CBCS SCHEME

USN [redacted] **18ME53**

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022

Dynamics of Machines

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define static equilibrium. State two conditions for equilibrium **(04 Marks)**

b. In a slider crank mechanism, the force applied to the piston is 1 kN, when the crank is at 60° from IDC. The length of the crank is 100mm and connecting rod is 300 mm. Calculate the driving torque T_2 on the crank to attain equilibrium. **(16 Marks)**

OR

2 The dimensions of a four-link mechanism are AB = 500mm, BC = 660mm, CD = 560mm and AD = 1000mm. The link AB has an angular velocity of 10.5 rad/sec counterclockwise and an angular retardation of 26 rad/sec^2 at the instant when it makes an angle of 60° with AD, the fixed link. The mass of the links BC and CD is 4.2 kg/m length. The link AB has a mass of 3.54 kg, the center of which lies at 200mm from A and a moment of inertia of 88,500 kg-mm^2. Neglecting gravity and friction effects, determine the instantaneous value of the drive torque required to be applied on AB to overcome the inertia forces. **(20 Marks)**

Module-2

3 a. Justify the need of balancing of rotating parts for high speed engines. What is the difference between static and dynamic balancing? **(04 Marks)**

b. A shaft carries four masses A, B, C and D placed in parallel planes perpendicular to the shaft axis and in this order along the shaft. The masses B and C are 40 kg and 28 kg and both are at 160mm radius. While the masses in planes A and D are at 200 mm radius. Angle between B and C is 100°, B and A is 190°, both angles being measured in the same sense. Planes A and B are 250mm apart, B and C are 500mm apart. If the shaft is to be in complete balance, determine (i) Masses in planes A and D (ii) Distance between planes C and D (iii) Angular position of mass D. **(16 Marks)**

OR

4 The pistons of a 4 cylinder vertical inline engine reach their uppermost position at 90° interval in order of their axial position. Pitch of cylinder is 0.35m, crank radius is 0.12m, length of connecting rod is 0.42m. The engine runs at 600rpm. If the reciprocating parts of each engine has a mass of 2.5kg, find the unbalanced primary and secondary forces and couples. Take central plane of engine as reference plane. **(20 Marks)**

Module-3

5 a. Define the following terms with reference to governors:
(i) Sensitiveness (ii) Hunting (iii) Isochronism (iv) Governor power **(08 Marks)**

b. Each arm of a porter governor is 300mm long and is pivoted on the axis of the governor. Each ball has a mass of 6 kg and the mass of sleeve is 18kg, the radius of rotation of ball is 200mm when the governor begins to lift and 250mm when the speed is maximum. Determine the maximum and minimum speed and range of speed of the governor. **(12 Marks)**

OR

6 a. Define gyroscopic effect. With usual notations and diagram, derive an expression for the gyroscopic couple produced by a rotating disc. **(08 Marks)**

b. An aeroplane has engine speed 2000rpm clockwise when viewed from rear. It is flying at 240 kmph speed and turns towards lift and completes a quarter circle of 60m radius. The mass of the rotor engine and the propeller of the plane is 450kg with a radius of gyration of 320 mm. Determine the gyration couple on the aircraft and its effect. In what way the effect changes when the (i) Aeroplane turns towards right (ii) Engine rotates clockwise when viewed from the front (nose end) and the aeroplane turns right. **(12 Marks)**

Module-4

7 a. Define the following terms:
(i) Simple harmonic motion (ii) Natural frequency (iii) Resonance
(iv) Forces vibration (v) Phase difference **(10 Marks)**

b. Find the natural frequency of the following system shown in Fig.Q7(b). **(10 Marks)**

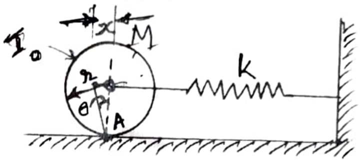


Fig.Q7(b)

OR

8 a. Set up the differential equation for a spring mass damper system and obtain complete solution for the over-damped system. **(10 Marks)**

b. A vibrating system consists of mass 25kg, a spring of stiffness 15 kN/m and a Damper. The damping provided is only 15% of critical value. Determine (i) Critical damping coefficient (ii) Damping factor (iii) Natural frequency (iv) Logarithmic decrement (v) Ratio of two consecutive amplitudes of vibration. **(10 Marks)**

Module-5

9 a. Define transmissibility and derive an expression for the transmissibility ratio and the phase angle for the transmitted force. **(10 Marks)**

b. A mass of 100 kg has been mounted on a spring-dash pot system having spring stiffness of 19600 N/m and damping coefficient 100 N-sec/mt. The mass acted upon by a harmonic force of 39N at the undamped natural frequency of the system; find
(i) Amplitude of vibration of the mass
(ii) Phase difference between the force and displacement
(iii) Forces transmissibility ratio. **(10 Marks)**

OR

10 a. Derive an expression for magnification factor or amplitude ratio for spring mass system with viscous damping subjected to harmonic force. **(10 Marks)**

b. A 54 N weight is suspended by a spring with a stiffness of 1100 N/m. It is forced to vibrate by a harmonic force of 5 N. Take viscous damping of 77 N-s/m and find,
(i) Resonant frequency (ii) Amplitude at resonance (iii) Phase angle at resonance.
(iv) Damped natural frequency (v) Frequency at which maximum amplitude of vibration occurs (vi) Maximum or Peak amplitude (vii) Phase angle corresponding to peak amplitude (viii) Speed at which maximum amplitude of vibration would occur. **(10 Marks)**

