15EE61

Sixth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Control System

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define Control System. Distinguish between open-loop and closed loop control system with an examples. **(06 Marks)**

b. For the mechanical system shown in Fig.Q1(b), write the differential equation relating to the force F(t). Also obtain the analogous electrical circuits based on i) Force-current analogy ii) Force-voltage analogy. **(10 Marks)**

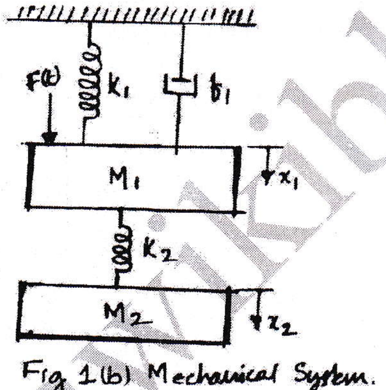

Fig.Q.1(b)

OR

2 a. Define servomotor. Compare AC servomotor and DC servomotor. **(04 Marks)**

b. Derive an expression for the transfer function of an armature controlled D.C. motor and also construct the block diagram of d.c. motor. **(12 Marks)**

Module-2

3 a. For the system represented by the following equations and find the transfer function X(s)/U(s) by the signal flow graph technique.

$$x = x_1 + \alpha_0 u ; \frac{dx_1}{dt} = -\alpha_1 x_1 + x_2 + \alpha_2 u ; \frac{du_2}{dt} = -\alpha_2 x_1 + \alpha_1 u$$ **(08 Marks)**

b. Using block diagram reduction technique. Obtain the transfer function of C(s)/R(s) as shown in Fig.Q.3(b). **(08 Marks)**

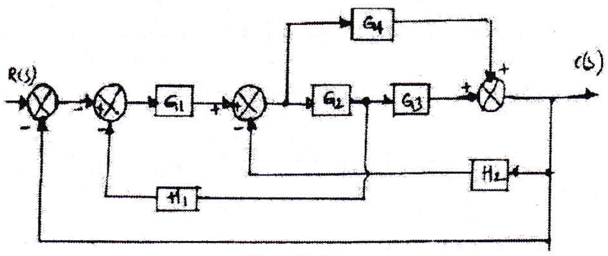

Fig.Q.3(b)

OR

4 a. State the Mason's gain formula. Find the transfer function $\frac{X_5}{X_1}$ of the system described by the signal flow graph (SFG) shown in Fig.Q.4(a). **(08 Marks)**

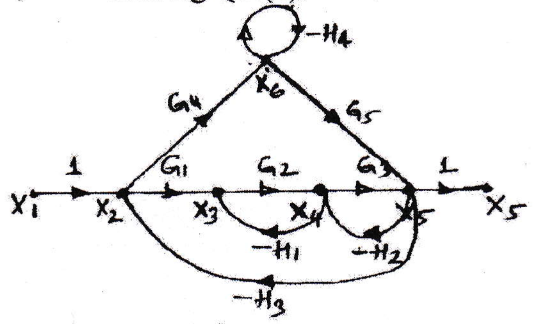

Fig.Q.4(a)

b. For the network shown in Fig.Q.4(b), construct the signal flow graph and determine the transfer function using Mason's gain formula. **(08 Marks)**

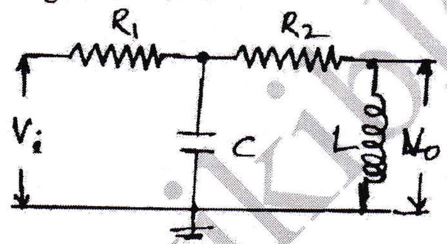

Fig.Q.4(b)

Module-3

5 a. Derive an expression for rise time and peak-time for a second order system excited by a step input (under-damped case). **(08 Marks)**

b. A unity feedback control system is characterized by an open-loop T.F. $G(s) = \frac{K}{s(s+\alpha)}$.

Where K and α are positive constant,
By what factor the amplifier gain K should be reduced so that the peak osvershoot of the unit step response reduces from 75% to 25%. **(08 Marks)**

OR

6 a. A unity feedback system having open-loop T.F. of $G(s) = \frac{K(2s+1)}{S(s+1)(s+1)^2}$. The input $r(t) = 1 + 6t$ is applied to the system. Determine the minimum value of K, if the steady state error is to be less than 0.1. **(04 Marks)**

b. A unity feedback control system has $G(s) = \frac{K(s+4)}{s(s+1)(s+2)}$ using Routh Hurwitz criterion. Find the range of K for which system to be stable and also determine the frequency of oscillations. **(06 Marks)**

c. What are the difficulties encountered while assessing the R-H criteria and how do you eliminate these difficulties? Explain with examples. **(06 Marks)**

Module-4

7 a. What do you means by (i) breakaway point and (ii) break in point. How can they be determined with an example? **(04 Marks)**

b. Sketch the roots locus plot for the system $G(s)H(s) = \dfrac{K}{s(s+2)(s+4)}$. Determine the range of K for which the system will have damped oscillating response. **(07 Marks)**

c. Show that part of root locus for the open loop T.F. $G(s)H(s) = \dfrac{K(s+2)}{S(s+1)}$ is a circle. **(05 Marks)**

OR

8 a. Derive an expression for resonant peak and resonant frequency for a second order system. **(06 Marks)**

b. Sketch the Bode-plot for the open-loop transfer function $G(s)H(s) = \dfrac{K}{s(s+1)(0.1s+1)}$ and determine the value of K for which system is to be stable. Also find the gain margin and phase margin. **(10 Marks)**

Module-5

9 a. State and explain the Nyquist stability criterion. **(06 Marks)**

b. Sketch the Nyquist plot and comment on the stability of the closed loop system whose open-loop transfer function is $G(s)H(s) = \dfrac{K(s-4)}{(s+1)^2}$. **(10 Marks)**

OR

10 a. Explain the pharse lag compensator with neat circuit diagram and derive expression for the transfer function of a lag compensator. **(06 Marks)**

b. What are the limitations of single stage phase lead control? **(04 Marks)**

c. Write notes on PID controller. **(06 Marks)**

Solution of VTU Question Paper [Jan 2020]

Control Systems [15EE61]

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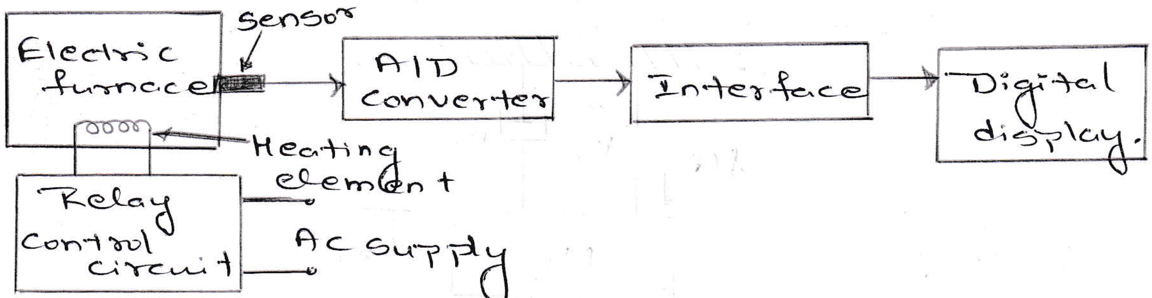
01.a. Define control system. Distinguish between open-loop and closed loop control system with an example (6 marks)

A system consists of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity, then the system is called control system.

(*) Open loop control system

Any physical system which does not automatically correct the variation in its output, is called an open loop system. Consider a system which is designed to control temperature.



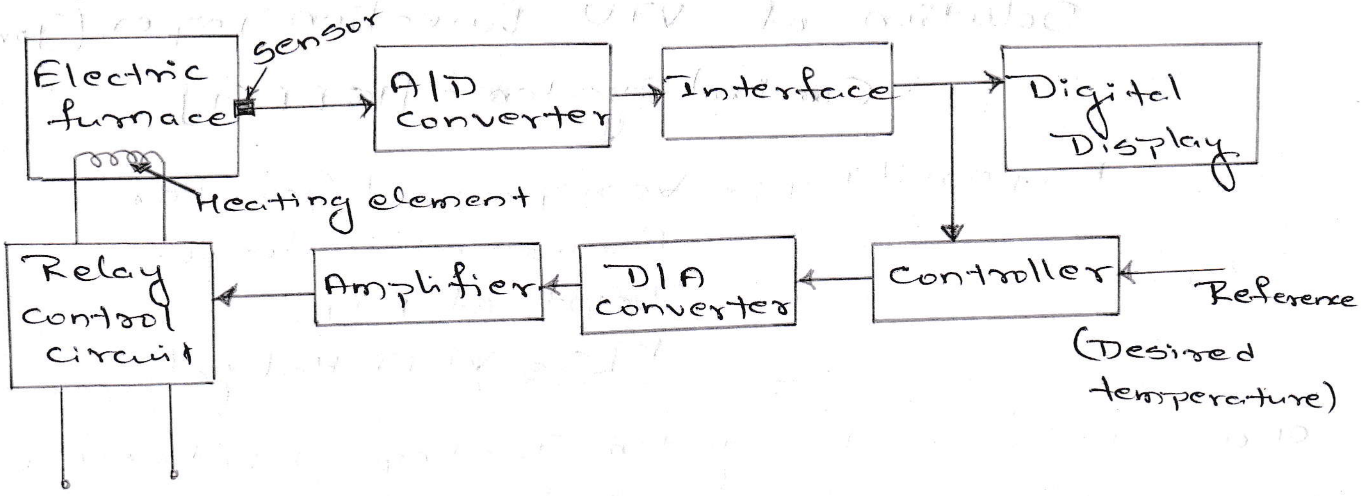
In the above system sensor measures the temperature and send it to display unit through A/D converter and interface. Depending on the temperature the operator need to turn on or off the heating element to maintain the temperature at desired value.

Advantages → Simple, low cost.

Disadvantages → Inaccurate, unreliable.

