Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN | | | | | | | | | | | **18MATDIP41**

Fourth Semester B.E.(CBCS) Examination
Additional Mathematics - II
(Common to all Branches)

Time: **3 Hrs** Max.Marks: **100**

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1 (a) Find the rank of the following matrix by applying elementary row transformations **(06 Marks)**

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix}$$

(b) Solve the following system of linear equations by Gauss elimination method: **(07Marks)**
$5x + 10y + z = 28; x + y + z = 6; 4x + 8y + 3z = 29.$

(c) Find all the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$ **(07Marks)**

OR

2 (a) Reduce the matrix into its echelon form and hence find its rank **(06 Marks)**

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

(b) Find all the eigenvalues and the corresponding eigenvectors of **(07Marks)**

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$$

(c) Solve the system of linear equations $3x + y - z = 3; 2x - 8y + z = -5; x - 2y + 9z = 8,$ by applying Gauss elimination method. **(07Marks)**

Module-2

3 (a) Find a real root of $x\, log_{10} x - 1.2 = 0$, correct to three decimal places lying in the interval $(2,3)$, by using Regula-Falsi method. **(06 Marks)**

(b) Use an appropriate interpolation formula to compute $f(2.18)$ for the following data **(07Marks)**

x	1.7	1.8	1.9	2.0	2.1	2.2
f(x)	5.474	6.050	6.686	7.389	8.166	9.025

(c) Use Weddle's rule to evaluate $\int_{-\pi/2}^{\pi/2} cos\, x\, dx$, by dividing $[-\pi/2, \pi/2]$ into six equal parts. **(07Marks)**

OR

4 (a) Find a real root of the equation $x^3 + x^2 + 3x + 4 = 0$ near $x = -1$ correct to four decimal places, by using Newton- Raphson method. **(06 Marks)**

(b) Use an appropriate interpolation formula to compute $f(42)$ for the following data: **(07Marks)**

x	40	50	60	70	80	90
$f(x)$	184	204	226	250	276	304

(c) Using Simpson's $(1/3)^{rd}$ rule, evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ by taking 6 equidistant ordinates. **(07Marks)**

Module-3

5 (a) Solve : $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} - 2y = 0.$ **(06 Marks)**

(b) Solve : $(D^2 - 7D + 10)y = (1 + e^x)^2$. **(07Marks)**

(c) Solve : $(D^2 + 2D + 3)y = \sin x$. **(07Marks)**

OR

6 (a) Solve: $(D^3 - 2D^2 + 4D - 8)y = 0.$ **(06 Marks)**

(b) Solve $\frac{d^2y}{dx^2} - 4y = cosh(2x - 1) + 3^x.$ **(07Marks)**

(c) Solve $(\mathrm{D}^2 + \mathrm{a}^2)y = \cos a\, x.$ **(07Marks)**

Module-4

7 (a) Form the partial differential equation by eliminating the arbitrary constants from **(07Marks)**
$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$

(b) Form the partial differential equation by eliminating the arbitrary functions from
$z = f(y + 2x) + g(y - 3x)$

(c) Solve $\frac{\partial^2 z}{\partial x \partial y} = sin\, x\, sin\, y$, for which $\frac{\partial z}{\partial y} = -2\, sin\, y$, when $x = 0$ and $z = 0$ when y is odd. **(07Marks)**

OR

8 (a) Form the partial differential equation by eliminating the arbitrary function from $f(xy + z^2, x + y + z) = 0.$ **(06 Marks)**

(b) Form the partial differential equation by eliminating the arbitrary functions from $z = xf_1(x + y) + f_2(x + y).$ **(07Marks)**

(c) Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y}$ for which $\frac{\partial z}{\partial x} = log\, x$ when $y = 1$and $z = 0$ when $x = 1.$ **(07Marks)**

Module-5

9 (a) State the axiomatic definition of probability. For any two arbitrary events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

(b) The probability that a team wins a match is $3/5$. If this team play 3 matches in a tournament, what is the probability that the team (i) win and (ii) loose, all the matches. **(07Marks)**

(c) In an UG class of a reputed engineering college, 70% are boys and 30% are girls; 5% of boys and 3% of the girls are irregular to the classes. What is the probability of a student selected at random is irregular to the classes and what is the probability that the irregular student is a girl? **(07Marks)**

OR

10 (a) State and prove Bayes's theorem. **(06 Marks)**

(b) The chance that a doctor will diagonise a disease correctly is 60%. The chance that a patient will die after correct diagonise is 40% and the chance of death by wrong diagonise is 70%. If a patient dies, what is the chance that his disease was correctly diagonised? **(07Marks)**

(c) Three students A,B,C write an entrance examination, their chances of passing are $1/2, 1/3$ and $1/4$ respectively. Find the probability that (i) atleast one of them passes (ii) atleast two of them passes (iii) all of them passes. **(07Marks)**

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Module - I

1. a) Find the rank of the following matrix by applying elementary row transformations.

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix}$$

Soln: Let $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix}$

$R_1 \leftrightarrow R_2$, $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$, $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \end{bmatrix}$

$R_3 \to 5R_3 - 4R_2$,

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

The above matrix is in the echelon form and has 3 non-zero rows.

$\therefore$ Rank of $A = \rho[A] = 3$.

1. b) Solve the following system of linear equations by Gauss - elimination method

$5x + 10y + z = 28, \quad x + y + z = 6, \quad 4x + 8y + 3z = 29$

Sol 2: The above system of equations can be written in the matrix form $AX=B$

$$\begin{bmatrix} 5 & 10 & 1 \\ 1 & 1 & 1 \\ 4 & 8 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 28 \\ 6 \\ 29 \end{bmatrix}$$

The augmented matrix is

$$[A:B] = \begin{bmatrix} 5 & 10 & 1 & : & 28 \\ 1 & 1 & 1 & : & 6 \\ 4 & 8 & 3 & : & 29 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$, $$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 5 & 10 & 1 & : & 28 \\ 4 & 8 & 3 & : & 29 \end{bmatrix}$$

$R_2 \to R_2 - 5R_1$
$R_3 \to R_3 - 4R_1$, $$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 5 & -4 & : & -2 \\ 0 & 4 & -1 & : & 5 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$, $$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 5 & -4 & : & -2 \\ 0 & 0 & 11 & : & 33 \end{bmatrix}$$

Here matrix A is in upper triangular form.