* * * * *

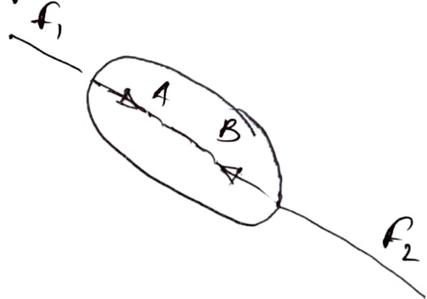
18ME53

Solution - Question Paper 2021-22

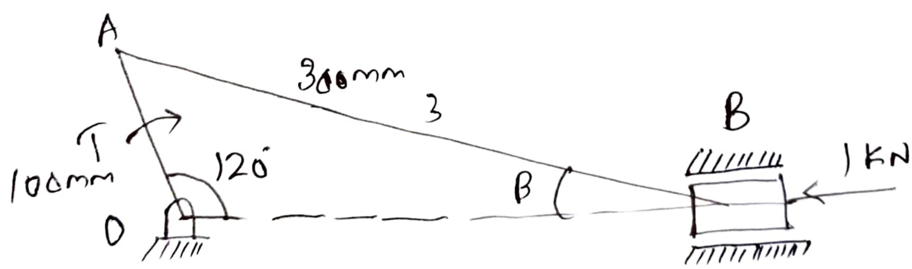
1 a) Static Equilibrium: A Body is in static equilibrium if it remains in its state of rest or motion. - 1

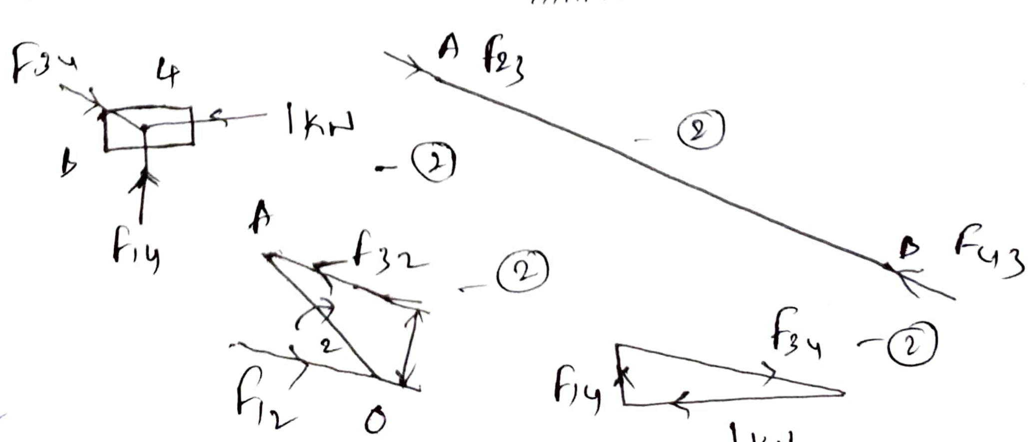
Equilibrium of two Members.

A member under the action of two force will be in equilibrium if, 1) the forces are of same magnitude
2) the forces act along the same line. - 3 - 4
3) the forces are in opposite direction.



1 b)





- 2 - 2 - 2 - 2 8

MSt

From force triangle $F_{34} = 1\,KN$

Also $F_{34} = F_{43} = F_{23} = F_{32}$ — ③

Member 2 will be in equilibrium if

F_{12} is equal parallel and opposite to F_{32} and — ③ — (16)

$T = F_{32} \cdot h = 1 \times 75 = 75$ KNmm (CCW)

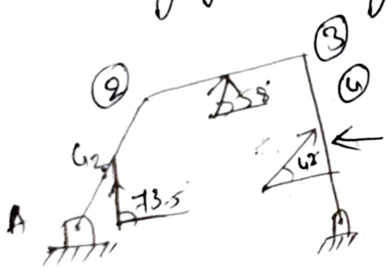
$h = 75$ on measurement — ②

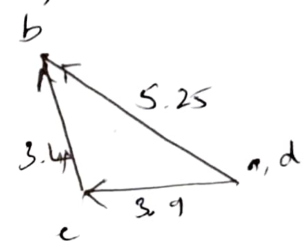
The input torque has to be equal and opposite to this couple. — ②

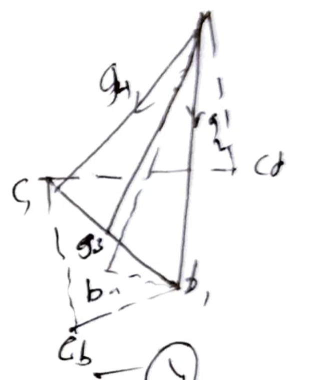
$T = 75$ KNmm $= 75$ Nm (CW)

2)

Draw the configuration of ABCD,







— ④

V_b or $ab = \omega_{ba} \times AB = 10.5 \times 0.5 = 5.25\ m/s$

V_c or $bc = 3.4\ m/s$ and V_c or $dc = 3.9\ m/s$

From acceleration diagram,

$$f^c_{ba} = \frac{(ab)^2}{AB} = \frac{(5.25)^2}{0.5} = 55.1\ m/s^2$$

$$f^t_{ba} = \alpha \times AB = 26 \times 0.5 = 13\ m/s^2 \quad ④$$

$$f^c_{cb} = \frac{(bc)^2}{BC} = \frac{(3.4)^2}{0.66} = 17.5\ m/s^2$$

$$f^c_{cd} = \frac{(dc)^2}{DC} = \frac{(3.9)^2}{0.56} = 27.2\ m/s^2$$

MSB

$m_2 = 3.54\,kg$, $m_3 = 0.66 \times 4.2 = 2.77\,kg$,

$m_4 = 0.56 \times 4.2 = 2.35\,kg$.

