6th Semester BE (CBCS) EC/TC Model Question Papers

15EC61

Visvesvaraya Technological University, Belagavi

MODEL QUESTION PAPER

6th Semester, B.E (CBCS) EC/TC

Course: 15EC61 - Digital Communication

Max Marks: 80 **Time: 3 Hours**

Note: (i) Answer Five full questions selecting any one full question from each Module.

(ii) Question on a topic of a Module may appear in either its 1st or/and 2nd question.

		Module 1	
1	(a)	Define Hilbert Transform. State the properties of it.	4
	(b)	Define the complex envelope of bandpass signals. Obtain the canonical representation of bandpass signals	6
	(c)	Derive the power spectral density of polar NRZ signals and plot the spectrum.	6
		OR	
2	(a)	Define the Pre-envelope. Show the spectral representations of pre-envelopes for low pass signals.	4
	(b)	Derive the expression for the complex low pass representation of bandpass systems.	7
	(c)	Given the data stream 1110010100. Sketch the transmitted sequence of pulses for each of the following line code. (i) Unipolar NRZ (ii) Polar NRZ (iii) Unipolar RZ (iv) bipolar RZ (v) Manchester code.	5
		Module 2	
3	(a)	Explain the Geometric representation of signals and express the energy of the signal in terms of the signal vector.	5
	(b)	Explain the Gram-Schmidt orthogonalization procedure.	5
	(c)	Explain the matched filter receiver with the relevant mathematical theory.	6
		OR	
4	(a)	Obtain the decision rule for Maximum likelihood decoding and explain the correlation receiver.	7
	(b)	The waveforms of four signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ described below. $s_1(t) = 1, \quad 0 < t < T/3$, $s_2(t) = 1, \quad 0 < t < 2T/3$, $s_3(t) = 1, \quad T/3 < t < T$,	9

Scheme & Soln. Prepared by

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		$s_4(t) = 1$, $0 < t < T$, and zero otherwise. Using the Gram-Schmidt orthogonalization procedure, find an orthonormal basis for this set of signals and construct the corresponding signal-space diagram.	
		Module 3	
5	(a)	Define binary phase shift keying. Derive the probability of error of BPSK.	7
	(b)	Define M-ary QAM. Obtain the constellation of QAM for M=4 and draw the signal space diagram	4
	(c)	Given the input binary sequence 1100100001. Sketch the waveforms of the inphase and quadrature components of a modulated wave and next sketch the QPSK signal.	5
		OR	
6	(a)	Describe the QPSK signal with its signal space characterization. With a neat block diagram explain the generation and detection of QPSK signals.	6
	(b)	Obtain the expression probability of symbol error of coherent binary FSK.	7
	(c)	Illustrate the operation of DPSK for the binary sequence 10010011	3
		Module 4	
7	(a)	With a neat block diagram Explain the digital PAM transmission through bandlimited baseband channels and obtain the expression for ISI.	5
	(b)	What are adaptive equalizers? Explain the linear adaptive equalizer based on the MSE criterion.	6
	(c)	The binary sequence 10010110010 is the input to the precoder whose output is used to modulate a duobinary transmitting filter. Obtain the precoded sequence, transmitted amplitude levels, the received signal levels and the decoded sequence.	5
		OR	
8	(a)	What is eye pattern? What is the Nyquist criterion for zero ISI? Given an example of the pulse with zero ISI.	5
	(b)	Explain the design of bandlimited signals with controlled ISI. Describe the time domain and frequency domain characteristics of a duobinary signal.	5
	(c)	What is channel equalization? With a neat diagram explain the concept of equalization using a linear transversal filter.	6
		Module 5	
9	(a)	Draw the 4 stage linear feedback shift register with 1st and 4th stage is connected to Modulo-2 adder. Output of Modulo-2 is connected to 1st stage input. Find the output PN sequence and obtain the autocorrelation sequence.	6
	(b)	With a neat block diagram explain the frequency hopped spread spectrum.	7
	(c)	Explain the effect of dispreading on narrowband interference.	3
		OR	
10	(a)	Explain the generation of direct sequence spread spectrum signal with the relevant waveforms and spectrums.	6
	(b)	With a neat block diagram explain the CDMA system based on IS-95.	7
	(c)	Write a short note on application of spread spectrum in wireless LANs.	3

① Course : 15EC61 – Digital Communication.

Model Question Paper and Solutions. 37

Module - 1

Max Marks : 80.

1. a) Define Hilbert Transform. State the properties of it. – 4M

→ Hilbert Transform (HT) of a signal $x(t)$ is a signal $\hat{x}(t)$, whose frequency components lag the frequency components of $x(t)$ by $90°$. In other words, $\hat{x}(t)$ has exactly the same frequency components present in $x(t)$ with same amplitude except there is a $90°$ phase delay. HT does not involve a domain change. HT of a signal $x(t)$ is another signal denoted by $\hat{x}(t)$ in the same domain (i.e., time domain with the same argument t).

For example. HT of a signal

$$x(t) = A\cos(2\pi f_0 t + \theta)$$

is given by
$$\hat{x}(t) = A\cos(2\pi f_0 t + \theta - 90°)$$
$$\hat{x}(t) = A\sin(2\pi f_0 t + \theta).$$

– 2M.

Properties of Hilbert Transform

HT has number of useful properties of its own.

① A signal $x(t)$ and its HT $\hat{x}(t)$ have the same magnitude spectrum.

$$|x(f)| = |\hat{x}(f)|.$$

② If $\hat{g}(t)$ is HT of $g(t)$, then HT of $\hat{g}(t)$ is $-g(t)$

w.k.t. HT is equivalent to passing $g(t)$ through a linear two port device with a transfer function of $-j\,\mathrm{sgn}(f)$

A twice HT is therefore equivalent to passing g(t) through cascade of a linear two port devices.

The overall transfer function of a cascaded linear two port devices is given by.

$$[-j\,sgn(f)]^2 = -1 \quad \text{for all } f.$$

Hence we can say that HT of $\hat{g}(t)$ is $-g(t)$.

(3) A signal g(t) & its HT $\hat{g}(t)$ are orthogonal over entire time interval $(-\infty, \infty)$ - 2M

81.

2) 26

1. b) Define the complex envelope of bandpass signals. Obtain the canonical representation of bandpass signals — (6 M).

→ If the pre envelope of a signal is bandpass, it can be used and studied easier if we shift it to low freq. The complex envelope is just the low-pass part of the analytic signal. The lowpass version of Preenvelope Signal is called as complex envelope.

It is given as. $$S_+(t) = \tilde{S}(t)\, e^{j2\pi f_c t}.$$

where $S_+(t)$ is the pre envelope of signal $s(t)$.

and $\tilde{S}(t)$ is the complex envelope of the signal $s(t)$.

— 2 M

Canonical Representation

We can express the bandpass signal $s(t)$ in terms of its complex envelope as

$$s(t) = Re\left[\tilde{s}(t)\, e^{j2\pi f_c t}\right]. \quad — (1)$$

$$\tilde{s}(t) = s_I(t) + j\, s_Q(t). \quad — (2).$$

representing eqn (1) in canonical form.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad — 1M$$

where $s_I(t)$ - inphase component

$s_Q(t)$ - quadrature component.

Complex envelope $\tilde{s}(t)$ may be pictured as a time varying phasor positioned at the origin of the (s_I, s_Q) plane in fig a below.

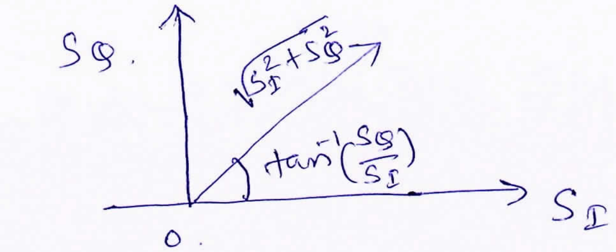

fig 1: phasor representation

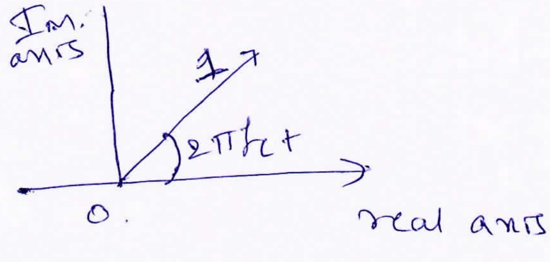

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fig 2: phasor representation of $e^{j2\pi f_c t}$.

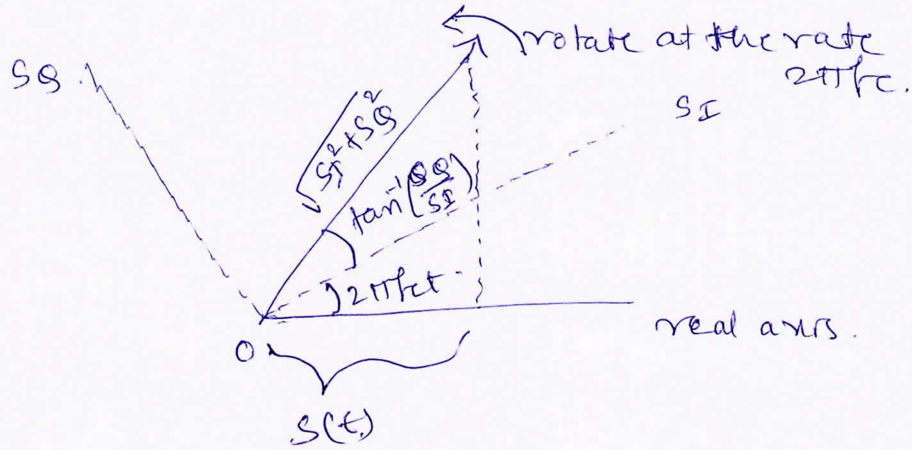

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fig. 3: phaso respresentation for $s(t)$.

Picture shows $\hat{s}(t)$ moves in (S_I, S_Q) plane while at the same time the plane itself rotates about the origin.

The original bandpass signal $s(t)$ is the projection of this time varying phasor on a real axis.

Alternatively we may define $\hat{s}(t)$ in polar form as

$$\hat{s}(t) = a(t)\, e[j\phi(t)]$$

where $a(t)$ and $\phi(t)$ are both real-valued low-pass functions.

$$\hat{s}(t) = a(t)\cos[2\pi f_c(t) + \phi(t)]$$

③ 24 where $a(t)$ = natural envelope of the band pass signal.

$\phi(t)$ = phase of the signal. — 1

$$\therefore \quad a(t) = |\hat{s}(t)| = |s_+(t)| = \sqrt{s_I^2(t) + s_Q^2(t)}$$

and $$\phi(t) = \tan^{-1}\left(\frac{s_Q(t)}{s_I(t)}\right).$$

and $$s_I(t) = a(t)\cos[\phi(t)]$$

$$s_Q(t) = a(t)\sin[\phi(t)] \quad - 1M$$

1. c). Derive the power spectral density of polar NRZ signals and plot the spectrum — (6M).

→ In NRZ polar format: A_k uses the values $+a$ for symbol 1 and $-a$ for symbol 0. and $+a$ & $-a$ are equiprobable.

$$A_k = \begin{cases} +a, & \text{symbol } 1 \\ -a & \text{symbol } 0 \end{cases} \quad - 1M$$

For $n=0$, we get $$R_A(0) = E[A_k^2]$$

$$R_A(0) = \sum_{i=1}^{2} [A_k A_k]_i P_i = a^2\left(\tfrac{1}{2}\right) + (-a^2) \times \tfrac{1}{2}$$

$$R_A(0) = a^2 \quad - 1M$$

where $R_A(n)$ is the autocorrelation function of data

For $n \neq 0$, the product $A_k A_{k-n}$ terms are $+a \times +a$, $+a \times -a$, $-a \times +a$, $-a \times -a$, we get

$$R_A(n) = \sum_{i=1}^{4} (A_k A_{k-n})_i P_i$$

$$= a^2\left(\tfrac{1}{4}\right) + \left(-\tfrac{a^2}{4}\right) - \tfrac{a^2}{4} + \tfrac{a^2}{4} = 0.$$

$$\therefore \quad R_A(n) = \begin{cases} a^2, & n=0 \\ 0, & n \neq 0 \end{cases} \quad - 1M$$

∴ The PSD with $T = T_b$ is

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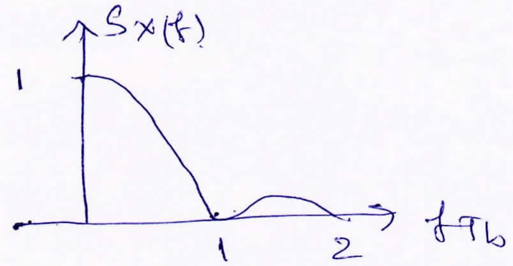
$$S_X(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}.$$

$$= \frac{1}{T_b} \cdot T_b^2\, \mathrm{sinc}^2(fT_b)\, R_A(0)$$

$$= \frac{1}{T_b}\, \mathrm{sinc}^2(fT_b) \cdot a^2$$

$$\therefore S_X(f) = a^2 T_b\, \mathrm{sinc}^2(fT_b).$$ — 1M

The PSD of $S_X(f)$ is drawn as shown below



— 2M

fig: PSD of NRZ polar format

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(4)

2a) Define the Pre-envelope. Show the spectral representations of pre-envelopes for low pass signals. (4M)

→ Pre envelope is an analytical signal of the positive part of g(t). Pre envelope of a signal g(t) is defined as

$$g_+(t) = g(t) + j\,\hat{g}(t)$$

where g(t) is a real valued signal, means it has both positive & negative frequencies.

$\hat{g}(t)$ – is the HT of g(t).

Pre envelope for positive frequency.

$$G_+(f) = G(f) + \text{sgn}(f)\,G(f)$$

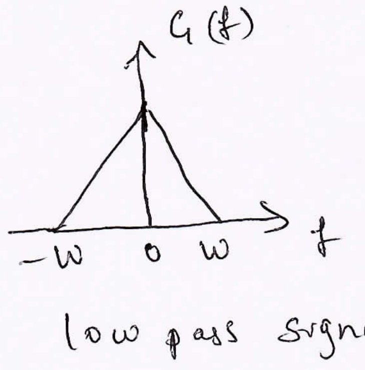
w.k.t. $\text{sgn}(f) = \begin{cases} 1 & \text{for } f>0 \\ 0 & f=0 \\ -1 & \text{for } f<0 \end{cases}$

$$\therefore\ G_+(f) = \begin{cases} 2G(f) & f>0 \\ G(0) & f=0 \\ 0 & f<0 \end{cases}$$ — 1M

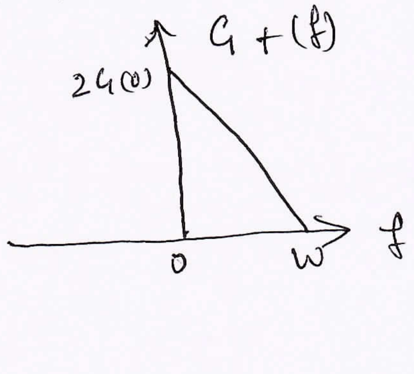
Pre envelope for negative frequency

$$g_-(t) = g(t) - j\,\hat{g}(t)$$

$$G_-(f) = \begin{cases} 0 & f>0 \\ G(0) & f=0 \\ 2G(f) & f<0 \end{cases}$$ — 1M



low pass signal.



— 1M

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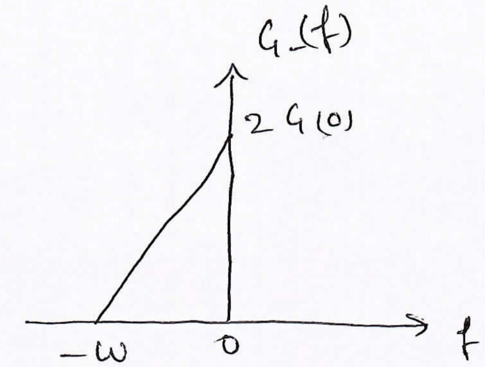
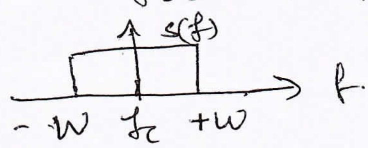


— 1M

2 b). Derive the expression for the complex low pass representations of bandpass systems. — (7M)

→. Let $s(t)$ is a narrowband signal.