Now convert the above matrix into equations

$$x+y+z=6 \quad — (1)$$

$$5y-4z=-2 \quad — (2)$$

$$11z=33 \quad — (3)$$

$$\boxed{z=3}$$

From (2), $5y-12=-2 \Rightarrow \boxed{y=2}$

From (1), $x+2+3=6 \Rightarrow \boxed{x=1}$

$\therefore x=1,\ y=2,\ z=3.$

1. c) Find all the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$

Solution: Let $A = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} (-5-\lambda) & 9 \\ -6 & (10-\lambda) \end{vmatrix} = 0$$

$$(-5-\lambda)(10-\lambda) + 54 = 0 \Rightarrow \lambda^2 - 5\lambda + 4 = 0$$

$\therefore$ $\lambda = 1, 4$ are the Eigen values of A

The Eigen vectors are found from the Equation $[A - \lambda I][X] = [0]$

$$\begin{bmatrix} (-5-\lambda) & 9 \\ -6 & (10-\lambda) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} (-5-\lambda)x + 9y &= 0 \\ -6x + (10-\lambda)y &= 0 \end{aligned}\right\} - (*)$$

Case (i): When $\lambda = 1$. we have $-6x + 9y = 0$

$-6x + 9y = 0$

Consider $-6x + 9y = 0 \Rightarrow -6x = -9y$

$\Rightarrow \frac{x}{9} = \frac{y}{6} = k$ $\therefore x = 9k, y = 6k$

where k is arbitrary. If $k = 1$ then $x = 9, y = 6$

$\therefore$ Eigen vector when $\lambda = 1$ is $X_1 = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$

Case (ii): When $\lambda = 4$

we have $-9x + 9y = 0$ and $-6x + 6y = 0$

Consider $-9x = -9y \Rightarrow \frac{x}{9} = \frac{y}{9} = k \Rightarrow x = k, y = k$

If $k = 1$ then $x = 1, y = 1$

$\therefore$ Eigen vector when $\lambda = 4$ is $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2 a) Reduce the matrix into its echelon form and hence find its rank $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$

Solution: Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$

$R_2 \to 3R_2 - 2R_1$
$R_3 \to R_3 - 2R_1$ $\Rightarrow A \sim \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$

$R_3 \to R_3 - 2R_2 \Rightarrow A \sim \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

The above matrix is in echelon form and has two non zero rows

$\therefore$ Rank of $A = \rho[A] = 02$.

2 b) Find all the Eigen values and corresponding Eigen vectors of $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$

Solution: The characteristic Equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} (-1-\lambda) & 1 & 2 \\ 0 & (-2-\lambda) & -1 \\ 0 & 0 & (-3-\lambda) \end{vmatrix} = 0$$

$$(-1-\lambda)[(-2-\lambda)(-3-\lambda) - 0] - 1[0] + 2[0] = 0$$

$$(-1-\lambda)[6 + 2\lambda + 3\lambda + \lambda^2] = 0$$

$$-6 - 5\lambda - \lambda^2 - 6\lambda - 5\lambda^2 - \lambda^3 = 0$$

$$-\lambda^3 - 6\lambda^2 - 11\lambda - 6 = 0$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$\lambda = -1, -2, -3$ are the Eigen values of A

Eigen vectors are found from the equation

$$[A-\lambda I][X]=[0]$$

$$\begin{bmatrix} (-1-\lambda) & 1 & 2 \\ 0 & (-2-\lambda) & -1 \\ 0 & 0 & (-3-\lambda) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} (-1-\lambda)x+y+2z&=0 \\ 0x+(-2-\lambda)y-z&=0 \\ (-3-\lambda)z&=0 \end{aligned}\right\} \quad -\ (*)$$

Case (i): When $\lambda=-1$, we have
$$y+2z=0 \quad -\ (1)$$
$$-y-z=0 \quad -\ (2)$$
$$-2z=0 \quad -\ (3)$$

Here $z=0$, $y=0$ and let $x=k$, arbitrary

$\therefore$ Eigen vector when $\lambda=-1$ is $X_1=\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$

Case (ii):

When $\lambda=-2$, we have

$$x+y+2z=0 \quad -\ (4)$$
$$-z=0 \quad -\ (5)$$
$$-z=0 \quad -\ (6)$$

Here $z=0$, Let $y=k_1$ then

from (4) $\Rightarrow$ $x+k_1+0=0 \Rightarrow x=-k_1$

$\therefore$ Eigen vectors when $\lambda=-2$ is

$$X_2=\begin{bmatrix} -k_1 \\ k_1 \\ 0 \end{bmatrix}.$$

Case (iii): When $\lambda=-3$, we have

$$2x+y+2z=0 \quad -\ (7)$$
$$y-z=0 \quad -\ (8)$$
$$0=0 \quad -\ (9)$$

Let $z = k$, arbitrary

from (8), $y - z = 0 \Rightarrow y = z = k$

from (7), $2x + k + 2k = 0$

$2x = -3k, \; x = -3/2\, k$

$\therefore$ Eigen vector when $\lambda = -3$ is

$$X_3 = \begin{bmatrix} -3/2\, k \\ k \\ k \end{bmatrix}.$$

2. c) Solve the system of linear equations

$3x + y - z = 3$; $2x - 8y + z = -5$; $x - 2y + 9z = 8$

by Gauss elimination method

Solution: Above system of equations can be written in the matrix form $AX = B$

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

$$A \quad X = B.$$

The augmented matrix is

$$[A:B] = \begin{bmatrix} 3 & 1 & -1 & : & 3 \\ 2 & -8 & 1 & : & -5 \\ 1 & -2 & 9 & : & 8 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$, $[A:B] \sim \begin{bmatrix} 1 & -2 & 9 & : & 8 \\ 2 & -8 & 1 & : & -5 \\ 3 & 1 & -1 & : & 3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$, $[A:B] \sim \begin{bmatrix} 1 & -2 & 9 & : & 8 \\ 0 & -4 & -17 & : & -21 \\ 0 & 7 & -28 & : & -21 \end{bmatrix}$

