

CBCS SCHEME

18CV52

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Fifth Semester B.E. Degree Examination, Feb./Mar. 2022 Analysis of Indeterminate Structures

Time: 3 hrs.

Max. Marks: 100

- Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Assume missing data suitably.

Module-1

- 1 Analyze the continuous beam shown in Fig.Q.1 by slope deflection method. Draw BMD and SFD. (20 Marks)

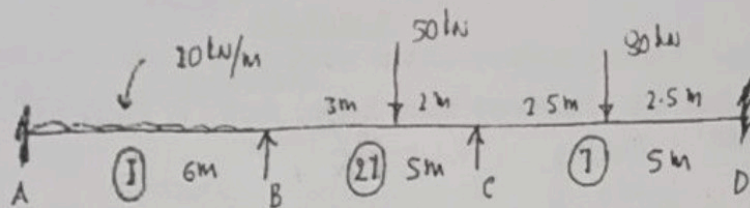


Fig.Q.1

OR

- 2 Analyze the portal frame shown in Fig.Q.2 by slope deflection method. Draw BMD. (20 Marks)

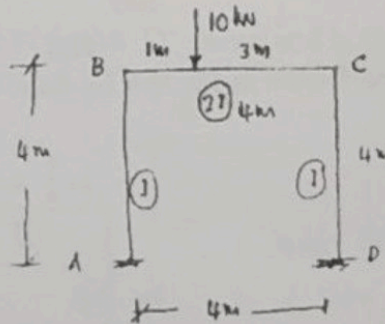


Fig.Q.2

Module-2

- 3 Analyze the beam shown in Fig.Q.3 by moment distribution method. Draw BMD EI is constant. (20 Marks)

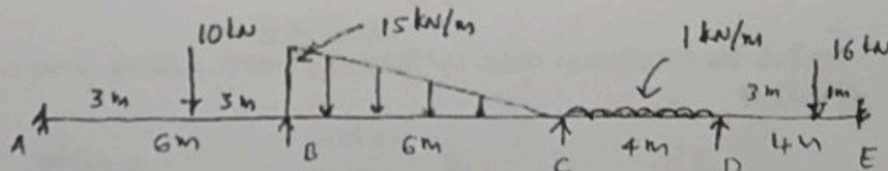


Fig.Q.3

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written e.g. 42+8=50, will be treated as malpractice.

- OR
- 4 Analyze the portal frame by moment-distribution method draw BMD. (20 Marks)

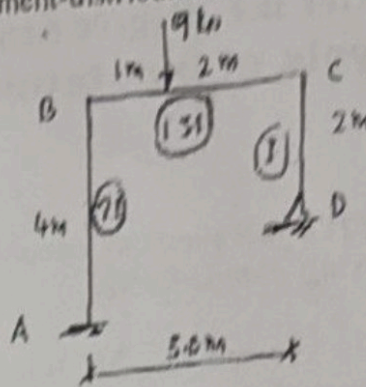


Fig.Q.4

Module-3

- 5 Analyze the continuous beam loaded shown in Fig.Q.5 by Kani's rotation method. Draw BMD. (20 Marks)

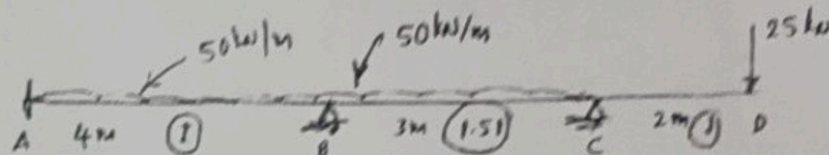


Fig.Q.5

OR

- 6 Analyze the frame shown in Fig.Q.6 by Kani's method. Take the advantage of symmetry. (20 Marks)

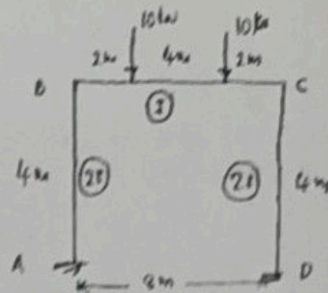


Fig.Q.6

Module-4

- 7 Analyze the continuous beam by flexibility matrix method (system approach). Draw BMD. (Fig.Q.7). (20 Marks)

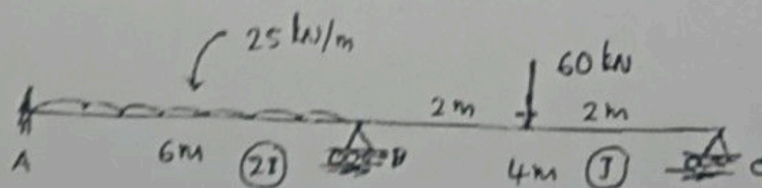


Fig.Q.7

OR

- 8 Analyze the L-frame shown in Fig.Q.8 by flexibility matrix method. Draw BMD (system approach). (20 Marks)

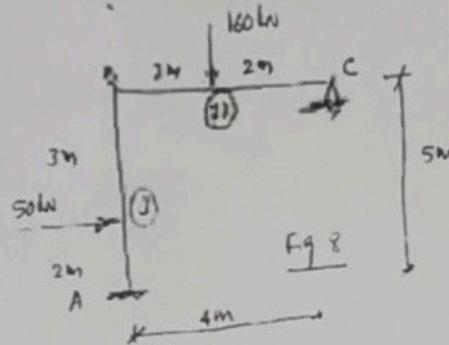


Fig.Q.8

Module-5

- 9 Analyze the continuous beam by stiffness matrix method (system approach) shown in Fig.Q.9. Draw BMD EI is constant. (20 Marks)

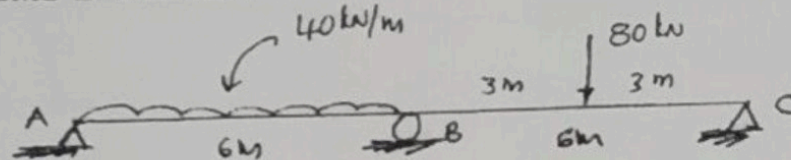


Fig.Q.9

OR

- 10 Find the forces in the members of a joint 'O' shown in Fig.Q.10 by stiffness matrix method. (system approach). (20 Marks)