(*) Closed loop control system

Control system in which the output has an effect upon the input quantity in order to maintain the desired output value are called closed loop system

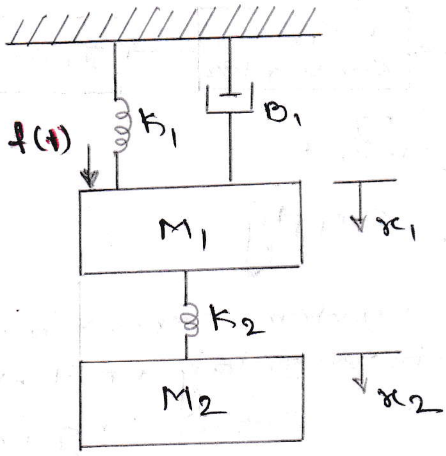


In above diagram the controller compairs the actural temperature with the desired temperature and based on this data it switch on or off the heating element automatically

Advantages → Accurate, reliable, less affected by noise

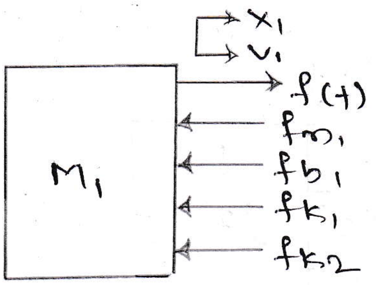
Disadvantages → Complex, costly, stability problems.

01.b For the mechanical system shown below, write the differential equation relating to the force f(t). Also obtain the analogous electrical circuit based on (i) Force current analogy (ii) Force voltage analogy. (10 marks)



There are two nodes. So we need to draw two free body diagram. Mass M_1 is taken as node 1 and Mass M_2 is taken as node 2.

Free body diagram of Mass M_1 (node 1)



Force balance equation

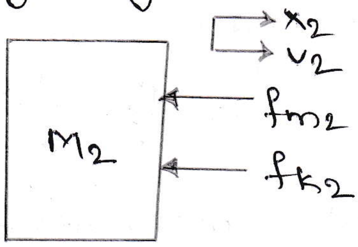
$$f(t) = f_{m1} + f_{b1} + f_{k1} + f_{k2}$$

$$f(t) = M \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + K_2 (x_1 - x_2)$$

Put $\frac{dx_1}{dt} = V_1$ and $\frac{d^2 x_1}{dt^2} = \frac{dv_1}{dt}$

So $$f(t) = M \frac{dv_1}{dt} + B V_1 + K_1 \int v_1 \, dt + K_2 \int (v_1 - v_2) dt \longrightarrow (1)$$

Free body diagram of Mass M_2 (node 2)



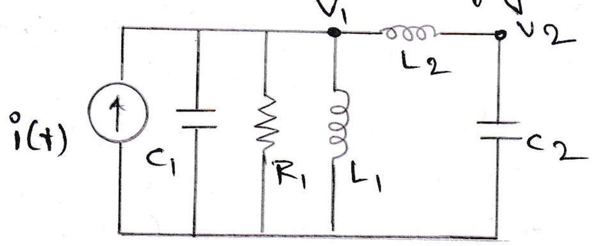
Force balance equation

$$0 = f_{m2} + f_{k2}$$

$$0 = M_2 \frac{d x_2^2}{dt^2} + K_2 (x_2 - x_1)$$

or $$0 = M_2 \frac{dv_2}{dt} + K_2 \int (v_2 - v_1) dt \longrightarrow (2)$$

→ Force current analogy



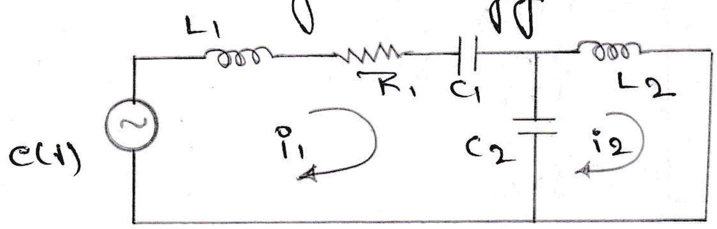
Applying KCL to node 1

$$i(t) = C_1 \frac{dv_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1} \int v_1 \, dt + \frac{1}{L_2} \int (v_1 - v_2) dt \longrightarrow (3)$$

Applying KCL to node 2.

$$0 = C_2 \frac{dv_2}{dt} + \frac{1}{L_2} \int (v_2 - v_1) dt \longrightarrow (4)$$

→ Force Voltage analogy



Applying KVL to loop 1

$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int (i_1 - i_2) dt \rightarrow (5)$$

Applying KVL to loop 2

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1) dt \longrightarrow (6)$$

02.a Define servomotor. Compare AC servomotor and DC servomotor. (4 marks)

The motors used in automatic control system or in servomechanism are called servomotors. They are used to convert electrical signal to angular motion.

DC servomotor	AC servomotor.
01. Higher power output	01. Relatively lesser power output than a DC servomotor for same size
02. Characteristic is linear	02. Characteristic is non linear.
03. Fast response due to low electrical and mechanical time constant	03. The response is relatively slower due to higher electrical and mechanical time constant
04. Suitable for large power applications	04. Suitable for low power applications.

02.b Derive an expression for the transfer function of an armature controlled D.C motor and also construct the block diagram of D.C motor. (12 marks)

Following assumptions are made to derive an expression for the transfer function of an armature controlled DC motor.

- Flux is proportional to the current through the field winding

$$\phi = K_f I_f = \text{constant}$$

- Torque developed is proportional to flux and armature current.

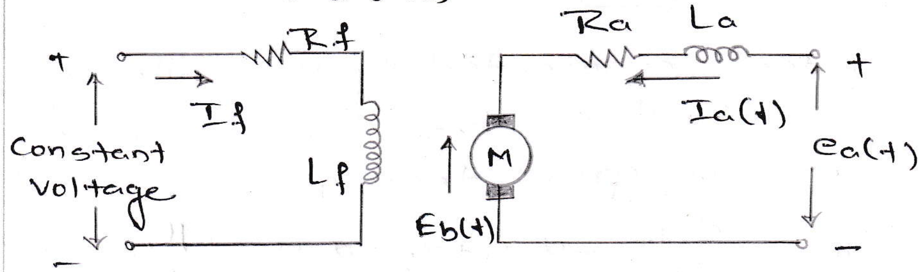
$$T = K_m' \phi I_a = K_m' K_f I_f I_a.$$

- Back emf is proportional to shaft velocity

$$E_b = K_b \omega_n \quad \text{where } \omega_n = \frac{d}{dt}\theta(t)$$

taking Laplace transform

$$E_b = K_b s \theta(s)$$



we can write

$$e_a(t) = E_b(t) + I_a(t) R_a + L_a \frac{d}{dt} I_a(t)$$

taking Laplace transform with zero initial condition

$$E_a(s) = E_b(s) + I_a(s)\left(R_a + sL_a\right)$$

or $$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a}$$

Substitute for E_b

$$I_a = \frac{E_a - K_b s \theta}{R_a + sL_a}$$

we have $T = K_m' K_f I_f I_a.$

$$\therefore T = K_m' K_f I_f \left[\frac{E_a - K_b s\theta}{R_a + sL_a}\right]$$

Also $T = (J_m s^2 + sB_m)\theta$

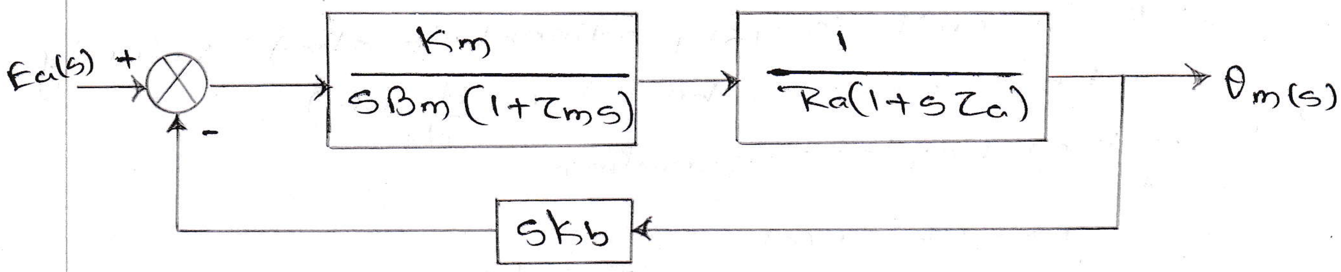
$$\therefore [J_m s^2 + sB_m]\theta = K_m' K_f I_f \left(\frac{E_a - K_b s\theta}{R_a + sL_a}\right)$$

$$\frac{K_m' K_f I_f E_a}{R_a + sL_a} = \frac{K_m' K_f K_b I_f s\theta}{R_a + sL_a} + \left(J_m s^2 + sB_m\right)\theta$$

$$\therefore \frac{\theta}{E_a} = \frac{K_m / sR_a B_m (1 + s\tau_m)(1 + s\tau_a)}{1 + K_m sK_b / sR_a B_m (1 + s\tau_m)(1 + s\tau_a)}$$

where $\tau_m = J_m / B_m$, $\tau_a = L_a/R_a$, $K_m = K_m' K_f$

Block diagram



Q3 a. For the system represented by the following equations and find the transfer function $X(s)/U(s)$ by the signal flow graph technique.

$$X = x_1 + \alpha_0 u \; ; \; \frac{dx_1}{dt} = -\alpha_1 x_1 + x_2 + \alpha_2 u \; ; \; \frac{dx_2}{dt} = -\alpha_2 x_1 + \alpha_1 u.$$

There is a printing mistake in Question Paper. Last equation is $\dfrac{dx_2}{dt} = -\alpha_2 x_1 + \alpha_1 u$.

We have

$X = x_1 + \alpha_0 u$. $\quad\quad \because \dfrac{dx}{dt} = \dot{x}$

$\dot{x}_1 = -\alpha_1 x_1 + x_2 + \alpha_2 u$

$\dot{x}_2 = -\alpha_2 x_1 + \alpha_1 u$.