$f_{g_2} = 22.6\,m/s^2 \angle 253.5^\circ$ $f_{g_3} = 52.0\,m/s^2$ $f_{g_4} = 25.7\,m/s^2 \angle 222^\circ$

$F_2 = m_2 f_{g_2} = 80N \angle 73.5$

$F_3 = m_3 f_{g_3} = 144N \angle 58^\circ$

$F_4 = m_4 f_{g_4} = 60N \angle 42^\circ$

$\alpha_2 = 26\,rad/s^2$ — (4)

$\alpha_3 = \dfrac{f^t_{cb}}{CB} = \dfrac{225}{0.66} = 34.1\,rad/s^2$ CCW

$\alpha_4 = \dfrac{f^t_{cd}}{CD} = \dfrac{44.3}{0.56} = 79.1\,rad/s^2$ CCW

$k_2^2 = \dfrac{I_g}{m_2} = \dfrac{885000}{354} = 25000\,mm^2$

$k_3^2 = \dfrac{l^2}{12} = \dfrac{(660)^2}{12} = 36300\,mm^2$ — (4)

$k_4^2 = \dfrac{l^2}{12} = \dfrac{(560)^2}{12} = 26133\,mm^2$ — (20)

$h_2 = \dfrac{k^2 \alpha}{f_{g_2}} = \dfrac{25000 \times 26}{22600} = 28.8\,mm$ — (8)

$h_3 = \dfrac{36300 \times 34.1}{52000} = 23.8\,mm$

$h_4 = \dfrac{26133 \times 79.1}{25700} = 80.4\,mm$

MSA

— (4)

$r_2 = 325\,mm$, $r_3 = 297\,mm$ $r_4 = 373\,mm$.

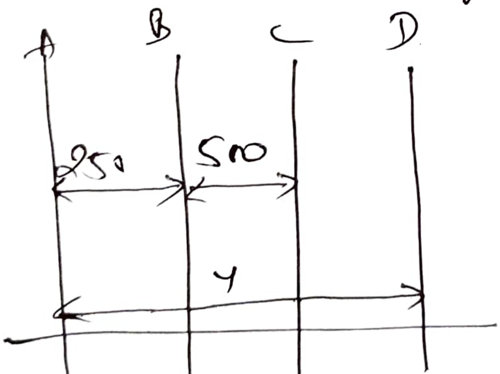
The need of balancing of rotating parts for high speed engines.

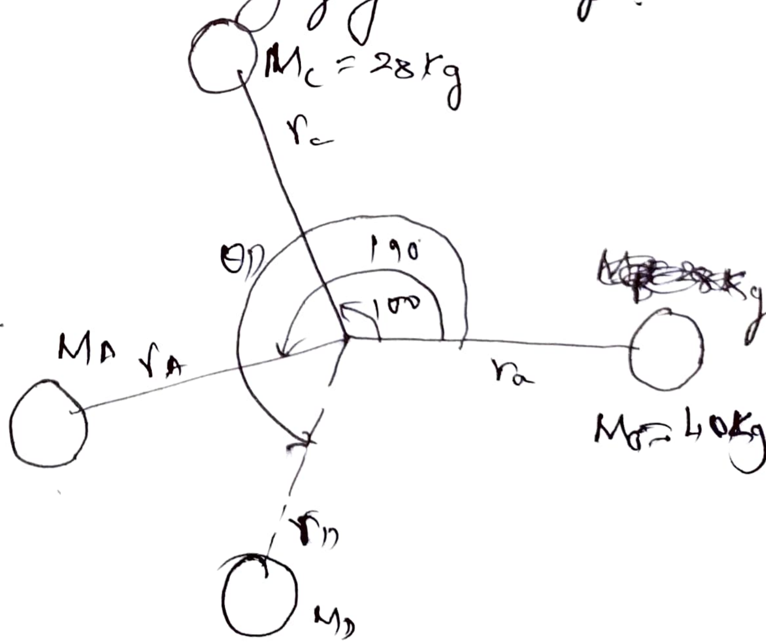
The rotating parts need to be balanced because when the parts rotate at high speed the centrifugal forces are developed on the rotating parts and uneven forces leads to vibrations of the total body which in turn causes wear at the shaft contact points and to the bearings. – (2)

Static balancing :– When the centre of gravity of an object is on the axis of rotation. Static balance has no tendency to rotate due to the force of gravity. – (1) – (4)

Dynamic balancing :– Dynamic balancing is when the rotation does not produce any resultant centrifugal force or couple. – (4)

3 b)



