Staff: Plasio. F Dias.

Subject: Microwaves & Antennas

Subject Code: 18EC63.

Max. Marks: 100.

Sem: 6

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Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T., HALIYAL (U.K.)

USN | | | | | | | | | | **15EC71**

Seventh Semester B.E. Degree Examination, Dec.2019/Jan.2020
Microwaves and Antennas

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. List four applications of Reflex Klystron. **(04 Marks)**

b. Derive transmission line equations in voltage and current forms. **(06 Marks)**

c. A transmission line is terminated in a resistive load of 1000Ω and has $L = 9\mu H/m$ and $C = 100pF/m$. Calculate reflection coefficient and standing wave ratio. **(06 Marks)**

OR

2 a. Define reflection coefficient. Derive an expression for reflection coefficient at load in terms of characteristic impedance and load impedance. **(08 Marks)**

b. Explain microwave system with the aid of a diagram. **(08 Marks)**

Module-2

3 a. For a two port network with mismatched load derive an expression for input reflection coefficient. **(06 Marks)**

b. Draw the diagram of Magic–Tee. Derive S–matrix of Magic Tee. **(10 Marks)**

OR

4 a. What is a reciprocal device? Write five point comparison among [S], [Z] and [Y] matrices. **(06 Marks)**

b. Given $[z] = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$. Find S–matrix. **(05 Marks)**

c. Explain coaxial line fixed alternator with a diagram. **(05 Marks)**

Module-3

5 a. Derive characteristic impedance of micro-strip lines. **(08 Marks)**

b. Define the following terms with respect to antennas :
i) Beam area
ii) Radiation intensity
iii) Beam efficiency
iv) Directivity. **(08 Marks)**

OR

6 a. Describe ohmic skin losses and radiation losses in micro-strip lines. **(10 Marks)**

b. A parabolic reflector antenna is circular in cross section with a diameter of 1.22m. If the maximum effective aperture is 55% of the physical aperture, calculate gain of the antenna in dB at 20 GHz. **(06 Marks)**

Module-4

7 a. Prove that directivity for a source with unidirectional pattern of $U_m \cos^n\theta$, where 'n' can be any number, can be expressed as $D = 2(n + 1)$. (06 Marks)

b. Obtain filed expression of two isotropic point sources of same amplitude and phase. (10 Marks)

OR

8 a. State and explain power theorem. (06 Marks)

b. Derive an expression for radiation resistance of short electric dipole. (10 Marks)

Module-5

9 a. Find directivity and radiation resistance of a loop antenna with diameter 2λ. (06 Marks)

b. Write a short note on Helical antenna geometry. (06 Marks)

c. What is the directivity in dB of a rectangular horn antenna, which has physical aperture of $81\lambda^2$, with aperture efficiency 89%? (04 Marks)

OR

10 a. Derive radiation resistance of a small single turn circular loop antenna with uniform phase current. (08 Marks)

b. Draw the structure of a pyramidal horn antenna. Use the principle of equality of path length and bring out the optimum horn dimensions. (08 Marks)

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Department of Electronics and Communication Engg

Subject : Microwaves and Antennas

Subject code : 18EC63

Sem : 6

Question Paper : 2

Max. marks : 100

Module – 1

1 (a) List four applications of Reflex klystron (04m)

(b) Derive transmission line equation in voltage and current forms. (8m)

(c) A transmission line is terminated in resistive load of 1000 Ω and has L = 9 μH/m and C = 100 pF/m. Calculate reflection coefficient and standing wave ratio. (8m)

OR

2 (a) Define reflection coefficient. Derive an expression for reflection coefficient at load interms of characteristic impedance and load impedance (08m)

(b) Explain microwave system with the aid of a diagram (4m)

(c) A telephone line has R = 6 ohms/km, L = 2.2 mh/km C = 0.005 μf/km and G = 0.05 μmho/km. Determine Zo, α, β at 1 KHz. If the line length is 100 km, determine the attenuation and phase shift of the signal. Calculate the phase velocity of the signal. (8m)

P.P. Dias

Module - 2

3 (a) For a two port network with mismatch load derive expression for input reflection coefficient. (8M)

(b) Draw the diagram of Magic Tee. Derive S matrix of Magic Tee (8M)

(c) Explain phase shift property of S matrix with diagram (4M)

OR

4 (a) What is reciprocal device? Write five point comparison among [S], [Z] and [Y] matrices. (8M)

(b) Explain coaxial line fixed attenuator with Diagram (4M)

(c) Write short notes on (i) Coaxial cable (ii) Phase shifters (8M)

Module - 3

5 (a) Derive characteristic impedance of Microstripline (8M)

(b) Define the following term with respect to antennas (i) Beam Area (ii) Directivity iii) Radiation Intensity (iv) Beam efficiency and Radiation resistance (8M)

(c) Explain parallel stripline. (4M)

P.T.O.

Module - 3

OR

6 (a) Describe ohmic skin losses and radiation losses in microstrip lines (8m)

(b) A parabolic reflector antenna is circular in cross section with diameter of 1.22 m. If the maximum effective aperture is 55% of the physical aperture. Calculate the gain of the antenna in dB at 20 GHz. (8m)

(c) Explain Radio communication link (4m)

Module - 4

7 (a) Prove that directivity for a source with unidirectional pattern of $U_m \cos^n \theta$, where 'n' can be any number can be expressed as $D = 2(n+1)$ (6m)

(b) Obtain field expression of two isotropic point sources of same amplitude and phase (8m)

(c) Explain pattern multiplication (6m)

OR

8 (a) Derive an expression for radiation resistance of short electric dipole. (8m)

(b) Explain thin Linear Antenna (6m)

(c) Compare fields of small loop and short electric dipole. (6m)

Module - 5

9 (a) Find the directivity and radiation resistance of loop antenna with diameter of 2λ (6M)

(b) Write short note on Helical Antenna geometry (6M)

(c) Explain Horn Antenna. What is the directivity in dB of a rectangular horn antenna, which has physical aperture of $81\lambda^2$, with aperture efficiency $\epsilon_{ap} = 0.6$. (8M)

OR

10 (a) Derive radiation resistance of a small single turn circular loop antenna with uniform phase current. (10M)

(b) Draw the structure of a pyramidal horn Antenna. Use the principle of equality of path lengths and bring out the optimum horn dimensions. (10M)

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PTO.

Scheme and Solution

Subject: Microwave and Antennas

Max. Marks: 1

Subject Code: 18EC63

1 (a) Four applications of Reflex Klystron

1×4 = 4M
List

Reflex klystron is used in applications where variable frequency is desirable
- Radio Receivers
- Parametric amplifiers
- Local oscillators of microwave receivers
- As signal source in microwave generators.

(b) Transmission line Equation in voltage and current forms.

Diagram + Derivation
4 + 4 = 8M

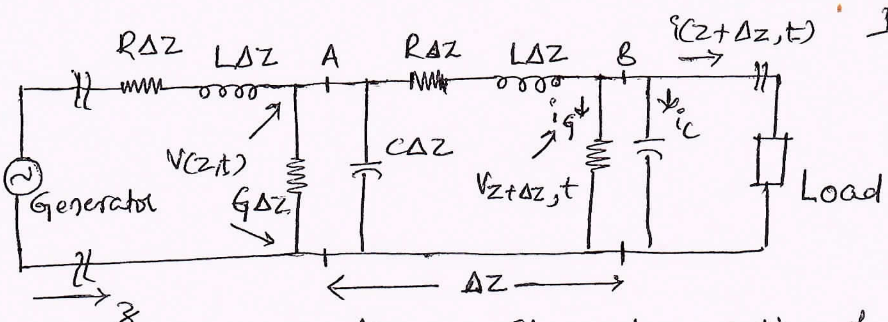


fig 1. Elementary section of Transmission line

By KVL,

$$V(z,t) = i(z,t) R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t} + V(z,t) + \frac{\partial V(z,t)}{dz} \cdot \Delta z$$

$$-\frac{\partial v}{\partial z} = Ri + L \cdot \frac{\partial i}{\partial t}$$

By KCL

$$i(z,t) = v(z+\Delta z, t) G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z, t)$$

$$-\frac{\partial i}{\partial z} = GV + C \cdot \frac{\partial v}{dt}$$

$$\frac{\partial^2 v}{\partial z^2} = RGV + (RC + LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial t^2} = RGi + (RC+LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}$$

$$v(z,t) = \operatorname{Re} V(z) e^{j\omega t}$$

$$i(z,t) = \operatorname{Re} I(z) \cdot e^{j\omega t}$$

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z}$$

$$\gamma = \alpha + j\beta$$

(c)

$R_L = 1000\,\Omega$

$L = 9\,\mu H/m$

$C = 100\,pF/m$

$\Gamma = ?$

$\rho = ?$

Data + Solution + Formula

$1 + 4 + 3 = 8M$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{9 * 10^{-6}}{100 * 10^{-12}}} = 0.03 * 10^3 = 30\,\Omega$$

Reflection coefficient

$$\Gamma \text{ or } K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{1000-20}{1000+20} = \frac{980}{1020} = 0.960$$

Voltage standing wave ratio

$$\rho = VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.96}{1-(0.96)} = \frac{1.96}{0.04} = 49$$

PRD.

