CBCS SCHEME

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18EE734

Seventh Semester B.E. Degree Examination, Feb./Mar. 2022
Advanced Control Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define the concept of i) State ii) State variable iii) State vector. List the advantages of advanced control theory over conventional control theory. **(10 Marks)**

b. Obtain the state model of the system by foster's form whose transfer function is :

$$T(s) = \frac{s^3 + 3s^2 + 2s}{s^3 + 12s^2 + 47s + 60}.$$ **(10 Marks)**

OR

2 a. Obtain the state model of the electrical networks shown in Fig.Q2(a), choosing $V_1(t)$ and $V_2(t)$ as state variable.

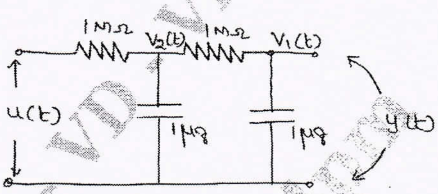

Fig.Q2(a) **(10 Marks)**

b. Derive the state model of the system by Jordan – Canonical form, described by the differential equation : $D^3y + 4D^2y + 5Dy + 2y = 2D^2u + 6Du + 5u$. **(10 Marks)**

Module-2

3 a. For the matrix 'A', obtain Eigen values, Eigen vectors and Modal matrix. Also prove that $M^{-1}AM$ forms a diagonal matrix.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}.$$ **(10 Marks)**

b. Find the state transition matrix for

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$, by using Cayley Hamilton theorem. **(10 Marks)**

OR

4 a. Obtain the complete time response of the system given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

Where u(t) is unit step input a t = 0 and $X^T(0) = [1 \quad 0]$. **(10 Marks)**

b. Determine the controllability and observability of the given system model by Kalman's test.

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t) \qquad y = [10 \;\; 5 \;\; 1] X(t).$$ **(10 Marks)**

Module-3

5 a. An observable system is described by $\dot{x}(t) = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u(t)$ $\quad y(t) = [0\ 0\ 1]\, X(t)$.

Design a state observer so that eigen values are at -4, $(-3 \pm j1)$. **(10 Marks)**

b. Consider the system defined by $\dot{x}(t) = Ax(t) + Bu(t)$ where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

By using state feedback control $u = -Kx$, it is desired to have the closed loop poles at $s = -2 \pm j4$ and $s = -10$. Determine the state feedback gain matrix 'k' by direct substitution method. **(10 Marks)**

OR

6 a. Determine the observer gain matrix for the state model $\dot{x} = Ax$, $y = cx$, described by $A = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$, $C = [1\ \ 0]$. The desired eigen values are at $\lambda_1 = -5$, $\lambda_2 = -5$. **(10 Marks)**

b. It is desired to place closed loop poles at $s = -3$ and $s = -4$ with control $u = -kx$. Determine state feedback gain matrix and control signal. $A = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = [1\ \ 0]$. **(10 Marks)**

Module-4

7 a. Explain the concept of i) Dead zone ii) Saturation iii) Non-Linear friction. **(10 Marks)**

b. Determine the kind of singularity for the differential equation given $\ddot{y} - 8\dot{y} + 17y = 34$ and draw the nature of phase Trajectory. **(10 Marks)**

OR

8 a. Discuss the procedure to draw phase Trajectory by using isoclines method for the given linear 2nd order servo described by the equation $\ddot{c} - 2\xi\omega_n \dot{c} + \omega_n^2 c = 0$, where $\xi = 0.15$, $\omega_n = 1$, $c(0) = 1.5$ and $\dot{c} = 0$. Determine singular point and construct phase trajectory. **(10 Marks)**

b. Define singular point. Explain different types of singular point with its phase Trajectory. **(10 Marks)**

Module-5

9 a. Show that the following quadratic form is positive definite
$V(x) = 8x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 4x_1x_3 - 2x_2x_3$. **(06 Marks)**

b. Determine the stability of the system described by $\dot{x} = Ax$; $A = \begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}$. **(10 Marks)**

c. Explain Asymptotic stability in large. **(04 Marks)**

OR

10 a. Use Krasovskii's theorem to show that the equilibrium state $x = 0$ of system described by $\dot{x}_1 = -3x_1 + x_2$, $\dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in large. **(10 Marks)**

b. Investigate the following non-linear system using direct method of Liapnov
$\dot{x}_1 = x_2$, $\dot{x}_2 = -x_1 - x_1^2 x_2$. **(10 Marks)**

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Solution of VTU Question Paper [Feb/Mar-2022]

Advanced Control Systems [18EE734]

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01.a. Define the concept of (i) State (ii) state variable (iii) State vector. List the advantages of advanced control theory over conventional control theory [10 marks]

Soln :- **State** :- State of a dynamic system is defined as a minimal set of variables such that the knowledge of these variables at $t = t_0$ together with the knowledge of the inputs for $t \geq t_0$ completely determines the behaviour of the system for $t > t_0$.

State variable :- The variables involved in determining the state of a dynamic system $X(t)$ are called the state variables. These are normally the energy storing elements of the system.

State Vector :- The 'n' state variables are necessary to discribe the complete behaviour of the system can be considered as 'n' components of a vector $X(t)$ called the state vector. at time t, $X(t)$ is the vector sum of all the state variables.

(*) Advantages of advanced control theory over conventional control theory.

01. The method takes into account the effect of all initial conditions.
02. It can be applied to nonlinear as well as time varying systems.
03. It can be conveniently applied to multiple input multiple output systems.
04. Any type of input can be considered for designing the system.
05. The variables selected need not necessarily be the physical quantities of the system.