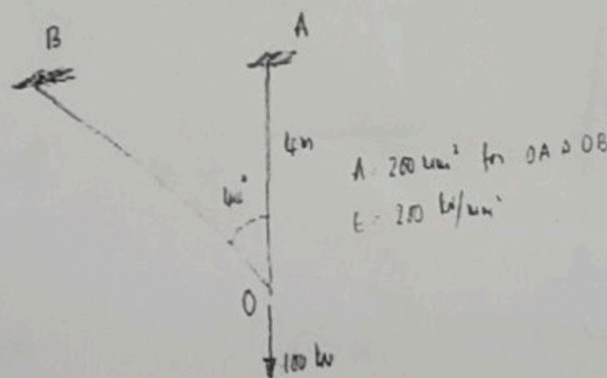
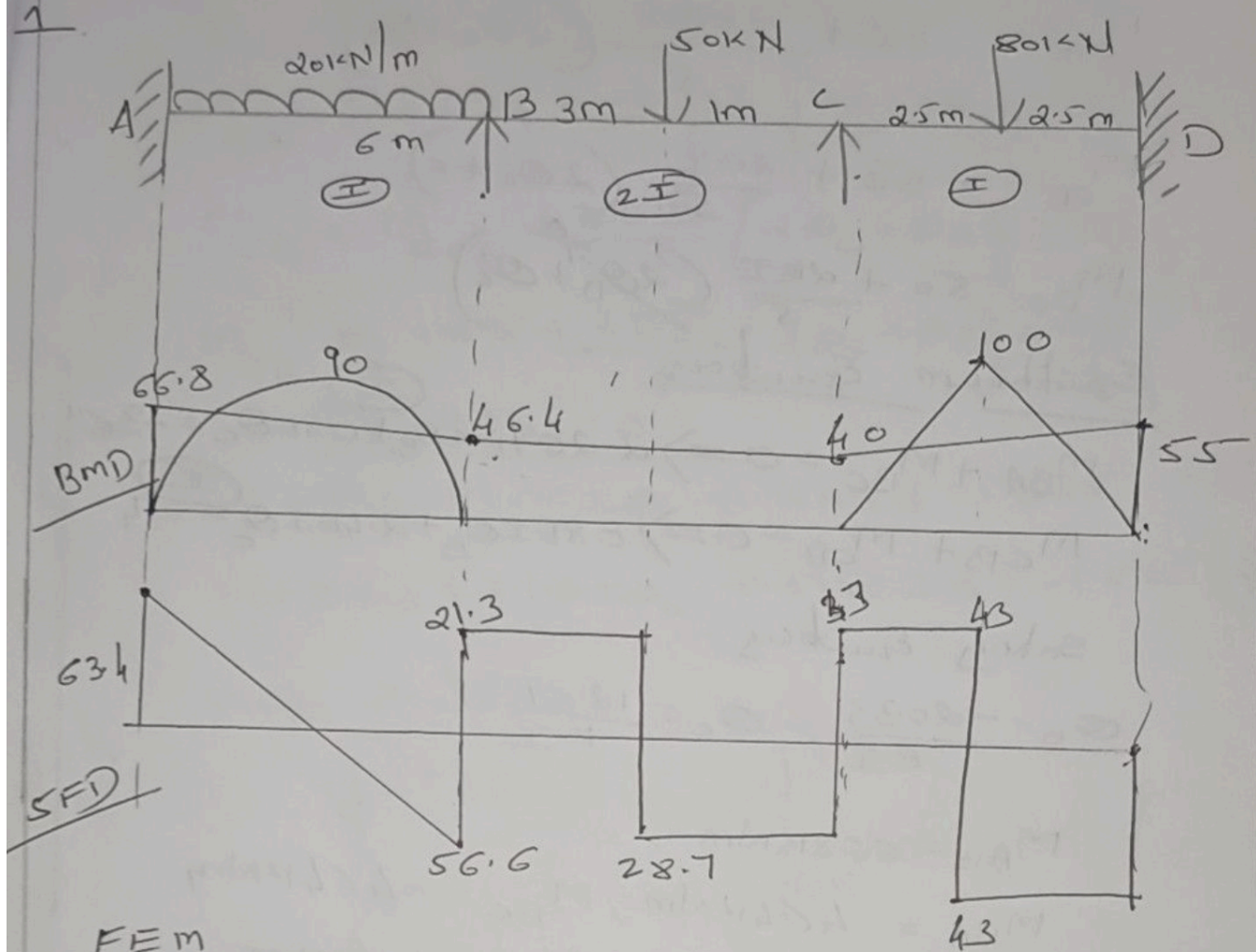


Fig.Q.10

Module-1



FEM

$$M_{FAB} = -60 \text{ kNm}$$

$$M_{FBA} = 60 \text{ kNm}$$

$$M_{FBC} = -24 \text{ kNm}$$

$$M_{FCB} = 36 \text{ kNm}$$

$$M_{FCD} = -50 \text{ kNm}$$

$$M_{FDC} = 50 \text{ kNm}$$

Equations

$$\theta_A = \theta_D = 0$$

Equilibrium Equations

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2\theta_A + \theta_B]$$

$$M_{AB} = -60 + \frac{2EI}{6} [0 + \theta_B]$$

$$M_{BA} = 60 + \frac{2EI}{6} [2\theta_B + 0]$$

$$M_{BC} = M_{FBC}^{(-24)} + \frac{2EI(2)}{5} (2\theta_B + \theta_C)$$

$$M_{CB} = 36 + \frac{2 \times 2EI}{5} (2\theta_C + \theta_B)$$

$$M_{CD} = -50 + \frac{2EI}{5} (2\theta_C + 0)$$

$$M_{DC} = 50 + \frac{2EI}{5} (2\theta_D + \theta_C)$$

Equilibrium Equations.

