First/Second Semester B.E. Degree Examination, June/July 2019
Engineering Physics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define SHM and mention any two examples. Derive the differential equation for SHM using Hooke's law. (07 Marks)

b. With a neat diagram, explain the construction and working of Reddy's tube. Mention any three applications of shock waves. (09 Marks)

c. For a particle executing SHM, its acceleration is found to be $15 cm/s^2$ when it is at 3cm from its mean position. Calculate time period. (04 Marks)

OR

2 a. Explain the basics of conservation of mass, momentum and energy. (06 Marks)

b. What are forced oscillations? Derive the expressions for steady state amplitude and phase angle in case of forced oscillations. (10 Marks)

c. A 20g oscillator with natural angular frequency 10 $rads^{-1}$ is vibrating in damping medium. The damping force is proportional to the velocity of the vibrator. Calculate the value of damping required for the oscillations to be critically damped. (Given damping coefficient is 0.17). (04 Marks)

Module-2

3 a. State and explain Hooke's law. Explain the nature of elasticity with the help of stress-strain diagram. (08 Marks)

b. Define bending moment. Derive the expression for bending moment interms of moment of inertia. (08 Marks)

c. Calculate the torque required to twist a wire of length 1.5m, radius 0.0425×10^{-2}m, through an angle $\left(\frac{\pi}{45}\right)$ radian, if the value of rigidity modulus of its material is $8.3 \times 10^{10} N/m^2$. (04 Marks)

OR

4 a. Define Poisson's ratio. Obtain the relation between y, n and σ where the symbols have their usual meaning. (08 Marks)

b. What are Torsional Oscillations? Mention any two applications of Torsional Pendulum. Derive the expression for couple per unit twist of a solid cylinder. (08 Marks)

c. Calculate the force required to produce an extension of 1mm in steel wire of length 2m and diameter 1mm (Young's modulus for steel $Y = 2 \times 10^{11} N/m^2$). (04 Marks)

Module-3

5 a. State and prove Gauss Divergence Theorem. (08 Marks)

b. Define fractional Index change (Δ). Derive the expression for Numerical aperature and acceptance angle of an optical fiber. (08 Marks)

c. A circular coil of radius 10cm having 50 turns carries a current of 5A. Determine the magnetic field produced by the coil at a distance of 3cm from the centre. Also determine magnetic field produced by the coil at its centre. **(04 Marks)**

OR

6 a. Derive wave equation in terms of electric field using Maxwell's equations for free space. **(08 Marks)**

b. Describe different types of optical fibers with neat diagrams. Mention any two mechanisms involved in fiber loss. **(08 Marks)**

c. Calculate the V-number for a fiber of core-diameter 40μm and with refractive indices of 1.55 and 1.5 respectively for core and cladding. When the wavelength of the propagating wave is 1400nm. Also calculate the number of modes that the fiber can support for propagation. Assume that the fiber is in air. **(04 Marks)**

Module-4

7 a. Starting from Schrodinger's time independent wave equation, derive the expression for energy eigen value and eigen function for an electron in one dimensional potential well of infinite height. **(10 Marks)**

b. Explain the construction and working of CO_2 LASER with the help of energy level diagram. **(06 Marks)**

c. The average output power of laser source emitting a laser beam of wavelength 632.8nm. Find the number of photons emitted per second by the laser source. **(04 Marks)**

OR

8 a. Define the terms population inversion and meta-stable state. Derive the expression for energy density of radiation at equilibrium interms of Einstein's coefficients. **(10 Marks)**

b. Using Heisenberg's uncertainty principle, show that electrons do not reside inside the nucleus. **(06 Marks)**

c. An electron is bound in an 1-D potential well of infinite height and of width 1 A. Calculate its energy values in the ground state and also in the first two excited states. **(04 Marks)**

Module-5

9 a. Define Fermi energy. Explain the variation of Fermi factor with temperature. **(08 Marks)**

b. What is Hall effect? Obtain the expression for Hall coefficient, and express Hall voltage interms of Hall coefficient. **(08 Marks)**

c. The dielectric constant of sulphur is 3.4. Assuming a cubic lattice for its structure, calculate the electronic polarizability of sulphur (given, density of sulphur = 2.07 g/cc and atomic weight = 32.07). **(04 Marks)**

OR

10 a. Mention the assumptions of Quantum free electron theory. Discuss two success of quantum free electron theory. **(08 Marks)**

b. Define the term internal field in case of solid dielectrics with one-dimensional equation. Explain polar and non-polar dielectrics with examples. **(08 Marks)**

c. The intrinsic charge carrier concentration of germanium is $2.4 \times 10^{19}/m^3$, calculate its resistivity if mobility of electrons and holes respectively are $0.39 m^2/vs$ and $0.19 m^2/vs$. **(04 Marks)**

KLS VDIT, HALIYAL (1)

ENGINEERING PHYSICS 18PHY12/22

I/II Semester B.E. June/July-2019

Question Paper Solutions:

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Module-I

1 a. Define SHM and mention any two examples. Derive the differential equation for SHM using Hooke's law.

Simple Harmonic Motion (SHM):

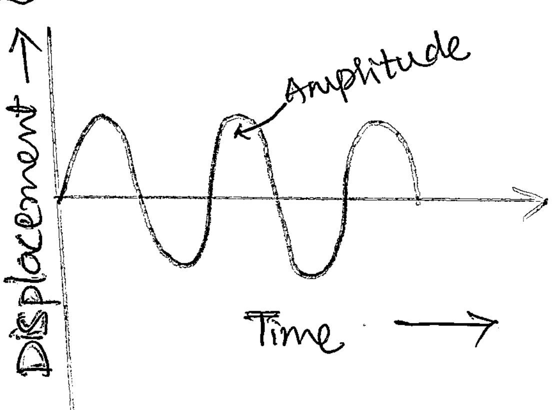
Defn/: A motion in which the displacement varies simultaneously with time is known as simple harmonic motion.

SHM is the oscillatory motion of a body where the restoring force is propootional to the negative of the displacement.

Examples of SHM:

1. simple pendulum.
2. vibrations of mass held by a two stretched springs.
3. vibrations of a stretched string.

Differential Equation of motion for SHM:



Let a body be initiated to an oscillatory motion after being displaced from its equilibrium position & left free.

For such oscillations, the lonely force acting on the body will be the restoring force F.

Wl.K.t, for a vibrating body,

$$F = -Kx$$

where $x \rightarrow$ is displacement and

K is the force constant.

If 'm' is the mass of the body, then, as per Newton's second law of motion. $F = ma$

$$F = m\frac{d^2x}{dt^2}$$

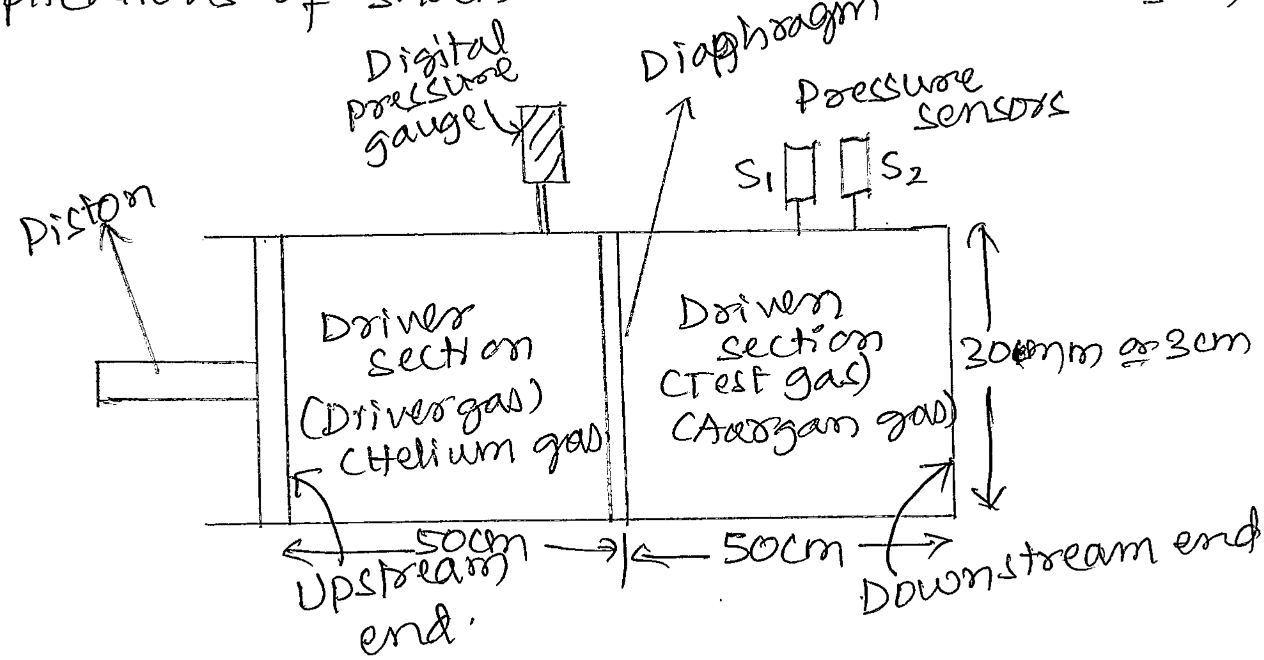
$$m \cdot \frac{d^2x}{dt^2} = -Kx \quad \text{—— (1)}$$

$$\frac{d^2x}{dt^2} = -\frac{K}{m}x \quad \text{—— (2)}$$

$$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0 \quad \text{—— (3)}$$

The above equation represents the equation of motion for a body executing free vibration.

1 b. With a neat diagram, explain the construction and working of Reddy's shock tube. Mention any three applications of shock waves. (09 M).



Construction:

1. Reddy shock tube (RST) consists of a steel tube of length 100 cm.

2. A diaphragm of thickness 0.1cm divides the tube into two compartments of length 50cm fitted with piston called Driver section filled with driver gas (Helium). The other compartment of length 50 cm is called Driven section filled with driver gas (Aargan gas).

3. Sensor S fitted to driver section measures the rupture pressure P_2, temperature T_2.

4. Two sensors S_1 & S_2 separated by a distance Δx fitted to driven section measures the pressure P_4, P_5 & temp T_4 & T_5 resp.

Working:

1. Driver section is filled with a gas at high pressure(P_2) and driven section is filled with gas of low pressure (P_1).

~~2. Diaphragm is~~

2. Diaphragm is ruptured to produce shock waves by pushing the piston and the rupture pressure P_2 & temperature is measured using sensor S.
3. The time "t" taken by the shock wave to travel the distance "x" is measured using sensors S_1, S_2 & CRO. The speed of the shock waves is calculated using $V = \frac{x}{t}$.
4. Then, if "a" is the speed of sound at laboratory temperature, the Mach number of shock waves is calculated using

$$\boxed{M = \frac{V}{a}}$$

5. The Mach number increases with increase of the thickness of the diaphragm.

1 C. For a particle executing SHM, its acceleration is found to be $15 cm/s^2$ when it is at 3cm from its mean position. Calculate time period. 4M

Given data:

acceleration, $a = 15 cm/s^2$

distance, ~~x~~ $y = 3cm$

In SHM

$$a = \omega^2 y$$

$$\omega^2 = \frac{a}{y} = \frac{15}{3} = 5 \ sec^{-2}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

2 a. Explain the basis of conservation of mass, momentum and energy. 6M

Conservation means the maintenance of certain quantities unchanged during physical processes. Conservation laws apply to closed system. A closed system is the one that does not exchange any matter with the outside and is not acted on by outside forces.

