CBCS SCHEME

USN 18EC52

Fifth Semester B.E. Degree Examination, Feb./Mar.2022
Digital Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that the sampling of DTFT of a sequence x(n) result in N-point DFT with a neat diagram. (10 Marks)

b. Find the 4-point DFT of the sequence $x(n) = \{1, 0, 0, 1\}$ using matrix method and verify the answer by taking the 4-point IDFT of the result. (10 Marks)

OR

2 a. Derive the circular Time shift property. (06 Marks)

b. Compute the circular convolution of the following sequences using DFT and IDFT method $x_1(n) = \{1, 2, 3, 4\}$ and $x_2(n) = \{4, 3, 2, 1\}$. (09 Marks)

c. If $W(n) = \frac{1}{2} + \frac{1}{2}\cos\left[\frac{2\pi}{N}\left(n - \frac{N}{2}\right)\right]$, what is the DFT of the window sequence $y(n) = x(n).w(n)$? Relate the answer in terms of X(K). (05 Marks)

Module-2

3 a. Find the output y(n) of a filter whose impulse response is $h(n) = \{1, 1, 1\}$ and the input signal $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap-add method. Assume the length of each block N is 6. (10 Marks)

b. What do you mean by computational complexity? Compare the direct computation and FFT algorithms. In the direct computation of 32-point DFT of x(n), How many
   (i) Complex multiplications
   (ii) Complex additions.
   (iii) Real multiplications.
   (iv) Real additions and
   (v) Trigonometric function evaluations are required. (10 Marks)

OR

4 a. Develop 8-point DIT-FFT Radix-2 algorithm and draw the signal flow graph. (10 Marks)

b. Given $x(n) = n + 1$ for $0 \le n \le 7$. Find X(K) using DIF-FFT algorithm. (10 Marks)

Module-3

5 a. What are the different design techniques available for the FIR filters? Explain Gibbs phenomenon. Explain the four window techniques for the designing of FIR filters. (10 Marks)

b. A low pass filter is to be designed with the following desired frequency response.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , \text{for } -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0 & , \frac{3\pi}{4} \le \omega \le \pi \end{cases}$$

Determine $H(e^{j\omega})$ for M = 7 using Hamming window. (10 Marks)

Scheme & Solution Prepared by Prof. SURAJ KADLE

OR

6 a. A FIR filter is given by,

$$y(n) = x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)$$

Draw the lattice structure. (10 Marks)

b. Based on the frequency-sampling method, determine the coefficients of a linear-phase FIR filter of length M = 15 which has a symmetric unit sample response and a frequency response that satisfies the conditions.

$$H\left(\frac{2\pi}{15}K\right) = 1; \quad K = 0, 1, 2, 3$$
$$= 0.4; \quad K = 4$$
$$= 0 \quad ; \quad K = 5, 6, 7$$

(10 Marks)

Module-4

7 a. The normalized transfer function of a 2nd order Butterworth filter is given by,

$$H_2(S) = \frac{1}{S^2 + 1.414S + 1}$$

Convert the analog filter into digital filter with cut-off frequency of 0.5π rad/sec using bilinear transformation. Assume T = 1 sec. (10 Marks)

b. A filter is given by the difference equation $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$. Draw direct form – I and direct form – II realizations. Also obtain the transfer function of the filter. (10 Marks)

OR

8 a. Derive mapping function used in transforming analog filter to digital filter by bilinear transformation, preserves the frequency selectivity and stability properties of analog filter. (10 Marks)

b. Design an IIR digital Butterworth filter that when used in the analog to digital with digital to analog will satisfy the following equivalent specification.
   (i) Low pass filter with –1 dB cut off 100π rad/sec.
   (ii) Stop band attenuation of 35 dB at 1000π rad/sec.
   (iii) Monotonic in stop band and pass band.
   (iv) Sampling rate of 2000 rad/sec.
   (v) Use bilinear transformation. (10 Marks)

Module-5

9 a. With the block diagram, explain Digital Signal processors based on the Harvard architecture. (10 Marks)

b. Discuss briefly the following special digital signal processor hardware units:
   (i) Multiplier and Accumulator (MAC) unit.
   (ii) Shifters.
   (iii) Address Generators. (10 Marks)

OR

10 a. Discuss the following IEE Floating-point formats:
   (i) Single precision format.
   (ii) Double precision format. (10 Marks)

b. With the diagram, explain the basic architecture of TMS320C54X family processor. (10 Marks)

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Digital Signal Processing - 18EC52.

Feb./Mar. - 2022 Scheme & Solution.

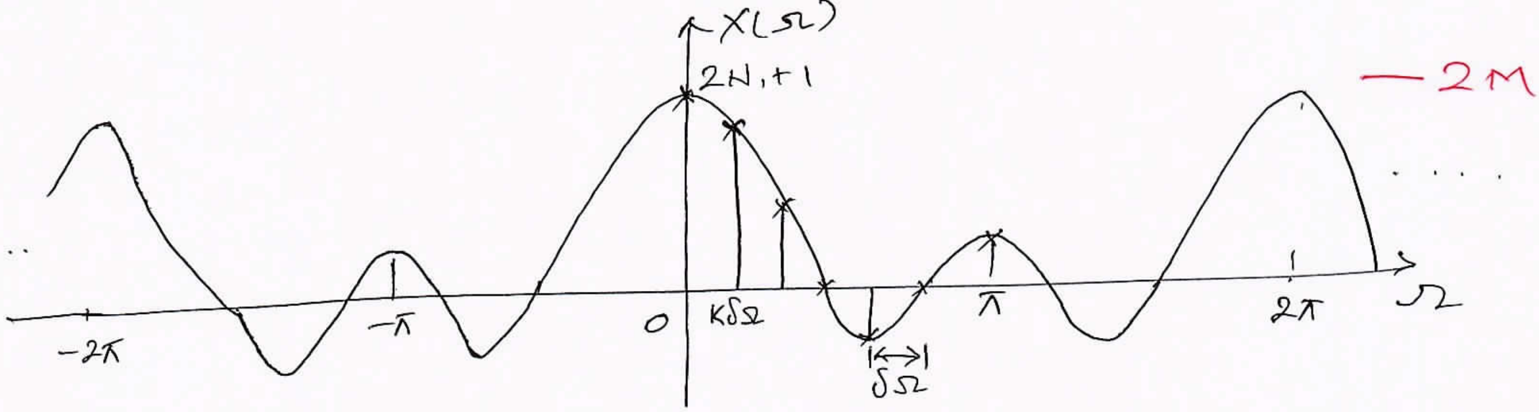
Module - 1.

1.a) Prove that the sampling of DTFT of a sequence $x[n]$ result in N-point DFT with a neat diagram. [Total - 10M]

→ Let us consider an aperiodic discrete-time signal $x[n]$, whose Discrete-Time Fourier Transform is defined as,

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \quad \text{——} \quad (1) \quad \text{— 1M}$$

The spectra of a aperiodic discrete-time signal will be a continuous quantity as shown in fig 1.a.



Suppose that we sample $X(\Omega)$ periodically in frequency at a spacing of $\delta\Omega$ radians between successive samples. Since $X(\Omega)$ is periodic with period 2π, samples only in the fundamental frequency range are necessary. For convenience, we take N-equidistant samples in the interval, $0 \le \Omega \le 2\pi$ with spacing, $\delta\Omega = \frac{2\pi}{N}$. — 1M

We consider the selection of N, the number of samples in the frequency domain as, $\Omega = \frac{2\pi}{N} K$. —— (2) — 1M

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using ② in ①

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, 2, \cdots N-1.$$

Summation can be subdivided into an infinite number of summations where each sum contains N terms. Thus,

$$X\left(\frac{2\pi}{N}k\right) = \cdots + \sum_{n=-N}^{-1} x[n]\, e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}} + \sum_{n=N}^{2N-1} x[n] e^{-j\frac{2\pi kn}{N}} + \cdots$$

which can also be written as,

$$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x[n]\, e^{-j\frac{2\pi kn}{N}} \quad \text{— 2M}$$

changing from $n \longrightarrow n+lN$ & interchanging the order.

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x[n+lN]\right] e^{-j\frac{2\pi kn}{N}} \quad \text{———— ③}$$

The signal, $x_p[n] = \sum_{l=-\infty}^{\infty} x[n+lN]$ is obtained by the — 1M
Periodic repetition of $x[n]$ every N samples, and is clearly periodic with fundamental period N.

$$\therefore\ X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$\boxed{X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}, \quad 0 \le k \le N-1} \quad \text{— 2M}$$

which is the N-point DFT of $x[n]$.

Therefore, by sampling of DTFT of a sequence $x[n]$, DFT $X(k)$ is obtained.

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07

1.b) Find the 4-point DFT of the sequence $x[n] = (1, 0, 0, 1)$ using matrix method and verify the answer by taking the 4-point IDFT of the result. [Total - 10 M]

$\rightarrow$ Given, $x[n] = [1, 0, 0, 1]$.

DFT, $X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi kn}{N}}$, $0 \le k \le N-1$ — 2M

Matrix method is given by, $X_N = [W_N]\, x_N$

for $N=4$, $X_4 = [W_4]\, x_4$ where, W_4 is twiddle factor matrix

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$ — 1M

$$\therefore \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ — 2M

$$= \begin{bmatrix} 1+0+0+1 \\ 1+0+0+j \\ 1+0+0-1 \\ 1+0+0-j \end{bmatrix} = \begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

$\therefore \boxed{X(k) = \{2, 1+j, 0, 1-j\}}$

Verification,

IDFT by matrix method is given by, $x_N = \frac{1}{N}[W_N^*]\, X_N$ — 1M

$x_4 = \frac{1}{4}[W_4^*]\, X_4$ W_4^* is conjugate of twiddle factor matrix. — 1M

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$ — 2M

$$= \frac{1}{4}\begin{bmatrix} 2+1+\cancel{j}+0+1-\cancel{j} \\ 2+\cancel{j}-1+0-\cancel{j}-1 \\ 2-1-\cancel{j}+0-1+\cancel{j} \\ 2-\cancel{j}+1+0+\cancel{j}+1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$\therefore \boxed{x[n] = [1, 0, 0, 1]}$ — 1M

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"OR"

2.a) Derive the circular time shift property. [Total - 6M]

→ Circular-Time Shift Property.