Q1.b Obtain the state model of the system by foster's form whose transfer function is

$$T(s) = \frac{s^3+3s^2+2s}{s^3+12s^2+47s+60} \qquad \text{[10 marks]}$$

Soln:-

$$s^3+12s^2+47s+60 \overline{\big)\, s^3+3s^2+2s\,} \big(1$$
$$\underline{s^3+12s^2+47s+16}$$
$$-9s^2-45s-60$$

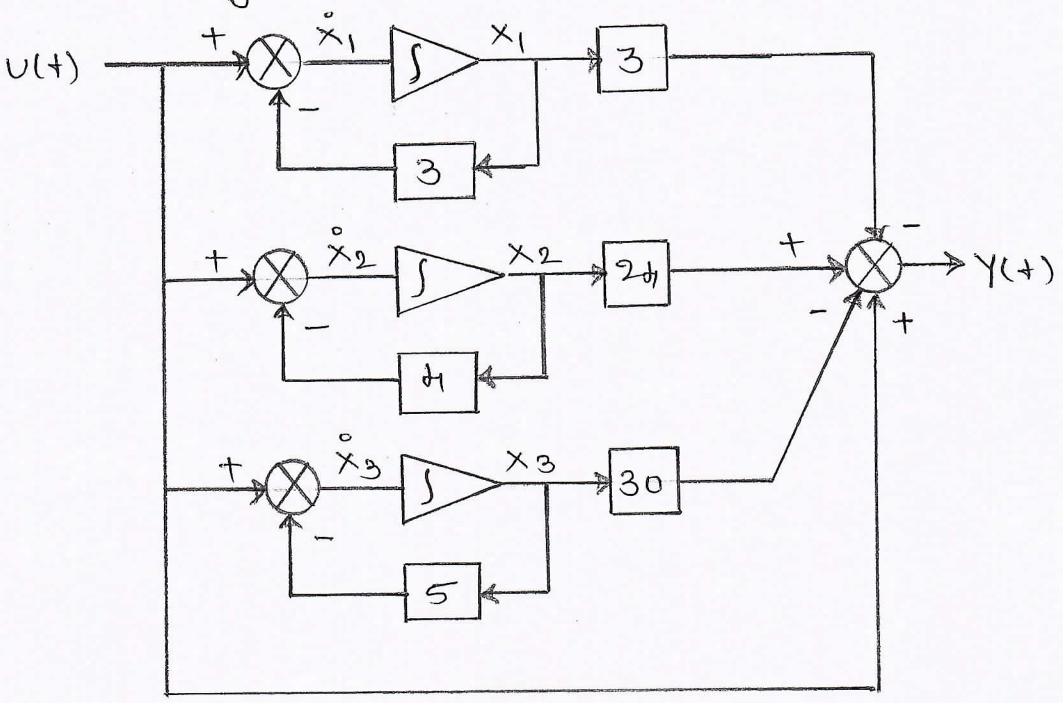
$$\therefore T(s) = 1 - \frac{9s^2+45s-60}{s^3+12s^2+47s+60}$$

$$= 1 - \left[\frac{9s^2+45s-60}{(s+3)(s+4)(s+5)}\right]$$

$$= 1 - \frac{A}{s+3} - \frac{B}{s+4} - \frac{60}{s+5}$$

$$T(s) = 1 - \frac{3}{s+3} + \frac{24}{s+4} - \frac{30}{s+5}$$

State diagram



$$\dot{x}_1 = u - 3x_1$$
$$\dot{x}_2 = u - 4x_2$$
$$\dot{x}_3 = u - 5x_3$$
$$y = -3x_1 + 24x_2 - 30x_3 + u.$$

State model

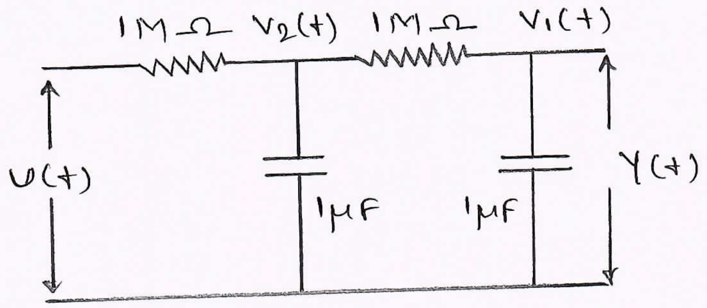
$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

where

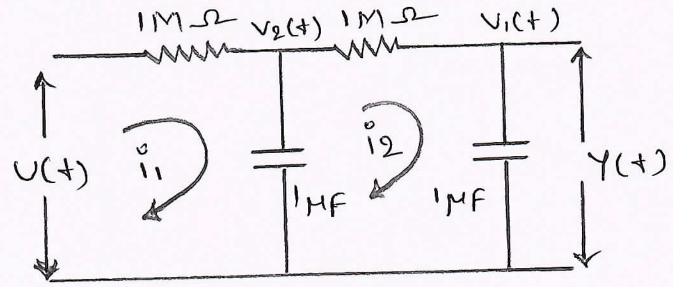
$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 24 & -30 \end{bmatrix} \qquad D = [1]$$

Q2.a. Obtain the state model of the electrical system shown, choosing $V_1(t)$ and $V_2(t)$ as state variables [10 marks]



Soln:- Select two currents as shown below



We can write

$$i_1(t) = \frac{U(t) - V_2(t)}{1\times10^6} \longrightarrow (1)$$

$$V_2(t) = \frac{1}{C}\int (i_1(t) - i_2(t))\, dt$$

i,e $$i_1 - i_2 = C\frac{dV_2(t)}{dt} \longrightarrow (2)$$

$$i_2(t) = \frac{V_2(t) - V_1(t)}{1\times10^6} \longrightarrow (3)$$

and $$V_1(t) = \frac{1}{C}\int i_2\, dt$$

$$\therefore\ i_2(t) = C\frac{dV_1(t)}{dt} \longrightarrow (4)$$

Elliminating i_1 and i_2 from above equation and $C = 1\times10^{-6}$ F

Substituting (1) and (3) in (2) we get

$$C\frac{dv_2}{dt} = \left[\frac{u(t)-v_2(t)}{1\times10^{6}}\right] - \left[\frac{v_2(t)-v_1(t)}{1\times10^{6}}\right]$$

$$\therefore \frac{dv_2}{dt} = \frac{1}{C}\left[\frac{u(t)}{1\times10^{6}}\right] + \frac{1}{C}\,\frac{v_1(t)}{1\times10^{6}} - \frac{1}{C}\left[\frac{2v_2(t)}{1\times10^{6}}\right]$$

$$\therefore \frac{dv_2}{dt} = v_1(t) - 2v_2(t) + u(t) \longrightarrow (5)$$

using equation (3) in equation (4)

$$C\frac{dv_1}{dt} = \frac{v_2 - v_1}{1\times10^{6}}$$

$$\therefore \frac{dv_1}{dt} = -v_1(t) + v_2(t)$$

Select $x_1(t) = v_1(t)$ and $x_2(t) = v_2(t)$

$$\dot{x}_1 = -x_1 + x_2$$

$$\dot{x}_2 = x_1 - 2x_2 + u(t)$$

and $y(t) = v_1(t) = x_1$

Hence the state model is

$$\dot{X} = AX + BU \quad \text{and} \quad Y = CX + DU$$

where

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = [0]$$

02.b. Derive the state model of the system by Jordan-Canonical form, described by the differential equation:

$$D^3y + 4D^2y + 5Dy + 2y = 2D^2u + 6Du + 5u$$

[10 marks]