1. Law of conservation of mass:

"The total mass of any isolated system remains unchanged and is independent of any chemical and physical changes that could occur within the system".

It is expressed as,

$$\rho_1 u_1 = \rho_2 u_2$$

2. Law of conservation of momentum:

"In a closed system, the total momentum remains a constant".

It can also be stated as 'when two objects before collision equals to the total momentum of the two objects after the collision".

It is expressed as.

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

3. Law of conservation of energy:

"The total energy of a closed system remains constant and is independent of any changes occuring within the system".

It is expressed as,

$$\boxed{h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}}$$

2 b. ~~Explain~~ What are forced oscillation? Derive the expression for steady state amplitude and phase angle in case of forced oscillations.

Definition:

It is a steady state of sustained vibrations of a body vibrating in a resistive medium under the action of an external periodic force which acts independently of the restoring force.

Theory of forced vibrations:

Consider a body of mass "m" executing vibrations in a damping medium acted upon by an external periodic force $F.\sin(pt)$
where $p \rightarrow$ is the angular frequency of the external force.

If "x" is the displacement of the body at any instant of time "t", then, The damping force which acts in a direction opposite to the movement of the body is equated to the term $F = -r\left(\frac{dx}{dt}\right)$ —① where $r \rightarrow$ is damping constant, and the restoring force is equated to the term.

$$\boxed{F = -Kx} \quad —②$$

where K is the force constant.

The net force acting on the body is resultant of all the three forces.

$\therefore$ Resultant force $= -r.\frac{dx}{dt} - Kx + F.\sin(pt)$ —③

By ~~2nd~~ Newton's 2nd law of motion.

Resultant force = ma

Resultant force $= m.\frac{d^2x}{dt^2}$ —④

$\therefore$ From eqn ③ & ④, we get

$$m.\frac{d^2x}{dt^2} = -r.\frac{dx}{dt} - Kx + F.\sin(pt)$$

$$m.\frac{d^2x}{dt^2} + r.\frac{dx}{dt} + Kx = F.\sin(pt)$$

This is the eqn of motion for forced vibrations.

Dividing throughout by "m", we get

$$\frac{d^2x}{dt^2} + \frac{r}{m}.\frac{dx}{dt} + \frac{K}{m}x = \frac{F}{m}.\sin(pt) \quad —⑤.$$

Let $\boxed{\frac{r}{m} = 2b}$

$r \rightarrow$ is damping constant
$m \rightarrow$ mass of oscillating body.

The natural frequency of vibration of the body "ω" is given by

$$\omega = \sqrt{\frac{K}{m}}$$

Squaring $\omega^2 = \frac{K}{m}$

∴ Equn (3) can be written as

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = \frac{F}{m}\cdot\sin(pt) \quad \text{—} (4)$$

As per the procedures followed to solve differential equations, the above equn has a solution of the form

$$\boxed{x = a\cdot\sin(pt-\alpha)} \quad \text{—} (5)$$

where a & α are the unknowns to be found.

However, since equn (5) represents a simple harmonic motion, a & α must represent resp. the amplitude and phase of the vibrating body.

Diff. equn (5) w.r.t. "t" we get,

$$\frac{dx}{dt} = ap\cdot\cos(pt-\alpha) \quad \text{—} (6)$$

Differentiating again w.r.t "t" we get

$$\frac{d^2x}{dt^2} = -ap^2\cdot\sin(pt-\alpha) \quad \text{—} (7)$$

Substituting in equn (4), we get

$$-ap^2\sin(pt-\alpha) + 2bap\cdot\cos(pt-\alpha) + \omega^2 a\cdot\sin(pt-\alpha)$$
$$= \frac{F}{m}\cdot\sin(pt) \quad \text{—} (8)$$

The right side of the above eqn can be written as,

$$\left(\frac{F}{m}\right)\sin(pt-\alpha+\alpha)$$

Substituting in eqn (8) & Simplifying we get,

$$[-ap^2.\sin(pt-\alpha)+a\omega^2.\sin(pt-\alpha)+2bap.\cos(pt-\alpha)]$$
$$=\left(\frac{F}{m}\right).\sin(pt-\alpha).\cos\alpha+\left(\frac{F}{m}\right).\cos(pt-\alpha).\sin\alpha$$

By equating the coefficients of $\sin(pt-\alpha)$ from both sides, we get

$$-ap^2+a\omega^2=\frac{F}{m}.\cancel{\sin}\cos\alpha \quad \text{—— (9)}$$

Similarly, by equating the coefficients of $\cos(pt-\alpha)$ from both sides, we get

$$2bap=\frac{F}{m}.\sin\alpha \quad \text{—— (10)}$$

Squaring & adding eqn (9) & (10), we get

$$[a(\omega^2-p^2)]^2+(2bap)^2=\left(\frac{F}{m}\right)^2[\cos^2\alpha+\sin^2\alpha]$$

$$a^2[(\omega^2-p^2)^2+4b^2p^2]=\left(\frac{F}{m}\right)^2$$

$$\boxed{a=\frac{(F/m)}{\sqrt{4b^2p^2+(\omega^2-p^2)^2}}} \quad \text{—— (11)}.$$

The above eqn represents the amplitude of the forced vibrations.

Substituting equ^n (11) in equ^n (5), the solution of the equation for forced vibration can be written as,

$$x = \frac{(F/m)}{\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}} \cdot \sin(pt - \alpha) \quad \text{—} (12)$$

Phase of forced vibrations:

Dividing equ^n (10) by equ^n (9), we get

$$\tan\alpha = \frac{2bap}{a(\omega^2 - p^2)} = \frac{2bap}{}$$

$$\tan\alpha = \frac{2bp}{(\omega^2 - p^2)}$$

∴ The phase α of the forced vibration is given by

$$\boxed{\alpha = \tan^{-1}\left[\frac{2bp}{\omega^2 - p^2}\right]} \quad \text{—} (13)$$

2 C. A 20gm oscillator with natural frequency 10 rad-s^{-1} is vibrating in damping medium. The damping force is proportional to the velocity of the vibrator. Calculate the value of damping required for the oscillations to be critically damped. (Given: damping constant coefficient is 0.17).

Given data:

Mass of oscillator, $m = 20 \text{ gm} = 0.02 \text{ Kg}$

Natural angular frequency, $\omega = 10 \text{ rad/sec.}$

Damping force $\propto \frac{dx}{dt}$

damping force $= r.\frac{dx}{dt}$ $\quad$ $r \rightarrow$ is damping constant

Damping coefficient, $r = 0.17 \text{ Kg/s}$.

To find: To identify the case under which the decay of oscillations takes place.

Soln:

The equation of motion for damped oscillations, is,

$$m.\frac{d^2x}{dt^2} + r.\frac{dx}{dt} + Kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}.\frac{dx}{dt} + \frac{K}{m}.x = 0$$

Let $\frac{r}{m} = 2b$ ——— (1)

and $\frac{K}{m} = \omega^2$ ——— (2)

where ω, is the natural angular frequency.

W.K.T, the nature of damping is decided by comparing is decided by comparing the value of b^2 with that of ω^2

Now, $b = \frac{r}{2m}$ From equn (1)

$b = \frac{0.17}{2 \times 0.02} = 4.25$

$b = 4.25$

or $b^2 = 18.06$

From data, $\omega = 10$ rad/s. or $\omega^2 = 100$

comparing values of b^2 & ω^2, we find that

$b^2 < \omega^2$

$\therefore$ It is the case of underdamping.

3 a. State and explain Hooke's law. Explain the nature of elasticity with the help of stress-strain diagram.

Hooke's law:

Statement:

It states that "within elastic limit stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

constant of proportionality is called modulus of elasticity

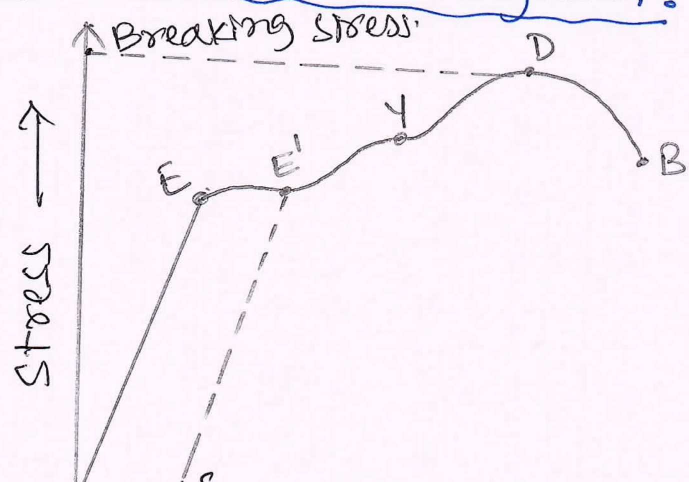
$\therefore$ Modulus of elasticity $E = \frac{\text{Stress}}{\text{Strain}}$

S.I. unit : N/m^2

CGS unit : $dyne/cm^2$

Dimensions : $[M^1 L^{-1} T^{-2}]$

Stress-strain diagram:



E - Elastic limit

Y → Yield point

B - Breaking point

S → set point

D → ultimate point

Stress - strain can be explained as follows:

* OE portion is a straight line ~~with~~ which indicates that stress is proportional to strain i.e. wire obeys Hooke's law up to point E.
* The stress corresponding to point E is maximum stress to which wire can be loaded without any permanent elongation. This is called as elastic limit of wire.
* EE' portion is curved towards strain axis which shows that increase in strain is more than what is warrantied (expected) by corresponding increase in stress.
* Stress is not proportional to strain Hooke's law is not obeyed
* At any point between E and E', if all the load is removed then some permanent elongation occurs in the wire, this is called set.
* when wire is again subjected to a load, a new straight line SE' is obtained indicating Hooke's law.
* Some portion after point Y is almost parallel to strain axis, i.e. strain increases without increase in stress just like wire flows. This is called plastic flow. The point at which this flow begins is called yield point Y.

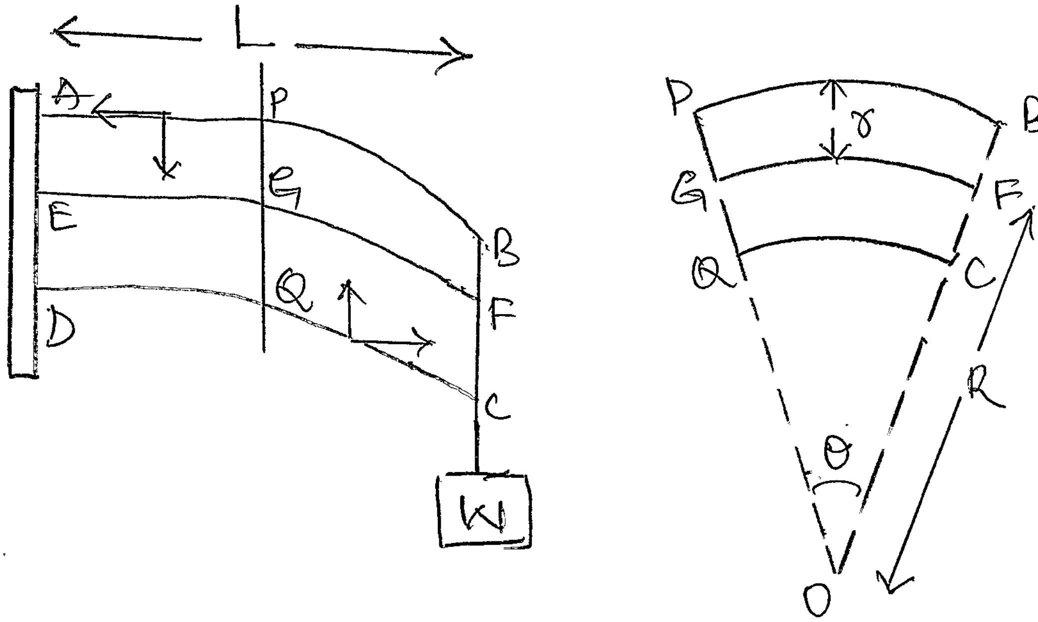
* With the plastic limit, the wire becomes thin and thin. Some weak points called neck are formed in the wire. At weakest point, wire breaks.