2 (a) Reflection coefficient

Defn + Derivation
3 + 5 = 8m

It is the ratio of the reflected voltage, at the receiving end which is $V_- e^{\gamma l}$, to the incident voltage at the receiving end, which is $V_+ e^{\gamma l}$

Reflection coefficient = $\dfrac{\text{reflected voltage or current}}{\text{incident voltage or current}}$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{-I_{ref}}{I_{inc}}$$

Derivation: Incident voltage & current

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$

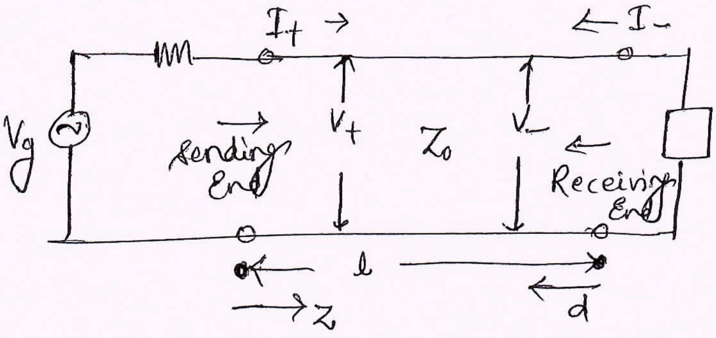
$$I = I_+ e^{-\gamma z} + I_- e^{+\gamma z}$$

$$I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

$$V_l = V_+ e^{-\gamma l} + V_- e^{\gamma l}$$

$$I_l = \frac{1}{Z_0} (V_+ e^{-\gamma l} - V_- e^{\gamma l})$$

$$Z_l = \frac{V_l}{I_l} = Z_0 \frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}}$$



$$\Gamma_l = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_L - Z_0}{Z_l + Z_0}$$

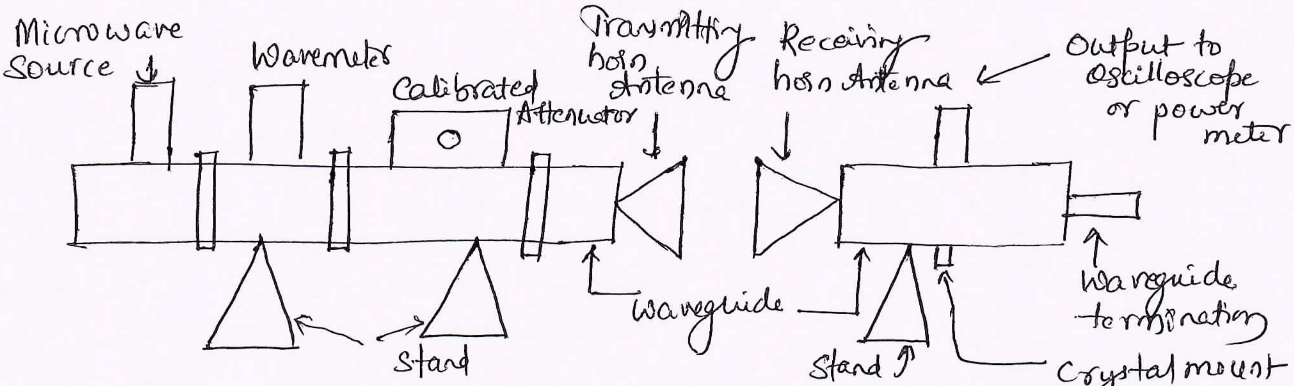
P.F. Bijan.

(b) Microwave system

- consists of transmitter subsystem
  - receiver subsystem

Diagram + Explanation
2 + 2 = 4m

- Transmitter subsystem involves
  - microwave oscillator
  - waveguides
  - transmitting antenna
- Receiver subsystem involves
  - receiving antenna
  - transmission line or waveguide
  - microwave amplifier
  - receiver



(c)

$R = 6$ ohms/km
$L = 2.2$ mh/km
$C = 0.005$ μf/km
$G = 0.05$ μmho/km
Length (line) = 100 km

$Z_0 = ?$
$\alpha = ?$
$\beta = ?$
$f = 1$ KHz
attenuation = ?
phase shift = ?

Data + formula + Solution
1 + 3 + 4 = 8m

$W = 2\pi * 1000$

$= 6280$ rad/sec

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{6+j6280 * 2.2 * 10^{-3}}{0.05 * 10^{-6} + j6280 * 0.005 * 10^{-6}}}$$

$$= 678.23 - j140.35 \text{ ohm}$$

$$\gamma = \sqrt{((R+j\omega L)(R+j\omega C))}$$

$$= \sqrt{(6+j6280 * 2.2 * 10^{-3})(0.05 * 10^{-6} + j6280 * 0.0005 * 10^{-6})}$$

$$= 0.0045 + j0.0213 = \alpha + j\beta$$

$\alpha = 0.0045$ Np/km

& $\beta = 0.0213$ rad/km

for 100 km length,

attenuation $= 0.45$ Np

$= 8.686 * 0.45$

$= 3.91$ dB

phaseshift $= 2.13$ rad.

Phase velocity $= \frac{\omega}{\beta} = \frac{6280 \text{ (rad/sec)}}{0.0213 \text{ (rad/km)}} = 294.84 * 10^3$ km/sec

3 (a) Two port network with mismatch load

Expression for input reflection coefficient

Explanation + Diagram + Derivation

2 + 2 + 4 = 8m

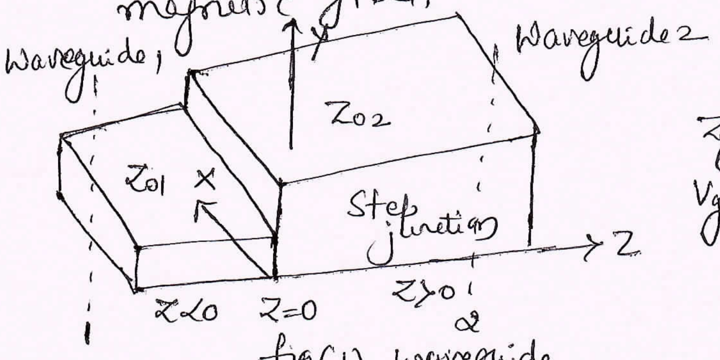
Ex: Two port junction
- waveguide step junction
- coaxial to waveguide transition
- two port network model.

During Propagation of microwave, through junction

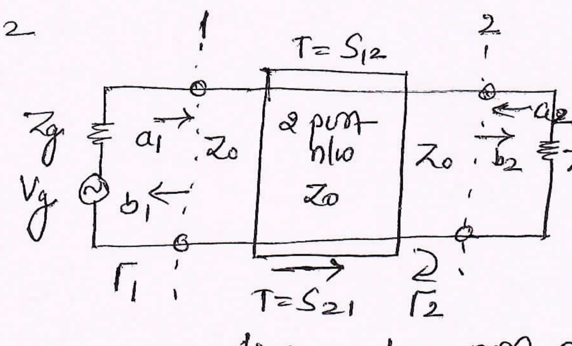
→ Evanescent modes are excited at each discontinuity, which contain reactive energy. Evanescent modes decay very fast from junction & become negligible after distance of order of one wavelength

→ Terminal reference planes 1 & 2 are choosen beyond this distance.

→ Equivalent voltage and currents at these positions are proportional to the total transverse electric & magnetic field.



fig(1) waveguide step junction



fig(2) two port network model

→ normalized load Z_L/Z_0

→ generator impedance Z_g/Z_0

Load reflection coefficient $\Gamma_2 = \frac{a_2}{b_2} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$

& P.A.D...

$$\Gamma_2 = \frac{a_2}{b_2} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

$$b_1 = S_{11} a_1 + S_{12} a_2 = S_{11} a_1 + S_{12} b_2 \Gamma_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2 = S_{21} a_1 + S_{22} b_2 \Gamma_2$$

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22}\Gamma_2}$$

For Mismatch load, input reflection coefficient

$$\Gamma_1 \neq S_{11}$$

For reciprocal network $S_{12} = S_{21}$

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2}$$

If the junction is lossless,

$$S_{11} S_{11}^* + S_{12} S_{12}^* = 1 \quad \text{——— (1)}$$

$$S_{22} S_{22}^* + S_{12} S_{12}^* = 1 \quad \text{——— (2)}$$

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0 \quad \text{——— (3)}$$

For lossless, reciprocal network

from (1) & (2) $\quad |S_{11}| = |S_{22}| \quad$ ——— (4)

from (3) & (4)

$$|S_{12}| = \sqrt{(1 - |S_{11}|^2)}$$

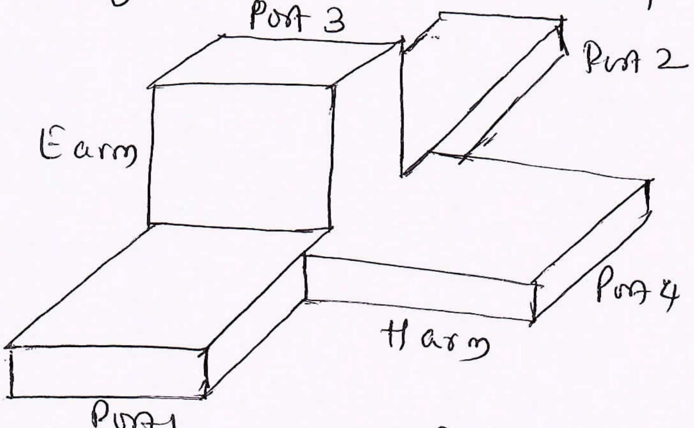
The input reflection coefficient

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2}$$

3(b) Magic Tee + Derivation of S matrix

Diagram + Derivation

3 + 5 = 8M



- Magic Tee is combination of E and H plane Tee
- It is called hybrid Tee.
- Consists of 4 ports.