$S(f)$ its fourier transform.



we also assume that $f_c > W$.

Let the signal $s(t)$ be applied to a linear-time-invariant bandpass system with impulse response $h(t)$ and freq. response $H(f)$.

The system BW $2B$ is usually narrower than or equal to the input signal BW $2W$.

Representing the bandpass impulse response $h(t)$ in terms of 2 quadrature components.

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t) \quad \text{— (1).}$$

— 1M

Corresponding complex impulse response is

$$\hat{h}(t) = h_I(t) + j\,h_Q(t). \quad \text{—— (2).}$$

Complex envelope $\hat{s}(t)$ of a bandpass signal $s(t)$ is

$$s(t) = \text{Re}\left[\hat{s}(t)\, e^{j2\pi f_c t}\right]$$

— 1M

Expressing $h(t)$ in terms of $\hat{h}(t)$

(5)

$$h(t) = Re\left[\hat{h}(t)\, e^{j2\pi f_c t}\right] \quad -(3)$$

then

$$2h(t) = \hat{h}(t)\, e^{j2\pi f_c t} + \hat{h}^*(t)\, e^{-j2\pi f_c t} \quad - 1M$$

Here $\hat{h}^*(t)$ is complex conjugate of $\hat{h}(t)$ – (4)

Applying FT to bothsides of eqn (4) and using complex conjugation property, we get.

$$2H(f) = \hat{H}(f-f_c) + \hat{H}^*(-f-f_c) \quad -(5)$$

Since $\hat{H}^*(f-f_c) = 0$ for $f>0$ then.

$$\hat{H}(f-f_c) = 2H(f), \quad f \geq 0. \quad - 2M$$

The above eqn shows that, for a band-pass freq. response $H(f)$, we can find complex low pass freq. response $\hat{H}(f)$ by taking i) $H(f)$ for $f>0$, ii) shifting it to the origin, iii) Scaling it by a factor of 2.

$\hat{H}(f)$ is decomposed into in-phase and quadrature phase Components as

$$\hat{H}(f) = \hat{H}_I(f) + j\hat{H}_Q(f)$$

where

$$\hat{H}_I(f) = \frac{1}{2}\left[\hat{H}(f) + \hat{H}^*(f)\right]$$

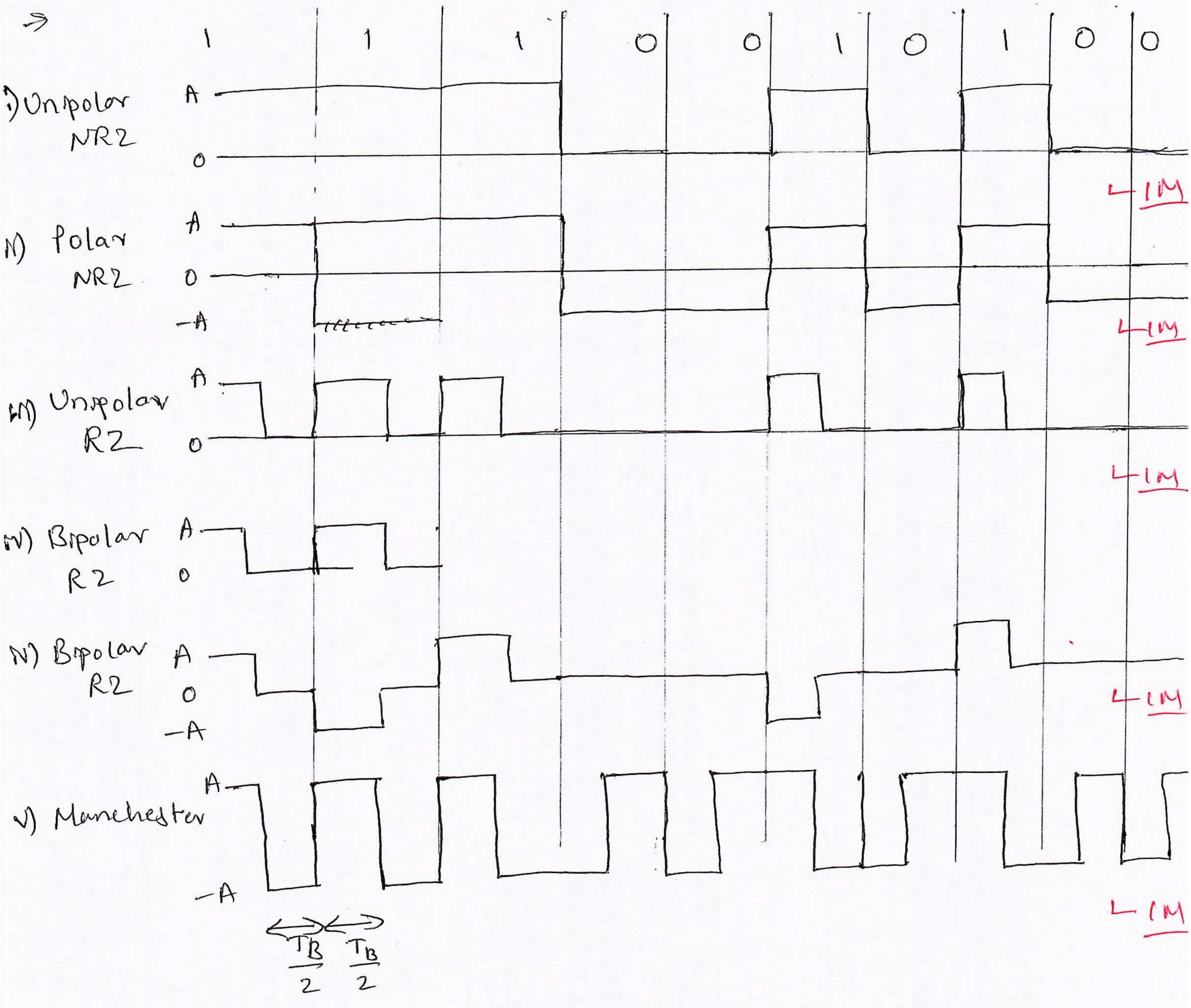
and

$$\hat{H}_Q(f) = \frac{1}{2j}\left[\hat{H}(f) - j\hat{H}^*(f)\right]$$

Using IFT, we get.

$$\hat{h}(t) = \int_{-\infty}^{\infty} \hat{H}(f)\, e^{j2\pi f t}\, df. \quad - 2M$$

2 c). Given the data stream 1110010100. Sketch the transmitted sequence of pulses for each of the following line code. i) Unipolar NRZ ii) Polar NRZ iii) Unipolar RZ iv) bipolar RZ v) Manchester — 5M.



3\. a) Explain the Geometric representation of signals and express the energy of the signal in terms of the signal vector — (5M).

→ A set of orthonormal vectors are mutually orthogonal and ~~all~~ have unit enegy. whose inner product with itself results in 1. and with other vectors results in 0.

The essence of geometric representation of signals is to represent any set of M energy signals $s_i(t)$ as linear combinations of N orthonormal basis functions, where $N \leq M$. That is, given a set of real-valued energy signals $s_1(t), s_2(t), \ldots s_M(t)$, each of duration T seconds.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t), \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \ldots M \end{cases}$$

where the co-efficients

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt. \begin{cases} i = 1, 2, \ldots M \\ j = 1, 2, \ldots N \end{cases} \quad \text{— 1M}$$

The real valued basis functions $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ form an orthonormal set, by which it means

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad \text{— 1M}$$

The set of coefficients $\{s_{ij}\}_{j=1}^{N}$ may be viewed as an N-dimensional signal vector, denoted by s_i.

We may state that each signal in the set $\{s_i(t)\}$ is completely determined by the signal vector.

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$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} ; \quad i = 1, 2 \ldots M.$$

The length of a signal vector s_i is denoted by $\|s_i\|$. The squared length of any signal vector s_i is defined as to be the inner product of s_i with itself as shown by $\|s_i\|^2 = s_i^T s_i = \sum_{j=1}^{N} s_{ij}^2, \quad i = 1, 2 \ldots M.$ — 1M

where s_{ij} is the j^{th} element of s_i.

The energy content of a signal and its representation as a vector can be ~~realated~~ given as.

$$E_i = \int_0^T s_i^2(t)\, dt \quad , i = 1, 2 \ldots M.$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\, \phi_j(t) \right] \left[\sum_{k=1}^{N} s_{ik}\, \phi_k(t) \right] dt.$$

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\, s_{ik} \int_0^T \phi_j(t)\, \phi_k(t)\, dt.$$

We know that $\phi_j(t)$ & $\phi_k(t)$ form an orthonormal set

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2 \qquad \text{— 2M}$$

8p. ① (SM).

3.b). Explain the Gram-Schmidt Orthogonalisation procedure.

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→. Suppose we have a set of M energy signals denoted by $s_1(t), s_2(t), \ldots s_M(t)$. Starting with $s_1(t)$ chosen from this set arbitrarily, the 1st basis function is denoted by

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

where E is the energy of signal $s_1(t)$.

$$s_1(t) = \sqrt{E_1}\,\phi_1(t).$$

$$s_1(t) = s_{11}\,\phi_1(t)$$

the coefficient $s_{11} = \sqrt{E_1}$ — 1M

Next using signal $s_2(t)$, we define the coefficient s_{21} as

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt.$$

we may thus introduce a new intermediate function

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t).$$ — 1M

which is orthogonal to $\phi_1(t)$. over the interval $0 \leq t \leq T$

the second basis function is defined as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$$

$$\phi_2(t) = \frac{s_2(t) - s_{21}\,\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}$$ — 1M

where E_2 is energy of signal $s_2(t)$

w.k.t, $\int_0^T \phi_2^2(t)\,dt = 1$

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in which case above eqn yields.

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0.$$

This shows $\phi_1(t)$ & $\phi_2(t)$ form an orthonormal pair

∴ In general we define $g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\,\phi_j(t)$.

where the co-efficients $S_{ij} = \int_0^T S_i(t)\,\phi_j(t)\,dt$, $j = 1, 2 \ldots i-1$ — 1M

we can now define the set of basis functions

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}} \qquad i = 1, 2, \ldots N.$$

which form an orthonormal set.

The dimension N is less than or equal to the no. of given signals M. depending on one of two possibilities.

1) The signals $S_1(t), S_2(t), \ldots S_M(t)$ form a linearly independent set, in which case $N = M$.

2) The signals $S_1(t), S_2(t), \ldots S_M(t)$ are not linearly independent, in which case $N < M$.

— 1M

Q1. (8)

3. c) Explain the Matched filter receiver with the relevant Mathematical theory. — (6M).. 33

→ Consider a linear filter with impulse response $h_i(t)$ with the received signal $x(t)$ as the i/p.

The resulting filter o/p $y_i(t)$ is given by the convolution of $x(t)$ & $h_i(t)$ re.,

$$y_i(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h_i(t-\tau)\, d\tau \quad \text{—} ①$$

evaluating this integral over the duration of transmitted symbol $0 \leq t \leq T$.

$$y_i(t) = \int_{t=0}^{T} x(t)\, h_i(T-t)\, dt \quad \text{—} ②$$

— 1M

w.k.t. the o/p of i^{th} correlator is defined by

$$x_i = \int_{0}^{T} x(t)\, \phi_i(t)\, dt \quad \text{—} ③$$

For $y_i(t)$ to equal x_i, we find from eqn ② that this condition is satisfied provided that we have to choose

$$h_i(T-t) = \phi_i(t) \quad \text{for } 0 \leq t \leq T,\ i = 1, 2, \ldots M$$

Equivalently, we may express the condition imposed on the desired impulse response of the filter as

$$h_i(t) = \phi_i(T-t) \quad \text{for } 0 \leq t \leq T,\ i = 1, 2, \ldots M$$

— ④.

— 2M

From eqn ④, it is possible to get the N components of the observation vector X, nearly x_i; $i = 1, 2, \ldots N$ that can be used to get the estimate $\hat{m}$ of the i^{th} transmitted symbol m_i

The scheme is depicted in below figure.

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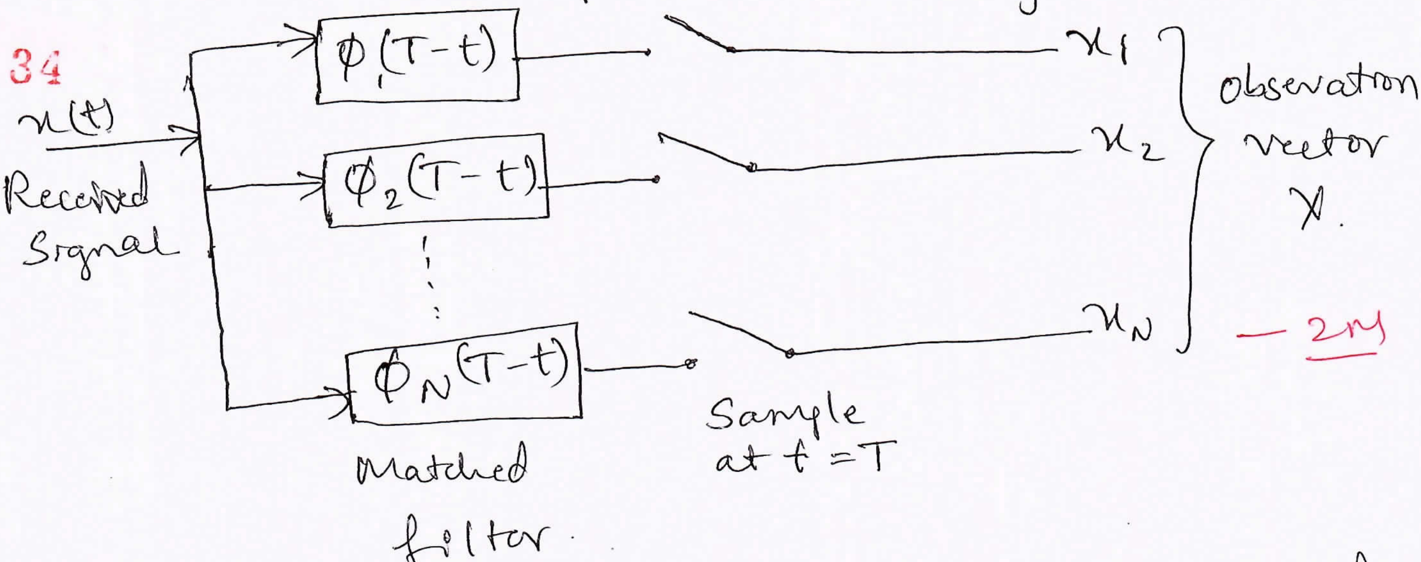

— 2M

A filter whose response is a time reversal and delayed version of its i/p signal as in eqn (4), is known as a filter matched to $\phi_j(t)$.

Optimum detector using a matched filter is called matched filter receiver.

— 1M

Q1. (a)

4.a). Obtain the decision rule for maximum likelihood decoding & explain the correlation receiver. — (7M)

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→ The likelihood function for AWGN channel is given by

$$f_x(X/m_k) = (\pi N_0)^{-N/2} \exp\left[\frac{-1}{N_0}\sum_{j=1}^{N}(x_j - s_{kj})^2\right] \quad -(1)$$

$k = 1, 2, \ldots M.$

The corresponding value of metric is the natural log of likelihood function and is given by.

$$\ln[f_x(X/m_k)] = -\frac{N_0}{2}\ln(\pi N_0) - \frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{kj})^2 \quad -(2)$$

— 1M

$k = 1, 2 \ldots M.$

From above eqn (2) it is seen that log-likelihood fun is maximum when the RHS of eqn (2) is maximum.

