CBCS SCHEME

18EC32

USN

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Network Theory

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Using source transformation technique find the current through 5Ω resistor for the circuit shown in Fig.Q.1(a) **(06 Marks)**

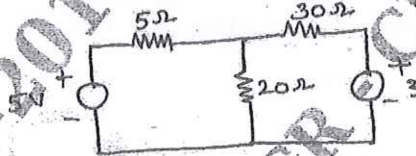

Fig.Q.1(a)

b. Use Mesh Analysis to determine the Mesh currents i_1, i_2 and i_3 for the network shown in Fig.Q.1(b). **(06 Marks)**

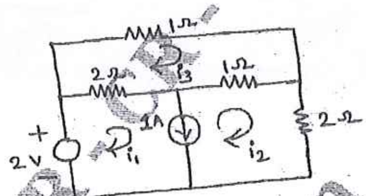

Fig.Q.1(b)

c. Find the power delivered by 1A current source using nodal analysis for the circuit shown in Fig.Q.1(c). **(08 Marks)**

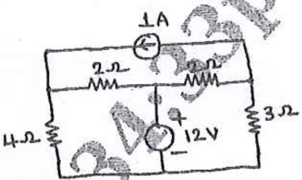

Fig.Q.1(c)

OR

2 a. Three Impedances are connected in delta, obtain the star equivalent of the network. **(06 Marks)**

b. Use Mesh Analysis to find the power delivered by the dependent voltage source in the circuit shown in Fig.Q.2(b). **(06 Marks)**

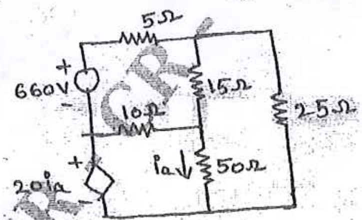

Fig.Q.2(b)

c. Determine all the node voltages for the circuit shown in Fig.Q.2(c) using nodal analysis. **(08 Marks)**

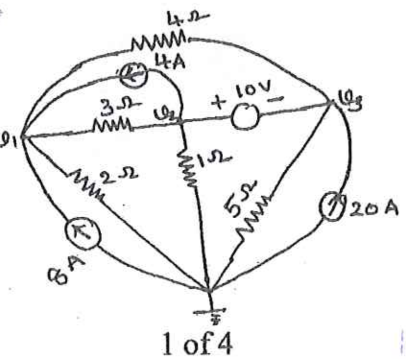

Fig.Q.2(c)

Module-2

3 a. State and explain superposition theorem **(06 Marks)**

b. Use Millman's Theorem to find the current flowing through $(2 + j3)\Omega$ impedance for the circuit shown in Fig.Q.3(b). **(08 Marks)**

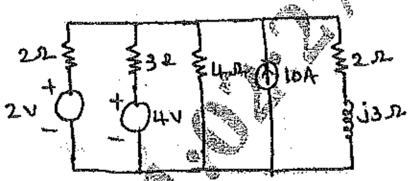

Fig.Q.3(b)

c. State and prove Norton's theorem. **(06 Marks)**

OR

4 a. Find the Thevenin's equivalent for the circuit shown in Fig.Q.4(a) with respect to terminals X-Y. **(08 Marks)**

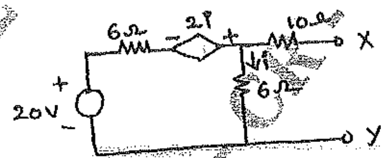

Fig.Q.4(a)

b. Find the condition for maximum power transfer in the AC circuit, where both R_L and X_L are varying. **(06 Marks)**

c. Determine the current through the load resistance using Norton's Theorem for the circuit shown in Fig.Q.4(c). **(06 Marks)**

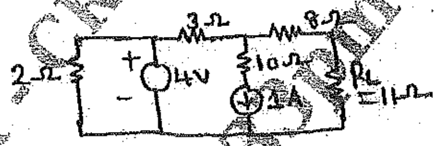

Fig.Q.4(c)

Module-3

5 a. Explain the behavior of R, L, C elements at the time of switching at $t = 0$, at $t = 0^+$ and $t = \infty$. **(07 Marks)**

b. In the network shown in Fig.Q.5(b). Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$. Assume that the capacitor is initially uncharged. **(07 Marks)**

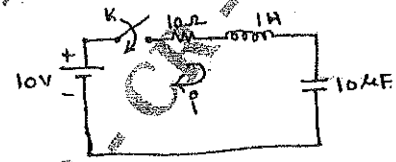

Fig.Q.5(b)

c. In the network shown in Fig.Q.5(c) find, i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$. The swich k is closed at $t = 0$ with zero current in the inductor. **(06 Marks)**

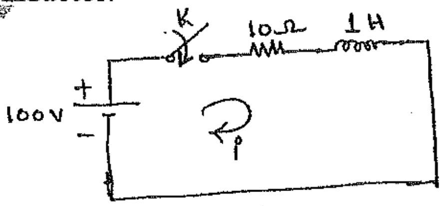

Fig.Q.5(c)

OR

6 a. In the network shown in Fig.Q.6(a). The switch k is changed from position a to b at $t = 0$, the steady state is reached at position a. Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$. Assume that the capacitor is initially uncharged. **(10 Marks)**

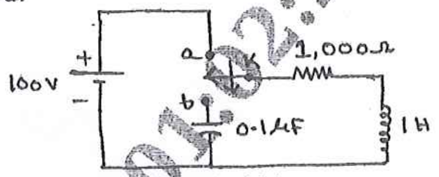

Fig.Q.6(a)

b. For the network shown in Fig.Q.6(b). The network is in steady state with switch k is closed. At $t = 0$, the switch is opened. Determine the voltage across the switch V_k and $\frac{d}{dt}V_k$ at $t = 0^+$. **(10 Marks)**

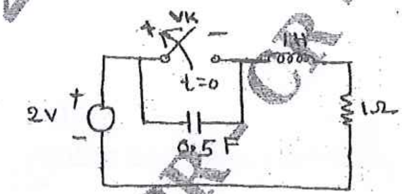

Fig.Q.6(b)

Module-4

7 a. Obtain Laplace transform of
i) Step function
ii) Ramp function
iii) Impulse function. **(09 Marks)**

b. Find the Laplace transform of the periodic signal x(t) as shown in Fig.Q.7(b). **(11 Marks)**

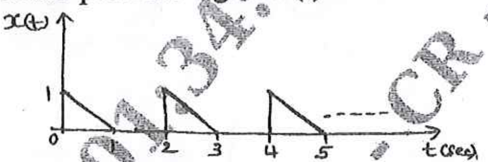

Fig.Q.7(b)

OR

8 a. In the series RL circuit shown in Fig.Q.8(a), the source voltage is $v(t) = 50 \sin 250t$V. Using Laplace transform determine, the current when switch K is closed at $t = 0$. **(10 Marks)**

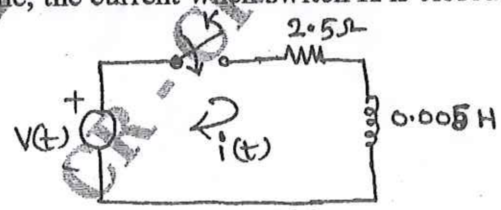

Fig.Q.8(a)

b. Find the Laplace transform of the non-sinusoidal periodic waveform shown in Fig.Q.8(b)

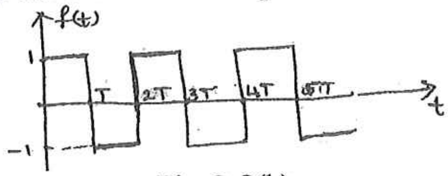

Fig.Q.8(b) **(10 Marks)**

Module-5

9 a. Define Z parameters. Determine Z parameters interms of Y parameters. **(06 Marks)**

b. Determine h parameters of the circuit shown in Fig.Q.9(b) **(07 Marks)**

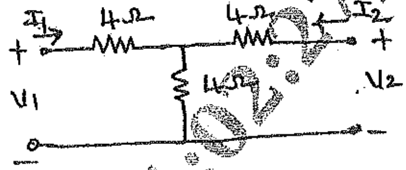

Fig.Q.9(b)

c. For the network shown in Fig.Q.9(c). Find the transmission parameters. **(07 Marks)**

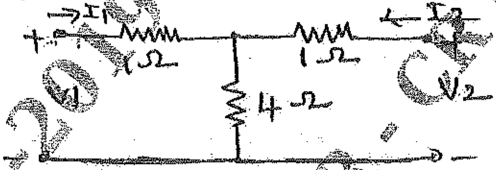

Fig.Q.9(c)

OR

10 a. Define Q-factor, selectivity and Band width. **(03 Marks)**

b. A series RLC circuit has a resistance of 10Ω, an inductance of 0.3H and a capacitance of 100μF. The applied voltage is 230V. Find: i) The resonant frequency ii) lower and upper cut off frequencies iii) current at resonance iv) currents at f_1 and f_2 v) Voltage across. the inductance at resonance. **(07 Marks)**

c. Derive the expression for the resonant frequency of the circuit shown in Fig.Q.10(c). Also show that the circuit will resonate at all frequency if $R_L = R_C = \sqrt{\frac{L}{C}}$. **(10 Marks)**

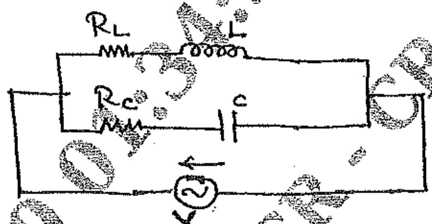

Fig.Q.10(c)