* The maximum stress upto which wire can be loaded ~~or~~ wire can bear is called as breaking stress. The corresponding point in the graph is breaking point B.

* The region between yield points to ultimate tensile strength. It is called strain-hardening, because in this region the curve shape depends on the strain rate.

* In the same way, the region between ultimate tensile strength to breaking point is called strain-hardening region.

3 b. Define bending moment. Derive the expression for bending moment interms of moment of inertia. $\boxed{BM = \frac{Y}{R} \cdot I_g}$ 8M



Consider a beam ABCD of length 'L', fixed at the end AD. When a load "W" is attached at the end BC. The upper portion AB gets elongated & experiences an inward force. The lower portion DC gets contracted & experiences an outward force. Whereas the neutral filament EF is neither elongated nor contracted.

The beam experiences two opposite couples; as a result the beam comes to rest.

At equilibrium condition:

Bending moment of the beam = Restoring couple acting on the beam

Consider small portion of the beam PBCQ which gets curved due to load. Let PB, QC and GF be the outer, inner and neutral filaments resp.

Let "O" be the centre and R is the radius of curvature of the arc of neutral filament GF and "r" be the distance of PB from GF.

Then, the length of filament, $GF = R\theta$,

θ is the angle subtended by GF at "O".

The length of the outer filament, $PB = (R+r)\theta$

$$\therefore \text{Linear strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{(R+r)\theta - R\theta}{R\theta}$$

$$= \frac{r}{R}$$

$$\text{But, Young's modulus } (Y) = \frac{\text{Linear Stress}}{\text{Linear Strain}}$$

$$\therefore \text{Linear stress} = \text{Young's modulus} \times \text{Linear strain}$$

$$= Y \cdot \frac{r}{R}$$

Also Force = (Linear) Stress × area of cross section $= Y \cdot \frac{r}{R} \cdot a$

Moment of the force = Force × distance

$$= Y \cdot \frac{r}{R} a \cdot r = \frac{Y}{R} \cdot ar^2$$

$\therefore$ The total moment of the force acting on the upper & lower surfaces of the beam $= \frac{Y}{R} \sum ar^2$

But, Bending moment of the beam = Total moment of the force acting on the upper & lower surfaces of the beam

$$= \frac{Y}{R} \sum ar^2 \quad \text{But } \sum ar^2 = I_g.$$

I_g be the geometrical moment of inertia.

$$\text{Bending moment of the beam} = \frac{Y}{R} \cdot I_g$$

3 C. Calculate the torque required to twist a wire of length 1.5 cm, radius 0.0425×10^{-2} m, through an angle $\left(\frac{\pi}{45}\right)$ radians, if the value of rigidity modulus of its material is 8.3×10^{10} N/m². 4M

Given:

Length of the wire, $L = 1.5$ m

Radius, $R = 0.0425 \times 10^{-2}$ m

Angle of twist, $\theta = \left(\frac{\pi}{45}\right)$ radian.

Rigidity modulus of the material, $\eta = 8.3 \times 10^{10}$ N/m²

To find:

Torque required to produce the twist, $\tau = ?$

Solution:

We know that

Couple per unit twist is given by

$$C = \frac{\pi \eta R^4}{2L}$$

$$C = \frac{\pi \times 8.3 \times 10^{10} \times (0.0425 \times 10^{-2})^4}{2 \times 1.5}$$

$$C = 2.8357 \times 10^{-3}$$

But, torque required

$$\tau = C \cdot \theta$$

$$\tau = 2.8357 \times 10^{-3} \times \frac{\pi}{45}$$

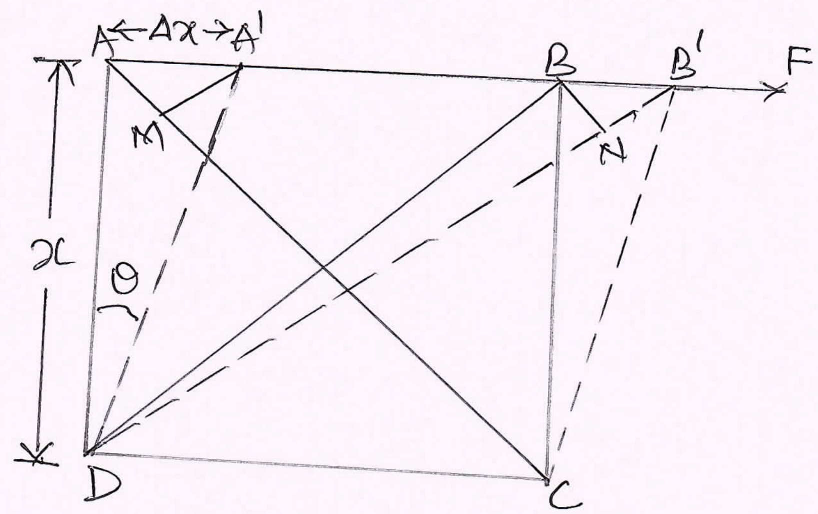
$$\boxed{\tau = 1.98 \times 10^{-4} \text{ N-m}}$$

∴ Torque required to produce the twist $= 1.98 \times 10^{-4}$ N-m.

4 a. Define Poisson's ratio. Obtain the relation between Y, η and σ, where the symbols have their usual meaning.

Definition of Poisson's ratio:

"Within the elastic limits of a body, the ratio of lateral strain to the longitudinal strain is a constant & is called Poisson's ratio. It is represented by "σ"



Let a force F acting tangentially on the upper surface ABQP of a cube of each side "x", displace it through a distance "Δx". A'B'QD be the new position of the upper surface.

$$\text{Shear stress} = \frac{\text{Force}}{\text{area}} = \frac{F}{x^2} = T, \text{ Tensile stress/Tension}$$

$$\text{Shear strain} = \frac{\text{Displacement of upper surface}}{\text{sides of cube}}$$

$$\boxed{\theta = \frac{\Delta x}{x}} \quad \text{—(1)}$$

$$\text{Rigidity modulus} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\boxed{\eta = \frac{T}{\theta}} \quad \text{—(2)}$$

Due to applied tangential tensile stress acts along the diagonal DB & compressive stress along the diagonal AC.

If α is the longitudinal strain coefficients &

$\beta \rightarrow$ is the lateral " "

Then elongation of the diagonal DB due to the tensile stress $= DB \cdot T \cdot \alpha$ — (3)

Also, elongation of the diagonal DB due to the compressive stress $= DB \cdot T \cdot \beta$ — (4)

~~Also~~, $\therefore$ Total elongation in length of DB $= DB \cdot T \cdot (\alpha+\beta)$ — (5)

Draw $BN \perp DB'$, then NB' is the elongation in length of DB

From the $\Delta^{le} BNB'$, $NB' = BB' \cdot \cos(BB'N) = BB' \cdot \cos 45^\circ$

$$= \Delta x \cdot \frac{1}{\sqrt{2}} \quad \text{— (6)}$$

From equ^n (5) & (6), we get

$$DB \cdot T(\alpha+\beta) = \Delta x \cdot \frac{1}{\sqrt{2}}$$

But $DB = \sqrt{2}\, x$

$$\therefore \sqrt{2}\, x \cdot T(\alpha+\beta) = \Delta x \cdot \frac{1}{\sqrt{2}}$$

From equ^n (2) $\frac{\Delta x}{x} = \theta$ & equ^n (3) $T = \eta\theta$

$$\therefore \sqrt{2}\, \eta\theta(\alpha+\beta) = \theta \cdot \frac{1}{\sqrt{2}}$$

$$\eta(\alpha+\beta) = \frac{1}{2}$$ on re-arranging, we get

$$\eta = \frac{1}{2\alpha(1+\beta/\alpha)}$$

$$\boxed{\eta = \frac{Y}{2(1+\sigma)}}$$ $\because \frac{1}{\alpha} = Y$ & $\frac{\beta}{\alpha} = \sigma$

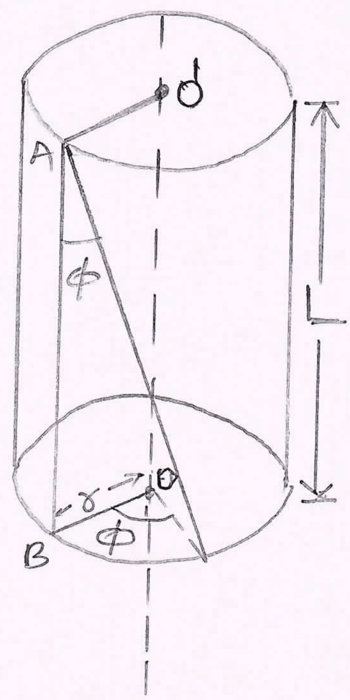
4 b. What are Torsional oscillations? Mention any two applications of Torsional pendulum. Derive the expression for couple per unit twist of a solid cylinder.

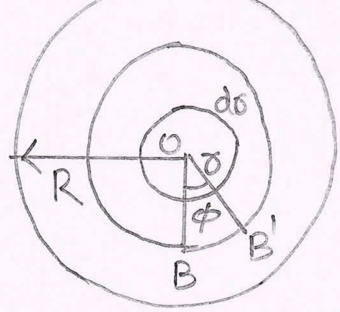
Oscillations of structural elements & parts of machines, manifested in periodically varying torsional strain.

Applications of Torsional pendulum:

1. The moment of inertia of irregular bodies.
2. The rigidity modulus of the material using the torsional pendulum of the material wire with regular body.

Expression for Torsion of a cylinder rod: $C = \frac{n\pi R^4}{2L}$





Consider a cylindrical wire of radius "R" and length "L" which is fixed at upper end and other end is twisted.

Let OO' be its axis

The wire is imagined to be made up of a large number of concentric hollow cylindrical wires / cylindri~~cal~~ of each thickness "dr".

If the rod is twisted at its lower end, then the concentric layers slide one over the other. This movement will be zero at the fixed end, & it gradually increases along the downward direction.

Let the line AB of the hollow cylindrical wire of radius "r" & thickness "dr" gets twisted to AB'. If OO' be the axis of the wire & "ϕ" be the angle of twisting, then from the sector OBB', arc $\boxed{BB' = r\phi}$ — ①

Consider the flat surface AB sheared to AB' due to twisting.