Soln:- Take Laplace transform on both sides.

$$s^3Y(s) + 4s^2Y(s) + 5sY(s) + 2Y(s) = 2s^2U(s) + 6sU(s) + 5U(s)$$

Transfer function

$$\frac{Y(s)}{U(s)} = \frac{2s^2+6s+5}{s^3+4s^2+5s+2}$$

Factorise the denominator as.

$$\frac{Y(s)}{U(s)} = \frac{2s^2+6s+5}{(s+1)^2(s+2)} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$$

$$A(s+2) + B(s+1)(s+2) + C(s+1)^2 = 2s^2+6s+5$$

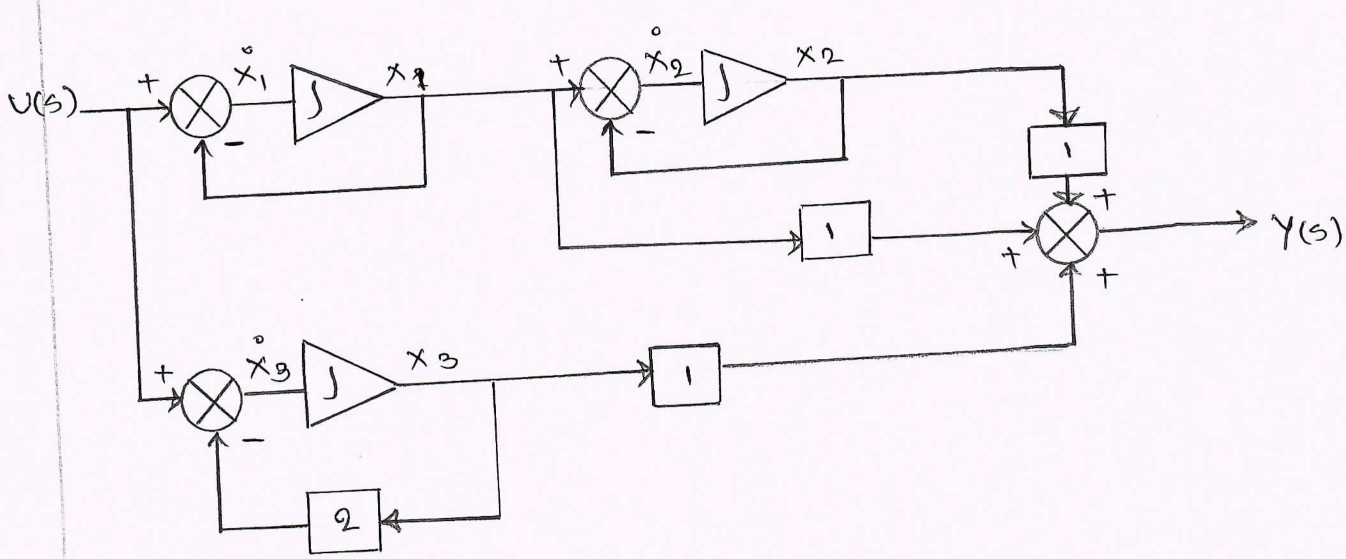
$$As + 2A + Bs^2 + 3Bs + 2B + Cs^2 + 2Cs + C = 2s^2+6s+5$$

$$B + C = 2, \quad A + 3B + 2C = 6, \quad 2A + 2B + C = 5$$

Solving $A = 1$, $B = 1$, $C = 1$

$$\therefore \frac{Y(s)}{U(s)} = \frac{1}{(s+1)^2} + \frac{1}{(s+1)} + \frac{1}{(s+2)}$$

State diagram.



So $\dot{x}_1 = u - x_1$, $\dot{x}_2 = x_1 - x_2$, $\dot{x}_3 = u - 2x_3$

and $y = x_1 + x_2 + x_3$.

State model

$$\dot{X} = AX + BU \quad \text{and} \quad Y = CX + DU$$

where

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad D = [0]$$

03. a. For the matrix 'A' obtain Eigen values, Eigen vectors and modal matrix. Also prove that $M^{-1}AM$ forms a diagonal matrix. [10 marks]

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}$$

Soln:-

$$|\lambda I - A| = \left| \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix} \right| = 0$$

$$= \begin{vmatrix} \lambda & -1 & 1 \\ 6 & \lambda+11 & -6 \\ 6 & 11 & \lambda-5 \end{vmatrix} = 0$$

$\therefore \lambda(\lambda+11)(\lambda-5) + 36 + 66 - 6(\lambda+11) + 6(\lambda-5) + 66\lambda = 0$

$\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$

Solving $\lambda_1 = -1$ $\lambda_2 = -2$ $\lambda_3 = -3$ Eigen values.

To find Eigen vectors,

For $\lambda_1 = -1$

$$[\lambda_1 I - A] = \begin{bmatrix} -1 & -1 & 1 \\ 6 & 10 & -6 \\ 6 & 11 & -6 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda_2 = -2$

$$[\lambda_2 I - A] = \begin{bmatrix} -2 & -1 & 1 \\ 6 & 9 & -6 \\ 6 & 11 & -7 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 3 \\ 6 \\ 12 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

For $\lambda_3 = -3$

$$[\lambda_3 I - A] = \begin{bmatrix} -3 & -1 & 1 \\ 6 & 8 & -6 \\ 6 & 11 & -8 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 2 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 9 \end{bmatrix}$$

Modal matrix $M = [M_1 \ M_2 \ M_3]$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 6 \\ 1 & 4 & 9 \end{bmatrix}$$

$$M^{-1} A M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 6 \\ 1 & 4 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 6 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Diagonal matrix

Q3.b Find the state transition matrix for

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$ by using Cayley Hamilton method.

[10 marks]

Soln:- The function to be evaluated is

$f(A) = e^{At}$ hence $p(\lambda) = e^{\lambda t}$

find the eigen values.