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Solution and Scheme for 18EC32, Dec 2019/ Jan 2020, Prepared and Submitted by

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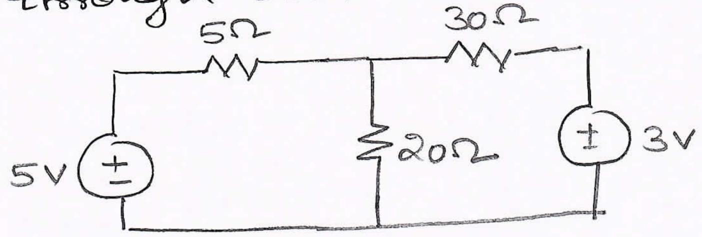
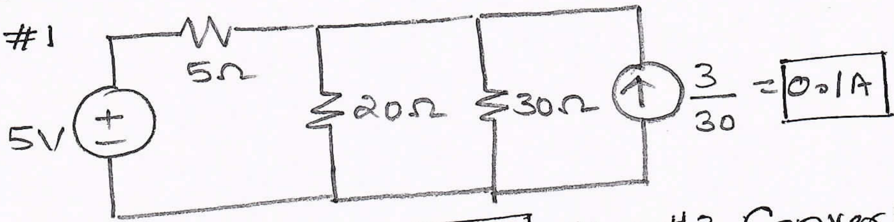
Module - 1

1 a> Using Source Transformation, find the current through 5Ω. [6M]



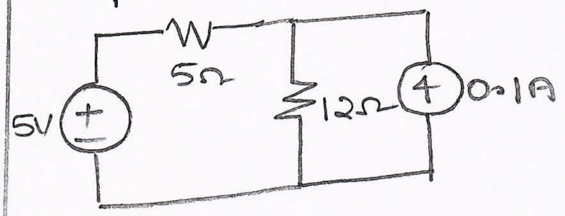
Sol^n: As current through 5Ω is required, by using source transformation, 5V, 5Ω should not be considered for transformation, hence 3V, 30Ω can be converted into eq. current source.

Step #1

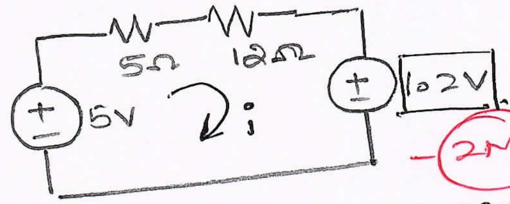


2M

Step #2 $20 \| 30$ gives $\boxed{12\Omega}$



Step #3, Convert 12Ω & 0.1A to voltage source



2M

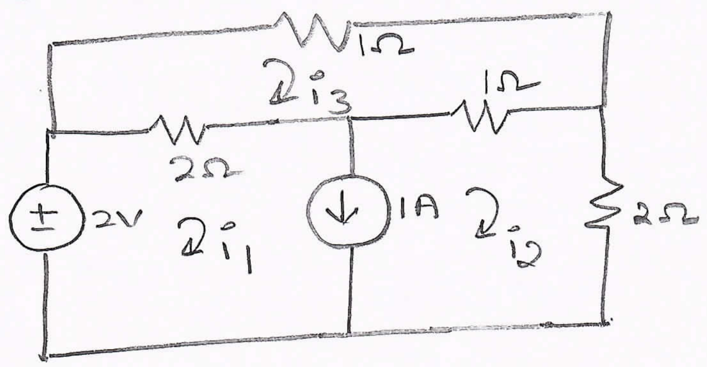
As, we have single loop in step #3, same current will flow through 5Ω, 12Ω, ∴ by KVL

$-5 + 5i + 12i + 1.2 = 0$, ∴ $\boxed{i = \frac{3.8}{17} = 0.22A}$, ∴

2M

$\boxed{i_{5\Omega} = 0.22A}$

1b> Determine the 3 mesh currents i_1, i_2 and i_3



Q. No	
1b Continued	

There are 3 mesh M_1, M_2 and M_3, with 3 current i_1, i_2 and i_3.

As, we can observe, A supermesh is formed b/w M_1 and M_2,

$\therefore$ Constraint eqn: $\boxed{i_1 - i_2 = 1A}$ — ① 1M

KVL to supermesh gives

$-2 + 2i_1 + 3i_2 - 3i_3 = 0$, $\boxed{2i_1 + 3i_2 - 3i_3 = 2}$ — ② 2M

KVL to Normal mesh [i.e mesh 3]

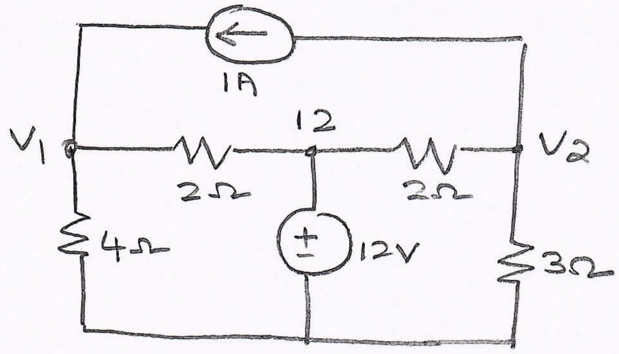
$\boxed{-2i_1 - i_2 + 4i_3 = 0}$ — ③ 1M

Solving eqn ①, ② and ③, we get

$\boxed{i_1 = 1.54 A}$, $\boxed{i_2 = 0.54 A}$, $\boxed{i_3 = 0.90 A}$ 2M

1c

Using Nodal Analysis, Find the power delivered by 1A current source



Let the two node voltages be V_1 and V_2

KCL to node V_1 gives

$$\frac{V_1}{4} + \frac{V_1 - 12}{2} = 1, \Rightarrow V_1\left[\frac{1}{4} + \frac{1}{2}\right] = 7$$

$\therefore \boxed{V_1 = 9.33 \text{ volts}}$ 2M

KCL to node V_2 gives

$$\frac{V_2}{3} + \frac{V_2 - 12}{2} + 1 = 0, \quad V_2\left[\frac{1}{3} + \frac{1}{2}\right] = 5$$

$\boxed{V_2 = 6.02 \text{ volts}}$ 2M

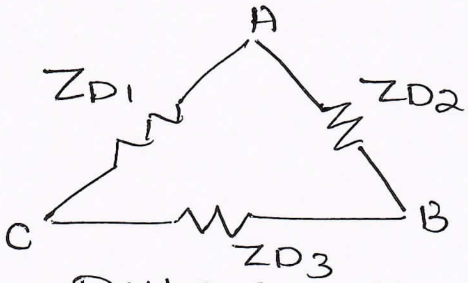
$\therefore$ Power delivered by 1A source is

$P = VI = (V_2 - V_1)(1A) = \boxed{-3.31 \text{ Watts}}$ 4M

Q. No

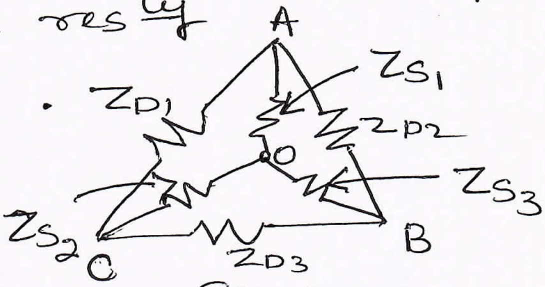
2a) Three impedances are connected in Delta, Obtain the Star equivalent of the network.

Sol^n Let, the three impedance of delta network are Z_{D1}, Z_{D2}, Z_{D3}, Let the eq. Star impedance as Z_{S1}, Z_{S2}, Z_{S3} res^ly



Delta (Fig1)

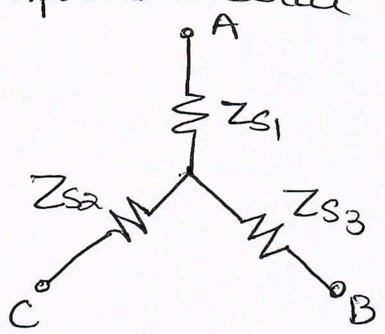
=



Star (Fig2) imposed on delta

Proof:

=



Star (Fig3)

#1] Comparing Fig1 and 3

#2] Resistance b/w node A & C is

→ In Delta

$= Z_{D1} \parallel (Z_{D2}+Z_{D3})$

$$= \frac{Z_{D1}[Z_{D2}+Z_{D3}]}{Z_{D1}+Z_{D2}+Z_{D3}} \quad -(1)$$

#3] Resistance b/w node A & C in Star

$$= Z_{S1}+Z_{S2} \quad -(2)$$

3M

#4] Equating (1) and (2)

$$\boxed{\frac{Z_{D1}[Z_{D2}+Z_{D3}]}{Z_{D1}+Z_{D2}+Z_{D3}} = Z_{S1}+Z_{S2}} \quad -(3)$$

#5] Resistance between C and B of Δ and Y are equated as it was done in eq^n (1) & (2) we get

$$\boxed{\frac{Z_{D3}[Z_{D1}+Z_{D2}]}{Z_{D1}+Z_{D2}+Z_{D3}} = Z_{S2}+Z_{S3}} \quad -(4)$$

Q. No

2a) Continued

#6] Res b/w A and B terminals of Δ and and equating them gives us

$$\boxed{\frac{Z_{D2}[Z_{D1}+Z_{D3}]}{Z_{D1}+Z_{D2}+Z_{D3}} = Z_{S1}+Z_{S3}} \quad -(5)$$

