CBCS SCHEME

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18CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Verify that, for any three propositions p, q, r the compound proposition. $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology or not. **(06 Marks)**

b. Test for validity of following argument.
If Ravi goes out with friends, he will not study
If Ravi do not study, his father becomes angry
His father is not angry
$\therefore$ Ravi has not gone out with friends **(07 Marks)**

c. Give direct and indirect proof of following statement "Product of two odd integers is an odd integer". **(07 Marks)**

OR

2 a. For any three propositions p, q, r, prove that $[\sim p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

b. Check for validity of following argument,
If a triangle has two equal sides then it is isosceles. If a triangle is isocelcs than it has two equal angles.
A certain triangle ABC does not have two equal angles
$\therefore$ The triangle ABC does not have two usual sides **(07 Marks)**

c. Consider the following open statement on set of all real numbers as universe:
$p(x) : x \geq 0$ $q(x) : x^2 \geq 0$ $r(x) : x^2 - 3x - 4 = 0$ $s(x) : x^2 - 3 > 0$
Then find truth value of i) $\exists x\ p(x) \wedge q(x)$ ii) $\forall x,\ p(x) \rightarrow q(x)$ iii) $\forall x,\ q(x) \rightarrow s(x)$ iv) $\forall x,\ r(x) \vee s(x)$ **(07 Marks)**

Module-2

3 a. By mathematical induction prove that
$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{2}\, n\,(2n-1)\,(2n+1)$$ **(06 Marks)**

b. Find coefficient of i) x° in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$
ii) $x^{11}\, y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ **(07 Marks)**

c. A total amount of Rs.1500 is to be distributed to three students A, B, C. In how many ways distribution can be done in the multiples of Rs.100 if
i) Every students sets at least Rs.300
ii) A must get at least Rs.500, B and C must set at least Rs.400 each. **(07 Marks)**

OR

4 a. By mathematical induction prove that for any positive integer n the number $11^{n+2} + 12^{2n+1}$ is divisible by 133 **(06 Marks)**

b. How many positive integers n can be formed from the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000. **(07 Marks)**

c. A certain question paper has 3 parts A, B, C with four questions in Part A, Five in B and Six in C. It is required to answer seven questions by selecting at least two from each part. In how many different ways student can answer seven questions. **(07 Marks)**

Module-3

5 a. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$ and f be a function from A to B defined by $f = \{(1, 7)\ (2, 7), (3, 8)\ (4, 6)\ (5, 9)\ (6, 9)\}$. Then find $f^{-1}(6)$, $f^{-1}(9)$. If $B_1 = \{7, 8\}$, $B_2 = \{8, 9, 10)$ find $f^{-1}(B_1)$, $f^{-1}(B_2)$. **(06 Marks)**

b. Let $A = \{1, 2, 3, 4\}$ and R be a relation on A defined by xRy if and only if x divides y. Then i) Write R as ordered pairs ii) Draw diagram iii) Write matrix of R. **(07 Marks)**

c. If f, g, h are functions from R to R defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2+2}$. Then verify that $f o(goh) = (fog)\ oh$ **(07 Marks)**

OR

6 a. If 30 dictionaries in a library contain total 61,327 pages then prove that at least one of the dictionaries must have at least 2045 pages. **(06 Marks)**

b. For any three nonempty sets A, B, C prove that

i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

ii) $AX\ (B \cap C) = (A \times B) \cap (A \times C)$ **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ define a partial order R on A by xRy if and only if x divides y. Draw Hasse diagram of R. **(07 Marks)**

Module-4

7 a. For the integers 1, 2, …n, there are 11660 derangements where 1, 2, 3, 4, 5 appear in first five positions then find value of n. **(06 Marks)**

b. Determine number of integers between 1 and 300 which are i) divisible by exactly two of 5, 6, 8 ii) at least two of 5, 6, 8. **(07 Marks)**

c. Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given $a_6 = 1$, $a_1 = 2$ **(07 Marks)**