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 2 \\ 0 & \lambda-1 & 0 \\ -1 & 0 & \lambda-3 \end{vmatrix} = 0$$

$\therefore \lambda(\lambda-1)(\lambda-3) + 2(\lambda-1) = 0$

$(\lambda-1)(\lambda^2 - 3\lambda + 2) = 0$

$\lambda_1 = 1, \ \lambda_2 = 1 \quad \lambda_3 = 2$

$\lambda_1 = 1$ is repeated 2 times.

$$R(\lambda) = \alpha_0 + \alpha_1\lambda + \alpha_2\lambda^2 = P(\lambda) = e^{\lambda t}$$

for $\lambda_1 = 1$

$$\alpha_0 + \alpha_1 + \alpha_2 = e^t \longrightarrow (1)$$

for $\lambda_2 = 1$ as it is repeated use

$$\frac{d}{d\lambda} P(\lambda)\Big|_{\lambda=\lambda_2} = \frac{d}{d\lambda} R(\lambda)\Big|_{\lambda=\lambda_2}$$

i.e $$\frac{d}{d\lambda} e^{\lambda t}\Big|_{\lambda=\lambda_2} = \frac{d}{d\lambda}(\alpha_0 + \alpha_1\lambda + \alpha_2\lambda^2)\Big|_{\lambda=\lambda_2}$$

$$te^{\lambda t}\Big|_{\lambda=\lambda_2} = \alpha_1 + 2\alpha_2\lambda\Big|_{\lambda=\lambda_2}$$

$$\therefore \alpha_1 + 2\alpha_2 = te^t \longrightarrow (2)$$

for $\lambda_3 = 2$

$$\alpha_0 + 2\alpha_1 + 4\alpha_2 = e^{2t} \longrightarrow (3)$$

Solving equation (1), (2) and (3)

from (2) $\alpha_1 = te^t - 2\alpha_2$

from (1) $\alpha_0 = e^t - \alpha_1 - \alpha_2 = e^t - te^t + 2\alpha_2 - \alpha_2$

$= e^t(1-t) + \alpha_2$

using in (3)

$$e^t(1-t) + \alpha_2 + 2te^t - 4\alpha_2 + 4\alpha_2 = e^{2t}$$

$$\alpha_2 = e^{2t} - e^t + te^t - 2te^t = e^{2t} - e^t - te^t$$

$$\alpha_1 = te^t - 2e^{2t} + 2e^t + 2te^t = 3te^t + 2e^t - 2e^{2t}$$

and $\alpha_0 = e^t - te^t + e^{2t} - e^t - te^t = -2te^t + e^{2t}$

$$f(A) = R(A) = \alpha_0 I + \alpha_1 A + \alpha_2 A^2$$

$$\therefore f(A) = e^{At} = \alpha_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix} + \alpha_2 \begin{bmatrix} -2 & 0 & -6 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^t - e^{2t} & 0 & 2e^t - 2e^{2t} \\ 0 & e^t & 0 \\ -e^t + e^{2t} & 0 & 2e^{2t} - e^t \end{bmatrix}$$

This is required state transition matrix.

Q4. a. Obtain the complete time response of the system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

where $u(t)$ is unit step input at $t=0$ and $x^T(0) = [1 \quad 0]$. [10 marks]

Soln.:- From a given state model.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

State transition matrix $\phi(t)$ is given by

$$\phi(t) = e^{At} = L^{-1}[sI - A]^{-1}$$

$$(sI - A) = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$|sI - A| = (s-1)^2$$

$$Adj(sI - A) = \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{Adj(sI-A)}{|sI-A|}$$

$$= \begin{bmatrix} \frac{s-1}{(s-1)^2} & 0 \\ \frac{1}{(s-1)^2} & \frac{s-1}{(s-1)^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$e^{At} = L^{-1}[(sI - A)^{-1}] = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$ZIR = e^{At} X(0)$$

$$= \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

$$ZSR = L^{-1}\left[\phi(s)\, B\, U(s)\right]$$

$$\phi(s) = [sI - A]^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad U(s) = \begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$\therefore \phi(s)\, B\, U(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{(s-1)^2} + \frac{1}{(s-1)} \end{bmatrix} \begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s(s-1)} \\ \frac{1}{s}\left(\frac{1}{(s-1)^2} + \frac{1}{(s-1)}\right) \end{bmatrix}$$

Using partial fraction

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

$$A + B = 0$$

$$A = -1$$

$$B = 1$$

$$= \frac{-1}{s} + \frac{1}{s-1}$$

$$\frac{1}{s}\left[\frac{1+s-1}{(s-1)^2}\right] = \frac{1}{(s-1)^2}$$

$$\frac{1}{(s-1)^2} = \frac{A}{s-1} + \frac{B}{(s-1)^2}$$

$$A(s-1) + B = 1$$

$$A = 0$$

$$B = 1$$

$$= \frac{1}{(s-1)^2}$$

So $$\phi(s)\, B\, U(s) = \begin{bmatrix} -\frac{1}{s} + \frac{1}{s-1} \\ \frac{1}{(s-1)^2} \end{bmatrix}$$

$$L^{-1}\left[\phi(s)\,B\,U(s)\right] = \begin{bmatrix} -1+e^{t} \\ te^{t} \end{bmatrix} = ZSR$$

total response $x(t) = ZIR + ZSR$

$$= \begin{bmatrix} e^{t} \\ te^{t} \end{bmatrix} + \begin{bmatrix} -1+e^{t} \\ te^{t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -1+2e^{t} \\ 2te^{t} \end{bmatrix}$$

Q4.b. Determine the controllability and observability of the given system model by Kalman's test.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$Y = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix} x(t)$$ [10 marks]

Soln.:- From the given model

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}$$

for controllability. $Q_c = [B : AB : A^2B]$

$$AB = \begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix} \qquad A^2B = \begin{bmatrix} 1 \\ -12 \\ 61 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -12 \\ 1 & -12 & 61 \end{bmatrix}$$

$|Q_c| = -84 \neq 0$ hence rank of $Q_c = n = 3$

The system is completely controllable

For observability $Q_o = [C^T : A^T C^T : A^{T^2} C^T]$

$$A^T C^T = \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix} \qquad A^{T^2} C^T = \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 10 & -6 & 6 \\ 5 & -1 & 5 \\ 1 & -1 & 5 \end{bmatrix}$$

$|Q_o| = 96 \neq 0$ hence rank of $Q_o = n = 3$

The system is completely observable

Q5. a. An observable system is described by

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u(t), \quad y(t) = [0 \ 0 \ 1] \, x(t)$$

Design a state observer so that the eigen values are at $-4, (-3 \pm j1)$. [10 marks]

Soln:- It is given in problem statement, that system is Observable so there is no need to check for Observability

Let $K_e = \begin{bmatrix} K_{e1} \\ K_{e2} \\ K_{e3} \end{bmatrix}$ be observer gain matrix

