Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN | | | | | | | | | | |

18MAT41

Fourth Semester B.E. Degree Examination

Complex Analysis, Probability and Statistical Methods

(Common to all Programmes)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Use of statistical tables allowed.

Module-1

1 (a) Show that $w = f(z) = z + e^z$ is analytic and hence find dw/dz. **(06 Marks)**

(b) Derive Cauchy-Riemann equation in cartesian form. **(07Marks)**

(c) Find the analytic function $f(z) = u + iv$, given $v = [r - (1/r)]\sin\theta, r \neq 0$. **(07Marks)**

OR

2 (a) If $f(z) = u(x,y) + iv(x,y)$ is an analytic function, show that the family of curves $u(x,y) = c_1$ and $v(x,y) = c_2$, c_1 & c_2 being constants, intersect each other orthogonally. **(06 Marks)**

(b) If $f(z)$ is analytic, show that $[(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]\ |f(z)|^2 = 4|f'(z)|^2$. **(07Marks)**

(c) Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. Also, determine the corresponding analytic function. **(07Marks)**

Module-2

3 (a) State and prove Cauchy's theorem. **(06 Marks)**

(b) Find the image in the w- plane bounded by the lines $x = 1, y = 1, x + y = 1$under the transformation $w = z^2$. **(07Marks)**

(c) Find the bilinear transformation which maps the points $z = 0,1,\infty$ into the points $w = -5,-1,3$, respectively. What are the invariant points under this transformation? **(07Marks)**

OR

4 (a) Evaluate: $\int_0^{2+i} (\bar{z})^2\, dz$ along the line $x = 2y$. **(06 Marks)**

(b) Evaluate: $\int_C \frac{e^z}{(z+1)(z-2)} dz$ where C is the circle $|z| = 3$. **(07Marks)**

(c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0,1,\infty$ respectively. What are the invariant points under this transformation? **(07Marks)**

Module-3

5 (a) A random variable X has the following probability function for various values of x: **(06 Marks)**

$X(=x_i)$	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Find (i) the value of k (ii) $P(x < 1)$ (iii)$P(x \geq -1)$.

(b) The probability of germination of a seed in a packet of seeds is found to be 0.7. If 10 seeds are taken for experimenting on germination in a laboratory, find the probability that (i) 8 seeds germinate (ii) at least 8 seeds germinate (iii) at most 8 seeds germinate. **(07Marks)**

(c) If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D. 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2100 hours (ii) less than 1950 hours (iii) between 1900 and 2100 hours. **(07Marks)**

OR

6 (a) The probability density function of a random variable $X(=x)$ is $f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0, & otherwise \end{cases}$. **(06 Marks)**
Find (i) the value of k (ii) $P(1 < x < 2)$ and (iii)$P(x \leq 1)$.

(b) In a certain factory manufacturing the razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades, in a consignment of 10,000 packet. **(07Marks)**

(c) The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes (ii) between 5 and 10 minutes. **(07Marks)**

Module-4

7 (a) The following are the marks of 8 students in Statistics and Mathematics. **(06 Marks)**

Marks in Statistics	25	43	27	35	54	61	37	45
Marks in Mathematics	35	47	20	37	63	54	28	40

Calculate the rank correlation coefficient.

(b) Fit a best fitting curve in the form $y = ax^b$ for the following data: (07Marks)

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.1	6.8	7.5

Calculate the value of y when $x = 3.5$.

(c) With usual notation, compute $\bar{x}, \bar{y}$ and r from the following lines of regression: (07Marks)

$y = 0.516x + 33.73$ and $x = 0.512\ y + 32.52$.

OR

8 (a) In a bivariate distribution, it is found that $\sigma_x = \sigma_y$ and the acute angle between the lines of regression is $tan^{-1}(3)$. Find the correlation coefficient. (06 Marks)

(b) Find the coefficient of correlation between the industrial production and export, using the following table; (07Marks)

Production (in Lakh tons)	55	56	58	59	60	60	60
Exports (in Lakh tons)	35	38	38	39	44	43	45

(c) Following are the measurements of air velocity (x) and evaporation coefficient (y) of burning fuel droplets in an impulse engine: (07Marks)

x	20	60	100	140	180	220
y	0.18	0.37	0.35	0.78	0.56	0.75

Find a best fitting parabola $y = ax^2 + bx + c$ to the above data and hence estimate y when $x = 99$.