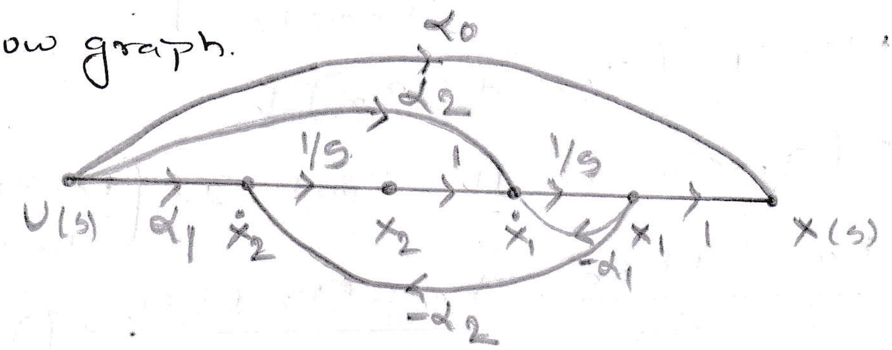
These are state space equations.

$X = x_1 + \alpha_0 u \Rightarrow$ Output equation

$$\left.\begin{aligned} \dot{x}_1 &= -\alpha_1 x_1 + x_2 + \alpha_2 u \\ \dot{x}_2 &= -\alpha_2 x_1 + \alpha_1 u \end{aligned}\right\} \Rightarrow \text{State equations.}$$

and $\dot{x}_1 = x_2$

Signal flow graph.



Number of forward paths $K = 3$

$P_1 = \alpha_1 \times 1/s \times 1 \times 1/s \times 1 = \alpha_1/s^2$

$P_2 = \alpha_2 \times 1/s \times 1 = \alpha_2/s$

$P_3 = \alpha_0$

Individual loop gain

$L_1 = -\alpha_1/s$

$L_2 = -\alpha_2/s^2$

There is no combination of two non-touching loops.

$$\Delta = 1-[L_1+L_2] = 1-\left(-\alpha_1/s - \alpha_2/s^2\right] = \frac{s^2+\alpha_1 s+\alpha_2}{s^2}$$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$\Delta_3 = 1-(L_1+L_2) = \frac{s^2+\alpha_1 s+\alpha_2}{s^2}$$

Mason's gain formula.

$$\text{transfer function} = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3\right]$$

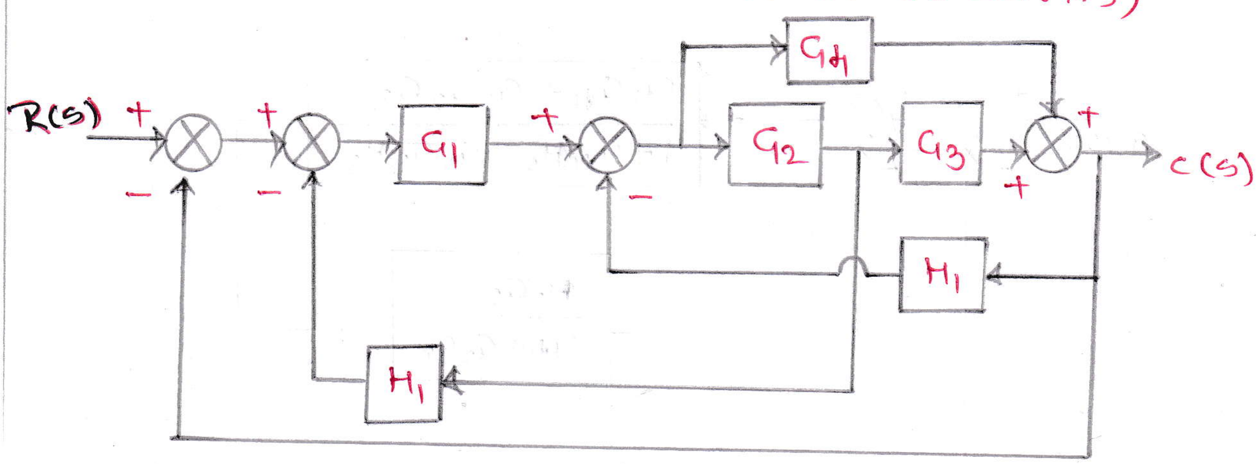
$$= \frac{1}{\Delta}\left[\frac{\alpha_1}{s^2} + \frac{\alpha_2}{s} + \alpha_0\left(\frac{s^2+\alpha_1 s+\alpha_2}{s^2}\right)\right]$$

$$= \frac{1}{\Delta}\left[\frac{\alpha_1 + \alpha_2 s + \alpha_0 s^2 + \alpha_1\alpha_0 s + \alpha_0\alpha_2}{s^2}\right]$$

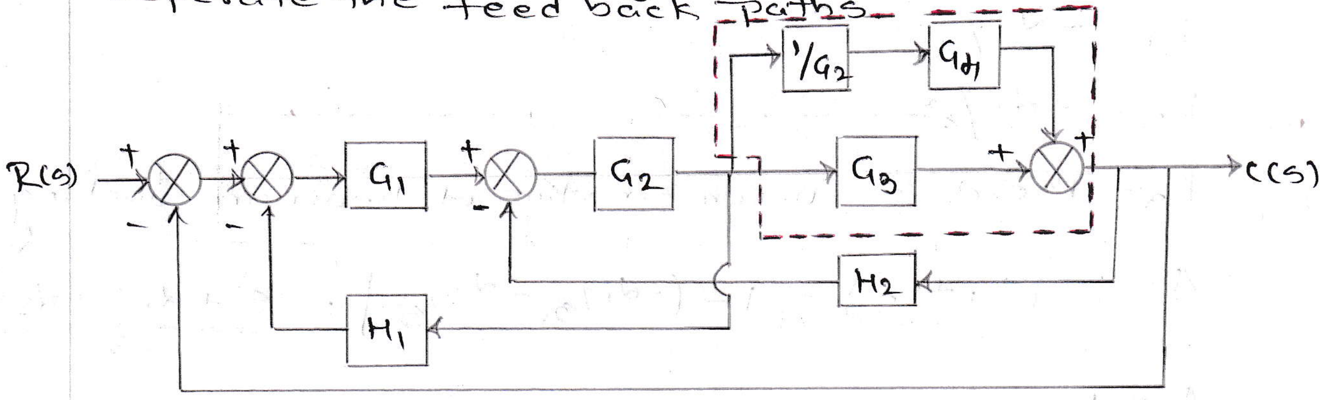
$$= \frac{s^2}{s^2+\alpha_1 s+\alpha_2}\left[\frac{\alpha_0 s^2 + (\alpha_0\alpha_1+\alpha_2)s + \alpha_0\alpha_2 + \alpha_1}{s^2}\right]$$

$$\frac{X(s)}{U(s)} = \frac{\alpha_0 s^2 + (\alpha_0\alpha_1+\alpha_2)s + \alpha_0\alpha_2 + \alpha_1}{s^2 + \alpha s + \alpha_2}$$

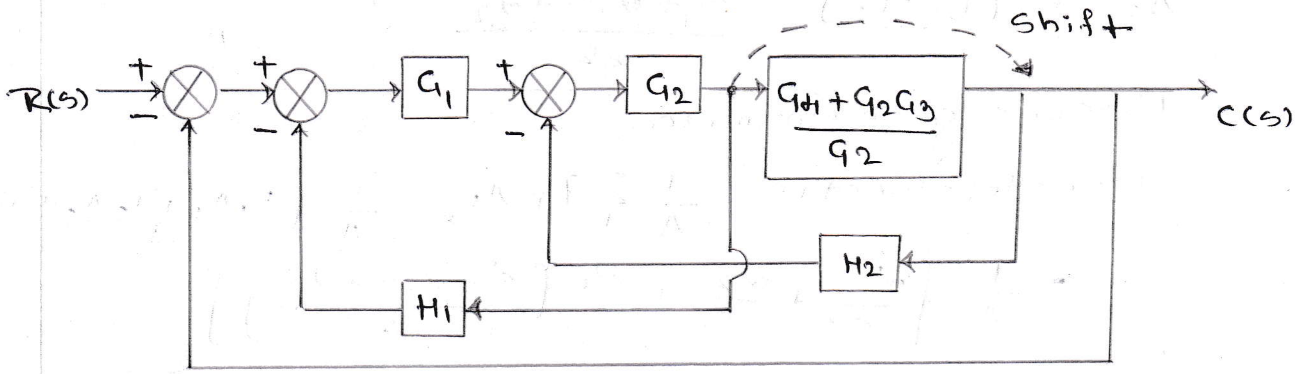
Q3.b. Using block diagram reduction technique. Obtain the transfer function $C(s)/R(s)$. (8 marks)

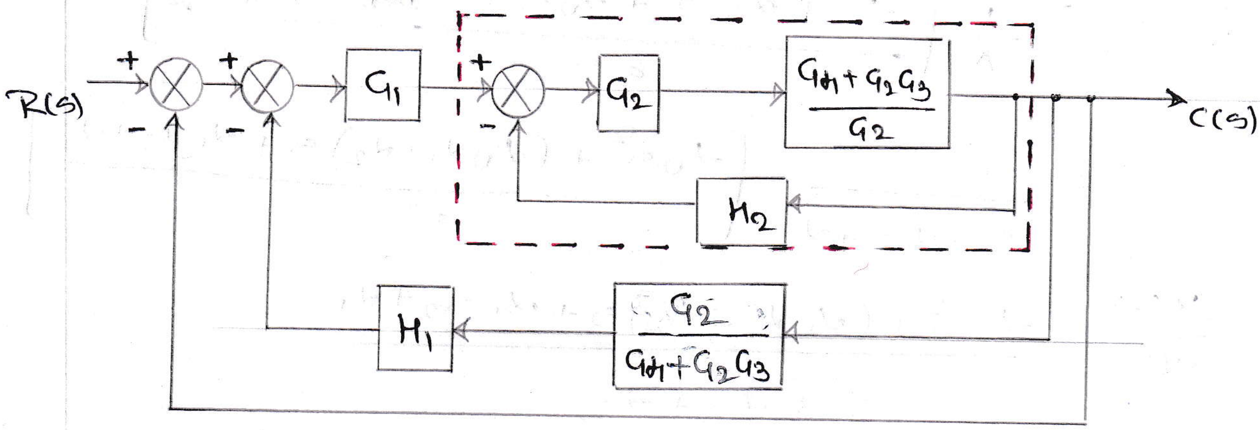


Shift the take away point of G_4 after G_2 and seperate the feed back paths.