OR

8 a. Out of 30 students of a hostel 15 study history, 8 study economics, 6 study geography and 3 study all the three subjects. Show that 7 or more study none of the subjects. **(06 Marks)**

b. An apple, a banana, a mango, and an orange to be distributed to 4 boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish apple, B_3 does not want banana or mango B_1 refuses orange. In how many ways distribution can be made so that all of them are happy. **(07 Marks)**

c. Solve $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given $a_0 = 2$. **(07 Marks)**

Module-5

9 a. Show that following graphs in the Fig.Q.9(a)(i) and Fig.Q.9(a)(ii) are isomorphic

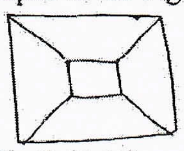

Fig.Q.9(a)(i)

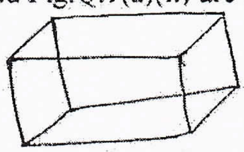

Fig.Q.9(a)(ii)

(06 Marks)

b. Define with an example to each i) Complement of a graph ii) Vertex degree iii) Rooted tree iv) Prefix code **(07 Marks)**

c. Apply merge sort to the list
-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3 **(07 Marks)**

OR

10 a. Prove that a tree with n vertices has $(n - 1)$ edges. **(06 Marks)**

b. Determine number of vertices in following graph G:

i) G has 9 edges and all vertices have degree 3

ii) G has 10 edges with 2 vertices of degree 4 and all other have degree 3 **(07 Marks)**

c. Obtain optimal prefix code for the message ROAD IS GOOD. **(07 Marks)**

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THIRD SEMESTER B.E DEGREE EXAMINATION FEB 202

DISCRETE MATHEMATICAL STRUCTURES
(18CS36)

Time: 3 hrs. MAX. Marks: 10

Solutions/Answers

Module - 1.

1a) $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$p \to (q \to r)$ A	$(p \to q) \to (p \to r)$ B	$A \to$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

From the truth table, we can see that the given proposition is always True for all possible values. ∴ It is tautology.

1b) Let p : Ravi goes out with friends.
q : Ravi will study.
r : Ravi's father becomes angry.

Given argument is

$$\begin{array}{l} p \to \neg q \\ \sim q \to r \\ \sim r \\ \hline \therefore \sim p \end{array} \Rightarrow \begin{array}{l} p \to r \\ \sim r \\ \hline \therefore \sim p \end{array}$$

∵ Rule of Syllogism

∵ Modus Tollens Rule.

∴ This is a valid argument.

1C) Given a statement is,

"If x is odd and y is odd then xy is odd"

Let P : x is odd

q : y is odd

r : xy is odd.

Given statement in symbolic form : $(P \wedge q) \rightarrow r$

Direct Proof : Let $P \wedge q$ be true.

$\Rightarrow$ p is true and q is true.

$\Rightarrow$ x is odd & y is odd

$\Rightarrow$ $x = 2k+1$ & $y = 2l+1$ $\quad k, l \in \mathbb{Z}$.

$\Rightarrow$ $xy = (2k+1)(2l+1)$

$= 4kl + 2k + 2l + 1$

$= 2(2kl + k + l) + 1$

$= 2m + 1$ where $m = 2kl + k + l \in \mathbb{Z}$.

$\therefore$ xy is odd.

Indirect Proof :

We know that

$(P \wedge q) \rightarrow r \Leftrightarrow -r \rightarrow \sim(P \wedge q)$

Let $\neg r$ be true $\Rightarrow$ xy is not odd.

$\Rightarrow$ xy is even.

$\Rightarrow$ x is even and y is odd is true and.

$\Rightarrow$ $-P$ is true & q is true

$\Rightarrow$ $-(P \wedge q)$ is true.

or x is odd & y is even

P is true & $\neg q$ is true

$\Rightarrow$ $\neg P \vee \neg q$ is true.

or x is even & y is even

$\neg P$ is true

$\neg q$ is true

$\sim q \vee \sim q$ is true.

$\Rightarrow$ $\sim(P \wedge q)$ is true.

$\therefore$ $\sim r \rightarrow \sim(P \wedge q)$ is true.