$$M_{BA} + M_{BC} = 0 \Rightarrow 2.267EI\theta_B + 0.8EI\theta_C = -36 \quad \textcircled{I}$$

$$M_{CB} + M_{CD} = 0 \Rightarrow 0.8EI\theta_B + 2.4EI\theta_C = -14 \quad \textcircled{II}$$

Solving Equations

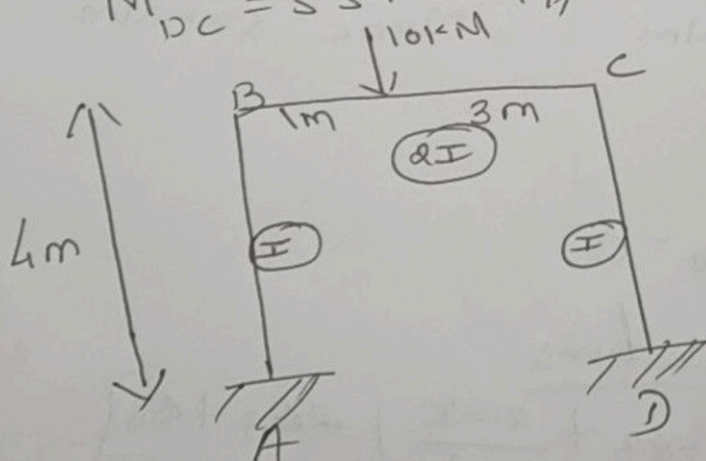
$$\theta_B = \frac{-2033}{EI} \quad \theta_C = \frac{12.61}{EI}$$

$$M_{AB} = 66.8 \text{ kNm}$$

$$M_{BA} = 46.412 \text{ kNm}, \quad M_{BC} = -46.412 \text{ kNm}$$

$$M_{CB} = -40.1 \text{ kNm}, \quad M_{CD} = 40.1 \text{ kNm}$$

$$M_{DC} = 55.14 \text{ kNm}$$



$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0.$$

$$M_{FBC} = -5.625 \text{ kNm}$$

$$M_{FCB} = 1.875 \text{ kNm}$$

5-1) Equations

$$\theta_A = \theta_D = 0$$

$$M_{Bc} = M_{Fbc} + \frac{2EI}{u} \left[2\theta_B + \theta_C \right] - \frac{0.75 \times EI \Delta}{u}$$

$$M_{Bc} = -1.5625 + \frac{2EI \times 2}{4} \left[2\theta_B + \theta_C \right] - \frac{0.75 \Delta}{u}$$

$$M_{cB} = M_{FCB} + \frac{2EI}{u} \left[2\theta_C + \theta_B \right] - \frac{0.75 \Delta}{u}$$

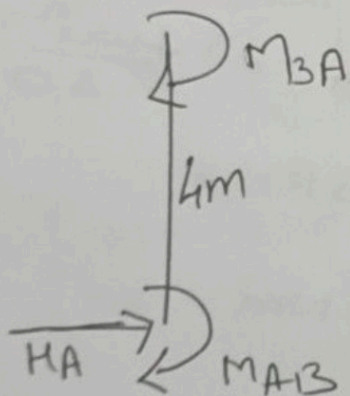
$$M_{cB} = 1.875 + \frac{4EI}{4} \left[2\theta_C + \theta_B \right] - \frac{0.75 \Delta}{u}$$

$$M_{BA} + M_{Bc} = 0$$

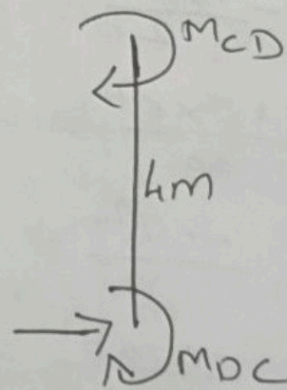
$$4EI\theta_B + 1.0EI\theta_C - 0.75EI\Delta = 5.625 \quad \text{--- (1)}$$

$$1.0EI\theta_B + 3.0EI\theta_C - 0.375EI\Delta = -1.875 \quad \text{--- (2)}$$

Sway Analysis.