The desired characteristic polynomial is

$$(s-(-3+j1))\,(s-(-3-j1))\,(s-(-4)) = [(s+3)^2+1]\,(s+4)$$

$$= (s^2+6s+10)(s+4) = s^3 + 6s^2 + 10s + 4s^2 + 24s + 40$$

$$= s^3 + 10s^2 + 34s + 40.$$

Now consider

$$|sI - (A + K_e C)| = \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \left\{ \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} + \begin{bmatrix} K_{e1} \\ K_{e2} \\ K_{e3} \end{bmatrix} [0 \ 0 \ 1] \right\} \right|$$

$$= \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \left\{ \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & Ke_1 \\ 0 & 0 & Ke_2 \\ 0 & 0 & Ke_3 \end{bmatrix} \right\} \right|$$

$$= \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 2 & -Ke_1 \\ 3 & -1 & 1-Ke_2 \\ 0 & 2 & -Ke_3 \end{bmatrix} \right|$$

$$= \begin{vmatrix} s-1 & -2 & Ke_1 \\ -3 & s+1 & -1+Ke_2 \\ 0 & -2 & s+Ke_3 \end{vmatrix}$$

$$= (s-1)\left((s+1)(s+Ke_3) - (-2)+(-1+Ke_2)\right] + 2\left[(-3)(s+Ke_3)\right]$$
$$+ Ke_1(6)$$

$$= s^3 + \left[Ke_3 + 1 - 1\right] s^2 + \left[Ke_3 - 2 + 2Ke_2 - Ke_3 - 1 - 6\right] s$$
$$+ \left[-Ke_3 + 2 - 2Ke_2 - 6Ke_3 + 6Ke_1\right]$$

Comparing the coefficients.

$Ke_1 = 25.1 \quad Ke_2 = 21.5 \quad Ke_3 = 10$

∴ Desired observer gain matrix Ke is

$$Ke = \begin{bmatrix} 25.1 \\ 21.5 \\ 10 \end{bmatrix}$$

05.b. Consider the system defined by $\dot{x}(t) = Ax(t) + Bu(t)$ where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. By using state feedback control $u = -Kx$, it is desired to have the closed loop poles at $s = -2 \pm j4$ and $s = -10$. Determine the state feedback gain matrix 'K' by direct substitution method. [10 marks]

Soln.:- First check for controllability

$$Q_c = [B : AB : A^2B]$$

$$AB = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix} \qquad A^2B = \begin{bmatrix} 1 \\ -6 \\ 31 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

$|Q_c| = -1$

rank of $Q_c = n = 3$

So system is controllable.

The desired characteristic equation is

$= (s-(-10))\,(s-(-2+j4))\,(s-(-2-j4))$

$= (s+10)\,(s+2-j4)\,(s+2+j4)$

$= (s+10)\,((s+2)^2+16) = (s+10)\,(s^2+4s+20)$

$= s^3+4s^2+20s+10s^2+40s+200$

$= s^3+14s^2+60s+200.$

Let the desired state feedback gain matrix K be

$K = [K_1, K_2, K_3]$

$$|sI - A + BK| = \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [K_1 \; K_2 \; K_3] \right|$$

$$= \left| \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 5 & s+6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_1 & K_2 & K_3 \end{bmatrix} \right|$$

$$= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1+K_1 & 5+K_2 & s+6+K_3 \end{vmatrix}$$

$$= s^3 + (6+K_3)s^2 + (5+K_2)s + (1+K_1)$$

Comparing with desired characteristic equation

$6+K_3 = 14 \Rightarrow K_3 = 8$

$5+K_2 = 60 \Rightarrow K_2 = 55$

$1+K_1 = 200 \Rightarrow K_1 = 199$

$\therefore K = [8 \quad 55 \quad 199]$

Q6. a. Determine the observer gain matrix for the state model $\dot{X} = AX$, $Y = CX$, described by

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

. The desired eigen values are at $\lambda_1 = -5$, $\lambda_2 = -5$. [10 marks]

Soln:- Observability test

$$Q_c = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$|Q_c| = 1 \neq 0$ rank of $Q_c = 2 = n$

So the system is completely state controllable

The desired characteristic equation is

$$(s-(-5))(s-(-5)) = (s+5)(s+5) = s^2 + 10s + 25$$

Let observer gain matrix $K_e = \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix}$

$$|sI - A + K_e C| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} K_{e1} & 0 \\ K_{e2} & 0 \end{bmatrix} \right|$$

$$= \begin{vmatrix} s+1+K_{e1} & -1 \\ -1+K_{e2} & s-2 \end{vmatrix}$$

$$= (s+1+K_{e1})(s-2) - (-1)(-1+K_{e2})$$

$$= s^2 + s + K_{e1}s - 2s - 2 - 2K_{e1} - 1 + K_{e2}$$

$$= s^2 + (3+K_{e1})s + (-3-2K_{e1}+K_{e2})$$

Comparing the coefficients with desired characteristic equation

$$3 + K_{e1} = 10 \Rightarrow K_{e1} = 7$$

$$-3 + 2K_{e1} + K_{e2} = 25 \Rightarrow K_{e2} = 14$$

The observer gain matrix K_e is given as

$$K_e = \begin{bmatrix} 7 \\ 14 \end{bmatrix}$$

06.b. It is desired to place closed loop poles at $s = -3$ and $s = -4$ with control $u = -Kx$. Determine state feedback gain matrix and control signal

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$ [10 marks]

Soln.- Controllability test

$$Q_c = [B \quad AB]$$

$$AB = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}$$ $|Q_c| = -4 \neq 0$ rank of $Q_c = 2$

so system is completely controllable

The given equation is not in the controllable canonical form, because B is other than $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. so use method of transformation matrix T to get state feedback gain matrix.

$$T = Q_c W$$

$$|sI - A| = \begin{vmatrix} s & -1 \\ 1 & s+3 \end{vmatrix} = s^2 + 3s + 1 \qquad a_1 = 3 \quad a_2 = 1$$

$$Q_c = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix} \qquad W = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$T = Q_c W = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Desired characteristic equation is

$$(s+3)(s+4) = s^2 + 7s + 12 \qquad \delta_1 = 7 \quad \delta_2 = 12$$

we have $K = [\delta_2 - a_2 \quad \delta_1 - a_1]\, T^{-1}$

$$= [12 - 1 \quad 7 - 3] \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1}$$

$$= [11 \quad 4] \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$K = [6.5 \quad 2]$$

Control signal $U = -Kx = -\begin{bmatrix} 6.5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

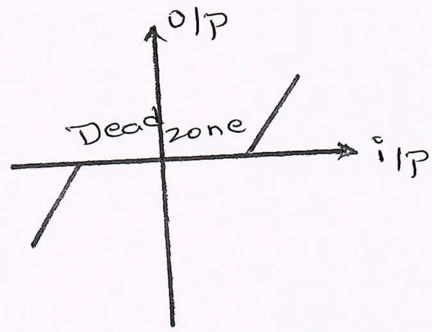
$$U = -6.5x_1 - 2x_2$$

07.a. Explain the concept of (i) Dead zone (ii) Saturation (iii) Non-linear friction. [10 marks]

Soln.:- (i) Dead zone

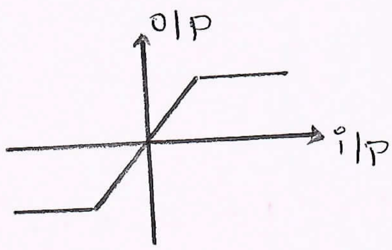
Some systems donot respond to very small input signals. For a particular range of input, the output is zero. This is called dead zone.