Second term is negative and we get maximum when magnitude of summation term is minimum.

This leads to the decision rule as.

choose $\hat{m} = m_i$, if. $\sum_{j=1}^{N}(x_j - s_{kj})^2$ is minimum for $k = i$

Further. $$\sum_{j=1}^{N}(x_j - s_{kj})^2 = \|X - S_k\|^2 \quad -(3)$$

— 1M

which is the square of the distance between the received signal point X and signal point S_k.

If $\ln[f_x(X/m_i)]$ is maximum for $k = i$, then this indicates that X is nearer to "S_i" than other signal points.

Hence decision rule can be simplified by

38 expanding left-hand side equation.

$$\sum_{j=1}^{N}(x_j - s_{kj})^2 = \sum_{j=1}^{N} x_j^2 - 2\sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} s_{kj}^2, \text{ — 1M}$$

$k = 1, 2, \ldots M.$

In RHS, 1^{st} term is the energy of received signal and can be ignored

3^{rd} term gives the energy of the signal $s_k(t)$ and.

2^{nd} term $\sum_{j=1}^{N} x_j s_{kj}$ is the inner product of received vector X and signal vector s_k.

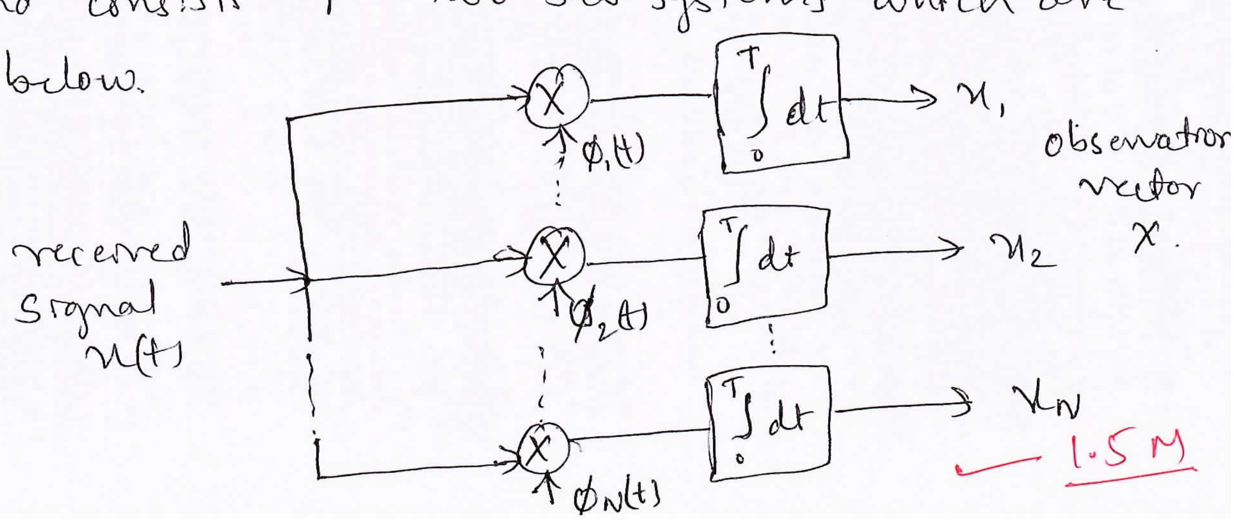
From eqn ③ X is minimum if $k = i$ & Y_k is maximum

i.e., $Y_k = \sum_{j=1}^{N} x_j - s_{kj} - \frac{1}{2} E_k$. — 1M

Correlation Receiver.

For an AWGN channel, when the transmitted signals are equally likely, the optimum receiver which minimizes average probability of error is a correlation receiver.

Correlation receiver works on maximum likelihood decision and consists of two sub-systems which are as shown below.



— 1.5M

Q7. 4a (10)

Fig. (a) shows the Bank of Correlators.

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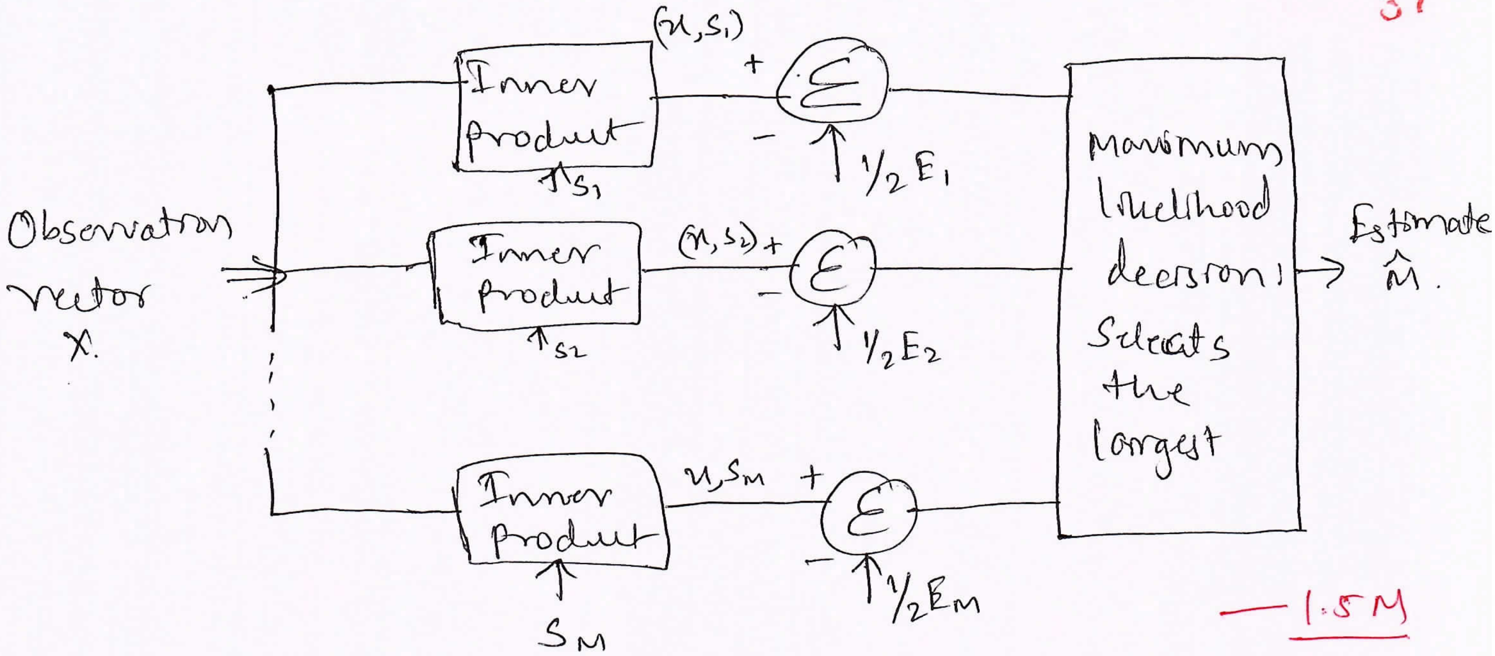


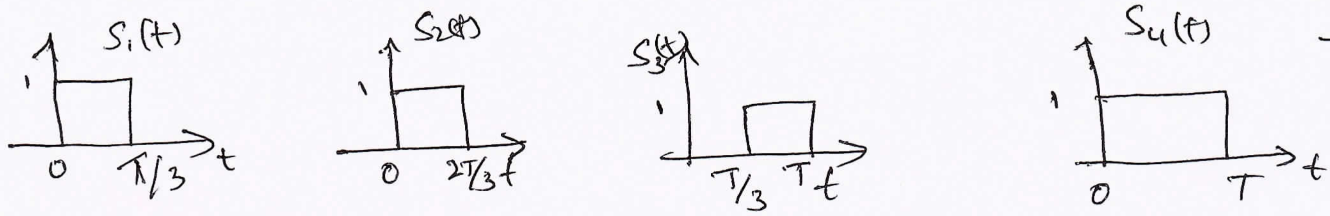
— 1.5M

fig (b) shows the computation of $\hat{m}$

Fig (b) estimates inner product of 'x' with signal vector S_i, $i = 1, 2 \ldots M$, from which the signals half energy $\frac{1}{2}E_i$ is substracted and finally maximum likelihood decision is employed to get the estimate $\hat{m}$.

Q (11)

4.b) Consider four signals $s_1(t)$, $s_2(t)$, $s_3(t)$ & $s_4(t)$ as shown in fig. below. Use GSO procedure to find the orthonormal basis functions for this set of signals. — 9M



→ First condition to apply this GSO procedure is to check whether the given signals are linearly dependent or independent.

By observation of waveforms / signals

$$s_4(t) = s_1(t) + s_3(t).$$

Therefore given set is not linearly independent.

$$\phi_i(t) = \frac{s_i(t)}{\sqrt{E_i}}$$ where E_i is the energy of signal $s_i(t)$

$$E_i = \int_0^T s_i^2(t)\,dt \quad , \quad i = 1, 2, \ldots M.$$

w.k.t.
$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}$$

where $g_i(t)$ is an intermediate function

given as
$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t).$$

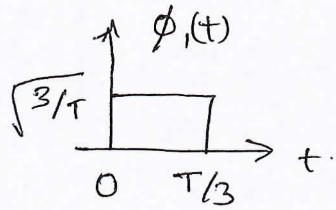
$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \quad , \quad j = 1, 2, \ldots i-1$$ — 1M

1)
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

28

$$E_1 = \int_0^{T/3} s_1^2(t)\,dt = \int_0^{T/3} 1^2\,dt = T/3.$$

$$\therefore \phi_1(t) = \frac{s_1(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\, s_1(t)$$



— 2M

2) $$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}.$$

$$g_2(t) = s_2(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t).$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

To calculate s_{21}

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt.$$

$$= \int_0^{T/3} 1\cdot\sqrt{\frac{3}{T}}\,dt. = \sqrt{\frac{3}{T}} \times [1]_0^{T/3}$$

$$= \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{3}{T} \times \left(\frac{T}{3}\right)^2}$$

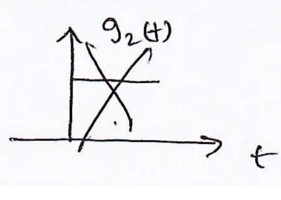
$$s_{21} = \sqrt{\frac{T}{3}}$$

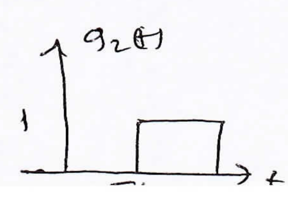
$\therefore g_2(t)$ between 0 to $T/3$. is given by.

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t) = 1 - \sqrt{\frac{T}{3}}\cdot\sqrt{\frac{3}{T}} = 1 - 1 = 0.$$

$g_2(t)$ between $T/3$ to $2T/3$ is given by

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t) = 1 - 0 = 1$$



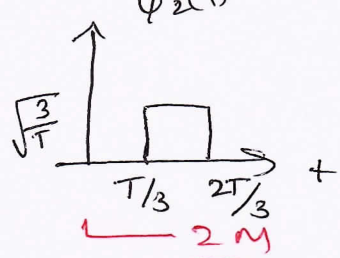


Calculation of Energy E_2 is

$$E_2 = \int_0^T g_2^2(t)\,dt = \int_{T/3}^{2T/3} g_2^2(t)\,dt = \int_{T/3}^{2T/3} 1^2\,dt$$

$$\therefore E_2 = \left[1\right]_{T/3}^{2T/3} = \frac{2T}{3} - \frac{T}{3} = \frac{T}{3}.$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\, g_2(t).$$



2M

3).
$$\phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)\,dt}}$$

$$g_3(t) = S_3(t) - \sum_{j=1}^{i-1} S_{ij}\,\phi_j(t).$$

$$g_3(t) = S_3(t) - S_{31}\phi_1(t) - S_{32}\phi_2(t).$$

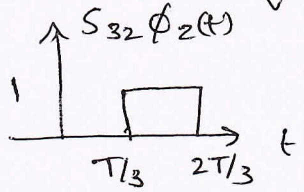
To calculate S_{31} & S_{32} we have.

$$S_{31} = \int_0^T S_3(t)\,\phi_1(t)\,dt = \int_0^{T/3} S_3(t)\,\phi_1(t)\,dt.$$

$$S_{31} = \int_0^{T/3} 0 \times \sqrt{3/T}\,dt = 0$$

$$S_{32} = \int_0^T S_3(t)\,\phi_2(t)\,dt = \int_{T/3} S_3(t)\,\phi_2(t)\,dt$$

$$S_{32} = \int_{T/3}^{2T/3} 1 \times \sqrt{3/T}\,dt = \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{T}{3}}$$



~~$g_3(t)$~~ $g_3(t)$ between 0 to $T/3$ is

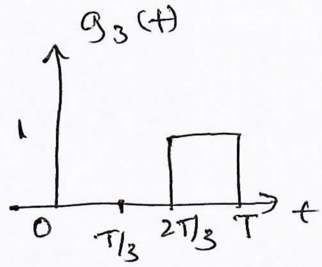
$$g_3(t) = 0 - 0 - 0 = 0.$$

$g_3(t)$ between $T/3$ to $2T/3$ is

$$g_3(t) = 1 - 0 - 1 = 0.$$

$g_3(t)$ between $2T/3$ to T

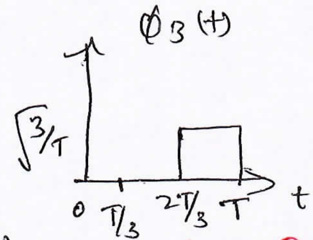
$$g_3(t) = 1 - 0 - 0 = 1.$$



$$E_3 = \int_0^T g_3^2(t)\,dt = \int_{2T/3}^{T} g_3^2(t)\,dt = \int_{2T/3}^{T} 1^2\,dt$$

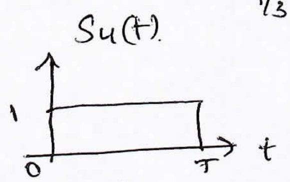
$$E_3 = T - \frac{2T}{3} = T/3$$

$$\phi_3(t) = \frac{g_3(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\,g_3(t)$$



2M

4) $$\phi_4(t) = \frac{g_4(t)}{\sqrt{\int_0^T g_4^2(t)\,dt}}$$



$$g_4(t) = S_4(t) - S_{41}\phi_1(t) - S_{42}\phi_2(t) - S_{43}\phi_3(t)$$

$$S_{41} = \int_0^T S_4(t)\,\phi_1(t)\,dt = \int_0^{T/3} 1 \times \sqrt{3/T}\,dt$$

$$S_{41} = \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{T}{3}}$$

$$S_{41}\phi_1(t) = \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}} = 1$$

Similarly, $S_{42}\phi_2(t) = S_{43}\phi_3(t) = 1$

$\therefore$ $g_4(t)$ between 0 to $T/3$

$$g_4(t) = 1 - 1 - 0 - 0 = 0.$$

between $T/3$ to $2T/3$ $\Rightarrow$ $g_4(t) = 1 - 0 - 1 - 0 = 0$

between $2T/3$ to T $\Rightarrow$ $g_4(t) = 1 - 0 - 0 - 1 = 0.$

$\therefore$ $g_4(t) = 0$, Hence $\phi_4(t) = 0.$

2M

(13)

Module - 3

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5. a) Define binary phase shift keying. Derive the probability of error of BPSK. — (7M)

→. In BPSK, the signals $S_1(t)$ and $S_2(t)$ that are used to represent binary symbols '1' and '0' respectively are defined as

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b.$$

and

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b.$$

Signals $S_1(t)$ and $S_2(t)$ differ only in phase by 180^0 — 1M and are referred to as antipodal signals.

It is seen that both signals can be represented in terms of only single basis function. $\phi_1(t)$ having unit energy.