If "θ" be the shearing angle, then arc $\boxed{BB' = L\theta}$ — ②

From eqn ① & ②, we have

$$L\theta = r\phi$$

∴ shearing angle, $\boxed{\theta = \frac{r\phi}{L}}$ — ③

Now, the cross sectional area of the layer under consideration is $2\pi r \cdot dr$

If F is the shearing force, then the shearing stress T is given by

$$T = \frac{\text{Force}}{\text{area}} = \frac{F}{2\pi r \cdot dr}$$

∴ shearing force, $\boxed{F = T \cdot (2\pi r \cdot dr)}$ — ④

If "θ" is the angle through which the layer is sheared then the rigidity modulus,

$$\eta = \frac{\text{shearing stress}}{\text{Shearing strain}} = \frac{T}{\theta}$$

$$\boxed{T = \eta\theta} \text{ — ⑤}$$

But, shearing stress = Rigidity modulus x shearing ~~angle~~ strain

$$= \eta \cdot \frac{r\phi}{L}$$

If "a" be the area of cross section of (hollow) wire, then shearing force acting on it is given by

Shearing force = shearing stress x area of cross section

$$= \eta \cdot \frac{r\phi}{L} \cdot a$$

Also, area over which shearing force acts,

$$a = \text{circumference} \times \text{thickness}$$

$$a = 2\pi r \cdot dr$$

$\therefore$ shearing force acting $= \eta \frac{r\phi}{L} \cdot 2\pi r \cdot \cancel{d\phi}\, dr.$

But, moment of the force acting on the wire about axis OO' = Force x distance

$$= \eta \frac{r\phi}{L} \cdot 2\pi r \cdot dr \cdot r$$

$$= \eta \frac{r\theta}{L} \cdot 2\pi r^2 \cdot dr \quad \text{—— (6)}$$

This is regarding only one layer of the cylinder. The total moment of the force acting on the cylindrical wire $= \int_0^R \eta \frac{r\theta}{L} \cdot 2\pi r^3 dr.$

i.e. Twisting couple acting on the wire due to torque,

$$C = \frac{2\pi\eta\theta}{L}\left[\frac{r^4}{4}\right]_0^R$$

$$C = \frac{\eta\pi\theta R^4}{2L}$$

$\therefore$ couple per unit twist is,

$$C = \frac{\text{couple acting on the wire}}{\text{Angle of twisting}}$$

$$= \frac{\pi\eta R^4\theta/2L}{\theta} = \frac{\pi\eta R^4}{2L}$$

$$C = \left(\frac{\pi\eta R^4}{2L}\right)$$

4 c. Calculate the force required to produce an extension of 1mm in steel wire of length 2m and diameter 1mm (Young's modulus for steel $= 2\times10^{11}\,Nm^{-2}$)

Given data:

Extension to be produced, $x = 10^{-3}\,m$

Length of the wire, $L = 2m$

Diameter, $d = 1mm = 10^{-3}\,m$

Young's modulus, $Y = 2\times10^{11}\,Nm^{-2}$

To find:

The force required to produce the extension, $F = ?$

Solution:

Radius of the wire, $R = \frac{d}{2} = \frac{10^{-3}}{2} = 0.5\times10^{-3}\,m$

Young's modulus of the material of the wire is given by,

$$Y = \frac{FL}{ax}$$

where, a is the area of cross section of the wire.

∴ Substituting $a = \pi R^2$, the equation for Y becomes

$$Y = \frac{FL}{\pi R^2 x}$$

$$F = \frac{\pi R^2 Y x}{L}$$

$$F = \frac{\pi \times (0.5\times10^{-3})^2 \times 2\times10^{11} \times 10^{-3}}{2}$$

$$\boxed{F = 78.54\,N.}$$

∴ Force required to produce the extension $= 78.54\,N$.

5 a. State and Prove Gauss Divergence theorem.

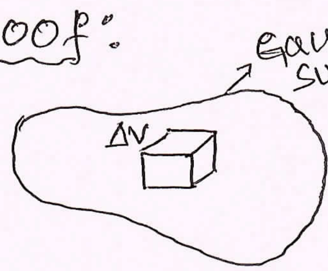
Statement:

The integral of the normal component of the flux density over any closed surface in an electric field is equal to the volume integral of the divergence of the flux throughout the space enclosed by the surface.

It is represented mathematically as,

$$\oint_S \vec{D}\cdot d\vec{s} = \int_V \nabla\cdot\vec{D}\cdot dV \quad \text{--- ①}$$

Proof:



Consider a gaussian surface in a region with certain charge density (Figure).

Inside the surface, consider a differential volume element ΔV.

Let ΔQ be the charge within the element.

If "ρ" is the charge density, and since "ρ" can vary continuously in the volume, we have

$$\rho_v = \lim_{\Delta V \to 0}\left[\frac{\Delta Q}{\Delta V}\right] = \frac{dQ}{dV} \quad \text{--- ②}.$$

$$\therefore \quad dQ = \rho_v \cdot dV$$

If Q is the total charge enclosed by the gaussian surface, then

$$Q = \int dQ = \int_V \rho_v \cdot dV$$

But, we know that

$$\nabla \cdot \vec{D} = \rho \quad \text{(By Maxwell's first equation)}$$

$$Q = \int_V \nabla \cdot \vec{D} \cdot dv \quad \text{—— (3)}$$

Now, by applying Gauss's law to the Gaussian surface, we have

$$\oint_S \vec{D} \cdot d\vec{s} = Q \quad \text{—— (4)}$$

∴ By eqn (3) & (4), we have

$$\boxed{\oint_S \vec{D} \cdot d\vec{s} = \int_V \nabla \cdot \vec{D} \cdot dv} \quad \text{—— (5)}$$

This is Gauss Divergence theorem or simply the divergence theorem.

5 b Define fractional index change (Δ). Derive the expression for Numerical aperture and acceptance angle of an optical fiber.

Fractional index change (Δ):

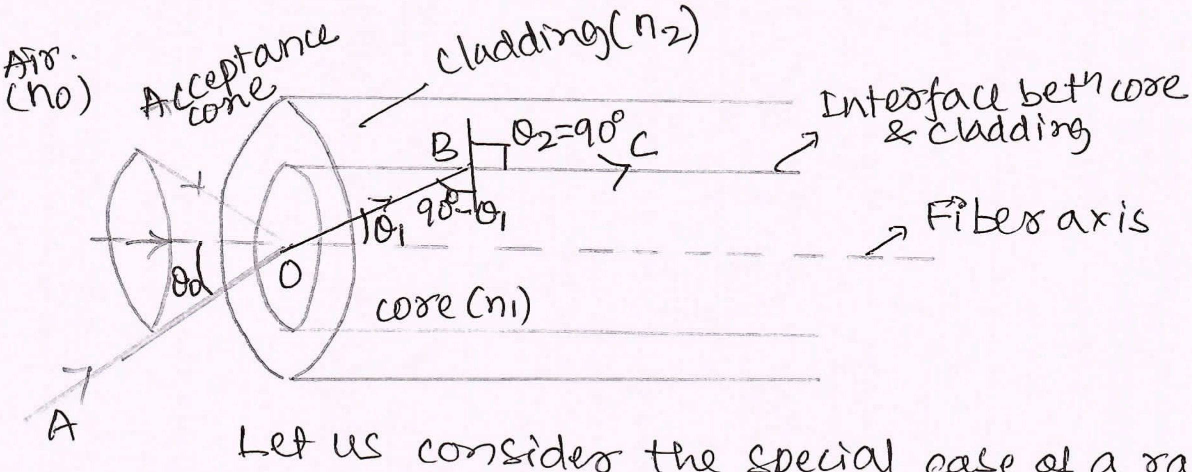
It is defined as the ratio of the difference in refractive indices of core and cladding to the refractive index of the core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$n_1 \to$ R.I. of core

$n_2 \to$ R.I. of cladding.

Derivation of Numerical aperture:



Let us consider the special case of a ray which suffers critical incidence at the core cladding surface.

The ray, to begin with travels along AO entering into the core at an angle θ_0 to the fiber axis. Then it is refracted along OB at an angle θ_1 in the core and further proceeds to fall at critical angle of incidence (equal to $90° - \theta_1$) at B on the interface between core & cladding.

Let n_0, n_1 & n_2 be the refractive indices of surrounding medium, core & cladding of the fiber respectively.

Now, for refraction at the point of entry of the ray AO into the core, we have by at position "O".

Applying Snell's law

$$\boxed{n_0 \cdot \sin\theta_0 = n_1 \sin\theta_1} \longrightarrow (1)$$

At the point "B" on the interface

angle of incidence $= 90^\circ - \theta_1$

angle of refraction $= \theta_2 = 90^\circ$

Again applying Snell's law, we have

$$n_1 \cdot \sin(90^\circ - \theta_1) = n_2 \cdot \sin\theta_2$$

$$n_1 \cdot \cos\theta_1 = n_2 \sin 90^\circ \qquad (\sin 90^\circ = 1)$$

$$n_1 \cdot \cos\theta_1 = n_2$$

$$\boxed{\cos\theta_1 = \frac{n_2}{n_1}} \longrightarrow (2)$$

Rewriting eqn (1) we have.

$$n_0 \cdot \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sqrt{(1 - \cos^2\theta_1)}$$

From eqn (2), we have.

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$= \frac{n_1}{n_0} \times \frac{1}{n_1} \sqrt{n_1^2 - n_2^2}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \longrightarrow (3)$$

If the medium surrounding the fiber is air, then $\boxed{n_0 = 1}$

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$N.A = \sqrt{n_1^2 - n_2^2}$$

If θ_i is the angle of incidence of an incident ray, then the ray will be able to propagate.

$$\theta_i < \theta_0$$

$$\sin\theta_i < \sin\theta_0$$

$$\sin\theta_i < \sqrt{n_1^2 - n_2^2}$$

This is the condition for propagation.

$$\sin\theta_i < N.A.$$

5 c. A circular coil of radius 10 cm having 50 turns carries a current of 5A. Determine the magnetic field produced by the coil at a distance of 3cm from the centre. Also determine magnetic field produced by the coil at its centre.

Given data:

Radius $r = 10cm = 0.1m$, turns, $n = 50$ current, $I = 5A$.

$B = ?$ $x = 5cm = 0.03m$ $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Magnetic field along axis of circular coil.

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I n r^2}{(r^2 + x^2)^{3/2}}$$

$$= 10^{-7} \times \frac{2 \times \pi \times 5 \times 50 \times (0.1)^2}{[(0.1)^2 + (0.03)^2]^{3/2}}$$

$$\boxed{B = 4.42 \times 10^{-7}\ T}$$

5 a Derive wave equation in terms of electric field using Maxwell's equations for free space.

Let us consider the two curl equations of Maxwell's.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

But, we know that $D = \varepsilon E$ and $B = \mu H$

$$\therefore \nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{—(1)}.$$

and $$\nabla \times \vec{H} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{—(2)}$$

Let us obtain the wave equation in terms of the electric field, for which we must obtain an equation relating the spatial co-ordinate of $\vec{E}$ to its time co-ordinate. This is done by eliminating $\vec{H}$ between the two equations in the following way.

Taking ~~past~~ curl for both sides of equ^n (3), we have

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}[\nabla \times \vec{H}] \quad \text{—(3)}$$

As per vector analysis,

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 E$$

$$= \nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2 E \quad \text{—(4)}.$$

[Since, as per Maxwell's equations.
$\nabla \cdot \vec{D} = \rho$ or $\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$.]