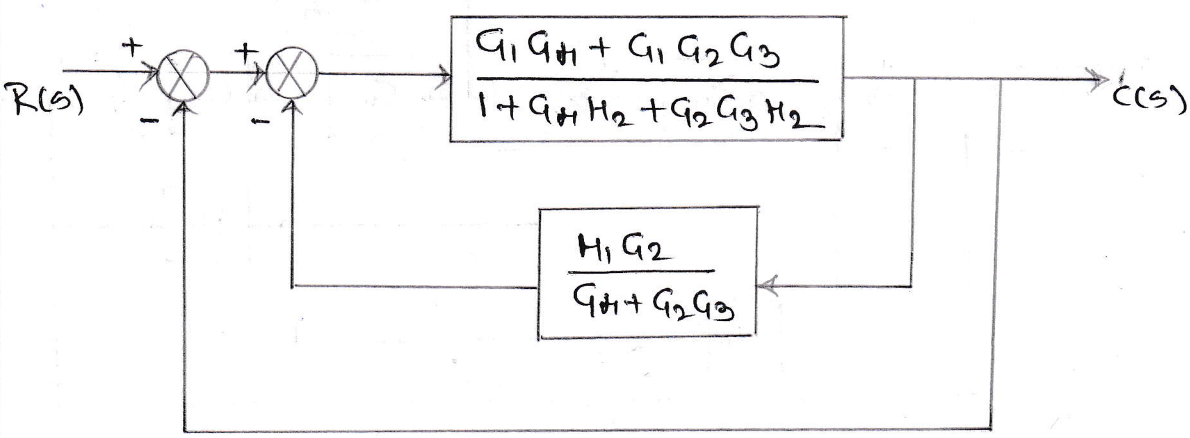


$$G_4/G_2 + G_3 = \frac{G_4 + G_2G_3}{G_2}$$

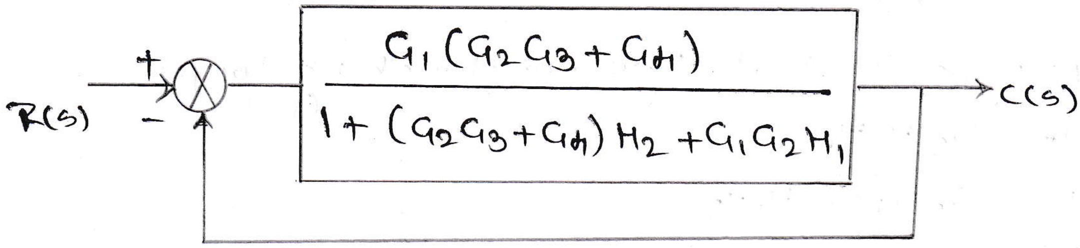




$$\frac{G_4 + G_2G_3}{1 + (G_4 + G_2G_3)H_2} = \frac{G_4 + G_2G_3}{1 + G_4H_2 + G_2G_3H_2}$$



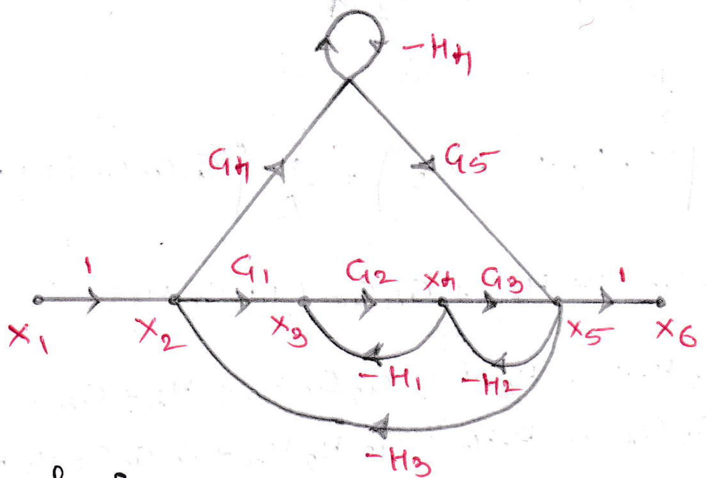
$$\frac{\dfrac{G_1G_2G_3 + G_4G_1}{1+(G_2G_3+G_4)H_2}}{1 + \dfrac{G_1[G_2G_3+G_4]}{1+(G_2G_3+G_4)H_2} \times \dfrac{H_1G_1}{G_2G_3+G_4}} = \frac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1}$$



$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1+G_1G_2G_3+G_1G_4}$$

Q4. a. State the Mason's gain formula. Find the transfer function X_5/X_1 of the system discribed by the signal flow graph shown below. (8 marks)



Number of forward paths $K=2$

- Forward path gain

$P_1 = G_1G_2G_3$

$P_2 = G_4G_5$

- Individual loop gain

$L_1 = -G_2H_1$

$L_2 = -G_3H_2$

$L_3 = -G_1G_2G_3H_3$

$L_4 = -H_4$

$L_5 = -G_4G_5H_3$

- Gain product of two non touching loops.

$L_{14} = G_2H_1H_4$

$L_{15} = G_2G_4G_5H_1H_3$

$L_{24} = G_3H_2H_4$

$L_{34} = G_1G_2G_3H_3H_4$

- Calculation of Δ

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + (L_{14} + L_{15} + L_{24} + L_{34}]$$

$$\Delta = 1 + G_2H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_4G_5H_3 + G_2H_1H_4 + G_2G_4G_5H_1H_3 + G_3H_2H_4 + G_1G_2G_3H_3H_4$$

L_4 is non touching to P_1

$\therefore \Delta_1 = 1-(-H_4) = 1+H_4$

L_1 is non touching to P_2

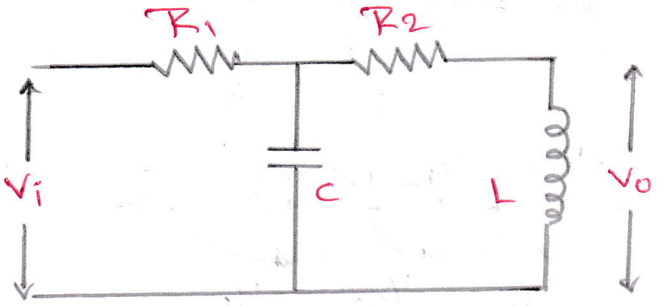
$\therefore \Delta_2 = 1-(-G_2H_1) = 1+G_2H_1$

Mason's gain formula.

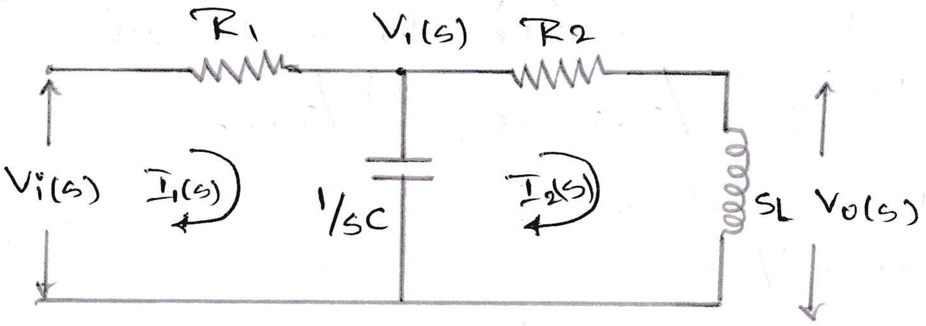
$$\frac{X_6}{X_1} = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right] = \frac{1}{\Delta}\left[G_1G_2G_3(1+H_4) + G_4G_5(1+G_2H_1)\right]$$

$$= \frac{G_1G_2G_3(1+H_4) + G_4G_5(1+G_2H_1)}{1 + G_2H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_4G_5H_3 + G_2H_1H_4 + G_2G_4G_5H_1H_3 + G_3H_2H_4 + G_1G_2G_3H_3H_4}$$

04.b For the network shown below, construct the signal flow graph and determine the transfer function using Mason's gain formula (8 marks)



Laplace transform of the given network is as shown below.



Select nodes as $V_i(s)$, $I_1(s)$, $V_1(s)$, $I_2(s)$ and $V_o(s)$

we can write

$$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} \longrightarrow (1)$$

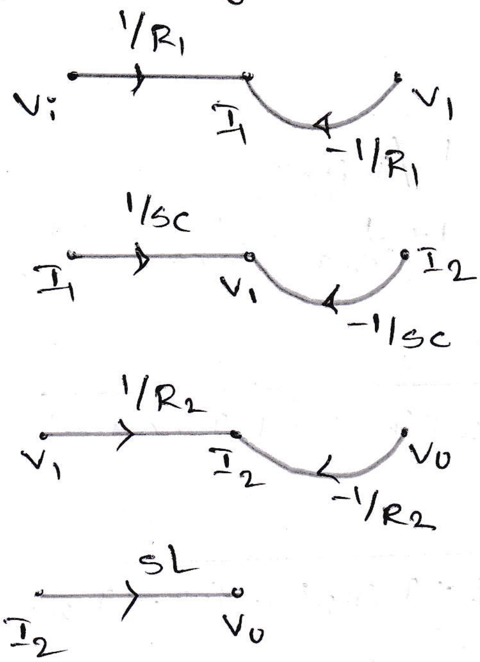
$$V_1(s) = (I_1(s) - I_2(s))\frac{1}{sC} \longrightarrow (2)$$

$$I_2(s) = \frac{(V_1(s) - V_o(s))}{R_2} \longrightarrow (3)$$

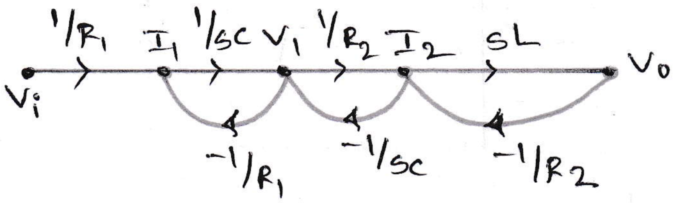
$$V_o(s) = I_2 sL \longrightarrow (4)$$

Signal flow graph's for above equations.