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t). \quad 0 \le t \le T_b.$$

$$\therefore \left.\begin{aligned} S_1(t) &= \sqrt{E_b}\,\phi_1(t) \\ S_2(t) &= -\sqrt{E_b}\,\phi_1(t) \end{aligned}\right\} 0 \le t \le T_b.$$ — 1M

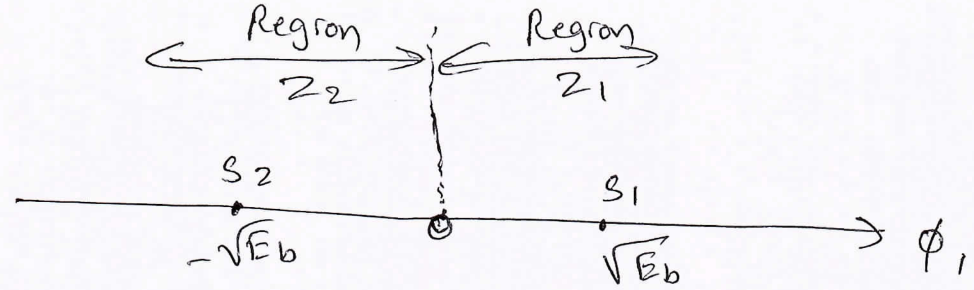
BPSK system is explained (characterized) by a set of antipodal signals can be represented in one-dimensional signal space (i.e... N=1)

The co-ordinates of the message points are

$$S_{11} = \int_{t=0}^{T_b} S_1(t)\,\phi_1(t)\,dt = +\sqrt{E_b}. \text{ for } S_1(t)$$

$$S_{21} = \int_{t=0}^{T_b} S_2(t)\,\phi_1(t)\,dt = -\sqrt{E_b} \text{ corresponding to } S_2(t)$$

40



— 1M

fig. - Signal Constellation diagram.

Probability of error calculation

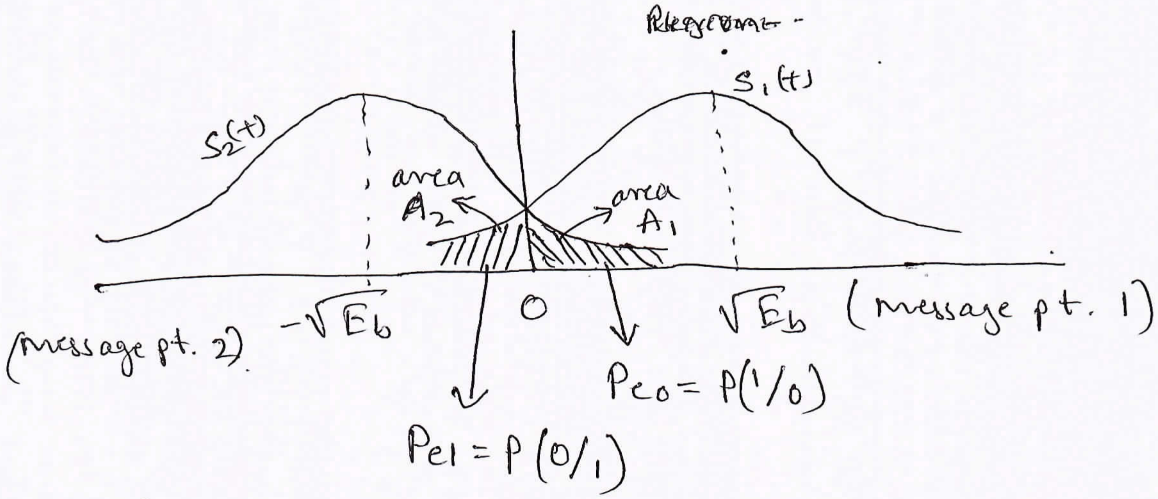


fig : Signal with Noise (AWGN) present.

Avg. probability of error $= P_{e1} + P_{e0}$.

The observation vector X_1 is given by.

$$X_1 = \int_0^T x(t)\,\phi_1(t)\,dt.$$

The error is of two types

1) $P_e(0/1)$ i.e., transmitted as '1' but received as '0'.

2) $P_e(1/0)$ i.e, transmitted as '0' but received as '1'.

— 1M

We assume that in most cases we have the areas A_1 & A_2 are same.

$$P_e(1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty \exp\left[-\frac{(x_1-\mu_1)^2}{2\sigma^2}\right] dx_1$$

μ = mean value $= -\sqrt{E_b}$ $\qquad \sigma^2$ = variance = $N_0/$

14

Threshold value = Th = 0.

$$\therefore P_{e0} = P_e(1/0) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} \int_0^{\infty} \exp\left[-\frac{[x_1-(-\sqrt{E_b})]^2}{2\frac{N_0}{2}}\right] dx_1$$

$$P_{e0} = \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left(-\frac{(x_1+\sqrt{E_b})^2}{N_0}\right] dx_1$$

put $z = \frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}$ $\quad dx_1 = \sqrt{N_0}\, dz.$

$$\therefore P_{e0} = \frac{1}{\sqrt{\pi \cancel{N_0}}} \int_{z=\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp(-z)^2\, dz . \sqrt{\cancel{N_0}} \quad -(1)$$

w.k.t $\quad \frac{1}{2}\operatorname{erfc}(u) = \frac{1}{\sqrt{\pi}} \int_u^{\infty} e^{(-z)^2} dz. \quad -(2)$

From eqn (1) & (2).

$$P_{e0} = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{N_0}}$$

Similarly $\quad P_e(0/1) = \frac{1}{2\pi\sigma^2} \int_{-\infty}^{0} \exp -\frac{(x_2-\mu)^2}{2\sigma^2} dx_2$

$$P_e(0/1) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{N_0}} \quad - 2M$$

Total probability of error $P_e = P_e(1/0)\, P_e(0) + P_e(0/1)\, P_e(1)$

Assuming probability of 1's & 0's are equal

$$P_e = \frac{1}{2}\left[P_e(1/0) + P_e(0/1)\right]$$

$$P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{N_0}} \quad - 1M$$

5 b). Define M-ary QAM. Obtain the constellation of QAM for M=4 and draw the signal space diagram. — (4M)

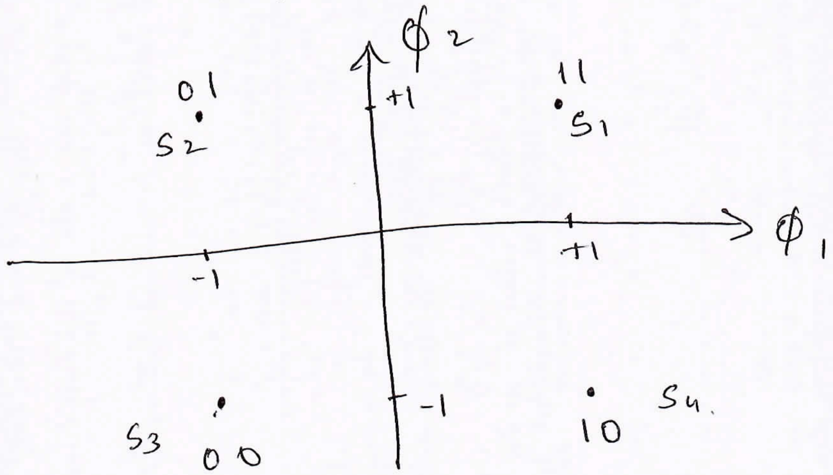
→ M-ary QAM is a hybrid form of modulation i.e., in this modulation technique the carrier experiences amplitude as well as phase modulation.

Therefore this also called as M-ary Quadrature Amplitude phase-shift keying (QASK).

In M-ary QAM a constant amplitude constraint is removed so as to permit the in-phase & quadrature components to be independent.

Some message points amplitude is increased (AM) and they are placed far apart from other message points so that area of decision region increases and therefore chances of falling the message points in other area reduces. and hence P_e reduces. — 2M

Constellation diagram for M=4.



— 2M

Q. (15)

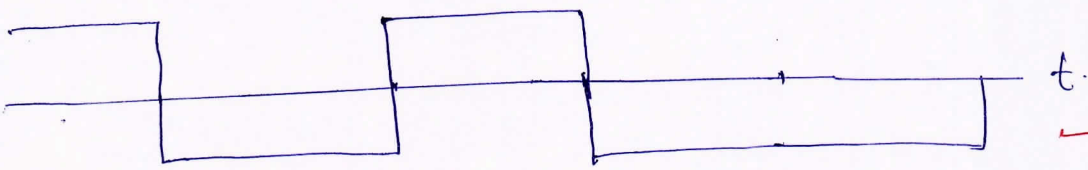
5c). Given the input binary sequence 1100100001. Sketch the waveforms of the inphase and quadrature components of a modulated wave and next sketch the QPSK signal → (5M).

→

Binary Sequence ⇒ 1 1 0 0 1 0 0 0 0 1

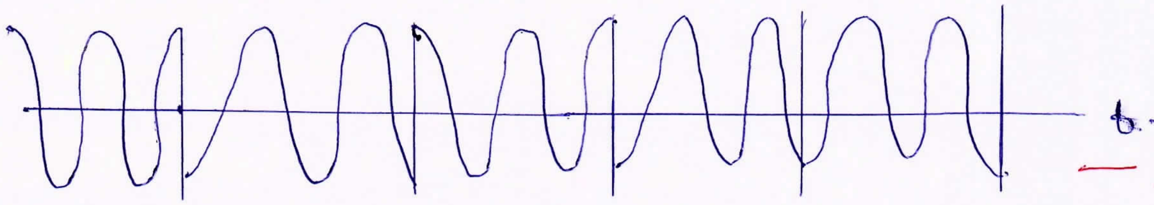
Inphase ⇒ 1 0 1 0 0

I(t) ⇒



1M

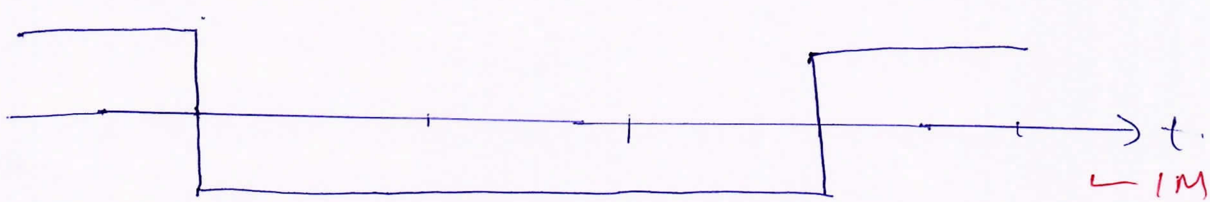
I(t) cos(2πfct) ⇒



1M

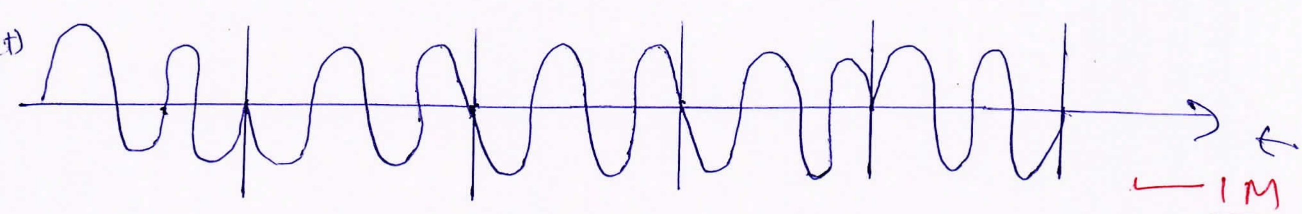
Quadrature ⇒ 1 0 0 0 1

Q(t) ⇒



1M

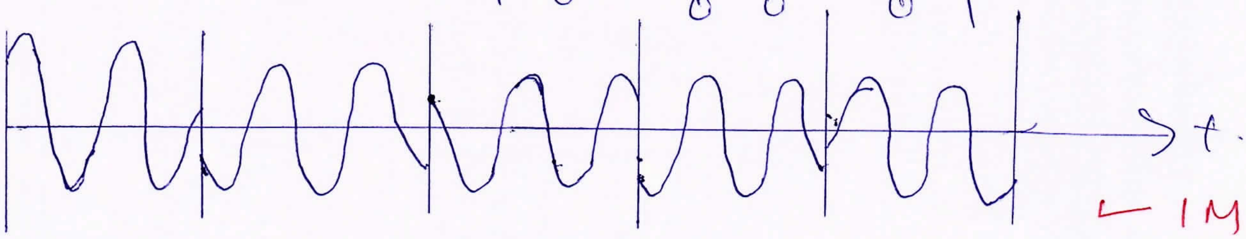
Q(t).Sin(2πfct) ⇒



1M

S(t) = QPSK.

11 00 10 00 01



1M

Q7. (16)

6 a) Describe the QPSK signal with its signal space characterization. With a neat block diagram explain the generation and detection of QPSK signals. — (6M)

→ In digital Communication, QPSK is one such modulation scheme which minimizes BW required for transmission.

In QPSK, the phase of the carrier takes one of the four discrete values like $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$, which are equally spaced in interval $(0, 2\pi)$.

QPSK signal is given by.

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere.} \end{cases}$$

Each phase can represent a pair of binary bits known as dibits.

It is possible to represent four signals $s_i(t)$, $i = 1, 2, 3, 4$, using 2 orthonormal basis functions $\phi_1(t)$ & $\phi_2(t)$ and they are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t \quad 0 \le t \le T. \quad \text{— 1M}$$

∴ QPSK signals can be represented in a 2-dimensional signal space (N=2) with four signal vectors, they are

$$S_1(t) = \sqrt{\frac{2E}{T_b}} \cos\left[2\pi f_c t + \frac{\pi}{4}\right] \quad \text{for } i=1 \text{ --- } 10$$

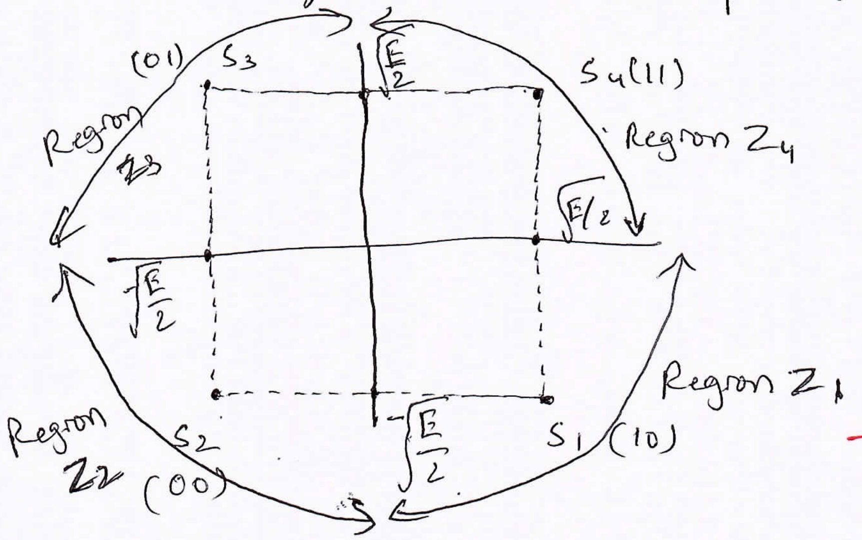
$$S_2(t) = \sqrt{\frac{2E}{T_b}} \cos\left[2\pi f_c t + 3\frac{\pi}{4}\right] \quad \text{for } i=2 \text{ --- } 00$$

$$S_3(t) = \sqrt{\frac{2E}{T_b}} \sin\left[2\pi f_c t + 5\frac{\pi}{4}\right] \quad \text{for } i=3 \text{ --- } 01$$

$$S_4(t) = \sqrt{\frac{2E}{T_b}} \sin\left[2\pi f_c t + \frac{7\pi}{4}\right] \quad \text{for } i=4 \text{ --- } 11 \quad \text{— 1M}$$

26 A QPSK can be regarded as superposition of two BPSK waves that use carriers $\cos 2\pi f_c t$ & $\sin 2\pi f_c t$.

Four message points will be placed 90° apart, which is shown below. All message points will be placed at distance $= \sqrt{E/2}$



— 1M

fig Signal space representation for coherent QPSK.