$\therefore$ From equ^n (3) & equ^n (4), we have

$$\nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu.\frac{\partial}{\partial t}[\nabla\times\vec{E}]$$

Using equ^n (1) in the right side, we have

$$\nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu.\frac{\partial}{\partial t}\left[\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

or $$\nabla^2\vec{E} - \mu\varepsilon.\frac{\partial^2\vec{E}}{\partial t^2} = \mu.\frac{\partial\vec{J}}{\partial t} + \nabla\left(\frac{\rho_v}{\varepsilon}\right) \quad \text{--- (5)}$$

The left side of the above equation in the characteristic form of a wave equation. The solution of such an equation represents a propagation wave. The right side represents the sources which are responsible for the wave field. i.e. the charges & currents.

Hence, equ^n (5) represents the wave equ^n in $\vec{E}$ for a medium with constant μ & ε. i.e. a homogeneous and isotropic medium.

If we consider free space, i.e. space where there are no charges or currents, then

$\rho_v = 0$ and $J = 0$

Equ^n (5) becomes

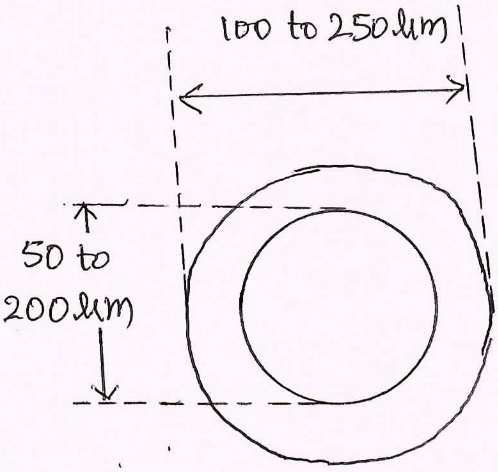
$$\boxed{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{--- (6)}$$

Applications:

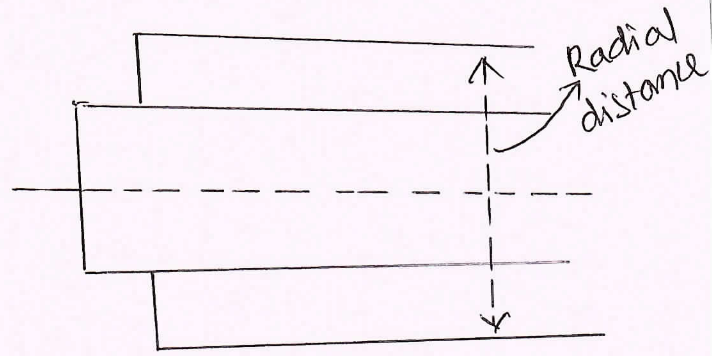
Single mode fibers are used in long distance communication because they have higher bandwidth.

Ex: Submarine cable system.

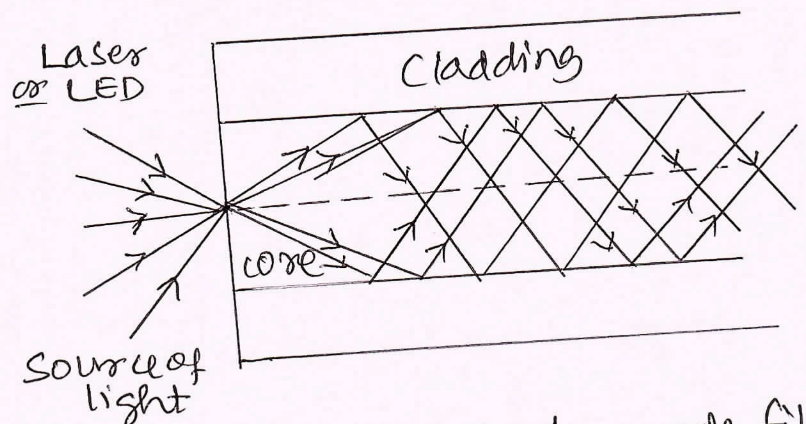
② Step-Index Multimode fiber:



Geometry



Refractive index profile.



Ray propagation.

It is similar to single mode fiber, but core has large diameter. The diameter value of the core is about 50 μm to 200 μm and external diameter of cladding is 100 μm to 250 μm. But the core is comparatively larger in diameter. It can propagate large number of modes as shown in figure.

Laser or LED is used as source of light. It has an application in data links.

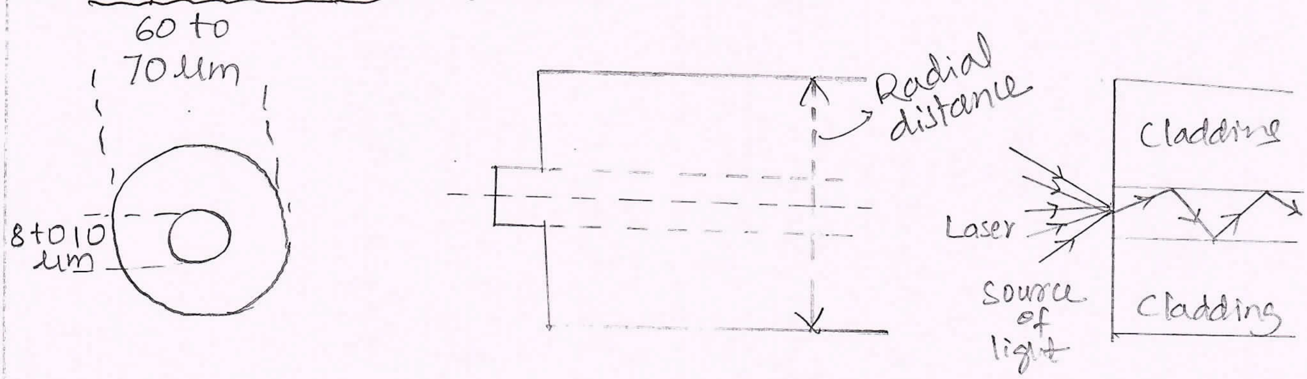
6 b. Describe different types of optical fibers with neat diagrams. Mention any two mechanisms involved in fiber loss.

The optical fibers are classified into 3 types namely

1. Step index sing mode fiber
2. Step index multimode fiber
3. Graded " " " .

This classification is done depending on the refractive index profile and the number of modes, that the fiber can guide.

① Step index single mode fiber:



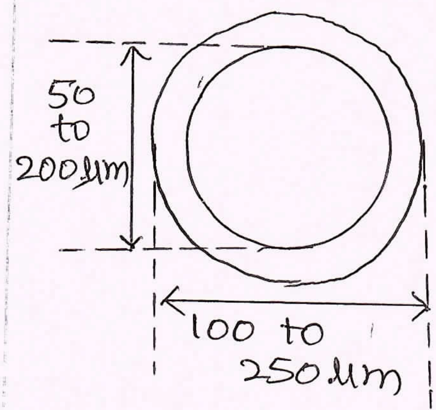
A single mode fiber has a core material of uniform R.I value. Similarly cladding also has a material of uniform index but of lesser value. This results in a sudden increase in the value of R.I. from cladding in core. Thus its R.I. profile takes the shape of a step. The diameter value of the core is about 8 to 10 um and external of cladding is 60 to 70 um.

Because of its narrow core, it can guide just a single mode as shown in figure (↑).

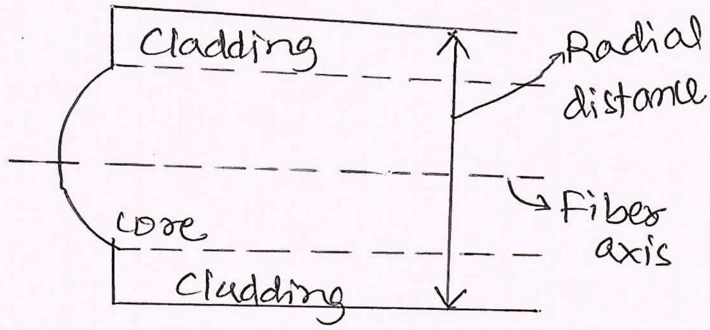
Hence it is also called single mode fiber.

Applications: They are used in submarine. It is used in short distance communication due to lower bandwidth.

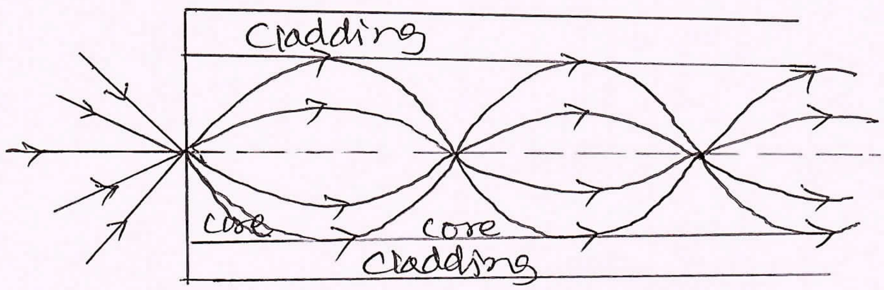
(3) Graded Index Multimode fibers (GRIN):



Geometry



Refractive Index profile.



Ray propagation.

It is also called GRIN. The geometry of the GRIN multimode fiber is similar to that of step index multimode fiber. Its core material has a special feature, that is R.I. value decreases in the radially outwards direction from the axis and becomes equal to that of the cladding at the interface. But the R.I.q the cladding remains uniform. The refractive index profile as shown in figure.

The incident rays bends and takes a periodic path along the axis. The rays have different paths with same period. Laser or LED is used as source of light. It is the expensive of all. It is used to telephone trunks between central offices.

Applications:

(i) Used in the telephone trunks between central offices.

(ii) It is used to short distance communication due to lower bandwidth. They transmitt transformation to shorter distance.

6 c. Calculate the V-number for a fiber of core-diameter 40 µm and with refractive indices of 1.55 & 1.5 resp. for core & cladding. When the wavelength of the propagating wave is 1400 nm. Also calculate the number of modes that the fiber can support for propagation. Assume that the fiber is in air.

Given data:

Refractive index of core, $n_1 = 1.55$

Refractive index of Cladding, $n_2 = 1.50$

Diameter of the core, $d = 40 \times 10^{-6}$ m

Wavelength of propagating light, $\lambda = 1400 \times 10^{-9}$ m

$n_0 = 1$

To find:

V-number, $V = ?$

Solution:

V-number is given by

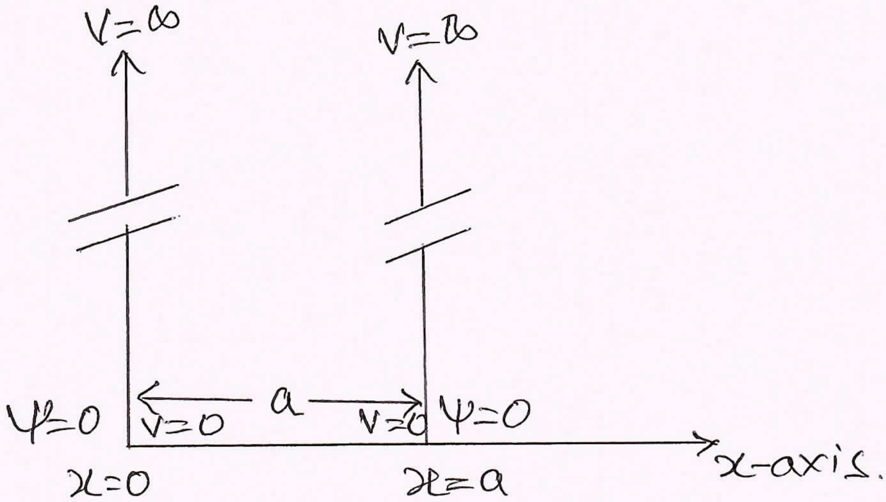
$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

$$= \frac{\pi \times 40 \times 10^{-6}}{1400 \times 10^{-9}} \sqrt{1.55^2 - 1.5^2}$$

$$\boxed{V = 35}$$

∴ The number of modes, the fiber support $= \frac{V^2}{2} = \frac{(35)^2}{2} = 612$

7 Q. Starting from Schrodinger's time independent wave equation, derive the expression for energy eigen value and eigen function for an electron in one-dimensional potential well of infinite height.



