MODEL QUESTION PAPER - 2
FOURTH Semester BE Degree (CBCS) Examination 2020-21

Additional Mathematics-II [18MATDIP41]

Time: 3 hours **Max Marks: 100**

Note: Answer any FIVE full questions, choosing ONE full question from each module

1 a) Find the rank of the following matrix by elementary row Echelon form (6M)

$$A=\begin{bmatrix}0 & 2 & 3 & 4\\ 2 & 3 & 5 & 4\\ 4 & 8 & 13 & 12\end{bmatrix}$$

b) Test for consistency and solve: (7M)

$$x+2y+3z=14;\ 4x+5y+7z=35;\ 3x+3y+4z=21$$

c) Find Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ (7M)

OR

2 a) Find the Eigen values & the Eigen vector corresponding to the largest eigen value of the matrix $A=\begin{bmatrix}1 & 2\\ 5 & 4\end{bmatrix}$ (6M)

b) Apply Gauss elimination method to solve the system of equation: (7M)

$2x+5y+7z=52,\ 2x+y-z=0,\ x+y+z=9.$

c) Test for consistency & solve the following system of equations: (7M)

$5x_1+x_2+3x_3=20;\ 2x_1+5x_2+2x_3=18;\ 3x_1+2x_2+x_3=14.$

3. a) The area of a circle(A) corresponding to diameter(D) is given below (6M)

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

b) Find the real root of equation $xe^x=2$, using N-R method correct to four decimal places. (7M)

c) Evaluate $\int_4^{5.2} \log_e x\, dx$ taking six equal strips by applying Weddle's Rule. (7M)

OR

4 a) Using Regula – Falsi method, find the real root of the equation $cosx = 3x - 1$. (6M)

b) Find the number of men getting wages below Rs.35 from the following table: (7M)

Wages (Rs)	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

c) Compute the value of $\int_{0.2}^{1.4}(\sin x - \log_e x + e^x)dx$ by using simpson's 1/3 rd rule by taking 6 equal strips (7M)

5 a) Solve: $(D^3 + 6D^2 + 11D + 6)y = 0$. (6M)

b) Solve: $y'' + 9y = cos2x.Cosx$. (7M)

c) Solve: $\frac{d^2y}{dx^2} - 4y = \cos h(2x - 1) + 3^x$. (7M)

OR

6 a) Solve: $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$ (6M)

b) Solve: $(D^2 + 4)y = sin^2x$. (7M)

c) Solve: $y'' + 4y' - 12y = e^{2x} - 3sin2x$. (7M)

7 a) Form the PDE by eliminating the arbitrary constants, $ax^2 + by^2 + z^2 = 1$. (6M)

b) Form the PDE by eliminating the arbitrary functions, $f(x^2 + 2yz, y^2 + 2zx) = 0$. (7M)

c) Solve: $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the conditions that $z = 1$& $\frac{\partial z}{\partial x} = y$ when $x = 0$. (7M)

OR

8) a) Form the PDE by eliminating the arbitrary constants: $z = a\log(x^2 + y^2) + b$. (6M)

b) Form the PDE by eliminating the arbitrary functions, $\varphi(x + y + z, x^2 + y^2 - z^2) = 0$. (7M)

c) Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ For which $\frac{\partial z}{\partial y} = -2siny$ when $x = 0$ and $z = 0$ if y is an add multiple of $\pi/2$. (7M)

9) a) Prove Baye's theorem on Conditional Probability. (6M)

b) If A and B are events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$ and $P(\overline{A}) = 5/8$ find $P(A), P(B)$ and $P(A \cap \bar{B})$ (7M)

c) In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%,15%,25%,40% of the total production. Out of these 5%,4%,3%,2% are defective. If a bolt drawn at random was found defective what is the probability that it was manufactured by A or D? (7M)

OR

10) a) If A and B are any events of S which are not mutually exclusive then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (6M)

b) Three machines A, B and C produce respectively 60%,30%,10% of the total number of items of a factory. The percentages of defective output of these machines are respectively2%,3%,4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. (7M)

c) The probability that a team wins a match is 3/5. If this team play 3 matches in a tournament, what is the probability that the team (7M)

AY: 2020-21

Department: MATHEMATICS
Subject with Sub. Code: Additional Mathematics-2/18MATDIP41
Semester / Division: 4 **Name of Faculty: Prof Hazel S Mathias**

Q.No.	MODULE – 01 **Solution and Scheme**
1. a)	Find the rank of the following matrix by elementary row echelon form $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$
Ans:	$R_1 \leftrightarrow R_2 \quad A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ $R_3 \to -2R_1 + R_3 \quad A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \end{bmatrix}$ $R_3 \to -R_2 + R_3 \quad A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\frac{1}{2}R_1, \ \frac{1}{2}R_2 \quad A \sim \begin{bmatrix} 1 & 3/2 & 5/2 & 2 \\ 0 & 1 & 3/2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ The matrix A in the row echelon form is having two nonzero rows; Thus $\rho(A) = 2$.
1. b)	Test for consistency and solve: $x + 2y + 3z = 14$; $4x + 5y + 7z = 35$; $x + 3y + 4z = 21$
Ans:	$[A : B] = \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 4 & 5 & 7 & : & 35 \\ 3 & 3 & 4 & : & 21 \end{bmatrix}$ is the augmented matrix

Q.No	Solution

$R_2 \to -4R_1 + R_2$

$R_3 \to -3R_1 + R_3$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & -3 & -5 & : & -21 \\ 0 & -3 & -5 & : & -21 \end{bmatrix}$$

$$R_3 \to -R_2 + R_3 \quad [A|B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & -3 & -5 & : & -21 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

We've $\rho(A) = 2$, $\rho(A:B) = 2$ i.e $r = 2$. Also $n = 3$.