Magic Tee

from characteristics.

$$S_{13} = S_{31} = 1/\sqrt{2} = S_{24} = S_{42}$$

$$S_{34} = 0$$

$$S_{14} = S_{41} = 1/\sqrt{2} = S_{24} = S_{42}, \; S_{34} = 0$$

$$S_{12} = S_{21} = 0$$

Symmetry property,

$$S_{14} = S_{41} = S_{24} = S_{42}$$

$$S_{31} = S_{13} = -S_{23} = -S_{32}$$

$$S_{34} = S_{43} = 0, \quad S_{21} = S_{12} = 0$$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

Apply unitary property row 1, 2

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{—— (1)}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

subtracting

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

$$\text{or } |S_{11}| = |S_{22}|$$

Unitary property applied to row 3, 4

$$2|S_{13}|^2 = 1 \quad \text{or} \quad S_{13} = 1/\sqrt{2}$$

$$2|S_{14}|^2 = 1 \quad \text{or} \quad S_{14} = 1/\sqrt{2}$$

Substitute in (1)

$$|S_{11}|^2 + |S_{12}|^2 + \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$\text{or } |S_{11}|^2 + |S_{12}|^2 = 0$$

Valid if $S_{11} = S_{22} = 0$

$$S_{12} = 0$$

$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix}$$

$$S_{13} = \frac{1}{\sqrt{2}} = |S_{14}|$$

$$\therefore [S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

3 (c)

Phase shift property

Diagram + Explanation
2 + 2 = 4M

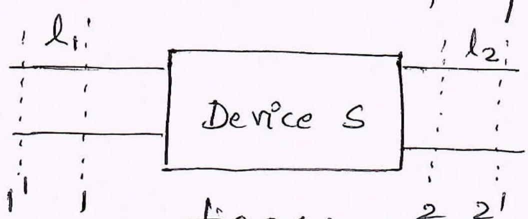


fig 3(c)

- S parameters of network are defined with respect to the position of the port or reference planes.
- Two port network with unprimed reference plane 1, 2 as in fig 3(c)

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

- If reference planes 1 & 2 are shifted outwards to 1' and 2' by electrical phase shift $\phi_1 = \beta_1 l_1$ and $\phi_2 = \beta_2 l_2$

Then new variables

$$a_1 e^{j\phi_1}, \; b_1 e^{-j\phi_1}, \; a_2 e^{j\phi_2}, \; b_2 e^{-j\phi_2}$$

new S matrix S' is given by

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

8
BPD.

7

4 (a) Reciprocal device,

Det^n + 5 points
3 + 5 = 8m

comparison of [S], [Z] & [Y] matrices

- If the port 1 and 2 are the interchanged for a two port network and performance of microwave device is still same, then the network is called as reciprocal network.
- Z matrix for reciprocal network is symmetric.

Comparison between [S] and [Z] matrices

$$[S] = ([Z] - Z_0[U])([Z] + Z_0[U])^{-1}$$
$$= ([Z]/Z_0 - [U])([Z]/Z_0 + [U])^{-1}$$

- Relation betn [S] and [Y] matrices

$$[S] = ([U] - [Y]/Y_0)([U] + [Y]/Y_0)^{-1}$$
$$[\bar{Z}] = (U-S)^{-1}(U+S), \quad \bar{Z} = Z/Z_0$$

- Number of elements are equal.
- For reciprocal device both [Z] and [S] satisfy reciprocity properties.

$$Z_{ij} = Z_{ji}, \quad S_{ij} = S_{ji}$$

- If [Z] is symmetrical, [S] is also symmetrical.

Advantages of [S] over [Z] & [Y].

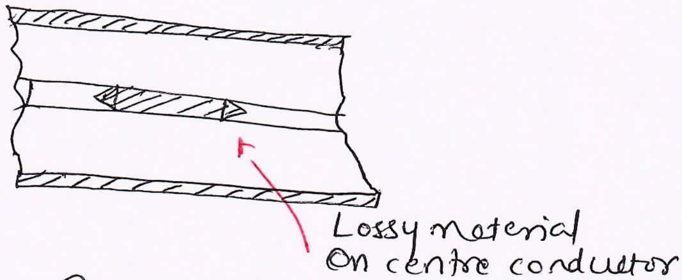
- In microwave techniques, source remains constant in power. Beside frequency measurements, the only other possible measurement parameters are VSWR, power & phase. These are essentially measurements of b/a, $|a|^2$ and $|b|^2$. Such direct correspondance is not possible with [Z] or [Y] representation.
- Unitary property of [S] helps, quick check of power balance of lossless structure. No, such immediate check is possible with [Z] or [Y]
- [S] is defined for given set of reference planes only. If the reference planes are changed, S coefficients vary only in phase. This is not case in [Z] or [Y], because voltage & current are functions of complex impedance and therefore both magnitude & phase change in [Z] & [Y]

? points.

4 (b) Coaxial line fixed attenuator:

Diagram + Explanation

$\Rightarrow 2 + 2 = 4m$



(a) Coaxial line fixed attenuator

- Uses a film with losses on centre conductor to absorb some of the power.
- Attenuators are passive devices, used to control power levels in a microwave system, by partially absorbing the transmitted signal wave.
- Attenuators are designed using resistive films (aquadag)
- Thin dielectric strip coated with resistive film, is construction details of fixed waveguide type attenuator.
- Dielectric strip is placed at the centre of the waveguide parallel to maximum E field.
- Induced current on the resistive film, due to incident wave results in power dissipation, leading to attenuation of microwave energy.
- Dielectric strip is tapered at both ends, upto a length of more than half wavelength to reduce reflections.

4 (c) i) Coaxial Cable.

ii) Phase shifters

Diagram + Explanation

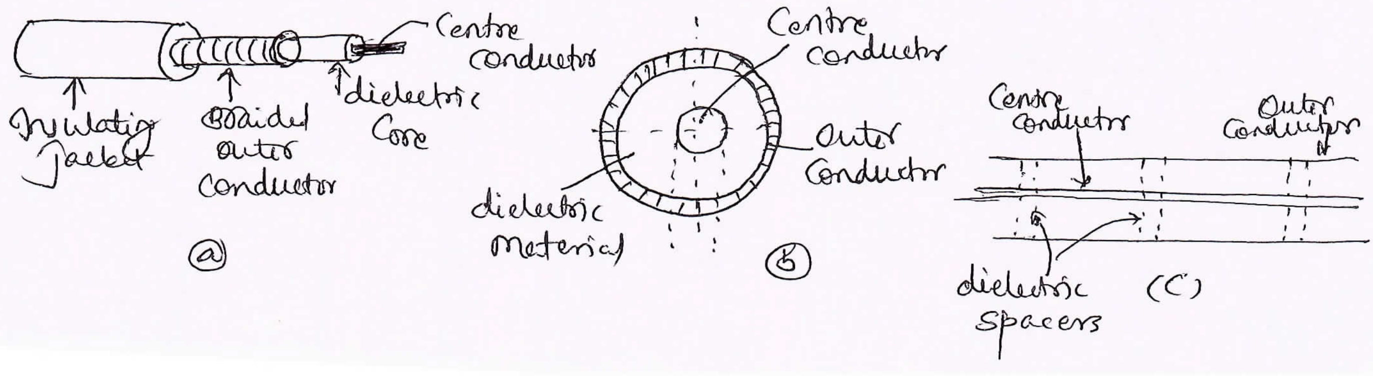
$(2+2) + (2+2)$

$= 4 + 4$

$= 8\ m$

i) Coaxial Cables: Types of Cables, with increasing order of shielding

a) Flexible b) Semi Rigid c) Rigid



• Coaxial cables used for interconnecting microwave components.

• Outer conductor of the coaxial line used to guide the signal through TEM mode.

• Outer conductor shields the external or internal signal leakage through it.

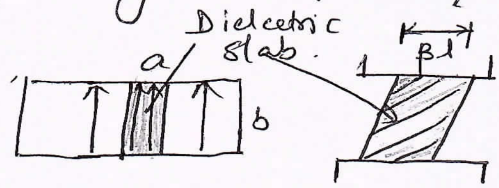
• Standard characteristic impedance 50Ω, 75Ω.

Flexible Cables → Low loss solid or foam polyethylene dielectrics
Outer single or double braid is connected for electromagnetic shielding by using knitted metal wire mesh.

Rigid Cable → Air dielectric
Conductors are supported by small dielectric spacers, such that they do not produce significant discontinuities of the signal flow.

Semi Rigid Cables → Solid dielectric is present.
Thin copper outer conductor exist

Coaxial Cables are used in the frequency range from dc to microwaves.
Attenuation increases with frequency, so upper frequency is limited.



(ii) Phase shifters:
• Two port device
• Produces a variable change in phase of the wave transmitted through it.

• Realized by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum E field.

• Differential phase change is produced due to the change of wavevelocity through the dielectric slab compared to that through empty waveguide

• Two ports are matched by reducing the reflections of the wave from dielectric slab tapered at both ends.

R. S. Dias.

Module - 3

5 (a) characteristic impedance of microstripline.

Explanation + Equation + Derivation
$\boxed{4+2+2=}$

- Characteristic impedance of a microstrip line is a function of the strip width, stripline thickness, the distance between the line and ground plane and the homogeneous dielectric constant of the board material.