$R_3 \to 4R_3 + 7R_2$,

$$[A:B] \sim \begin{bmatrix} 1 & -2 & 9 & : & 8 \\ 0 & -4 & -17 & : & -21 \\ 0 & 0 & -231 & : & -231 \end{bmatrix}$$

Here matrix A is in upper triangular form

Now equations are

$$x - 2y + 9z = 8 \quad \text{— (1)}$$
$$-4y - 17z = -21 \quad \text{— (2)}$$
$$-231z = -231 \quad \text{— (3)}$$

$$\boxed{z = 1}$$

Equation (2) $\Rightarrow$ $-4y - 17 = -21 \Rightarrow \boxed{y = 1}$

Equation (1) $\Rightarrow$ $x - 2 + 9 = 8 \Rightarrow \boxed{x = 1}$

$\therefore x = 1, \ y = 1, \ z = 1$

Module - 02

3.a) Find a real root of $x\log_{10}x - 1.2 = 0$, correct to 3 decimal places lying in the interval $(2,3)$ by using Regula-Falsi method.

Sol:

Let $f(x) = x\log_{10}x - 1.2$

Here $f(2) = -0.6$ and $f(3) = 0.23 > 0$

$\therefore$ root lies in $(2,3)$ and root is in the neighbourhood of 3. Let us find 'a' and 'b' for applying the method such that $(b-a)$ is small enough.

$f(2.7) = 2.7\log_{10}(2.7) - 1.2 = -0.0353$

$f(2.8) = 2.8\log_{10}(2.8) - 1.2 = 0.052$

$\therefore$ Root lies in $(2.7, 2.8)$

$\therefore a = 2.7,\ b = 2.8$

I iteration: $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

$$x_1 = \frac{2.7f(2.8) - 2.8f(2.7)}{f(2.8) - f(2.7)}$$

$$x_1 = \frac{2.7(0.052) - 2.8(-0.0353)}{0.052 + 0.0353} = 2.7404$$

Now $f(x_1) = f(2.7404) = 2.7404 \cdot \log_{10}(2.7404) - 1.2$

$= -0.00021 < 0$

$\therefore$ Root lies in $(2.7404, 2.8)$

$a = 2.7404 \quad b = 2.8$

II iteration: $x_2 = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \dfrac{2.7404f(2.8) - 2.8f(2.7404)}{f(2.8) - f(2.7404)}$

$x_2 = 2.7406$

$\therefore$ Required root $\boxed{x = 2.7404}$

3. b) Use an appropriate interpolation formula to compute $f(2.18)$ for the following data

x	1.7	1.8	1.9	2	2.1	2.2
$f(x)$	5.474	6.050	6.686	7.389	8.166	9.025

Solution: Here $x = 2.18$ is near the end value 2.2

Hence Newton's backward interpolation formula is appropriate

The backward difference table is as follows

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1.7	5.474					
		0.576				
1.8	6.050		0.06			
		0.636		0.007		
1.9	6.686		0.067		0	
		0.703		0.007		0.001
2	7.389		0.074		0.001	
		0.777		0.008		
2.1	8.166		0.082			
		0.859				
2.2	9.025					

Here $y_n = 9.025$, $\nabla y_n = 0.859$ $\nabla^2 y_n = 0.082$, $\nabla^3 y_n = 0.008$

$\nabla^4 y_n = 0.001$, $\nabla^5 y_n = 0.001$

By Newton's Backward Interpolation formula

$$y = f(x) = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n$$

$$+ \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_n + \frac{r(r+1)(r+2)(r+3)(r+4)}{5!}\nabla^5 y_n$$

Where $r = \frac{x - x_n}{h} = \frac{2.18 - 2.2}{0.1} = -0.2$

$\therefore$

$$y = f(2.18) = 9.025 + (-0.2)(0.859) + \frac{(-0.2)(-0.2+1)}{2}(0.082)$$

$$+ \frac{(-0.2)(-0.2+1)(-0.2+2)}{6}(0.008)$$

$$+ \frac{(-0.2)(-0.2+1)(-0.2+2)(-0.2+3)}{24}(0.001)$$

$$+ \frac{(-0.2)(-0.2+1)(-0.2+2)(-0.2+3)(-0.2+4)}{120}(0.001)$$

$$= 9.025 - 0.1718 - 0.00656 - 0.000384 + 0.0004328$$

$$- 0.0172$$

$$f(2.18) = 8.8466$$

$$\therefore \quad \boxed{f(2.18) = 8.8466}$$

3. c) Use Weddle's rule to evaluate $\int_{-\pi/2}^{\pi/2} \cos x \, dx$ by dividing $[-\pi/2, \pi/2]$ into 6 equal parts

Solution: $a = -\pi/2$, $b = \pi/2$, $n = 6$. Let $y = \cos x$

Length of each part, $h = \frac{b-a}{n} = \frac{\pi/2 + \pi/2}{6} = \frac{\pi}{6} = 30^\circ$