QPSK generation

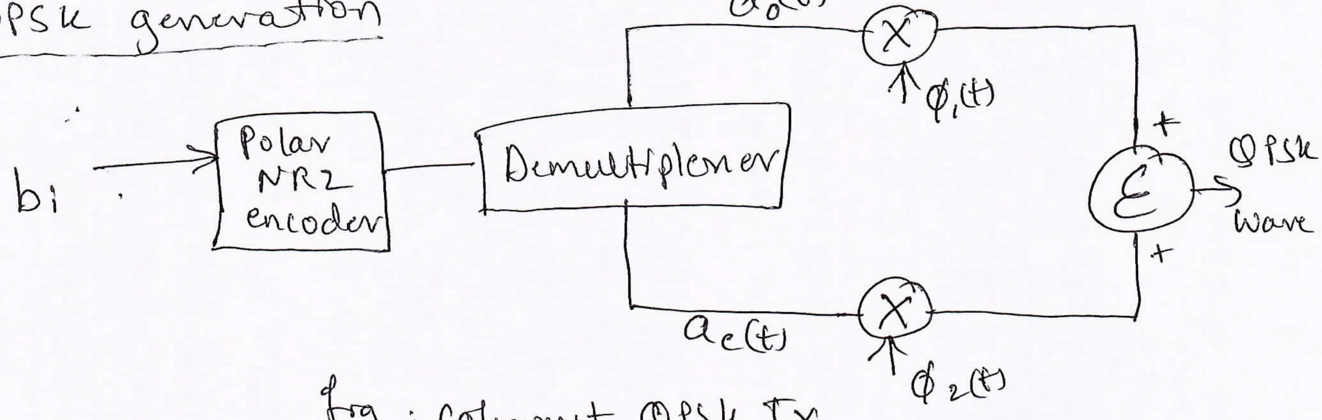


fig : Coherent QPSK Tx

'b_i' is the input binary sequence with a bit interval of Tb. seconds.

Demultiplexer is serial to parallel converter. Thus using Demux a binary sequence is divided into an odd indexed sequence $a_o(t)$ & an even indexed sequence $a_e(t)$. These sequences phase modulate two carriers with same freq. but that are in phase quadrature. Then the two signals are added to get QPSK wave.

— 1.5M

Detection of QPSK signal.

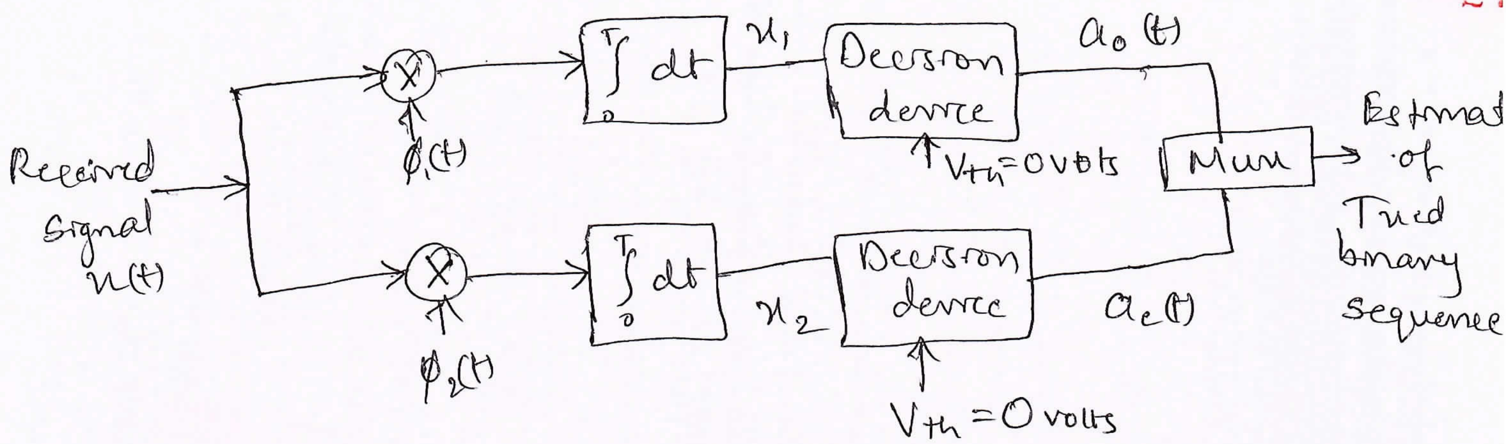


fig: Coherent detection of QPSK signal.

It consists of a pair of correlators with locally generated pair of $\phi_1(t)$ & $\phi_2(t)$. let x_1 and x_2 are the correlator outputs in response to the received signal $x(t)$

A decision is made by comparing x_1 and x_2 with a threshold of 0 volts.

If inphase channel o/p, $x_1 > 0$, a decision is made in favour of symbol '1', otherwise it is in favour of symbol '0'.

If $x_2 > 0$, a decision is made in favour of symbol '1', otherwise it is in favour of symbol '0'

The binary sequences are multiplexed to get an estimate of original binary sequence. — 1.5M

6b). obtain the expression for probability of symbol error of Coherent binary FSK. — 7M.

→ we receive signals at receiver alongwith gaussian noise. ie,

$$x_1(t) = s_1(t) + w_1(t) \text{ and}$$
$$x_2(t) = s_2(t) + w_2(t).$$

let us assume here Y as new random variable whose sample value y is equal to difference between x_1 & x_2

$$y = x_1 - x_2$$

the conditional mean value of the random variable Y depends on which binary symbol was transmitted.

when symbol '1' was transmitted, mean value is given by

$$E[Y/1] = E[X_1/1] - E[X_2/1]$$
$$= E[\sqrt{E_b}] - E[0]$$
$$= \sqrt{E_b}$$ — 1M

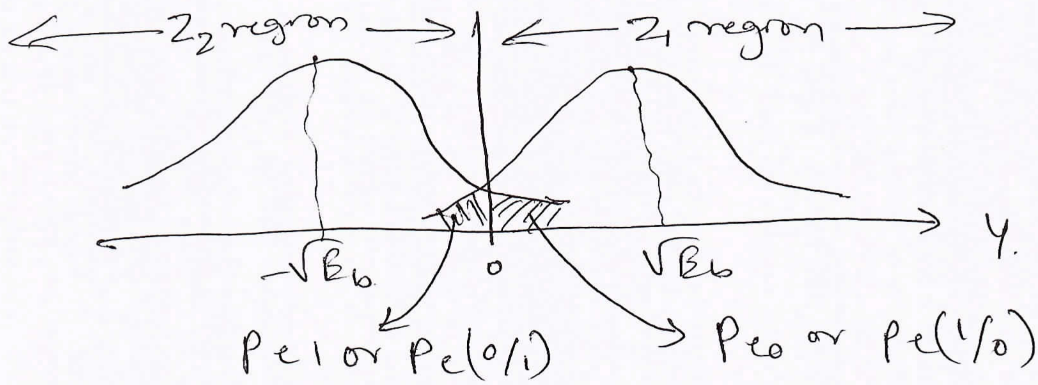
Similarly when symbol '0' was transmitted

$$E[Y/0] = E[X_1/0] - E[X_2/0]$$
$$= E[0] - E[+\sqrt{E_b}]$$
$$= -\sqrt{E_b}.$$ — 1M

the variance of Y is independent of which symbol was sent.

$\therefore$ $Var[Y] = Var[x_1] + Var[x_2]$

$$Var[Y] = \frac{N_0}{2} + \frac{N_0}{2} = N_0$$ — 1M



The observation vector x has two elements x_1 & x_2 given by

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt \;\&$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt.$$

where $x(t)$ is the received signal

w.k.t. gaussian function is given by — 1M

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int \exp\left[\frac{-(x-\mu_x)^2}{2\sigma^2}\right] dx.$$

66

18 Here we have considered Y. as gaussian variable

$$\therefore P_e(1/0) = P_{e0} = \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(Y-(-\sqrt{E_b}))^2}{2N_0}\right] dy.$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(Y+\sqrt{E_b})^2}{2N_0}\right] dy.$$

w.k.t. $\frac{1}{2}\operatorname{erfc}(u) = \frac{1}{\sqrt{\pi}} \int_u^\infty \exp(-z)^2\, dz.$ — (1)

— 1M

$$Z = \frac{Y+\sqrt{E_b}}{\sqrt{2N_0}}$$

for $Y=0$ $\quad Z = \frac{\sqrt{E_b}}{\sqrt{2N_0}}$

$y=\infty \quad Z=\infty \quad$ & $dy = \sqrt{2N_0}.\, dz.$

$$\therefore \quad P_{e0} = \frac{1}{\sqrt{2\pi N_0}} \int_{Z=\sqrt{\frac{E_b}{2N_0}}}^{\infty} \exp(-z)^2 \sqrt{2N_0}\, dz$$

$$P_{e0} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \exp(-z)^2\, dz. \quad \text{— (2).}$$

Comparing eqn (1) & (2).

$$P_{e0} = P_e(1/0) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}}$$ — 1M

Similarly

$$P_{e1} = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}}$$

$\therefore$ Total probability $= P_e = \frac{1}{2}\left[P_{e0} + P_{e1}\right]$

$$\therefore \quad P_e = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{2N_0}}$$ — 1M

//.

6 c) Illustrate the operation of DPSK for the binary Sequence 10010011 — (3M)

→

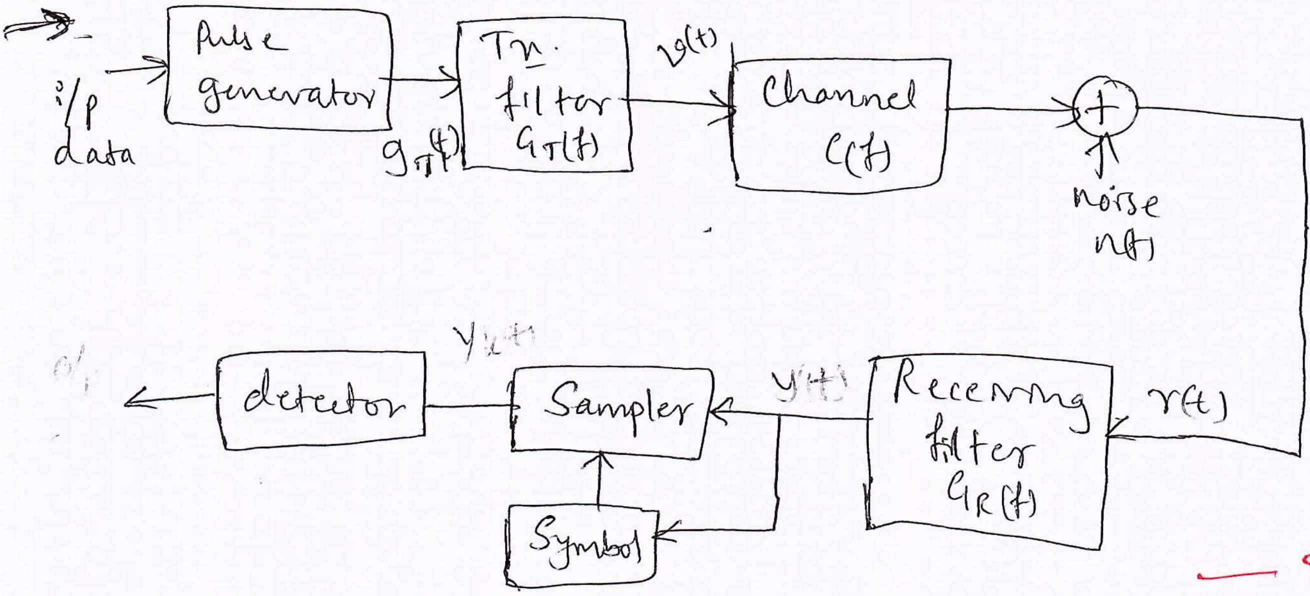
b_k =		1	0	0	1	0	0	1	1
d_{k-1} =		1	1	0	1	1	0	1	1
d_k =	1 (reference bit)	1	0	1	1	0	1	1	1
Transmitted phase =	0	0	π	0	0	π	0	0	0

— 1M (for d_{k-1}), — 1M (for d_k), — 1M (for transmitted phase)

Module - 4

7 a). With a neat block diagram, Explain the digital PAM transmission through bandlimited baseband channels & obtain the expression for ISI. — (5M).

→



— 2M

P. Ikbu 1a

i) The sys consists of a transmitting filter having an impulse response $g_T(t)$. The linear filter channel with AWGN, a receiving filter with an impulse response of $g_R(t)$, a sampler that periodically samples the o/p of the receiving filter and a symbol detector.

The sampler requires the extraction of a timing signal from the received signal, which serves as a clock that specifies the appropriate time instants for sampling the o/p of the receiving filter.

The baseband signal at the o/p of the txing filter is expressed as

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t-nT)$$

The channel o/p or received signal is given as

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t-nT) + n(t)$$

$h(t)$ – impulse response of the cascade of the txing filter and the channel.

$$\therefore h(t) = c(t) * g_T(t).$$ — 1M

$n(t)$ – represents the AWGN.

If $g_R(t)$ is matched to $h(t)$, then o/p SNR will be maximum

The o/p of the receiving filter is given as

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t-nT) + w(t)$$

where $x(t) = h(t) * g_R(t)$.

$$x(t) = g_T(t) * c(t) * g_R(t)$$

& $w(t) = n(t) * g_R(t)$. — 1M

To recover the info. symbols $\{a_n\}$, o/p of receiving filter is sampled periodically

Thus the sampler produces

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n\, x(mT - nT) + w(mT)$$

or

$$y_m = \sum_{n=-\infty}^{\infty} a_n\, x_{m-n} + w_m.$$

$$y_m = x_0 a_m + \sum_{n \neq m} a_n\, x_{m-n} + w_m.$$

The 1st term is the desired symbol a_m, scaled by gain parameter x_0.

The 2nd term $\sum_{n=-\infty} a_n x_{m-n}$ represents the effect of the other symbols at the sampling instant $t = mT$, called the intersymbol interference (ISI).

3rd term represents AWGN. — 1M

7.b). What are adaptive equalizers? Explain the linear adaptive equalizer based on the MSE. criterion → (6M).

→ For channels whose freq. response characteristics are unknown and time variant, we may use a linear filter with adjustable parameters, which are updated on a periodic basis to compensate for the channel distortion. Such a filter, having parameters that are adjusted periodically, is called as adaptive equalizers.

An adaptive equalizer is a filter that compensates for dispersion effects of a channel.