Since $\rho(A) = \rho[A:B] = 2 < 3$ (i.e $r < n$) the system is consistent and will have infinite solutions. Here $(n-r) = 1$ and hence one of the variables can take arbitrary values.

We now have, $x + 2y + 3z = 14$ — ①

$-3y - 5z = -21$ — ②

Let $z = k$ be arbitrary,

from ② $-3y - 5k = -21$ OR $y = \frac{21-5k}{3} = 7 - \frac{5k}{3}$

Now from ① $x + 2\left(7 - \frac{5k}{3}\right) + 3k = 14 \quad \therefore x = \frac{k}{3}$

Thus $x = \frac{k}{3}$, $y = 7 - \frac{5k}{3}$, $z = k$ represent infinite solutions since k is arbitrary.

1.c) Find eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Ans:- The eigen values are obtained from the characteristic equation $|A - \lambda I| = 0$

i.e $$\begin{vmatrix} (1-\lambda) & 1 & 3 \\ 1 & (5-\lambda) & 1 \\ 3 & 1 & (1-\lambda) \end{vmatrix} = 0$$

Q.No	Solution

On expanding,

$(1-\lambda)[(5-\lambda)(1-\lambda)-1]-1[1(1-\lambda)-3]+3[1-3(5-\lambda)]=0$

i.e $(1-\lambda)[\lambda^2-6\lambda+4]-1[-2-\lambda]+3[-14+3\lambda]=0$

i.e $\lambda^2-6\lambda+4-\lambda^3+6\lambda^2-4\lambda+2+\lambda-42+9\lambda=0$

i.e $-\lambda^3+7\lambda^2-36=0$ OR $\lambda^3-7\lambda^2+36=0$.

$\lambda=-2$ is a root by inspection. Now by synthetic division,

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & 0 & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

$\Rightarrow \lambda^2-9\lambda+18=0$ OR $(\lambda-3)(\lambda-6)=0$ OR $\lambda=3,6$

$\therefore \lambda=-2,3,6$ are the eigen values.

We now form the system of equations,

$$\left.\begin{array}{l} (1-\lambda)x+1.y+3.z=0 \\ 1.x+(5-\lambda)y+1.z=0 \\ 3.x+1.y+(1-\lambda).z=0 \end{array}\right\} \quad \text{—— (1)}$$

Case i) : Let $\lambda=-2$ and solve from (1)

$3x+1y+3z=0$ —(i)

$1.x+7y+1z=0$ —(ii)

$3x+1.y+3z=0$

Applying the rule of cross multiplication to (i) & (ii) we get,

$$\frac{x}{\begin{vmatrix}1 & 3\\ 7 & 1\end{vmatrix}}=\frac{-y}{\begin{vmatrix}3 & 3\\ 1 & 1\end{vmatrix}}=\frac{z}{\begin{vmatrix}3 & 1\\ 1 & 7\end{vmatrix}}$$

i.e $\dfrac{x}{-20}=\dfrac{-y}{0}=\dfrac{z}{20}$ $\therefore (x,y,z)=(1,0,-1)$ is the eigen vector corresponding to $\lambda=-2$.

Q.No **Solution**

Case ii) Let $\lambda = 3$ and we've from (1),

$$-2x + 1y + 3z = 0 \quad \text{—— (iii)}$$
$$1x + 2y + 1z = 0 \quad \text{—— (iv)}$$
$$3x + 1y - 2z = 0$$

Applying the rule of cross multiplication to (iii) & (iv)

we get $$\frac{x}{\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}}$$

i.e $$\frac{x}{-5} = \frac{-y}{-5} = \frac{z}{-5}$$ $\therefore (x, y, z) = (1, -1, 1)$ is the eigen vector corresponding to $\lambda = 3$.

Case iii) Let $\lambda = 6$ and we've from (1)

$$-5x + 1 \cdot y + 3z = 0 \quad \text{—— (v)}$$
$$1x + (-1)y + 1z = 0 \quad \text{—— (vi)}$$
$$3x + 1 \cdot y - 5z = 0$$

Applying the rule of cross multiplication to (v) & (vi)

we get, $$\frac{x}{\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -5 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -5 & 1 \\ 1 & -1 \end{vmatrix}}$$

i.e $$\frac{x}{4} = \frac{-y}{-8} = \frac{z}{4}$$ $\therefore (x, y, z) = (1, 2, 1)$ is the eigen vector corresponding to $\lambda = 6$.

Thus $-2, 3, 6)$ are the eigen values and the corresponding eigen vectors are $(1, 0, -1)$; $(1, -1, 1)$; $(1, 2, 1)$.

2. a) Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$$

Q.No	Solution

Ans:- The characteristic equation of A is $|A-\lambda I|=0$

i.e $\begin{vmatrix} 1-\lambda & 2 \\ 5 & 4-\lambda \end{vmatrix} = 0$

On expanding the determinant we've,

$(1-\lambda)(4-\lambda)-(5)(2)=0$

i.e $4-5\lambda+\lambda^2-10=0$ OR $\lambda^2-5\lambda-6=0$

OR $\lambda=6$ and $\lambda=-1$ are the eigen values of A.

$\lambda=6$ is the largest eigen value of A.

Now we form the equations $[A-\lambda I][x]=[0]$, $x=\begin{bmatrix} x \\ y \end{bmatrix}$

i.e $\begin{bmatrix} 1-\lambda & 2 \\ 5 & 4-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

OR $\left.\begin{aligned} (1-\lambda)x+2y&=0 \\ 5x+(4-\lambda)y&=0 \end{aligned}\right\}$ — ①

Let $\lambda=6$. ① becomes, $-5x+2y=0$

$5x-2y=0$

consider $5x=2y$ OR $\frac{x}{2}=\frac{y}{5}=k$ (say) $\Rightarrow x:y=2:5$

Hence the eigen vector corresponding to the eigen value $\lambda=6$ is $[2 \;\; 5]^T$.