The values of x and $y = \cos x$ are given by

x	$x_0 = -\pi/2 = -90^\circ$	$x_1 = x_0 + h = -60^\circ$	$x_2 = x_1 + h = -30$	$x_3 = x_2 + h = 0$	$x_4 = x_3 + h = 30$	$x_5 = x_4 + h = 60$	$x_6 = x_5 + h = \pi/2 = 90^\circ$
$y = \cos x$	$y_0 = \cos x_0 = 0$	$y_0 = \cos x_1 = 0.5$	$y_2 = \cos x_2 = 0.8660$	$y_3 = \cos x_3 = 1$	$y_4 = \cos x_4 = 0.8660$	$y_5 = \cos x_5 = 0.5$	$y_6 = \cos x_6 = 0$

Weddle's rule is given by

$$\int_a^b y \, dx = \frac{3h}{10} \sum_{i=0}^{n} k y_i$$

$$\therefore \int_{-\pi/2}^{\pi/2} \cos x \, dx = \frac{3h}{10} \sum_{i=0}^{6} k y_i$$

$$= \frac{3h}{10} \left[k y_0 + k y_1 + k y_2 + k y_3 + k y_4 + k y_5 + k y_6 \right]$$

$$= \frac{3h}{10} \left[y_0 + 5 y_1 + y_2 + 6 y_3 + y_4 + 5 y_5 + y_6 \right]$$

$$= \frac{3(\pi/6)}{10} \left[0 + 5(0.5) + 0.8660 + 6 + 0.8660 + 5(0.5) + 0 \right]$$

$$= 12.0011$$

4. a) Find a real root of the equation $x^3+x^2+3x+4=0$ near $x=-1$ correct to four decimal places, by using Newton - Raphson method.

<u>Solution:</u> Let $f(x)=x^3+x^2+3x+4$.

We have to find the root near $x=-1$.

$\therefore$ Let $x_0=-1$. Also $f'(x)=3x^2+2x+3$

By Newton - Raphson formula.

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

<u>I-iteration:</u>

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=-1-\frac{f(-1)}{f'(-1)}=-1-\frac{1}{3-2+3}=-1.25$$

<u>II- iteration:</u>

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=-1.25-\frac{f(-1.25)}{f'(-1.25)}$$

$$=-1.25-\frac{[(-1.25)^3+(-1.25)^2+3(-1.25)+4]}{[3(-1.25]^2+2(-1.25)+3]}$$

$x_2=-1.2229$

<u>III - iteration.</u>

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}=-1.2229-\frac{f(-1.2229)}{f'(-1.2229)}$$

$$=-1.2229-\frac{[(-1.2229)^3+(-1.2229)^2+3(-1.2229)+4]}{[3(-1.2229)^2+2(-1.2229)+3]}$$

$x_3=-1.2229$

$\therefore$ Required root is $\boxed{x=-1.2229}$

4. b) Use an appropriate interpolation formula to compute $f(42)$ for the following data

x	40	50	60	70	80	90
$f(x)$	184	204	226	250	276	304

Here we shall find $f(42)$ using Newton's forward interpolation formula.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$x_0=40$	$y_0=184$			
		$\Delta y_0=20$		
$x_1=50$	$y_1=204$		$\Delta^2 y_0=2$	
		$\Delta y_1=22$		$\Delta^3 y_0=0$
$x_2=60$	$y_2=226$		$\Delta^2 y_1=2$	
		$\Delta y_2=24$		$\Delta^3 y_1=0$
$x_3=70$	$y_3=250$		$\Delta^2 y_2=2$	
		$\Delta y_3=26$		$\Delta^3 y_2=0$
$x_4=80$	$y_4=276$		$\Delta^2 y_3=2$	
		$\Delta y_4=28$		
$x_5=90$	$y_5=304$			

We have Newton's forward interpolation formula

$$y=f(x)=y_0+r\Delta y_0+\frac{r(r-1)}{2!}\Delta^2 y_0+\cdots$$

Where $r=\frac{x-x_0}{h}=\frac{42-40}{10}=0.2$

$$y=f(42)=184+(0.2)(20)+\frac{(0.2)(0.2-1)\,2}{2}$$

$$=187.84$$

$\therefore \boxed{f(42)=187.84}$

4.c) Using Simpson's $\frac{1}{3}$rd rule, evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ by taking 6 equidistant ordinates

Solution: Here $a=0,\ b=3,\ n=6$.

$\therefore\ h = \frac{b-a}{n} = \frac{3-0}{6} = 0.5$. Let $y = \frac{1}{(1+x)^2}$

The values of x and y are as follows

x	$x_0=0$	$x_1=x_0+h$ $=0.5$	$x_2=x_1+h$ $=1$	$x_3=x_2+h$ $=1.5$
$y=\frac{1}{(1+x)^2}$	$y_0=\frac{1}{(1+x_0)^2}$ $=1$	$y_1=\frac{1}{(1+x_1)^2}$ $=4/9$	$y_2=\frac{1}{(1+x_2)^2}$ $=\frac{1}{4}$	$y_3=\frac{1}{(1+x_3)^2}$ $=\frac{4}{25}$

$x_4=x_3+h$ $=2$	$x_5=x_4+h$ $=2.5$	$x_6=x_5+h=3$
$y_4=\frac{1}{(1+x_4)^2}$ $=1/9$	$y_5=\frac{1}{(1+x_5)^2}$ $=4/49$	$y_6=\frac{1}{(1+x_6)^2}$ $=1/16$

By Simpson's $1/3$rd rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots)+2(y_2+y_4+\cdots)\right]$$

$$\therefore \int_0^3 \frac{dx}{(1+x)^2} = \frac{0.5}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{0.5}{3}\left[(1+1/16)+4\left(4/9+4/25+\frac{4}{49}\right)+2\left(1/4+1/9\right)\right]$$

$$= \frac{0.5}{3}\left[1.0625+0.6860*4+0.7222\right]$$

$$= 0.7547$$

$$\boxed{\therefore \int_0^3 \frac{dx}{(1+x)^2} = 0.7547}$$

Module-03

5. a) Solve $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} - 2y = 0$

Solution: We have $(D^3 - 3D - 2)y = 0$

A.E. is $m^3 - 3m - 2 = 0$.