(1)



overall signal flow graph



- Forward path gain.

  there is only one forward path

  $P_1 = sL / R_1 R_2 sC = L / R_1 R_2 C$

- Individual loop gain

  $L_1 = -1/sCR_1$

  $L_2 = -1/sCR_2$

  $L_3 = -sL/R_2$

- Gain product of two nontouching loops.

  $L_{13} = L / C R_1 R_2$

- Calculation of Δ

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_{13})$$

$$= 1 - \left(\frac{-1}{sCR_1} - \frac{1}{sCR_2} - \frac{sL}{R_2} \right) + \frac{L}{CR_1R_2}$$

$$\Delta_1 = 1$$

- Mason's gain formula.

$$\frac{V_o}{V_i} = \frac{1}{\Delta}(P_1 \Delta_1) = \frac{L / R_1 R_2 C}{1 + \frac{1}{sCR_1} + \frac{1}{sCR_2} + \frac{sL}{R_2} + \frac{L}{CR_1R_2}}$$

$$= \frac{sL}{sR_1R_2C + R_2 + R_1 + s^2LR_1C + sL}$$

$$\frac{V_o}{V_i} = \frac{sL}{s^2LR_1C + s(L + R_1R_2C) + (R_1 + R_2)}$$

Q5. a Derive an expression for rise time and peak time for a second order system excited by a step input (under damped case) (8 marks)

Unit step response of 2nd order system for under damped case is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

at $t = t_r$, $c(t) = c(t_r) = 1$

$$\therefore c(t) = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1$$

$$\frac{-e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

Since $-e^{-\xi\omega_n t_r} \neq 0$ so $\sin(\omega_d t_r + \theta) = 0$

when $\phi = 0, \pi, 2\pi, 3\pi \ldots$ $\sin\phi = 0$

$$\therefore \omega_d t_r + \theta = \pi$$

$$\omega_d t_r = \pi - \theta$$

$\therefore$ rise time $t_r = \dfrac{\pi - \theta}{\omega_d}$ in sec

where $\theta = \tan^{-1} \dfrac{\sqrt{1-\xi^2}}{\xi}$ in radians.

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

• Peak time

we have $c(t) = 1 - \dfrac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$

expression for t_p is obtained by

$$\left.\frac{dc(t)}{dt}\right|_{t=t_p} = 0$$

$$\therefore \frac{dc(t)}{dt} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} (-\xi\omega_n) \sin(\omega_d t + \theta) + \left[-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t + \theta)\, \omega_d\right]$$

Put $\omega_d = \omega_n \sqrt{1-\xi^2}$

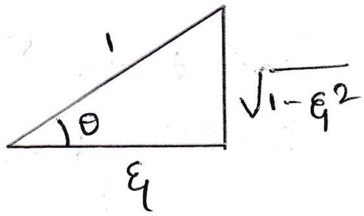
$$= \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \xi\omega_n \sin(\omega_d t + \theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \omega_n \sqrt{1-\xi^2} \cos(\omega_d t + \theta)$$

$$= \frac{e^{-\xi \omega_n t}\,\omega_n}{\sqrt{1-\xi^2}} \left[\xi \sin(\omega_d t + \theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\left[\cos\theta \sin(\omega_d t+\theta) - \sin\theta \cos(\omega_d t+\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\left[\sin(\omega_d t+\theta)-\theta\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t)$$



$$\therefore \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t_p) = 0$$

Since $e^{-\xi\omega_n t} \neq 0$ $\quad \sin(\omega_d t_p) = 0$

or $\omega_d t_p = \pi$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

Q5.b A unity feedback control system is characterized by an open loop T.F. $G(s) = K/s(s+\alpha)$. where K and α are positive integer contants. By what factor the amplifier gain K should be reduced so that the peak overshoot of the unit step response reduces from 75 % to 25 %. (8 marks)

Closed loop T.F $= \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{K/s(s+\alpha)}{1+K/s(s+\alpha)} = \dfrac{K}{s(s+\alpha)+K}$

$$T(s) = \frac{K}{s^2+\alpha s+K}$$

Characteristic equation $= s^2+\alpha s+K = 0$

Compare with $s^2+2\xi\omega_n s+\omega_n^2 = 0$

$\omega_n^2 = K \quad \therefore \omega_n = \sqrt{K}$

and $\quad 2\xi\omega_n = \alpha \quad \therefore \xi = \alpha/2\omega_n = \alpha/2\sqrt{K}$

We know that

$$\% M_p = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} \times 100$$

$$0.75 = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}}$$

$$-0.287 = \frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}}$$

$(0.287)^2(1-\xi_1^2) = \pi^2 \xi_1^2$

$0.0827(1-\xi_1^2) = \pi^2 \xi_1^2$

$\xi_1^2 = 8.309 \times 10^{-3}$

or $\xi_1 = 0.0911$ for $M_p = 0.75$

$0.25 = e^{-\pi \xi_2 / \sqrt{1-\xi_2^2}}$

$-1.386 = -\pi \xi_2 / \sqrt{1-\xi_2^2}$

$1.9218(1-\xi_2^2) = \pi^2 \xi_2^2$

$\xi_2^2 = 0.1629$

$\xi_2 = 0.403$ for $M_p = 0.25$

Let $K = K_1$ for ξ_1 and $K = K_2$ for ξ_2 with α constant

$0.0911 = \dfrac{\alpha}{2\sqrt{K_1}}$ and $0.403 = \dfrac{\alpha}{2\sqrt{K_2}}$

$\dfrac{0.0911}{0.403} = \sqrt{\dfrac{K_2}{K_1}}$ or $K_2 = K_1 \times 0.0511$

So K should be reduced by 0.0511.

06.a. A unity feedback system having closed loop T.F of $G(s) = \dfrac{K(2s+1)}{s[5s+1)(s+1)^2}$. The input $r(t) = 1+6t$ is applied to the system. Determine the minimum value of K, if the steady state error is to be less than 0.1. (4 marks)

Printing Mistake

Given that, input $r(t) = 1+6t$

taking Laplace transform

$R(s) = 1/s + 6/s^2$

error signal

$$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{1}{s}+\frac{6}{s^2}}{1+\frac{K(2s+1)}{s(5s+1)(s+1)^2}}$$

$$= \frac{\frac{1}{s}+\frac{6}{s^2}}{\frac{s(5s+1)(s+1)^2+K(2s+1)}{s(5s+1)(s+1)^2}}$$

$$= \frac{1}{s}\left[\frac{s(5s+1)(s+1)^2}{s(5s+1)(s+1)^2+K(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(s+1)^2}{s(5s+1)(s+1)^2+K(2s+1)}\right]$$

The steady state error ess can be obtained from final value theorem.

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\left\{\frac{1}{s}\left[\frac{s(5s+1)(s+1)^2}{s(5s+1)(s+1)^2+K(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(s+1)^2}{s(5s+1)(s+1)^2+K(2s+1)}\right]\right\}$$

$$e_{ss} = 0 + \frac{6}{K} = \frac{6}{K}$$

Given that $e_{ss} < 0.1$

$\therefore\ 0.1 = 6/K$ or $K = 6/0.1 = 60$.

minimum value of $K = 60$ to $e_{ss} < 0.1$.

06.b. A unity feedback control system has $G(s) = \dfrac{K(s+a)}{s(s+1)(s+2)}$ Using Routh Hurwitz criterion. Find the range of K for which system to be stable and also determine the frequency of oscillations. (6 marks)

$$\text{Closed loop T.F} = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s+a)/s(s+1)(s+2)}{1+\dfrac{K(s+a)}{s(s+1)(s+2)}}$$

$$= \frac{K(s+a)}{s(s+1)(s+2)+K(s+a)}$$

Characterisic equation $s(s^2+3s+2)+Ks+aK=0$

$$s^3+3s^2+(2+K)s+aK=0$$

Routh array

s^3	1	$2+K$
s^2	3	aK
s^1	$\dfrac{6+3K-aK}{3}$	0
s^0	aK	

from s^0 row $aK > 0$ or $K > 0$

from s^1 row $6-K > 0$ or $K < 6$

range of K for stability is $0 < K < 6$.

When $K=6$ system will be marginally stable then from s^2 row.

$$3s^2+4K=0$$

Put $K=6$

$$3s^2+4*6 = 3s^2+24=0$$

$$s^2=-24/3$$

$$s=\sqrt{-8} = \pm 2.82i$$

$\therefore$ frequency of oscillation is 2.82 rad/sec

06.c What are the difficulties encountered while assessing the R-H criteria and how do you eliminate these difficulties? Explain with examples (6 marks)

01. First element of any of the Routh's array is zero and the same remaining row contains at least one non zero element

Example - $s^5+2s^4+3s^3+6s^2+2s+1=0$

s^5	1	3	2
s^4	2	6	1
s^3	0ϵ	1.5	0
s^2	$\frac{6\epsilon-3}{\epsilon}$	1	0
s^1	$\dfrac{\frac{1.5(6\epsilon-3)}{\epsilon}-\epsilon}{\frac{(6\epsilon-3)}{\epsilon}}$		
s^0	1		

"Replace '0' by small number ϵ"

let $\epsilon \to 0$

s^5	1	3	2
s^4	2	6	1
s^3	0	1.5	0
s^2	$-\infty$	1	0
s^1	1.5	0	0
s^0	1		

02. All the elements of a row in Routh's array are zero

Example-

s^5	a	b	c
s^4	d	e	f
s^3	0	0	0

Row of all zeros.

form auxilary equation using coefficients of a row which is just above the row of all zeros

$A(s) = ds^4 + es^2 + f$

Take a derivative of A(s) w.r.t s.

$$\frac{dA(s)}{ds} = 4ds^3 + 2es$$

Replace zeros row by the coefficients of $\frac{dA(s)}{ds}$ and continue

s^5	a	b	c
s^4	d	e	f
s^3	4d	2e	0

07.a. What do you mean by (i) breakaway point and (ii) break in point. How can they be determined with an example? (4 marks)

The break away or breakin points either lie on real axis or exist as complex conjugate pairs. If there is a root locus on real axis between 2 poles then there exist a breakaway point. If there is a root locus on real axis between 2 zeros then there exist a break in point. If there is a root locus on real axis between pole and zero then there may be or may not be breakin or breakaway point

Let the characteristic equation be in the form

$$B(s) + K A(s) = 0$$

$$\therefore K = \frac{-B(s)}{A(s)}$$

The breakaway or breakin point is given by roots of the equation $dK/ds = 0$. The roots of $dK/ds = 0$ are actual breakaway or breakin point provided for this value of root, the gain K should be positive and real.