So by $(P \wedge q) \rightarrow r$ is true.

2a> Consider

$LHS = [\sim P \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

First consider.

$\Leftrightarrow -p \wedge (-q \wedge r) \Leftrightarrow (-p \wedge -q) \wedge r)$ Associative law

$\Leftrightarrow [\neg(p \vee q)] \wedge r \Leftrightarrow \boxed{r \wedge [\neg(p \vee q)]}$ Demorgan's law & Commutative law

and.

$(q \wedge r) \vee (p \wedge r) \Leftrightarrow (r \wedge q) \vee (r \wedge p)$ commutative law

$\Leftrightarrow r \wedge (q \vee p)$ Distributive law

$\Leftrightarrow \boxed{r \wedge (p \vee q)}$ Commutative law

$\therefore \{r \wedge (\sim p \vee q) \vee (r \wedge (p \vee q)\}$

$\Leftrightarrow r \wedge \{[\neg(p \vee q)] \vee (p \vee q)\}$ Distributive law.

$\Leftrightarrow r \wedge T_0$ Inverse law.

$\Leftrightarrow r.$ //.

2b. Let $P(x)$: x has two equal sides.

$q(x)$: x is isosceles.

$r(x)$: x has two equal angles.

a : triangle ABC.

Given $\forall x,\ P(x) \rightarrow q(x)$

$\forall x,\ q(x) \rightarrow r(x)$.

$\sim r(a)$

$\therefore \sim P(a)$

$\Rightarrow$ Universal Specification

$P(a) \rightarrow q(a)$

$q(a) \rightarrow r(a)$

$\sim r(a)$

$\therefore \sim P(a)$

$P(a) \rightarrow q(a)$

$q(a) \rightarrow r(a)$

$\therefore P(a) \rightarrow r(a)$ $\because$ law of Syllogism.

$\sim r(a)$

$\therefore \sim P(a)$ This is valid argument. in view of Modus Tollens //

2c)

i) $\exists x, P(x) \wedge q(x)$.

we know that, there exists a real no $x=1$, for which both $P(x)$ and $q(x)$ are true.

$\exists x, P(x) \wedge q(x)$ is a true statement.

Its truth value is 1.

ii) $\forall x, P(x) \rightarrow q(x)$.

for every real no x, $q(x)$ is true.

$\therefore \forall x, P(x) \rightarrow q(x)$ is true

$\therefore$ Its truth value is 1.

iii) $\forall x, q(x) \rightarrow s(x)$

WKT, $s(x)$ is false, and
$q(x)$ is true for $x=1$

Thus $\forall x, q(x) \rightarrow s(x)$ is false.

$\therefore$ It's truth value is 0

iv) $\forall x, r(x) \vee s(x)$

$r(x)$ is true only for $x=4$ & $x=-1$

$r(x)$ and $s(x)$ are false for $x=1$

Thus, $r(x) \vee s(x)$ is not always true.

$\therefore \forall x, r(x) \vee s(x)$ is false.

$\therefore$ It's truth value is 0

3a) Let $S(n) = 1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$

Basis step: we know that,

$S(1): 1^2 = \frac{1}{3} \times 1 \times 3.$

$1 = 1$ which is true.

Inductive step: we assume that $S(n)$ is true for $n=k$, where $k \geq 1$

then, $1^2 + 3^2 + 5^2 + \cdots + (2k-1)^2 = \frac{1}{3} k(2k-1)(2k+1)$

Adding $(2k+1)^2$ on both sides,

$1^2+3^2+5^2+\cdots+(2k-1)^2+(2k+1)^2=\frac{1}{3}k(2k+1)(2k+1)$

$= \frac{1}{3}(2k+1)[k(2k-1)+3(2k+1)]$

$= \frac{1}{3}(2k+1)[2k^2-k+6k+3]$

$= \frac{1}{3}(2k+1)(2k^2+5k+3)$

$= \frac{1}{3}(2k+1)(k+1)(2k+3)$

This stmt is true for $S(k+1)$.