$$H_A + H_D = 0$$

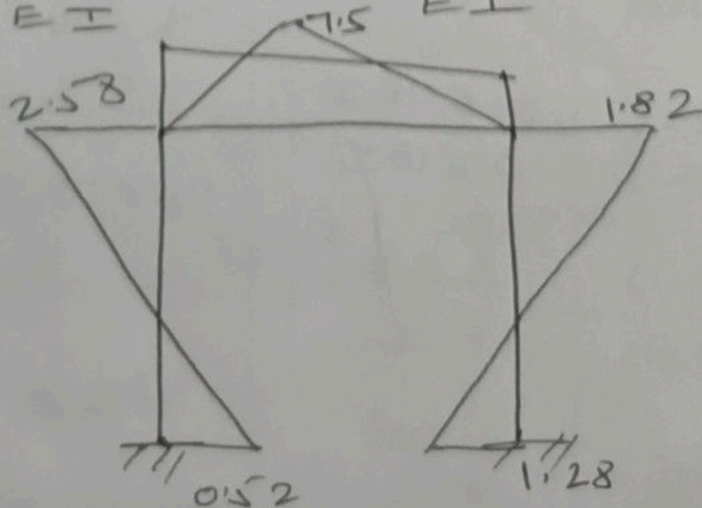


$$3.0EI\theta_B + 1.54EI\theta_C - 2.25EI\Delta = 0 \quad \text{--- (3)}$$

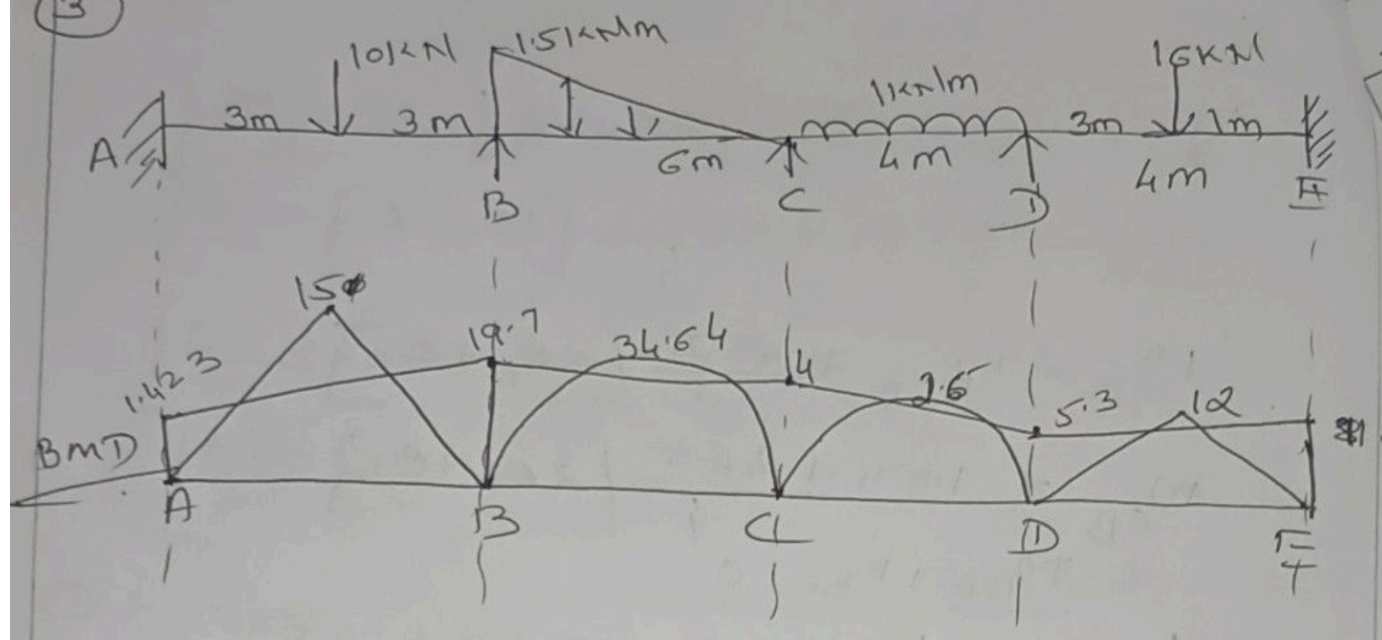
Solving Equations (1) (2) & (3)

$$\theta_B = \frac{2.05}{EI}, \quad \theta_C = \frac{-1.05}{EI}, \quad \Delta = \frac{2.03}{EI}$$

BMD



3



$$M_{FAB} = -7.5 \text{ kNm} = \frac{-wL}{8}$$

$$M_{FBA} = +7.5 \text{ kNm} = \frac{+wL}{8}$$

$$M_{FBC} = -\frac{15 \times 6^2}{20} = -27 \text{ kNm} = \frac{-wL^2}{20}$$

$$M_{FCB} = \frac{15 \times 6^2}{30} = 18 \text{ kNm} = \frac{+wL^2}{30}$$

$$M_{FDC} = \frac{-wL^2}{12} = -1.33 \text{ kNm}$$

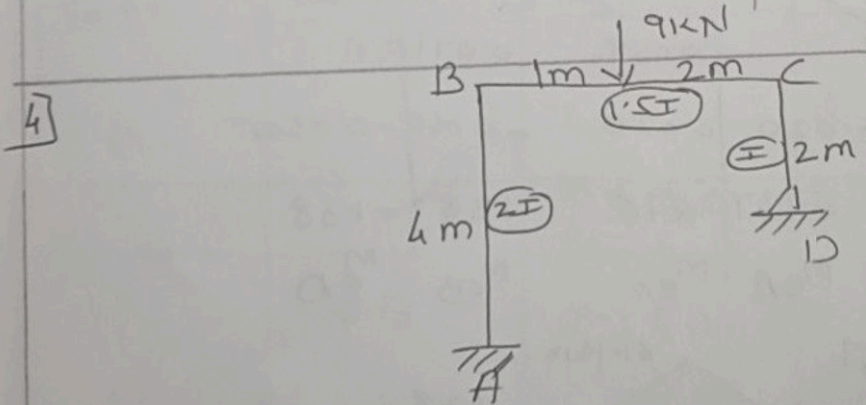
$$M_{FDE} = -\frac{wab^2}{L^2} = -3 \text{ kNm}$$

$$M_{FED} = \frac{wa^2b}{L^2} = +9 \text{ kNm}$$

Distribution Factor

Joint	Member	ΣK K	ΣK	DF
A	BA	0.67EI	1.334EI	0.5
	BC	0.67EI		
B	BC	0.67EI	1.667EI	0.5
	CB	0.67EI		
C	CD	4EI	2EI	0.59
	DC	EI		
D	DC	EI	2EI	0.5
	DE	EI		

Joint	A	B	C	D	E
member	A B	BA BC	CB CD	DC DE	ED
DF	0	0.5 0.5	0.4 0.6	0.5 0.5	0
FEM	-7.5	7.5 -27	18 -133	-133 -3	
Balance		9.75 9.75	-666 -9.98	0.835 0.835	
CO	4.875	3.334	4.875 0.4175	4.97	0.487
Balance		1.667 1.667	-2.117 -3.170	2.496 2.496	
CO	0.8335	-1.0585 0.835	1.248	-1.585	1.248
Balance		0.529 0.529	-0.832 1.248	1.585 0.792	1.2
CO	0.2045	-0.416 0.264	0.396	-0.623	0.396
Balance		0.208 0.208	-0.26 0.395	0.3115 0.3115	
CO	0.104	-0.1321 0.109	0.155	-0.09	0.155
Balance		0.066 0.066	0.103 -0.141	0.09 0.09	
Final	-1.426	19.72 -19.72	14.19 -14.19	-1.53 +1.53	11.216
	M_{AB}	M_{BA} M_{BC}	M_{CB} M_{CD}	M_{DC} M_{DE}	M_{ED}