Methods of determining characteristic impedance

→ field Equation method :⇒ Needs digital computer & complicated.
- Gives accurate value

→ Comparative method or indirect method:
- Deriving characteristic impedance from well known equation & ~~designer~~ making some changes.

Well known Equation of Z_0 of a wire over ground transmission line

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d} \quad \text{for } h \gg d \qquad (1)$$

ϵ = dielectric constant, medium
h = height from the center of the wire to the ground plane.
d = diameter of the wire.

If Effective or Equivalent values of the relative dielectric constant ϵ_r of medium and diameter 'd' of wire, is determined, Z_0 can be calculated.

Effective dielectric constant ϵ_{re}

for homogeneous dielectric medium

propagation delay time / unit length

$$T_d = \sqrt{\mu\epsilon}$$

$\mu \leftarrow$ permeability of the medium
$\epsilon \leftarrow$ permittivity of the medium

$T_{df} = \sqrt{\mu_0 \epsilon_0} = 3.333$ ns/m or 1.016 ns/ft

$\mu_0 = 4\pi \times 10^{-7}$ H/m or 3.83×10^{-7} H/ft

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m or 2.69×10^{-12} F/ft

relative permeability 1

Propagation delaytime for line in non magnetic medium,

$$T_d = 1.106\sqrt{\epsilon_r} \quad \text{ns/ft}$$

Effective relative dielectric constant for a microstrip line can be related the relative dielectric constant of the board material,

Empirical Equation,

$$\epsilon_{re} = 0.475\epsilon_r + 0.67 \quad \text{—— (2)}$$

$\epsilon_r \leftarrow$ relative dielectric constant of the board material.

$\epsilon_{re} \leftarrow$ effective relative dielectric constant for microstripline.

$$d = 0.67w\left(0.8 + \frac{t}{w}\right) \quad \text{—— (3)}$$

$d \leftarrow$ diameter of the wire overground

$w \leftarrow$ width of the microstripline

$t \leftarrow$ thickness of the microstripline

The limitation of ratio of thickness to width is between 0.1 and 0.8.

Substitute (2) for dielectric constant and Equation (3) for equivalent diameter in Equation (1)

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] \quad \text{for } (h < 0.8w) \quad \text{—— (A)}$$

ϵ_r = relative dielectric constant of the board material.

h = height from microstripline to the ground.

w = width of the microstripline

t = thickness of the stripline

R.P. Dyas.

Equation (4) is the equation of the characteristic impedance for a narrow microstripline.

Velocity of Propagation is,

$$v = \frac{c}{\sqrt{\epsilon_{re}}} = \frac{3 \times 10^8}{\sqrt{\epsilon_{re}}} \text{ m/sec.}$$

Characteristic impedance of wide microstripline, derived by ~~Assadourian~~ Assadourian,

$$Z_0 = \frac{h}{\omega}\sqrt{\frac{\mu}{\epsilon}} = \frac{377h}{\sqrt{\epsilon_r}\,\omega}$$

5 (b) Antenna Parameters:

(Formula + Explanations
2M × 4 = 8M
(parameters)

(i) Beam Area: (Ω_A)

The Beam Area Ω_A is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its maximum value over and was zero else where. Thus power radiated $= P(\theta,\phi)\Omega_A$ Watts.

$$\Omega_A = \iint_{4\pi} P_n(\theta, \phi)\cdot d\Omega \quad \text{Sr Beam Area.}$$

$d\Omega = \sin\theta \cdot d\theta \cdot d\phi \cdot \text{Sr.}$

Beam area or beam solid angle or Ω_A of antenne is given by the integral of the normalized power pattern over a sphere (4π Sr)

Beam area of an antenne, described approximately interms of angles subtended by half power points of the main lobe, in two principal planes.

$$\text{Beam Area} \simeq \Omega_A \cong \theta_{HP}\,\phi_{HP}$$

θ_{HP} or ϕ_{HP} are the half power beamwidths in the two principal planes.
minor lobes neglected.

PRO

5 (3) (ii) Directivity (D)

- Directivity of an antenna is equal to the ratio of the maximum power density $P(\theta,\phi)_{max}$ (Watts/m²) to its average value over a sphere, as observed in the far field of an antenna.

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}}$$ Directivity from pattern

- Directivity is dimensionless ratio ≥ 1

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\cdot d\Omega} = \frac{4\pi}{\Omega_A}$$ Directivity from beam Area Ω_A.

Directivity is the ratio of the area of a sphere (4π Sr) to the beam area Ω_A of the antenna.

Smaller the beam area, the larger the directivity D.

For Isotropic Antenna, $D = 1$

Actual Antenna, $D > 1$

Short dipole with Beam Area $\Omega_A = 2.67\pi$ Sr

$\underline{D = 1.5}$ (1.76 dBi)

(iii) Radiation Intensity

Power radiated from an antenna per unit solid angle is called radiation intensity 'U' (Watts per steradian or per square degree).

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}} = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

Poynting vector S depends on the distance from the antenna (varying inversely as the square of the distance), the radiation intensity $U(\theta,\phi)$, as a function of angle, to its maximum value.

P.P. Bhise.

(iv) Beam Efficiency & Radiation resistance:

Beam efficiency is the ratio of the main beam area to the (total) beam area is called (main) beam efficiency ε_m.

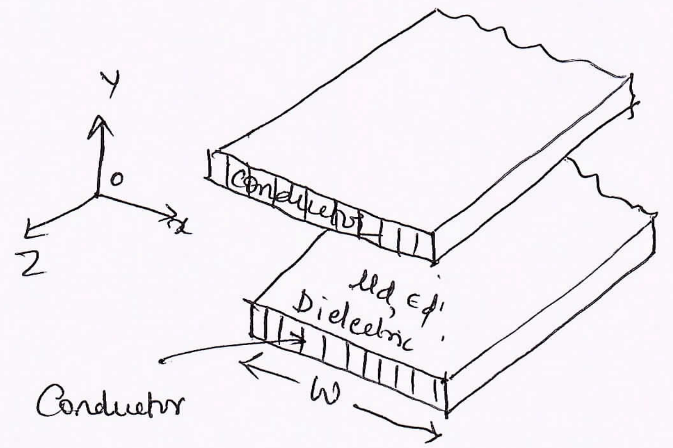
$$\text{Beam Efficiency} = \varepsilon_m = \frac{\Omega_m}{\Omega_A} \quad \text{(dimensionless)}$$

Radiation resistance: R_r

From Circuit point of View, antenna appears to the transmissionlines as a resistance R_r, called radiation resistance. It is a resistance coupled from space to the antenna terminals.

(c) Parallel stripline.

Diagram + Explanation
2 + 2 = 4M



- Parallel stripline consists of two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness.

Plate width is w, separation distance is 'd'
relative dielectric constant of the slab is ε_{rd}

Distributed parameters: $L = \frac{\mu_c d}{w}$ H/m $\qquad R = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$ Ω/m

$C = \frac{\varepsilon d w}{d}$ F/m $\qquad q = \frac{\sigma d w}{d}$ ℧/m

Characteristic Impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{377\, d}{\sqrt{\varepsilon_{rd}} \cdot w} \quad \text{for } w \gg d$$

6 (a) Def^n + Formula + Explan
3 + 2 + 3 = 8

Ohmic Skin Losses and Radiation Losses in Microstripline

For a low loss dielectric substrate, major attenuation factor at microwave frequencies arises due to the current on the strip. The current distribution in the transverse plane is fairly uniform with minimum value at the central axis and shoots upto a maximum at the edges of the strip.

~~Assume~~ uniform current distribution in the region $-W/2 < x < W/2$ or $|x| < W/2$, attenuation constant due to ohmic loss of a wide line ($W/h > 1$) is

$$\alpha_c \cong \frac{8.686}{Z_0 W} \sqrt{\frac{\pi f \mu}{\sigma}} \ \text{dB/m} \quad \text{—— (1)}$$

or $$\alpha_c = \frac{8.686 \, R_s}{Z_0 W} \ \text{dB/cm} \quad \text{for } \frac{W}{h} > 1$$

where $R_s = \sqrt{\frac{\pi f \mu}{\sigma}}$ is the surface skin resistance in Ω/square

$R_s = \frac{1}{\delta \sigma}$ is Ω/square

$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$ is the skin depth in cm.

For narrow microstrip line with $W/h < 1$, equation (1) may not hold good, because current distribution is not uniform.

Radiation Losses:

Radiation losses depend on the substrate thickness (geometry) and dielectric constant.

Radiated power loss is given as $P_{rad} = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 \frac{F_{\epsilon re}}{Z_0} P_t$

$h \leftarrow$ distance between ground plane and microstrip.

$F_{\epsilon re} \leftarrow$ radiation factor

$Z_0 \leftarrow$ characteristic impedance.

Radiation.

$\lambda_0 \leftarrow$ free space wavelength

$$f(\epsilon_{re}) = \frac{\epsilon_{re}+1}{\epsilon_{re}} - \frac{\epsilon_{re}-1}{2\epsilon_{re}\sqrt{\epsilon_{re}}} \ln \frac{\sqrt{\epsilon_{re}}+1}{\sqrt{\epsilon_{re}}+1}$$

$\epsilon_{re} \leftarrow$ Effective dielectric constant.