07. b. Sketch the root locus plot for the system $G(s)H(s) = \dfrac{K}{s(s+2)(s+4)}$. Determine the range of K for which the system will have damped oscillating response (7 marks)

Step 01. Location of poles and zeros

m = no. of zeros = 0

n = no. of poles = 3 $\quad P_1 = 0,\ P_2 = -2,\ P_3 = -4$

Step 02. Root locus on real axis.

Part of real axis between 0 to −2 and −4 to −∞ is part of root locus.

Step 03. Angle of asymptotes and centroid.

Angle of asymptotes $= \dfrac{\pm 180(2q+1)}{n-m}$ $\qquad q = 0, 1, 2, 3.$

$q=0$ angle $= \dfrac{\pm(180)}{3} = \pm 60^\circ$

$q=1$ angle $= \dfrac{\pm(180((2\times1)+1)}{3} = \pm 180^\circ$

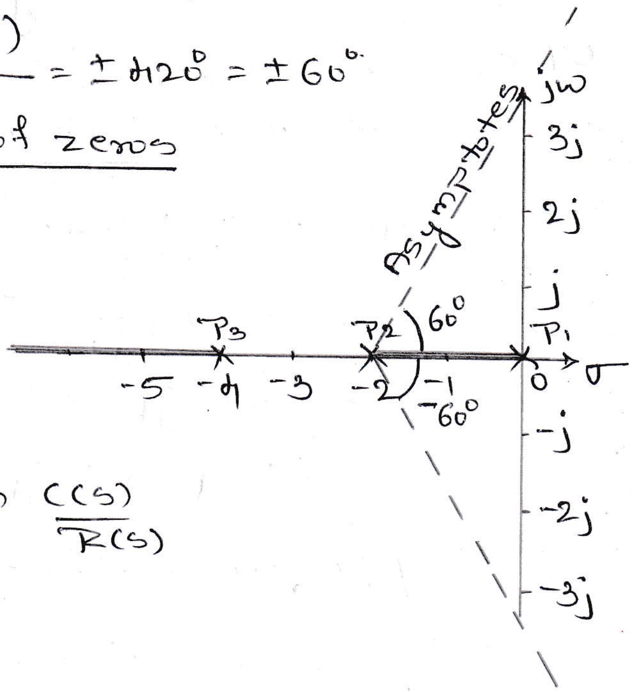
$q=2$ angle $= \dfrac{\pm(180((2\times2)+1)}{3} = \pm 300^\circ$

$q=3$ angle $= \dfrac{\pm(180((2\times3)+1)}{3} = \pm 420^\circ = \pm 60^\circ$

Centroid $= \dfrac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

$= \dfrac{(0-2-4)-0}{3}$

Centroid $= -2$.



Step 04. Break away point

Closed loop transfer function $\dfrac{C(s)}{R(s)}$

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+2)(s+4)+K}$$

characteristic equation

$$s^3 + 6s^2 + 8s + K = 0$$

or $K = -s^3 - 6s^2 - 8s$

$dK/ds = -3s^2 - 12s - 8 = 0$

Solving $\quad s = -0.84 \quad s = -3.15$

when $s = -0.84$ $K = 3.079$ real and positive Valid Break away Point

when $s = -3.15$ $K = -3.079$ negative invalid

Step 05. Angle of arrival or departure

No complex pole or zero, so no need to calculate angle of arrival or departure.

Step 06. Intersection with $j\omega$ axis.

Characteristic equation $s^3 + 6s^2 + 8s + K = 0$

Put $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

Equating real and imaginary parts.

$-\omega^3 + 8\omega = 0$ $\quad$ $-6\omega^2 + K = 0$

$\omega^2 = 8$ $\quad$ $K = 6\omega^2 = 6 \times 8$

$\omega = \pm 2.82$ $\quad$ $K = 48.$

Range of K

$$0 < K < 48$$

Plot see the graph.

07. c. Show that part of root locus for the open loop T.F $G(s)H(s) = \dfrac{K(s+2)}{s(s+1)}$ is circle (5 marks)

$$G(s)H(s) = \frac{K(s+2)}{s(s+1)}$$

Put $s = \alpha + j\beta$

$$= \frac{K(\alpha + j\beta + 2)}{(\alpha + j\beta)(\alpha + j\beta + 1)}$$

$$\angle G(s)H(s) = \frac{\tan^{-1}(\beta/\alpha+2)}{\tan^{-1}(\beta/\alpha) + \tan^{-1}(\beta/\alpha+1)}$$

$$= \tan^{-1}(\beta/\alpha+2) - \tan^{-1}\left(\frac{\beta/\alpha + \beta/\alpha+1}{1 + \left(\frac{\beta}{\alpha}\right)(\beta/\alpha+1)}\right)$$

$$= \tan^{-1}\left(\frac{\beta}{\alpha+2}\right) - \tan^{-1}\left(\frac{\beta(\alpha+1) + \alpha\beta}{\alpha(\alpha+1) + \beta^2}\right)$$

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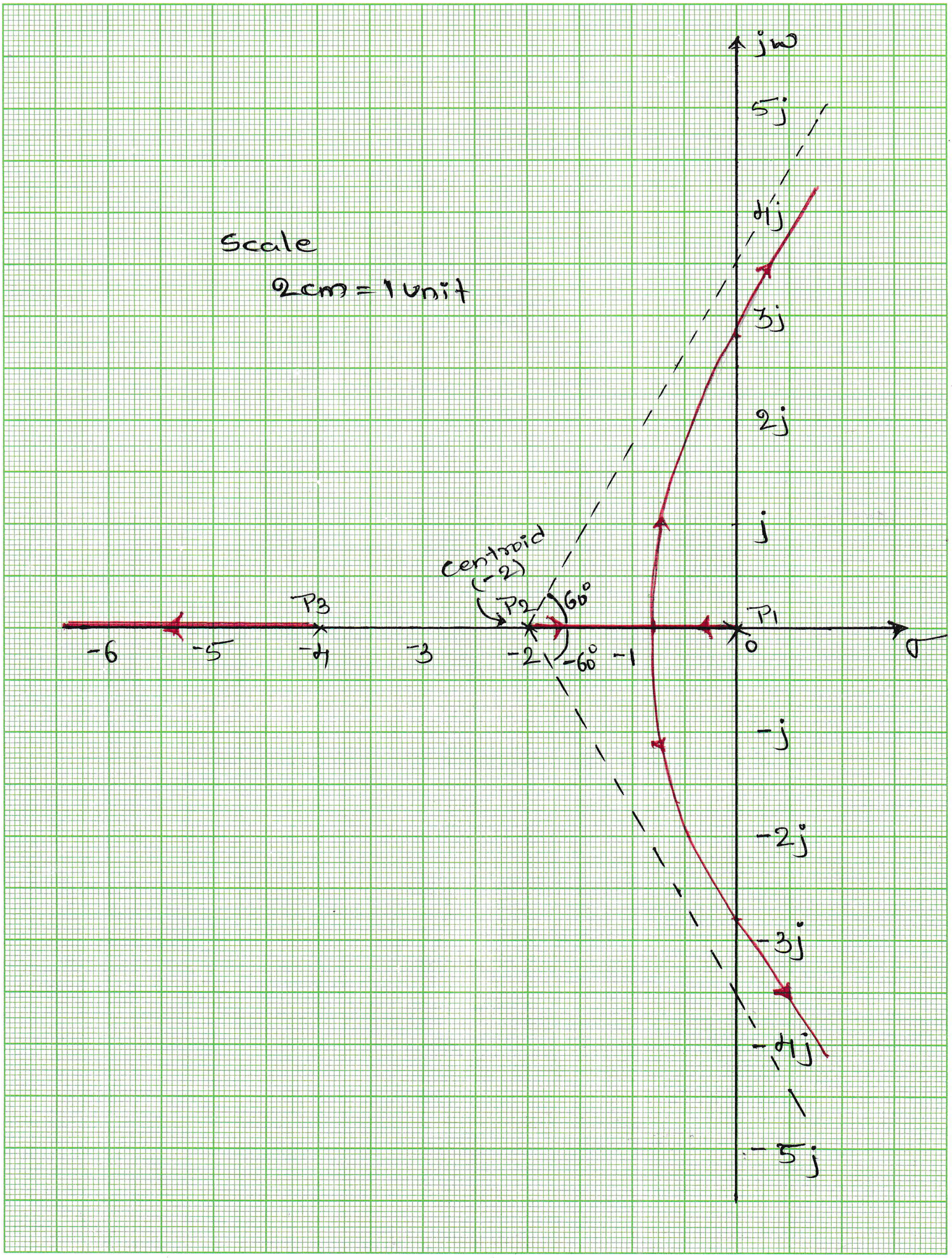
07.b. Root locus

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$



$$\angle G(s)H(s) = \tan^{-1}\left(\frac{\frac{\beta}{\alpha+2} - \frac{\beta(\alpha+1)+\alpha\beta}{\alpha(\alpha+1)+\beta^2}}{1-\left(\frac{\beta}{\alpha+2}\right)\left(\frac{\beta(\alpha+1)+\alpha\beta}{\alpha(\alpha+1)+\beta^2}\right)}\right)$$