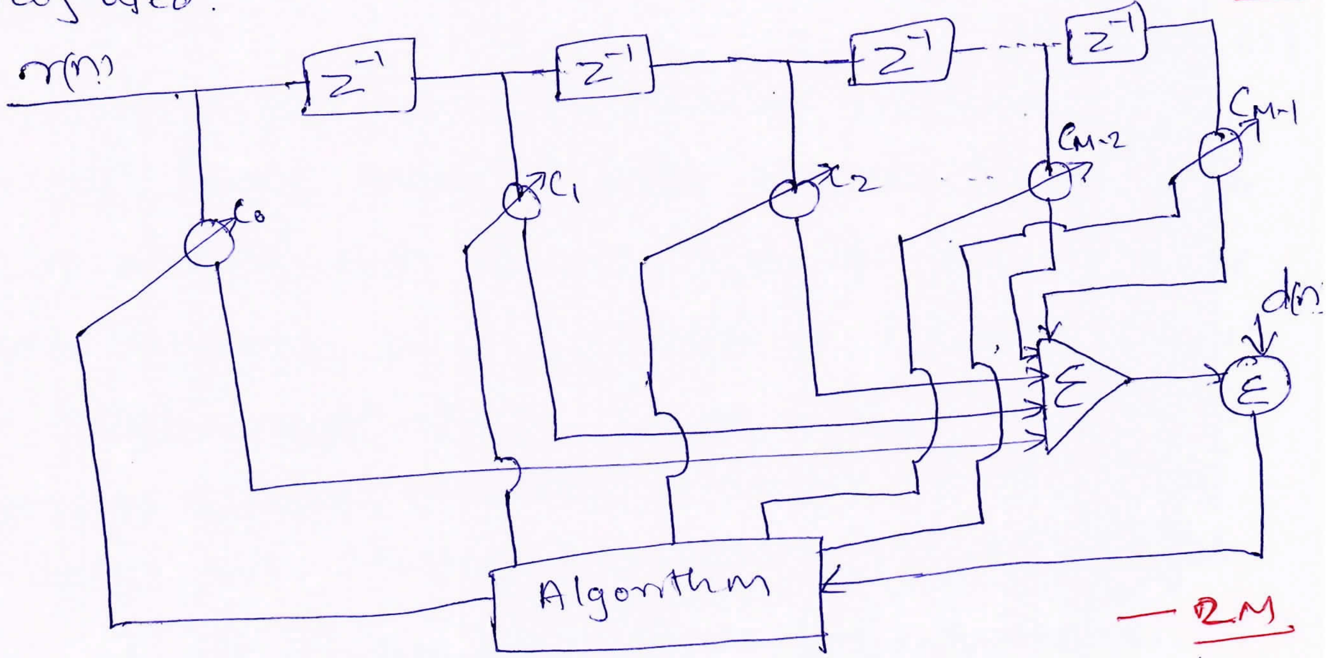
Adaptive equalization does this by adjusting its coefficients continuously during the transmission of data.

Adaptive equalization can be achieved in two ways

1) Pre channel equalization 2) Post channel equalization.

Post channel is well established technique and pre channel is rarely used.

— 1M



— 2M

Fig: Components of adaptive equalization

It has tapped delay line filters with set of delay elements, set of adjustable multipliers connected to the delay line taps & a summer for adding multiplier o/p.

All this information is given to the algorithm

$$y(nT) = \sum_{i=0}^{M-1} C_i \, x(nT - iT)$$

C_i - weight of i^{th} tap. , total no. of taps are M.

In conventional FIR filter, the tap weights are constant to particular designed response, In adaptive equalizer the C_i's are variable and are adjusted by an algorithm. — 1M

Adaptive equalizers works in two modes.
1) Training Mode 2) Decision directed mode.

In training mode, A known sequence d(n) is transmitted and synchronized version of it is generated in the receiver applied to adaptive equalizer.

This training sequence is selected as maximum length PN sequence.

The difference between the resulting response $y(nT)$ and desired response $d(nT)$ is error signal. This error signal is used to ~~elcom~~ estimate the direction in which the coefficients of filter are to be optimized using algorithm.

$$\text{error signal } e(nT) = y(nT) - d(nT)$$ — 1M

Once error signal is minimum, squared error is observed, adjust the tap weight and optimize the equalizer

In Decision directed mode, the actual info-bearing signal is transmitted.

Q7.b

(ii) Then we apply the output of decision device to the equalizer.

In this mode it tracks relatively slow variations in the channel characteristics which are adjusted by algorithms like Least Mean Square (LMS). — 1M

29

7 c) The binary sequence 10010110010 is the input to the precoder whose output is used to modulate a duobinary transmitting filter. obtain the precoded sequence, transmitted amplitude levels, the received signal levels and the decoded sequence. → (5M)

22

→

B_k	⇒		1	0	0	1	0	1	1	0	0	1	0
Precoded Sequence	⇒	1	0	0	0	1	1	0	1	1	1	0	0 — 1M
Polar representation	⇒	+1	-1	-1	-1	+1	+1	-1	+1	+1	+1	-1	-1
~~Received Signal levels~~	⇒	+1	-1	-1	-1	+1	+1	-1	+1	+1	+1	-1	-1 ← 1M
Duobinary Coded output	⇒		0	-2	-2	0	2	0	0	2	2	0	-2 ← 1M
Received sequence	⇒		0	-2	-2	0	2	0	0	2	2	0	-2
Decoded binary sequence	⇒		1	0	0	1	0	1	1	0	0	1	0 ← 2M

8 a). What is eye pattern? What is the Nyquist criterion for zero ISI? Given an example of the pulse with zero ISI. → (5M)

→ The ISI and other signal degradations can be viewed conveniently on an oscilloscope through what is known as eye pattern.

If we display the received signal on the vertical input with the horizontal sweep rate set at $1/T$, the resulting oscilloscope display is called an eye pattern.

Careful analysis of the eye pattern is imp. in analyzing the degradation mechanism.

The interior region of eye pattern is called eye opening.

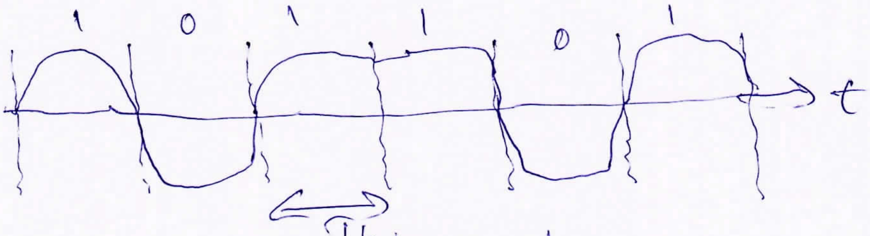
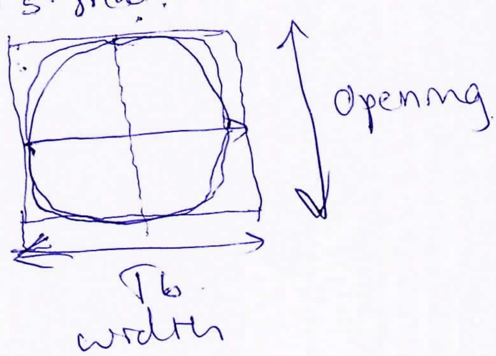


fig: received signal.



— 2M

Fig: Eye pattern.

We set superposition of successive symbol intervals to produce eye pattern

When the effect of ISI is excessive, traces from the upper portion of the eye pattern cross traces from lower portion with the result that the eye is completely closed.

Nyquist Criterion

The pulse shaping function $p(t)$ with FT given by $P(f)$ that satisfies.

Q. 23

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b.$$

has $$p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i = k \\ 0 & i \neq k \end{cases} \rightarrow (1).$$

A pulse p(t) that satisfies the condition of eqn (1) is called a nyquist pulse, and the condition itself is refered to as Nyquist criterion for distortionless binary baseband data transmission. — 2M

The ISI can be minimised by controlling p(t) in time domain & P(f) in freq. domain.

One of the functions that gives zero-ISI is

$p(t) = \text{sinc}(2B_0 t)$, where $B_0 = \frac{1}{2T_b}$ is called Nyquist BW.

Nyquist BW is defined as the minimum transmission BW for zero ISI. — 1M

Q.P. (24) 50

8. b) Explain the design of bandlimited signals with controlled ISI. Describe the time domain and freq. domain characteristics of a duobinary signal. → (5M)

→ As we know that raised cosine spectrum $P(f)$ and associated $p(t)$ results in a BW larger than the theoretical minimum B_0 or W.

If we desire to reduce the pulse BW further, we must somehow increase the width of $p(t)$. The wider signals have narrower BW.

The desire to widen the pulse width may cause ISI with adjacent pulses.

However, in the binary baseband transmission, with just two symbols 0 and 1, a controlled amount of ISI may be permissible, since there are only a few similar interference patterns.

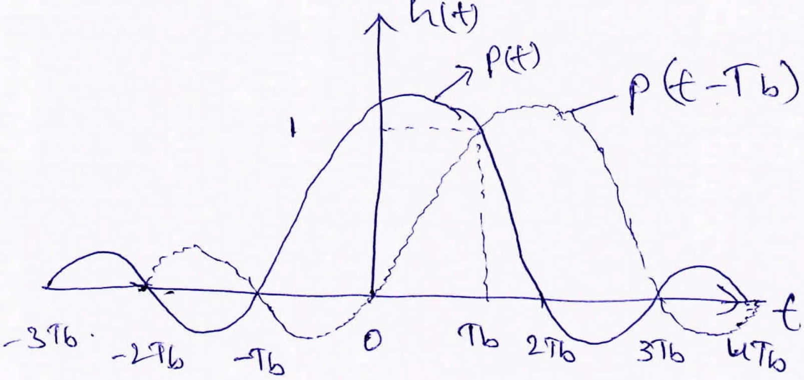
Defining a pulse by $P(nT_b) = \begin{cases} 1 & n=0,1 \\ 0 & \text{for all other } n \end{cases}$

1M

In Duobinary coding some controlled amount of ISI is introduced into the data stream to be transmitted at fixed number of adjacent sampling instants rather than eliminating it.

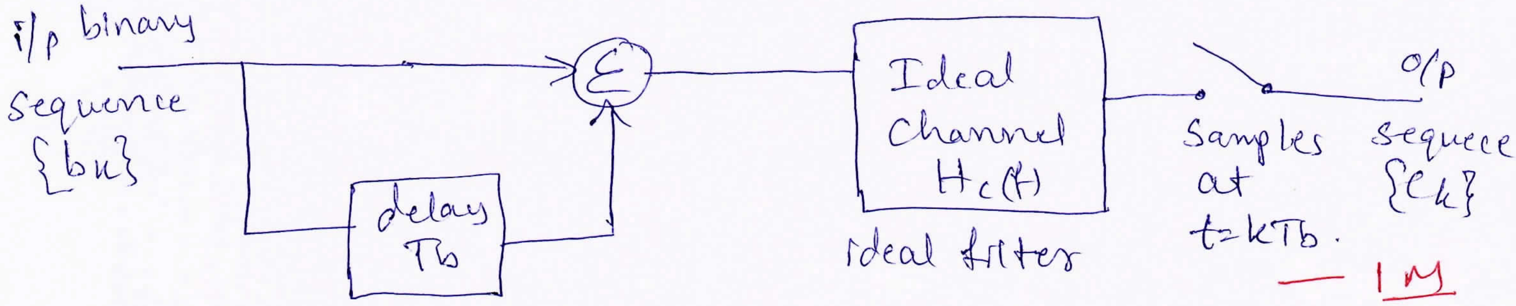
We introduce the correlation i.e., Correlated interface between the pulses and by changing detection procedure at the receiver effect of ISI is cancelled at the detector.

- We consider duobinary filter whose impulse response h(t) has a shape given below.



— 1M

fig ① Impulse response of duobinary filter.



— 1M

fig ② Block diagram of duobinary encoder.

Duobinary encoder has the digital filter and ideal filter.

When sequence $\{b_k\}$ is applied to encoder, the o/p will contain levels $-2, 0, 2$.

$H_c(f)$ is the freq. response of ideal filter or channel.

For every impulse applied we get a pair of impulses spaced at Tb sec.

The freq. response of delay element is $e^{-j2\pi f T_b}$. The freq. response of digital filter is $1 + e^{-j2\pi f T_b}$.

∴ Overall transfer function of this filter is

$$H(f) = H_c(f) + H_c f\, e^{-j2\pi f T_b}.$$

$$= 2H_c(f)\left[\frac{e^{j\pi f T_b} + e^{-j\pi f T_b}}{2}\right] \times e^{-j2\pi f T_b}$$

(25) $H(f) = 2H_c(f)\cos(\pi f T_b)\, e^{-j\pi f T_b}$.

Ideal channel transfer function is

$$H_c(f) = \begin{cases} 1 & |f| \le 1/2T_b \\ 0 & |f| > 1/2T_b \end{cases}$$

$$\therefore \; H(f) = \begin{cases} 1 + e^{-j2\pi f T_b} & |f| \le B_0 \\ 0 & |f| > B_0 \end{cases}$$

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\, e^{-j\pi f T_b} \\ 0 \end{cases}$$

— 2M.

8f- 26

30

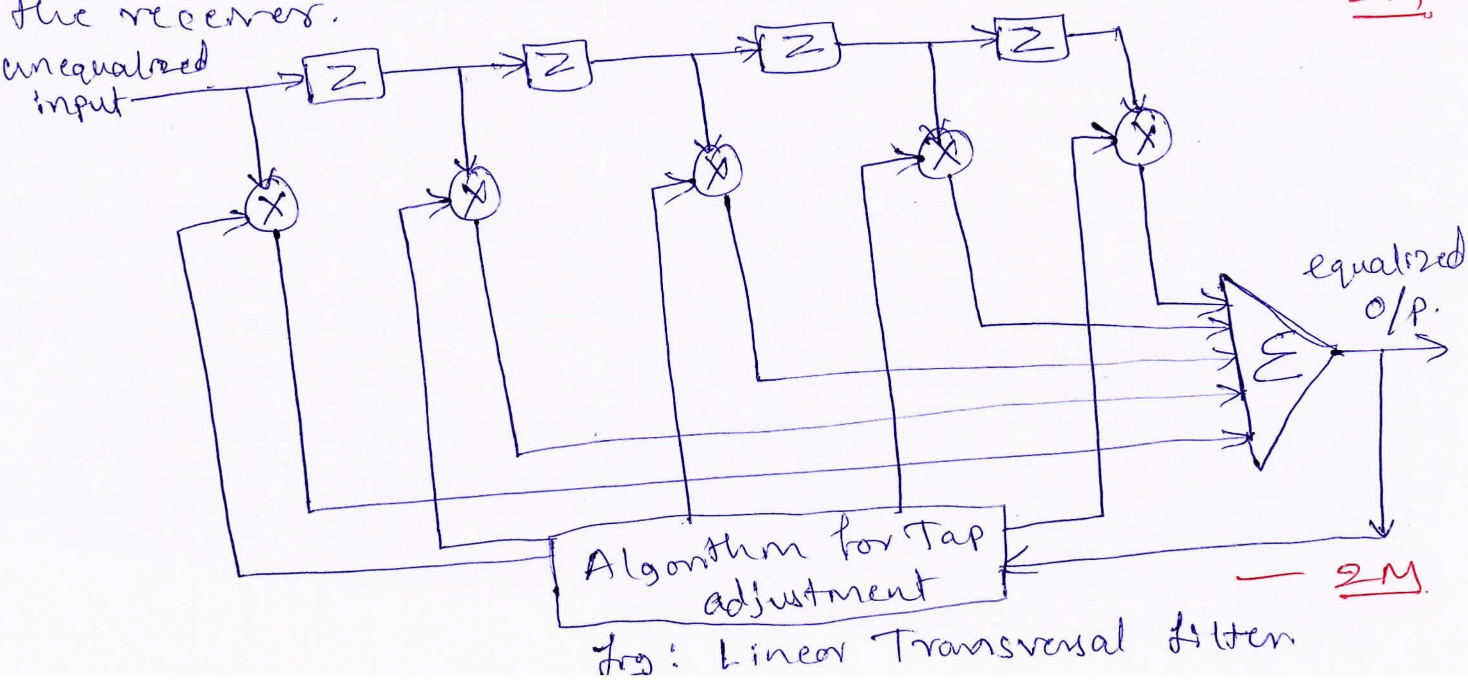
Q 16) what is channel equalization? with a neat diagram explain the concept of equalization using a linear transversal filter. — (6 M).

→ In digital communication systems, data transmitted from the transmitter to the receiver over a channel is distorted by the addition of some random noise in the channel, If the freq. response of the channel is not flat and its phase response is not linear the signal is distorted. To make the amplitude response of the channel flat the channel equalization technique is used. The device used at the receiver is called as equalizer. It is also called as Inverse channel or Inverse filter.

whatever the effect of channel introduces on the signal, we can nullify the effect using equalizer.

To ~~pe~~ compensate the effect of transmitting filter, channel and receiving filter we can use equalizer in the receiver.

