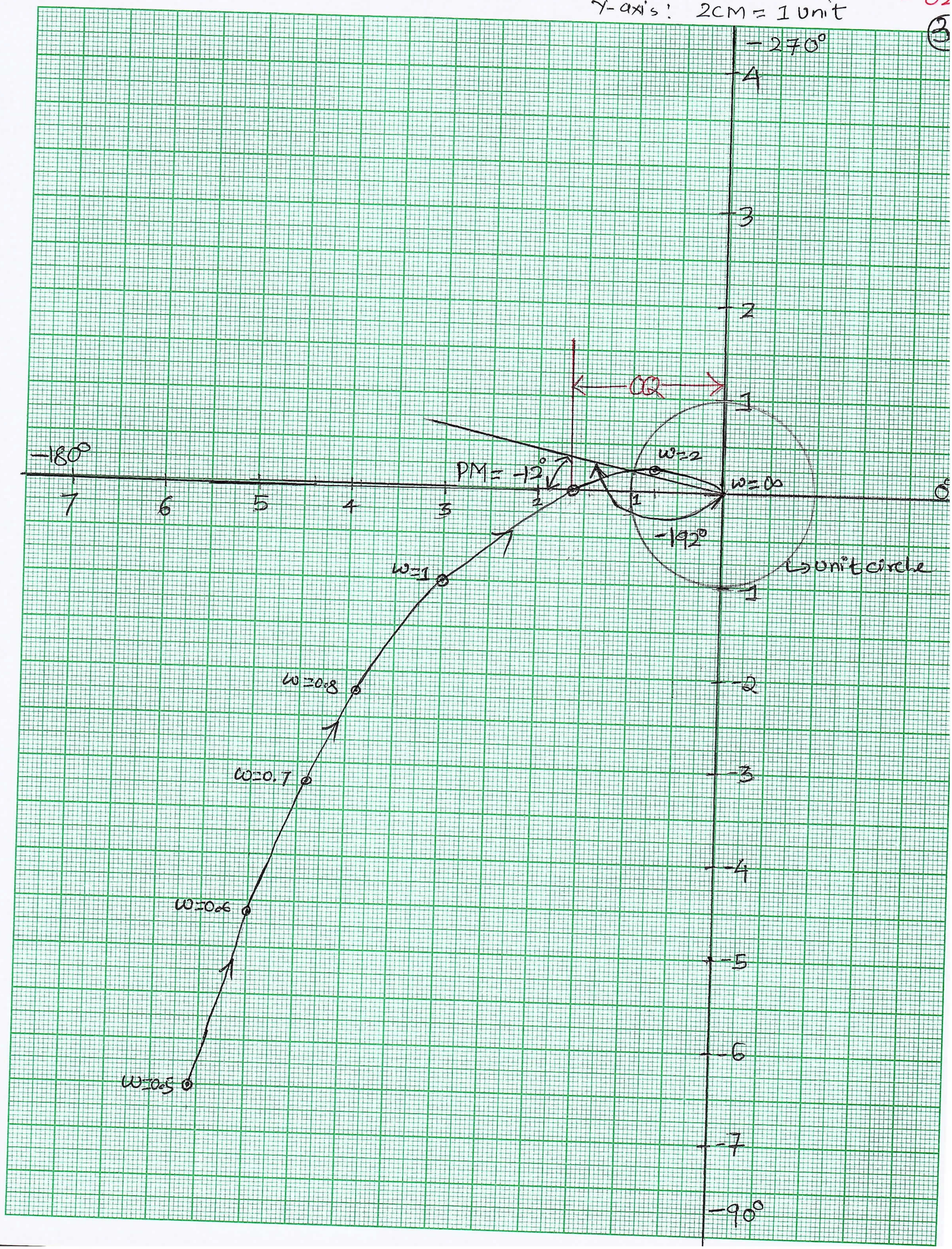
Q9.b

Scale: X-axis : 2CM = 1 unit

Y-axis : 2CM = 1 unit

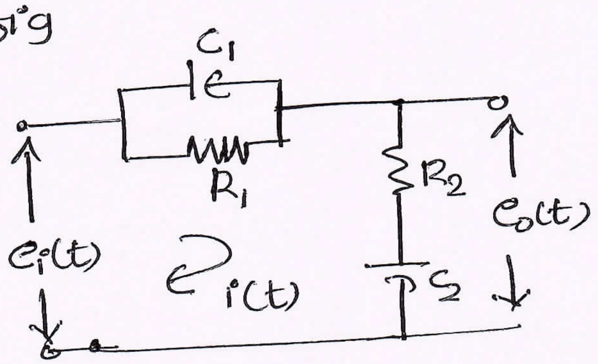
–02

(3)