Module-5

9 (a) Determine (i) marginal distribution (ii) covariance between the discrete random variables X and Y, of the jont probsbility distribution: (06 Marks)

X \ Y	3	4	5
2	1/6	1/6	1/6
5	1/12	1/12	1/12
7	1/12	1/12	1/12

(b) The mean life time of a sample of 100 fluorescent tube lights manufactured by a company is found to be 1570 hrs with a standard deviation of 120 hrs. Test the hypothesis that the mean life-time of the lights produced by the company is 1600 hrs at 0.01 level of significance. (07Marks)

(c) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure $5, 3, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4$. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 d.f. is 2.201). **(07Marks)**

OR

10 (a) Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type I and Type II errors. **(06 Marks)**

(b) It is claimed that a random sample of 49 tyres has a mean life of 15,200 kms. Is the sample drawn from a population whose mean is 15,150 kms and whose standard deviation is 1,200 kms? Test the significance at 0.05 level. **(07Marks)**

(c) Fit a binomial distribution for the data **(07Marks)**

No. of Heads	0	1	2	3	4
Frequency	122	60	15	2	1

and also test the goodness of fit given that $\chi^2_{0.05} = 7.815$ for 3 d.f.

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Fourth Semester B.E. Degree Examination

Complex Analysis, Probability Dist and Statistical Methods.

Time: 3 Hrs. Max. Marks: 100

Module - 1.

1a. $W = z + e^z \Rightarrow u + iv = (x+iy) + e^{x+iy}$ — (1m)

— (1m)

$u = x + e^x \cos y$; $v = y + e^x \sin y$

$u_x = 1 + e^x \cos y$; $u_y = -e^x \sin y$; $v_x = e^x \sin y$; $v_y = 1 + e^x \cos y$ — (1m)

$\therefore u_x = v_y$ and $u_y = -v_x$. — (2m)

$\therefore W = z + e^z$ is analytic.

$\frac{dw}{dz} = f'(z) = u_x + iv_x = 1 + e^x \cos y + i(e^x \sin y) = 1 + e^{x+iy}$

$$\boxed{\frac{dw}{dz} = 1 + e^z.}$$ — (1m)

1b. Let $f(z)$ be analytic at a point $z = x+iy$

$f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$

$\delta x, \delta y$ be the increments in x & y and $\delta z = \delta x + i\delta y$ be the corresponding increments in 'z'.

Case i:- Let $\delta y = 0$ $\therefore \delta z = \delta x$, $\delta z \to 0 \Rightarrow \delta x \to 0$ — (3)

$\therefore f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$ ------ (1)

Case ii:- Let $\delta x = 0$ $\therefore \delta z \to 0 \Rightarrow i\delta y = 0$ i.e. $\delta y \to 0$

$\therefore f'(z) = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}$ ------ (2) Equating the real & imaginary — (3)

Parts of eqⁿ 1 & 2 we get, — (1)

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \; ; \; \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}}$$

7 M.

1c. Let, $V = \left(r - \frac{1}{r}\right)\sin\theta$

we need to show $V_{rr} + \frac{1}{r}V_r + \frac{1}{r^2}V_{\theta\theta} = 0$.

$V_r = \left(1 + \frac{1}{r^2}\right)\sin\theta$; $V_{rr} = -\frac{2}{r^3}\sin\theta$ — (2)

$V_\theta = \left(r - \frac{1}{r}\right)\cos\theta$; $V_{\theta\theta} = -\left(r - \frac{1}{r}\right)\sin\theta$.

Hence, — (1)

$-\frac{2}{r^3}\sin\theta + \frac{1}{r}\sin\theta + \frac{1}{r^3}\sin\theta - \frac{1}{r}\sin\theta + \frac{1}{r^3}\sin\theta = 0$

Thus the given V is harmonic. consider the C-R equation in polar form. To find u.

$r u_r = V_\theta$; $r V_r = -u_\theta$. — (2)

$u_r = \left(1 - \frac{1}{r^2}\right)\cos\theta$; $u_\theta = -\left(r + \frac{1}{r}\right)\sin\theta$.

$u = \int\left(1 - \frac{1}{r^2}\right)\cos\theta\, dr + f(\theta)$ and $u = -\int\left(r + 1/r\right)\sin\theta\, d\theta + g(r)$

$u = (r + 1/r)\cos\theta + f(\theta)$ and $u = (r + 1/r)\cos\theta + g(r)$.

we must have $f(\theta) = 0$ & $g(r) = 0$. — (2)

Thus $u = (r + 1/r)\cos\theta$.

we have,

$f(z) = u + iv = (r + 1/r)\cos\theta + i(r - 1/r)\sin\theta$.

Putting $r = z$ and $\theta = 0$.

$$\boxed{f(z) = (z + 1/z)}$$

7M.