4

$$M_{FAB} = M_{FBA} = 0$$

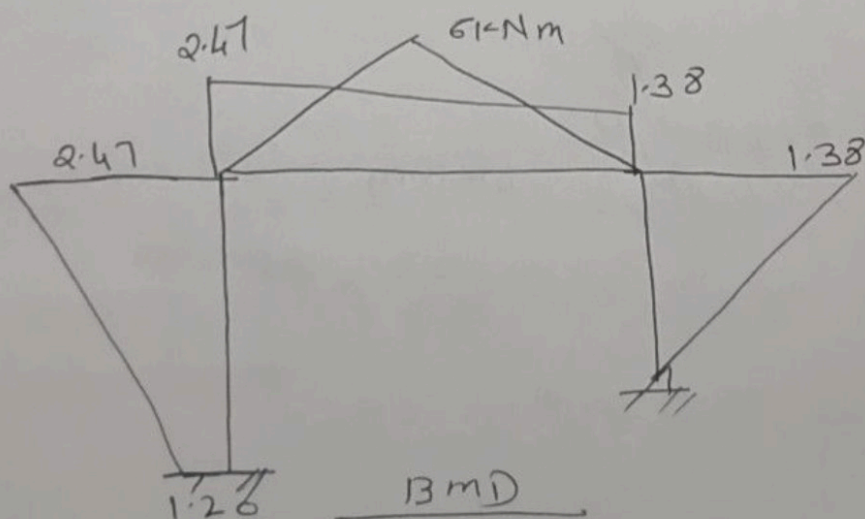
$$M_{FBC} = \frac{-w a b^2}{u^2} = -4 \text{ kNm}$$

$$M_{FCB} = \frac{w a^2 b}{u^2} = \frac{9 \times 1 \times 2}{3^2} = 2.0 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

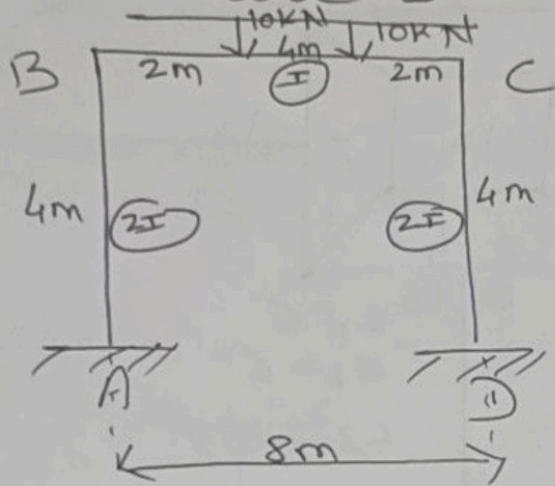
Joint	member	K	ΣK	DF
B	BA	$2EI$	$4EI$	0.5
	BC	$2EI$		0.5
C	CB	$2EI$	$3.5EI$	0.57
	CD	$1.5EI$		0.43

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.57	0.43	1
FEM	0	0	-4	2	0	0
Total Balance	0	2	2	-1.14	-0.86	
CO	1			1		
Balance		0.285	0.285	-0.57	-0.43	
CO	0.1425			0.1425		
Balance		0.1425	0.1425	-0.081	0.0612	
CO	0.071			0.071		
Balance		0.020	0.020	-0.040	-0.0305	
Final Moment	1.2133	2.467	-2.16	1.38	-1.38	
	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	



6

Module - 3



FEM

$$M_{FAB} = M_{FBA} = M_{FCD} = F_{FDC} = 0$$

$$M_{FBC} = -\frac{wab^2}{4^2} - \frac{wab^2}{4^2}$$

$$= -\frac{10 \times 2 \times 6^2}{8^2} - \frac{10 \times 6^2 \times 2}{8^2}$$

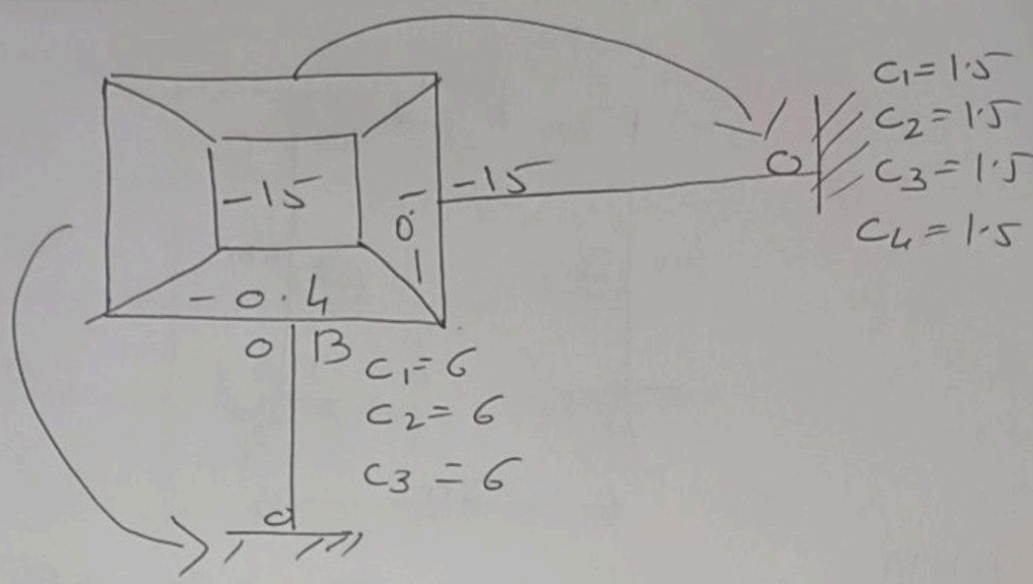
$$M_{FBC} = -15 \text{ kNm}$$

$$M_{FCB} = \frac{wa^2b}{4^2} + \frac{wa^2b}{4^2} = \frac{10 \times 2^2 \times 6}{8^2} + \frac{10 \times 6^2 \times 2}{8^2}$$