Consider a particle of mass "m" free to move in one dimension along +ve x-direction between $x=0$ to $x=a$. The potential energy outside, this region is infinite and within the region is zero. The particle is in bound state.

Such a configuration of potential in space is called infinite potential well. It is also called particle in a box.

The schrodinger's equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0.$$

This is Schrodinger's time independent equn for a particle.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \quad \text{--- (1)} \qquad (\because V=\infty).$$

For outside, the equation holds good.

If $\psi = 0$ & $|\psi|^2 = 0$

That is particle cannot be found outside the well & also at the walls.

The Schrodinger's equ^n inside the well is

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E)\psi = 0} \quad \text{--- (2)}$$

This is Eigen-value equation.

Putting $\frac{8\pi^2 m}{h^2}E = K^2$ --- (3)

Equation (2) becomes.

$$\boxed{\frac{d^2\psi}{dx^2} + K^2\psi = 0} \quad \text{--- (4)}$$

The general solution of the quadratic equ^n (4) is of the form.

The solution of this equ^n.

$$\psi = C \cdot \cos Kx + D \cdot \sin Kx \quad \text{--- (5)}$$

where C & D are constants determined from boundary condition as follows.

~~Case (i)~~: $\psi(x) = 0$ at $x = 0$.

From equ^n (5)

case (i) At $x=0$ at $\psi=0$

Equation (5) becomes

$$0 = C \cdot \cos(0) + D \cdot \sin(0)$$

$$\therefore \boxed{C=0} \quad \text{—— (6)}$$

case (ii) $x=a$ at $\psi=0$

$$0 = C \cdot \cos(Ka) + D \cdot \sin(Ka)$$

From equn (6), we have $\boxed{C=0}$

$$\therefore D \cdot \sin(Ka) = 0 \quad \text{—— (7)}$$

$D \neq 0$, Because the wave concept vanishes.

$$\sin(Ka) = 0$$

$$Ka = \sin^{-1}(0)$$

$Ka = n\pi$ where ~~n0~~ $n = 0, 1, 2, 3 \ldots\ldots$

$$\boxed{K = \frac{n\pi}{a}} \quad \text{—— (8)}$$

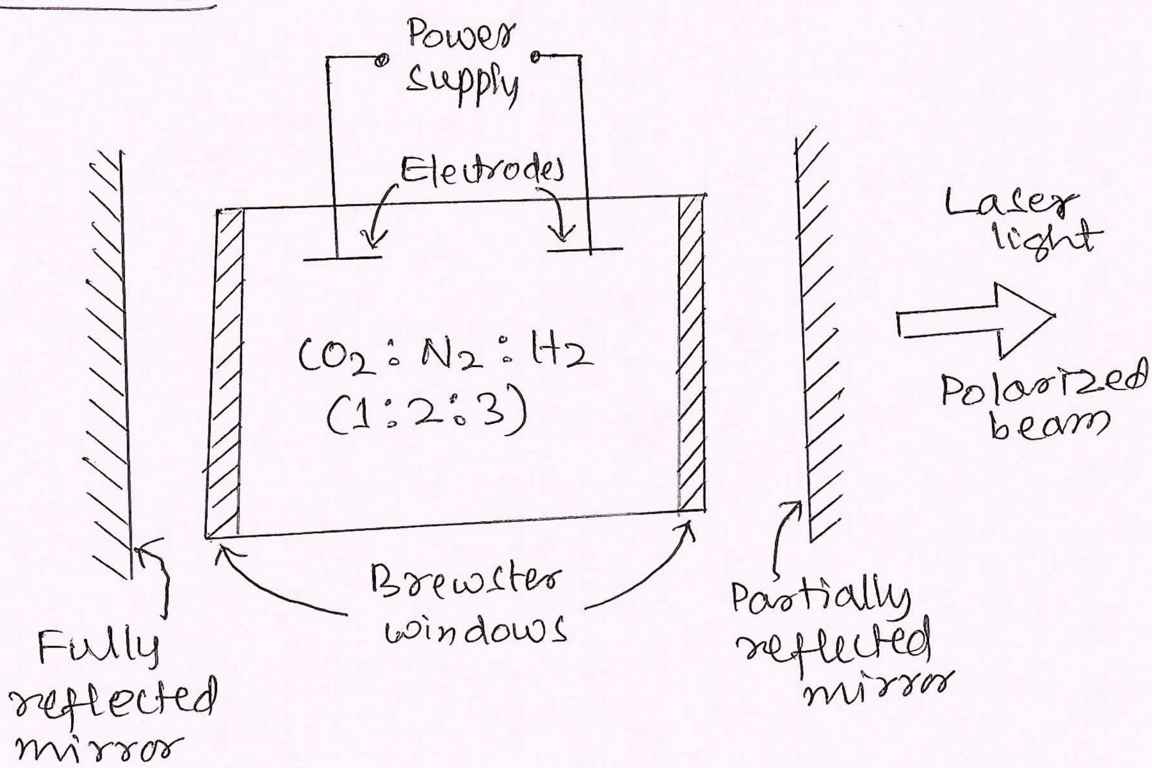
Using this equation (5), we have

$$\boxed{\psi_n = D \cdot \sin\left(\frac{n\pi}{a}\right)x} \quad \text{—— (9)}$$

which gives permitted wave function.

7 b. Explain the construction and working of CO_2 Laser with the help of energy level diagram.

CO_2 Laser:



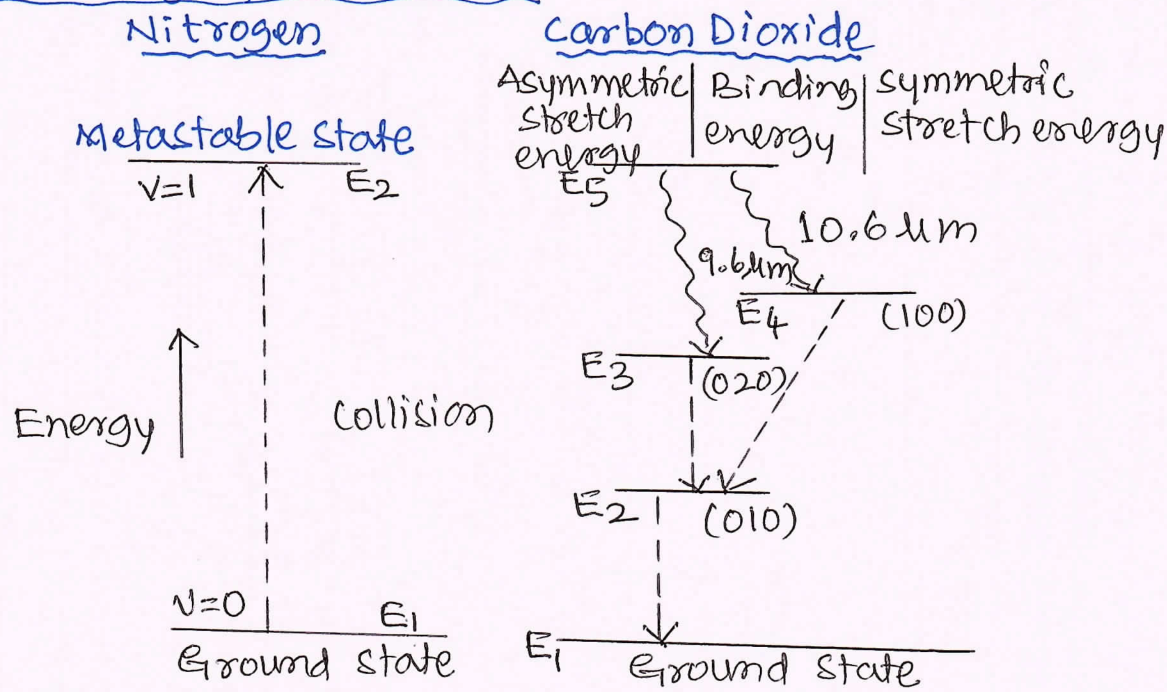
Construction:

* The schematic diagram of CO_2 laser is as shown in the figure. This laser is invented by CKN. Patel an Indian engineer.
* It consists of a glass discharge tube of length 1 m & 2.5 cm in diameter filled with a mixture of CO_2, N_2 & H_2 in the ratio 1:2:3.
* High DC voltage can be applied to the gas between the electrodes.
* Ends of the tube is filled with (NaCl) brewster windows to get polarized laser beam.
* Two concave focal silicon mirrors coated with aluminium are provided at the

ends of the tube which acts as a optical resonators.

* Cold water is circulated through a tube surrounding the discharge tube.

Working of CO_2 Laser:



Principle: CO_2 laser works on the principle of stimulated emission.

Working:

* CO_2 laser is a five level molecular gas laser which produces continuous or pulsed laser beam.
* It works on the principle of stimulated emission between the rotational sublevels up an upper & lower vibrational levels of CO_2 molecules.
* Ionization takes place due to electric discharge when high DC voltage is applied between electrodes producing electrons.
* The accelerated ēs excite both N_2 & CO_2 atoms to their higher energy levels V=1 & C_5 from their ground states 0 & C_1 due to collision as follows:

$$e_1 + N_2 \longrightarrow N_2^* + e_2$$ and
$$e_1 + CO_2 \longrightarrow CO_2^* + e_2$$

where e_1 & e_2 are the energies of $\bar{e}$ before and after collision.

* N_2^* molecule in excited state level collide with CO_2 molecules in their ground state C_1 & excite it to metastable state C_5 by resonant energy transfer as level C_5 of CO_2 is same as level $v=1$ of N_2 is given by
$$N_2^* + CO_2 \longrightarrow CO_2^* + N_2$$

* As this process continues due to electric discharge pumping, population inversion takes place between C_5 & C_4 and C_5 & C_3.
* The transition/de-excitation takes place as follows:

$C_5 \to C_4$ producing laser 10.6 um (IR region)

$C_5 \to C_3$ " " 9.6 um (IR region)

$C_4 \to C_2$

$C_3 \to C_2$ Radiation less transitions.

$C_2 \to C_1$

* Due to high thermal conductivity of He, it removes heat from mixture & de-populate the lower states C_3 & C_2 quickly.
* Laser beam is amplified by using optical resonators.
* The laser outputs is 100 KW for continuous mode & 10 KW in pulsed mode.

7 c. The average output power of laser source emitting a laser beam of wavelength 632.8nm. 4M
Find the number of photons emitted per second by the laser source

Given data:

wavelength, $\lambda = 632.8\ nm = 632.8 \times 10^{-9} m$

Number of photons emitted $= N = ?$

time $= 1$ sec

Planck's constant, $h = 6.63 \times 10^{-34}$ J/K

velocity of light, $c = 3 \times 10^{8}$ m/s.