#7] Adding three eqns (3), (4) and (5) gives,

$$\boxed{2[Z_{S1}+Z_{S2}+Z_{S3}] = 2\frac{[Z_{D1}\cdot Z_{D3}+Z_{D3}Z_{D2}+Z_{D2}Z_{D1}]}{Z_{D1}+Z_{D2}+Z_{D3}}} \quad -(6)$$

eqn (6) - (3) gives.

$$\boxed{Z_{S3} = \frac{Z_{D2}\times Z_{D3}}{Z_{D1}+Z_{D2}+Z_{D3}}} \quad -(7)$$

3M

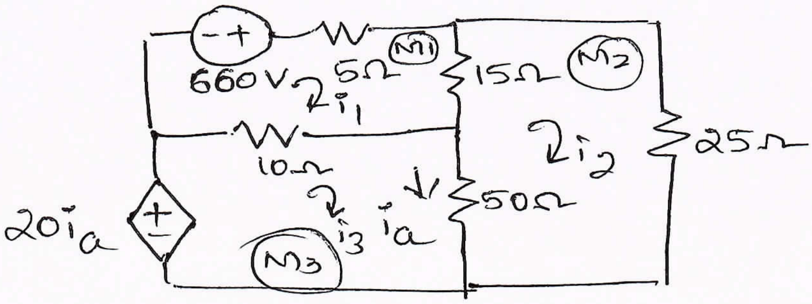
eqn (6) - (4) gives

$$\boxed{Z_{S1} = \frac{Z_{D1}\times Z_{D2}}{Z_{D1}+Z_{D2}+Z_{D3}}} \quad -(8)$$

eqn (6) - (5) gives.

$$\boxed{Z_{S2} = \frac{Z_{D1}\times Z_{D3}}{Z_{D1}+Z_{D2}+Z_{D3}}} \quad -(9)$$

2b) Using Mesh Analysis, find the power delivered by the dependent source.



Q.No

2b Continued

From Fig, $i_a = i_3 - i_2$

KVL M1

$30i_1 - 15i_2 - 10i_3 = 660$ — (1) — 1M

KVL M2

$-15i_1 + 40i_2 + 50i_2 - 50i_3 = 0$

$-15i_1 + 90i_2 - 50i_3 = 0$ — (2) — 1M

KVL to M3

$-20i_a + 60i_3 - 10i_1 - 50i_2 = 0$

$-20i_3 + 20i_2 + 60i_3 - 10i_1 - 50i_2 = 0$

$-10i_1 - 30i_2 + 40i_3 = 0$ — (3) — 2M

$i_1 = 42A, \quad i_2 = 22A, \quad i_3 = 27A$

$P_{20i_a} = 20i_a \times (-i_3)$

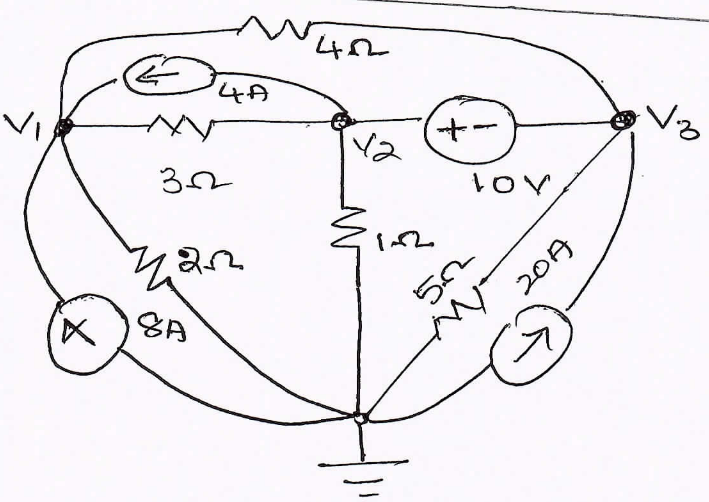
$= 20(i_3 - i_2) \times (-i_3)$

$\boxed{P_{20i_a} = -2700 \text{ Watts}}$ — 2M

AK

Q.No

2c)



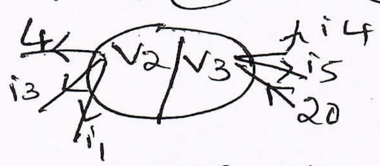
KCL at V_1

$$\frac{V_1}{2}+\frac{V_1-V_2}{3}+\frac{V_1-V_3}{4}=12 \; ; \; V_1[1.08]-V_2[0.33]-V_3[0.25]=1$$ -(

KCL at V_2 and V_3 cannot be applied directly, as it is forming supernode,

$\therefore$ Constraint eqn gives $\boxed{V_2-V_3=10}$ -(2) -(4M)

Combining node (2) and (3) and applying KCL,



$$V_2+\frac{V_2-V_1}{3}+4+\frac{V_3-V_1}{4}+\frac{V_3}{5}+(-20)=0$$

$V_1[-0.58]+V_2[1.33]+V_3[0.45]=16$ -(3) -(4M)

Solving eqn (1), (2) and (3) we get

$\boxed{V_1=18.15\text{ V}}$, $\boxed{V_2=17.43\text{ V}}$ and $\boxed{V_3=7.43\text{ V}}$

Module-2

3a) State and Explain Super position theorem.

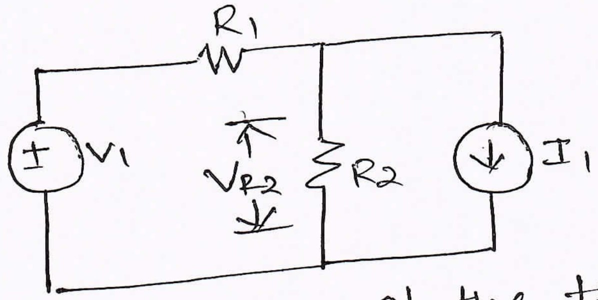
Statement: In any linear circuit, containing multiple sources, the response in any branch of the network is the algebraic sum of the respons obtained by considering each source as acting seperately, during which, all other sources are disabled except any dependent sources.

-(1M)

Q. No

3a Continued

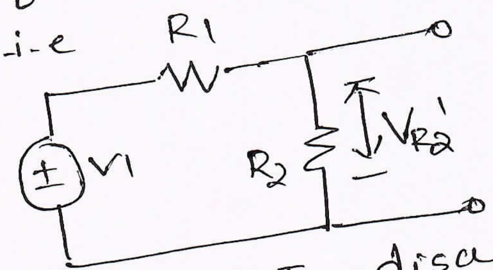
Explanation: Consider the following ckt, **-5M**



Let us assume that, Voltage across R2 is required,

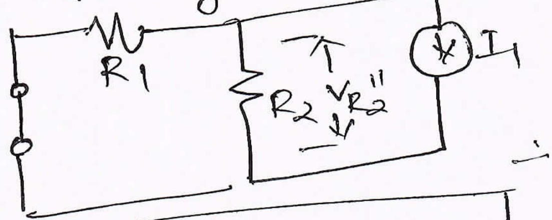
Step 1] Keep one of the two sources active, Let V_1 be active, I_1 is disabled, (open ckt), and find voltage across R_2, call it as V_{R2}'

i.e



$$V_{R2}' = \frac{V_1 \times R_2}{R_1 + R_2} \quad -① $$

Step 2] Enable I_1, disable V_1 (short ckt), find voltage across R_2, call it as V_{R2}''



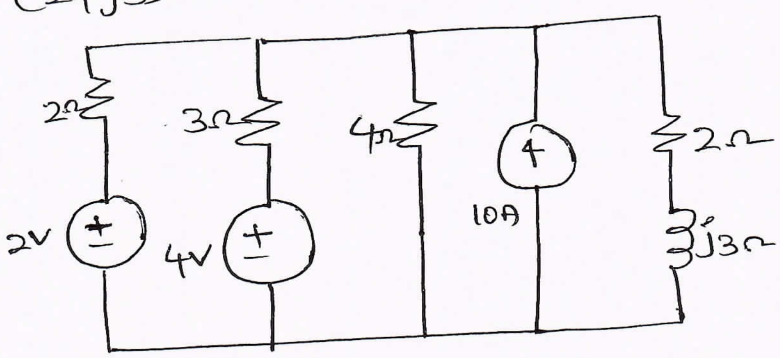
If current through R_2 is

$$I_2 = \frac{I_1 \times R_1}{R_1 + R_2}$$

$$\therefore V_{R2}'' = I_2 \times R_2 \quad - ②$$

$$\therefore \boxed{V_{R2} = V_{R2}' + V_{R2}''}$$

3b) Using Millman's theorem, find current through $(2+j3)\Omega$

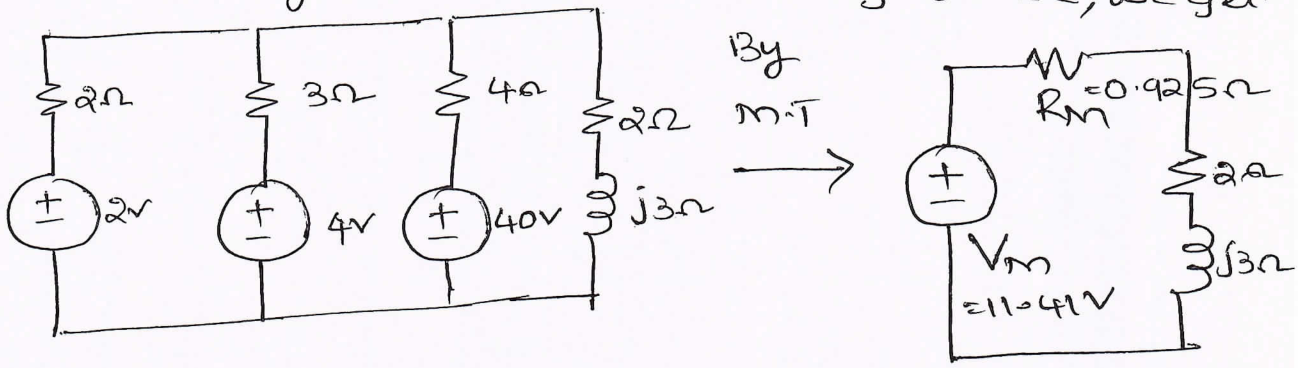


AK

Q-No

3b Continued

Transforming 4Ω and 1A to voltage source, we get



where

$$V_m = \frac{(2)\times\frac{1}{2} + 4\times\frac{1}{3} + 40\times\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 11.41V$$