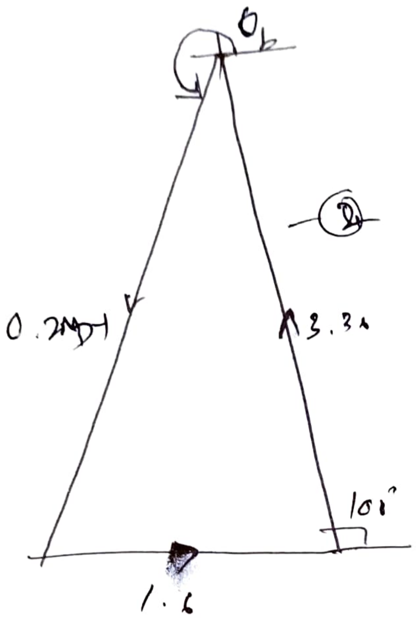


– (3)

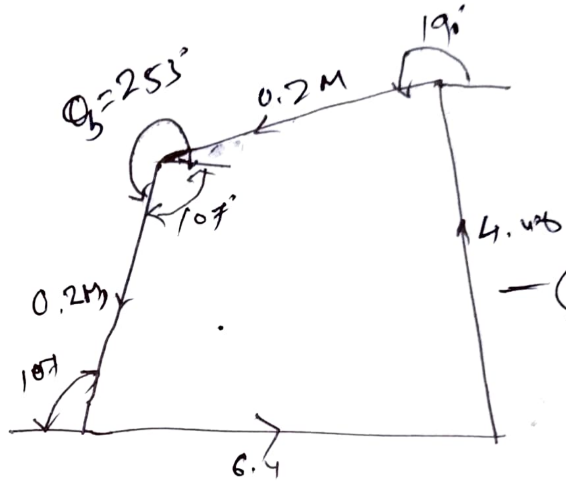
MSD

Plane	Mass (M) kg	Radius r m	Force ÷ ω^2	Distance from Plane A	Couple ÷ ω^2
A	M_A	0.2	$0.2 M_A$	0	0
B	40	0.16	6.4	0.25	1.6
C	28	0.16	4.48	0.75	3.36
D	M_D	0.2	$0.2 M_D$	y	$0.2 M_D y$

— (2)



Couple polygon



Scale 2.5 : 1

Force polygon

— (2)

1) Draw oa = 1.6, Parallel to OM_B in the Same direction of OM_A.

2) From a draw ob = 3.36, Parallel to OM_C.

3) Join b to o, Now, ob × Scale = 0.2 $M_D y$ — (16)

$$ob \times \frac{1}{5} = 0.2 M_D y$$

$$M_D y = 17.45 \quad \theta = 253°$$ — (2)

→ oc × Scale = 0.2 M_D.

$$9.35 \times \frac{1}{2.5} = 0.2 M_D = M_D = 18.7 \text{ kg}$$ — (3)

$$y = \frac{17.45}{M_D} = \frac{17.45}{18.7} = 0.933 \text{m}$$

$$bc \times Scale = 0.2 M_A$$

$$M_A = 22.9 \text{ kg}.$$

MSR

$r = 0.12$ m, $l = 0.42$ m

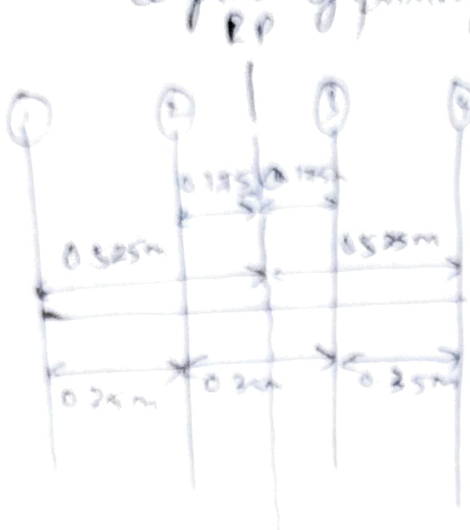
N = 600 rpm, M = 2.5 kg, $n = \frac{l}{r} = \frac{0.42}{0.12} = 3.5$

Analytical Method

Vertical Component of primary force $F_{PV} = M\omega^2 r \sum \cos\theta$

$= M\omega^2 r(\cos 0 + \cos 90 + \cos 180 + \cos 270)$

$= 0$



Horizontal Component of primary force

$F_{PH} = M\omega^2 r \sum \sin\theta$

$= M\omega^2 r(\sin 0 + \sin 90 + \sin 180 + \sin 270) = 0$

Primary forces are completely balanced.

(4)

Primary couples

Vertical component of primary couple $C_{PV} = M\omega^2 r \sum a \cos\theta$

$= M\omega^2 r(-0.525\cos 0 - 0.175\cos 90 + 0.175\cos 180 + 0.525\cos 270)$

$= -0.7 M\omega^2 r$

(2)

Horizontal comp of primary couple

$C_{PH} = M\omega^2 r \sum a \sin\theta$

$= M\omega^2 r(-0.525\sin 0 - 0.175\sin 90 + 0.175\sin 180 + 0.525\sin 270)$

$= -0.7 M\omega^2 r$

$UPC = \sqrt{\{(-0.7)M\omega^2 r\}^2 + (-0.7 M\omega^2 r)^2} = 0.7 M\omega^2 r\sqrt{2}$

(3)

$= 0.7 \times 2.5 \times \left(\frac{2\pi 600}{60}\right)^2 \times 0.12 \times \sqrt{2} = 1172.49$ Nm

P.T.O.