$m = -1$ is the first root by inspection

$$m=-1 \left| \begin{array}{cccc} 1 & 0 & -3 & -2 \\ 0 & -1 & 1 & 2 \\ \hline 1 & -1 & -2 & 0 \end{array} \right.$$ by synthetic division.

$\therefore$ Equation is $m^2 - m - 2 = 0 \Rightarrow m = -1, -2$

$\therefore y = (c_1 + c_2 x)e^{-x} + c_3 e^{-2x}$

5. b) Solve $(D^2 - 7D + 10)y = (1+e^x)^2$

A.E. is $(m^2 - 7m + 10) = 0 \quad \therefore m = 2, 5$

$\therefore$ C.F. $y_c = c_1 e^{2x} + c_2 e^{5x}$

P.I. (y_p): Now $(D^2 - 7D + 10)y = (1+e^x)^2 = 1 + 2e^x + e^{2x}$

$$\therefore y_p = \frac{1}{D^2 - 7D + 10} + 2\frac{e^x}{D^2 - 7D + 10} + \frac{e^{2x}}{D^2 - 7D + 10}$$

Type(I), $a=0$ Replace D by $a=0$ | Type(I), $a=1$ Replace D by 1 | Type(I), $a=2$ Replace D by 2 But denominator=0

$$= \frac{1}{10} + \frac{2e^x}{(1-7+10)} + \frac{xe^{2x}}{2D-7}$$

$$= \frac{1}{10} + \frac{2e^x}{4} + \frac{xe^{2x}}{2(2)-7}$$

$$y_p = \frac{1}{10} + \frac{e^x}{2} - \frac{xe^{2x}}{3}$$

$\therefore$ G.S. is $y = y_c + y_p$

$$y = c_1 e^{2x} + c_2 e^{5x} + \frac{1}{10} + \frac{e^x}{2} - \frac{xe^{2x}}{3}.$$

5. c) Solve $(D^2+2D+3)y = \sin x$

Solution $y = y_c + y_p = C.F + P.I$

Solution: y_c (C.F): Now $(D^2+2D+3)y=0$

A.E. is $m^2+2m+3=0$

$m = -1 \pm 2i$

$\therefore y_c = e^{x}(c_1\cos 2x + c_2 \sin 2x)$

y_p (P.I): Let $(D^2+2D+3)y = \sin x$

$y = \dfrac{\sin x}{D^2+2D+3}$, Type ② where $a=1$ Replace D^2 by $-a^2=-1$

$\therefore y_p = \dfrac{\sin x}{-1+2D+3}$

$= \dfrac{\sin x}{2D+2} \times \dfrac{(2D-2)}{(2D-2)}$

$= \dfrac{(2D-2)\sin x}{4D^2-4}$, Again replace D^2 by $-a^2=-1$

$= \dfrac{(2D-2)\sin x}{4(-1)-4}$

$= \dfrac{2D\sin x - 2\sin x}{-8}$

$y_p = \dfrac{2\cos x - 2\sin x}{-8} = \dfrac{1}{4}\left[\sin x - \cos x\right]$

$\therefore$ G.S. $y = y_c + y_p$

$y = e^{x}(c_1\cos 2x + c_2\sin 2x) + \frac{1}{4}[\sin x - \cos x]$

6.a) Solve $(D^3 - 2D^2 + 4D - 8)y = 0$

Solution: Auxilary Equation is $m^3 - 2m^2 + 4m - 8 = 0$

$$m^2(m-2) + 4(m-2) = 0$$
$$(m-2)(m^2+4) = 0$$
$$m = 2, \quad m^2 = -4$$
$$m = 2, \quad m = \pm 2i$$

$\therefore \boxed{y = C_1 e^{2x} + C_2 \cos 2x + C_3 \sin 2x}$ is the g.s.

6.b) Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$.

Solution: G.S. $y = y_c + y_p = C.F + P.I$

C.F. (y_c): Let $\frac{d^2y}{dx^2} - 4y = 0 \Rightarrow (D^2-4)y = 0$

A.E. is $m^2 - 4 = 0 \Rightarrow m = 2, -2$

$$\therefore y_c = C_1 e^{2x} + C_2 e^{-2x}$$

P.I. (y_p): Now $(D^2-4)y = \cosh(2x-1) + 3^x$

$$= \frac{1}{2}\left[e^{2x-1} + e^{-(2x-1)}\right] + 3^x$$

$$y_p = \frac{1}{2}\left[\frac{e^{2x-1}}{D^2-4} + \frac{e^{-(2x-1)}}{D^2-4}\right] + \frac{3^x}{D^2-4}$$

$$y_p = \frac{1}{2}\left[y_{p_1} + y_{p_2}\right] + y_{p_3} \quad \text{——} \ (1)$$

Now $y_{p_1} = \frac{e^{2x-1}}{D^2-4}$, Type (1) $a=2$
Replace D by $a=2$.
But $Dr = 0$

$$= \frac{x e^{2x-1}}{2D}$$

$y_{p_1} = \frac{x e^{2x-1}}{4}$, Again replace D by 2

$y_{p_2} = \frac{e^{-(2x-1)}}{D^2-4}$, Type(1), $a=-2$
Replace D by $a=-2$
But $Dr=0$

$y_{p_2} = x\,\frac{e^{-(2x-1)}}{2D} = \frac{x\,e^{-(2x-1)}}{-4}$, Replace D by $a=-2$

and
$y_{p_3} = \frac{3^x}{D^2-4} = \frac{e^{\log 3^x}}{D^2-4} = \frac{e^{(\log 3)x}}{D^2-4}$, Type(1) $a=\log 3$
Replace D by $\log 3$