Thus $S(k+1)$ is true for whenever $S(k)$ is true where $k\geq 1$. Hence By Mathematical Induction, $S(n)$ is true for $\forall n\geq 1$.

3b) i) x^0 in the expansion of $(3x^2-\frac{2}{x})^{15}$

ii) $x^{11}y^4$ in the expansion of $(2x^3-3xy^2+z^2)^6$

i) $(3x^2-\frac{2}{x})^{15} = \sum_{r=0}^{15}\binom{15}{r}(3x^2)^r.(-\frac{2}{x})^{15-r}$

$= \sum_{r=0}^{15}\binom{15}{r}3^r.(-2)^{15-r}.x^{2r}.(\frac{1}{x})^{15-r}$

$= \sum_{r=0}^{15}\binom{15}{r}3^r.(-2)^{15-r}.x^{2r}.x^{r-15}$

$= \sum_{r=0}^{15}\binom{15}{r}.3^r.(-2)^{15-r}.x^{3r-15}$

Taking $r=5$ in the above expansion, the co-efficient of x^0 is $= \binom{15}{5}3^5.(-2)^{10}$

$= 15C_5 . 3^5 (-2)^{10}$

$= 74,72,42,496.$

ii) The general term in the expansion of $(2x^3-3xy^2+z^2)^6$ is

$= \binom{6}{n_1\, n_2\, n_3}(2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$

$= \binom{6}{n_1\, n_2\, n_3}2^{n_1}(-3)^{n_2}.x^{3n_1}.x^{n_2}.y^{2n_2}.z^{2n_3}$

For $n_3 = 0$, $n_2 = 2$, $n_1 = 3$ we have

$$\binom{6}{3,2,0} \; 2^3 . (-3)^2 . x^{11} y^4$$

$\therefore$ Co-eff of $x^{11} y^4$ is $2^3 . (-3)^2 . \binom{6}{3,2,0}$

$$= 8 \times 9 \times \frac{6!}{3! . 2! \, 0!}$$

$$= 4{,}320 \; //$$

3c> There are 15 objects. (15 hundred Rs notes), to be distributed among 3 students A, B, C.

i> Every student gets atleast Rs. 300.

Distribute Rs. 300 to every student. $(300 \times 3) = 900$

Remaining '6' notes should be distributed among 3, students.

$r = 6$, $n = 3$.

This can be done in $^{n+r-1}C_r$ ways.

$$= {}^{3+6-1}C_6$$

$$= {}^{8}C_6 \text{ ways.}$$

$$= \frac{8!}{6! . (8-6)!}$$

$$= \frac{\cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!} \; 2!}$$

$$= 28 \; //$$

ii>

	A	B	C
a)	500	400	600
b)	500	500	500
c)	500	600	400
d)	600	400	500
e)	600	500	400
f)	700	500	400

6 - ways.

By Direct method.

By using combination with Repitition,

Distribute Rs 500 to A, Rs 400 to B, C each.

Remaining 2 notes of 100 should be distributed among 3-students A, B, C.

$r = 2, \quad n = 3.$

No of ways of distributing $= {}^{n+r-1}C_r$

$= {}^{3+2-1}C_2 = {}^4C_2$

$= \frac{4!}{2! \times 2!}$

$= \frac{4^{2} \times 3 \times \cancel{2!}}{\cancel{2!} \times 2}$

$= 6$ ways //.

4a) We know that, $11^{n+2} + 12^{2n+1}$

$A_1 = 11^{1+2} + 12^{2+1}$

$= 11^3 + 12^3$

$= 1331 + 1728 = 3059$

Thus, An is divisible by 133 for $n=1$

Induction step:

Assume that An is divisible by 133, for $n = k \geq 1$

Now we find that,

$A_{k+1} = 11^{k+3} + 12^{2(k+1)+1}$

$= (11^{k+2} \times 11) + (12^{2k+1} \times 12^2)$

$= (11^{k+2} \times 11) + (12^{2k+1} \times 144)$

$= (11^{k+2} \times 11) + \{12^{2k+1} \times (11 + 133)\}$

$= (11^{k+2} + 12^{2k+1}) \times 11 + (12^{2k+1} \times 133)$

$= (A_k \times 11) + (12^{2k+1} \times 133)$

This representation shows that A_{k+1} is divisible by 133, when A_k is divisible by 133.