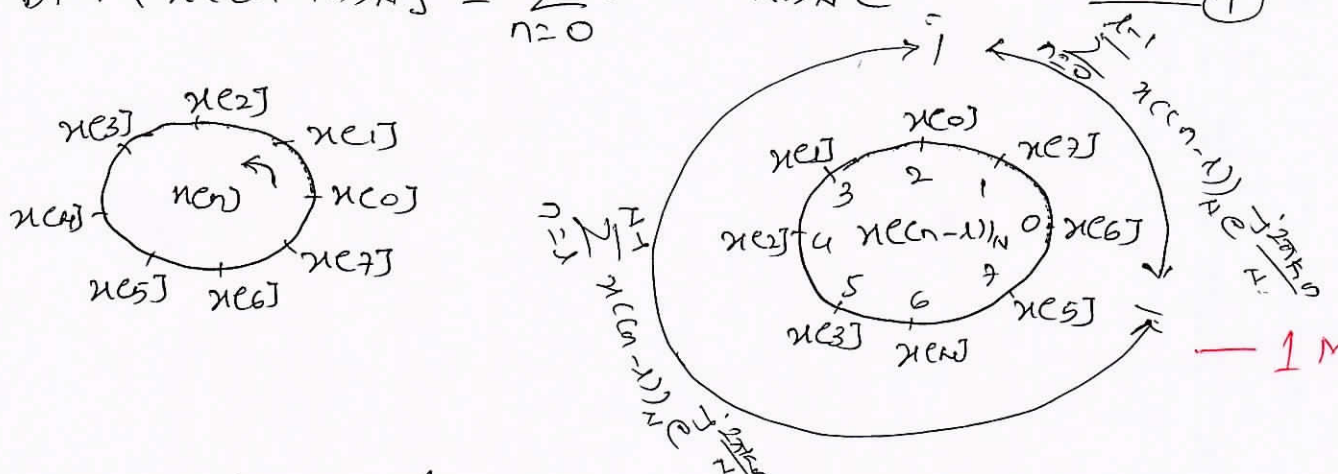
Statement:-

If, $x[n] \xleftrightarrow[N]{DFT} X(k)$

then, $x((n-l))_N \xleftrightarrow[N]{DFT} X(k)\, e^{-j\frac{2\pi k l}{N}}$ — 1M

Shifting the sequence circularly by 'l' samples is equivalent to multiplying its DFT by a complex exponential sequence $e^{-j\frac{2\pi k l}{N}}$

$$DFT\{x((n-l))_N\} = \sum_{n=0}^{N-1} x((n-l))_N\, e^{-j\frac{2\pi k n}{N}} \quad \text{— (1)}$$



— 1M

On the basis of circular shift we can split the summation in eqⁿ (1) shifted sequence for $l=2$, the DFT of circularly shifted sequence $x((n-l))_N$ can be split into two parts.

$$DFT\{x((n-l))_N\} = \sum_{n=l}^{N-1} x((n-l))_N\, e^{-j\frac{2\pi k n}{N}} + \sum_{n=0}^{l} x((n-l))_N\, e^{-j\frac{2\pi k n}{N}} \quad \text{— (2)}$$

— 1M

$x((n-l))_N = x(N-l+n)$ ∴ Second summation can be expressed as,

$$\sum_{n=0}^{l-1} x((n-l))_N\, e^{-j\frac{2\pi k n}{N}} = \sum_{n=0}^{l-1} x(N-l+n)\, e^{-j\frac{2\pi k n}{N}}, \quad \text{let } m = N-l+n.$$

then, $n=0, \quad m = N-l$ &

$n = l-1, \quad m = N-1$

$$\therefore \sum_{n=0}^{l-1} x((n-l))_N e^{-j\frac{2\pi kn}{N}} = \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l-N)}{N}}$$

$$= \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}} \quad \text{— (3)} \quad \because e^{\frac{j2\pi kN}{N}} = 1.$$

consider the first summation,

$$\sum_{n=l}^{N-1} x((n-l))_N e^{-j\frac{2\pi kn}{N}} \quad , \text{ put } n-l=m$$

when, $n=l$, $m=0$ & — 2M

$n=N-1$, $m=N-1-l$.

$$\therefore \sum_{n=l}^{N-1} x((n-l))_N e^{-j\frac{2\pi kn}{N}} = \sum_{m=0}^{N-1-l} x[m] e^{-j\frac{2\pi k(m+l)}{N}} \quad \text{— (4)}$$

$\therefore$ Using (3) & (4) in (2)

$$DFT\{x((n-l))_N\} = \sum_{m=0}^{N-1-l} x[m] e^{-j\frac{2\pi k(m+l)}{N}} + \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}}$$

$$= \sum_{m=0}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}}$$

$$DFT\{x((n-l))_N\} = \sum_{m=0}^{N-1} x[m] e^{-j\frac{2\pi km}{N}} \; e^{-j\frac{2\pi kl}{N}}$$

$$\boxed{DFT\{x((n-l))_N\} = X(k)\, e^{-j\frac{2\pi kl}{N}}} \quad \text{— 1M}$$

2.b) Compute the circular convolution of the following sequences using DFT & IDFT method. [Total - 9M]
$x_1[n] = [1, 2, 3, 4]$ & $x_2[n] = [4, 3, 2, 1]$.

$\longrightarrow$ $X_1(k) = DFT\{x_1[n]\}$ & $X_2(k) = DFT\{x_2[n]\}$

$Y(k) = X_1(k) X_2(k)$ — 1M

$y[n] = IDFT\{Y(k)\}$.

DFT, $X_{1N} = [W_N]\, x_{1N}$

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix}$$ — 2M

$\therefore X_1(k) = \{10, -2+2j, -2, -2-2j\}$.

DFT, $X_{2N} = [W_N]\, x_{2N}$

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+3+2+1 \\ 4-3j-2+j \\ 4-3+2-1 \\ 4+3j-2-j \end{bmatrix}$$ — 2M

$X_2(k) = \{10, 2-2j, 2, 2+2j\}$

$Y(k) = \{100, 8j, -4, -8j\}$ — 1M

IDFT, $y_N = \frac{1}{N}[W_N^*]\, Y_N$

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 100 \\ 8j \\ -4 \\ -8j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 100+\cancel{8j}-4-\cancel{8j} \\ 100-8+4-8 \\ 100-\cancel{8j}-4+\cancel{8j} \\ 100+8+4+8 \end{bmatrix}$$ — 2M

$$= \frac{1}{4} \begin{bmatrix} 96 \\ 88 \\ 96 \\ 120 \end{bmatrix}$$

$\boxed{\therefore y(n) = [24, 22, 24, 30].}$ — 1M

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2.c) If $w[n] = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{N}\left(n - \frac{N}{2}\right)\right)$, what is the DFT of the window sequence $y[n] = x[n].w[n]$? Relate the answer in terms of $X(k)$. [Total – 5M]

→ Given, $w[n] = \frac{1}{2} + \frac{1}{2}\cos\left[\frac{2\pi}{N}\left(n - \frac{N}{2}\right)\right]$, $0 \le n \le N-1$

$$w[n] = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}e^{j\frac{2\pi}{N}(n-\frac{N}{2})} + \frac{1}{2}e^{-j\frac{2\pi}{N}(n-\frac{N}{2})}\right] \quad \text{— 1M}$$

$$w[n] = \frac{1}{2} + \frac{1}{4}e^{j\frac{2\pi n}{N}}e^{-j\pi} + \frac{1}{4}e^{-j\frac{2\pi n}{N}}e^{j\pi}$$

$$w[n] = \frac{1}{2} + \frac{1}{4}W_N^{-n}(-1) + \frac{1}{4}W_N^{n}(-1)$$

$$\therefore w[n] = \frac{1}{2} - \frac{1}{4}W_N^{-n} - \frac{1}{4}W_N^{n} \quad \text{— 2M}$$

Now, $y[n] = x[n].w[n]$

$$y[n] = \frac{1}{2}x[n] - \frac{1}{4}x[n]W_N^{-n} - \frac{1}{4}x[n]W_N^{n}$$

we know that, $DFT\{x[n]W_N^{-ln}\} = X((k-l))_N$ — 1M

Hence, $\boxed{Y(k) = \frac{1}{2}X(k) - \frac{1}{4}X((k-1))_N - \frac{1}{4}X((k+1))_N}$ — 1M

Module - 2.

3.a) Find the output $y[n]$ of a filter whose impulse response is $h[n] = [1, 1, 1]$ and the input signal, $x[n] = [3, -1, 0, 1, 3, 2, 0, 1, 2, 1]$ using overlap-add method. Assume the length of each block N is 6. [Total – 10M]

→ $h[n] = [1, 1, 1]$

$x[n] = [3, -1, 0, 1, 3, 2, 0, 1, 2, 1]$.

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$N=6$, $M=3$,

$N = L+M-1 \quad \therefore L=4.$ — 1M

Segmented input data blocks:

$x_1[n] = [3, -1, 0, 1, 0, 0]$ $\quad h[n] = [1, 1, 1, 0, 0, 0]$

$x_2[n] = [3, 2, 0, 1, 0, 0]$ — 2M

$x_3[n] = [2, 1, 0, 0, 0, 0]$

Output data blocks using 6-point circular convolution:-

$y_1[n] = h[n] ⑥ x_1[n]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$y_1[n] = [3, 2, 2, 0, 1, 1]$

$y_2[n] = h[n] ⑥ x_2[n]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$y_2[n] = [3, 5, 5, 3, 1, 1]$

$y_3[n] = h[n] ⑥ x_3[n]$ — 4M

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y_3[n] = [2, 3, 3, 1, 0, 0]$

$\therefore y_1[n] = [3, 2, 2, 0, 1, 1]$

$\qquad y_2[n] = [3, 5, 5, 3, 1, 1]$ — 2M

$\qquad\qquad y_3[n] = [2, 3, 3, 1, 0, 0]$

$$\boxed{y[n] = [3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1]}$$ — 1M

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3.b) What do you mean by computational complexity? Compare the direct computation and FFT algorithms. In the direct computation of 32-point DFT of x[n], How many
i) Complex multiplications,
ii) Complex additions.
iii) Real Multiplications.
iv) Real additions, and
v) Trigonometric function evaluations are required. [Total - 10M]

→ In the DFT computation, computational complexity mean, the numbes of arithematic calculations involved to calculate N-point DFT. — 1M

Comparison :-

	Direct computation	FFT Algorithms
i) No. of complex multiplications for N-point DFT.	N^2	$\frac{N}{2}\log_2 N$ — 2M
ii) No. of complex additions for N-point DFT.	$N^2 - N$	$N \log_2 N$

For N = 32 using direct computation.
i) Complex multiplications — N^2 — 1024
ii) Complex additions — $N^2 - N$ — 992
iii) Real multiplications — $4N^2$ — 4096
iv) Real additions — $4N^2 - 2N$ — 4032.
v) Trigonometric function evaluations — $2N^2$ — 2048.

— 7 M

"OR"

4.a) Develop 8-point DIT-FFT Radix-2 algorithm and draw the signal flow graph. [Total-10M]

→ $X(k)$ can be obtained from $F_1(k)$ & $F_2(k)$ which are two 4-point DFT's. consider, for $N=8$.