$y_{p_3} = \frac{e^{(\log 3)x}}{(\log 3)^2-4}$

$y_{p_3} = \frac{3^x}{(\log 3)^2-4}$

$\therefore$ Equation (1) is $y_p = \frac{1}{2}(y_{p_1}+y_{p_2}) + y_{p_3}$

$y_p = \frac{x}{8}e^{2x-1} - \frac{x}{8}e^{-(2x-1)} + \frac{3^x}{(\log x)^2-4}$

$\therefore$ G.S. $y = y_c + y_p$

$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{8}e^{2x-1} - \frac{x}{8}e^{-(2x-1)} + \frac{3^x}{(\log x)^2-4}$

6 c) Solve $(D^2+a^2)y = \cos ax$

Solution: G.S. $y = y_c + y_p$

y_c: Let $(D^2+a^2)y=0$
A.E. is $m^2+a^2=0 \Rightarrow m = \pm ai$

$y_c = c_1\cos ax + c_2 \sin ax$

P.I. (y_p): Let $(D^2+a^2)y = \cos ax$

Type ② Replace D^2 by $-a^2$

But denominator is zero

$\therefore$

$$\therefore y_p = \frac{x \cos ax}{2D} = \frac{x}{2}\int \cos ax \, dx$$

$$y_p = \frac{x}{2a} \sin ax$$

$\therefore$ G.S. $y = y_c + y_p = C_1 \cos ax + C_2 \sin ax + \frac{x}{2a} \sin ax.$

Module - 04

7.a) Form the partial differential equation by eliminating the arbitrary constants from

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Solution: Let $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ — ①

differentiate ① partially w.r.t x and y

$$2\frac{\partial z}{\partial x} = \frac{2x}{a^2} + 0 \Rightarrow \frac{\partial z}{\partial x} = \frac{x}{a^2} \quad — ②$$

$$2\frac{\partial z}{\partial y} = 0 + \frac{2y}{b^2} \Rightarrow \frac{\partial z}{\partial y} = y/b^2 \quad — ③$$

Use ② and ③ in Equation ①

$$2z = x\left(\frac{x}{a^2}\right) + y\left(\frac{y}{b^2}\right)$$

$$2z = x\left(\frac{\partial z}{\partial x}\right) + y\left(\frac{\partial z}{\partial y}\right)$$ is the required

partial differential equation.

7. b) Form the partial differential Equation by eliminating the arbitrary function from
$$z = f(y+2x) + g(y-3x)$$

Solution: Let $z = f(y+2x) + g(y-3x)$ — ①

differentiate Eq² ① partially w.r.t x and y

$$\frac{\partial z}{\partial x} = p = f'(y+2x)\cdot 2 - 3g'(y-3x)$$

$$\frac{\partial z}{\partial y} = q = f'(y+2x) + g'(y-3x)$$

Also $\dfrac{\partial^2 z}{\partial x^2} = 4f''(y+2x) + 9g''(y-3x) = r$ — ②

$$\frac{\partial^2 z}{\partial y^2} = t = f''(y+2x) + g''(y-3x) \quad \text{— ③}$$

$$\frac{\partial^2 z}{\partial x \partial y} = s = 2f''(y+2x) - 3g''(y-3x) \quad \text{— ④}$$

Now consider Eq² ② + 3 × Eq² ④

$$r + 3s = 10f''(y+2x) \quad \text{— ⑤}$$

Again now consider Eq² ④ + 3 × Eq² ③

$$s + 3t = 5f''(y+2x) \quad \text{— ⑥}$$

Now dividing ⑤ by ⑥

$$\frac{r+3s}{s+3t} = \frac{10}{5} = 2$$

$$r + 3s = 2s + 6t$$

$\boxed{r + s - 6t = 0}$ is the required partial differential Equation.

7. c) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\pi/2$.

Solution: Here we first find z by integration and apply the given conditions to determine the arbitrary functions occuring as constant of integration.

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \sin x \cdot \sin y$$

Integrating w.r.t. x treating y as constant

$$\frac{\partial z}{\partial y} = \int \sin x \sin y \, dx + f(y)$$

$$= -\cos x \cdot \sin y + f(y) \quad \text{—— (1)}$$

Integrating w.r.t. y treating x as constant

$$z = \int -\cos x \sin y \, dy + \int f(y)\,dy + g(x)$$

$$z = \cos x \cos y + \int f(y)\,dy + g(x) \quad \text{—— (2)}$$

Also, by data, $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$

Equation (1) $\Rightarrow$ $-2\sin y = -\sin y + f(y)$

$$f(y) = -\sin y$$

Also $\int f(y)\,dy = \int -\sin y \, dy = \cos y$

$\therefore$ Equation (2) is $z = \cos x \cos y + \cos y + g(x)$ —— (3)

Using the condition $z=0$ if $y = (2n+1)\pi/2$ in (3)

we have, $0 = \cos x \cdot \cos(2n+1)\frac{\pi}{2} + \cos(2n+1)\frac{\pi}{2} + g(x)$

$0 = 0 + 0 + g(x) \Rightarrow g(x) = 0$

Thus the solution of the PDE is

$$\boxed{z = \cos x \cos y + \cos y = \cos y(\cos x + 1)}$$

8. a) Form the partial differential equation by eliminating the arbitrary function from $f(xy+z^2, x+y+z)=0$

Solution: The above function is in the form $f(u,v)=0$

where $u = xy+z^2$, $v = x+y+z$

$$\frac{\partial u}{\partial x} = y + 2z\frac{\partial z}{\partial x}, \quad \frac{\partial v}{\partial x} = 1 + \frac{\partial z}{\partial x}$$

$$\frac{\partial u}{\partial y} = x + 2z\frac{\partial z}{\partial y}, \quad \frac{\partial v}{\partial y} = 1 + \frac{\partial z}{\partial y}$$