– 4M

$$R_m = \frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 0.925\Omega$$

∴ Current through $(2+j3)\Omega$

$= \boxed{2.71 \angle -45 \ A}$

– 4M

3c) Statement and Proof of Norton's Theorem:

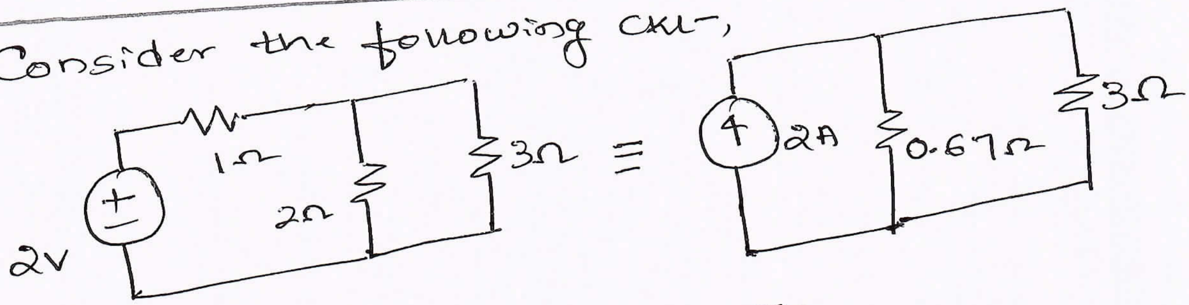
Soln: Statement: Any combination of linear bilateral ckt elements and active sources, regardless of the connection or complexity, connected to a given load Z_L, can be replaced by a simple two terminal network, consisting of a single current source of I_N amperes and a single impedance Z_N in parallel with it, across the two terminals of the load Z_L. – 2M

Proof: Consider the following ckt diagram, here let us understand the method followed to find current through 3Ω, using conventional method and other by Nortons Theorem

Q.No

3c) continued

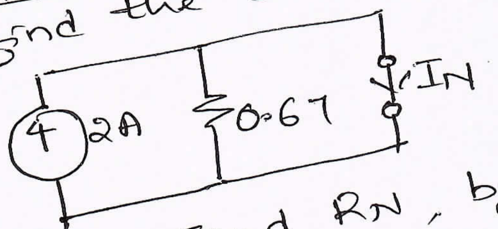
Consider the following ckt,



$$\therefore I_{3\Omega} = \frac{2 \times 0.67}{3.67} \approx 0.365 A$$

Now by, Nortons theorem

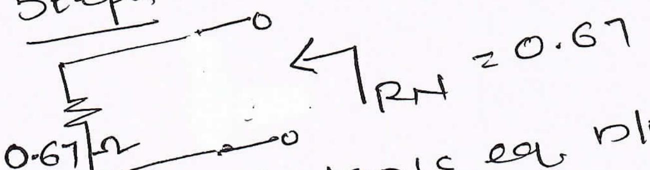
Step1: Remove 3Ω, replace it by short ckt, and find the current I_N.



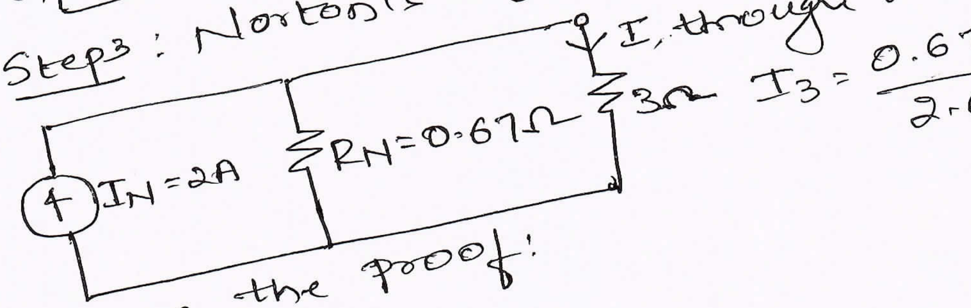
$\boxed{I_N = 2A}$.

4M

Step2: Find R_N, by disabling source



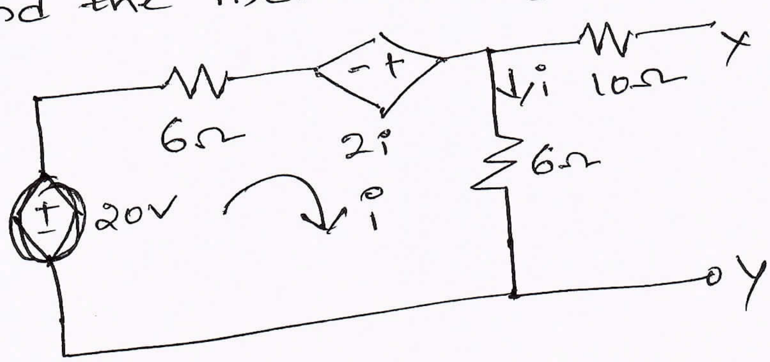
Step3: Norton's eq. n/w is



$$I_3 = \frac{0.67 \times 2}{2.67} = 0.365 A$$

hence the proof!

4a) Find the Thevenins equivalent w.r.t points x-y



Q-No

4a) Continued

To obtain V_{th}, as we have dependent source we need open ckt voltage across, X - Y.
KVL to left side mesh, we get

$-20 + 6i - 2i + 6i = 0, \therefore \boxed{i = 2A}$

As, no current flows through 10Ω,
$\therefore$ KVL to outer path of right side gives

$-6i + 0 \times 10 + V_{oc} = 0$

$\therefore V_{oc} = 6i \Rightarrow \boxed{V_{oc} = 6 \times 2 = 12V}$

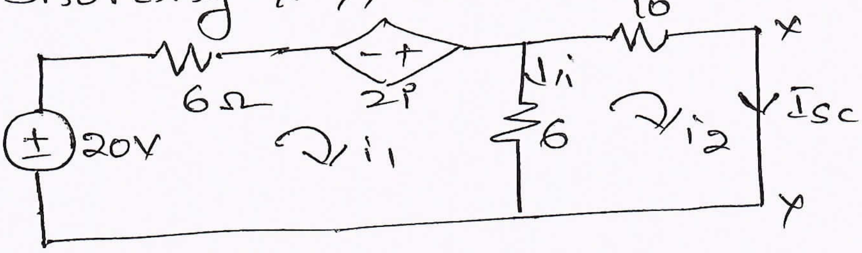
4M

$\therefore \boxed{V_{Th} = 12V}$

To, find R_{Th}, we need Isc through X-Y, $\therefore$

$R_{Th} = \frac{V_{oc}}{I_{sc}}$

Shorting X-Y, terminals we get



here $i = i_1 - i_2$

KVL to M1

$-20 + 6i_1 - 2i_1 + 2i_2 + 6i_1 - 6i_2 = 0$

$\therefore \boxed{10i_1 - 4i_2 = 20}$ — (1)

KVL to M2

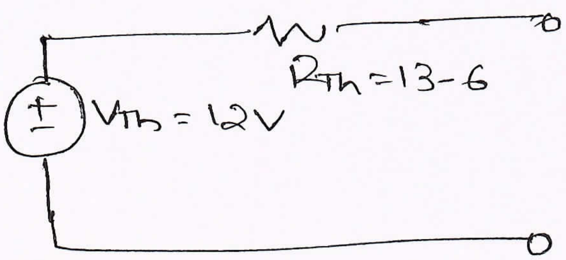
$\boxed{-6i_1 + 16i_2 = 0}$ — (2)

Solving (1) & (2)

$i_2 = I_{sc} = \frac{120}{136}$

$\therefore R_{Th} = \frac{12}{\frac{120}{136}} = 13.6\Omega$

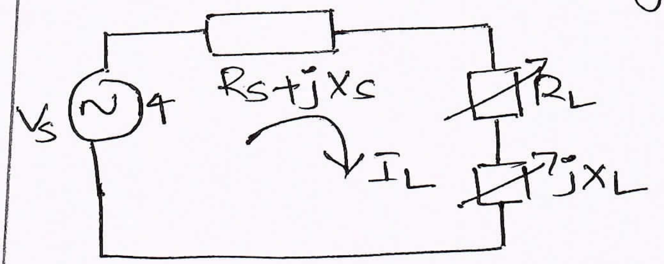
4M



Q.No

4b) Find the condition for maximum power transfer in AC ckt, where both R_L and X_L are varying

Soln: Consider the following ckt-



From above ckt,

$$I_L = \frac{V_s}{(R_s+jX_s)+(R_L+jX_L)} = \frac{V_s}{(R_s+R_L)+j(X_s+X_L)}$$

$$\therefore |I_L| = \frac{V_s}{\sqrt{(R_s+R_L)^2+(X_s+X_L)^2}}$$

$$\therefore P = |I_L|^2 \times R_L$$

$$= \frac{V_s^2}{(R_s+R_L)^2+(X_s+X_L)^2} \times R_L$$

$$= \frac{V_s^2 \times R_L}{(R_s+R_L)^2}$$ — 3M

If, R_L, varies, then, $\frac{dP}{dR_L}$ will give us, maximum power as

$$\frac{dP}{dR_L} = V_s^2 \times \frac{(R_s+R_L)^2 \times 1 - 2(R_s+R_L)}{(R_s+R_L)^2} = 0$$

$$\therefore (R_s+R_L)^2 - 2(R_s+R_L) = 0$$

$$\therefore \boxed{R_L = R_s}$$ — 3M

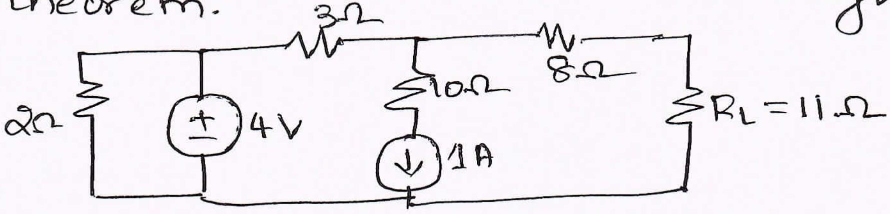
$\therefore$ $Z_L = (R_s - jX_s)$ i.e. $Z_L = Z_s^*$

AK

Pra...