NKB

Secondary force

$\Sigma M r \cos 2\theta = 0 \quad \Sigma M r \sin 2\theta = 0$

Vertical component of secondary force:

$$F_{SV} = \frac{M w^2 r}{n} \Sigma \cos 2\theta$$

$$= \frac{M w^2 r}{n} (\cos 0 + \cos 180 + \cos 0 + \cos 180) = 0$$

Horizontal component of secondary force — (3)

$$F_{SH} = \frac{M w^2 r}{n} \Sigma \sin 2\theta$$

$$= \frac{M w^2 r}{n} (\sin 0 + \sin 180 + \sin 0 + \sin 180) = 0$$

∴ Sec forces are completely balanced. — (9)

Secondary couple — (20)

$\Sigma M r a \cos 2\theta = 0 \quad \Sigma M r a \sin 2\theta = 0$

Vertical component of secondary couple

$$C_{SV} = \frac{M w^2 r}{n} \Sigma a \cos 2\theta$$ — (2)

$$= \frac{M w^2 r}{n} (-0.525 \cos 0 - 0.175 \cos 180 + 0.175 \cos 0 + 0.525 \cos 180)$$

$$= -0.7 \frac{M w^2 r}{n}$$

Horizontal component of secondary couple

$$= C_S = -0.7 \frac{M w^2 r}{n}$$ — (3)

$$= 0.7 \times 2.5 \times \left(\frac{2 \pi\, 600}{60}\right)^2 \times \frac{0.12}{3.5}$$

$= 236.87$ Nm

MSB

Solution and Scheme	Marks

Sensitiveness: For maintaining constant speed of rotation, the movement of sleeve should be as large as possible and the corresponding change of equilibrium speed as small as possible.

$$S = \frac{N_{max} - N_{min}}{\left(\frac{N_{max} + N_{min}}{2}\right)} \quad - (2)$$

Hunting:- Hunting is condition in which the speed of the engine controlled by the governor fluctuates continously above and below the mean speed. It is caused by a governor that is too sensitive. - (2)

Isochronism: A governer is said to be isochronism, when the equilibrium speed is constant for all raddii of rotation of the balls within the working range. (8)

Governer power:- It is product of the mean value of the effort and distance through which sleeve moves, - (2)

Power = mean effort × lift of sleeve. (2)

3 b) $m = 6kg$, $M = 18kg$, $r_1 = 0.2m$, $r_2 = 0.25m$, $N_1 = ?$

$N_2 = ?$

Since $\alpha = \beta$ $\tan\alpha = \tan\beta = 1$ $\quad N_2 - N_1 = ?$

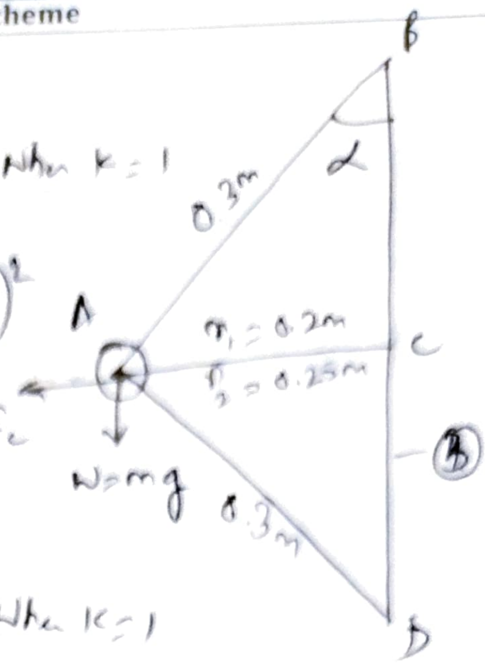
$r_1 = 0.2m$. - (4)

$h_1 = \sqrt{0.3^2 - 0.2^2} = 0.2236m$

$r_2 = 0.25m$ $\quad h_2 = \sqrt{0.3^2 - 0.25^2} = 0.165m$.

MSD

g) $N_1^2 = \left(\frac{m+M}{m}\right)\frac{g}{h_1}\left(\frac{60}{2\pi}\right)^2$ when $k=1$



$= \left(\frac{6+18}{6}\right) \times \frac{9.81}{0.2236}\left(\frac{60}{2\pi}\right)^2$ — (4)

$N_1 = 126.5$ rpm Min speed

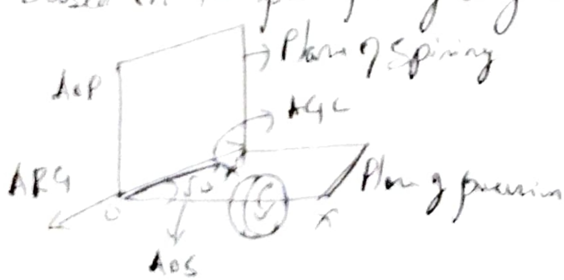
$N_2^2 = \left(\frac{m+M}{m}\right)\frac{g}{h_2}\left(\frac{60}{2\pi}\right)^2$ when $K=1$

$= \left(\frac{6+18}{6}\right) \times \frac{9.81}{0.1658} \times \left(\frac{60}{2\pi}\right)^2$ — (3) — (12)