2a Consider $u(x,y) = C_1$ Diff w.r.t 'x' treating y as a function of x.

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dx} = 0$ OR $\frac{dy}{dx} = -\frac{\partial u}{\partial x}\Big/\frac{\partial u}{\partial y} = m_1$ — (3)

lly $v(x,y) = C_2$; $\frac{dy}{dx} = -\frac{\partial v}{\partial x}\Big/\frac{\partial v}{\partial y} = m_2$

$\therefore m_1 m_2 = \dfrac{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}}$ ----- (1)

But $f(z) = u + iv$ is analytic & we have C-R equations.

ie $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ using these in eq (1) — (1)

$m_1 m_2 = \dfrac{\frac{\partial v}{\partial y}\cdot -\frac{\partial u}{\partial y}}{\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}} = -1$. Hence the curves intersect orthogonally at every pt of intersection. — (2)

6M.

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2b. Let, $f(z) = u + iv \Rightarrow |f(z)|^2 = u^2 + v^2 = \phi$ (say)

$\phi = u^2 + v^2$ Diff Partially w.r.t 'x' twice ---- (1)

$\phi_{xx} = 2[u u_{xx} + u_x^2 + v v_{xx} + v_x^2]$ ---- (1)

$\left.\right\}$ ---- (4)

My

$\phi_{yy} = 2[u u_{yy} + u_y^2 + v v_{yy} + v_y^2]$ ---- (2)

$\phi_{xx} + \phi_{yy} = 4[u_x^2 + v_x^2]$ —— (1)

$|f'(z)|^2 = u_x^2 + v_x^2$

$\therefore \boxed{\phi_{xx} + \phi_{yy} = 4|f'(z)|^2}$ —— (1)

7m.

2c. Let,

$u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$

$u_x = \cos x \cosh y - 2\sin x \sinh y + 2x + 4y$

$u_{xx} = -\sin x \cosh y - 2\cos x \sinh y + 2$

$u_y = \sin x \sinh y + 2\cos x \cosh y - 2y + 4x$

$\left.\right\}$ — (4m)

$u_{yy} = \sin x \cosh y + 2\cos x \sinh y - 2$

Thus,

$u_{xx} + u_{yy} = 0$. $\therefore u$ is a harmonic. —— (1m)

Consider,

$f'(z) = u_x + iv_x$ But $v_x = -u_y$ (C-R equation) —— (1m)

$f'(z) = u_x - iu_y$ putting $x = z$ & $y = 0$

$f'(z) = [u_x]_{(z,0)} - i[u_y]_{(z,0)}$

$f'(z) = \cos z + 2z - i(2\cos z + 4z)$ Integrating w.r.t 'z' —— (1m)

$\boxed{f(z) = (1-2i)(z^2 + \sin z) + C}$.

7m.

Module-2

3a. **Statement:** If $f(z)$ is analytic at all points inside and on a simple closed curve C. then $\int_C f(z)dz = 0$ — (1)

Proof: Let,

$f(z) = u+iv$ then

$$\oint_C f(z)\,dz = \int_C (u+iv)(dx+idy) = \int_C (udx - vdy) + i\int_C (vdx + udy)$$

By Green's theorem, If M and N are two real valued functions of (x,y) having continuous first order p. derivatives in a region bounded by the curve C.

$$\int_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$ — (4)

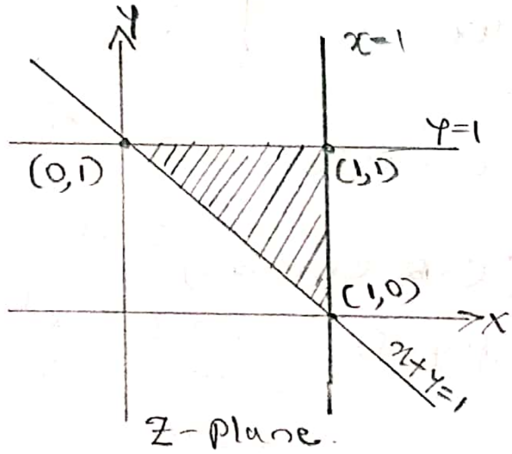
$$\therefore \int_C f(z)\,dz = \iint_R \left(\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\right) dx\,dy + i\iint_R \left(\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\right) dx\,dy$$