$x[n] = [x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]]$

$f_1[n] = x[2n] = [x[0], x[2], x[4], x[6]], \quad 0 \le n \le \frac{N}{2}-1$

$f_2[n] = x[2n+1] = [x[1], x[3], x[5], x[7]], \quad 0 \le n \le \frac{N}{2}-1$

— 1M

$\therefore X(k) = F_1(k) + W_N^k F_2(k)$

$X(k+\frac{N}{2}) = F_1(k) - W_N^k F_2(k) \qquad 0 \le k \le \frac{N}{2}-1$ — 1M

For second stage of decimation,

$V_{11}[n] = f_1[2n] \qquad 0 \le n \le \frac{N}{4}-1 \qquad V_{21}[n] = f_2[2n] \qquad 0 \le n \le \frac{N}{4}-1$

$V_{12}[n] = f_1[2n+1] \qquad\qquad V_{22}[n] = f_2[2n+1]$

$\therefore$ — 1M

$F_1(k) = V_{11}(k) + W_{N/2}^k V_{12}(k)$

$F_1(k+\frac{N}{4}) = V_{11}(k) - W_{N/2}^k V_{12}(k)$

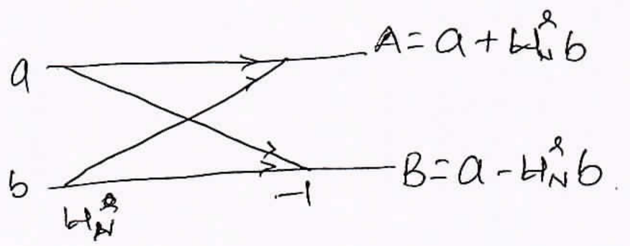
&

$F_2(k) = V_{21}(k) + W_{N/2}^k V_{22}(k)$

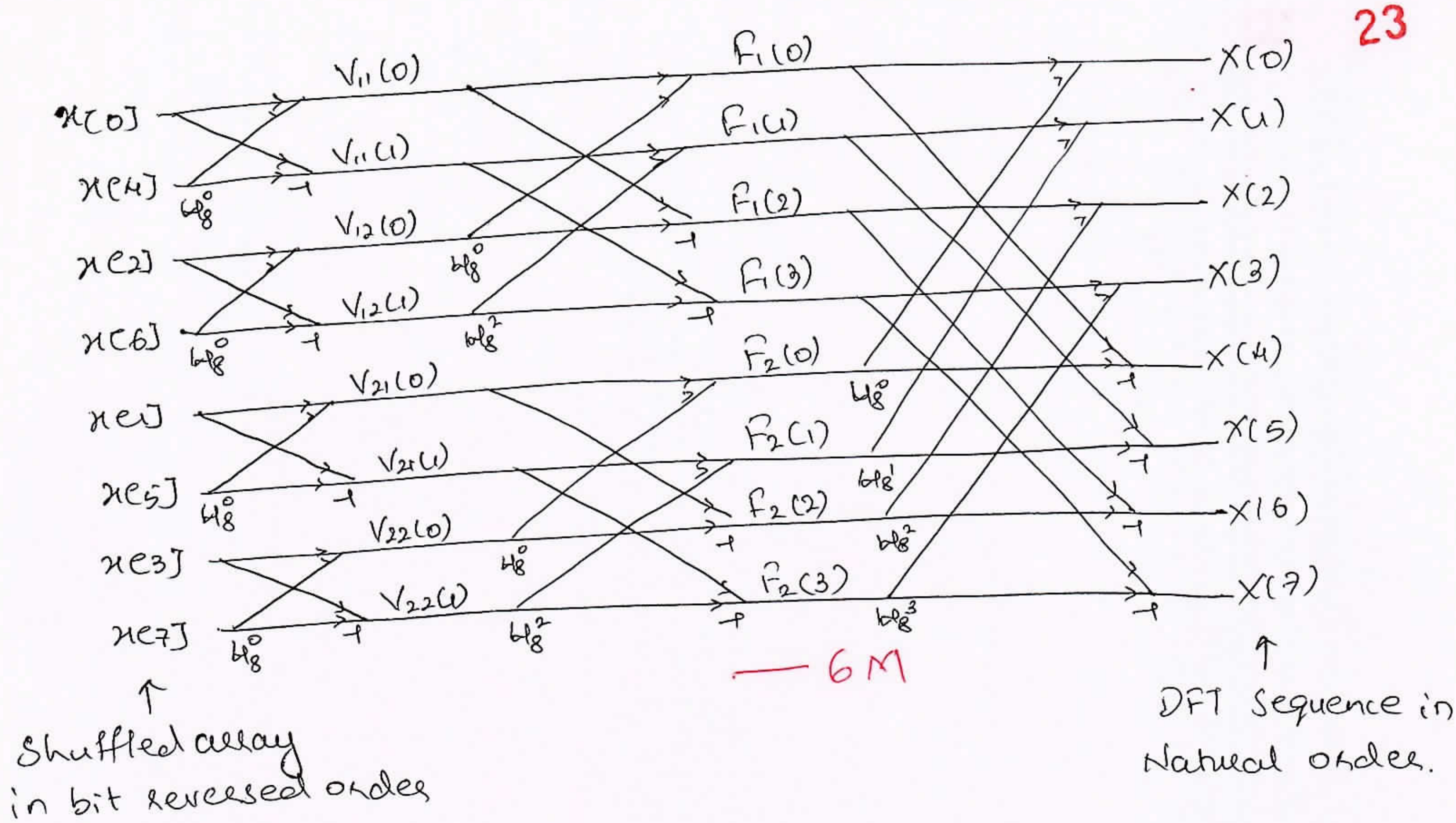
$F_2(k+\frac{N}{4}) = V_{21}(k) - W_{N/2}^k V_{22}(k)$

$0 \le k \le \frac{N}{4}-1$

For 2-point computation,



— 1M

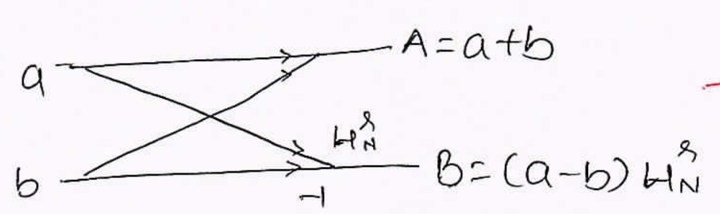


4.b) Given, $x[n] = n+1$ for $0 \le n \le 7$. Find $X(k)$ using DIF-FFT algorithm. [Total - 10M].

$\longrightarrow$ $x[n] = n+1$, $0 \le n \le 7$

$x[n] = [1, 2, 3, 4, 5, 6, 7, 8]$ — 1M

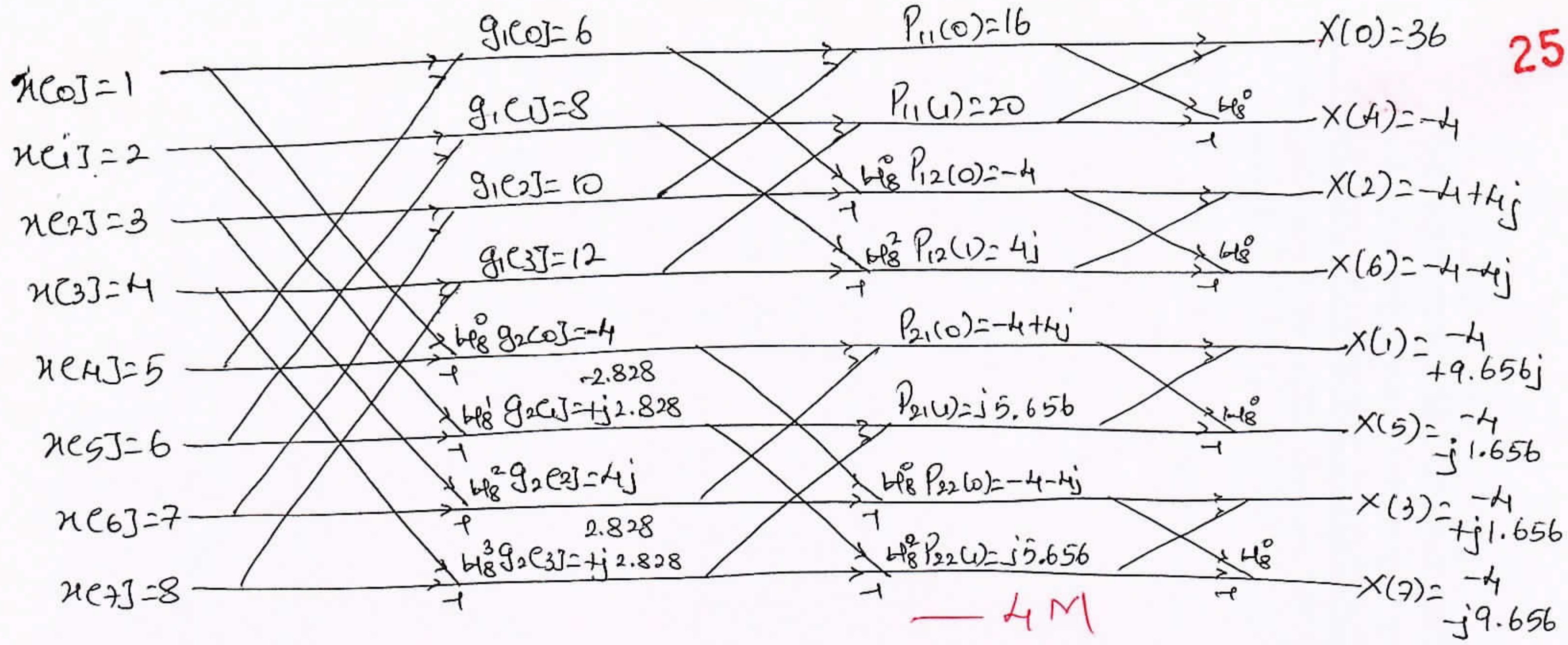
For DIF-FFT algorithm,



$W_8^0 = 1 = W_4^0 = W_2^0$ $\qquad$ $W_8^2 = -j = W_4^1$ — 1M

$W_8^1 = 0.707 - j0.707$ $\qquad$ $W_8^3 = -0.707 - j0.707$.

26/03



$g_1[0] = x[0] + x[4] = 1 + 5 = 6$, $\quad g_1[1] = x[1] + x[5] = 2 + 6 = 8$

$g_1[2] = x[2] + x[6] = 3 + 7 = 10$, $\quad g_1[3] = x[3] + x[7] = 4 + 8 = 12.$

$g_2[0] = (x[0] - x[4])W_8^0 = (1-5)\times 1 = -4$, $\quad g_2[1] = (x[1] - x[5])W_8^1 = -2.828 + j2.828.$

$g_2[2] = (x[2] - x[6])W_8^2 = 4j$, $\quad g_2[3] = (x[3] - x[7])W_8^3 = 2.828 + j2.828.$

$P_{11}(0) = g_1[0] + g_1[2] = 6 + 10 = 16$, $\quad P_{11}(1) = g_1(1) + g_1(3) = 8 + 12 = 20.$

$P_{12}(0) = [g_1(0) - g_1[2]]W_8^0 = -4$, $\quad P_{21}(1) = [g_1[1] - g_1(3)]W_8^2 = 4j$ — 2M

$P_{21}(0) = g_2[0] + g_2[2] = -4 + 4j$, $\quad P_{21}(1) = g_2(1) + g_2(3) = j5.656$

$P_{22}(0) = [g_2(0) - g_2(2)]W_8^0 = -4 - 4j.$ $\quad P_{22}(1) = [g_2(1) - g_2(3)]W_8^2 = j5.656.$

$X(0) = P_{11}(0) + P_{11}(1) = 16 + 20 = 36$, $\quad X(4) = [P_{11}(0) - P_{11}(1)]W_8^0 = -4$

$X(2) = P_{12}(0) + P_{12}(1) = -4 + 4j$, $\quad X(6) = [P_{12}(0) - P_{12}(1)]W_8^0 = -4 - 4j$

$X(1) = P_{21}(0) + P_{21}(1) = -4 + 9.656j$, $\quad X(5) = [P_{21}(0) - P_{21}(1)]W_8^0 = -4 - 1.656j$

$X(3) = P_{22}(0) + P_{22}(1) = -4 + 1.656j$, $\quad X(7) = [P_{22}(0) - P_{22}(1)]W_8^0 = -4 - 9.656j$

$\therefore$

$$\boxed{X(k) = \{36, -4+9.656j, -4+4j, -4+1.656j, -4, -4-1.656j, -4-4j, -4-9.656j\}}$$

— 1M

Module - 3

5. a) What are the different design techniques available for the FIR filters? Explain Gibbs phenomenon. Explain the four window techniques for the designing of FIR filters. [Total - 10M]

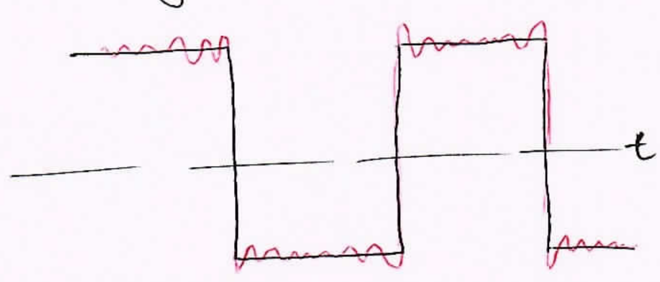
→ There are two design techniques available for the FIR filters, namely :-

i) FIR filter design using Window functions. — 1M

ii) FIR filter design using Frequency Sampling Method.