$N_2 = 146.9$ rpm = Max speed

Range $N_2 - N_1 = 146.9 - 126.5$
$= 20.4$ rpm. — (2)

6 a) Gyroscopic effect: For measuring or maintaining orientation based on the principle of angular momentum — (2)



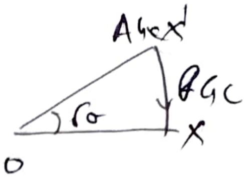
Consider a disc spinning with angular velocity about axis of spin OX in anticlockwise direction when seen from the front as shown in fig. — (2)

MSD

let I = mass moment of inertia of disc OX

ω = Angular velocity of the disc

$\therefore$ Angular momentum of the disc = $I.\omega$



$\vec{OX} - \vec{OX'} = XX'$ — (2)

$XX' = ox \times \delta\theta = I\omega\delta\theta$.

Rate of change of angular momentum — (8)

$= I\omega \frac{d\theta}{\delta t}$ — (2)

$C = \lim_{\delta t \to 0} I.\omega.\frac{d\theta}{\delta t} = I\omega.\frac{d\theta}{\delta t}$ N.m.

$C = I\omega\omega_p$.

6b) $m = 450\,kg$, $k = 0.32\,m$

$\omega = \frac{2\pi \times 2400}{60} = 209.4\ rad/s$ $\omega_p = \frac{v}{r} = \frac{66.67}{60} = 1.11\ rad/s$

$v = \frac{240 \times 10^3}{3600} = 66.67\ m/s$ — (2) $I = mk^2 = 450 \times (0.32)^2$

$= 46.08\ kgm^2$ — (3)

Effect → $C = I\,\omega\,\omega_P = 46.08 \times 209.4 \times 1.11$

Figure Shows the aeroplane in space $= 10.73\ kNm$. ~~= 10.73 kNm~~.

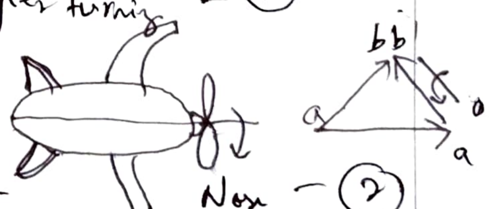
When turns left a) oa is angular momentum before turning

b) ob is angular momentum after turning — (2)

c) ab is applied couple

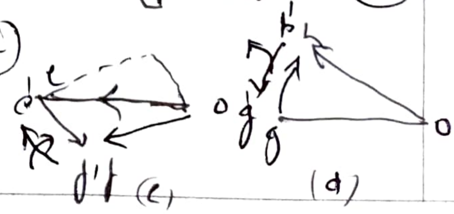
(12)

1) Nose is depressed & tail is raised in figure 'C' while right turn



— (2)

2) When aeroplane takes right turn, the tail is depressed and nose is raised — (2)



MSD

Solution and Scheme | Marks

7a) SHM - The motion of a body to & fro about a fixed point is called SHM.

$x = A\sin\omega t$ — ②
$\dot{x} = A\omega\cos\omega t$
$\ddot{x} = -A\omega^2\sin\omega t$

Natural Frequency:- When no external forces acts on the system after giving it an initial displacement the body vibrates. — ②

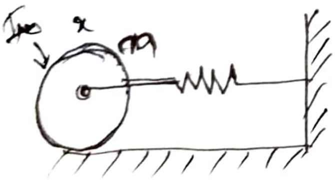
$\omega_n = \sqrt{k/m}$ rad/s.

Resonance:- The vibration of a system when the frequency of external forces is equal to the natural frequency of the system. — ②

Forced vibration:- If a system is subjected to an external force, the resulting vibration is known as forced vibration — ②

Phase difference:- It is the angle between two rotating vectors representing SHM of the same frequency. — ② — ⑩

7b)



A circular cylinder of mass m & radius r as shown in figure.

According to Newton's method,

$F \propto a$ $F = ma$ $F = I\ddot{\theta}$ = For angular displacement $I\ddot{\theta} = -$Restoring Torque

$I = \frac{1}{2}Mr^2$, $F = -\frac{1}{2}Mr\ddot{\theta}$ — ④

$M\ddot{x} + kx = F$

$M\ddot{x} + kx = -\frac{1}{2}Mr\ddot{\theta}$

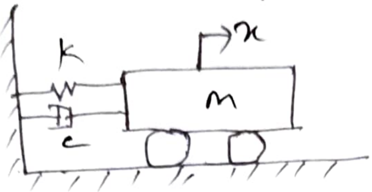
$M\ddot{x} + kx = -\frac{1}{2}M\ddot{x}$ — ④ 10.

$\ddot{x} + \frac{k}{3M/2}x = 0$ $\omega_n = \sqrt{\frac{2}{3}\frac{k}{M}}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3M}}$ — ②

Nsp

Question Number	Solution	Marks Allocated
8 a)	Differential equation for spring mass damper system.	