Power, $P = \frac{\text{Energy}}{\text{time}} = \frac{E}{t}$

$P = \frac{E}{t}$

$E = P.t$ — (1)

$E = Nh\nu$

$E = \frac{Nhc}{\lambda}$ — (2)

$P.t = \frac{Nhc}{\lambda}$

$N = \frac{P.t.\lambda}{hc} \Rightarrow N = \frac{P.\lambda}{thc}$

$\boxed{N = 1.59 \times 10^{6}}$

8 a. Define the terms population inversion and meta-stable state. Derive the expression for energy density of radiation at equilibrium interms of Einstein's coefficients.

Population inversion:

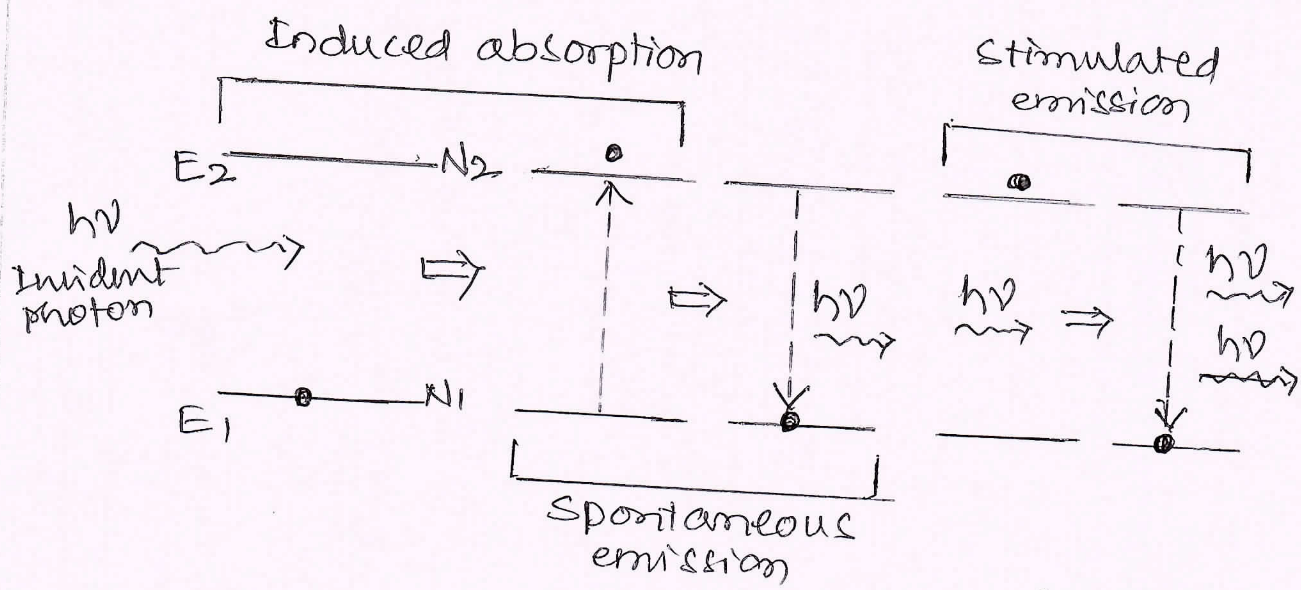
If a photon having energy $E_2 - E_1$ is incident on such a gas, the probability of induced absorption is more than the probability of stimulated emission,

The number of atoms in the higher energy state must be made greater than the number of atoms in the lower energy state. This state, in which there is a larger number of atoms in the higher energy state than the lower, is called population inversion.

Metastable state:

It is state in which the atoms or molecules stays ~~in~~ for long duration in the order of 10^{-2} to 10^{-3} sec. ~~and~~

Energy density of radiations in terms of Einstein's coefficients:



Consider two energy states E_1 & E_2 of a system of atoms ($E_2 > E_1$). Let there be N_1 atoms with energy E_1 and N_2 atoms with energy E_2 per unit volume of the system.

N_1 & N_2 are called number density of atoms in the state ① & 2 resp.

Let radiations with a continuous spectrum of frequencies be incident upon the system.

Let there be radiation of frequency ν, such that $\nu = \left(\frac{E_2 - E_1}{h}\right)$ and

Let E_ν be the energy density of radiations of frequency ν.

Then $E_\nu \cdot d\nu$ will be the energy density of radiations whose frequency lies in the range ν and $\nu + d\nu$.

Let us consider the absorption and the two emission process can be case.

(i) Induced absorption:

In the case of induced absorption, an atom in the level E_2 can go the level E_2, when it absorbs a radiations of frequency ν, such that

$$\nu = \left[\frac{E_2 - E_1}{h}\right] \quad \text{--- (1)}$$

The number of such absorptions per unit time per unit volume, is called rate of absorption. The rate of absorption depends upon,

(i) the number density of lower energy state i.e. N_1 &

ii) the energy density i.e. $E\nu$.

$\therefore$ Rate of induced absorption $\propto N_1 E\nu$

$$\text{Rate of I.A} = B_{12} N_1 E\nu \quad \text{--- (2)}$$

where B_{12} is the constant of proportionality, called Einstein's coefficient for induced absorption.

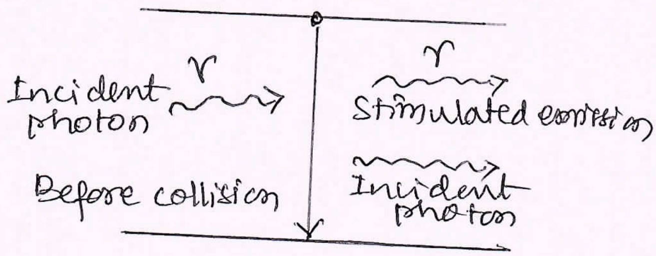
(ii) Spontaneous emission:

In case of spontaneous emission, an atom in the higher energy level E_2 undergoes transition to the lower energy level E_1 voluntarily by emitting a photon. The number of such spontaneous emission per unit volume, is called rate of spontaneous emission, which is proportion to only the number density in the higher energy state, i.e. N_2

$$\therefore \text{Rate of spontaneous emission} = A_{21} N_2 \quad \text{--- (3)}$$

where A_{21} is coeff. of spontaneous emission.

(iii) Stimulated Emission:



The number of stimulated emission per unit time per unit volume is called rate of stimulated emission.

Rate of stimulated emission $\propto N_2 E\nu$ — (4)

Rate of stimulated emission $= B_{21} N_2 E\nu$ — (4)

where B_{21} is coefficient of stimulated emission.

At thermal equilibrium condition,

Rate of induced absorption = Rate of spontaneous emission + Rate of stimulated emission

Using equ^n (2), (3) & (4), we have

$$B_{12} N_1 E\nu = A_{21} N_2 + B_{21} N_2 E\nu \quad \text{— (5)}$$

$$E\nu \left[B_{12} N_1 - B_{21} N_2\right] = A_{21} N_2$$

$$E\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Dividing both Nr & Dr by $B_{21} N_2$, we get

$$E\nu = \frac{A_{21} N_2 / B_{21} N_2}{(B_{12} N_1 - B_{21} N_2)/B_{21} N_2}$$

$$E\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \cdot \frac{N_1}{N_2} - 1}\right] \quad \text{— (6)}$$

According to Boltzmann's law,

$$\frac{N_1}{N_2} = e^{\left(\frac{E_2 - E_1}{KT}\right)} = e^{h\nu/KT} \quad \text{— (7)}$$

Equ^n (6) becomes.

$$E\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \cdot e^{h\nu/KT} - 1}\right] \quad \text{— (8)}$$

According to Planck's law, equ^n for $E\nu$ is.

$$E\nu = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{h\nu/KT} - 1}\right] \quad \text{— (9)}$$

Now comparing equ^n (8) & (9) terms by term on the basis,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{or} \quad \boxed{B_{12} = B_{21}}$$

8 b. Using Heisenberg's principle, show that electrons do not reside inside the nucleus.

Electrons donot present inside the nucleus : using Heisenberg's uncertainty principle:

Electron to be present in the nucleus, maximum uncertainty in position $\Delta x = 10^{-14} m$

$\Delta x = 10^{-14}$ (Diameter of nucleus)

According to Heisenberg's Uncertainty principle, we have

The minimum uncertainty in momentum is given by

$$\Delta x . \Delta p \geq \frac{h}{4\pi}$$

$\Delta x \rightarrow$ uncertainty in the position.

$\Delta p_x \rightarrow$ " " " momentum.

$$\Delta x . \Delta P_x \geq \frac{h}{4\pi}$$

$$\Delta P_x \geq \frac{h}{4\pi . \Delta x}$$

$$\Delta P_x \geq \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}}$$

$$\Delta P_x \geq 5.275 \times 10^{-21} \text{ Kg.m/s.}$$

$$= P \text{ (say)}.$$

The minimum energy of the electron in the nucleus is given by

$$\therefore \ E \geq \frac{p^2}{2m}$$

$$\geq \frac{(5.275\times10^{-21})^2}{2\times 9.1\times10^{-31}}$$

$$E \geq 1.527\times10^{-11}\ J$$

$$\geq \frac{1.527\times10^{-11}}{1.6\times10^{-19}}\ eV$$

$$E \geq 95.45\times10^{6}\ eV$$

$$E \geq 95.45\ MeV$$

$$\boxed{E = 95\ MeV}$$

But the maximum kinetic energy of the electrons (β- particle) emitted from the nucleus does not exceed 4 MeV.

Hence electrons cannot present inside the nucleus of an atom.

8 C. An electron is found in an 1-D. potential well of infinite height and of width 1 A°. Calculate its energy values in the ground state and also in the first two excited state.

Given data:

Width of the potential well, $a = 1A^0 = 10^{-10} m$

$m \to 9.1 \times 10^{-31} kg$ $h = 6.63 \times 10^{-34} J.s.$

To find:

Values of energy of the e^- in ground state, $E_1 = ?$

" " " first " $E_2 = ?$

" " " Second " $E_3 = ?$

Solution:

The energy of the e^- in an ~~one~~ 1-D infinite potential well is given by

$$E_n = \frac{n^2 h^2}{8ma^2}$$

For ground state, n=1

$$E_1 = \frac{h^2}{8ma^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E_1 = 6.0314 \times 10^{-18} J$$

$$E_1 = \frac{6.0314 \times 10^{-14}}{1.6 \times 10^{-19}} eV$$

$$\boxed{E_1 = 37.64 \, eV}$$

For n=2, First excited state

$$E_2 = \frac{2^2 h^2}{8ma^2} = 4 \times 37.64 = 4(E_1)$$

$$E_2 = 2.4125 \times 10^{-17} J = 150.54 \, eV$$

For n=3, Second excited state

$$E_3 = 9 \times E_1 =$$

$$E_3 = 9 \times 6.0314 \times 10^{-18} = 5.4282 \times 10^{-17} J$$

$$\boxed{E_3 = 338.7 eV}$$

9 a. Define Fermi energy. Explain the variation of Fermi factor with temperature. (8M)

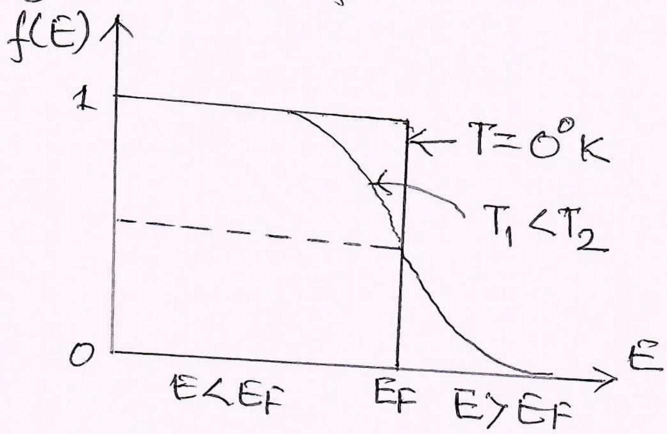
Fermi Energy:

The energy corresponding to the highest occupied level at zero degree absolute is called the fermi energy.