Gibbs Phenomenon:-

Gibbs' phenomenon occurs near a jump discontinuity in the signal. It says that no matter how many terms are included in Fourier series, there will always be an error in the form of an overshoot near the discontinuity. The overshoot always be about 9% of the size of the jump.



— 1M

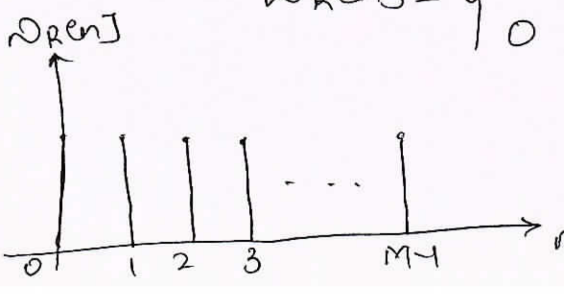
Window Functions:-

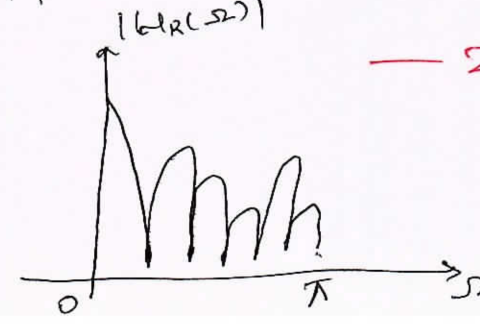
i) Rectangular window:-

The rectangular window of length 'M' is given as,

$$w_R[n] = \begin{cases} 1, & \text{for } 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

— 2M



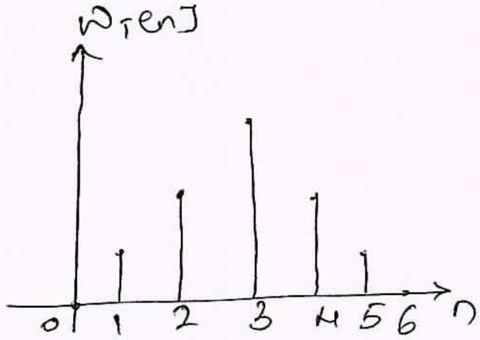


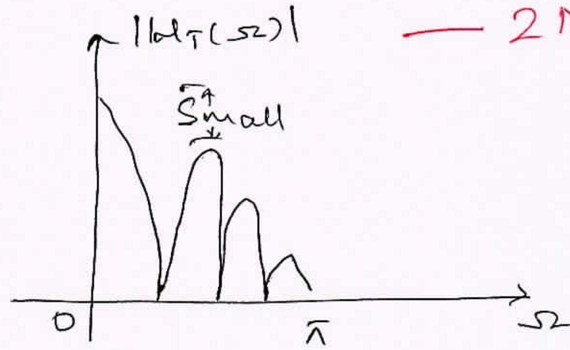
Here, main lobe width should be as small as possible and side lobe height should also be as small as possible. As the length 'M' increases the main lobe becomes narrower, but the area under the side lobe remains same.

ii) Bartlett Window (Triangular window):-

Bartlett window is defined as,

$$w_T[n] = \begin{cases} 1 - \dfrac{2\left|n - \frac{M-1}{2}\right|}{M-1}, & \text{for } 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$





— 2M

iii) Hamming Window:-

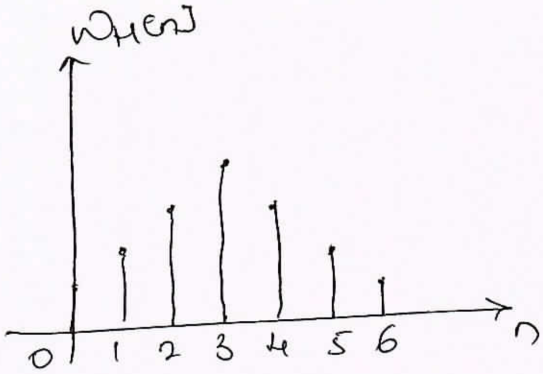
Hamming window is most commonly used window in speech processing, it is defined as,

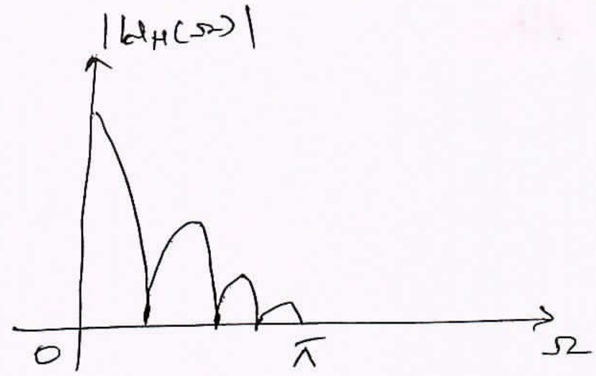
— 2M

$$w_H[n] = \begin{cases} 0.54 - 0.46 \cos \dfrac{2\pi n}{M-1}, & \text{for } 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

Hamming window has a bell like shape, its first & last samples are not zero. Advantage of using Hamming is utilizing full length of the window.

Hamming window offers reduced side lobe height but increased main lobe width.

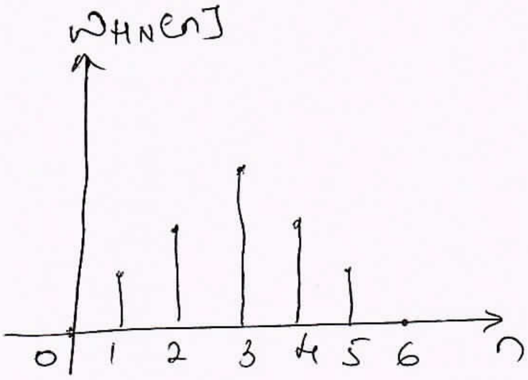


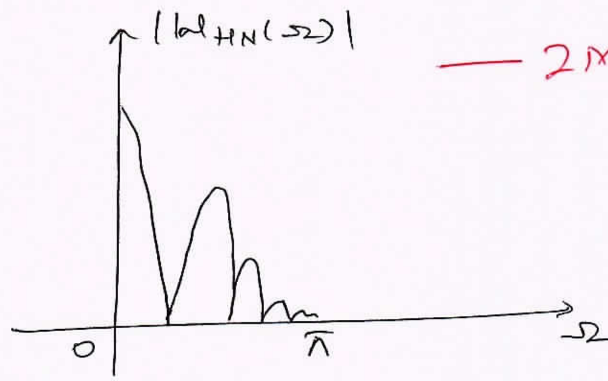


iv) <u>Hanning Window</u>:-

Hanning window has shape similar to that of Hamming. Its first and last samples are zero.

$$w_{HN}[n] = \begin{cases} \frac{1}{2}\left(1-\cos\frac{2\pi n}{M-1}\right), & \text{for } 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$





— 2M

Hanning window offers narrow main lobe, but first few sidelobes are significant, then sidelobes reduce rapidly.

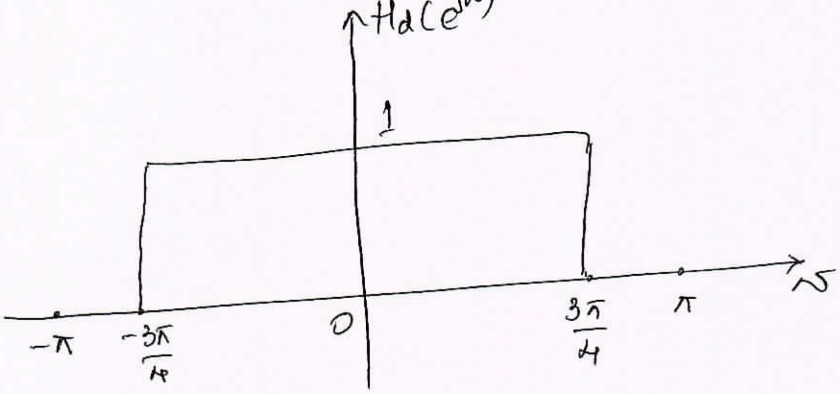
5. b) A low pass filter is to be designed with the following desired frequency response,

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & \text{for } -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le \omega \le \pi \end{cases}$$

[Total – 10M]

Determine $H(e^{j\omega})$ for $M=7$ using Hamming window.

$$\rightarrow \quad H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & \text{for } -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le \omega \le \pi \end{cases}$$



— 1M

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{3\pi/4} e^{-j3\omega}\, e^{j\omega n}\, d\omega$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{3\pi/4} e^{j\omega(n-3)}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega(n-3)}}{j(n-3)}\right]_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}}$$ — 2M

$$\therefore h_d[n] = \begin{cases} \dfrac{\sin \frac{3\pi(n-3)}{4}}{\pi(n-3)}, & \text{for } n \ne 3 \\ \dfrac{3}{4}, & \text{for } n = 3 \end{cases}$$ — 2M

and, window function for $M = 7$

$$w_H[n] = \begin{cases} 0.54 - 0.46\cos\frac{2\pi n}{M-1}, & \text{for } 0 \le n \le M-1, \quad 0 \le n \le 6 \\ 0, & \text{otherwise} \end{cases}$$ — 1M

$w_H[n] = [0.08, 0.31, 0.77, 1, 0.77, 0.31, 0.08]$.

— 2M

$h_d[n] = [0.07, -0.15, 0.22, 0.75, 0.22, -0.15, 0.07]$

$\therefore h[n] = h_d[n]\, w_H[n]$

$h[n] = [0.00056, -0.0465, 0.1694, 0.75, 0.1694, -0.0465, 0.00056]$.