∴ By induction, the given result is true

4b>

Here 'n' must be of the form, with 7-digits formed by, 3, 4, 4, 5, 5, 6, 7

$n = x_1 x_2 x_3 x_4 x_5 x_6 x_7$

If 'n' wants to exceed 5,000,000

then $x_1 = 5, 6$ or 7.

Suppose $x_1 = 5$, Then its arrangement of 6-digits which contains two 4's and one each of 3, 5, 6, 7

$\therefore$ The no of such arrangements $= \dfrac{6!}{2! \times 1! \times 1! \times 1! \times 1!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$

$= 360$

Next suppose $x_1 = 6$,

then its arrangement of 6-digits, which contains two 4's & 2's 5's & each of 3, ~~5~~, 7

$\therefore$ The no of such arrangements $= \dfrac{6!}{2! \cdot 2! \cdot 1! \cdot 1! \times 1!} =$

$= \dfrac{6 \times 5 \times \overset{2}{\cancel{4}} \times 3 \times \cancel{2!}}{\cancel{2} \times \cancel{2!}}$

$= 180$

Next suppose $x_1 = 7$,

$\therefore$ then no of arrangements, $= \dfrac{6!}{2! \cdot 2! \times 1! \cdot 1! \cdot 1!}$

$= 180$

According By the sum rule

$\therefore$ The no of arrangements of which n - exceeds 5,000,000 $= 360 + 180 + 180$

$= 720$ ways

4c) Question paper has 3-parts. A, B, C
with 4- Questions - in Part A
5- Questions - in Part B
6- Questions - in Part C

It is required to answer - 7- Questions, by selecting atleast two questions from each part.

∴ Different possible ways in which a student can make a selection are

i) 2 questions from part A, 2 from B, 3 from C
ii) 2 " from A, 3 from B, 2 from C
iii) 3 " from A, 2 from B, 2 from C.

i) The no of selection $= 4C_2 \times 5C_2 \times 6C_3$
$= \underline{1200 \text{ ways}}$

ii) No. of selection $= 4C_2 \times 5C_3 \times 6C_2$
$= \underline{900 \text{ ways}}$

iii) No of Selection $= 4C_3 \times 5C_2 \times 6C_2$
$= \underline{600 \text{ ways}}$

∴ The total no of possible Selections
$= 1200 + 900 + 600 = \underline{2700} \text{ ways}$

MODULE - 3

5a)

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$

$f: A \to B$, defined by $f = \{(1,7)(2,7)(3,8)(4,6), (5,9)(6,9)\}$.

$f^{-1}(6) = \{x \in A \mid f(x) = 6\} = \{4\}$.
$f^{-1}(9) = \{x \in A \mid f(x) = 9\} = \{5, 6\}$.

for $B_1 = \{7, 8\}$.

$f(x) \in B_1$ when $f(x) = 7$, and $f(x) = 8$

Here $f(x) = 7$ when $x = 1$, $x = 2$

$f(x) = 8$ when $x = 3$.

$\therefore f^{-1}(B_1) = \{1, 2, 3\}$

111ly $B_2 = \{8, 9, 10\}$.

$f(x) = 8$ when $x = 3$

$f(x) = 9$ when $x = 5, 6$.

$f(x) = 10$ for no values of 'x'

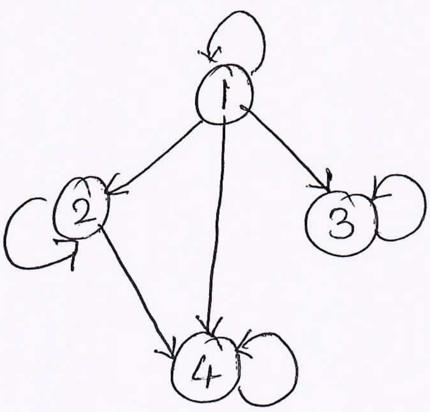
$\therefore f^{-1}(B_2) = \{x \in A \mid f(x) \in B_2\} = \{3, 5, 6\}$.