Dependence of Fermi factor on temperature & effects on occupancy of Energy levels:

The dependence of fermi factor on temp & effect on occupancy of energy levels is as shown in figure.

Let us consider the different cases of distribution as follows.



(i) Probability of occupation for E<EF at T=0°K:

When T=0°K and E<EF, we have for the probability.

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1} = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

$\boxed{f(E) = 1}$ for $E<E_F$

Hence, f(E)=1 means the energy level is certainly occupied and E<EF applies to all the energy levels below EF.

∴ At T=0°K, all the energy levels below the fermi level are occupied.

(ii) Probability of occupation for $E > E_F$ at $T = 0^\circ K$:

When $T = 0^\circ K$ and $E > E_F$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1} = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} = 0$$

$\boxed{f(E) = 0}$ for $E > E_F$

$\therefore$ $T = 0^\circ K$, all the energy levels above Fermi levels are unoccupied.

(iii) Probability of occupation at ordinary temp: ($E = E_F$ at $T > 0^\circ K$)

At ordinary temperature, though the values of probability remains 1 for $E < E_F$, it starts decreasing from 1, as the values of E becomes closure to E_F.

The value of f(E) becomes ½ at $E = E_F$

For $E = E_F$ at $T > 0^\circ K$

$$\therefore f(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1} = \frac{1}{e^{0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

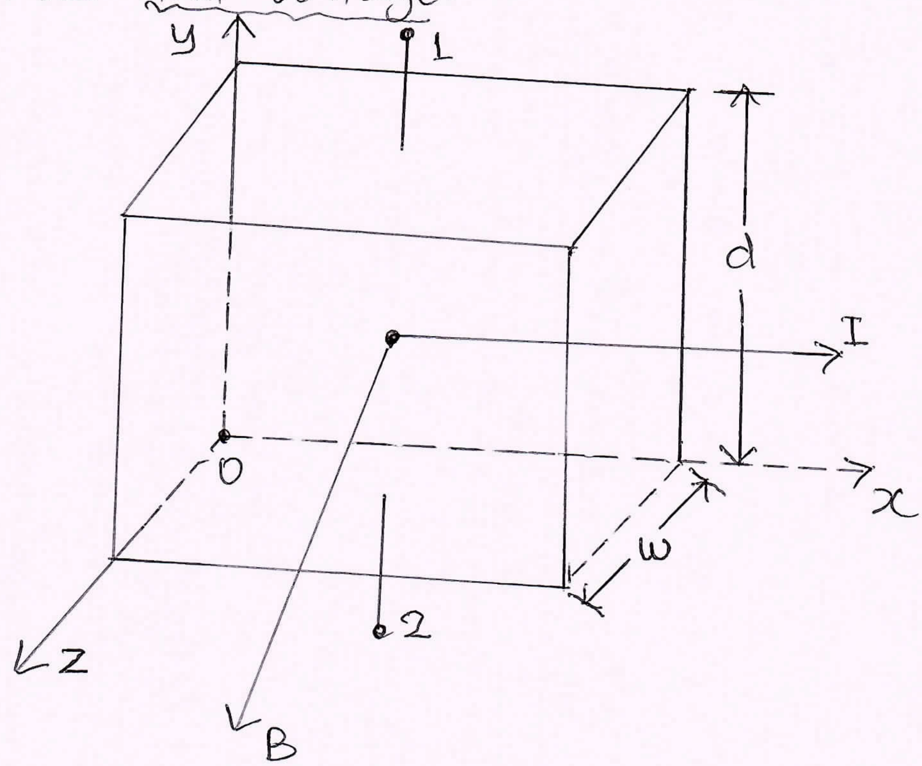
$\boxed{f(E) = 0.5}$ for $E = E_F$

Further, for $E > E_F$, the probability value falls off to zero rapidly.

9 b. What is Hall effect? obtain the expression for Hall coefficient, and express Hall voltage interms of Hall coefficient.

Hall effect:

When magnetic field is applied $\perp^{lar}$ to the direction of the current in a conductor, a potential difference develops along a direction $\perp^{lar}$ to both current and magnetic field. This effect is known as Hall effect. The potential difference developed is known as Hall voltage.



Consider a rectangular conductor of cross-section $w \times d = A$ in which current "I" flows along x-axis. When a magnetic field "B" is applied along z-axis.

Hall voltage V_H develops along Y-axis. between the terminals 1 & 2.

If E_H be the Hall electric field and "v" be the drift velocity (of charges), then under equilibrium conditions.

Force on charges due to Hall electric field =
= Force on charges due to magnetic field.

$$q \cdot E_H = q \cdot v \cdot B$$

$$\therefore \; E_H = vB \qquad \text{Also} \quad E_H = \frac{V_H}{d}$$

$$\therefore \; \frac{V_H}{d} = vB \qquad \text{or} \quad \boxed{V_H = vBd} \longrightarrow (1).$$

Also, w.k.t $I = nqAv$ or $\boxed{v = \frac{I}{nqA}} \longrightarrow (2)$

$\therefore$ From equn (1) & (2), we get

$$V_H = \frac{I \cdot B \cdot d}{nqA}$$

But, $A = w \times d$

$$\therefore \; \boxed{V_H = \frac{I \cdot B \cdot d}{nqw \cdot d}} \Rightarrow \boxed{V_H = \frac{I \cdot B}{nqw}} \longrightarrow (3)$$

The quantity $\frac{I}{nq}$ is the coefficient of charge density & is called the Hall coefficient "R_H".

$$\boxed{R_H = \frac{I}{nq}} \longrightarrow (4)$$

From equn (3) & (4), we get

$$V_H = R_H \cdot \frac{I \cdot B \cdot d}{A}$$

$$\boxed{R_H = V_H \cdot \frac{A}{I \cdot B \cdot d}} \longrightarrow (5)$$

R_H & charge density can be determined as V_H, B, d and A are all measurable quantities.

9 The dielectric constant of sulphur is 3.4.
c. Assuming a cubic lattice for its structure. Calculate the electronic polarizibility of sulphur (Given: Density of sulphur = 2.07 gm/cc and atomic weight = 32.07)

Given data:

Dielectric constant; $\varepsilon_r = 3.4$

Density, $D = 2.07\, gm/cc = 2.07 \times 10^3\, Kg/m^3$

Atomic weight $= 32.07$

Crystal structure of sulphur is cubic.

To find:

Electronic polarizability, $\alpha_e = ?$

Solution:

Since the crystal structure of sulphur is cubic we can apply Claussius - Mossotti equation,

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}$$

$$\therefore \quad \alpha_e = \frac{3\varepsilon_0}{N}\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] \quad \text{—— (1)}$$

Now, N be the number of atoms/unit volume can be written as

$$N = \frac{N_A \times 10^3 \times D}{\text{atomic weight}}$$

$$N = \frac{6.025 \times 10^{26} \times 2.07 \times 10^3}{32.07}$$

$$\therefore \quad N = 3.89 \times 10^{28} / m^3$$

By substituting the values for various parameters, Equn (1) becomes.

$$\alpha_e = \frac{3 \times 8.854 \times 10^{-12}}{3.89 \times 10^{28}}\left[\frac{3.4 - 1}{3.4 + 2}\right]$$

$$\alpha_e = 3.035 \times 10^{-40}\ F\text{-}m^2$$

$\therefore$ Electronic polarizability of sulpher is $3.035 \times 10^{-40}\ F\text{-}m^2$.

10 a. Mention the assumptions of Quantum free electron theory. Discuss two success of quantum free electron theory. (8M)

Assumptions of QFET:

1. The valence $\bar{e}$ are free to move inside the metal.
2. The energies of $\bar{e}$ are quantized and the the distribution of $\bar{e}$ in the allowed descrete energy levels is according to Pauli's exclusion principle which prohibits more than one $\bar{e}$ in single quantum state
3. The $\bar{e}$ cannot escape from the nucleus due to potential barrier inside in an atom.
4. The electrostatic force of attraction between the free electrons with lattice ions and force of repulsion amongst the free electrons are negligible.

Success of QFET:

1. According to QFET, both experimental and theoretical values of Lorentz number are in good agreement with each other.
2. The concept of electrical conductivity & thermal conductivity are explained by this theory.
3. The concept of photoelectric effect, compton effect & blackbody radiation are explained by this theory.
4. The QFET gives the correct dependence of resistivity temperature.
5. Electrons energy is distributed according to Fermi-Dirac statistics.
6. Electron energies are discrete & quantized.

10 b. Define the term internal field in case of solid diectrics with one-dimensional equation. Explain polar and non-polar dieelectrics with examples. (8M)

Internal field in case of solid dielectrics:

In case of solids & liquids subjected to external electric field the atoms are surrounded on all sides by other polarized atoms and the internal intensity of the electric field at a given point of the material is, not equal to the intensity of the applied field E.

The internal field Ei is defined as the electric field acting at the location of a given atom, is given by the sum of the electric field created by the neighbouring atoms & the applied field. In evaluation of the bulk polarization, the additional effects of the surrounding polarized atoms are to be taken into account.

The effective field intensity Ei in the dielectrics is given by

$$\boxed{E_i = E + E'}$$

where E' is the field due to neighbouring atoms.

Polar and Non-polar dielectrics:

Whether a molecule is a polar or non-polar can be judged from its structure.

Symmetric molecules are non-polar, since the centres of gravity of +ve & -ve charges coincide with each other.

Ex: Thus monoatomic molecules He, Ne, Ar & Xe are non-polar. Molecules consisting of two identical atoms linked by a homopolar bonds, such as H_2, N_2, C_2 are non-polar.

Polar dielectrics:

Asymmetric molecules are polar. The molecules of ionic compounds with a heteropolar bond such as potassium iodide (KI) have a high dipole moment and are polar.

10 C. The intrinsic charge carrier concentration of germanium is $2.4\times10^{10}/m^3$, calculate its resistivity, if mobility of electrons & holes respectively are $0.39\ m^2/V\text{-}s$ and $0.19\ m^2/V\text{-}s$.

Given data:

Intrinsic carrier concentration for germanium,

$$n_i = 2.4\times10^{19}/m^2$$

Electron mobility, $\mu_e = 0.39\ m^2/V\text{-}s$

Hole mobility, $\mu_h = 0.19\ m^2/V\text{-}s$

To find:

Resistivity of the sample, $\rho = ?$

Solution:

Equation for conductivity σ_i

$$\sigma_i = n_i e(\mu_e + \mu_h)$$

But resistivity, $\rho_i = \frac{1}{\sigma_i}$

$$\rho_i = \frac{1}{n_i e(\mu_e+\mu_h)}$$

$$= \frac{1}{(2.4\times10^{10})\times1.6\times10^{-19}(0.39+0.19)}$$

$$\boxed{\rho_i = 0.449\ \Omega\text{-}m}$$

∴ Resistivity of germanium is 0.449 m.

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