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Frequency response,

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)}\left\{h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h[n]\cos\omega\left(n-\frac{M-1}{2}\right)\right\}$$

$M=7$,

$$H(e^{j\omega}) = e^{-j\omega 3}\left[h(3) + 2\sum_{n=0}^{2} h[n]\cos\omega(n-3)\right]$$

$$H(e^{j\omega}) = e^{-j3\omega}\left(h[3] + 2h[0]\cos(-3\omega) + 2h[1]\cos(-2\omega) + 2h[2]\cos(-\omega)\right]$$

$$\boxed{H(e^{j\omega}) = e^{-j3\omega}\left(0.75 + 0.000112\cos(3\omega) + (-0.093)\cos(2\omega) + 0.3388(\cos\omega)\right]}$$

— 2M

∴ Pseudomagnitude response,

$$|H(e^{j\omega})| = 0.75 + 0.000112\cos(3\omega) - 0.093\cos(2\omega) + 0.3388\cos(\omega)$$

and, phase response

$$\angle H(e^{j\omega}) = \begin{cases} -3\omega, & |H(e^{j\omega})| > 0 \\ -3\omega + \pi, & |H(e^{j\omega})| < 0. \end{cases}$$

"OR"

6.a) A FIR filter is given by,

$$y[n] = x[n] + \frac{2}{5}x[n-1] + \frac{3}{4}x[n-2] + \frac{1}{3}x[n-3]$$

Draw the lattice structure. [Total - 10M]

→ $y[n] = x[n] + \frac{2}{5}x[n-1] + \frac{3}{4}x[n-2] + \frac{1}{3}x[n-3]$.

Applying Z-transform.

$$Y(z) = X(z) + 0.4z^{-1}X(z) + 0.75z^{-2}X(z) + 0.33z^{-3}X(z)$$

— 1M

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$$H(z) = \frac{Y(z)}{X(z)} = 1 + 0.4 z^{-1} + 0.75 z^{-2} + 0.33 z^{-3}$$ — 1 M

filter is of order, $m=3$

$\therefore a_3(1) = 0.4$, $a_3(2) = 0.75$, $a_3(3) = 0.33$ — 1M

i> for, $m=3$, $K_3 = a_3(3) = 0.33$

$$a_2(i) = \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2}, \quad i=1,2$$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2} = \frac{0.4 - (0.33)(0.75)}{1-(0.33)^2} = 0.169$$ — 2 M

$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2} = \frac{0.75 - (0.33)(0.4)}{1-(0.33)^2} = 0.693$$

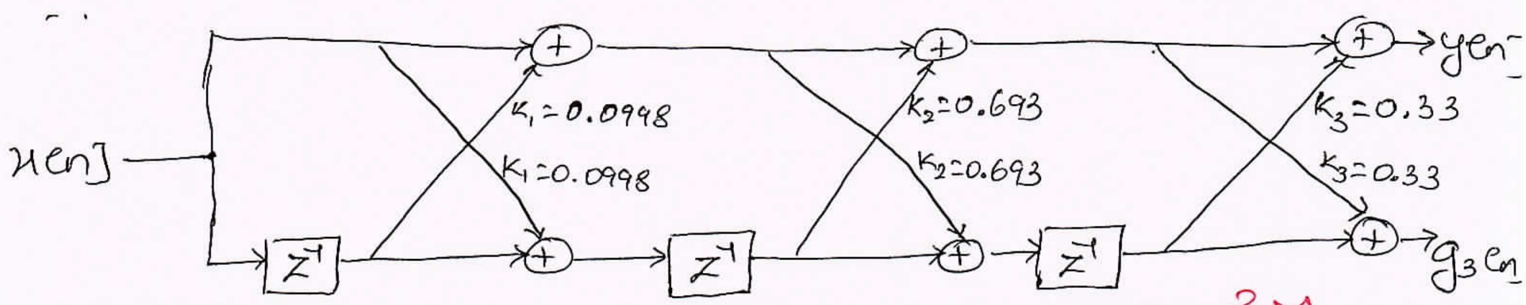
ii> for, $m=2$, $K_2 = a_2(2) = 0.693$

$$a_1(i) = \frac{a_2(i) - a_2(2)a_2(2-i)}{1-K_2^2}, \quad i=1$$ — 1 M

$$a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} = \frac{0.169 - (0.693)(0.169)}{1-(0.693)^2} = 0.0998$$

iii> for, $m=1$, $K_1 = a_1(1) = 0.0998$ — 1 M

$\therefore$



— 3M

6.b) Based on the frequency-sampling method, determine the coefficients of a linear-phase FIR filter of length $M=15$ which has a symmetric unit sample response and a frequency response that satisfies the conditions.

$$H\left(\frac{2\pi}{15}k\right) = 1 \; ; \quad k = 0,1,2,3$$
$$= 0.4 \; ; \quad k = 4$$
$$= 0 \; ; \quad k = 5,6,7.$$

[Total - 10M]

$\longrightarrow$

We know that,

$$h[n] = \frac{1}{M}\left\{H(0) + 2\sum_{k=1}^{\frac{M-1}{2}} Re\left[H\left(\frac{2\pi}{15}k\right) e^{j\frac{2\pi kn}{M}}\right]\right\} \quad \text{— 1M}$$

for, $M=15$

$$h[n] = \frac{1}{15}\left\{1 + 2\sum_{k=1}^{7} Re\left[H\left(\frac{2\pi}{15}k\right) e^{j\frac{2\pi kn}{15}}\right]\right\} \quad \text{— 1M}$$

for, $n=0$, $h[0] = \frac{1}{15}\left\{1 + 2\sum_{k=1}^{7} H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{2\pi kn}{15}\right)\right\}$

$h[0] = \frac{1}{15}\left\{1 + 2[1\times 1 + 1\times 1 + 1\times 1 + 0.4\times 1]\right\} = \underline{0.52}.$

for, $n=1$, $h[1] = \frac{1}{15}\left\{1 + 2\sum_{k=1}^{7} H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{2\pi k}{15}\right)\right\}.$

— 3M

$h[1] = \frac{1}{15}\left\{1 + 2[1\times 0.91 + 1\times 0.66 + 1\times 0.30 + 0.4\times(-0.10)]\right\} = \underline{0.31}$

for, $n=2$, $h[2] = \frac{1}{15}\left\{1 + 2\sum_{k=1}^{7} H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{4\pi k}{15}\right)\right\}$

$h[2] = \frac{1}{15}\left\{1 + 2[1\times 0.66 + 1\times -0.10 + 1\times -0.80 + 0.4\times -0.97]\right\} = \underline{-0.017}$

for, $n=3$, $h[3] = \frac{1}{15}\left\{1 + 2\sum_{k=1}^{7} H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{6\pi k}{15}\right)\right\}.$

$h[3] = \frac{1}{15}\left\{1 + 2[1\times 0.3 + 1\times -0.8 + 1\times -0.8 + 0.4\times 0.3]\right\} = \underline{-0.090}$

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05

for, $n=4$, $h(4)=\frac{1}{15}\left\{1+2\sum_{k=1}^{7}H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{8\pi k}{15}\right)\right\}$

$h(4)=\frac{1}{15}\left\{1+2[1\times-0.1+1\times-0.9+1\times0.3+0.4\times0.91]\right\}=\underline{0.021}$

for, $n=5$, $h(5)=\frac{1}{15}\left\{1+2\sum_{k=1}^{7}H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{10\pi k}{15}\right)\right\}$

$h(5)=\frac{1}{15}\left\{1+2[1\times-0.5+1\times-0.5+1\times1+0.4\times-0.5]\right\}=\underline{0.04}$

for, $n=6$, $h(6)=\frac{1}{15}\left\{1+2\sum_{k=1}^{7}H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{12\pi k}{15}\right)\right\}$ — 3 M

$h(6)=\frac{1}{15}\left\{1+2[1\times-0.8+1\times0.3+1\times0.3+0.4\times-0.8]\right\}=\underline{-0.00026}$

for, $n=7$, $h(7)=\frac{1}{15}\left\{1+2\sum_{k=1}^{7}H\left(\frac{2\pi}{15}k\right)\cos\left(\frac{14\pi k}{15}\right)\right\}$

$h(7)=\frac{1}{15}\left\{1+2[1\times-0.97+1\times0.91+1\times-0.8+0.4\times0.66]\right\}=\underline{-0.0128}$

Therefore, because of symmetric unit sample response

$h[n]=h[M-1-n]$, $M=15$

$h[0]=0.52=h[14]$

$h[1]=0.31=h[13]$

$h[2]=-0.017=h[12]$

$h[3]=-0.090=h[11]$ — 2M

$h[4]=0.021=h[10]$

$h[5]=0.04=h[9]$

$h[6]=-0.00026=h[8]$

$h[7]=-0.0128=h[7]$.

26/03

Module-4

07

7\. a) The normalized transfer function of a 2^{nd} order Butterworth filter is given by, $H_2(S) = \frac{1}{S^2+1.414S+1}$

Convert the analog filter into digital filter with cut-off frequency of 0.5π rad/sec using bilinear transformation. Assume T = 1 sec.

[Total - 10M]

→ $\Omega_d = 0.5\pi$ rad/sec & T = 1 sec.

i) Calculate the prewarped analog frequency as,

$$\Omega_a = \frac{2}{T}\tan\left(\frac{\Omega_d T}{2}\right) = \frac{2}{1}\tan\left(\frac{0.5\pi \times 1}{2}\right)$$ — 2M

$\therefore \Omega_a = 2$ rad/sec

ii) Performing the prototype transformation as,

$$H(s) = H_2(s)\Big|_{s=\frac{s}{\Omega_a}}$$

$$H(s) = \frac{1}{s^2+1.414s+1}\Big|_{s=\frac{s}{2}}$$

— 2M

$$H(s) = \frac{1}{(0.5s)^2+1.414\times 0.5s+1} = \frac{1}{0.25s^2+0.707s+1}$$

iii) Apply the bilinear transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$ — 1M

$$H(z) = \frac{1}{0.25s^2+0.707s+1}\Big|_{s=\frac{2}{1}\frac{z-1}{z+1}}$$

$$H(z) = \frac{1}{0.25\left(2\frac{z-1}{z+1}\right)^2+0.707\left(2\frac{z-1}{z+1}\right)+1}$$ — 1M

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09

$$H(z) = \frac{1}{\left(\frac{z-1}{z+1}\right)^2 + 1.414\left(\frac{z-1}{z+1}\right) + 1}$$

$$H(z) = \frac{(z+1)^2 \cancel{(z+1)}}{(z-1)^2 \cancel{(z+1)} + 1.414(z-1)(z+1)^{\cancel{2}} + (z+1)^2 \cancel{(z+1)}}$$

$$\therefore \quad H(z) = \frac{(z+1)^2}{(z-1)^2 + 1.414(z-1)(z+1) + (z+1)^2} \quad \text{— 2M}$$

$$H(z) = \frac{z^2 + 2z + 1}{3.414 z^2 + 0.707.}$$

$$\boxed{H(z) = \frac{1 + 2z^{-1} + z^{-2}}{3.414 + 0.707 z^{-2}}} \quad \text{— 2M}$$

7.b) A filter is given by the difference equation, $y[n] - \frac{1}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n] + \frac{1}{2} x[n-2]$. Draw direct form - I and direct form - II realizations. Also obtain the transfer function of the filter. [Total - 10M]

$\longrightarrow$ $y[n] - \frac{1}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n] + \frac{1}{2} x[n-2]$.