5b) Let $A = \{1, 2, 3, 4\}$ and 'R' be a relation on A defined by xRy, iff x divides y.

$\therefore R = \{1/1, 1/2, 1/3, 1/4, 2/2, 2/4, 3/3, 4/4\}$

i) $R = \{(1,1), (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,4)\ (3,3)\ (4,4)\}$

ii)



iii)

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5c)

$$f(x) = x^2, \quad g(x) = x+5, \quad h(x) = \sqrt{x^2+2}$$

i) $f \circ (g \circ h)$

$$g \circ h = g[h(x)] = g(\sqrt{x^2+2}) = \sqrt{x^2+2} + 5$$

$$f \circ (g \circ h)(x) = f((g \circ h)(x$$

$$= f(\sqrt{x^2+2} + 5)$$

$$= (\sqrt{x^2+2} + 5)^2$$

$$= (x^2+2) + 25 + 10\sqrt{x^2+2}$$

$$= x^2 + 27 + 10\sqrt{x^2+2}$$

//

ii) $g \circ (f \circ h)$

$$(f \circ g)(x) = f(g(x))$$

$$= f(x+5)$$

$$= (x+5)^2 = x^2 + 25 + 10x.$$

$$(f \circ g) \circ h(x) = (f \circ g)\, h(x)$$

$$= [h(x)]^2 + 25 + 10(h(x))$$

$$= (\sqrt{x^2+2})^2 + 25 + 10\sqrt{x^2+2}$$

$$= x^2 + 2 + 25 + 10\sqrt{x^2+2}$$

$$= x^2 + 27 + 10\sqrt{x^2+2}$$

//

$$\therefore f \circ (g \circ h)(x) = (f \circ g) \circ h(x).$$

6a)

30 - dictionaries. Total No. of pages = 61,237

Treating the pages as pigeons and dictionaries as pigeonholes, we find by using the generalized pigeonhole principle, that atleast one of the dictionaries, must contain $p+1$ or more pages.

where $p = \left\lfloor \frac{61327 - 1}{30} \right\rfloor = \lfloor 2044.2 \rfloor = 2044.$

6b) i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Let LHS $(A \cup B) \times C$

Let $(x, y) \in (A \cup B) \times C$

$\Rightarrow x \in A \cup B$ and $y \in C$

$\Rightarrow x \in A$ or $x \in B$ and $y \in C$

$\Rightarrow \{(x \in A)$ and $(y \in C)\}$ or $\{(x \in B)$ and $(y \in C)\}$

$\Rightarrow (x, y) \in (A \times C)$ or $(x, y) \in (B \times C)$

$\Rightarrow (x, y) \in (A \times C) \cup (B \times C)$ R.H.S.

$\therefore (A \cup B) \times C = (A \times C) \cup (B \times C).$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(x, y) \in [A \times (B \cap C)]$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B)$ or $(y \in C)$

$\Rightarrow \{(x \in A)$ and $(y \in B)\}$ or $\{x \in A$ and $y \in C\}$

$\Rightarrow \{(x, y) \in (A \times B)\}$ or $\{(x, y), (A \times C)\}$

$\Rightarrow \{x, y \in (A \times B) \cap (A \times C)\}$

$\Rightarrow (A \times B) \cap (A \times C)$ R.H.S.

$\therefore A \times (B \cap C) = (A \times B) \cap A \times C$

6c) $A = \{1, 2, 3, 4, 6, 8, 12\}$. Define partial order R

$R = \{xRy \text{ iff } x \text{ divides } y\}$.

$R = \{(1,1)\ (1,2)(1,3)\ (1,4), (1,6)\ (1,8)\ (1,12), (2,2)$
$(2,4)\ (2,6)\ (2,8)\ (2,12)\ (3,6)\ (3,12), (4,8)$
$(4,12)\ (6,12)\ (12,12), (3,3)\ (4,4)\ (6,6)\ (8,8)\}$

'R' is partial-order on set A.