Now $f(u,v)=0$

differentiating w.r.t x and y by applying chain rule

$$\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = 0 \Rightarrow \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = -\frac{\partial f}{\partial v}\frac{\partial v}{\partial x} \quad \text{--- (1)}$$

$$\frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} = -\frac{\partial f}{\partial v}\frac{\partial v}{\partial y} \quad \text{--- (2)}$$

Dividing (1) by (2)

$$\frac{\partial u/\partial x}{\partial u/\partial y} = \frac{\partial v/\partial x}{\partial v/\partial y}$$

$$\frac{y+2z\frac{\partial z}{\partial x}}{x+2z\frac{\partial z}{\partial y}} = \frac{1+\frac{\partial z}{\partial x}}{1+\partial z/\partial y}$$

$$\left(1+\frac{\partial z}{\partial y}\right)\left(y+2z\frac{\partial z}{\partial x}\right) = \left(1+\frac{\partial z}{\partial x}\right)\left(x+2z\frac{\partial z}{\partial y}\right)$$

$$x + 2z\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}z$$

$$= y + 2z\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}y + 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}z$$

Thus $\frac{\partial z}{\partial x}(x-2z) - \frac{\partial z}{\partial y}(y-2z) + (x-y) = 0$

(OR) $\boxed{p(x-2z) - q(y-2z) + (x-y) = 0}$

is the required P.D.E.

8. b) Form the partial differential equation by eliminating the arbitrary functions from

$$z = x f_1(x+y) + f_2(x+y). \quad \text{--- (1)}$$

Solution: Differentiating partially w.r.t x and y

$$\frac{\partial z}{\partial x} = x f_1'(x+y) + f_1(x+y) + f_2'(x+y)$$

$$\frac{\partial z}{\partial y} = x f_1'(x+y) + f_2'(x+y)$$

$$\frac{\partial^2 z}{\partial x^2} = x f_1''(x+y) + 2 f_1'(x+y) + f_2''(x+y) \quad \text{--- (2)}$$

$$\frac{\partial^2 z}{\partial x \partial y} = x f_1''(x+y) + f_1'(x+y) + f_2''(x+y) \quad \text{--- (3)}$$

$$\frac{\partial^2 z}{\partial y^2} = x f_1''(x+y) + f_2''(x+y) \quad \text{--- (4)}$$

Now using the RHS of (4) in (2) as well as in (3),

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} + 2 f_1'(x+y) \quad \text{--- (5)}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y^2} + f_1'(x+y) \quad \text{--- (6)}$$

Multiplying (6) by 2 and subtracting from (5),

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = -\frac{\partial^2 z}{\partial y^2}$$

Thus $$\boxed{\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0}$$

is the required P.D.E.

8. c) Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y}$ for which $\frac{\partial z}{\partial x} = \log_e x$ when $y=1$ and $z=0$ when $x=1$.

Solution: Since the condition is in terms of $\frac{\partial z}{\partial x}$, we shall use the fact that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

and write the given PDE as $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{x}{y}$

Integrating wrt y treating x as constant

$$\frac{\partial z}{\partial x} = x \int \frac{1}{y}\, dy + f(x) = x \log y + f(x) \quad \text{— (1)}$$

Integrating w.r.t x treating y as constant

$$z = \log y \int x\, dx + \int f(x)\, dx + g(y)$$

$$z = \frac{x^2}{2} \log y + \int f(x)\, dx + g(y) \quad \text{— (2)}$$

By data, $\frac{\partial z}{\partial x} = \log_e x$ when $y=1$, Using this in (1)

$$\log x = x \log 1 + f(x) \quad \text{or} \quad f(x) = \log x$$

Also $\int f(x)\, dx = \int \log x\, dx = x \log x - x$

Equation (2) is,

$$z = \frac{x^2}{2} \log y + x \log x - x + g(y) \quad \text{— (3)}$$

Also by data, $z=0$ when $x=1$, Using this in (3)

$$0 = \frac{1}{2} \log y - 1 + g(y) \quad \text{or} \quad g(y) = 1 - \tfrac{1}{2} \log y$$

Thus the solution is given by

$$\boxed{z = \frac{x^2}{2} \log y + x \log x - x + 1 - \log \sqrt{y}}$$

Module - 05

9. a) State the axiomatic definition of probability. For any two arbitrary events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution: Axiomatic definition of probability:

If S is the sample space and E is the set of all events then to each event A in E we associate a unique real number $P = P(A)$ known as the probability of the event A, if the following axioms are satisfied

(1) $P(S) = 1$

(2) For every event A in E, $0 \leq P(A) \leq 1$

(3) If $A_1, A_2, \ldots A_n$ are mutually exclusive events then

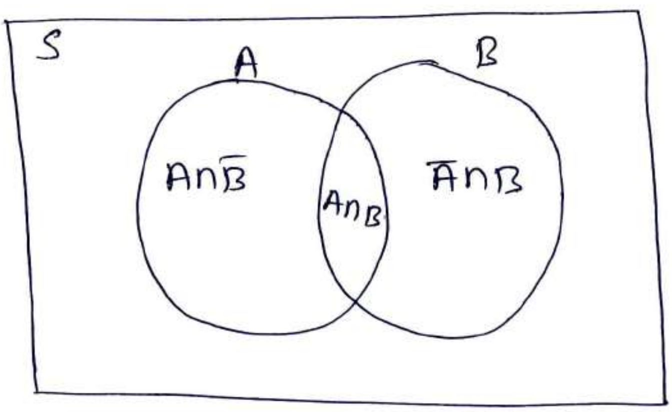
$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

Addition Theorem:

If A and B are any two events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof: We prove the result using the following Venn diagram.