2 b) Apply Gauss elimination method to solve the system of equations: $2x+5y+7z=52$

$2x+y-z=0$

$x+y+z=9$

Ans:- As it is convienient to have the leading coefficient as one we shall interchange the 1st and 3rd equations.

The augmented matrix will be

$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 2 & 1 & -1 & : & 0 \\ 2 & 5 & 7 & : & 52 \end{bmatrix}$

Q.No	Solution
	$R_2 \to -2R_1 + R_2$, $R_3 \to -2R_1 + R_3$ $\quad [A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right]$
	$R_3 \to 3R_2 + R_3 \quad [A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$
	Hence we've $x+y+z=9$ $-y-3z=-18$ $-4z=-20 \Rightarrow z=5$.
	By back substitution, $y=3$ and $x=1$ Thus $x=1, y=3, z=5$ is the required solution.
2.c)	Test for consistency and solve the following system of equations: $5x_1 + x_2 + 3x_3 = 20;\ 2x_1 + 5x_2 + 2x_3 = 18;\ 3x_1 + 2x_2 + x_3 = 14$.
Ans:	$[A:B] = \left[\begin{array}{ccc:c} 5 & 1 & 3 & 20 \\ 2 & 5 & 2 & 18 \\ 3 & 2 & 1 & 14 \end{array}\right]$ is the augmented matrix
	$R_2 \to -2R_1 + 5R_2$, $R_3 \to -3R_1 + 5R_3$ $\quad [A:B] \sim \left[\begin{array}{ccc:c} 5 & 1 & 3 & 20 \\ 0 & 23 & 4 & 50 \\ 0 & 7 & -4 & 10 \end{array}\right]$
	$R_3 \to -7R_2 + 23R_3 \quad [A:B] \sim \left[\begin{array}{ccc:c} 5 & 1 & 3 & 20 \\ 0 & 23 & 4 & 50 \\ 0 & 0 & -120 & -120 \end{array}\right]$
	We've $\rho(A) = 3$, $\rho(A:B) = 3$ i.e $r=3$. Also $n=3$. Since $\rho(A) = \rho(A:B) = 3$, the system is consistent and will have unique solution.

Q.No	Solution

We now have, $5x_1 + x_2 + 3x_3 = 20$ —— (i)

$23x_2 + 4x_3 = 50$ —— (ii)

$-120x_3 = -120 \quad \therefore x_3 = 1$

From (ii) we get $x_2 = 2$ and from (i) we get $x_1 = 3$.
Thus $x_1 = 3, x_2 = 2, x_3 = 1$ is the unique solution.

MODULE - 02

3-a) The area of a circle (A) corresponding to diameter (D) is given below.

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

Ans:- Here we've to find A when D = 105. As this value 105 is near to the end value 100, Newton's backward interpolation formula is appropriate

$x = D$	$y = A$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
$x_n = 100$	$y_n = 7854$				

We've Newton's backward interpolation formula:

$$y_r = y_n + r.\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \cdots$$

Q.No	Solution
	where $r = \frac{x-x_n}{h}$; $r = \frac{105-100}{5} = 1$
	From the table $\nabla y_n = 766$, $\nabla^2 y_n = 40$, $\nabla^3 y_n = 2$, $\nabla^4 y_n = 4$
	$\therefore f(105) = 7854 + 1(766) + \frac{(1)(2)}{2}(40) + \frac{(1)(2)(3)}{6}(2) + \frac{(1)(2)(3)(4)}{24} \times 4$
	$= 7854 + 766 + 40 + 2 + 4 = 8666$
	Thus the area (A) corresponding to diameter (D) = 105 is 8666.
3.b)	Find the real root of equation $xe^x = 2$, using N-R method correct to 4 decimal places.
Ans:	Let $f(x) = xe^x - 2$
	$f(0) = -2 < 0$, $f(1) = 0.7183 > 0$
	$\therefore$ A real root lies in $(0,1)$ and let $x_0 = 1$.
	Also, $f'(x) = xe^x + e^x = e^x(x+1)$
	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{0.1783}{e^1(2)}$
	$= 0.8679$.
	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8679 - \frac{f(0.8679)}{f'(0.8679)}$
	$\therefore x_2 = 0.8679 - \frac{[0.8679 . e^{0.8679} - 2]}{e^{0.8679}[0.8679+1]} = 0.8528$
	$x_3 = 0.8528 - \frac{[0.8528\, e^{0.8528} - 2]}{e^{0.8528}(1.8528)} = 0.8526$
	$x_4 = 0.8526 - \frac{[0.8526 . e^{0.8526} - 2]}{e^{0.8526}[0.8526+1]} = 0.8526$

Q.No	Solution
	Thus the required real root correct to 4 decimal places is 0.8526.

3.c) Evaluate $\int_4^{5.2} \log_e x \, dx$ taking 6 equal strips by applying weddle's rule.

Ans: The length of each strip $(h) = \frac{5.2-4}{6} = 0.2$; $n=6$.

The values of x and $y = \log_e x$ are tabulated.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Weddle's rule for $n=6$ is given by,

$$\int_a^b y\,dx = \frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6\right]$$

$$\int_4^{5.2} \log_e x\,dx = \frac{3(0.2)}{10}\left[1.3863+5(1.4351)+1.4816+6(1.5261)+1.5686+5(1.6094)+1.6487\right]$$

Thus $\int_4^{5.2} \log_e x\,dx = 1.8279$.

4.a) Using Regula-Falsi method, find the real root of the equation $\cos x = 3x-1$.