— 2 M



— 2 M.

fig: Linear Transversal filter

we know that in real channels, the ISI is limited to a finite no. of samples.

As a result, in practice for example, the channel equalizer is approximated by a finite duration impulse response (FIR) filter or transversal filter, with adjustable tap co-efficients $\{C_n\}$.

The time delay τ between adjacent taps may be selected as large as T, the symbol interval in which case the FIR equalizer is called a symbol spaced equalizer.

In this case the input to the equalizer is the sampled sequence given by

$$y_m = x_0 a_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} a_n x_{m-n} + w_m$$

— 1M

However, we note that when $1/T < 2W$, frequencies in the received signal that are above the folding freq. $1/T$ are aliased into freq. below $1/T$

In this case the equalizer compensates for the aliased channel-distorted signal.

when the time delay τ between the adjacent taps is such that $\frac{1}{\tau} \geq 2W \geq \frac{1}{T}$, no aliasing occurs. hence the inverse channel equalizer compensates for the true channel distortion

since $\tau < T$, channel equalizer is said to have fractionally spaced taps, and it is called a fractionally spaced equalizer.

Q.8.c (27)

In practice Z is often selected as $Z = \frac{I}{2}$, In this case, the sampling rate at the o/p of the filter $G_R(f)$ is $\frac{2}{T}$. — 1M

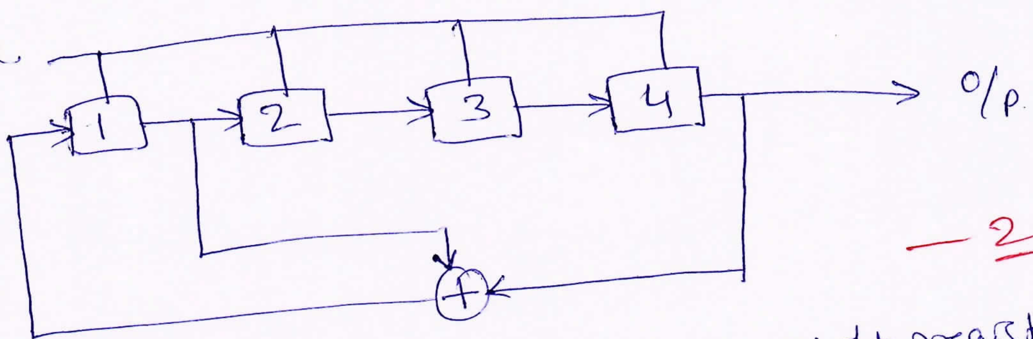
27

Q1 (28)

27

9 a) Draw the 4 stage linear feedback shift register with 1st & 4th stage is connected to modulo-2 adder. Output of Modulo-2 is connected to 1st stage input. Find the output PN sequence and obtain the autocorrelation Sequence. —— (6M)

→ Let the initial condition of shift registers as 1 0 0 0



— 2M

Fig! 4 stage linear Feed back shift register.

clk	$S_1 \oplus S_4$	S_2	S_3	S_4	o/p	
1	1	0	0	0	0	← initial state.
1	1	1	0	0	0	
1	1	1	1	0	0	
1	1	1	1	1	1	
1	0	1	1	1	1	
1	1	0	1	1	1	
1	0	1	0	1	1	
1	1	0	1	0	0	
1	1	1	0	1	0	
1	0	1	1	0	0	
1	0	0	1	1	1	
1	1	0	0	1	1	
1	0	1	0	0	0	
1	0	0	1	0	0	
1	0	0	0	1	1	
1	1	0	0	0	0	← initial sequence.

$N = 2^M - 1$

$N = 2^4 - 1 = 15.$

— 1M

$\therefore$ ~~the~~ PN sequence is

PN $\Rightarrow$ 0 0 0 1 1 1 1 0 1 0 1 1 0 0 1 — 1M

<u>Autocorrelation sequence</u>

PN $\Rightarrow$	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1
$c(n) \Rightarrow$	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1

mapping $0 \rightarrow 1$
$1 \rightarrow -1$

$$R(d) = \frac{1}{N} \sum_{n=1}^{N} c(n)\, c(n-d)$$

for $d = 0$ $\quad R(0) = \frac{1}{15} \sum_{n=1}^{15} c^2(n) = \frac{1}{15} \times 15 = 1$

for $d = 1$ $\quad R(1) = \frac{1}{15} \sum_{n=1}^{15} c(n)\, c(n-1)$

$\therefore c(n) =$	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1
$c(n-1) =$	-1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1
$c(n)\,c(n-1) =$	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1	-1	1	-1

$\therefore$ $R(1) = \frac{1}{15} \sum_{n=1}^{15} c(n)\, c(n-1) = \frac{1}{15}(-1)$

Similarly $\quad R(2) = R(3) = R(4) = \ldots = R(14) = \frac{-1}{15}$

$R(15) = R(0) = 1.$

$$\therefore R(d) = \begin{cases} 1, & d = 0, \pm N, \pm 2N, \ldots \\ -1/15, & \text{others} \end{cases}$$

— 2M

9b) with a neat block diagram explain the frequency hopped spread spectrum. → (7M).

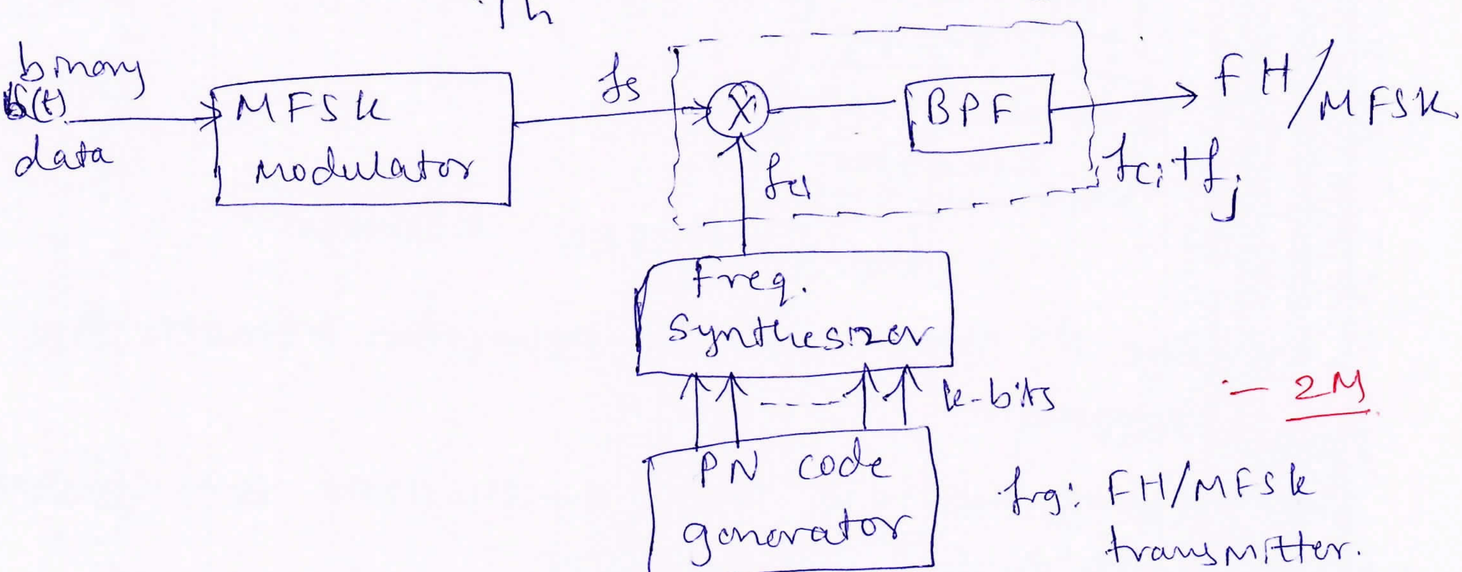
→ In FHSS communication system, the binary data sequence modulates the freq. of the carrier signal resulting in an FSK signal or groups of k binary bits (MFSK) and a set of 2^k sub carrier freq. are chosen to generate a multilevel FSK or M-ary FSK signal.

This type of spread spectrum in which the carrier freq. hops randomly from one freq. to another is called FHSS.

By choosing a large no. of randomly hopping discrete carrier freq. it is possible to have a modulated signal of wide BW.

FHSS system is divided into 2 types (1) Slow-freq. hopping SS system & (2) Fast-freq. hopping SS system. — 1M

It may be noted that carrier freq. changes every T_h seconds. Also, $R_h = \frac{1}{T_h}$ is called hopping rate.



— 2M

fig: FH/MFSK transmitter.

The 1st stage of an FH/MFSK transmitter is the freq. modulator and 2nd stage is freq. mixer.

Incoming binary sequence is applied to a serial to parallel converter to get blocks of k-bits

· Depending on k-bit binary pattern, any one of the 2^k discrete amplitude levels of M-ary PAM signal is obtained.

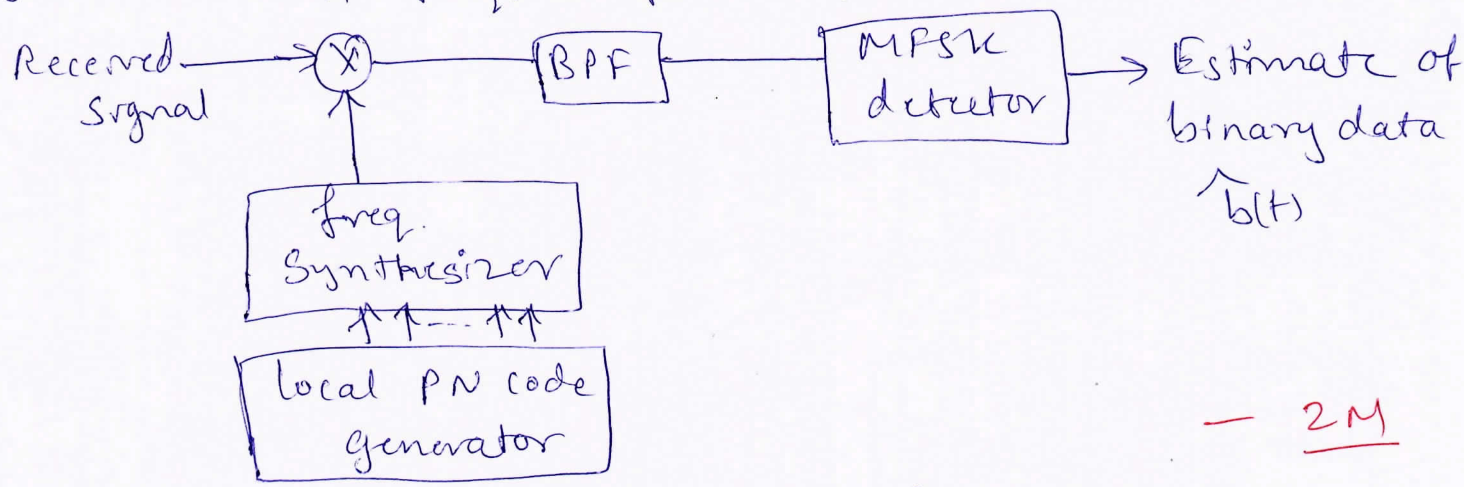
This M-ary PAM signal is applied to a voltage controlled Oscillator. (VCO)

For every amplitude level in the M-ary PAM signal, the VCO produces one of the 2^k discrete freq.

The O/P of the MFSK modulator is then mixed with the O/P of a freq. synthesizer.

As a result freq. hops over 2^k distinct values.

The BPF passes the sum freq for the transmission and rejects the diff. freq. components. — 1M



— 2M

fig: FH/MFSK - Receiver.

In the 1st stage, mixing operation removes the freq. hopping

The O/P of the local freq. synthesizer is in synchronization with that of transmitter.

The O/P of mixer is passed through a BPF which selects the diff. freq. component from the mixer.

9b
(30) The O/P of the BPF is the MFSK detector consists of a bank of M matched filters 3
An estimate of the original signal is obtained by Selecting the largest filter O/P. — 1M

9. c) Explain the effect of dispreading on narrowband interference. → (3M).

→ The effect of an interfering signal on the demodulation of the desired information-bearing signal is given as, suppose that the received signal is

$$x(t) = A_c b(t)\ c(t) \cos(2\pi f_c t) + i(t) \quad -(1).$$

where $i(t)$ is the interference signal.

The despreading operation at the receiver gives

$$x(t).\ c(t) = A_c b(t) \cos 2\pi f_c t + i(t).\ c(t) \quad -(2).$$

— 1M

Multiplication of $i(t)$ with $c(t)$ is to spread the BW of $i(t)$ to W Hz.

Let P_I be avg. Jamming signal power and T_c - bit interval of the PN sequence $c(t)$. T_c is also called as chip interval.

If we assume L_c to be the period of the PN sequence $c(t)$. then there are L_c chips in one bit interval of $b(t)$.

$$\therefore L_c T_c = T_b.$$

— 1M

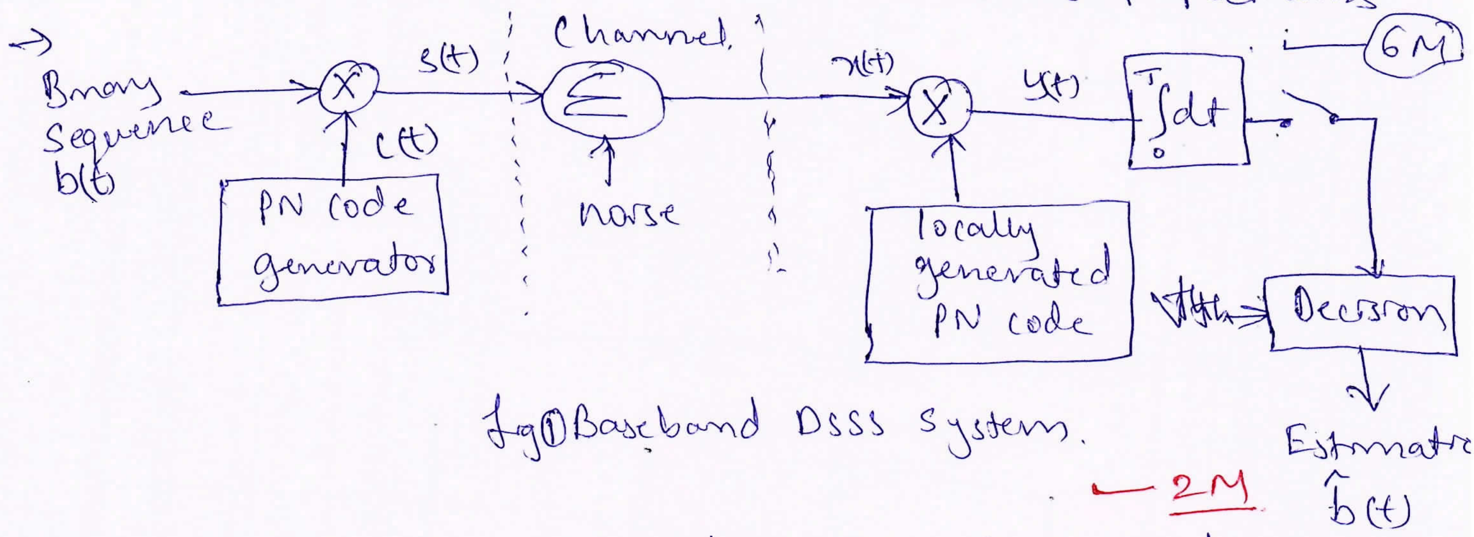
The total power in interfering signal at the output of demodulator is

$$\frac{P_I}{(W/R_b)} = \frac{P_I}{(T_b/T_c)} = \frac{P_I}{L_c}.$$

$\therefore$ power is reduced by an amount of L_c.
L_c can be also called as processing gain.

— 1M

10 a) Explain the generation of direct sequence spread Spectrum signal with the relevant waveforms & spectrums 31



fig① Baseband DSSS system.