Angle condition $\angle G(s)H(s) = -180^\circ$

$$-180 = \tan^{-1}\left(\frac{\frac{\beta}{\alpha+2} - \frac{\beta(\alpha+1)+\alpha\beta}{\alpha(\alpha+1)+\beta^2}}{1-\left(\frac{\beta}{\alpha+2}\right)\left(\frac{\beta(\alpha+1)+\alpha\beta}{\alpha(\alpha+1)+\beta^2}\right)}\right)$$

$$\therefore \frac{\beta}{\alpha+2} - \frac{\beta(\alpha+1)+\alpha\beta}{\alpha(\alpha+1)+\beta^2} = 0$$

$$\beta(\alpha(\alpha+1)+\beta^2) - \beta(\alpha+1+\alpha)(\alpha+2) = 0$$

$$\alpha^2+\alpha+\beta^2 - (2\alpha^2+\alpha+4\alpha+2) = 0$$

$$-\alpha^2-4\alpha-2+\beta^2 = 0$$

add and subtract 2

$$-\alpha^2-4\alpha-4+2+\beta^2 = 0$$

$$(\alpha-2)^2+\beta^2 = (\sqrt{2})^2$$

It is equation of circle with center at $(-2,0)$ and radius $\sqrt{2}$

08. a. Derive an expression for resonant peak and resonant frequency for a second order system. (6 marks)

- Resonant peak (M_r)

Consider the closed loop transfer function of second order system

$$\frac{C(s)}{R(s)} = M(s) = \omega_n^2 / s^2+2\xi\omega_n s+\omega_n^2.$$

Sinusoidal transfer function $M(j\omega)$ is obtained by letting $s=j\omega$

$$\therefore M(j\omega) = \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2\left(-\frac{\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n} + 1\right)}$$

$$= \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

let normalized frequency $u = \left(\frac{\omega}{\omega_n}\right)$

M = magnitude of closed loop transfer function

α = phase of closed loop transfer function

$$M = |M(j\omega)| = \left[\frac{1}{(1-u^2)^2 + (2\xi u)^2}\right]^{1/2}$$

$$M = \left[(1-u^2)^2 + 4\xi^2 u^2\right]^{-1/2}$$

$$\alpha = \angle M(j\omega) = -\tan^{-1}\frac{2\xi u}{1-u^2}$$

Resonant peak is the maximum value of M. The condition for maximum value of M can be obtained by $\frac{dM}{du} = 0$ when $u = u_r$

where $u_r = \frac{\omega_r}{\omega}$ = normalized resonant frequency

$$\frac{dM}{du} = -\frac{1}{2}\left[(1-u^2)^2 + 4\xi^2u^2\right]^{-3/2}\left[2(1-u^2)(-2u) + 8\xi^2 u\right]$$

$$= \frac{4u(1-u)^2 - 8\xi^2 u}{2\left[(1-u^2)^2 + 4\xi^2u^2\right]^{3/2}}$$

Replacing u by u_r

$$\frac{4u_r(1-u_r)^2 - 8\xi^2 u_r}{2\left((1-u_r^2)^2 + 4\xi^2 u_r^2\right)^{3/2}} = 0$$

or $4u_r(1-u_r)^2 - 8\xi^2 u_r = 0$

$$4u_r - 4u_r^3 - 8\xi^2 u_r = 0$$

or $u_r = \sqrt{1-2\xi^2}$

we have

$$M_r = \left.\frac{1}{\left[(1-u^2)^2 + 4\xi^2u^2\right]^{1/2}}\right|_{u=u_r}$$

$$= \frac{1}{\left((1-u_r^2)^2 + 4\xi^2 u_r^2\right)^{1/2}} = \frac{1}{\left[(1-(1-2\xi^2))^2 + 4\xi^2(1-2\xi^2)\right]^{1/2}}$$

$$= \frac{1}{(4\xi^4 + 4\xi^2 - 8\xi^4)^{1/2}} = \frac{1}{(4\xi^2 - 4\xi^4)^{1/2}}$$

$$= \frac{1}{2\xi\sqrt{1-\xi^2}}$$

∴ Resonant peak $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

- Resonant frequency (ω_r)

  normalized resonant frequency $u_r = \frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}$

∴ Resonant frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

08.b. Sketch the Bode-plot for the openloop transfer function $G(s)H(s) = K/s(s+1)(0.1s+1)$ and determine the value of K for which system is to be stable. Also find the gain margin and phase margin. (10 marks)

Sinusoidal transfer function is obtained by replacing s by $j\omega$. Put $K=1$

$$G(j\omega)H(j\omega) = \frac{1}{(j\omega)(j\omega+1)(0.1j\omega+1)}$$

Magnitude plot

Term	Corner frequency rad/sec	Slope db/dec	Change in slope
$1/j\omega$	—	-20	—
$1/1+j\omega$	$\omega_{c1}=1$	-20	$-20-20=-40$
$1/1+0.1j\omega$	$\omega_{c2}=10$	-20	$-40-20=-60$

Let lower frequency $\omega_l = 0.1$ rad/sec

higher frequency $\omega_h = 50$ rad/sec

Let $A = |G(j\omega)|$ in db

at $\omega = \omega_l = 0.1$ rad/sec

$$A = 20\log 1/\omega = 20\log 1/0.1 = 20\text{db}$$

at $\omega = \omega_{c1} = 1$ rad/sec

$$A = 20 \log 1/1$$

$$A = 0 \text{ db}$$

at $\omega = \omega_{c2} = 10$ rad/sec

$$A = \left(\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right) + A \text{ at } \omega_{c1}$$

$$= -40 \times \log \frac{10}{1} + 0$$

$$= -40 \text{db}$$

at $\omega = \omega_h = 50$ rad/sec

$$A = -60 \times \log \frac{50}{10} - 40$$

$$= -81.93 \text{db}.$$

Phase plot

$$\phi = \angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}(0.1\omega)$$

ω rad/sec	φ(ω) deg.
0.1	−96.28
0.2	−102.45
0.5	−119.42
1	−140.71
2	−168.74
3.16	−180
5	−195.25
7	−206.86
10	−219.29
20	−240.57
50	−257.5
100	−263.7

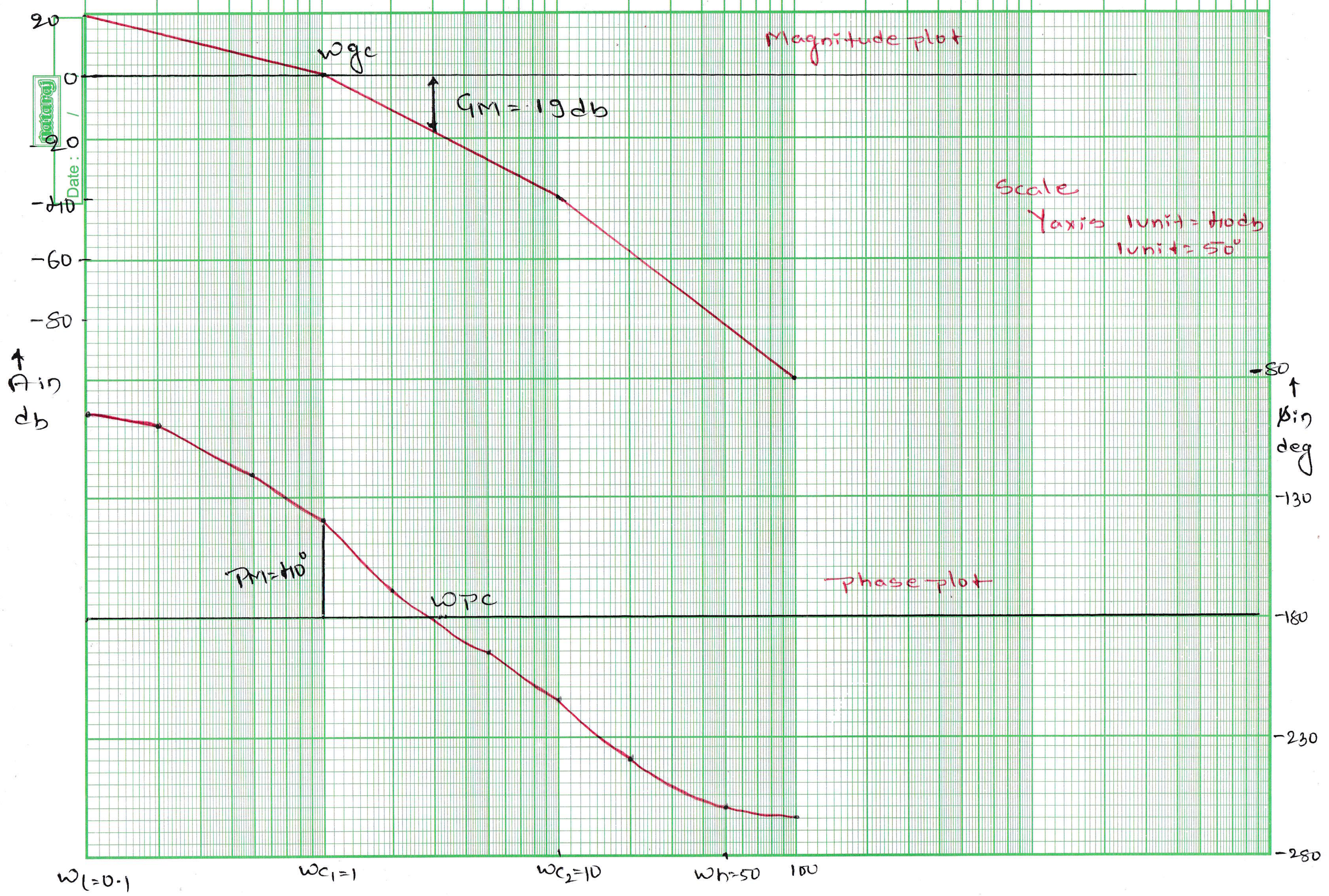
Gain margin GM = 19.98 db

Phase margin PM = 39.29°

at Limiting case GM = 0 db

$$\therefore 20 \log K = 20$$

or K = 10 for marginal stability.