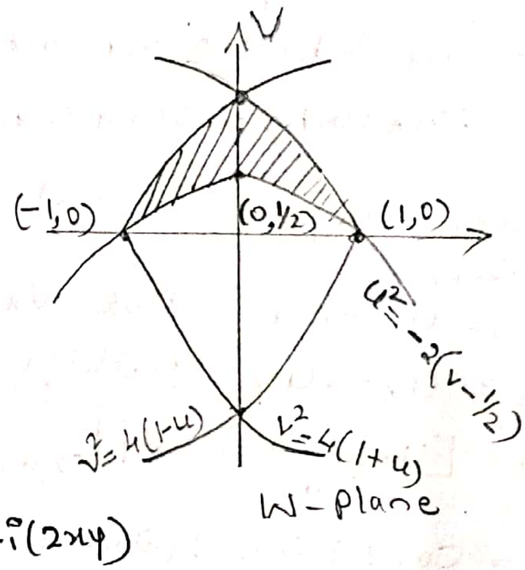
By C-R equation — (1)

$$\boxed{\oint_C f(z)\,dz = 0.}$$

6 M.

3b.





— (1)

let, $W = z^2$

That is $u+iv = (x+iy)^2 = (x^2-y^2) + i(2xy)$

$\therefore u = x^2 - y^2$ and $v = 2xy$ ----- (1) — (1)

Consider $x = 1$ eqn (1) becomes.

$u = 1 - y^2$, $v = 2y$

Substituting $v/2 = y$ in u we have,

$u = 1 - (v^2/4)$

ie $v^2 = 4(1-u)$. This is a parabola in a w-plane with vertex $(1,0)$ & symmetrical about the u-axis. — (1)

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Consider $y=1$ eqⁿ ① becomes

$u = x^2 - 1$, $v = 2x$ substituting $v/2 = x$ in u

we have,

$u = (v^2/4) - 1 \Rightarrow v^2 = 4(1+u)$. This is also Parabola — (1)

in w-plane with vertex $(-1,0)$ & symmetrical about the u-axis

consider, $x+y=1$ or $y = 1-x$ eqⁿ ① becomes

$u = -1+2x$ & $v = 2x(1-x)$

Substituting $2x = 1+u$, v becomes — (2)

$v = (1+u)\left(1 - \frac{1+u}{2}\right)$ or $v = \frac{1}{2}(1-u^2)$

ie $1-u^2 = 2v$ or $u^2 = -2[v-(1/2)]$. This is also parabola in the (1)

w plane with the vertex $(0, 1/2)$ Symmetrical about v-axis.

7M.

3c. Let,

$w = \frac{az+b}{cz+d}$ be the required bilinear transformation.

$z=0$, $w=-5$; $-5 = b/d \Rightarrow b = -5d$ ---- ①

ie $z=1$, $w=-1$; $-1 = \frac{a+b}{c+d} \Rightarrow a+b+c+d = 0$ ---- (2) — (2)

$z=\infty$, $w=3$; $w = \frac{z[a+(b/z)]}{z[c+(d/z)]} = \frac{a+(b/z)}{c+(d/z)}$

Now consider,

$z=\infty$, $w=3$; $3 = \frac{a+0}{c+0} \Rightarrow a = 3c$ ---- (3) — (2)

using ① and ③ in (2) we get,

$4c - 4d = 0$ or $c = d$. Choosing $d = 1$

we get,

$c=1$, $b=-5$, $a=3$

Thus the required bilinear transformation is $w = \frac{3z-5}{z+1}$ — (2)

The invariant points are obtained by taking $w = z$.

$z = \frac{3z-5}{z+1}$ or $z^2 - 2z + 5 = 0$ — (1)

hence, $\boxed{z = 1 \pm 2i}$ are the invariant points.

7M.

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4a. Let, $I = \int_{0}^{2+i} (\bar{z})^2 \, dz$.

we have, $(\bar{z})^2 = (x-iy)^2 = (x^2-y^2) - i(2xy)$ ------ (1) — (1)

And, $dz = dx + i\,dy$ ------ (2) and the line $x = 2y$

then $dx = 2\,dy$ — (2)

$z = 0$ to $2+i \Rightarrow (x,y)$ varies from $(0,0)$ to $(2,1)$

where, $0 \le y \le 1$

$\therefore I = \int_{y=0}^{1} [(4y^2-y^2) - i.4y^2](2dy + i dy)$ — (2)

$I = \int_{y=0}^{1} (10-5i) y^2 \, dy = \frac{5}{3}(2-i)$ — (1)

Thus, $I = \frac{5}{3}(2-i)$ along the given path.