Sensors, amplifiers shows this type of nonlinearity.



(ii) Saturation.

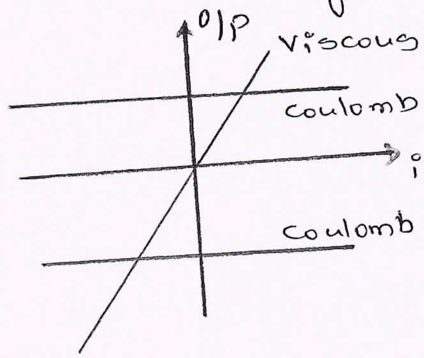
For small inputs, the output varies linearily with input. And when input exceeds a particular limit the output becomes constant



Output of electronic amplifiers, Electrical and hydraulic motors and transformers show this nonlinearity

(iii) Non linear friction.

All friction are nonlinear, except viscous friction. The nonlinear friction is called coulomb friction. The coulomb friction is a force acting in opposite direction of motion but it is constant in magnitude irrespective of velocity.



The friction existing between the brushes resting against the commutator in an DC machine is an example for nonlinear friction.

07.b Determine the kind of singularity for the differential equation given $\ddot{y} - 8\dot{y} + 17y = 34$ and draw the nature of phase trajectory. [10 marks]

Soln.:- $\ddot{y} - 8\dot{y} + 17y - 34 = 0$

The above equation can be rewritten as

$$\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 17y - 34 = 0$$

$$\frac{d^2(y-2)}{dt^2} - 8\frac{d}{dt}(y-2) + 17(y-2) = 0$$

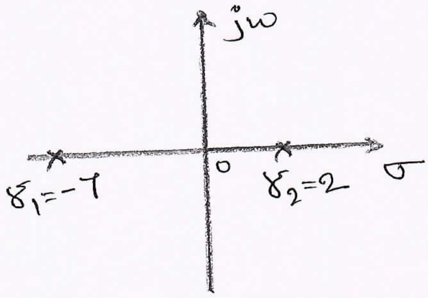
The characteristic equation is

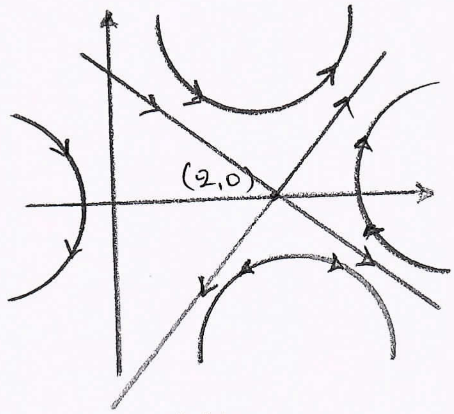
$$(s-2)^2 - 8(s-2) + 17(s-2) = 0$$

$$(s^2 - 4s + 4) - 8s + 16 + 17s - 34 = 0$$

$$s^2 + 5s - 14 = 0$$

$$r_1 = -7, \quad r_2 = 2$$





saddle point

08.a Discuss the procedure to draw phase trajectory by using isoclines method for the given linear 2nd order servo described by the equation $\ddot{c} - 2\xi\omega_n\dot{c} + \omega_n^2 c = 0$ where $\xi = 0.15$, $\omega_n = 1$, $c(0) = 1.5$ and $\dot{c} = 0$. Determine singular point and construct phase trajectory.

[10 marks]

Soln:- Consider the equation

$$\ddot{c} - 2\xi\omega_n\dot{c} + \omega_n^2 c = 0$$

substitute $\xi = 0.15$, $\omega_n = 1$

$$\ddot{c} - 0.3\dot{c} + c = 0.$$

taking laplace transform

$$s^2 C(s) - 0.3 s\, C(s) + C(s) = 0$$

$$s^2 - 0.3s + 1 = 0$$

Solving $s_1, s_2 = 0.15 \pm j0.9886$.

The roots are complex conjugate and located in right half of s plane. The singular point is unstable focus.