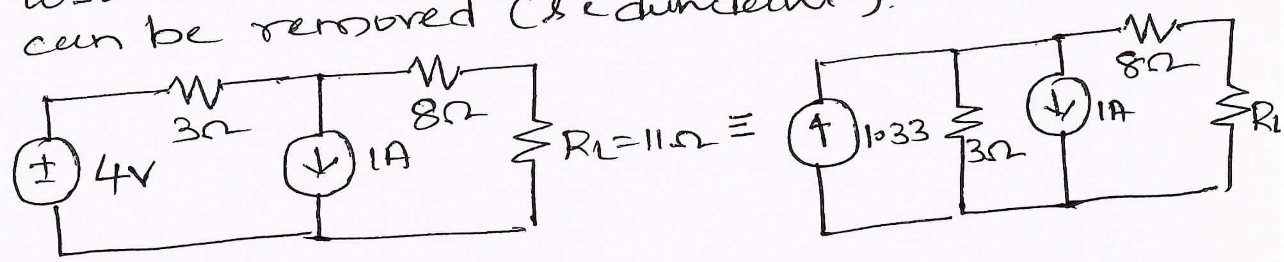
Q. No.

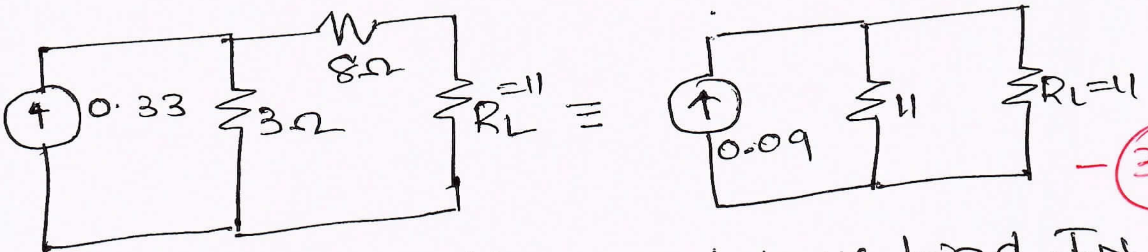
4C] Determine the Current through R_L, using No[rton's] theorem.



Sol^n: As, it is required to find current through R_L by N/T, Let us reduce the ckt complexity by source transformation.

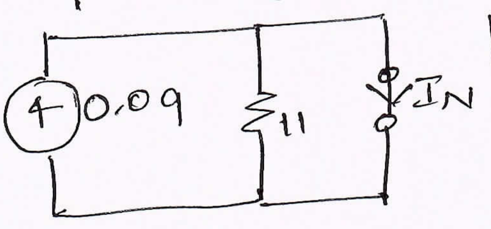
Step1: as 2Ω and 4V are parallel to each other, 2Ω can be removed (Redundant), ∴ Converting 4V, 3Ω to current source we get, and also 10Ω is in series with 1A current source, 10Ω can be removed (Redundant).





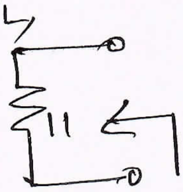
– (3M)

Step2: By Norton's theorem, let us find, I_N by replacing R_L by S/C, ∴



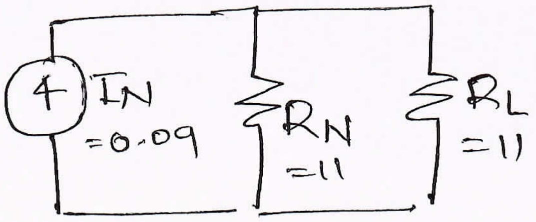
$\boxed{I_N = 0.09}$

Step3: To find R_N from R_L,



$R_N = 11Ω$

∴ Nortons eq. n/w is



$$\therefore I_{RL} = \frac{0.09 \times 11}{11+11}$$

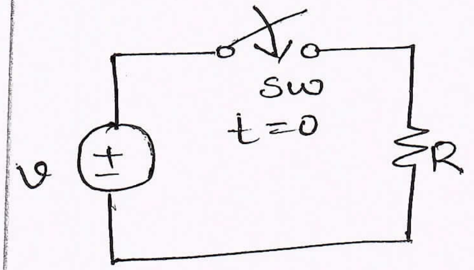
– (3M)

$\boxed{I_{RL} = 0.04\ A}$

AK

Q-No	
5a)	Explain the behaviour of R, L, C elements at th time of switching at $t=0$, $t=0+$ and $t=\infty$

i] For Resistor (R)

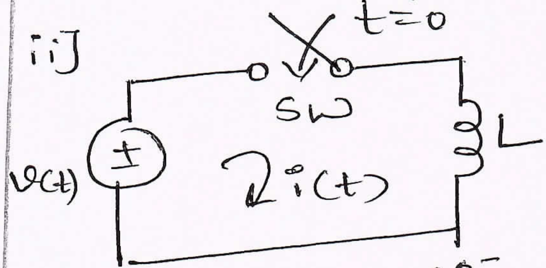


Time	SW	$i(t)$	$v(t)$
0	Closing	$i(0)=\frac{V(0)}{R}$	$V(0)=i(0)\times$
0+ & ∞	Remains closed	$i(0^+)$ & $i(\infty)$ $=\frac{V(t)}{R}$	$V(0^+)=i(t)$ & $V(\infty)$

Current through resistor and voltage across resistor changes instantly

1M

ii]



WKT

$$i_L(t)=\frac{1}{L}\int_{-\infty}^{t} V_L(t)\cdot dt$$

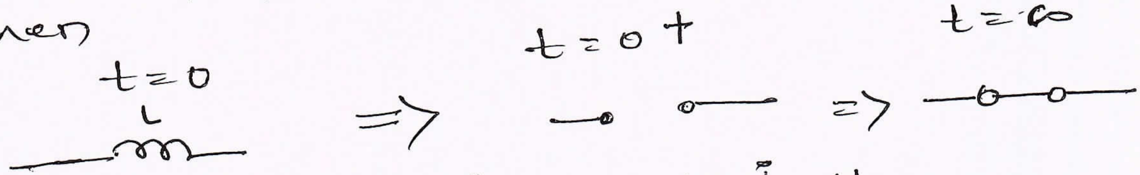
$$\therefore i_L(t)=\frac{1}{L}\int_{-\infty}^{0^-} v\cdot d\tau+\frac{1}{L}\int_{0^-}^{t} v\cdot d\tau.$$

$$i_L(t)=i_L(0^-)+\underbrace{\left\{\frac{1}{L}\int_{0^-}^{0^+} v\cdot d\tau\right\}}_{\to 0}$$

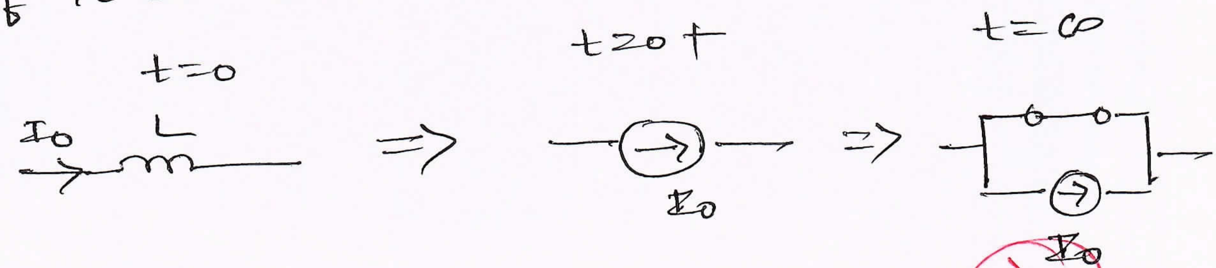
3M

$\therefore$ at $t=0+$

$\boxed{i_L(0^+)=i_L(0^-)}$ indicates current through inductor cannot change instantly, $\therefore$ For inductor at $t=0$, if Inductor has zero initial current then



if it carries initial current I_0 then



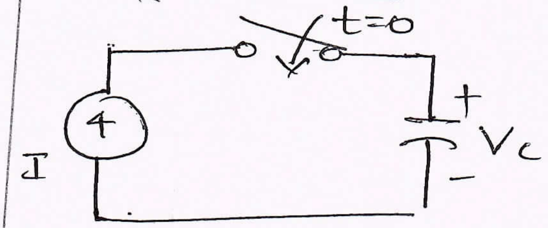
~~3M~~

Q. No

5a) Continued

For Capacitor

—x—



WKT

$$V_c = \frac{1}{C}\int_{-\infty}^{t} i(t)\,d\tau$$

$$\therefore V_c(t) = \frac{1}{C}\int_{-\infty}^{0^-} i(t)\,d\tau + \frac{1}{C}\int_{0^-}^{t} i(t)\,d\tau$$

$$\therefore V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i(t).\,d\tau$$