6 M.

4b. The points $z = a = -1$, $z = a = 2$ being $(-1,0)$ $(2,0)$ lies inside $|z| = 3$.

we shall resolve $\frac{1}{(z+1)(z-2)}$ into partial fractions. — (1)

Let, $\frac{1}{(z+1)(z-2)} = \frac{A}{(z+1)} + \frac{B}{z-2}$ or $1 = A(z-2) + B(z+1)$

Putting $z = 2$, $1 = B(3)$ $\therefore B = 1/3$

putting $z = -1$, $1 = A(-3)$ $\therefore A = -1/3$

hence, $\frac{e^{2z}}{(z+1)(z-2)} = \frac{1}{3}\left[\frac{e^{2z}}{z-2} - \frac{e^{2z}}{z+1}\right]$ — (3)

$\int_C \frac{e^{2z}}{(z+1)(z-2)} dz = \frac{1}{3}\left[\int_C \frac{e^{2z}}{z-2} dz - \int_C \frac{e^{2z}}{z+1} dz\right]$ ----- (1)

we have Cauchy's integral formula. — (1)

$\int_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$

Taking $f(z) = e^{2z}$ and $a = 2, -1$

$\int_C \frac{e^{2z}}{z-2} dz = 2\pi i f(2) = 2\pi i e^4$ and $\int_C \frac{e^{2z}}{z+1} dz = \frac{2\pi i}{e^2}$ — (2)

Substituting these in the RHS of (1)

$\int_C \frac{e^{2z}}{(z+1)(z-2)} = \frac{2\pi i}{3}\left[e^4 - \frac{1}{e^2}\right]$

7 M.

4c. $z = 1, i, -1$ and $w = 0, 1, \infty$

$z = 1$, $w = 0$ gives $a + b = 0$ ---- (1)

$z = i$, $w = 1$ gives $ai + b - ci - d = 0$ --- (2) — (4)

$z = -1$, $w = \infty$ gives $1/w = \dfrac{cz+d}{az+b} \Rightarrow -c + d = 0$ ---- (3)

Adding (2) & (3) — (2)

$ai + b - (1+i)c = 0$ ---- (4) solving (1) & (4)

we get,

$w = i\left[\dfrac{1-z}{1+z}\right]$ and the invariant points are $z = \dfrac{-(1+i) \pm \sqrt{6i}}{2}$. — (1)

7M.

Module 3.

5a. Let,

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	k	0.2	$2k$	0.3	k

we must have, $P(x_i) \geq 0$ and $\Sigma P(x_i) = 1$

for a probability Distribution.

$\Sigma P(x_i) = 1$ requires $4k + 0.6 = 1. \Rightarrow k = 0.1$ — (2)

i] $k = 0.1$ — (2)

ii] $P(x < 1) = P(-2) + P(-1) + P(0)$

$= 0.1 + 0.1 + 0.2$ — (2)

$= 0.4$.

iii] $P(x \geq -1) = 1 - P(x < -1) = 1 - 0.1 = 0.9$.

6M.

5b. Given

$p = 0.7$ then $q = 1 - p = 0.3$. and $n = 10$ — (1)

here,

$P(x) = nC_x p^x q^{n-x} \Rightarrow 10C_x (0.7)^x (0.3)^{10-x}$. — (2)

i] $P(x = 8) = 10C_8 (0.7)^8 (0.3)^2 = 0.23346$.

ii] $P(x \geq 8) = P(8) + P(9) + P(10) = 0.3828$. — (4)

iii] $P(x \leq 8) = 1 - P(x \geq 8) = 1 - 0.3828$

$P(x \leq 8) = 0.6172$.

7M.

5c. Given

$\mu = 2000$, $\sigma = 60$ — (1)

we have, S.n.v , $z = \frac{x-\mu}{\sigma} = \frac{x-2000}{60}$

i) To find $P(x>2100)$

If $x = 2100$, $z = 1.67$. — (2)

$P(x>2100) = P(z>1.67)$

$= P(z\geq 0) - P(0<z<1.67)$

$= 0.5 - \phi(1.67) = 0.0475$

$\Rightarrow 2500 \times 0.0475 = 119$.

ii) $P(x<1950)$ If $x = 1950$, $z = -5/6 = -0.83$ — (2)

$P(x<1950) = P(z<-0.83) = P(z>0.83)$

$= P(z\geq 0) - P(0<z<0.83)$

$= 0.5 - \phi(0.83) = 0.2033$.

∴ number of bulbs that are likely to last for more than 1950 hours is $2500 \times 0.2033 = 508$.

iii) $P(1900<x<2100)$

If $x = 1900$, $z = -1.67$ & if $x = 2100$, $z = 1.67$ — (2)

$P(-1.67<z<1.67) = 2P(0<z<1.67)$

$= 2\phi(1.67) = 0.905$

∴ number of bulbs that are likely to last betn 1900 & 2100 hours is 2263.