Ans:- Let $f(x) = \cos x + 1 - 3x$

In radians, $f(0) = 2 > 0$, $f(1) = -1.46 < 0$

A real root lies in $(0,1)$ and we expect the root in the neighbourhood of 1.

consider $f(0.6) = -0.0253 > 0$, $f(0.7) = -0.3352 < 0$

$\therefore$ the root lies in $(0.6, 0.7)$

Q.No	Solution

I iteration: $a = 0.6 \quad f(a) = -0.0253$

$b = 0.7 \quad f(b) = -0.3352$

$$x_1 = \frac{a f(b) - b.f(a)}{f(b) - f(a)} = \frac{0.6(-0.3352) - (0.7)(-0.0253)}{-0.3352 - (-0.0253)}$$

$\therefore x_1 = 0.607$

II iteration: $f(x_1) = f(0.607) = 0.00036 > 0$

$\therefore$ the root lies in $(0.607, 0.7)$

Now, $a = 0.607$, $f(a) = 0.00036$

$b = 0.7 \quad f(b) = -0.3352$

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0.607(-0.3352) - (0.7)(0.00036)}{-0.3352 - 0.00036}$$

$\therefore x_2 = 0.607$

Hence the real root correct to 3 decimals is 0.607.

4.b) Find the number of men getting wages below Rs. 35 from the following table:

Wages (Rs)	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

Ans: The given data is reconstituted with $f(x)$ representing the number of men, less than income of Rs. x. That is as follows;

Less than $10 = 9$, less than $20 = 9+30 = 39$

Less than $30 = 39+35 = 74$, less than $40 = 74+42 = 116$

Newton's backward interpolation formula is given by,

$$y_r = y_n + r \nabla y_n + \frac{r(r+1)}{2!} \nabla^2 y_n + \frac{r(r+1)(r+2)}{3!} \nabla^3 y_n + \cdots$$

where $r = \frac{x - x_n}{h} = \frac{35-40}{10} = -0.5$.

Q.No	Solution

We shall find $f(35)$ by constructing the backward difference table.

x	$f(x)=y$	∇y	$\nabla^2 y$	$\nabla^3 y$
10	9			
		30		
			5	
20	39			$\nabla^3 y_n = 2$
		35		
30	74		$\nabla^2 y_n = 7$	
		$\nabla y_n = 42$		
$x_n = 40$	$y_n = 116$			

$$f(35) = 116 + (-0.5)(42) + \frac{(-0.5)(0.5)}{2}(7) + \frac{(-0.5)(0.5)(1.5)}{6}(2)$$

$\therefore f(35) = 94$

Thus, the number of men getting wages below Rs. 35 is 94.

4.c) compute the value of $\int_{0.2}^{1.4} (\sin x - \log_e^x + e^x)\,dx$ using simpson's $1/3^{rd}$ rule taking six parts.

Ans:- Length of each part $(h) = \frac{1.4 - 0.2}{6} = 0.2$

Let $y = f(x) = \sin x - \log_e^x + e^x$

The values of $y = f(x)$ corresponding to equidistant values of x: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4 are tabulated correct to 4 decimals. (calculator in RAD mode).

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y	3.6295	2.7975	2.8976	3.1660	3.5598	4.0698	4.7042
	y_0	y_1	y_2	y_3	y_4	y_5	y_6.

Simpson's $1/3^{rd}$ rule for $n=6$ is given by

$$I = \int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

Q.No	Solution
	$I = \frac{0.2}{3}\left[(3.0295+4.7042)+4(2.7975+3.1660+4.0698)+2(2.8976+3.5598)\right]$
	$= 4.05211$
	Thus the required value of the given interval is 4.052.

Module -03

5.a) Solve:

$(D^3+6D^2+11D+6)y=0$

Ans: We've $(D^3+6D^2+11D+6)y=0$

AE is $m^3+6m^2+11m+6=0$

We shall find one root by inspection by taking values for $m=1, -1, 2, -2$ etc. Since all the terms are positive we've to try only negative values for m.

Putting $m=-1$ we've, $(-1)^3+6(-1)^2+11(-1)+6=0$

$\therefore m=-1$ is a root. The other roots can be found through the process of synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & 0 & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & \underline{|0} \end{array}$$

Hence we get $m^2+5m+6=0$

i.e $(m+2)(m+3)=0 \Rightarrow m=-2,-3$.

Hence $m=-1,-2,-3$ are the roots of the AE which are all real and distinct.

Thus $y=c_1e^{-x}+c_2e^{-2x}+c_3e^{-3x}$ is the general solution

5. b) Solve: $y''+9y = \cos 2x \cdot \cos x$.

Ans: We've $(D^2+9)y = \cos 2x \cdot \cos x$.

AE is $m^2+9=0 \Rightarrow m=\pm 3i$

$\therefore y_c = c_1\cos 3x + c_2 \sin 3x$

Q.No	Solution

$$y_p = \frac{1}{2}\frac{(\cos x + \cos 3x)}{D^2+9} = \frac{1}{2}\cdot\frac{\cos x}{D^2+9} + \frac{1}{2}\cdot\frac{\cos 3x}{D^2+9}$$

$$= p_1 + p_2 \text{ (say)}$$

$$p_1 = \frac{1}{2}\frac{\cos x}{D^2+9} = \frac{1}{2}\cdot\frac{\cos x}{-1^2+9} = \frac{\cos x}{16}, \quad D^2 \to -1^2 = -1$$

$$p_2 = \frac{1}{2}\cdot\frac{\cos 3x}{D^2+9} = \frac{1}{2}\cdot\frac{\cos 3x}{-3^2+9} \quad (Dr. = 0)$$

$$= \frac{1}{2}\cdot x\,\frac{\cos 3x}{2D} = \frac{x}{4}\int \cos 3x\,dx = \frac{x\sin 3x}{12}$$

complete solution : $y = y_c + y_p$ where $y_p = p_1 + p_2$

Thus $y = c_1\cos 3x + c_2\sin 3x + \frac{\cos x}{16} + \frac{x\sin 3x}{12}$.