From the figure

$$A = (A \cap \bar{B}) \cup (A \cap B)$$

$$B = (\bar{A} \cap B) \cup (A \cap B)$$

$P(A) = P(A \cap \bar{B}) + P(A \cap B)$, since $A \cap \bar{B}$ and $A \cap B$ are disjoint.

$P(B) = P(\bar{A} \cap B) + P(A \cap B)$, since $\bar{A} \cap B$ and $A \cap B$ are disjoint.

Now $P(A) + P(B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B) + P(A \cap B)$

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

Thus $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

9. b) The probability that a team wins a match is $3/5$. If this team play 3 matches in a tournament, what is the probability that the team (i) win and (ii) loose, all the matches.

Solution: Let W be the event of winning a match by the team.

$$P(W_1) = P(W_2) = P(W_3) = 3/5$$

$$\therefore P(\overline{W_1}) = P(\overline{W_2}) = P(\overline{W_3}) = 1 - 3/5 = 2/5$$

(i) Probability of winning all the matches

$$= P(W_1) \cdot P(W_2) \cdot P(W_3) = \frac{27}{125}.$$

(ii) Probability of losing all the matches

$$= P(\overline{W_1}) \cdot P(\overline{W_2}) \cdot P(\overline{W_3}) = \frac{8}{125}.$$

9. c) In an UG class of a reputed engineering college, 70% are boys and 30% are girls. 5% of boys and 3% of the girls are irregular to the classes. What is the probability of a student selected at random is irregular to the classes and what is the probability that the irregular student is a girl?

Solution: Probability of selecting a Boy $= P(B) = 70\% = 0.7$

Probability of selecting a girl $= P(G) = 30\% = 0.3$

Let I be the event of selecting an irregular student

$\therefore P(I/B) = 5\% = 0.05$, $P(I/G) = 3\% = 0.03$

Hence, $P(I) = P(B) \cdot P(I/B) + P(G) \cdot P(I/G)$

$$= (0.7)(0.05) + (0.3)(0.03)$$

$$= 0.044$$

probability of selecting an irregular student is 0.044.

Next we have to find $P(G/I)$ and by Baye's theorem

$$P(G/I) = \frac{P(G)\,P(I/G)}{P(I)} = \frac{(0.3)(0.03)}{0.044}$$

$$= 0.2045$$

$\therefore$ probability that the irregular student is a girl is 0.2045.

10. a) State and prove Baye's theorem

Statement:

Let $A_1, A_2, A_3, \cdots A_n$ be a set of exhaustive and mutually exclusive events of the sample space S with $P(A_i) \neq 0$ for each i. If A is any other event associated with A_i,

$A \subset \bigcup_{i=1}^{n} A_i$ with $P(A) \neq 0$ then

$$P(A_i/A) = \frac{P(A_i) \cdot P(A/A_i)}{\sum_{i=1}^{n} P(A_i) P(A/A_i)}$$

Proof: We have

$$S = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n \quad \text{and} \quad A \subset S$$

$$\therefore \quad A = S \cap A = (A_1 \cup A_2 \cup A_3 \cup \cdots A_n) \cap A$$

Using distributive law in the RHS

$$A = (A_1 \cap A) \cup (A_2 \cap A) \cup (A_3 \cap A) \cup \cdots \cup (A_n \cap A)$$

Since $A_i \cap A$ for $i = 1$ to n are mutually exclusive, we have by applying the addition rule of probability

$$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + \cdots + P(A_n \cap A)$$

Now applying multiplication rule onto each term in the RHS we have

$$P(A) = P(A_1) \cdot P(A/A_1) + P(A_2) P(A/A_2) + \cdots + P(A_n) \cdot P(A/A_n)$$

That is
$$P(A) = \sum_{i=1}^{n} P(A_i) P(A/A_i) \quad \text{—— (1)}$$

The conditional probability of A_i for any i given A is defined by

$$P(A_i/A) = \frac{P(A_i \cap A)}{P(A)} = \frac{P(A_i)\,P(A/A_i)}{P(A)}$$

Using ① in the denominator of RHS

$$\boxed{P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)}}$$

10. b) The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnose is 40% and the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was correctly diagnosed ?

Solution: Let A be the event of correct diagnosis and B be the event of wrong diagnosis

$\therefore P(A) = 0.6$ and $P(B) = 0.4$

Let E be the event that the patient dies.

$\therefore P(E/A) = 0.4$ and $P(E/B) = 0.7$

We have to find $P(A/E)$ and By Baye's theorem

$$P(A/E) = \frac{P(A)\cdot P(E/A)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B)}$$

$$= \frac{(0.6)(0.4)}{(0.6)(0.4) + (0.4)(0.7)} = 0.4615$$

$$\boxed{P(A/E) = 0.4615}$$

10. c) Three students A, B, C write an entrance examination, their chances of passing are $1/2$, $1/3$ and $1/4$ respectively. Find the probability that (i) at least one of them passes (ii) atleast two of them passes (iii) all of them passes.

<u>Solution:</u> Let E be the event of passing the examination by a student.

$\therefore P(A) = 1/2, \quad P(B) = 1/3, \quad P(C) = 1/4$

$P(\bar{A}) = 1/2, \quad P(\bar{B}) = 2/3, \quad P(\bar{C}) = 3/4.$

(i) probability of at least one of them passes

$= 1 -$ probability of none of them passing

$= 1 - 1/2 \cdot 2/3 \cdot 3/4 = 3/4.$

(ii) probability of at least 2 of them passing

$= P(A) \cdot P(B) \cdot P(\bar{C}) + P(B)\, P(C) \cdot P(\bar{A}) + P(C)\, P(A)\, P(\bar{B})$

$+ P(A)\, P(B)\, P(C)$

$= \frac{3}{24} + \frac{1}{24} + \frac{2}{24} + \frac{1}{24} = 7/24$

(iii) Probability of all of them passing

$= P(A)\, P(B)\, P(C)$

$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{24}.$