Q9c. Explain Lead Lag compensating network —06M—

A Lag-Lead compensator is a combination of lag and lead compensators. An electric network which acts as lag-lead compensator is shown in below fig



—02M—

* The Transfer function of above network is

$$\frac{E_o(s)}{E_i(s)} = \frac{(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_1}{\beta}s\right)(1+T_2\beta s)}$$

Where $T_1 = R_1C_1$, $T_2 = R_2C_2$, $\frac{\beta}{T_1}+\frac{1}{\beta T_2} = \frac{1}{R_1C_1}+\frac{1}{R_2C_1}$, $\alpha\beta=1$

* The phase lead portion involving T_1, adds phase lead angle while the phase lag portion involving T_2 provide attenuation near and above the gain crossover frequency —02M—

* Effects of Lag-Lead Compensator
  * Lag-Lead compensator is used when both fast response and good static accuracy are desired.
  * Use of Lag-Lead compensator increases the low frequency gain which improves the steady state
  * It increases bandwidth of the system, making the system response very fast.

—02M—

— OR —

Q10a What are the advantages of State-Space analysis — 04M —

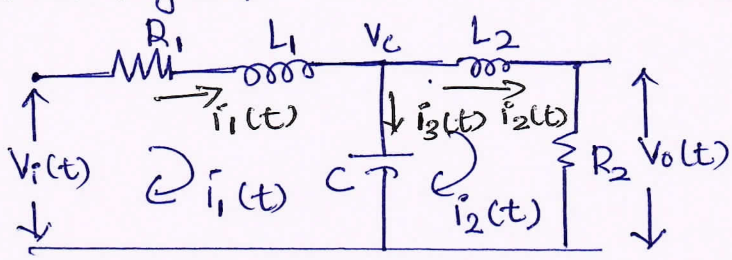
① State-space analysis includes the effect of all intial conditions — 01M —

② It is useful to determine the time-domain response of non-linear systems effectively — 01M —

③ state equations involving matrix algebra are highly compatible with simulation on digital computers. — 01M

④ MIMO systems can be easily represented and analyzed using state variables. — 01M —

Q10b Obtain the state equations for the electrical Networks shown in fig 10.b — 08M —



Step 1 : Label all branch current

step 2 : derivative relationship for storage element

$$i_3 = C\frac{dV_c(t)}{dt} \quad - ①$$

$$V_{L_1} = L_1\frac{di_1(t)}{dt} \quad - ②$$

$$V_{L_2} = L_2\frac{di_2(t)}{dt} \quad - ③$$

* State variable are i_1, i_2, V_c — 02M —

Step3 : Apply KCl at V_c

$$i_1(t) = i_3(t) + i_2(t)$$

$$\boxed{i_3 = i_1 - i_2} \quad -(4)$$

KVl to loop1

$$-V_i + R i_1(t) + V_{L_1} + V_c = 0$$

$$\boxed{V_{L_1} = V_i - V_c - R i_1} \quad -(5)$$

KVl to loop2 $\quad -V_c + V_{L_2} + V_0 = 0$

$$\boxed{V_{L_2} = V_c - i_2 R_2} \quad -(6)$$

— 03M —

Sub^n of 4, 5, 6 in 1, 2, 3

$$\dot{V}_c = \frac{1}{C} i_1 - \frac{1}{C} i_2 \qquad \dot{i}_2 = \frac{V_c}{L_2} - i_2 \frac{R_2}{L_2}$$

$$\dot{i}_1 = \frac{V_i}{L_1} - \frac{V_c}{L_1} - \frac{R i_1}{L_1}$$

— 02M —

Step4 output equation $V_0 = i_2 R_2$

State eq^n and output eq^n in vector-matrix form

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & 0 \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix} V_i$$

$$V_0 = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix}$$

— 01M —

Q1a. Compute the STM for the given system matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ using Laplace approach technique. -08M-

Step1 $[SI - A] = \begin{bmatrix} S-1 & 0 \\ -1 & S-1 \end{bmatrix}$ -01M-

Step2

$$[SI-A]^{-1} = \frac{Adj[SI-A]}{|SI-A|}$$

$$Adj[SI-A] = \begin{bmatrix} S-1 & 1 \\ 0 & S-1 \end{bmatrix}^T = \begin{bmatrix} S-1 & 0 \\ 1 & S-1 \end{bmatrix}$$

$$|SI-A| = (S-1)^2$$

$$\therefore [SI-A]^{-1} = \frac{\begin{bmatrix} S-1 & 0 \\ 1 & S-1 \end{bmatrix}}{(S-1)^2} = \begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix}$$

-04M-

Step3 State transition Matrix STM

$$e^{At} = \mathcal{L}^{-1}[SI-A]^{-1} = \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

-03M-