5.c) Solve: $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$

Ans:- We've $(D^2-4)y = \cosh(2x-1) + 3^x$

AE is $m^2 - 4 = 0$ OR $(m-2)(m+2) = 0 \therefore m = 2, -2$.

$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

$$y_p = \frac{\cosh(2x-1) + 3^x}{D^2-4} = \frac{1}{2}\left[\frac{e^{2x-1}}{D^2-4} + \frac{e^{-(2x-1)}}{D^2-4}\right] + \frac{3^x}{D^2-4}$$

$$= p_1 + p_2 + p_3 \text{ (say)}$$

$$p_1 = \frac{1}{2}\cdot\frac{e^{2x-1}}{D^2-4} = \frac{1}{2}\cdot\frac{e^{2x-1}}{2^2-4} \quad (Dr = 0)$$

$$= \frac{1}{2}x\,\frac{e^{2x-1}}{2D} = \frac{1}{2}\cdot\frac{x}{2}\cdot\frac{e^{2x-1}}{2} = \frac{x}{8}\cdot e^{2x-1}$$

$$p_2 = \frac{1}{2}\cdot\frac{e^{-(2x-1)}}{D^2-4} = \frac{1}{2}\cdot\frac{e^{-(2x-1)}}{(-2)^2-4} \quad (Dr = 0)$$

$$= \frac{1}{2}\cdot x\,\frac{e^{-(2x-1)}}{2D} = \frac{1}{2}\cdot\frac{x}{2}\cdot\frac{e^{-(2x-1)}}{-2} = \frac{-x}{8}\cdot e^{-(2x-1)}$$

Q.No	Solution
	$p_3 = \frac{3^x}{D^2-4} = \frac{(e^{\log 3})^x}{D^2-4} = \frac{e^{\log 3 \cdot x}}{D^2-4} = \frac{e^{\log 3 . x}}{(\log 3)^2-4}$
	$p_3 = \frac{3^x}{(\log 3)^2-4}$
	Complete solution: $y = y_c + y_p$ where $y_p = p_1 + p_2 + p_3$
	$\therefore y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{8} e^{2x-1} - \frac{x}{8} e^{-(2x-1)} + \frac{3^x}{(\log 3)^2 - 4}$
	Thus $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{4}\sinh(2x-1) + \frac{3^x}{(\log 3)^2-4}$
6. a)	Solve: $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$
Ans:	AE is $4m^4 - 4m^3 - 23m^2 + 12m + 36 = 0$
	If $m=2$: $64 - 32 - 92 + 24 + 36 = 124 - 124 = 0$.
	$\Rightarrow m=2$ is a root by inspection. Now by synthetic division,
	$\begin{array}{c\|ccccc} 2 & 4 & -4 & -23 & 12 & 36 \\ & 0 & 8 & 8 & -30 & -36 \\ \hline & 4 & 4 & -15 & -18 & \boxed{0} \end{array}$
	Now, $4m^3 + 4m^2 - 15m - 18 = 0$
	If $m=2$, $32 + 16 - 30 - 18 = 0$, Again $m=2$ is a root
	By synthetic division,
	$\begin{array}{c\|cccc} 2 & 4 & 4 & -15 & -18 \\ & 0 & 8 & 24 & 18 \\ \hline & 4 & 12 & 9 & \boxed{0} \end{array}$
	Now $4m^2 + 12m + 9 = 0$ OR $(2m+3)^2 = 0 \Rightarrow m = -3/2, -3/2$
	Hence the roots of the AE are $2, 2, -3/2, -3/2$.
	Thus $y_c = (c_1 + c_2 x)e^{2x} + (c_3 + c_4 x)e^{-3x/2}$ is the general solution.

Q.No	Solution
6 b)	Solve: $(D^2+4)y = \sin^2 x$.
	AE is $m^2+4=0 \Rightarrow m=\pm 2i$
	$\therefore y_c = c_1\cos 2x + c_2 \sin 2x$
	$y_p = \frac{\sin^2 x}{D^2+4} = \frac{1}{2}\frac{(1-\cos 2x)}{D^2+4} = \frac{1}{2}\cdot\frac{1}{D^2+4} - \frac{1}{2}\frac{\cos 2x}{D^2+4}$
	$= p_1 + p_2$
	$p_1 = \frac{1}{2}\cdot\frac{e^{0x}}{D^2+4} = \frac{1}{2}\cdot\frac{e^{0x}}{0^2+4} = \frac{1}{8}$
	$p_2 = -\frac{1}{2}\frac{\cos 2x}{D^2+4}$ $(D^2 \to -2^2)$; $p_2 = -\frac{1}{2}\frac{\cos 2x}{-4+4}$ $(Dr=0)$
	$\therefore p_2 = -\frac{1}{2}\cdot x\cdot\frac{\cos 2x}{2D} = -\frac{1}{4}\cdot x\int \cos 2x\,dx = \frac{-x\sin 2x}{8}$
	Complete solution : $y = y_c + y_p$ where $y_p = p_1 + p_2$
	Thus $y = c_1\cos 2x + c_2\sin 2x + \frac{1}{8} - \frac{x\sin 2x}{8}$
6 c)	Solve: $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$
Ans:-	solve $(D^2+4D-12)y = e^{2x} - 3\sin 2x$
	AE is $m^2+4m-12=0$ OR $(m+6)(m-2)=0 \Rightarrow m=2,-6$
	$\therefore y_c = c_1 e^{2x} + c_2 e^{-6x}$
	$y_p = \frac{e^{2x}}{D^2+4D-12} - \frac{3\sin 2x}{D^2+4D-12} = p_1 - p_2$ (say)
	$p_1 = \frac{e^{2x}}{D^2+4D-12} = \frac{e^{2x}}{4+8-12}$ $(Dr=0)$
	$p_1 = x\cdot\frac{e^{2x}}{2D+4} = x\cdot\frac{e^{2x}}{8} = \frac{xe^{2x}}{8}$
	$p_2 = \frac{3\sin 2x}{D^2+4D-12}$; $D^2 \to -4$
	$p_2 = \frac{3\sin 2x}{-4+4D-12} = \frac{3\sin 2x}{4(D-4)} = \frac{3(D+4)\sin 2x}{4(D^2-16)}$