Applying z-transform,

$$Y(z) - 0.25 z^{-1} Y(z) + 0.125 z^{-2} Y(z) = X(z) + 0.5 z^{-2} X(z) \quad \text{— 1M}$$

$$Y(z)\left[1 - 0.25 z^{-1} + 0.125 z^{-2}\right] = X(z)\left[1 + 0.5 z^{-2}\right]$$

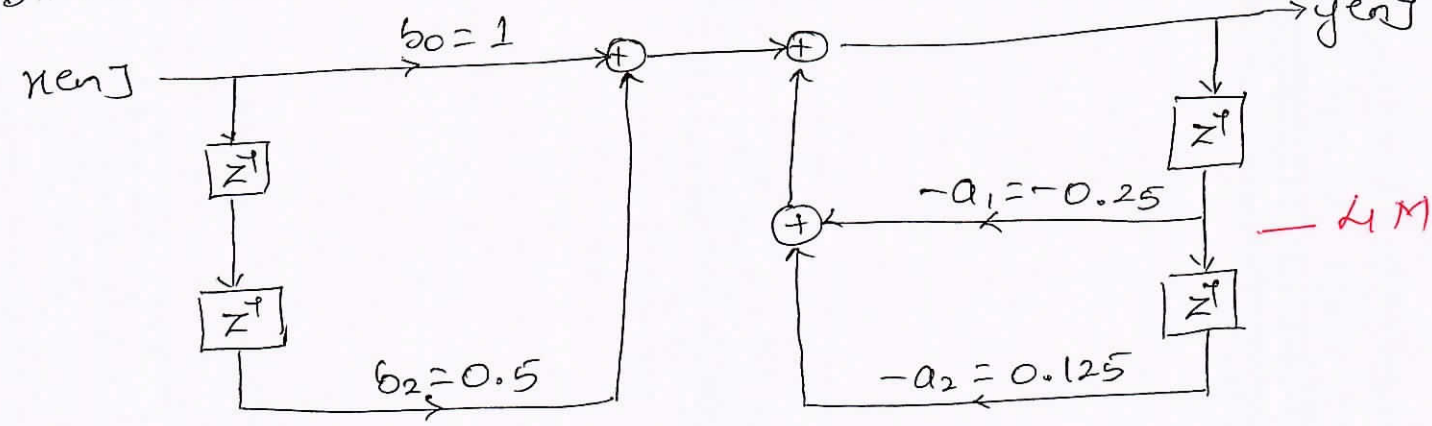
$$\therefore \quad \boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 0.5 z^{-2}}{1 - 0.25 z^{-1} + 0.125 z^{-2}}} \quad \text{— 1M}$$

26/03

$b_0 = 1, \quad b_2 = 0.5$ — 1M

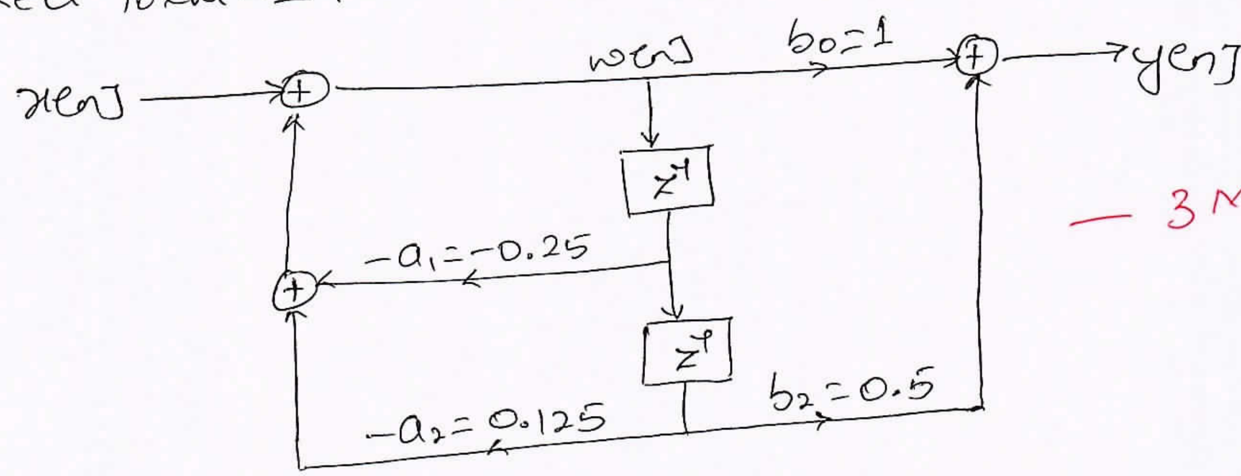
$a_1 = 0.25, \quad a_2 = -0.125$

Direct form - I.



— 4M

Direct form - II.



— 3M

"OR"

8.a) Derive mapping function used in transforming analog filter to digital filter by bilinear transformation, preserves the frequency selectivity and stability properties of analog filter. [Total - 10M]

→ Bilinear Transformation, converts an analog filter into a digital filter. Consider the curve shown in figure 8. the area under the curve can be determined using the following integration: — 1M

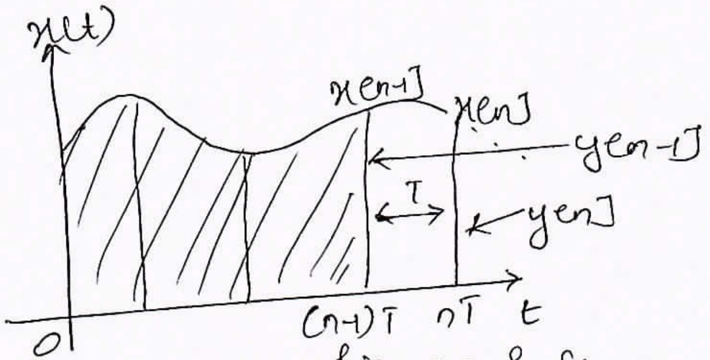


figure 8.a

$$y(t) = \int_0^t x(t)\, dt \quad \text{—} \; ①$$

— 1M

Applying Laplace transform on eqⁿ ① we have,

$$Y(s) = \frac{X(s)}{s} \quad \text{—} \; ②$$

∴ Laplace transfer function can be written as,

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s} \quad \text{—} \; ③$$

— 2M

Now, we examine the numerical integration method shown in figure 8.a, to approximate the integration of eqⁿ ① using the following difference equation.

$$y[n] = y[n-1] + \frac{x[n] + x[n-1]}{2} T \quad \text{—} \; ④$$

— 1M

T → Sampling period.

$y[n] = y[nT]$ — output sample that is the whole area under the curve.

$y[n-1] = y[nT-T]$ — previous output sample.

Applying Z-transform on both sides of eqⁿ ④

$$Y(z) = z^{-1} Y(z) + \frac{T}{2} \left(X(z) + z^{-1} X(z) \right).$$

Solving for the ratio $\frac{Y(z)}{X(z)}$,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{T}{2} \frac{1 + z^{-1}}{1 - z^{-1}} \quad \text{—} \; ⑤$$

— 1M

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comparing eq^n ③ & ⑤

$$\frac{1}{S} = \frac{T}{2}\,\frac{1+z^{-1}}{1-z^{-1}} = \frac{T}{2}\,\frac{z+1}{z-1} \quad — ⑥$$

Solving for S, gives the bilinear transformation.

$$\boxed{S = \frac{2}{T}\,\frac{z-1}{z+1}} \quad —— ⑦ \qquad — 1M$$

Bilinear transformation method is a mapping or trans--formation of points from the S-plane to z-plane. eq^n ⑦ can be alternatively written as,

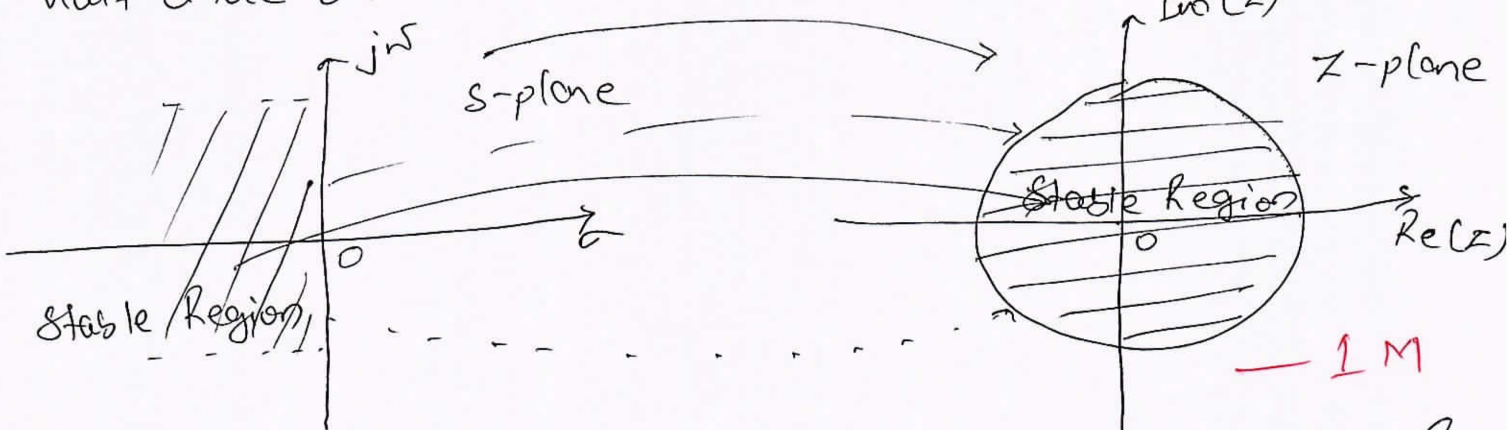
$$z = \frac{1 + \frac{ST}{2}}{1 - \frac{ST}{2}} \quad — ⑧$$

The general mapping properties are summarized as,

① The left half of S-plane is mapped onto the inside of the unit circle of the z-plane.

② The right half of S-plane is mapped onto the outside of the unit circle of the z-plane. — 2M

③ The positive jω axis in the splane is mapped onto the positive half circle, while negative jω axis is mapped onto the negative half circle on the unit circle.



— 1M

Q.b) Design an IIR digital Butterworth filter that when used in the analog to digital with digital to analog will satisfy the following equivalent specification.

i) Low pass filter with -1 dB cut off 100π rad/sec.

ii) Stop band attenuation of 35 dB at 1000π rad/sec.

iii) Monotonic in stop band and pass band.

iv) Sampling rate of 2000 rad/sec.

v) Use bilinear transformation.

[Total - 10M]

→ Given

Passband attenuation, $A_p = 1$ dB for $\Omega_p = 100\pi$ rad/sec
or $F_p = 50$ Hz.