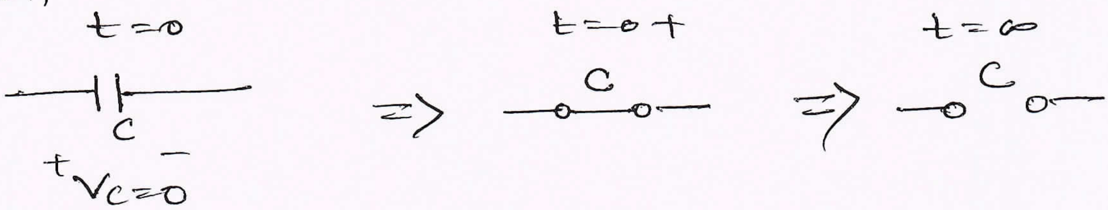
3M

For $t=0+$

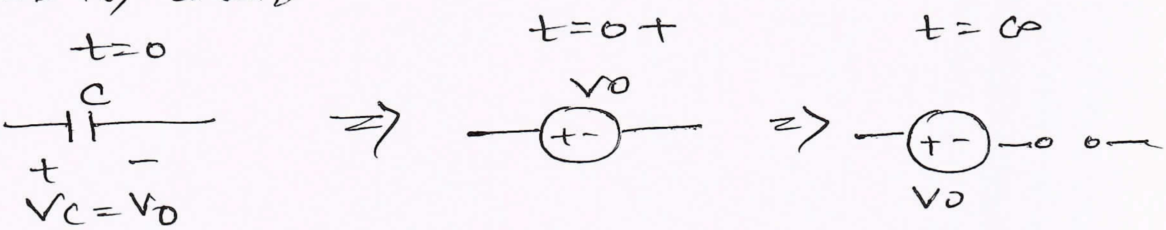
$$V_c(0^+) = V_c(0^-) + \cancel{\frac{1}{C}\int_{0^-}^{0^+} i(0^+)\,d\tau} \rightarrow 0$$

$$\therefore \boxed{V_c(0^+) = V_c(0^-)}$$ => Voltage across capacitor cannot change instantly.

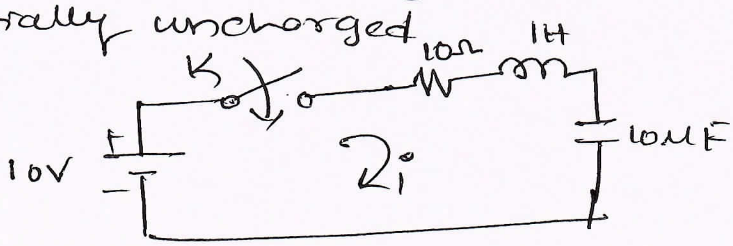
Case(i): If capacitor is initially not having voltage then



Case(ii): If capacitor is having some initial voltage across it, then

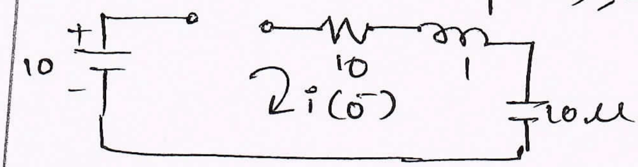


5b] Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0+$, assume capacitor is initially uncharged



5b] Continued.

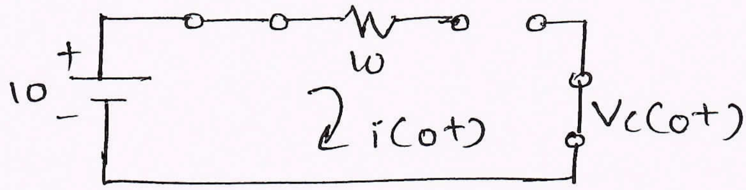
At $t=0^-$ (sw is open), ∴



∴ $i(0^-) = 0 = i_L(0^-)$

$v_c(0^-) = 0$

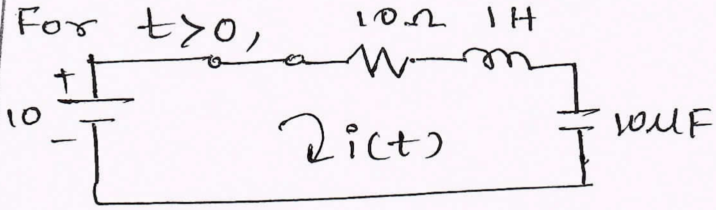
At $t=0^+$ (sw is closed)



∴ $i(0^+) = i_L(0^+) = 0$

$V_c(0^+) = 0$

For $t>0$,



By KVL

4M

$$-10 + 10\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{10\times10^{-6}}\int_0^t i(t)\,dt = 0 \quad \text{--- (1)}$$

For $t=0^+$

$$-10 + 10\,i(0^+) + \frac{di(0^+)}{dt} + 0 = 0$$

$$\therefore \boxed{\frac{di(0^+)}{dt} = 10 \text{ A/s}} \quad \text{--- (2)}$$

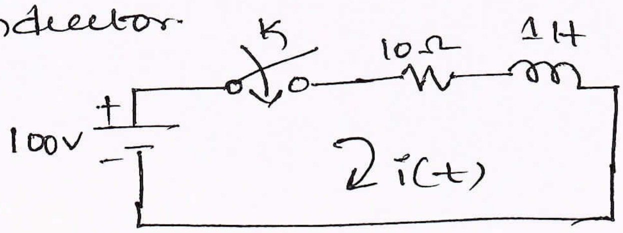
Diff eqn (1),

$$0 + 10\frac{di(0^+)}{dt} + \frac{d^2i(0^+)}{dt^2} + \frac{1}{10^{-5}}\,i(0^+) = 0$$

3M

$$\boxed{\frac{d^2i(0^+)}{dt^2} = -10\times10 = -100 \text{ A/s}^2} \quad \text{--- (2)}$$

5c] Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$, sw is closed at $t=0$, assume zero initial current through inductor.



Q.No

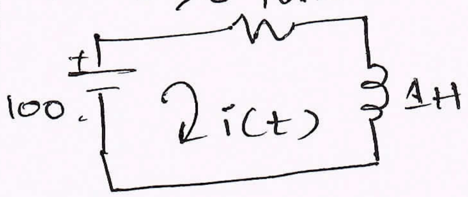
5c) Continued

At, $t=0^-$, Sw is open, $\therefore$ $i_L(0^-)=0=i_L(0^+)$

At, $t=0^+$, Sw is closed ($\therefore$ L ← O/C)

$i(0^+) = i_L(0^+) = 0$

For $t>0$



By KVL

$$-100 + 10i(t) + 1\frac{di(t)}{dt} = 0 \quad \text{①}$$

$$10i(t) + \frac{di(t)}{dt} = 100 \quad \text{②}$$

$\therefore$ at $t=0^+$, $\dfrac{di(0^+)}{dt} = 100 - 10i(0^+) = 100\ A/s$

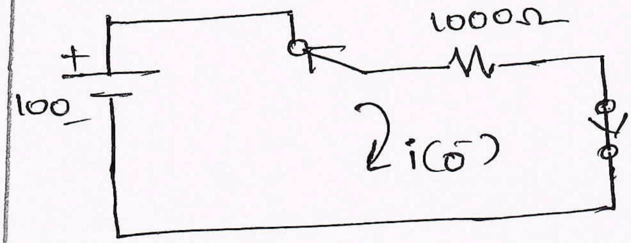
Diff eqn ②, and solving at $t=0^+$ 3M

$$10\frac{di(0^+)}{dt} + \frac{d^2i(0^+)}{dt^2} = 0, \therefore$$

$$\frac{d^2i(0^+)}{dt^2} = -10\times100 = -1000\ A/s^2$$

3M

6a) For $t=0^-$, (Sw ← a, S.S. is reached), $\therefore$

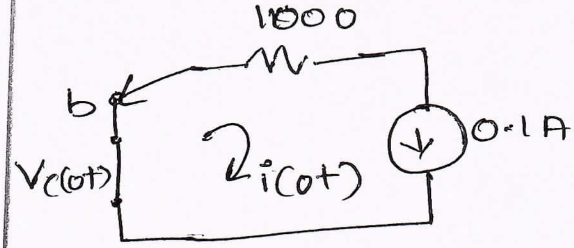


$V_c(0^-)=0$

$i(0^-) = i_L(0^-) = \dfrac{100}{1000} = 0.1A$

$\therefore i_L(0^+) = i_L(0^-) = 0.1A$

For $t=0^+$ (Sw ← b,



$i(0^+) = i_L(0^+) = 0.1A$

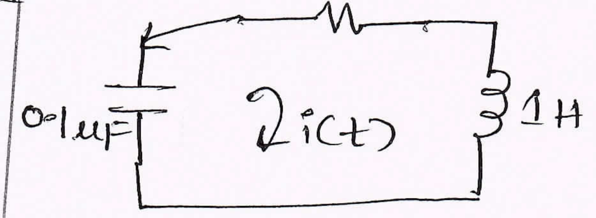
$V_c(0^+) = 0$

5M

Q.No

6a Continued

For $t>0$



By KVL

$$\frac{1}{0.1\mu F}\int_0^t i(t)\,dt + 1000\, i(t) + 1\frac{d\,i(t)}{dt} = 0 \quad (1)$$