$$= 15 \text{ kNm}$$

Rotational factors

Joint	member	K	ΣK	DF	RF
B	BA	$2EI$	$2.5EI$	0.8	-0.4
	BC	$0.5EI$		0.2	-0.1
C	CB	$0.5EI$	$2.5EI$	0.2	-0.1
	CD	$2EI$		0.8	-0.4



cycle 1 @ B

$$M'_{BA} = -0.4 [-15 + (0+0)] = 6 \text{ kNm}$$

$$M'_{BC} = -0.1 (-15 + (0+0)) = 1.5 \text{ kNm}$$

cycle-2 @ B

$$M'_{BA} = -0.4 (-1.5 + 0) = 6 \text{ kNm}$$

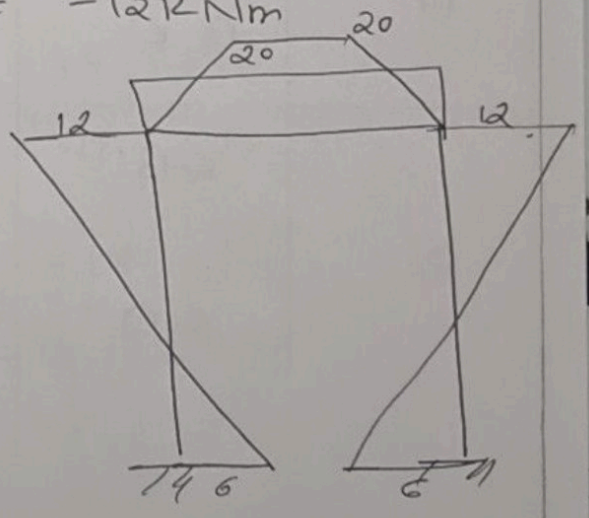
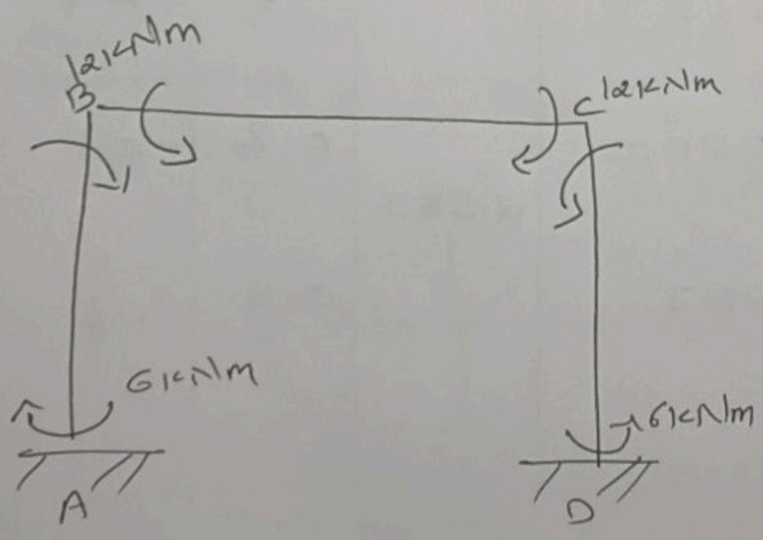
$$M'_{BC} = 1.5 \text{ kNm}$$

Final moments

$$M_{AB} = M_{FAB} + 2M'_{AB} + M''_{BA} = 0 + 2(0) + 6 = 6 \text{ kNm}$$

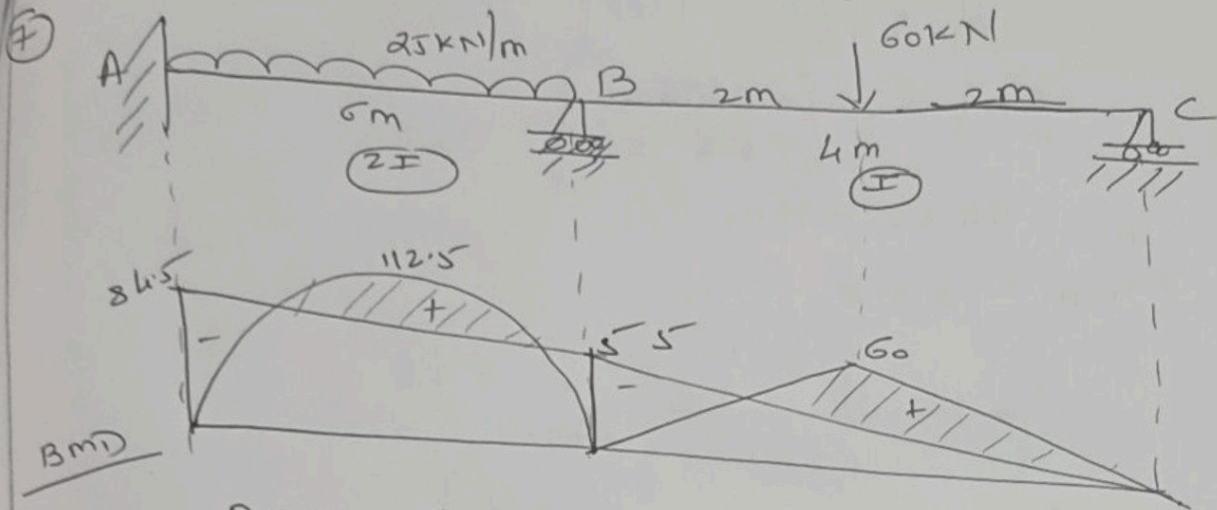
$$M_{BA} = M_{FBA} + 2M'_{BA} + M''_{AB} = 0 + 2(6) + 0 = 12 \text{ kNm}$$