We have $\frac{d^2c(t)}{dt^2} - 0.3\frac{d}{dt}c(t) + c(t) = 0$

let $x_1 = c(t)$ and $x_2 = \dot{c}(t)$

$\therefore \dot{x}_1 = \dot{c}(t) = x_2$

$\dot{x}_2 = \ddot{c}(t) = -c(t) + 0.3\dot{c}(t)$

$= -x_1 + 0.3x_2$

$$m = \frac{dx_2}{dx_1} = \frac{-x_1 + 0.3x_2}{x_2}$$

$m x_2 = -x_1 + 0.3x_2$

$(m - 0.3)x_2 = -x_1$

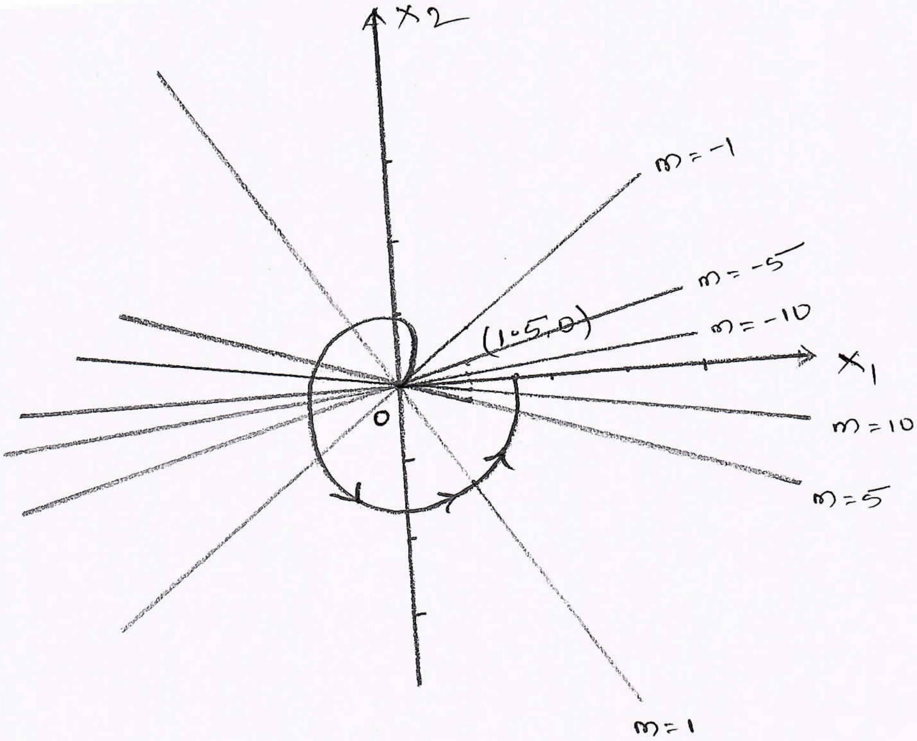
$x_2 = \frac{-x_1}{m - 0.3}$ equation of isocline.

Initial point $(1.5, 0)$

$$0 = \frac{-1.5}{m - 0.3}$$

$m = 0.3 - \infty = -\infty$ trajectory moves downwards.

m	Isocline equation	$\theta = \tan^{-1} m$
1	$x_2 = -1.42x_1$	45°
5	$x_2 = -0.21x_1$	78.69°
10	$x_2 = -0.10x_1$	84.28°
∞	$x_2 = 0$	90°
-1	$x_2 = 0.77x_1$	-45°
-5	$x_2 = 0.18x_1$	-78.69°
-10	$x_2 = 0.09x_1$	-84.28°
$-\infty$	$x_2 = 0$	-90°



08.b Define singular point. Explain different types of singular points with its phase trajectory. [10 marks]

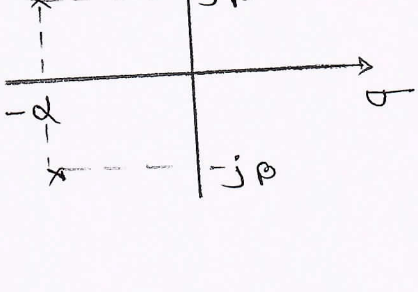
Sol^n:- Singular points are points in the state plane where $\dot{x}_1 = \dot{x}_2 = 0$. At these points the slope of the trajectory $\frac{dx_2}{dx_1}$ is indeterminate. These points can also be the equilibrium points of the nonlinear system depending wether the state trajectories can reach these or not.

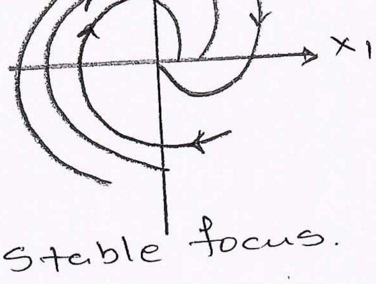
Singular points are roots of characteristic equation $s^2+2\xi\omega_n s+\omega_n^2 = (s-\gamma_1)(s-\gamma_2)=0$ Depending on the values of γ_1 and γ_2, six different types of singular points are obtained.

01. Stable system with complex roots

$\gamma_1 = -\alpha + j\beta \quad \gamma_2 = -\alpha - j\beta$

Output response $c(t) = c_1 e^{-\alpha t} \sin(\beta t + c_2)$



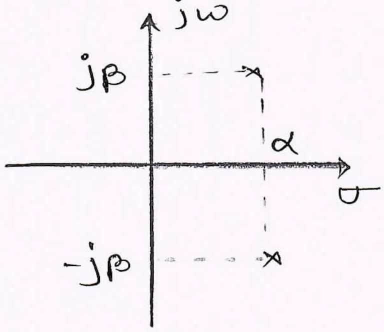


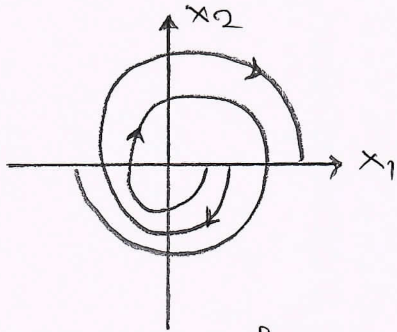
Stable focus.

02. Unstable system with complex roots.

$r_1 = \alpha + j\beta$, $r_2 = \alpha - j\beta$

Output response is $c(t) = c_1 e^{\alpha t} \sin(\beta t + c_2)$



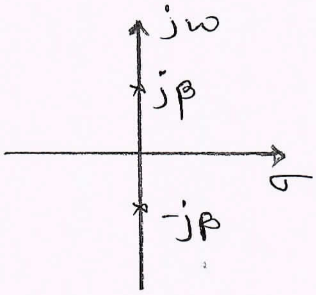


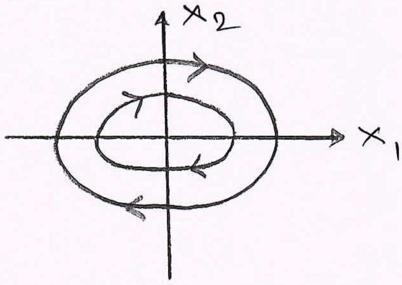
Unstable focus.

03. Marginally stable system with complex roots.

$r_1 = j\beta$ $r_2 = -j\beta$

Output response is $c(t) = c_1 \sin(\beta t + c_2)$



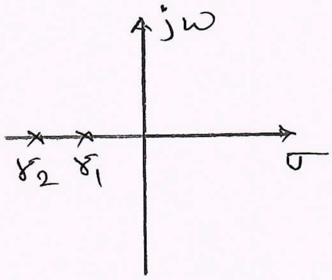


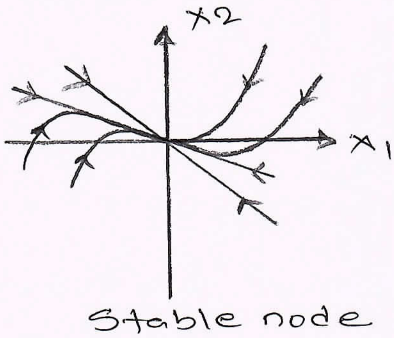
Vertex.