7M.

6a Let,

$f(x) \geq 0$ if $k \geq 0$. Also we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie $\int_0^3 kx^2\,dx = 1 = \left[\frac{kx^3}{3}\right]_0^3 = 1 \Rightarrow k = 1/9$. — (1)

i) $k = 1/9$. — (1)

ii) $P(1<x<2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_1^2 = \frac{7}{27}$ — (2)

iii) $P(x\leq 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_0^1 = \frac{1}{27}$. — (2)

6M.

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6b. Let,

p = Probability of a defective blade $= 0.002$

In a packet of 10, the mean number of defective blade is

$m = np = 10 \times 0.002 = 0.02$. — (1)

$P(x) = \frac{m^x e^{-m}}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$ Also, $e^{-0.02} = 0.9802$ — (2)

— (1)

$f(x) = \frac{9802(0.02)^x}{x!}$. — (1)

i] No defective $= f(0) = 9802$ — (1)

ii] One defective $= f(1) = 9802(0.02) \simeq 196$. — (1)

iii] Two defective $= f(2) = \frac{9802(0.02)^2}{2!} \simeq 2$.

7M.

6c. we have,

$f(x) = \alpha e^{-\alpha x}, \quad x > 0$

mean $= 1/\alpha = 5 \Rightarrow \alpha = 1/5$ — (1)

hence,

$f(x) = 1/5 \, e^{-x/5}$ — (1)

i] $P(x<5) = \int_0^5 f(x)\,dx = \int_0^5 \frac{1}{5} e^{-x/5}\,dx$

$= 1 - 1/e = 0.6321$ — (2)

$P(x<5) = 0.6321$

ii] $P(5<x<10) = \int_5^{10} f(x)\,dx = \int_5^{10} \frac{1}{5} e^{-x/5}\,dx$ — (2)

$= 1/e - 1/e^2 = 0.2325$ — (1)

$P(5<x<10) = 0.2325$

7M.

Module 4.

7a.

Marks in Statistics	Rank in S	Marks in Mathematics	Rank in M	$d=(S-M)$	$d^2=(S-M)^2$
25	8	35	6	2	4
43	4	47	3	1	1
27	7	20	8	-1	1
35	6	37	5	1	1
54	2	63	1	1	1
61	1	56	2	-1	1
37	5	28	7	-2	4
45	3	40	4	-1	1
					$\Sigma d^2 = 14$

— (3)

we have,

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)} \quad \text{and} \quad n = 8$$ — (1)

$$\rho = 1 - \frac{6\times 14}{8(8^2-1)} = 1 - \frac{84}{504} = 1 - 0.1666 = 0.833$$ — (2)

$$\boxed{\rho = 0.83}$$

6M.

7b. Consider,

$Y = ax^b$ Taking log on both side.

$\therefore \log_e Y = \log_e a + b\log_e x$ And — (2)

Let,

$Y = \log_e y$, $A = \log_e a$, $X = \log_e x$

The normal equations are

$\Sigma Y = nA + b\Sigma X$ — (1)

$\Sigma XY = A\Sigma X + b\Sigma X^2$ and $n = 6$.

x	y	$X=\log_e x$	$Y=\log_e y$	XY	X^2
1	2.98	0	1.09192	0	0
2	4.26	0.6931	1.4492	1.0044	0.4804
3	5.21	1.0986	1.6505	1.8132	1.2069
4	6.1	1.3863	1.8082	2.5067	1.9218
5	6.8	1.6094	1.9169	3.0850	2.5902
6	7.5	1.7917	2.0149	3.6100	3.2101
		$\Sigma X=6.5791$	$\Sigma Y=9.93162$	$\Sigma XY=12.019$	$\Sigma X^2=9.4094$

— (2)

The normal equation becomes.

$6A+6.5791b=9.93162$
$6.5791A+9.4094b=12.019.$ } solving — (2)

$A=1.09144$ & $b=0.5141$

$\log_e a=A \Rightarrow a=e^A=e^{1.09144}=2.978$ — 7M.

Thus, $y=2.9x^{0.5}$ is the required curve of fit $\Rightarrow y=5.42$

7c. W.K.T the regression lines passes through $\bar{x}$ & $\bar{y}$

$0.516\bar{x}-\bar{y}=-33.73.$
$\bar{x}-0.512\bar{y}=32.52$ } solving

we get, — (2)

$\bar{x}=67.66$ and $\bar{y}=68.64$

we shall now rewrite ~~the equation~~ of the regression lines to find the coefficients.