Q.No	Solution

$$p_2 = \frac{3\,(2\cos 2x + 4\sin 2x)}{-80}$$

Complete solution : $y = y_c + p_1 - p_2$

Thus $y = c_1 e^{2x} + c_2 e^{-6x} + \frac{x e^{2x}}{8} + \frac{3}{40}(\cos 2x + 2\sin 2x).$

MODULE-04

7.a) Form the PDE by eliminating the arbitrary constants from, $ax^2 + by^2 + z^2 = 1.$

Ans:- By data, $z^2 = 1 - ax^2 - by^2$ —— ①

Differentiating ① partially w.r.t. x and y.

$$2z.\frac{\partial z}{\partial x} = 2zp = -2ax \quad \therefore a = -\frac{zp}{x} \text{ —— ②.}$$

$$2z.\frac{\partial z}{\partial y} = 2zq = -2by \quad \therefore b = -\frac{zq}{y} \text{ —— ③}$$

Using (2) and (3) in (1) we obtain,

$$z^2 = 1 + \frac{zp}{x}.x^2 + \frac{zq}{y}.y^2$$

Thus $(z^2 - 1) = z(px + qy)$ is the required PDE.

7.b) Form the PDE by eliminating the arbitrary functions from, $f(x^2 + 2yz,\ y^2 + 2zx) = 0$

Ans:- We've by data $f(u, v) = 0$ where $u = x^2 + 2yz$, $v = y^2 + 2zx$

Differentiating $f(u, v) = 0$ partially w.r.t. x and y by applying chain rule we've,

$$\frac{\partial f}{\partial u}.\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}.\frac{\partial v}{\partial x} = 0 \text{ OR } \frac{\partial f}{\partial u}.\frac{\partial u}{\partial x} = -\frac{\partial f}{\partial v}.\frac{\partial v}{\partial x} \text{ —— ①}$$

$$\& \ \frac{\partial f}{\partial u}.\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}.\frac{\partial v}{\partial y} = 0 \text{ OR } \frac{\partial f}{\partial u}.\frac{\partial u}{\partial y} = -\frac{\partial f}{\partial v}.\frac{\partial v}{\partial y} \text{ —— ②}$$

Dividing ① by ② we obtain,

$$\frac{\partial u}{\partial x}\Big/\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}\Big/\frac{\partial v}{\partial y} \text{ —— ③}$$

Q.No	Solution

$$\frac{\partial u}{\partial x} = 2x + 2yp \quad ; \quad \frac{\partial v}{\partial x} = 2(z + xp)$$

$$\frac{\partial u}{\partial y} = 2(yq + z) \quad ; \quad \frac{\partial v}{\partial y} = 2y + 2xq$$

Hence from ③ $\dfrac{2(x+yp)}{2(yq+z)} = \dfrac{2(z+xp)}{2(y+xq)}$

OR $xy + x^2q + y^2p + xypq = yzq + xypq + z^2 + xzp$

Thus $(y^2 - zx)p + (x^2 - yz)q = (z^2 - xy)$ is the PDE.

7.c) Solve: $\dfrac{\partial^2 z}{\partial x^2} + 3 \cdot \dfrac{\partial z}{\partial x} - 4z = 0$ subject to the conditions that $z = 1$ and $\dfrac{\partial z}{\partial x} = y$ when $x = 0$.

Ans:- Let us suppose that z is a function of x only. The given PDE assumes the form of an ODE,

$(D^2 + 3D - 4)z = 0$ where $D = d/dx$

AE is $m^2 + 3m - 4 = 0$ OR $(m-1)(m+4) = 0$

$\therefore m = 1, \quad m = -4$

The solution of the ODE is given by $z = c_1 e^x + c_2 e^{-4x}$

Solution of the PDE is given by $z = f(y)e^x + g(y) \cdot e^{-4x}$ — ①

Differentiating partially w.r.t. x we get,

$$\frac{\partial z}{\partial x} = f(y) \cdot e^x + g(y) \cdot e^{-4x}(-4) \quad \text{—— ②}$$

By data, $z = 1$ and $\dfrac{\partial z}{\partial x} = y$ when $x = 0$

Hence ① and ② becomes $1 = f(y) + g(y)$ & $y = f(y) - 4g(y)$

By solving these simultaneously we get,

$f(y) = \frac{1}{5}(4+y)$ and $g(y) = \frac{1}{5}(1-y)$

we substitute these in ①

Thus $z = \frac{1}{5}(4+y)e^x + \frac{1}{5}(1-y)e^{-4x}$ is the required solution.