Damping force – $c\dot{x}$

Accelerating force – $m\ddot{x}$

Spring force – Kx

The equation of motion

$$m\ddot{x} + c\dot{x} + Kx = 0 \quad -(1)$$

To solve equation (1) Assume displacement

$$x = e^{st} \quad -(2)$$

$$\dot{x} = s e^{st} \quad -(3)$$

— (3)

Diff 3 w.r.t time

$$\ddot{x} = s^2 e^{st}$$

$$m(s^2 e^{st}) + c s e^{st} + k e^{st} = 0$$

$$e^{st} \neq 0$$

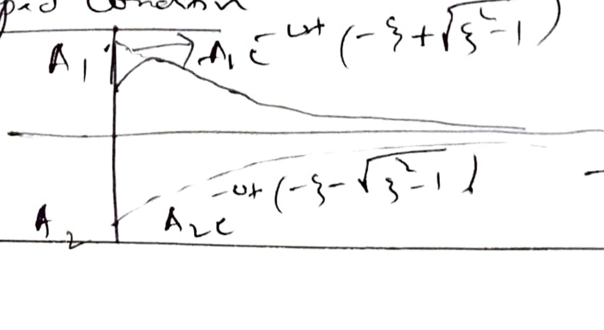
$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1 \quad b = c/m \quad c = k/m.$$

$$s_{1,2} = \frac{-c/m \pm \sqrt{(c/m)^2 - 4(k/m)}}{2} \quad -(3)$$

$$x = A_1 e^{\left(-\frac{c}{2m} \pm \sqrt{(c/m)^2 - (k/m)}\right)} + A_2 e^{\left(-c/m - \sqrt{(k/m)^2} - k/m\right)}$$

Over damped Condition



— (1)

MA

Subject Title :   Subject Code :

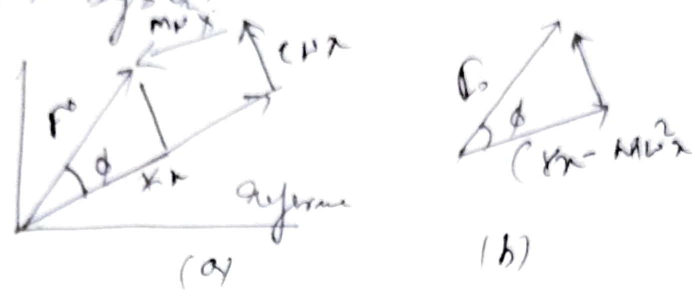
Question Number	Solution	Marks Allocated
	$\cancel{S_1 = +\omega_n\sqrt{-\xi+\sqrt{\xi^2-1}}}$ $\quad S_1 = \omega_n(-\xi + \sqrt{\xi^2-1})$	
	$\cancel{S_2 = +\omega_n\sqrt{\xi}}$ $\quad S_2 = \omega_n(-\xi - \sqrt{\xi^2-1})$	
	$x = x_0$ at $t=0$ to obtain A_1 & A_2	
	$\dot{x} = 0$ at $t = 0$	
	$\dot{x} = A_1(-\xi+\sqrt{\xi^2-1})\omega_n e^{+(\xi+\sqrt{\xi^2-1})\omega_n}$	
	$+ A_2(-\xi \mp \sqrt{\xi^2-1})\omega_n e^{(-\xi-\sqrt{\xi^2-1})\omega_n}$	
	A_1 & A_2 $\Rightarrow$, $x = \left[\dfrac{\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\right] x_0 e^{(-\xi+\sqrt{\xi^2-1})\omega_n}$ — (2)	10
	$+ \left(\dfrac{-\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\right) x_0 e^{(-\xi-\sqrt{\xi^2-1})\omega_n}$	
8 b)	$m = 25$ kg, $k = 15000$ N/m. ~~Assume~~ $F = 25 \times 9.81$ $= 245$ N $= F$	
	$C_c = 2m\omega_n$, $\omega_n = \sqrt{k/m} = \dfrac{1}{2\pi}\sqrt{\dfrac{15000}{25}}$ — (2)	
	$\omega_n = 3.89$ Hz.	
	$C_c = 2 \times 25 \times 3.89 = 194$ N.s/m — (2)	
	15% of $C_c = 29.23$ N.s/m.	
	$F = cx \quad c = \dfrac{F}{x} = \dfrac{245}{1} =$ $\quad x = 1$ Assume — (2)	10
	$= 245$ N.s/m.	
	damping factor $= \dfrac{c}{c_c} = \dfrac{245}{194} = 1.26$ — (2)	
	Log decrement $= \dfrac{\xi}{\sqrt{1-\xi^2}} 2\pi = \dfrac{1.26}{\sqrt{1-1.26^2}} 2\pi = 10.32$ (2)	
	MSD	

Subject Title :   Subject Code :

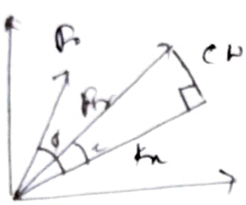
Question Number	Solution	Marks Allocated

9 a.

Transmissibility: The ratio of force transmitted to the foundation to that of impressed force acting upon the system.