04. Stable system with real roots.

r_1 and r_2 are real and distinct roots located on left half of splane

Output response $c(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$



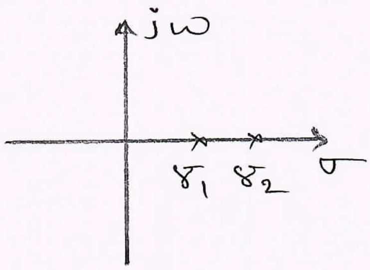


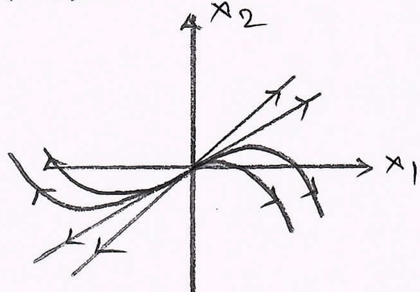
Stable node

05. Unstable system with real roots.

r_1 and r_2 are real and distinct roots located on right half of splane

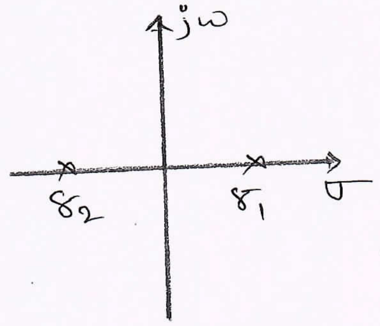
Output response $c(t) = c_1 e^{-r_1 t} + c_2 e^{-r_2 t}$

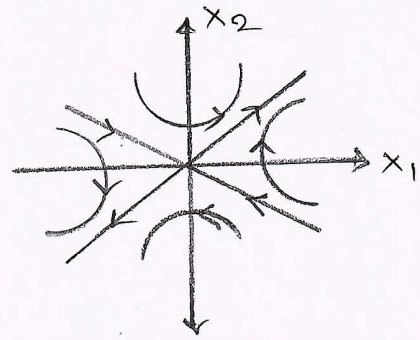




Unstable node

06. Unstable system with one positive and one negative real roots.





Saddle point.

09.a. Show that the following quadratic form is positive definite

$$V(x) = 8x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 4x_1x_3 - 2x_2x_3$$

[06 marks]

Sol^n.- $V(x) = 8x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 4x_1x_3 - 2x_2x_3$

Can be written as.

$$V(x) = x^T P x = [x_1 \; x_2 \; x_3] \begin{bmatrix} 8 & 1 & -2 \\ 1 & 1 & -1 \\ -2 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Applying sylvester's criterion

$$8 > 0$$

$$\begin{vmatrix} 8 & 1 \\ 1 & 1 \end{vmatrix} = 7 > 0$$

$$\begin{vmatrix} 8 & 1 & -2 \\ 1 & 1 & -1 \\ -2 & -1 & 4 \end{vmatrix} = 20 > 0$$

As all the successive principal minors of the matrix P are positive, $V(x)$ is positive definite.

09.b Determine the stability of the system described by

$$\dot{x} = Ax \; ; \; A = \begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}.$$

[10 marks]

Sol^n.- We have

$$A^T P + PA = -Q$$

let $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -P_{11} - 2P_{21} & -P_{12} - 4P_{22} \\ P_{11} - 4P_{21} & P_{12} - 4P_{22} \end{bmatrix} + \begin{bmatrix} -P_{11} - 2P_{12} & P_{11} - 2P_{22} \\ -P_{21} - 2P_{22} & P_{21} - 4P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2P_{11} - 4P_{12} & P_{11} - 5P_{12} - 2P_{22} \\ P_{11} - 5P_{12} - 2P_{22} & 2P_{12} - 5P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$-2P_{11} - 4P_{12} = -1$$

$$P_{11} - 5P_{12} - 2P_{22} = 0$$

$$2P_{12} - 5P_{22} = -1$$

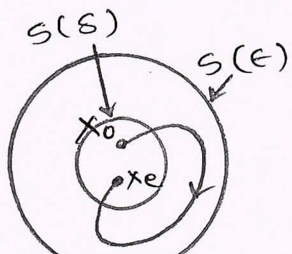
Solving $P_{11} = 37/78$ $P_{12} = P_{21} = 1/78$ $P_{22} = 16/78$

$$P = \begin{bmatrix} 37/78 & 1/78 \\ 1/78 & 16/78 \end{bmatrix}$$

P is positive definite. So system under consideration is asymptotically stable in the large.

09.c. Explain Asymptotic stability in large. [04 marks]

Soln.- An equilibrium state Xe of the system is said to be asymptotically stable if it is stable in the sense of Liapunov and if every solution starting within S(δ) converges without leaving S(ε) to Xe as 't' increases indefinitely.



If asymptotic stability holds for all states then the equilibrium state is said to be asymptotically stable in the large.

10.a Use Krasovskii's theorem to show that the equilibrium state $x=0$ of system described by $\dot{x}_1 = -3x_1 + x_2$, $\dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in large

[10 marks]

Soln.- We have $f_1(x) = \dot{x}_1 = -3x_1 + x_2$

$$f_2(x) = \dot{x}_2 = x_1 - x_2 - x_2^3$$

Jacobian matrix is given by

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & -1-3x_2^2 \end{bmatrix}$$

let $Q = J^T P + P J$

let $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ positive definite

$$\text{so } Q = \begin{bmatrix} -3 & 1 \\ 1 & -1-3x_2^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 1 & -1-3x_2^2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 2 \\ 2 & -2-6x_2^2 \end{bmatrix}$$

For system to be asymptotically stable Q must be negative definite or $-Q$ must be positive definite.

$$\therefore -Q = \begin{bmatrix} 6 & -2 \\ -2 & 2+6x_2^2 \end{bmatrix}$$

$6 > 0$

$|Q| = 12 + 36x_2^2 - 4 = 36x_2^2 + 8 > 0$

So $-Q$ is positive definite. Hence Q is negative definite. Thus the given system is asymptotically stable in large

10.b Investigate the following non-linear system using direct method of Liapnov

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 - x_1^2 x_2$$ [10 marks]

Soln.-

$f(0,t) \Rightarrow$ $0 = x_2$

$0 = -x_1 - x_1^2 x_2$

So $x_1 = 0$ $x_2 = 0$ is equilibrium point

Let Liapunov function be

$V = x_1^2 + x_2^2$ Positive definite

$$\dot{V} = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$$

$$= 2x_1x_2 + 2x_2(-x_1 - x_1^2x_2)$$

$$= 2x_1x_2 - 2x_1x_2 - 2x_1^2x_2^2$$

$= -2x_1^2x_2^2$ it is negative definite

∴ Origin of the system is asymptotically stable

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