Stopband attenuation, $A_s = 35$ dB for $\Omega_s = 1000\pi$ rad/sec
or $F_s = 500$ Hz.

i) Let us convert the analog frequencies to their discrete time values

$$\therefore f_p = \frac{F_p}{F_s} = \frac{50}{2000} = 0.025, \text{ Hence } \omega_p = 2\pi f_p = 2\pi \times 0.025$$

$$\therefore \omega_p = 0.157.$$ — 2M

$$\therefore f_s = \frac{F_s}{F_s} = \frac{500}{2000} = 0.25, \text{ Hence } \omega_s = 2\pi f_s = 2\pi \times 0.25$$

$$\omega_s = 1.57.$$

ii) Using frequency prewarping.

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

$\frac{2}{T}$ cancels out during calculations

$$\Omega = \tan \frac{\omega}{2}$$ — 2M

$$\Omega_p = \tan \frac{\omega_p}{2} = \tan \frac{0.157}{2} = 0.078 \text{ rad/sec}$$

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$\Omega_s = \tan\frac{\omega_s}{2} = \tan\frac{1.57}{2} = 0.99$ rad/sec.

equivalent analog filter according to bilinear transformat are,

$A_p = 1$ dB , $\Omega_p = 0.078$ rad/sec

$A_s = 35$ dB , $\Omega_s = 0.99$ rad/sec.

iii) Order of Butterworth filter,

$$N = \frac{1}{2}\frac{\log\left[\frac{10^{0.1A_s dB}-1}{10^{0.1A_p dB}-1}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{1}{2}\frac{\log\left[\frac{10^{0.1\times35}-1}{10^{0.1\times1}-1}\right]}{\log\left(\frac{0.99}{0.078}\right)}$$

$N = 1.84 \cong 2.$ — 1M

iv) to determine cut-off frequency.

$$\Omega_c = \frac{1}{2}\quad \frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{1/2N}} + \frac{\Omega_s}{\left(\frac{1}{A_s^2}-1\right)^{1/2N}}$$

$$\Omega_c = \frac{1}{2}\quad \frac{\Omega_p}{(10^{0.1A_p dB}-1)^{1/2N}} + \frac{\Omega_s}{(10^{0.1A_s dB}-1)^{1/2N}}$$

$$\Omega_c = \frac{1}{2}\quad \frac{0.078}{(10^{0.1\times1}-1)^{1/2\times2}} + \frac{0.99}{(10^{0.1\times35}-1)^{1/2\times2}}$$ — 2M

$\Omega_c = 0.121$ rad/sec.

v) System function of normalized lowpass butterworth filter for, $N=2$

$$H_{an}(s) = \frac{1}{s^2+\sqrt{2}\,s+1}.$$

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vi) System function of required analog filter.

$$S \longrightarrow \frac{\Omega p}{\Omega_{cp}} S$$

$$S \longrightarrow \frac{1}{0.121} S$$

$$H_a(s) = H_{an}(s)\Big|_{s \to \frac{s}{0.121}}$$

$$= \frac{1}{\left(\frac{s}{0.121}\right)^2 + \sqrt{2}\left(\frac{s}{0.121}\right) + 1}$$ — 1M

$$H_a(s) = \frac{0.0146}{s^2 + 0.171s + 0.0146}.$$

This is the system function of equivalent analog filter.

vii) H(z) using bilinear transformation.

$$S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$ since, $\frac{2}{T} = 1$.

$$S = \frac{1-z^{-1}}{1+z^{-1}}$$

$$H(z) = H_{an}(s)\Big|_{s = \frac{1-z^{-1}}{1+z^{-1}}}$$

$$= \frac{0.0146}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.171\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.0146}$$

$$H(z) = \frac{0.0123(1+z^{-1})^2}{1 - 1.66z^{-1} + 0.711z^{-2}}$$ — 2M

This is the system function of the required butterworth digital filter.

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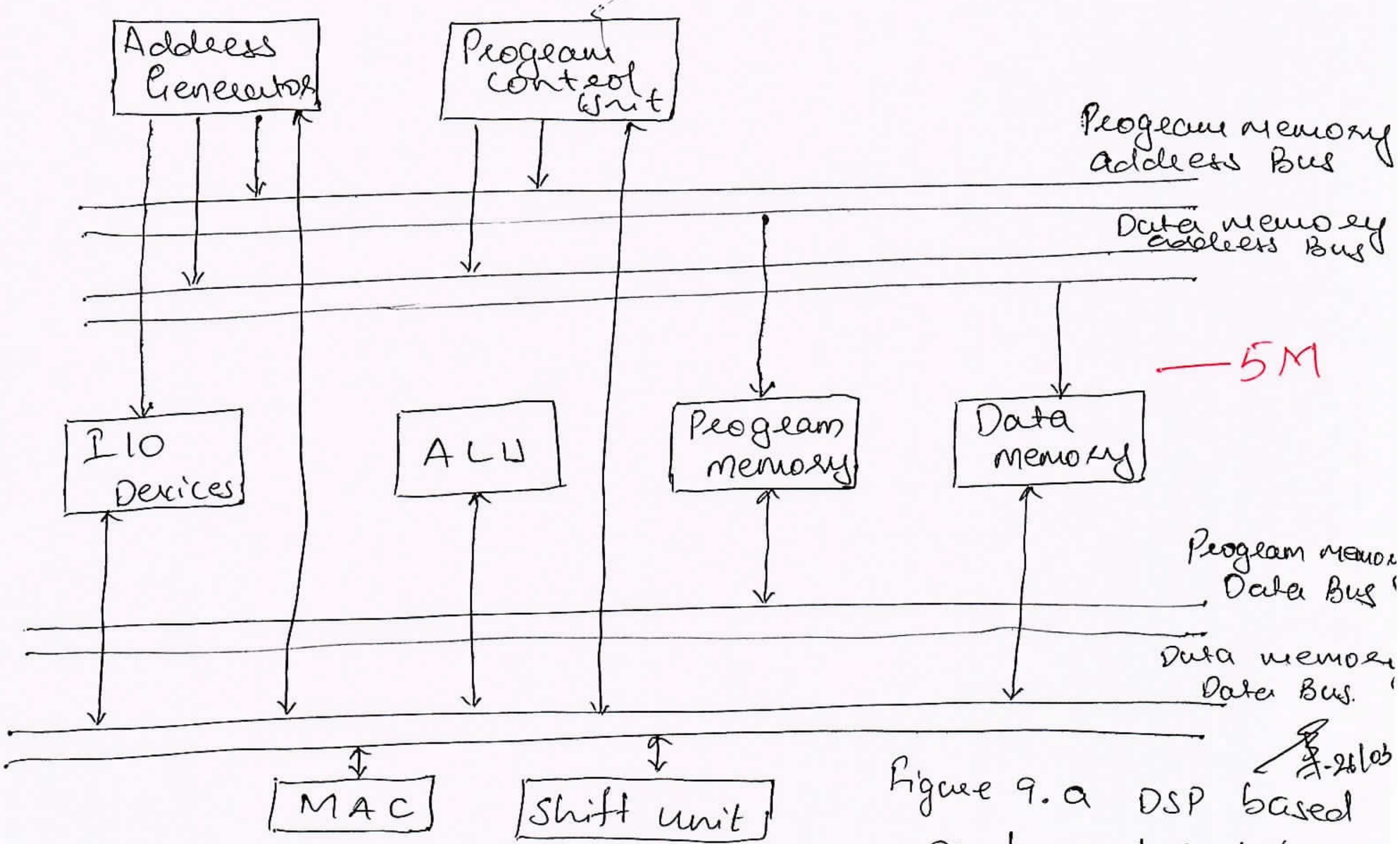
Module - 5

9. a) With the block diagram, explain Digital Signal Processors based on the Harvard Architecture.

→ Digital Signal Processor Architecture :- [Total - 10M]

Unlike microprocessors and microcontrollers DSP have special features that require operations such as FFT, filtering, convolution & correlation. Therefore, digital Signal Processors use a different dedicated hardware architecture.

To accelerate the execution speed of DSP, digital signal processors are designed based on Harvard Architecture. Figure 9.a shows DSP based on Harvard architecture it has two seperate memory spaces. One is dedicated to the program code, while the other is employed for data. — 2M



— 5M

Figure 9.a DSP based on harvard architecture.

To accommodate two memory spaces, two corres-ponding address buses and two data buses are used. The program memory and data memory have their own connections to the program memory bus and data memory bus respectively. This means that the Harvard architecture processor can fetch the program instruction and data in parallel at the same time. There is an additional unit called a multiplier and accumulator (MAC), which is the dedicated hardware used for the digital filtering operation. Shift unit, is used for the scaling operation for fixed-point implementation when the processor performs digital filtering. — 3M

9.6) Discuss briefly the following special digital signal processor hardware units:
i) Multiplier and Accumulator (MAC) unit.
ii) Shifters.
iii) Address Generators.

[Total – 10M]

→ i) Multiplier and Accumulator (MAC) unit:-

Multiplier and accumulator is a special hard-ware unit for enhancing the speed of digital filtering. This is dedicated hardware, and the corresponding instruction is generally referred to as MAC operation.

Figure 9.61 shows a typical hardware MAC, the multiplier has a pair of input registers, each holding the 16-bit input to the multiplier. — 2M

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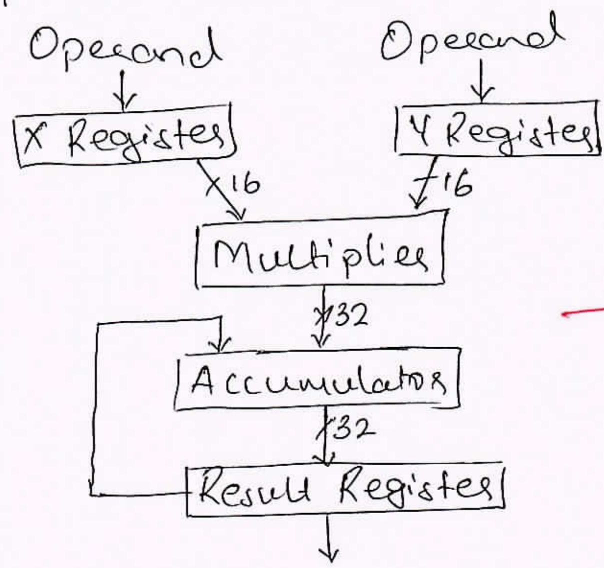
The result of multiplication is accumulated in a 32-bit accumulator unit. The result register holds the double precision data from the accumulator.