Quality factor of microstrip is very high

Value of Q of microstrip is limited by radiation losses of substrate and dielectric constant.

$$Q = 0.63h\sqrt{\sigma f}$$

$h \leftarrow$ distance between ground plane and microstrip

$\sigma \leftarrow$ conductivity of dielectric

$f \leftarrow$ Operating frequency

As open strip suffers from radiation losses, it is the one of the limitation of Microstripline.

6 (b) Parabolic Reflector:

Data + formula + Solution

2 + 3 + 3 = 8M

$d = 1.22$ m

$A_{em} = 0.55 A_p$.

$f = 20$ GHz

$A_p = \pi r^2$

$= \pi (0.61)^2$

$A_p = 1.168$

Since $A_{em} = 0.55 A_p$

$A_{em} = 0.55 \times 1.168$

$A_{em} = 0.6424$

$$\lambda = c/f = \frac{3 \times 10^8}{20 \times 10^9} = 0.015 \text{ m}$$

$$D = \frac{4\pi A_{em}}{\lambda^2}$$

Spro

$$D = \frac{4\pi * 0.642}{(0.015)^2}$$

$$D = 35.85 * 10^3$$

$$D = G = 35.85 * 10^3$$

$$G_{dB} = 10\log_{10}(35.856 * 10^3)$$

$$G_{dB} = 45.55 \text{ dB}.$$

6 (c) Radio communication link

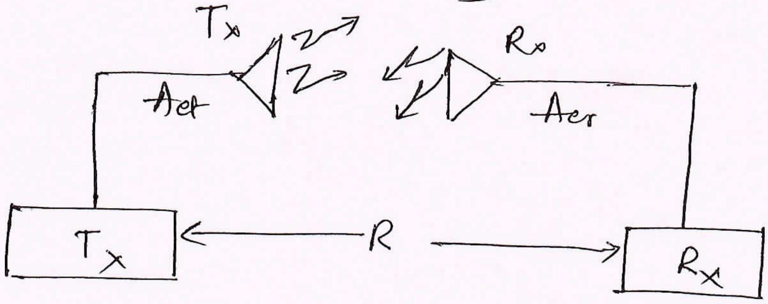


Diagram + Explanation
2 + 2 = 4M

Let us assume lossless matched Antenna. Let transmitter field power P_t to transmitting antenna of effective aperture A_{et} at a distance 'R'. The Receiving antenna with effective apperture A_{er} intercept some of the power radiated by transmitting antenna and deliver to receiver R. Assume transmitting antenna is isotropic then power per unit area available at receiving antenna is $S_r = P_t / 4\pi R^2$

If Antenna has gain G_t

$$S_r = \frac{P_t G_t}{4\pi R^2}$$

Power collected by receiving Antenna with A_{er}

$$P_r = S_r A_{er}$$

$$G_t = D_t = \frac{4\pi A_{et}}{\lambda^2}$$

$$P_r = \frac{P_t \, 4\pi \, A_{et} A_{er}}{4\pi R^2 \lambda^2}$$

$$P_r = \frac{P_t A_{et} A_{er}}{R^2 \lambda^2}$$

$$\frac{P_r}{P_t} = \frac{A_{et}\, A_{er}}{R^2 \lambda^2}$$ Friss transmission formula

$P_t \leftarrow$ Transmitting power W
$P_r \leftarrow$ Receiving power W
$A_{et} \leftarrow$ effective Aperture of T_x
$A_{er} \leftarrow$ effective aperture of R_x
$\lambda \leftarrow$ wavelength
$R \leftarrow$ distance between Antenna.

Module – 4

7 @ $U = U_m \cos^n \theta$ Data + formule + Soln

$D = 2n+1$ prove. $1+2+3 = 6m$

$0^\circ \le \theta \le 90^\circ$
Pattern are independent of azimuth angle ϕ.

$U = U_m \cos^n \theta$.
$n \leftarrow$ any number

$$P = \int_0^{2\pi}\int_0^{\pi/2} U_m \cos^n\theta \cdot \sin\theta \cdot d\theta \cdot d\phi$$

$$D = \frac{\cancel{4\pi}}{\cancel{2\pi}\int_0^{\pi/2} \sin\theta \cdot \cos^n\theta\, d\theta} = \frac{2}{-\left.\frac{\cos^{n+1}\theta}{n+1}\right|_0^{\pi/2}}$$

$$= 2(n+1)$$

for $n=1$

$D \underset{\text{approx}}{\simeq} 2.78 = 4.4$ dBi
$D \underset{\text{exact}}{\simeq} 4 = 6.0$ dBi

$D_{exact} - D_{approx} = 1.6$ dB

for $n = 2$

$D \underset{\text{approx}}{=} 4.94 \Rightarrow 6.9$ dBi
$D_{exact} = 6 \Rightarrow 7.8$ dBi
$D_{exact} - D_{approx} = 0.9$ dB

For $n=3$

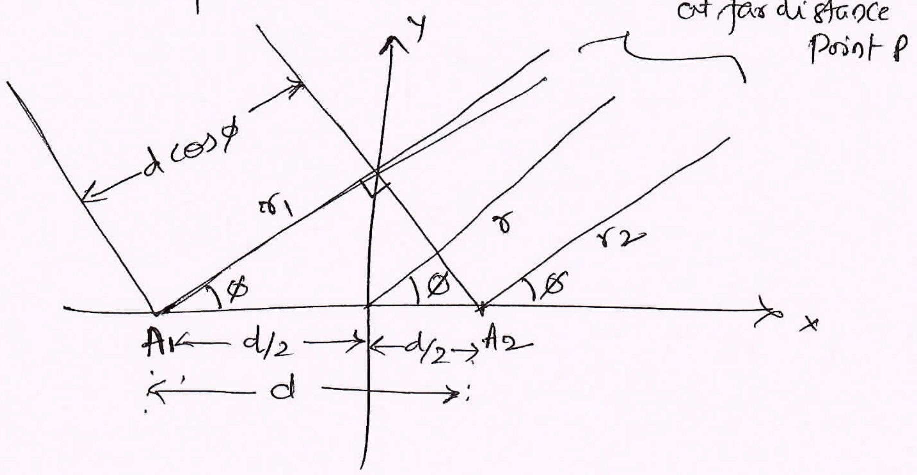
$D_{app.} = 7.3 \simeq 8.6$ dBi $D_{exact} = 8 \Rightarrow 9.0$ dBi, $D_{exact} - D_{approx} = 0.4$ dB

JPD.

(13)

7 (b) Two isotropic point sources of same amplitude and phase

Diagram + Formula + Explanation/ Derivation $2+3+3 = 8m$



Consider two isotropic point source A_1 & A_2 separated by distance 'd' & located symmetrically. Let Distance between point 'P' (far distance point) and A_1, A_2 are r_1 and r_2 respectively.

At far distance, $r_1 = r_1 = r_2$

Both sources are supplied with same current of same amplitude and phase. Radiation from source A_2 reaches earlier as compared to A_1, because A_1 has to travel extra path difference $d\cos\phi$.

$$P_d = d\cos\phi$$

Path difference interms of λ

$$P_d = \frac{d\cos\phi}{\lambda}$$

Phase difference $\psi = 2\pi P_d$

$$\psi = \frac{2\pi d\cos\phi}{\lambda}$$

$$\psi = \beta d\cos\phi$$

$$\psi = d_r\cos\phi$$

where $d_r = \beta d$

Consider origin of co-ordinate system as reference for phase

1. At distant point in the direction of ϕ the field from source 1 is retarted by $\frac{1}{2} d_r\cos\phi$.
2. At distant point in the direction of ϕ, the field from

source 2 is advanced by $\frac{1}{2} d r \cos\phi$

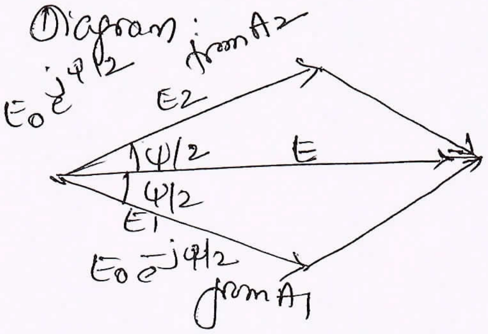
Let E_1 make the far field from source A_1
E_2 be the far field from source A_2.
Total field,

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = 2E_0 \cos(\psi/2)$$

$$E = 2E_0 \cos\left(\frac{d r \cos\phi}{2}\right)$$

Vector Diagram:



Let $d = \lambda/2$

$$\psi = d r \cos\phi$$

at $d = \lambda/2$

$$\psi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \cos\phi$$

$$\psi = \pi \cos\phi$$

$$E = 2E_0 \cos\left(\frac{\pi \cos\phi}{2}\right)$$

Normalized field

$$E_n = \cos\left(\frac{\pi\cos\phi}{2}\right)$$

Normalized field pattern

(a) Direction of maximum

$$\cos\left(\frac{\pi\cos\phi}{2}\right) = \pm 1$$

$$\frac{\pi\cos\phi}{2} = \pm n\pi \quad , n = 0, 1, 2$$

$$\phi = \cos^{-1}(\pm 2n)$$

if $n = 0$

$\phi = \pi/2, 2\pi/3$

$$\boxed{\phi_{max} = 90^\circ, 270^\circ}$$

(b) Direction of nulls

$$\cos\left(\frac{\pi\cos\phi}{2}\right) = 0$$

$$\frac{\pi\cos\phi}{2} = \pm(2n+1)\frac{\pi}{2}$$

$$\phi = \cos^{-1}(\pm 2n+1)$$

if $n = 0$, $\boxed{\phi_{null} = 0, 180^\circ}$

Direction of HPBW (HP Pd)