$$M_{BC} = M_{FBC} + 2M'_{BC} + M''_{CB} = -15 + 2(1.5) + 0 = -12 \text{ kNm}$$



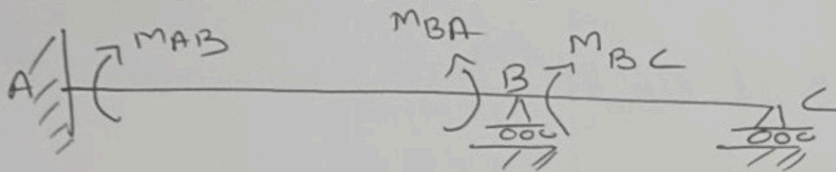
BMD

Module-4



$$DSI = 4 - 2 = 2$$

General co-ordinates



$$[D] - [DL] = [F][P]$$

$2 \times 2 \quad 2 \times 1$

$$DL_1 = \frac{wL^3}{24EI} = \frac{25 \times 6^3}{24 \times 2EI} = \frac{112.5}{EI}$$

$$DL_2 = \frac{wL^2}{16EI} = \frac{60 \times 4^2}{16EI} = \frac{60}{EI}$$

$$DL_2 = \frac{60 + 112.5}{EI} = \frac{172.5}{EI}$$

$$DL = \frac{1}{EI} \begin{bmatrix} 112.5 \\ 172.5 \end{bmatrix}$$

$$[F] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2.33 \end{bmatrix} \frac{1}{EI}$$

$$f_{11} = \frac{ML}{3EI} = \frac{1}{EI}$$

$$f_{21} = \frac{ML}{6EI} = \frac{0.5}{EI}$$

$$f_{12} = \frac{ML}{6EI} = \frac{0.5}{EI}$$

$$f_{22} = \left(\frac{ML}{3EI} \right)_{BA} + \left(\frac{ML}{3EI} \right)_{BC} = \frac{2.33}{EI}$$

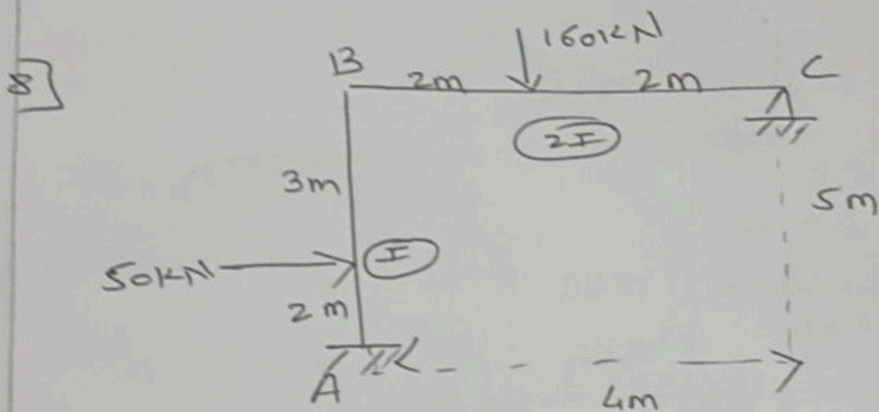
$$[D] - [DL] = [F] [P]$$

$$[P] = [F]^{-1} [DL]$$

$$[P] = \begin{bmatrix} -84.55 \\ -55.88 \end{bmatrix}$$

$$M_{AB} = 84.55 \text{ kNm}$$

$$M_{BA} = 55.88 \text{ kNm}$$



$$DSI = 5 - 3 = 2$$



$$[DL] \quad \theta_A = \frac{wb}{6EI} \left[\frac{L^2 - b^2}{L} \right] = \frac{50 \times 3}{6EI} \left(\frac{5^2 - 3^2}{5} \right) = \frac{80}{EI}$$

$$@ AB \quad \theta_B = \frac{wa}{6EI} \left[\frac{b^2 - a^2}{L} \right] = \frac{70}{EI} = \frac{50 \times 2}{6EI} \left[\frac{5^2 - 2^2}{5} \right]$$

$$@ BC \quad \theta_B = \frac{wL^2}{16EI} = \frac{160 \times 4^2}{16EI} = \frac{80}{EI}$$

$$DL_1 = \theta_A = \frac{80}{EI}$$

$$DL_2 = (\theta_B)_{AB} + (\theta_B)_{BC}$$

$$= \frac{80}{EI} + \frac{70}{EI} = \frac{150}{EI}$$

$$DL = \begin{bmatrix} DL_1 \\ DL_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 150 \end{bmatrix} \frac{1}{EI}$$

$$[F] = \begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 2.33 \end{bmatrix} \frac{1}{EI}$$

$$f_{11} = \frac{ML}{3EI} = \frac{1.67}{EI}$$

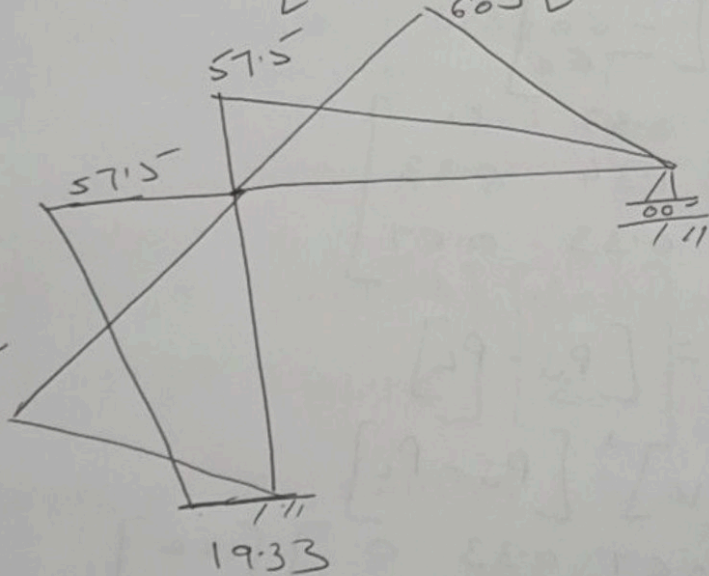
$$f_{21} = \frac{ML}{6EI} = \frac{0.83}{EI}$$