— 2M

Figure 9.61 MAC dedicated to DSP

ii) Shifters:-

In digital filtering, to prevent overflow, a scaling operation is required. A simple scaling-down operation shifts data to the right, while scaling-up operation shifts data to the left. Shifting data to the right is the same as dividing the data by 2. eg:- $011_2 = 3_{10}$ shifting right by 1 bit $001_2 = 1$. ($3/2 = 1.5$ truncating 1.5 gives 1). — 3M

Shifting data to the left is equivalent to multiplying the data by 2. Eg:- $110_2 = 6_{10}$ that is, $3 \times 2 = 6$.

iii) Address Generators:-

The DSP generates the addresses for each datum on the data buffer to be processed. A special hardware unit for circular buffering is used. Figure 9.52 shows the circular buffer for 8 data samples. — 1M

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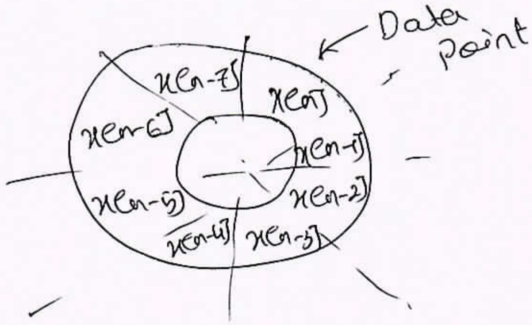
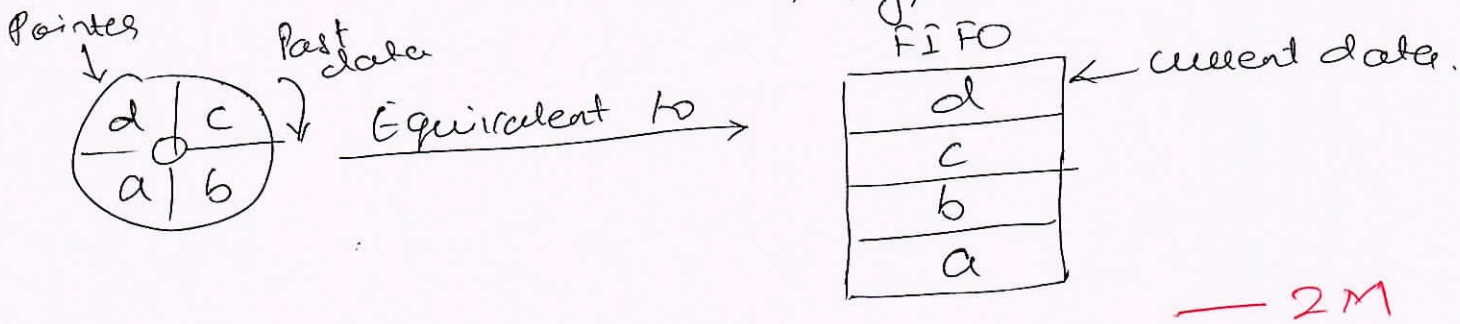


Figure 9.62 Data Buffer.

In circular buffer, a pointer is used to point always to the newest data sample. The circular buffer acts like a first-in/first-out (FIFO) buffer, but each datum on the buffer does not have to be moved.

Data flow: a, b, c, d, e, f, g, ...



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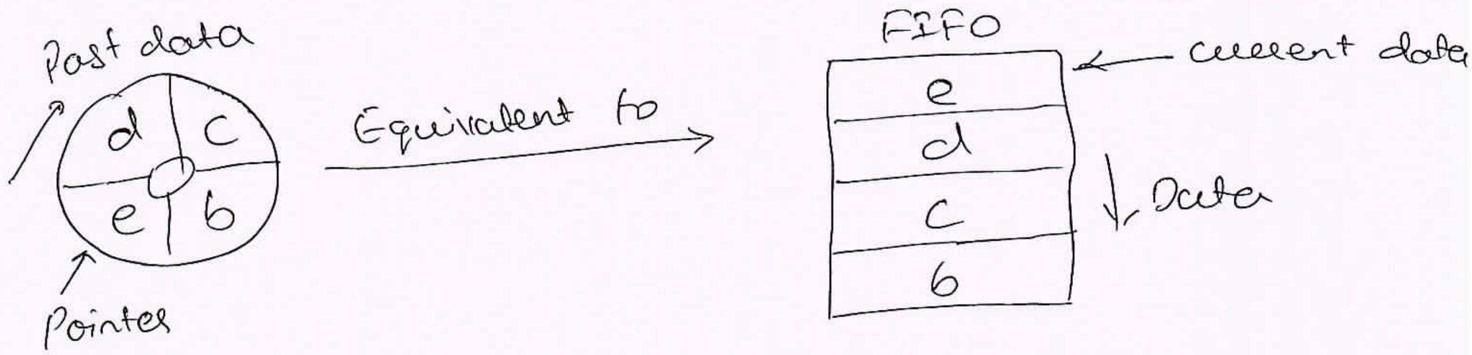

The circular buffer will change the pointer to the next position on arrival of new datum e and update old 'a' it costs pointer only one movement. However, FIFO has to move each of the other data by one position to accomodate one new data, for FIFO it will take 4 data movements.

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"OR"

10. a) Discuss the following IEEE floating-points formats:-

i) Single Precision format.

ii) Double Precision format. [Total – 10M]

→ i) Single Precision Format:-

IEEE floating-point formats are widely used in many modern digital signal processors. Figure 10.a1 shows IEEE single precision format for floating point numbers.

31	30 ... 23	22 ... 0
S	Exponent	Fraction

$$x = (-1)^S \times (1.F) \times 2^{E-127}$$ — 2M

Figure. 10.a1 IEEE single precision floating-point format.

Standard representation of IEEE single precision floating point format uses 23 fraction bits F, 8 exponent bits E, and 1 sign bit S, with a total of 32 bits each word. F is mantissa in 2's complement positive binary fraction represented from bit 0 to bit 22. — 2M Mantissa is in normalized range between +1 & −1. When S=1 number is negative & when S=0 the number is positive. Exponent E is in excess 127 form. The value of 127 is the offset from 8-bit exponent range from 0 to 255, so that E−127 will have range from −127 to 128.

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Example:-

0 10000000 00000000000000000000000 — 1M

$$x = (-1)^S \times (1.F) \times 2^{E-127} = (-1)^0 \times (1.0_2) \times 2^{128-127} = 2.0$$

ii) Double Precision Format:-

Figure 10.a2 shows IEEE double precision format.

31	30 … 20	19 … 0	31 … 0
S	Exponent	Fraction	Fraction
← Odd Register		→	← Even Register →

$$X = (-1)^S \times (1.F) \times 2^{E-1023}$$ — 2M

Figure 10.a2 IEEE double precision floating-point format.

IEEE double precision format requires 64-bit word. First bit is the sign bit S, the next eleven bits are the exponent bits E, and the final 52 bits are the fraction bits F. — 2M It increases the dynamic range of number representation, since there are eleven exponent bits; the double precision format also reduces the interval size in the mantissa normalized range of +1 to +2.

Example: 001000 ...... 00.110 0000

$S = 0$, $E = 2^9 = 512$ &

$1.F = 1.11_2 = 1.75$ — 1M

$\therefore X = (-1)^0 (1.75) \times 2^{512-1023} = 2.6104 \times 10^{-154}$.

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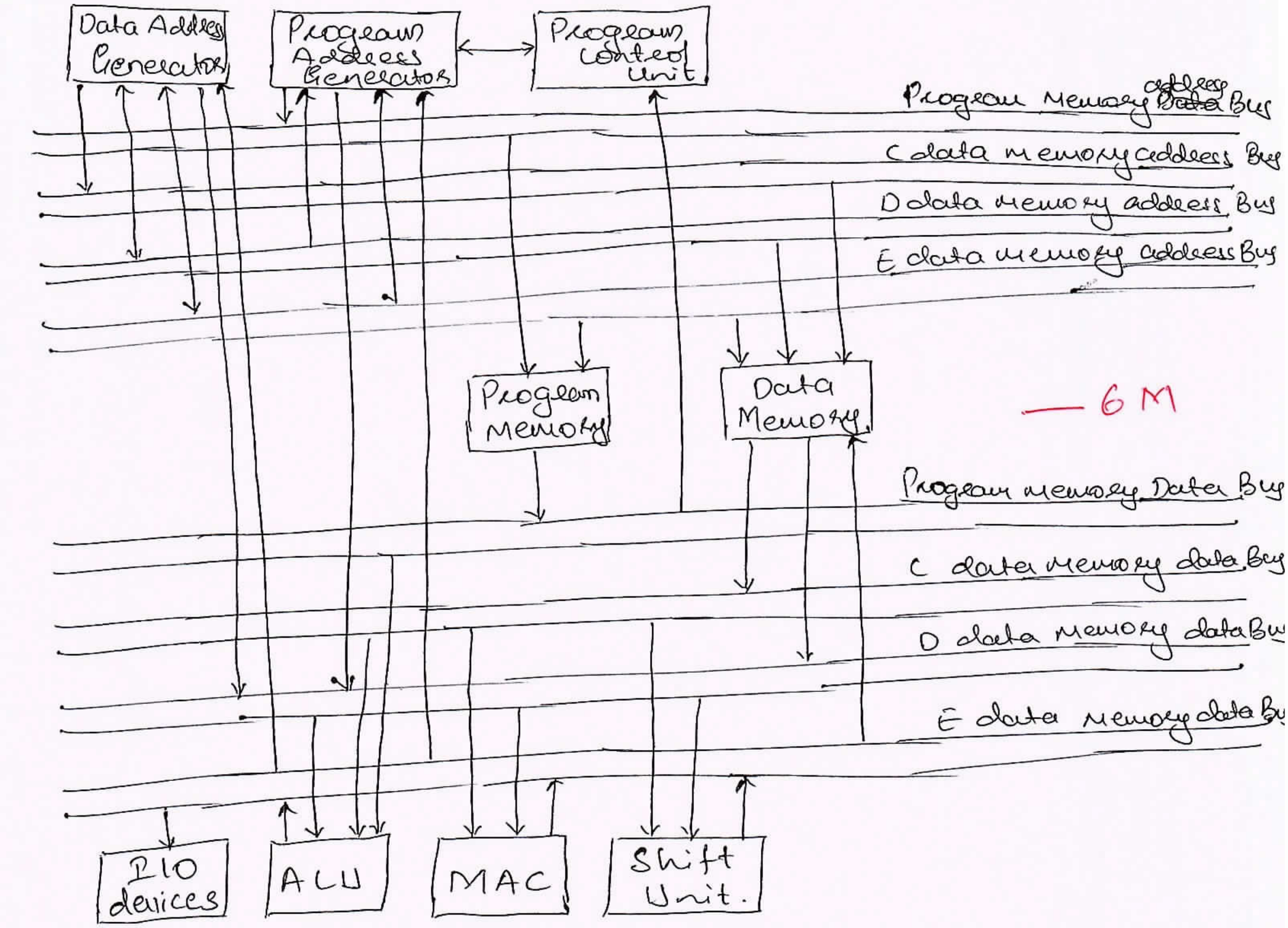
10.b) With the diagram, explain the basic architecture of TMS320C54X family processors. [Total - 10M]

→



— 6 M

Figure 10.6 Basic architecture of TMS320C54X family.

The fixed-point TMS320C54X families supporting 16-bit data have on-chip program & data memory which includes data RAM & program ROM. Four data & address buses are used. The program memory address & data bus are responsible for fetching the program instruction. C & D data memory address & data buses deal with fetching data from the data memory, while E data memory address & data bus are dedicated to moving data into data memory. — 2 M

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In addition, E memory data bus can access the I/O devices.

ALU can fetch data from the C, D, and program memory data buses and access the E memory data bus. It has two independent 40-bit accumulators, which are able to operate 40-bit addition. The multiplier can fetch data from C & D memory data buses and write data via the E memory data bus and is capable of operating 17-bit x 17-bit multiplications.

— 2M.

Program control unit fetches instructions via the program memory data bus. In order to speed up memory access, there are two address generators available. Advanced Harvard architecture is employed, where several instruc-tions operate at the same time for a given single instruction cycle. Processing performance offers 40 MIPS.

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