$y=0.516x+33.73$ ----- (1)
$x=0.512y+32.52$ ----- (2) } — (1)

From eq (1) & (2) — (2)

$r\cdot\frac{\sigma y}{\sigma x}=0.516$ and $r\frac{\sigma x}{\sigma y}=0.512$

Correlation Coefficient $r=\sqrt{0.516\times0.512}$ — (2)

$r=\pm0.367 \simeq \pm0.4$

Thus $r=0.4$. since both the regression coefficients are positive — 7M.

8a. If θ be the angle between the lines of regression we have,

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right) \quad \text{-----} (1)$$ — (1)

By the data, — (1)

$\theta = \tan^{-1}(3)$ or $\tan\theta = 3$ and $\sigma_x = \sigma_y$

hence eqⁿ (1) becomes. — (1)

$$3 = \frac{\sigma_x^2}{2\sigma_x^2}\left(\frac{1-r^2}{r}\right)$$ — (2)

$$r^2+6r-1=0$$

$$r = \frac{-6\pm\sqrt{40}}{2} \Rightarrow r = +0.1623 \text{ or } -6.1623$$ — (1)

Since, $|r|\le 1$ the required $\boxed{r = 0.1623}$ 6M.

8b. Solution: $n = 7$.

x	y	$z=x-y$	x^2	y^2	z^2
55	35	20	3025	1225	406
56	38	18	3136	1444	324
58	38	20	3364	1444	400
59	39	20	3481	1521	400
60	44	16	3600	1936	256
60	43	17	3600	1849	289
60	45	15	3600	2025	225
$\Sigma x=408$	$\Sigma y=282$	$\Sigma z=126$	$\Sigma x^2=23806$	$\Sigma y^2=11444$	$\Sigma z^2=2294$

— (3)

$$\bar{x} = \frac{\Sigma x}{n} = \frac{408}{7} = 58.28 \quad ; \quad \bar{y} = \frac{\Sigma y}{n} = \frac{282}{7} = 40.28$$

$$\bar{z} = \frac{\Sigma z}{n} = \frac{126}{7} = 18$$ — (2)

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{23806}{7} - (58.28)^2 = 4.298$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{11444}{7} - (40.28)^2 = 12.37$$

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$$\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{2294}{7} - (18)^2 = 3.71$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x\sigma_y} = \frac{4.298 + 12.37 - 3.71}{2 \times 2.073 \times 3.51}$$ — (2)

$\boxed{r = 0.89}$.

7M.

8c. The normal eqⁿ associated with $y = ax^2 + bx + c$ are as follows.

$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$ — (1)

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$ and $n = 6$.

x	y	xy	x^2y	x^2	x^3	x^4
20	0.18	3.6	72	400	8000	160000
60	0.37	22.2	1332	3600	216000	12960000
100	0.35	35	3500	10,000	1000000	100000000
140	0.78	109.2	15288	19600	2744000	384160000
180	0.56	100.8	18144	32400	5832000	1049760000
220	0.75	165	36300	48400	10648000	2342560000

→ (3)

$\Sigma x = 720$, $\Sigma y = 2.99$, $\Sigma xy = 435.8$, $\Sigma x^2 y = 74636$,

$\Sigma x^2 = 114400$, $\Sigma x^3 = 20448000$, $\Sigma x^4 = 2.49686 \times 10^{10}$ — (2)

$114400a + 720b + 6c = 2.99$

$20448000a + 114400b + 720c = 435.8$

$2.49686 \times 10^{10} a + 20448000b + 114400c = 74636$ — (1)

$a = -4.0299 \times 10^{-8}$; $b = 2.7596 \times 10^{-3}$, $c = 0.1679$

7M.

Module - 5

9a.

x\y	3	4	5
2	1/6	1/6	1/6
5	1/12	1/12	1/12
7	1/12	1/12	1/12

i) marginal distribution. —— (2)

x_i	2	5	7
$f(x_i)$	1/2	1/4	1/4

y_j	3	4	5
$g(y_j)$	1/3	1/3	1/3

—— (1)

$E(X) = \mu_x = \sum x_i f(x_i) = 1 + 1.25 + 1.75 = 4.$ —— (1)

$E(Y) = \mu_y = \sum y_j g(y_j) = 1 + 1.33 + 1.66 = 4$ —— (1)

$E(XY) = \sum x_i y_j J_{ij} = 16.$ —— (1)

ii) $Cov(X,Y) = E(XY) - \mu_x \mu_y = 16 - 4 \times 4 = 0$

$\boxed{Cov(X,Y) = 0}$

6M.