Magnitude plot
ωgc
GM = 19 db
Scale
Y axis 1 unit = 40db
1 unit = 50°
20
0
-20
-40
-60
-80
A in db
PM = 40°
ωpc
Phase plot
-80
ϕ in deg
-130
-180
-230
-280
ωl = 0.1
ωc1 = 1
ωc2 = 10
ωh = 50
100

09. a. State and explain the Nyquist stability criterion (6 marks)

Nyquist suggested to select a single valued function $F(s)$ as $1+G(s)H(s)$

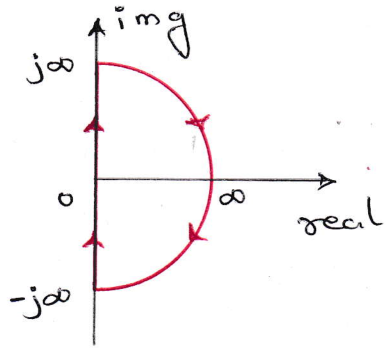
$$F(s) = 1+G(s)H(s)$$

we know that Poles of $1+G(s)H(s)$ = Poles of open loop transfer function.

Zeros of $1+G(s)H(s)$ = closed loop pole of the system.

For stability all zeros of $1+G(s)H(s)$ must be the left half of s plane. He suggested that rather than analyzing whether all zeros are on left half of s-plane, it is better to examine presence of any zero on right half of s-plane. So select a Nyquist path as below.

Map the points on Nyquist path into F-plane with the help of mapping function $1+G(s)H(s)$ to get $\Gamma'(s)$ locus. It is called Nyquist plot



Let N = encirclements of origin of F-plane by Nyquist plot

According to mapping theorem

$$N = P - Z$$

for system to be stable $Z = 0$

$\therefore N = P$ = Number of open loop poles. (for closed loop system)

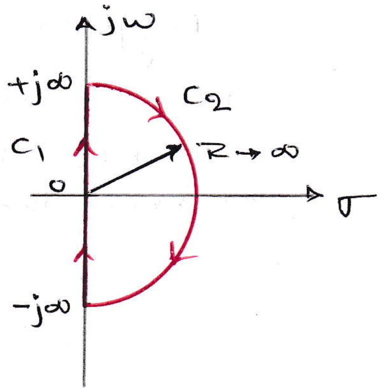
$N = P = 0$ for open loop system.

"Nyquist stability criterion states that for absolute stability of the system, the number of encirclements of new origin of F-plane by Nyquist plot must be equal to number of poles of $1+G(s)H(s)$"

09. b Sketch the Nyquist plot and comment on the stability of the closed loop system whose open loop transfer function is $G(s)H(s) = \dfrac{K(s-1)}{(s+1)^2}$

Choose entire right half of s plane as Nyquist contour.

Mapping of section C_1



$$G(j\omega)H(j\omega) = \frac{K(j\omega-4)}{(j\omega+1)^2}$$

$$= \frac{K(j\omega-4)}{(j\omega+1)^2}\,\frac{(1-j\omega)^2}{(1-j\omega)^2}$$

$$= \frac{K(6\omega^2-4)}{(1+\omega^2)^2} + j\,\frac{K\omega(9-\omega^2)}{(1+\omega^2)^2}$$

at $\omega = -\infty$ $\quad G(j\omega)H(j\omega) = 0+j0$

at $\omega = 0^-$ $\quad G(j\omega)H(j\omega) = -4K-j0$

at $\omega = 0^+$ $\quad G(j\omega)H(j\omega) = -4K+j0$

at $\omega = +\infty$ $\quad G(j\omega)H(j\omega) = 0-j0.$

Mapping of section C_2

Put $s = Re^{j\theta}$ $\quad R\to\infty$, θ varing from $+90$ to $-90°$

we can approximate $(s+1) = s$.

$$\text{so } \lim_{R\to\infty} \frac{K(Re^{j\theta}-4)}{(Re^{j\theta}+1)^2} = \lim_{R\to\infty} \frac{K}{Re^{j\theta}} = 0\,e^{-j\theta}$$

$$\theta = 90 \Rightarrow 0e^{-j90}$$

$$\theta = -90 \Rightarrow 0e^{j90}$$

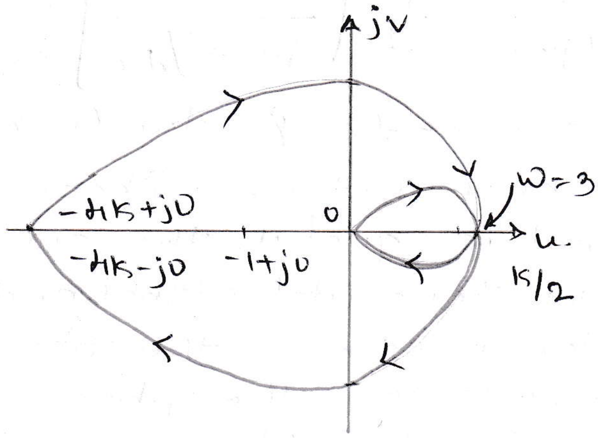
The plot turns around the origin from $\angle -90°$ to $0°$ to $\angle 90°$. The point of intersection of the Nyquist plot on the real axis is obtained by equating the imaginary part to zero.

i.e $$\frac{K\omega(9-\omega^2)}{(1+\omega^2)^2} = 0$$

$$\omega^2 = 9$$

$$\omega = \pm 3 \text{ rad/sec.}$$

$|G(j\omega)H(j\omega)|$ is $$\frac{K(6(3)^2-4)}{(1+3^2)^2} = K/2$$



$N = -1,\ P = 0$

$N = P - Z$

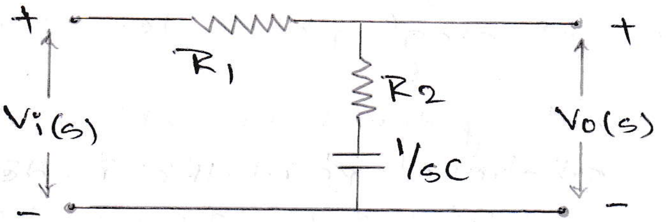
$-1 = 0 - Z$

$Z = 1$

system is unstable for $K < -2$ or $K > 1/4$.

10.a Explain the phase lag compensator with neat circuit diagram and derive expression for the transfer function of a lag compensator. (6 marks)

The lag compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied. The lag compensator circuit in the 's' domain is shown in figure below.



The transfer function of this lag compensator is

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left[\frac{s + 1/\tau}{s + 1/\alpha\tau}\right]$$

where $\tau = R_2 C$, $\alpha = \frac{R_1 + R_2}{R_2}$

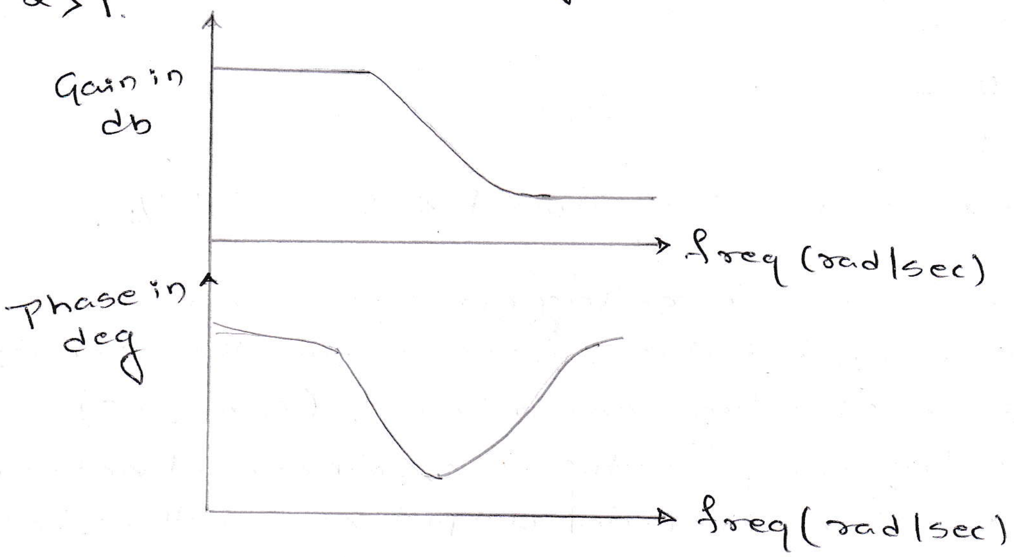
Here $\alpha > 1$, from the transfer function we can conclude that the lag compensator has one pole at $s = -1/\alpha\tau$ and one zero at $s = -1/\tau$. This means the pole will be nearer to origin in the pole-zero configuration of lag compensator.

Substitute $s = j\omega$ we get

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\left[\frac{j\omega + 1/\tau}{j\omega + 1/\alpha\tau}\right]$$

Phase angle $\phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\alpha\omega\tau)$

We know that, the phase angle of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function. So, in order to produce the phase lag at the output of this compensator, the phase angle of the transfer function should be negative. This will happen when $\alpha > 1$.



10.b What are the limitations of single phase lead control? (4 marks)

The limitations of single phase lead control are as follows:

01. If the original system is unstable or has low stability margin, the aditional phase-lead required to realize a certain desired phase margin may be excessive. This may require very small value of α and may result in large bandwidth.

02. If the original system is unstable or has a low stability margin, the phase curve of the Bode plot of the forward path transfer function has a steep negative slope at the gain crossover frequency and the single stage phase lead control is ineffective. It requires an amplifier with large gain which will be costly.

03. The maximum phase lead available from a single lead network is less than 60°. Thus, if a phase lead of more than 60° is required a multistage controller may be used.

10.c Write note on PID controller. (6 marks)

A PID controller produces an output signal consisting of three terms - one proportional to error signal. another one proportional to integral of error signal and the third one proportional to derivative of error signal. The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces or eliminates the steady state error. The derivative controller reduces the rate of change of error. PID controllers are commonly employed in process control industries. The determination of the proportional, integral, and derivative constents of the controller called tuning in process control depends on the dynamic response of the plant.