If 'R' is reflexive, antisymmetric, and transitive.

i) Reflexive: $\forall a \in A, (a,a) \in R$.

$(1,1)\ (2,2)\ (3,3)\ (4,4)\ (6,6)\ (8,8)\ (12,12) \in R$

Hence 'R' is Reflexive.

ii) AntiSymmetric:

if $(a,b) \in R$ and $a \neq b$, then we see that
$(b,a) \notin R$, $\forall a, b \in A$.

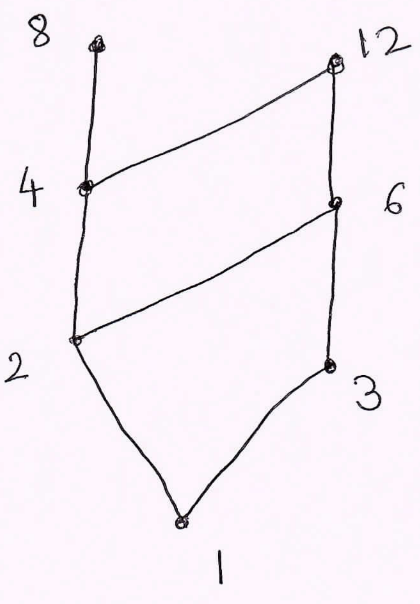
$\therefore$ R is antisymmetric.

iii) Transitive:

if $(a,b) \in R$ and $(b,c) \in R$ then we that $(a,c) \in R$

$\therefore$ R is transitive.

Thus 'R' is partial order on A. i.e (A, R) is poset



Hasse Diagram.

7a) For integers $1, 2 \ldots\ldots n$, $d_n = 11,660$.

$n = 1, 2, 3, 4, 5$.

The integers $1, 2, 3, 4, 5$ can be deranged in the first five places in d_5 ways;

The last $n-5$ integers in d_{n-5} ways.

Hence, the no of derangements

$$d_n = d_5 \times d_{n-5}.$$

$$11660 = d_5 \times d_{n-5}, \text{ So that}$$

$$d_{n-5} = \frac{11660}{d_5} = 265$$

$$= \frac{11660}{44} = 265$$

But $265 = d_6$, Thus $n-5 = 6$, So that $n = 11$

$\therefore n = 11$ //

7b) $S = \{1, 2 \ldots\ldots 300\}$.

Let A_1, A_2, A_3 be subsets of 'S' whose elements are divisible by 5, 6, 8 resply.

$S_0 = |S| = 300$

$|A_1| = \left\lfloor \frac{300}{5} \right\rfloor = 60$, $|A_2| = \left\lfloor \frac{300}{6} \right\rfloor = 50$ $|A_3| = \left\lfloor \frac{300}{8} \right\rfloor = 37$

$|A_1 \cap A_2| = \left\lfloor \frac{300}{30} \right\rfloor = 10$ $|A_1 \cap A_3| = \left\lfloor \frac{300}{40} \right\rfloor = 7$

$|A_2 \cap A_3| = \left\lfloor \frac{300}{24} \right\rfloor = 12$

$|A_1 \cap A_2 \cap A_3| = \left\lfloor 300/120 \right\rfloor = 2$.

$S_1 = |A| + |B| + |C|$
$= 147$

$S_2 = |A \cap B| + |B \cap C| + |A \cap C|$
$= 29$.

$S_3 = |A \cap B \cap C| = 2$

i) $E_2 = S_2 - {}^3C_1 \; S_3$

$= 29 - {}^3C_1 \times 2 = 23$.

ii) $L_2 = S_2 - {}^2C_1 S_3$.