$$f_{12} = \frac{ML}{6EI} = \frac{0.83}{EI}$$

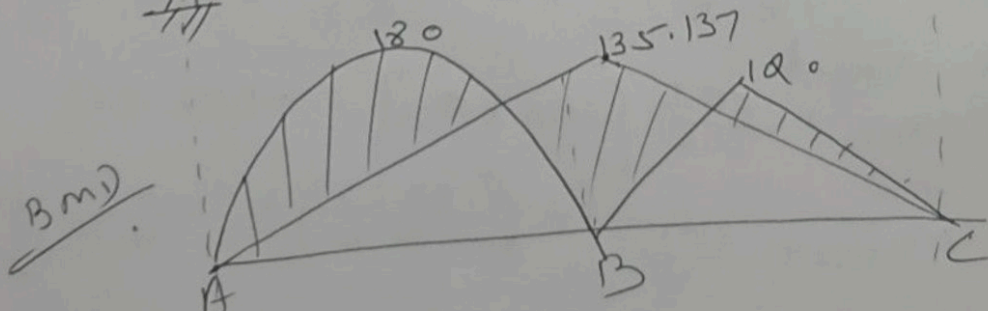
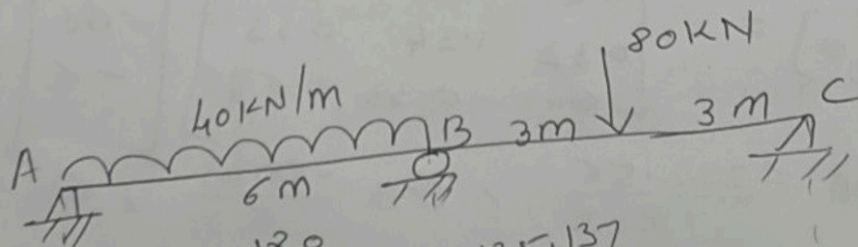
$$f_{22} = \left(\frac{ML}{3EI} \right)_{BA} + \left(\frac{ML}{3EI} \right)_{BC} = \frac{2.33}{EI}$$

$$[P] = [F]^{-1} [DL] = \frac{1}{EI} \begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 2.33 \end{bmatrix}^{-1} \begin{bmatrix} -80 \\ -150 \end{bmatrix} \frac{1}{EI}$$

$$[P] = \begin{bmatrix} -19.33 \\ -57.51 \end{bmatrix} \frac{1}{EI} = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix}$$



9



$$M_{FAB} = \frac{-wu^2}{12} = -120 \text{ kNm}$$

$$M_{FBA} = \frac{wu^2}{12} = \frac{40 \times 36}{12} = 120 \text{ kNm}$$

$$M_{FBC} = \frac{-wu^2}{8} = -60 \text{ kNm}, \quad M_{FCB} = 60 \text{ kNm}$$

$$\text{DoF} = 3$$



$$P_{\dot{u}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_L = \begin{bmatrix} P_{L1} \\ P_{L2} \\ P_{L3} \end{bmatrix} = \begin{bmatrix} -120 \\ +60 \\ 60 \end{bmatrix}$$

$$[P_{\dot{u}} - P_L] = \begin{bmatrix} 120 \\ -60 \\ -60 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.67 & 0.33 & 0 \\ 0.33 & 1.34 & 0.33 \\ 0 & 0.33 & 0.67 \end{bmatrix}$$

$$[K][\theta] = [P_{\dot{u}} - P_L]$$

$$[\theta] = [K]^{-1} [P_{\dot{u}} - P_L]$$

$$[\theta] = \begin{bmatrix} 0.67 & 0.33 & 0 \\ 0.33 & 1.34 & 0.33 \\ 0 & 0.33 & 0.67 \end{bmatrix} \begin{bmatrix} 120 \\ -60 \\ -60 \end{bmatrix}$$

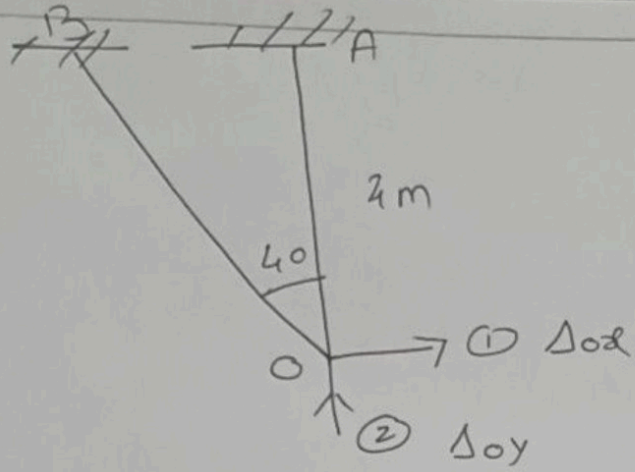
$$\theta_A = \frac{222.56}{EI}, \quad \theta_B = \frac{-88.23}{EI}, \quad \theta_C = \frac{-46.1}{EI}$$

Final Moment

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2\theta_A + \theta_B] = -1.03 \approx 0$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2 \left(\frac{-88.23}{EI} \right) + \left(\frac{222.56}{EI} \right) \right]$$

$$= 135.37 \text{ kNm}$$



$$[K][\Delta] = [P_s]$$

$$[P_s] = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

	$\frac{AB}{L}$	θ	$\frac{AE}{L} \cos^2 \theta$	$\frac{AE}{L} \sin^2 \theta$	$\frac{AE}{L} \cos \theta \sin \theta$
θ_A	100	90	0	100	0
θ_B	76.73	130	31.39	44.96	-37.57
			31.39	144.96	-137.57
			K_{11}	K_{22}	K_{33}

$$\begin{bmatrix} 31.39 & -37.57 \\ -37.57 & 144.16 \end{bmatrix} \begin{bmatrix} \Delta_{ox} \\ \Delta_{oy} \end{bmatrix} = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

Solving $\Delta_{ox} = -1.197$ $\Delta_{oy} = -1.0$

$$F_{0A} = -100 [-(1.197 \times 0) + (-1 \times 1)] = -100 \text{ kNm}$$

$$F_{0B} = -76.63 [(-1.19 \times -0.64) + (-1.0 \times 0.766)] = 0 \text{ kNm}$$

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