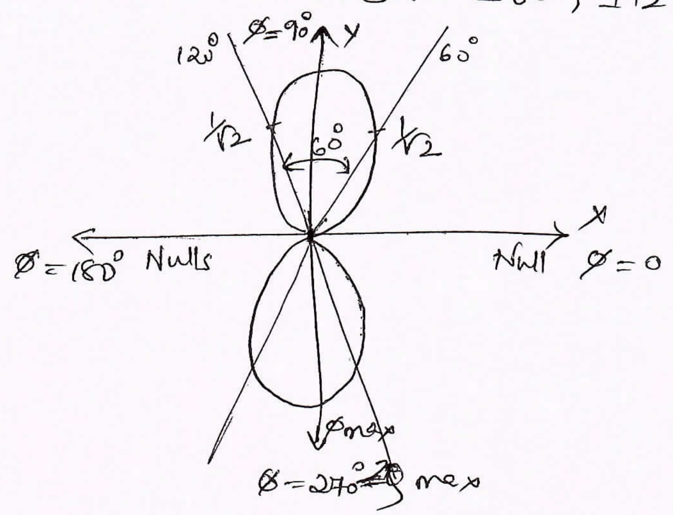
$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{4}$$

$$\phi = \cos^{-1}\pm(\pm 2n+1)\frac{1}{2}$$

if $n=0$,

$$\phi_{HPBW} = \pm 60^\circ, \pm 120^\circ$$



7 (c) Pattern multiplication

Diagram + Explanation
3 + 3 = 6m

The field pattern of an array of non isotropic but similar point sources is the product of pattern of individual source and the pattern of an array of isotropic point source having the same location, relative amplitudes and phase, as the nonisotropic point source.

If the field of non isotropic source

If the field of nonisotropic source and the array of isotropic source vary in phase with space angle that they have phase pattern that is not constant. The statement of principle of pattern multiplication may be extended to include this more general case.

Statement :- The total field pattern of an array of nonisotropic but similar sources is a product of individual source pattern and pattern of array of isotropic point sources each located at the phase centre of individual source and having same relative amplitude and phase while the total phase pattern is the sum of phase pattern of individual source and the array of isotropic point source.

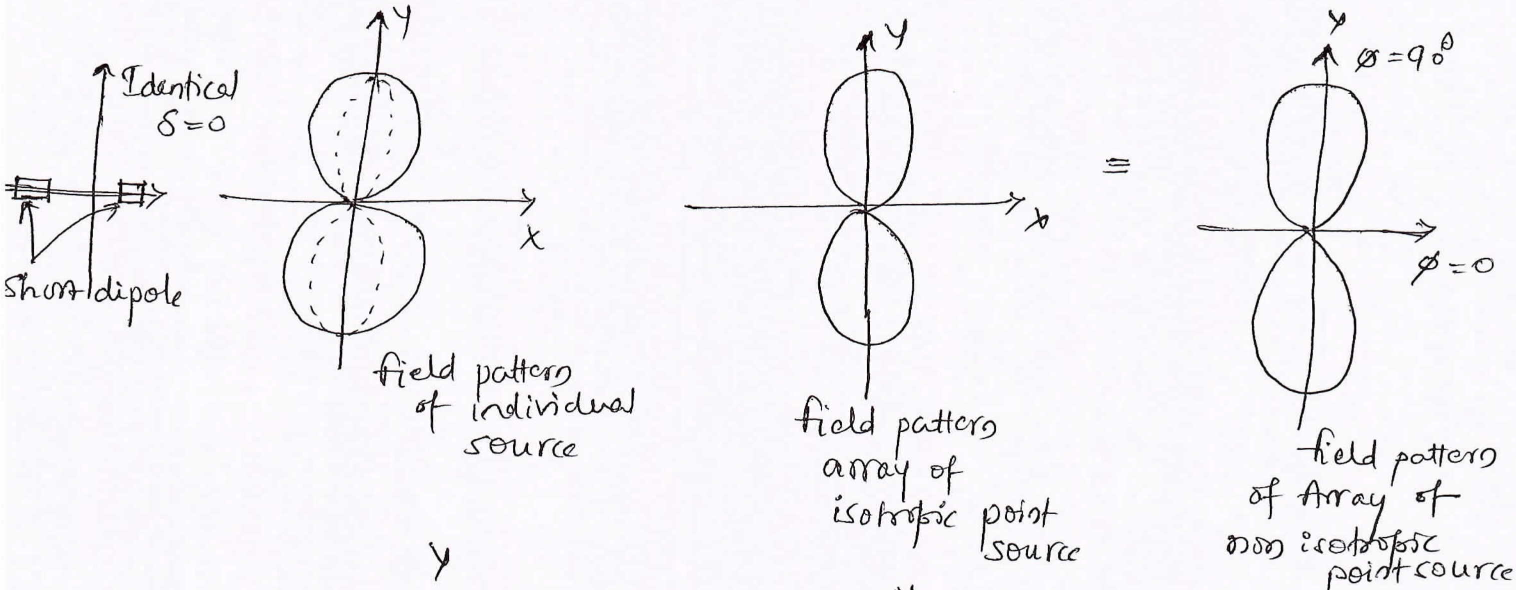
$$E = \underbrace{f(\theta,\phi)\ F(\theta,\phi)}_{\text{field pattern}} \lfloor \underbrace{f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{Phase pattern}}$$

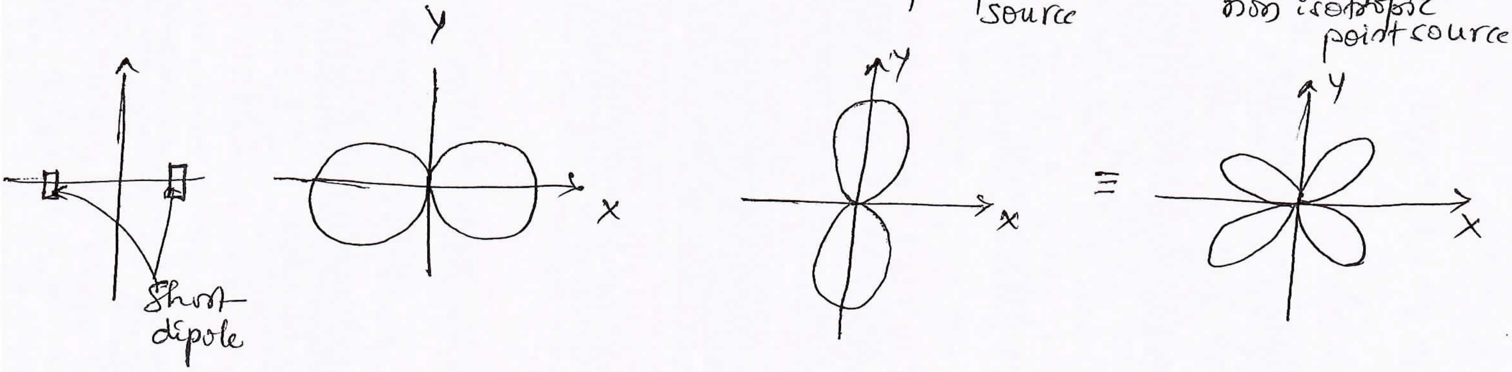
$f(\theta,\phi) \rightarrow$ field pattern of individual source

$f_p(\theta,\phi) \rightarrow$ phase pattern of individual source.

$F(\theta,\phi) \rightarrow$ field pattern of array of isotropic sources

$F_p(\theta,\phi) \rightarrow$ Phase pattern of array of isotropic sources.





PTO.

8 (a) Radiation resistance of short electric dipole.

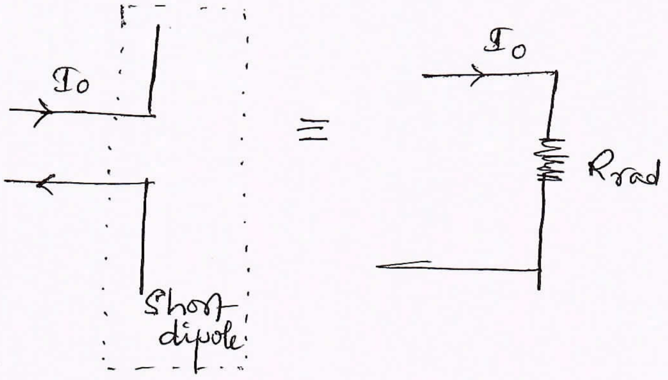


Diagram + formula + Derivation
2 + 3 + 3 = 8m

Calculate the total power radiated by integrating pointing vector of the far field over a sphere. This power is then equated to $I^2 R_{rad}$, where I is rms current.

$$P_{av} = \frac{1}{2} Re \{ \vec{E} \times H^* \}$$

$$= \frac{1}{2} Re \{ E_\theta H_\phi^* \hat{a}_r - E_r H_\phi^* \hat{a}_\theta \}$$

Radiation field is far field.