For $t=0^+$,
First term in eqn (1) is $V_C(0^+)=0$, $\therefore i(0^+)=0.1$

$$\therefore \frac{d\,i(0^+)}{dt} = -1000\times 0.1 = -100\ A/s$$

Diff eqn (1) again,

$$\frac{1}{10^{-7}}\, i(t) + 1000\frac{d\,i(t)}{dt} + \frac{d^2 i(t)}{dt^2} = 0$$

At $t=0^+$,

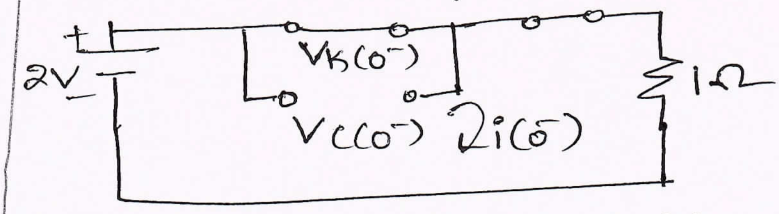
$$10^7\, i(0^+) + 1000\cdot(-100) + \frac{d^2 i(0^+)}{dt^2} = 0$$

$$\therefore \boxed{\frac{d^2 i(0^+)}{dt^2} = -9\times 10^5\ A/s^2}$$

5M

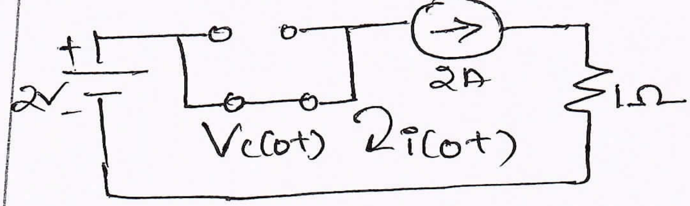
6b. At, $t=0^-$, (Sw ← ~~open~~ closed), $\therefore V_C(0^-)=V_K(0^-)=0$

$$i(0^-) = i_L(0^-) = \frac{2}{1} = 2A$$



5M

At, $t=0^+$, C ← S/C, L ← 2A, $\therefore$



$i(0^+) = i_L(0^+) = 2A$

$V_C(0^+)=0$, $V_K(0^+)=0$

here, $V_K = \frac{1}{C}\int i(t)\,dt \Rightarrow \frac{dV_K(t)}{dt} = \frac{i(t)}{C}$, For $t=0^+$

$$\frac{dV_K(0^+)}{dt} = \frac{i(0^+)}{C} = \frac{2}{0.5} = 4\ A/s$$

5M

Q.No

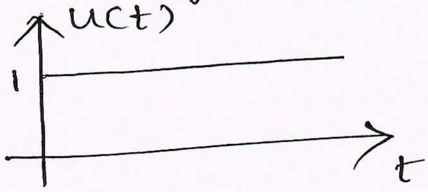
Module - 4

7a) i] Laplace Transform of a unit step function:

It is defined mathematically as

$u(t) = 1$ for $t \geq 0$

$= 0$ for $t < 0$



$$\therefore L\{u(t)\} = \int_0^\infty u(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty 1 \cdot e^{-st} \cdot dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty = \left[\frac{e^{-\infty}}{-s} - \frac{e^0}{-s}\right]$$

$$\boxed{L\{u(t)\} = \frac{1}{s}}$$

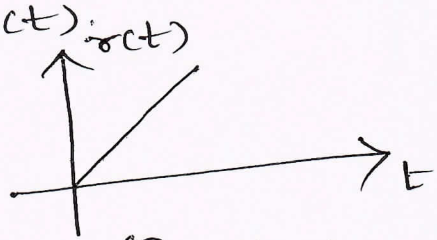
3M

ii] Ramp function $f(t) = r(t)$

It is given by

$r(t) = t$ for $t \geq 0$

$= 0$ for $t < 0$



$$L\{r(t)\} = \int_0^\infty r(t) \cdot e^{-st} dt = \int_0^\infty t \cdot e^{-st} \cdot dt$$

$$= \left[\frac{t \cdot e^{-st}}{-s}\right]_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} \cdot 1\, dt$$

$$= [0-0] + \frac{1}{s}\int_0^\infty e^{-st} \cdot dt$$

$$= \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{s}\left[0 - \frac{1}{-s}\right]$$

$$\therefore \boxed{L\{r(t)\} = \frac{1}{s^2}}$$

3M

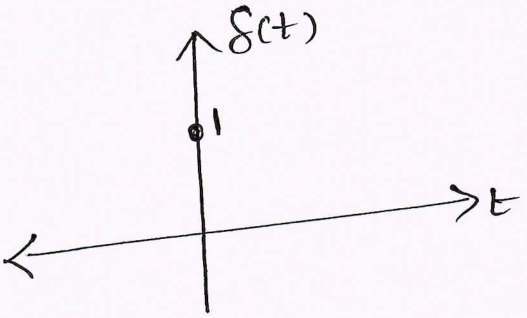
Q. No

7a) Continued

impulse function $\delta(t)$.

It is defined as $\delta(t) = 1$ for $t = 0$

$= 0$ for $t \neq 0$.



$$\delta(t) = \frac{d}{dt} u(t)$$

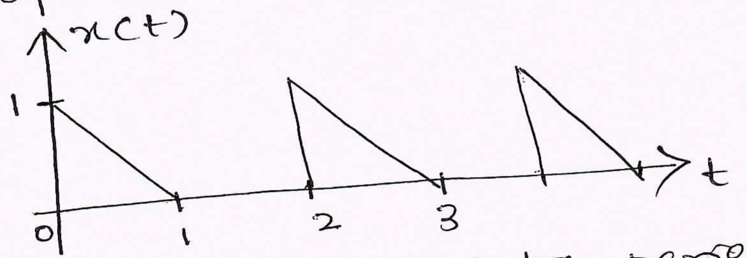
$$= s \cdot L\{u(t)\} - u(t)\Big|_{t=0^-}$$

$$= s \cdot \frac{1}{s} - 0$$

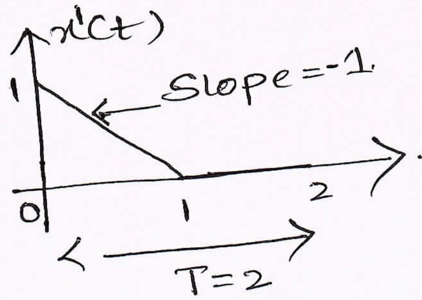
$$\boxed{L\{\delta(t)\} = 1}$$

3M

7b) Laplace transform of periodic signal $x(t)$.



Consider one complete period T



$$x'(t) = u(t) - r(t) + r(t-1)$$

$$\therefore X'(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}$$

8M

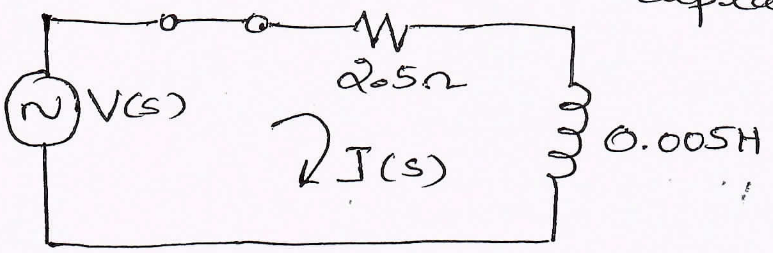
$$\therefore X(s) = \frac{X'(s)}{1 - e^{-Ts}} = \frac{\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}}{1 - e^{-2s}}$$

3M

Q. No

8a

Converting the ckt into Laplace domain we get



$$I(s) = \frac{12,500}{(2.5+0.005s)(s^2+(250)^2)}$$

$$= \frac{12500}{0.005[s+500][s^2+250^2]}$$

$$= \frac{2.5\times10^6}{(s+500)(s^2+250^2)}$$

5M

$$I(s) = \frac{A}{s+500} + \frac{Bs+C}{s^2+250^2}$$

By P.F, we get

$$= \frac{A(s^2+62500) + (s+500)(Bs+C)}{(s+500)(s^2+250^2)}$$

=

equating co-efficient on b/s of s

$A+B=0$ -(1), $500B+C=0$, $62500A+500C=2.5\times10^6$

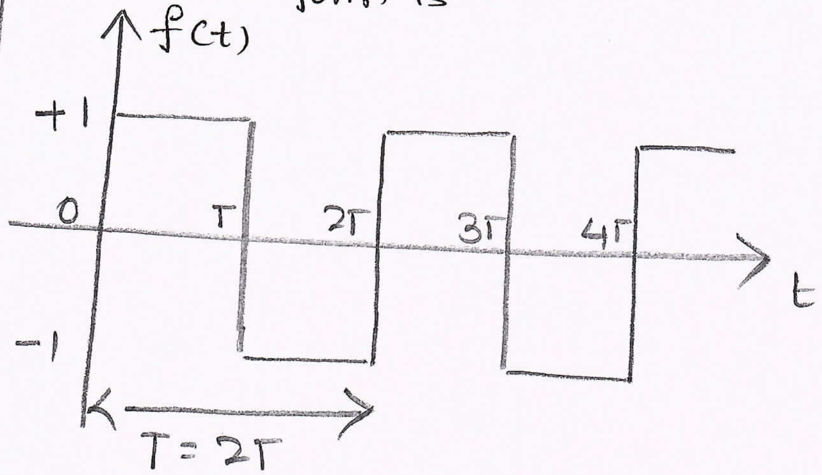
Solving $A=8$, $B=-8$, $C=4000$,

$$\therefore I(s) = \frac{8}{s+500} - \frac{8s}{s^2+250^2} + \frac{4000}{250} \times \frac{250}{s^2+250^2}$$