Q.No	Solution
8.a)	Form the PDE by eliminating the arbitrary constants: $z = a\log(x^2+y^2)+b$.
Ans:-	By data, $z = a\log(x^2+y^2)+b$ ——— ①
	$\therefore \frac{\partial z}{\partial x} = p = \frac{a}{x^2+y^2}\cdot 2x$ OR $p = \frac{2ax}{x^2+y^2}$ ——— ②
	$\frac{\partial z}{\partial y} = q = \frac{a}{x^2+y^2}\cdot 2y$ OR $q = \frac{2ay}{x^2+y^2}$ ——— ③
	Dividing ② by ③ we get, $\frac{p}{q} = \frac{x}{y}$
	Thus, $py - qx = 0$ is the required PDE.
8.b)	Form the PDE by eliminating the arbitrary functions. $\phi(x+y+z,\ x^2+y^2-z^2)=0$.
Ans:-	We've by data $\phi(u,v)=0$ ——— ①
	where $u = x+y+z$ and $v = x^2+y^2-z^2$
	Now, $\frac{\partial u}{\partial x} = 1+\frac{\partial z}{\partial x} = 1+p$; $\frac{\partial v}{\partial x} = 2x - 2z\cdot\frac{\partial z}{\partial x} = 2(x-zp)$
	$\frac{\partial u}{\partial y} = 1+\frac{\partial z}{\partial y} = 1+q$; $\frac{\partial z}{\partial y} = 2y-2z\cdot\frac{\partial z}{\partial y} = 2(y-zq)$
	Let us differentiate ① w.r.t. x and y by applying chain rule, i.e $\frac{\partial\phi}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial\phi}{\partial v}\cdot\frac{\partial v}{\partial x} = 0$ OR $\frac{\partial\phi}{\partial u}\cdot\frac{\partial u}{\partial x} = -\frac{\partial\phi}{\partial v}\cdot\frac{\partial v}{\partial x}$ ——②
	& $\frac{\partial\phi}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial\phi}{\partial v}\cdot\frac{\partial v}{\partial y} = 0$ OR $\frac{\partial\phi}{\partial u}\cdot\frac{\partial u}{\partial y} = -\frac{\partial\phi}{\partial v}\cdot\frac{\partial v}{\partial y}$ ——— ③
	Dividing ② by ③ we obtain,
	$\frac{\partial u}{\partial x}\Big/\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}\Big/\frac{\partial v}{\partial y}$
	i.e $\frac{1+p}{1+q} = \frac{2(x-zp)}{2(y-zq)}$ OR $(1+p)(y-zq) = (1+q)(x-zp)$
	i.e $y - zq + py - pqz = x - zp + qx - pqz$
	i.e $py + pz - qx - qz = x - y$
	Thus $p(y+z) - q(x+z) = x-y$ is the required PDE.

Q.No	Solution
8.c)	Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$

Ans: Here we first find z by integration and apply the given conditions to determine the arbitrary functions occuring as constants of integration.

The given PDE can be written as $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \sin x \sin y$

Integrating w.r.t. x treating y as constant,

$$\frac{\partial z}{\partial y} = \sin y \int \sin x \, dx + f(y)$$

i.e $\frac{\partial z}{\partial y} = -\sin y \cos x + f(y)$ —— ①

Integrating w.r.t. y treating x as a constant,

$$z = -\cos x \int \sin y \, dy + \int f(y) \, dy + g(x)$$

i.e, $z = (-\cos x)(-\cos y) + F(y) + g(x)$, where $F(y) = \int f(y) dy$

Thus $z = \cos x \cos y + F(y) + g(x)$ —— ②

Also by data, $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$ Using this in ①

$$-2\sin y = (-\sin y)\cdot 1 + f(y) \quad \text{OR} \quad f(y) = -\sin y.$$

Hence $F(y) = \int f(y) dy = \int -\sin y \, dy = \cos y.$

With this ② becomes $z = \cos x \cos y + \cos y + g(x)$ —— ③

Using the condition that $z=0$ if $y = (2n+1)\frac{\pi}{2}$ in ③

we have $0 = \cos x \cos(2n+1)\frac{\pi}{2} + \cos(2n+1)\frac{\pi}{2} + g(x).$

But, $\cos(2n+1)\frac{\pi}{2} = 0$. Hence $0 = 0 + 0 + g(x)$ OR $g(x) = 0$.

Thus the solution of the PDE is given by

$$z = \cos x \cos y + \cos y$$

OR $z = \cos y(\cos x + 1)$.

Q.No	Solution

MODULE-05

9. a) Prove Baye's theorem on conditional Probability.

Ans: Let $A_1, A_2, \dots A_n$ be a set of exhaustive and mutually exclusive events of the sample space S with $P(A_i) \neq 0$ for each i. If A is any other event associated with A_i $(A \subset \bigcup_{i=1}^{n} A_i)$ with $P(A) \neq 0$ then

$$P(A_i|A) = \frac{P(A_i)\,P(A|A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A|A_i)}$$

Proof: We've $S = A_1 \cup A_2 \cup \dots \cup A_n$ and $A \subset S$

$\therefore A = S \cap A = (A_1 \cup A_2 \cup \dots \cup A_n) \cap A$

Using distributive law in the RHS we've

$A = (A_1 \cap A) \cup (A_2 \cap A) \cup \dots \cup (A_n \cap A)$

Since $A_i \cap A$ for $i = 1$ to n are mutually exclusive, by applying the addition rule of probability,

$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + \dots + P(A_n \cap A)$

Now by applying multiplication rule onto each term in the RHS we've

$P(A) = P(A_1)\,P(A|A_1) + P(A_2)\,P(A|A_2) + \dots + P(A_n)\,P(A|A_n)$

That is $P(A) = \sum_{i=1}^{n} P(A_i)\,P(A|A_i)$ ——— ①

The conditional probability of A_i for any i given A, is defined by $P(A_i|A) = \dfrac{P(A_i \cap A)}{P(A)} = \dfrac{P(A_i)\,P(A|A_i)}{P(A)}$

Using ① in the denominator of the RHS we've,

$$P(A_i|A) = \frac{P(A_i)\,P(A|A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A|A_i)}$$

This proves Baye's theorem for conditional probability.