$$E_r = 0$$

$$E_\theta = \frac{I_o L \sin\theta}{4\pi \epsilon_o} e^{j\omega(t - r/c)} \left\{ \frac{j\omega}{c^2 r} \right\}$$

$$H_\phi = \frac{I_o L \sin\theta}{4\pi} e^{j\omega(t - r/c)} \left\{ \frac{j\omega}{cr} \right\}$$

$$\frac{E_\theta}{H_\phi} = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\Omega = 120\pi$$

$$E_\theta = \sqrt{\frac{\mu_o}{\epsilon_o}} H_\phi$$

$$S_r = \frac{1}{2} Re \left\{ \sqrt{\frac{\mu_o}{\epsilon_o}} H_\phi H_\phi^* \right\}$$

$$S_r = \frac{1}{2} Re \left\{ \sqrt{\frac{\mu_o}{\epsilon_o}} |H_\phi|^2 \right\}$$

$$S_r = \frac{1}{2} \frac{I_o^2 L^2 \sin^2\theta \, \omega^2}{(4\pi)^2 (cr)^2} \sqrt{\frac{\mu_o}{\epsilon_o}}$$

$$P = \iint S_r \, ds$$

$$= \int_0^{2\pi}\int_0^{\pi} S_r\, r^2 \sin\theta\, d\theta . d\phi$$

$$= \iint \frac{1}{2} \frac{I_0^2 L^2 \omega^2}{(4\pi)^2 (C\cancel{r})^2} \sqrt{\frac{\mu_0}{\epsilon_0}}\, \cancel{r^2} \sin^3\theta\, d\theta\, d\phi$$

$$P = \frac{1}{2} \frac{I_0^2 L^2 \omega^2}{(4\pi)^2 C^2} \sqrt{\frac{\mu_0}{\epsilon_0}} * 2\pi \int_0^{\pi} \sin^3\theta . d\theta$$

$$I = \int_0^{\pi} \sin^3\theta . d\theta = \frac{4}{3}$$

$$P = \frac{1}{2} \frac{I_0^2 L^2 \omega^2}{(4\pi)^2 C^2} \sqrt{\frac{\mu_0}{\epsilon_0}} * 2\pi * \frac{4}{3}$$

$$P = \frac{I_0^2 L^2 \beta^2}{3 * 4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$P = \frac{I_0^2 L^2 \beta^2}{12\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \omega,$$

or $\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\omega^2 I_0^2 L^2}{c^2\, 12\pi}$

$\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{(2\pi)^2 I_0^2}{12\pi} \left(\frac{L}{\lambda}\right)^2$

$P \propto \omega^2$, $P \propto I_0^2$

It radiates more & more power as frequency increases & reaches out far distance.

By circuit point of view, this power should be equal to $P = \frac{1}{2} I_0^2 R_{rad}$ [I_0 is peak current]

$$\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\beta^2 I_0^2 L^2}{12\pi} = \frac{1}{2} I_0^2 R_{rad}$$

Solving for R_{rad}

$$R_{rad} = \frac{120\pi \beta^2 L^2}{6\pi}$$

$$R_{rad} = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \qquad \because \beta = \frac{2\pi}{\lambda}$$

$$\boxed{R_{rad} = 790 \left(\frac{L}{\lambda}\right)^2}$$ current is uniform.

$$\boxed{R_{rad} = 790\, L^2 \lambda}$$

$\frac{L}{\lambda} = \frac{1}{10} = 8\Omega$ $\qquad$ $\frac{L}{\lambda} = \frac{1}{100} = 0.079\Omega$

It is assumed that with end loading on the dipole, the current is uniform. However with no end loading the current is zero at the end of dipole. If dipole is short the current distribution appears to be triangular distribution

$$I_{av} = \frac{1}{2} I_0$$

$$P = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}} \beta^2 I_{av}^2 L^2}{12\pi}$$

$$R_{rad} = 790 \left(\frac{I_{av}}{I_0}\right)^2 \left(\frac{L}{\lambda}\right)^2$$

But for short dipole $I_{av} = \frac{I_0}{2}$

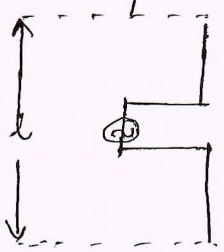
$$\boxed{R_{rad} = 197 \left(\frac{L}{\lambda}\right)^2 \Omega}$$ when current is not uniform & dipole

$$\boxed{R_{rad} = 790 \left(\frac{L}{\lambda}\right)^2 \Omega}$$ → Current is uniform

8 (b) Thin Linear Antenna

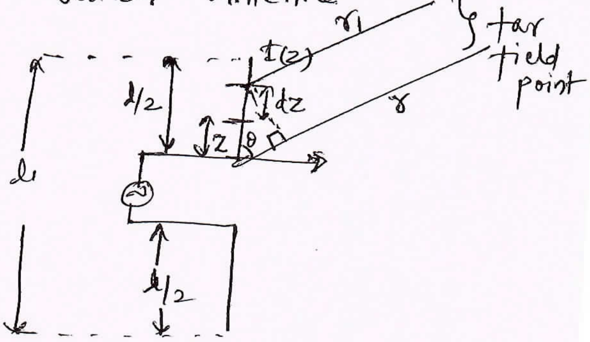
Diagram + Explanation
3+3=6M

A thin linear antenna or linear dipole is a piece of wire of length 'l' excited by voltage or current source at its centre.



$d << \frac{l}{100}$ current flow along length of wire

Radiation field of thin linear Antenna



Current distribution linear dipole

$$I(z) = \begin{cases} I_0 \sin\left(\beta\left(\frac{l}{2} - z\right)\right) & z > 0 \\ I_0 \sin\left(\beta\left(\frac{l}{2} + z\right)\right) & z < 0 \end{cases}$$

Diagram.

$$E_\theta = j\frac{60 [I_0] f(\theta)}{r}, \quad H_\phi = j\frac{[I_0] F(\theta)}{2\pi r}$$

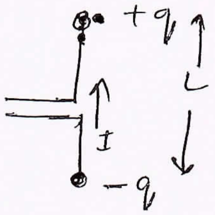
$f(\theta)$ ← relative variation of electric field.

$f(\theta) = \cos(\pi/2 \cos\theta)$

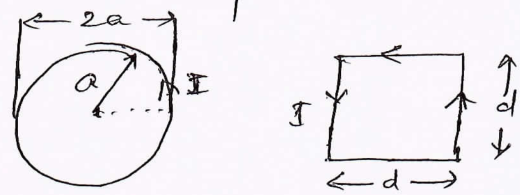
8 C Comparision of small loop and Short electric dipole

Diagram + formula/ Explanation $3+3=6$ M

- A short linear conductor is called short dipole when $L << \lambda$ & $d << L$



Small Loop



$d^2 = \pi a^2$

	Short Dipole	Small loop
E field	$E_\theta = j\dfrac{60\pi[I]\sin\theta}{\lambda}\cdot\dfrac{L}{r}$	$E_\phi = \dfrac{120\pi^2[I]\sin\theta}{r}\cdot\dfrac{A}{\lambda^2}$
	$H_\phi = j\dfrac{[I]\sin\theta}{2r}\,\dfrac{L}{\lambda}$	$H_\theta = \dfrac{\pi[I]\sin\theta}{r}\cdot\dfrac{A}{\lambda^2}$

9 (a) Loop Antenna

Data + formula + soln
1 + 2 + 3 = 6m

$D = ?$

$R_r = ?$

$d = 2\lambda$

Radiation resistance

$$R_r \simeq 31171 \left(\frac{A}{\lambda^2}\right)^2$$

$$R_r \simeq 31171 \frac{A^2}{\lambda^4}$$

$A = \pi a^2$

$d = 2\lambda$ (diameter)

$\therefore a = \lambda$

$$\therefore R_r \simeq 31171 \frac{(\pi a^2)^2}{\lambda^4}$$

$$\simeq 31171 \frac{(\pi)^2 (\cancel{\lambda^2})^2}{\cancel{\lambda^4}}$$

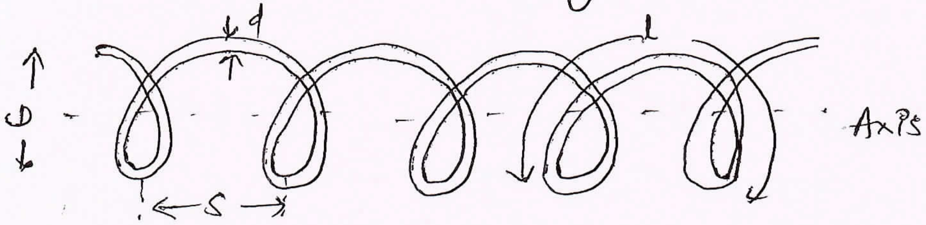
$$R_r = 307645.44 \ \Omega$$

Directivity is the same as that of an infinitesimal dipole

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} = 1.5$$

(b) Helical Antenna Geometry

Diagram + Explanation
3 + 3 = 6m



D = Diameter of helix

C = Circumference of helix πD

S = Spacing between turn

α = Pitch angle $\tan^{-1}(S/\pi D)$

l = length of 1 turn

$A = nS$ = axial length.

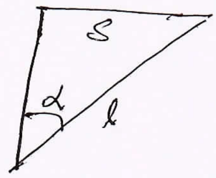
n = no. of turns.

$L = nl$ = length of conductor

d = diameter of helical conductor.