— 2M

The i/p binary sequence b(t) is the baseband signal in NRZ polar format. with each bit occupying a time of Tb seconds.

C(t) is a wideband code sequence which has noise like properties. This sequence is generated by a feedback shift registers. which is also expressed in NRZ polar format. But the duration of each bit (T_c) in c(t) is the period of the clock pulse that is applied to the generator.

Normally $T_c << T_b$ and hence bit rate of c(t) i.e., R_c also called as chip rate is much greater than $R_b = \frac{1}{T_b}$

The info-bearing signal b(t) is multiplied by the signal from the PN sequence generator i.e., c(t)

(32)

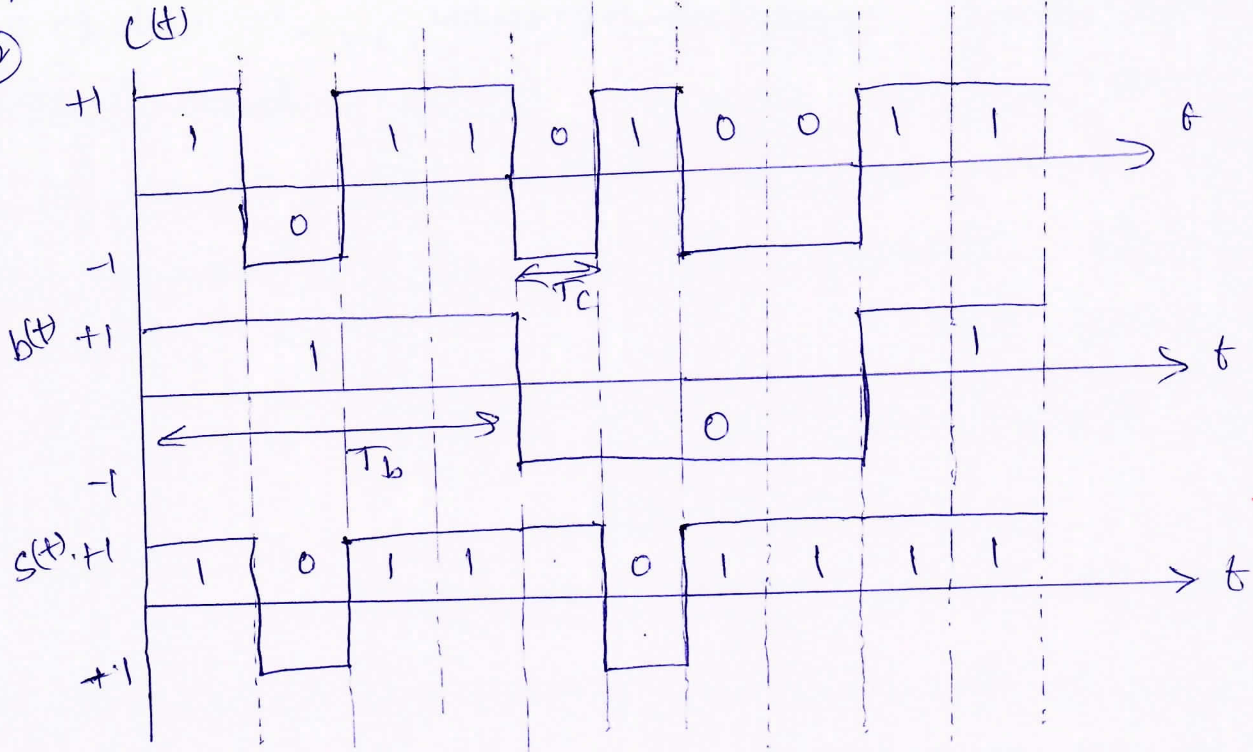
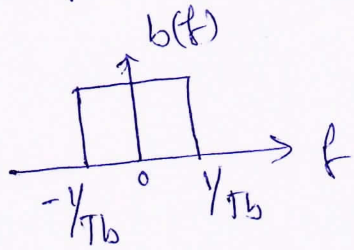
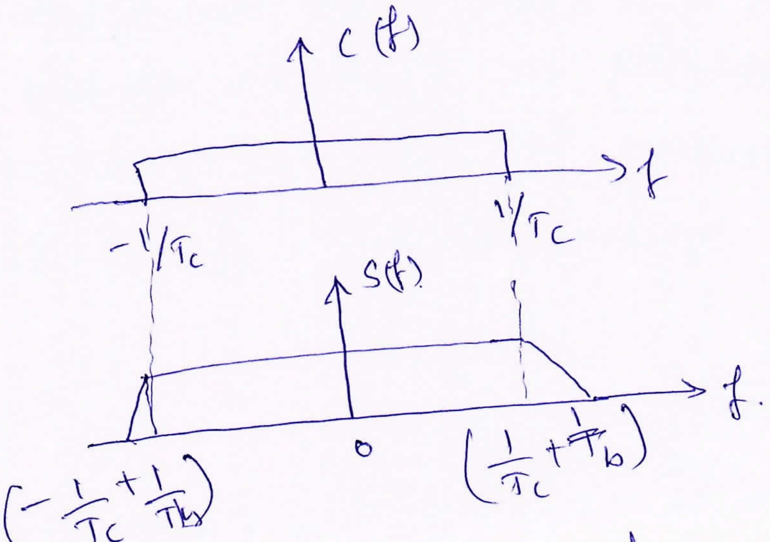


Fig ②. generation of DSSS signal

— 2M

This multiplication operation serves to spread the BW of the info-bearing signal into the wider BW occupied by PN sequence c(t).





— 1M

Fig ③. Convolution of power spectra.

38 The fig③ shows spectrum spreading, where $b(f)$ is narrowband and PN sequence $c(f)$ is wideband, and the BW of the spectrum $S(f)$ is nearly equals to that of $c(f)$. Therefore PN sequence performs the role of a spreading code.

Let the received signal be $x(t)$, then.

$$x(t) = s(t) + i(t).$$

where $i(t)$ is the interference signal,

The 1st step in the detection process is to multiply the received signal $x(t)$ by PN sequence $c(t)$. performed by product modulator.

$\therefore$ O/p of product modulator is

$$y(t) = x(t) \cdot c(t)$$

$$y(t) = [s(t) + i(t)] \, c(t).$$

$$y(t) = [b(t) \cdot c(t) + i(t)] \, c(t) \qquad - 1M$$

$$y(t) = b(t) \, c^2(t) + i(t) \cdot c(t). \qquad -①$$

$$y(t) = b(t) + i(t) \cdot c(t) \qquad [\because c^2(t) = 1]$$

Thus eqn ① it is clear that the spectrum of $s(t)$ is despread, resulting in spectrum of $b(t)$

On multiplication by $c(t)$, the spectrum of $i(t)$ is spread but spectral height of $i(t) \cdot c(t)$ is small.

Q8 (33)

10 b). With a neat block diagram explain the CDMA system based on IS-95 — (7M) 27

→ CDMA technique overcomes the problem of FDMA and TDMA, In situations where all the users are required to transmit simultaneously. but also occupy the same RF BW of the channel, CDMA can be used

DS-CDMA has been adopted as a multiple access method for a digital cellular voice communication in North America and was developed by Qualcomm as a standard. TIA designated the standard as IS-95 for use in the 800MHz. and 1900MHz bands

A major advantage of CDMA over other multiple access methods is that entire freq. band is available at each base station, The BW used for transmission from a base station to the mobile receiver is 1.25 MHz.

The signals transmitted in both forward & reverse links are DSSS signals having a chip rate of 1.2288×10^6 chips/s. 1M

* The Block diagram of IS-95 Forward link is given below

Speech encoder Code excited linear Predictor (CELP). generates data at variable rates of 9600, 4800, 2400, 1200 bps in frame intervals of 20msec.

Encoded speech is passed through convolutional encoder and block interleaver.

Forward Link

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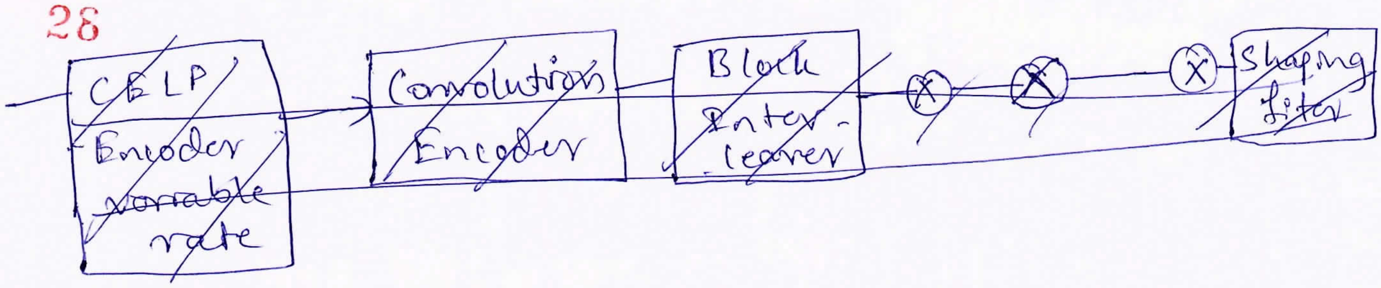


For lower speech data rates of 4800, 2400, or 1200 bps, the o/p symbols of convolutional encoder is seperated twice, 4 times or eight times. Thus the bit rate is fixed at 9600 bps.

The o/p of block interleaver is at 19.2bps and is multiplied by long code (of period $N = 2^{42} - 1$) of chip rate 1.2288×10^6 chips per second & decimated by a factor 64. — 1M

The long code identifies the mobile station in forward and reverse links uniquely.

Each channel user is assigned a walsh sequence of 64 bits uniquely from a set of 64 sequences.

One sequence is used to transmit pilot signal.

Pilot signal is used to estimate channel characteristics like signal strength and carrier phase and employed in coherent detection.

One sequence is used for time synchronization and some sequences for paging.

About remaining 61 channels are assigned to different users.

Each user multiplies the data sequences by 64 bit walsh sequence. — 1M

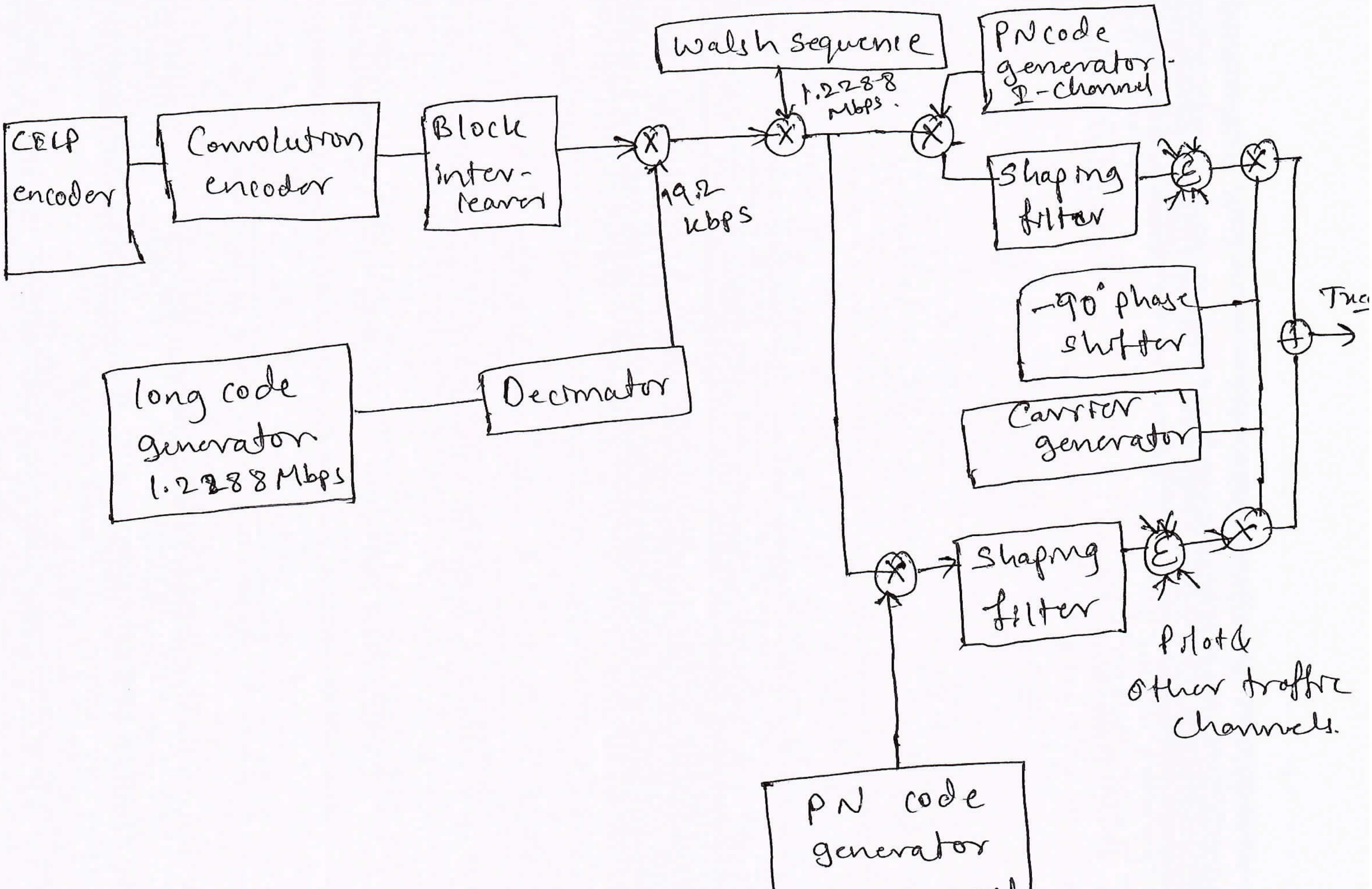
The resulting binary sequence is spread by multiplication with two PN sequences of length $N \simeq 2^{15}$ creating I and Q signal components, resulting in 4 phase signal.

Different base stations are identified by the offsets of these PN sequences.

At the Rx a RAKE demodulator is used to seperate the multipath components and combined before detection using Viterbi soft decision.

-1M

Demodulator uses non-coherent demodulation of the 64 orthogonal walsh sequences to recover the encoded data bits.



-3M

Fig: Block diagram of IS-95 Forward Link

Q1. (35)

10 b). Write a note on application of spread spectrum in wireless LAN's → (3M)

→ There are many applications of spread spectrum, some of them are.

1) Low-detectability signal Transmission

In this application, the information-bearing signal is txed at a very low power level relative to the background channel noise and thermal noise that is generated in the front end of a receiver.

The intended receiver can recover the weak info-bearing signal from the background noise with the help of processing gain and coding gain. The other receiver having lack of info. abount PN code sequence is unable to take the advantage of processing gain & coding gain.

2) Code division Multiple Access (CDMA)

The spread spectrum permits many DSSS signals to occupy the same channel BW. Here all the users has its own pseudorandom sequence.

Therefore it is possible to have several users' transmit messages simultaneously over the same channel BW.

3) Wireless LAN's — 1M

Spread spectrum signals have been used in the IEEE wireless LAN standards 802.11 & 802.11b, which operate in the 2.4GHz ISM unlicensed freq. band.

The available BW is subdivided into 14 overlapping 22MHz channels, although not all channels are used in all countries. In USA only channels 1 ~~to~~ 11

40

are used.

In the 802.11 standard an 11-chip barker sequence is modulated and transmitted at a chip rate of 11 MHz.

the 11-chip Barker sequence is

$$\{1, -1, 1, 1, -1, 1, 1, 1, -1, -1, -1\}$$

This sequence is desirable because its autocorrelation has sidelobes of less than or equal to 1, compared with the peak autocorrelation value of 11.

The Barker sequence is modulated either with BPSK or QPSK.

DSSS is also used in the higher speed (2nd generation) IEEE 802.11b wireless LAN standard, which operates in the same 2.4GHz ISM band.

— 2M