9b. $n = 100$, $\sigma = 120$

$H_0 : \mu = 1600$ } —(2)

$H_1 : \mu < 1600$

The mean life of sample of 100 tube lights is 1570.

$\therefore$ Difference $= 1600 - 1570 = 30$ —— (2)

$$Z = \frac{\text{difference}}{\sigma/\sqrt{n}} = \frac{30}{(120/10)} = 2.5$$ —— (1)

$Z = 2.5 < 2.58$

$\therefore$ The alternate hypothesis that $\mu < 1600$ is accepted at 1% level of significance. —— (2)

7M.

9c. $\bar{x} = \frac{\Sigma x}{n} = \frac{31}{12} = 2.5833$ —— (1)

$S^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2 = 9.538 \Rightarrow S = 3.088$ —— (3)

We have,

$t = \frac{\bar{x}-\mu}{S}\sqrt{n}$

Let us suppose that the stimulus administration is not accompanied with increase in blood pressure $\mu = 0$ —— (1)

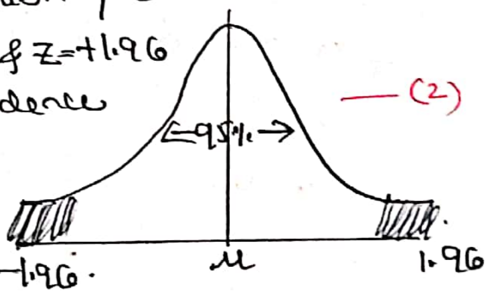
$t = \frac{2.5833-0}{3.088}\sqrt{12} = 2.9 > 2.201$

Hence the Hypothesis is rejected at 5% level of significance. We conclude with 95% confidence that the stimulus in general is accompanied with increase in blood pressure. —— (2)

7M.

10a. i) Null hypothesis: The hypothesis formulated for the purpose of its rejection under the assumption that it is true is called null hypothesis and it is denoted by Ho. —— (2)

ii) Confidence intervals: In this normal probability dat curve that 95% of the area lies between $z=-1.96$ & $z=+1.96$ so this area we can say that 95% of confidence limit that z lies between $(-1.96, +1.96)$. —— (2)



iii) Type I and Type II Error:

If a hypothesis is rejected while it should have been accepted it is known as Type I Error. (2)

If a hypothesis is accepted while it should have been rejected is known as Type II Error.

6M.

10b. $n = 49$, $\bar{x} = 15,200$ —— (1)

Null Hypothesis $H_0 = \mu = 15,150$ and $\sigma = 1200$ —— (1)

Alternate hypothesis; $H_1 : \mu \neq 15150$ —— (1)

level of significance $\alpha = 0.05$

$|Z| = \left|\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right| = \left|\frac{15200-15150}{1200/\sqrt{49}}\right| = 0.29166$ —— (2)

The value of Z is $= 0.29166 < 1.96$. —— (2)

Thus the null hypothesis is accepted at 5% level of significance.

7M.

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Proc. Given

No of Heads x	0	1	2	3	4
Frequency f	122	60	15	2	1.

$\mu = \frac{\sum fx}{\sum f} = 0.5$

Binomial Dist $\mu = np$ since $n=5$ — (2)

$\therefore p = \frac{\mu}{n} = \frac{0.5}{5} = 0.1$

$\therefore \boxed{p = 0.1}$ $\boxed{q = 0.9}$

$p(x) = {}^nC_x p^x q^{n-x}$

$p(x) = 5C_x (0.1)^x (0.9)^{5-x} \Rightarrow f(x) = 200 \times [5C_x (0.1)^x (0.9)^{5-x}]$ → (3)

$f[x=0] = 200 \times 5C_0 \times (0.1)^0 (0.9)^{5-0} = 118$

$f(x=1) = 200 \times 5C_1 (0.1)^1 (0.9)^4 = 66$

$f(x=2) = 200 \times 5C_2 (0.1)^2 (0.9)^3 = 15$

$f(x=3) = 200 \times 5C_3 (0.1)^3 (0.9)^2 = 1.62 \simeq 2$

$f(x=4) = 200 \times 5C_4 (0.1)^4 (0.9)^1 = 1$

O_i	122	60	15	2+1=3
E_i	118	66	15	3.

— (1)

$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= \frac{(122-118)^2}{118} + \frac{(60-66)^2}{66} + 0 + 0$ — (1)

$= \frac{16}{118} + \frac{36}{66} = 0.1355 + 0.5454$

$= 0.6809 < \chi^2_{0.05} = 7.815$

Therefore the fitness is considered good

7 M.

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