If one turn of helix is unrolled on flat plane, the relation between S, C, l & α

$C = \pi d$



$$\tan\alpha = \frac{S}{\pi d}$$

$$l^2 = S^2 + C^2$$

Special case

$\alpha = 0$, $S = 0$ Circular loop Antenna (N turn)

$\alpha = 90^0$, $D = 0$ linear Dipole Antenna (linear wire $L = nl$)

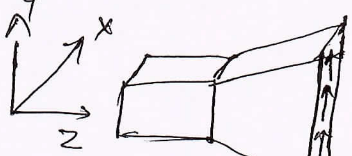
9 (c) Horn Antenna

Diagram + Explanation + Soln of problem

2 + 3 + 3 = 8m

Horn Antenna is regarded as flared out waveguide structure. The function of horn Antenna is to produce uniform phase with large aperture, then that of waveguide hence greater directivity.

Ex: E plane sectoral horn Antenna



→ Other Types are
- Rectangular Horn Antenna
- Circular Horn Antenna
  - (i) Conical Horn Antenna
  - (ii) Biconical horn Antenna
  - (iii) Exponentially tapered.

- Rectangular Horn Antenna
  - E plane sectoral horn Antenna
  - H plane sectoral horn Antenna
  - Pyramidal Horn Antenna
  - Exponential Tapered Horn Antenna

Problem

$$\epsilon_{ap} = 0.6$$

$$A_p = 81\lambda^2$$

$$D = \frac{4\pi \epsilon_{ap} A_p}{\lambda^2}$$

$$= \frac{4\pi (0.6)\, 81\lambda^2}{\lambda^2}$$

$$D = 610.72$$

$$D = 10\log_{10}(610.72) = 27.8\ dB$$

Explanation + Derivation
5+5 = 10 M

10 (a) Radiation resistance of small single turn circular loop.

Poynting vector is integrated over a large sphere yielding the total power P radiated.

Power is then equated to the square of the effective current on the loop times the radiation resistance R_r

$$P = \frac{I_0^2}{2} R_r$$

$I_0 \leftarrow$ Peak current in time on the loop.

Poynting vector

$$S = \frac{1}{2} Re \{ E \times H^* \}$$

But we have far field E_ϕ, H_θ

$$S_r = \frac{1}{2} Re \{ E_\phi H_\theta^* \}$$

for far field $\frac{E_\phi}{H_\theta} = 120\pi$

$$E_\phi = 120\pi H_\theta = Z_0 H_\theta$$

$$S_r = \frac{1}{2} Re \{ Z_0 H_\theta H_\theta^* \}$$

$$S_r = \frac{1}{2} Re \{ Z_0 |H_\theta|^2 \}$$

$$S_r = \frac{1}{2} |H_\theta|^2 * 120\pi$$

$$|H_\theta| = \frac{I_0 \beta a}{2r} J_1(\beta a \sin\theta)$$

$$|H_\theta|^2 = \left(\frac{I_0 \beta a}{2r}\right)^2 J_1^2(\beta a \sin\theta)$$

$$S_r = \frac{1}{2} \frac{(I_0 \beta a)^2}{4r^2} J_1^2(\beta a \sin\theta) * 120\pi$$

$$P = \int_0^{2\pi} \int_0^{\pi} S_r \, r^2 \sin\theta \cdot d\theta \cdot d\phi$$

Dias

$$P = 2\pi \int_0^{\pi} \frac{(I_0 \beta a)^2}{4r^2} J_1^2(\beta a \sin\theta) \, r \, 60\pi \sin\theta \cdot d\theta.$$

$$P = 30\pi^2 (I_0 \beta a)^2 \int_0^\pi J_1^2 (\beta a \sin\theta) \sin\theta \cdot d\theta$$

Case (i) for small loop

$$J_1^2(x) = \left(\frac{x}{2}\right) = \frac{x^2}{4}$$

$$P = 30\pi^2 (I_0 \beta a)^2 \frac{\beta a^2}{4} \int_0^\pi \sin^3\theta \cdot d\theta$$

$$= 30\pi^2 \frac{I_0^2 (\beta a)^4}{\cancel{4}} \times \frac{\cancel{4}}{3}$$

$$P = 10\pi^2 I_0^2 (\beta a)^4 \quad \text{——— (1)}$$

From circuit point of view,

$$P = \frac{1}{2} I_0^2 R_r \quad \text{——— (2)}$$

Equate (1) & (2)

$$\frac{1}{2} I_0^2 R_r = 10\pi^2 I_0^2 (\beta a)^4$$

$$R_r = 20\pi^2 (\beta a)^4$$

$$R_r = 197\, C\lambda^4$$

$$R_r = 197 \left(\frac{C}{\lambda}\right)^4$$

$$R_r = 20\pi^2 \left(\frac{2\pi}{\lambda}\right)^4 a^4$$

$$= 20\pi^2 (2)^4 \pi^2 \frac{\pi^2 a^4}{\lambda^4}$$

$$= 31171 \frac{A^2}{\lambda^4}$$

$$R_r \cong 31171 \left(\frac{A}{\lambda^2}\right)^2$$

$$R_r \cong 31200 \left(\frac{A}{\lambda^2}\right)^2$$

Case (ii) $\int_0^\pi J_1^2 (x \sin\theta) \sin\theta \cdot d\theta = \frac{1}{x} \int_0^{2x} J_2(y) \cdot dy$

y be any function $x' = \beta a$

$$P = 30\pi^2 \beta a^2 I_0^2 \frac{1}{\beta} \int_0^{2\beta a} J_2(y) \cdot dy \quad \text{——— (1)}$$

& PTO.

With respect to circuit

$$P = \frac{1}{2} I_0^2 R_r \quad \text{— (2)}$$

Equate (1) & (2)

$$30\pi^2 \beta a I_0^2 \int_0^{2\beta a} J_2(y) \cdot dy = \frac{1}{2} I_0^2 R_r$$

$$R_r = 60\pi^2 \beta a \int_0^{2\beta a} J_2(y) \cdot dy$$

$$R_r = 60\pi^2 C_\lambda \int_0^{2C_\lambda} J_2(y) \cdot dy$$

if $C_\lambda \geq 5$

$$\int_0^{2C_\lambda} J_2(y)\, dy \simeq 1$$

$$R_r = 60\pi^2 C_\lambda$$

$$R_r = 592\, C_\lambda$$

or $R_r = 60\pi^2\, 2\pi \frac{a}{\lambda}$

$$R_r = 3720 \left(\frac{a}{\lambda}\right)$$

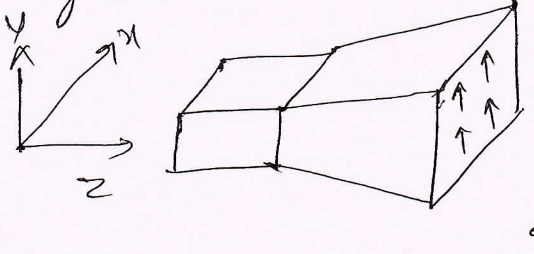
For values between $1/3$ and 5 the integral can be evaluated using transformation

$$\int_0^{2C_\lambda} J_2(y) \cdot dy = \int_0^{2C_\lambda} \left(J_0(y) \cdot dy - 2J_1(2C_\lambda)\right)$$

Diagram + Explanation + Solution
3+3+4=10m

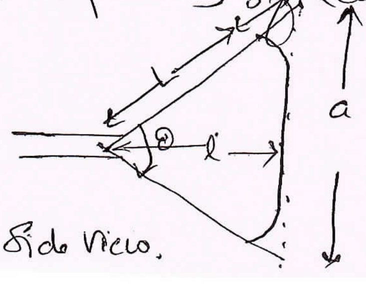
(6) Pyramidal Horn Antenna & optimum horn dimension

Diagram + Equation + Explanation
4+3+3=10m



The flare is both direction E & H is called pyramidal sectoral Horn Antenna.

Design Equation of Rectangular Horn Antenna.



Side View.

From Figure

$$\cos(\theta/2) = \frac{L}{L+\delta} \qquad \sin(\theta/2) = \frac{a/2}{L+\delta}$$

$$\tan(\theta/2) = \frac{a}{2L}$$

$\theta \leftarrow$ flare angle (θ_E for E plane, θ_H for H plane)
$a \leftarrow$ aperture (a_E for E plane, a_H for H plane)
$L \leftarrow$ Horn Length (m)
$\delta \rightarrow$ path length difference

$$L^2 + \frac{a^2}{4} = (L+\delta)^2 \qquad \delta << L \text{ we can ignore } \delta$$

$$L^2 + \frac{a^2}{4} = L^2 + \delta^2 + 2L\delta$$

$$\therefore L \cong \frac{a^2}{8\delta} \qquad \delta << L$$

$$\theta = 2\tan^{-1}\left(\frac{a}{2L}\right)$$

$$\theta = 2\cos^{-1}\left(\frac{L}{L+\delta}\right)$$

$$\delta_0 = \frac{L}{\cos(\theta/2)} - L \quad \text{— optimum } \delta \quad \text{— (1)}$$

$$L = \frac{\delta_0 \cos(\theta/2)}{1-\cos(\theta/2)} \quad \text{— optimum length. — (2)}$$

* Maximum directivity occurs at largest flare angle for which δ does not exceed cartesian value δ_0. The optimum horn dimensions are related as represented in Eqn (1) & (2)

If δ is sufficiently small fraction of wavelength, the field has nearly uniform phase over entire aperture. For constant length 'L' the directivity of horn increases as aperture area and flare angle θ are increased. However if aperture & θ becomes so large that δ is equivalent to 180°, then field at edge of aperture is in phase opposition to field on the axis.

The phase reversal at edge of aperture reduces directivity. It follows that maximum directivity occurs at largest flare angle for which δ does not exceed cartesian value. δ_0