— (2)



Out of these four forces kx & $c\omega$ are two common forces acting on the & also on the foundation.

$$F_{tr} = \sqrt{(kx)^2 + (c\omega x)^2} \quad — (2)$$

$$F_{tr} = x\sqrt{k^2 + (c\omega)^2}$$

$$\cancel{x = \frac{F_0}{\sqrt{(k)^2+(c\omega)^2}}} \qquad x = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

$$F_{tr} = \frac{F_0\sqrt{k^2+(c\omega)^2}}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}} \quad — (2)$$

— (10)

$$T_r = \frac{F_{tr}}{F_0} = \frac{\sqrt{1+(2\xi(\omega/\omega_n))^2}}{\sqrt{((\omega/\omega_n)^2)^2 + (2\xi\omega/\omega_n)^2}}$$

T_r is transmissibility ratio — (2)

MSD $\alpha = \tan^{-1}\left(\frac{c\omega}{k}\right)$ $\quad \phi = \tan^{-1}\left(\frac{2\xi\omega/\omega_n}{1-(\omega/\omega_n)^2}\right) - \tan^{-1}(2\xi\omega/\omega_n)$

Subject Title :

Subject Code :

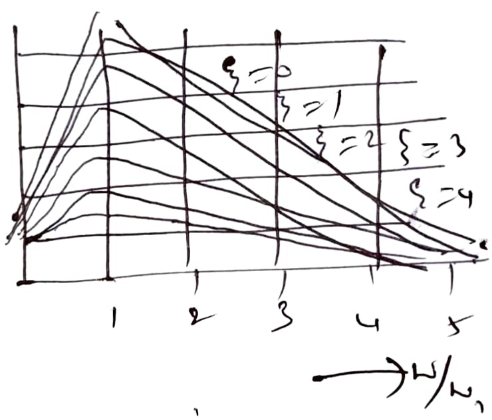
Question Number	Solution	Marks Allocated
9 b)	$m = 100\,kg,\ K = 19600\,N/m,\ C = 100\,N\text{-}s/m,\ F_0 = 39\,N$	
	$\omega_n = 14\,rad/s = \sqrt{K/m}$	
	$C = 2m\omega_n\xi$	
	$\xi = 0.0357$ damping ratio	
	i) Amplitude of mass	
	$X_{max} = \dfrac{X_0}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$	
	$X_0 = \dfrac{F_0}{K} = 1.98\times10^{-3}\,m \quad \omega = \omega_n$	
	$X_{max} = 0.028\,m.$ — (4)	
	ii) $\phi = \tan^{-1}\left(\dfrac{2\xi\,\omega/\omega_n}{1-(\omega/\omega_n)^2}\right) = 90^\circ$ — (2)	
	iii) $\epsilon = \sqrt{\dfrac{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$ — (4)	(10)
	$\epsilon = \sqrt{\dfrac{1+(2\times0.035)^2}{2\times0.0358}} = 14.04$	

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated
10(i) a)	Magnification factor: It is very important to study the steady state behaviour of the system Bequation. The ratio of steady state amplitude to the low frequency deflection — ②	10

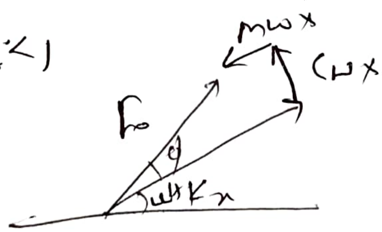
$$MF = \frac{X}{X_{st}} = \frac{1}{\sqrt{(1-(\omega/\omega_n)^2) + (2\xi\frac{\omega}{\omega_n})^2}}$$ — ②

$MF = \frac{X}{X_{st}}$

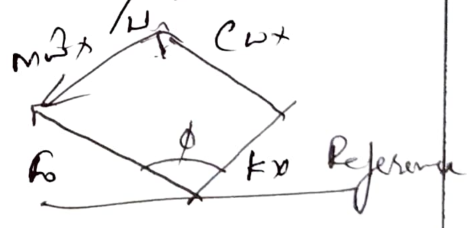


— ②

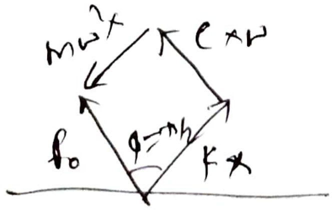
1) $\omega/\omega_n \ll 1$



2) $\omega/\omega_n \gg 1$ — ②



3) $\omega/\omega_n = 1$



— ②

Subject Title :

Subject Code :

Question Number	Solution	Marks Allocated
10 b).	$W = 54N$, $m = 5.5kg$, $K = 1100 N/m$, $c = 77 N\cdot s/m$	
	a) $\omega_n = \sqrt{K/m} = \sqrt{\frac{1100}{5.5}} = 14.1$ rad/sec	2
	b) $X_r = \frac{X_{st}}{2\xi}$ = $X_{st} = F_0/K = \frac{5}{1100} = \frac{0.0045}{155.1}$; $\xi = c/c_c = \frac{77}{2\times5.5\times14.1}$ $= 0.000029$ m	2
	c) Phase angle: $\phi = \tan^{-1}\left(\frac{2\xi\sqrt{\omega/\omega_n}}{1-(\omega/\omega_n)^2}\right) = 90°$	2
	d) $\omega_p = \omega_n\sqrt{1+2\xi^2}$ $= 14.1\sqrt{1+2(0.49)^2} = 7.15$ rad/s	2
	e) Peak amplitude - $X_p = 1.1750\times10^{-4}$ m	2
	f) Phase for peak amplitude $\tan\phi_p = \frac{\sqrt{1+2\xi^2}}{\xi}$ $= \frac{\sqrt{1+2(0.49)^2}}{0.49}$ $= 2.48$ rad. $\phi_p = 68°$	2
		10