Q.No	Solution
9.b)	If A and B are events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$ and $P(\bar{A}) = 5/8$ find $P(A)$, $P(B)$ and $P(A \cap \bar{B})$
Ans:-	$P(A) = 1 - P(\bar{A}) = 1 - (5/8)$ $\therefore P(A) = 3/8$
	We've $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	i.e $7/8 = 3/8 + P(B) - 1/4$
	$\therefore P(B) = 7/8 - 3/8 + 1/4$ OR $P(B) = 3/4$
	Also we've $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 3/8 - 1/4 = 1/8$
	$\therefore P(A \cap \bar{B}) = 1/8$.
9.c)	In a bolt factory there are 4 machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3%, 2% are defective. If a bolt drawn at random was found defective what is the probability that it was manufactured by A/D?
Ans:	By data A, B, C, D manufacture 20%, 15%, 25% & 40% of the total production. Hence we've
	$P(A) = 0.2$, $P(B) = 0.15$, $P(C) = 0.25$, $P(D) = 0.4$
	Let X be the event of selection of a defective bolt.
	Then $P(X \mid A) = 0.05$, $P(X \mid B) = 0.04$, $P(X \mid C) = 0.03$, $P(X \mid D) = 0.02$. We need to compute $P(A \cup D \mid X)$
	Since A and D are mutually exclusive we've,
	$P(A \cup D \mid X) = P(A \mid X) + P(D \mid X)$ —— ①
	We've by Baye's theorem,
	$P(A \mid X) = \dfrac{P(A).P(X \mid A)}{P(A).P(X \mid A) + P(B)P(X \mid B) + P(C)P(X \mid C) + P(D).P(X \mid D)}$
	$\therefore P(A \mid X) = \dfrac{(0.2)(0.05)}{(0.2)(0.05) + (0.15)(0.04) + (0.25)(0.03) + (0.4)(0.02)}$

Q.No	Solution

i.e $P(A|X) = \dfrac{0.01}{0.0315} \approx 0.3175$ —— (2)

Also $P(D|X) = \dfrac{P(D)\,P(X|D)}{0.0315} = \dfrac{(0.4)(0.02)}{0.0315} \approx 0.254$ —— (3)

Using (2) and (3) in (1)

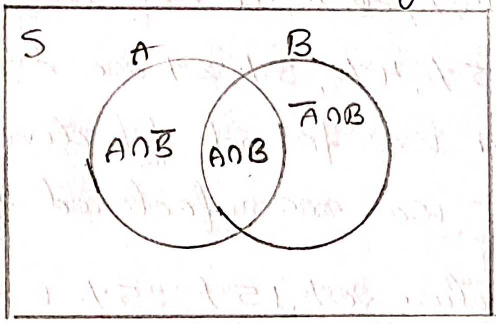
$P(A \cup D|X) = 0.3175 + 0.254 = 0.5715.$

Thus $P(A \cup D|X) = 0.5715.$

OR

10.a) If A and B are any events of S which are mutually exclusive then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Ans: We prove the result using the following Venn diagram.



From the figure $A = (A \cap \bar{B}) \cup (A \cap B)$

$B = (\bar{A} \cap B) \cup (A \cap B)$

$\Rightarrow P(A) = P(A \cap \bar{B}) + P(A \cap B)$, since $A \cap \bar{B}$ & $A \cap B$ are disjoint.

From the figure, $P(B) = P(\bar{A} \cap B) + P(A \cap B)$, since $\bar{A} \cap B$ and $A \cap B$ are disjoint.

Here we've used the 3rd axiom of probability that $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ if A_1 & A_2 are mutually exclusive.

Now, $P(A) + P(B) = [P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)] + P(A \cap B)$

That is $P(A) + P(B) = [P(A \cup B)] + P(A \cap B)$

Thus, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

10.b) Three machines produce (A, B, C) respectively 60%, 30% 10% of the total number of items of a factory. The

Q.No	Solution
	Percentages of defective output of these machines are respectively 2%, 3%, 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C.
Ans:	Let A, B, C stand for the events of selection of an item from machines A, B, C

$\therefore P(A) = \frac{60}{100} = 0.6,\ P(B) = \frac{30}{100} = 0.3,\ P(C) = \frac{10}{100} = 0.1$

Suppose D is the event of selection of a defective item then

$P(D|A) = \frac{2}{100} = 0.02,\ P(D|B) = \frac{3}{100} = 0.03,\ P(D|C) = \frac{4}{100} = 0.04$

To find the probability that a selected item is produced from the machine C, we need to find $P(C|D)$, we've by Baye's theorem,

$$P(C|D) = \frac{P(C).P(D|C)}{P(A).P(D|A) + P(B).P(B|B) + P(C).P(D|C)}$$

$$= \frac{(0.1)(0.04)}{(0.6)(0.02) + (0.3)(0.03) + (0.1)(0.04)}$$

Thus $P(C|D) = 0.16$.

10.c) The probability that a team wins a match is 3/5. If this team play 3 matches in a tournament, what is the probability that the team

(a) win all the matches b) win atleast one match

(c) win atmost one match (d) lose all the matches.

Ans: Let W be the event of winning a match by the team.

$P(W_1) = P(W_2) = P(W_3) = 3/5$

$\therefore P(\overline{W}_1) = P(\overline{W}_2) = P(\overline{W}_3) = 2/5$

Q.No	Solution	
	(a) Probability of winning all the matches $= P(W_1) \cdot P(W_2) \cdot P(W_3) = 27/125.$ (b) Probability of winning atleast one match $= 1 -$ Probability of losing all the matches $= 1 - P(\bar{W_1}) \cdot P(\bar{W_2}) \cdot P(\bar{W_3})$ $= 1 - 8/25 = 17/25$ (c) Probability of winning atmost one match. $= P(\bar{W_1}) P(\bar{W_2}) P(\bar{W_3}) + P(W_1) P(\bar{W_2}) P(\bar{W_3}) + P(\bar{W_1}) P(W_2) P(\bar{W_3})$ $+ P(\bar{W_1}) + P(\bar{W_2}) \cdot P(W_3)$. $= \frac{8}{125} + 3\left[\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}\right] = \frac{8}{125} + \frac{36}{125} = \frac{44}{125}$ (d) Probability of losing all the matches = $P(\bar{W_1}) \cdot P(\bar{W_2}) P(\bar{W_3}) = (2/5)(2/5)(2/5) = 8/125$.	