$$\therefore i(t) = 8e^{-500t} - 8\cos(250t) + 16\sin(250t)\ A$$

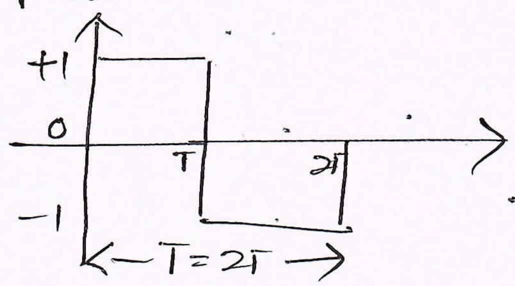
5M

Q.NO	
8b)	Given waveform is



5M

Considering one complete cycle, we get



$f'(t) = u(t) - u(t-T) - u(t-T) + u(t-2T)$

$f'(t) = u(t) - 2u(t-T) + u(t-2T)$

$\therefore F'(s) = \frac{1}{s} - \frac{2}{s} e^{-Ts} + \frac{1}{s} e^{-2Ts}$

5M

$\therefore F(s) = \frac{F'(s)}{1-e^{-Ts}} = \frac{\frac{1}{s}\left[1 - 2e^{-Ts} + e^{-2Ts}\right]}{1-e^{-2Ts}}$

Module-5

9a) Z-Parameter: The Z-parameters of a two port network is expressed by two port voltages V_1, V_2 as a function of two port current I_1, I_2. i.e

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Z-in terms of Y parameter — 2M

WKT $I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

Q. No

9a) Continue

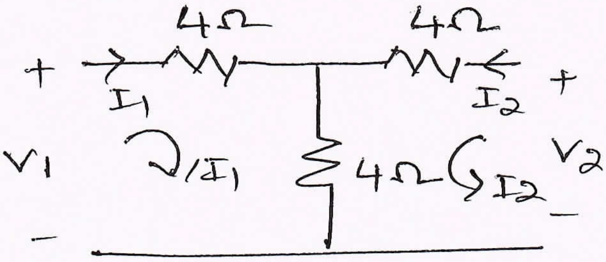
By cramers rule

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}}{\Delta y} I_1 - \frac{Y_{12}}{\Delta y} I_2$$

$$\therefore V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta y} = \frac{Y_{11}}{\Delta y} I_2 - \frac{Y_{21}}{\Delta y} I_1$$ — 2M

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta y} & -\frac{Y_{12}}{\Delta y} \\ -\frac{Y_{21}}{\Delta y} & \frac{Y_{11}}{\Delta y} \end{bmatrix}$$ — 2M

9b) h-parameters.



KVL to i/p side gives

$$-V_1 + 8I_1 + 4I_2 = 0$$

$$\boxed{V_1 = 8I_1 + 4I_2} \quad -(1)$$

KVL to o/p side

$$\boxed{V_2 = 4I_1 + 8I_2} \quad -(2)$$

4M

From (2), $\boxed{I_2 = \frac{1}{8}V_2 - \frac{1}{2}I_1}$

$$\therefore I_2 = -\frac{1}{2}I_1 + \frac{1}{8}V_2 \Rightarrow \boxed{h_{21} = -\frac{1}{2}}, \boxed{h_{22} = \frac{1}{8}}$$

$\therefore$ Using I_2 in V_1, we get

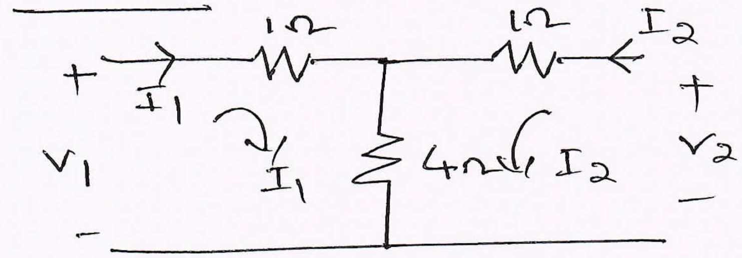
$$V_1 = 8I_1 + (-4/2)I_1 + \frac{1}{2}V_2$$

3M

$$V_1 = 6I_1 + \frac{1}{2}V_2 \Rightarrow \boxed{h_{11} = 6}, \boxed{h_{12} = \frac{1}{2}}$$

Q.No

9c) Given



KVL M1 : $V_1 = 5I_1 + 4I_2$ - (1)

1M

KVL M2 : $V_2 = 4I_1 + 5I_2$ - (2)

Solving I_1 from (2), $I_1 = \frac{1}{4}V_2 - \frac{5}{4}I_2$ (3)

$\therefore$ $\boxed{C = \frac{1}{4}}$ $\boxed{D = \frac{5}{4}}$

3M

Using (3) in (1)

$V_1 = \frac{5}{4}V_2 - \frac{25}{4}I_2 + 4I_2$; $V_1 = \frac{5}{4}V_2 - \frac{9}{4}I_2$

$\boxed{A = \frac{5}{4}}$ $\boxed{B = \frac{9}{4}}$

3M

10a) i) Quality factor: $Q = \frac{\omega_0 L}{R}$ but $\omega_0 = 2\pi f_0$.

$\therefore$ $Q = 2\pi f_0 \frac{L}{R}$;

1M

Energy stored in Inductor at resonance is $\frac{1}{2}LI_0^2$, $\times$ and $\div$ by $\frac{1}{2}I_0^2$, we get

$$Q = 2\pi f_0 \frac{L}{R} = 2\pi f_0 \left[\frac{\frac{1}{2}LI_0^2}{\frac{1}{2}RI_0^2}\right]$$

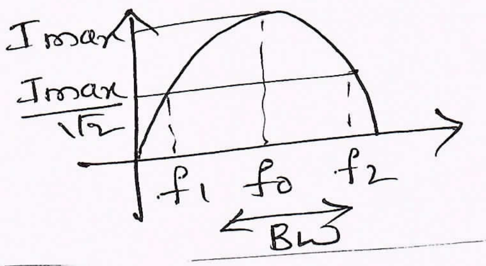
$$Q = 2\pi \left[\frac{\text{max energy Stored in L}}{\text{Total energy w.r.t time}}\right]$$

Q.NO

10a continued

ii) Selectivity: $S = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW} = \frac{f_0}{\frac{R}{2\pi L}}$ (1M)

iii) Band width: $BW = f_2 - f_1$, it gives the range of operatable frequency for getting resonance condn at f_0, by varying L and C



(1M)

10b) Given: $R = 10\Omega$, $L = 0.3H$, $C = 100\mu F$, $V_{in} = 230V$. I at f_1 and f_2 = ?. f_0 = ?, f_1 and f_2 = ?, I_0 = ?,

V_L = ?

(1M)

i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = 29.05\ Hz$

ii) $\Delta f = \frac{R}{4\pi L} = 2.65$, $\therefore f_1 = f_0 - \Delta f = 26.4\ Hz$ (2M)

$f_2 = f_0 + \Delta f = 31.7\ Hz$

(1M)

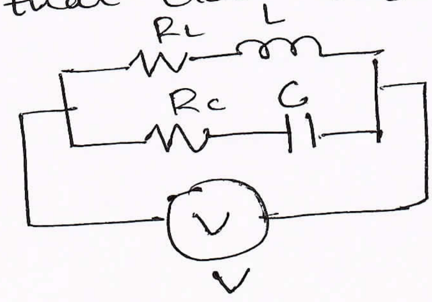
iii) $I_0 = \frac{V}{R} = \frac{230}{10} = 23A$

iv) Current at f_1 and f_2 (2M)

I at f_1 and f_2 = $0.707 \times I_0 = 16.26\ A$

v) $V_L = j Q_0 V = j\left(\frac{1}{R}\sqrt{\frac{L}{C}}\right) \cdot V = 1259\angle 90^\circ\ V.$ (1M)

10c) Show that ckt resonates at all frequency



Q.No	
10C Continued	Taking admittance of each branch

$$Y = \left(\frac{1}{R_L + jX_L}\right) + \left(\frac{1}{R_C - jX_C}\right)$$ — 2M

$$= \frac{R_C - jX_C + R_L + jX_L}{(R_L + jX_L)(R_C - jX_C)}$$

but $X_L \cdot X_C = \frac{L}{C}$

$$= \frac{(R_L + R_C) + j(X_L - X_C)}{R_L R_C + X_L X_C + j(R_C X_L - R_L X_C)}$$ — 2M

if we put $R_L = R_C = \sqrt{\frac{L}{C}}$ then

$$Y = \frac{(\sqrt{L/C} + \sqrt{\frac{L}{C}}) + j(X_L - X_C)}{\sqrt{\frac{L}{C}}\sqrt{\frac{L}{C}} + \frac{L}{C} + j(\sqrt{\frac{L}{C}})(X_L - X_C)}$$ — 2M

$$= \frac{2\sqrt{\frac{L}{C}} + j(X_L - X_C)}{2\sqrt{\frac{L}{C}} + j(\sqrt{\frac{L}{C}})(X_L - X_C)}$$ — 2M

$$= \frac{\cancel{2\sqrt{\frac{L}{C}} + j(X_L - X_C)}}{\sqrt{\frac{L}{C}}\left[\cancel{2\sqrt{\frac{L}{C}} + j(X_L - X_C)}\right]}$$

$$= \frac{1}{\sqrt{L/C}} = \frac{1}{R} = \sqrt{\frac{C}{L}}$$ — 2M