$= 29 - (2 \times 2) = 25$

7c) Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$,

given $a_0 = 1$, $a_1 = 2$

Characteristic eqn is. $k^2 - 2k + 2 = 0$

$$k = 1 \pm i$$

∴ The general solution is :

$$a_n = r^n [A \cos n\theta + B \sin n\theta]$$

where $r = \sqrt{1^2 + 1^2} = \sqrt{2}$ &

$$\theta = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\therefore \quad a_n = (\sqrt{2})^n \left[A \cos \frac{n\pi}{4} , B \sin \frac{n\pi}{4}\right]$$

Given $a_0 = 1$ & $a_1 = 2$.

$\Rightarrow$ $1 = A$, $2 = (\sqrt{2}) \left[A \cos \frac{\pi}{4} + B \sin \frac{\pi}{4}\right] = A + B$ —— ①

$\Rightarrow$ $A = 1$, & $B = 1$

∴ Soln is

$$a_n = (\sqrt{2})^n \left[\cos \frac{n\pi}{4} + \sin \frac{n\pi}{4}\right] \text{ By using ①}$$

8a)

Let 'S' denote the set of all students in a hostel.

A_1, A_2, A_3, who study History, Economics, Geography respectively. $|S| = 30$, $|A_1| = 15$, $|A_2| = 8$, $|A_3| = 6$

$$\therefore \quad S_1 = \Sigma |A_i| = A_1 + A_2 + A_3$$

$$= 15 + 8 + 6 = 29.$$

$$S_3 = |A_1 \cap A_2 \cap A_3| = 3.$$

$$|\bar{A_1} \cap \bar{A_2} \cap \bar{A_3}| = |S| - \Sigma |A_i| + \Sigma |A_i \cap A_j| - \Sigma |A_1 \cap A_2 \cap A_3|$$

$$= |S| - S_1 + S_2 - S_3$$

$$= 30 - 29 + S_2 - \cancel{S}\,3$$

$$= S_2 - 2$$

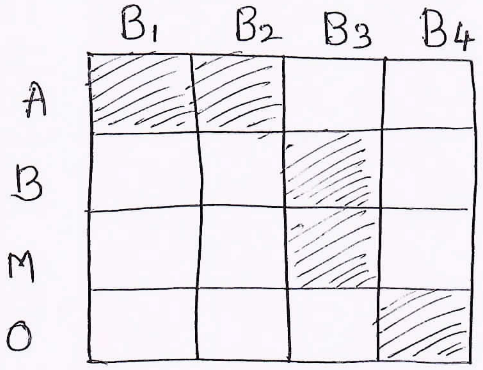
$|A_1 \cap A_2 \cap A_3| \subseteq (A_i \cap A_j)$ for $i, j = 1, 2, 3 \ldots$

$S_2 = \Sigma |A_i \cap A_j| \geq 3|A_1 \cap A_2 \cap A_3| = 9$

$|\bar{A_1} \cap \bar{A_2} \cap \bar{A_3}| \geq 9 - 2 \geq 7.$

$\therefore$ 7 or more study none of the subjects.

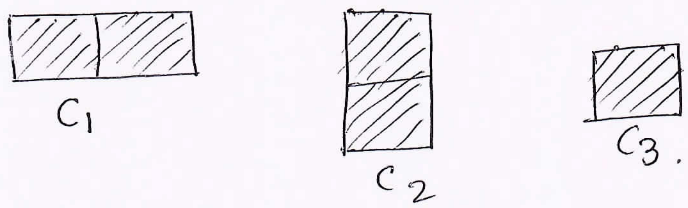
8b)



Let B_1, B_2, B_3, B_4 represent 4 - boys.

Let A, B, M, O represent Apple, Banana & Mango & Orange.

Let



$r(C, x) = r(C_1, x) \times r(C_2 . x) \times r(C_3, x)$

$= (1+2x) \times (1+2x) \times (1+x)$

$= 1 + 5x + 8x^2 + 4x^3.$

Here $r_1 = 5, \quad r_2 = 8, \quad r_3 = 4.$

$S_0 = n! = 4! = 24.$ $\qquad S_k = (n-k).k!$

$S_1 = (4-1)! \times r_1 = 30$

$S_2 = (4-2)! \times r_2 = 16$

$S_3 = (4-3)! \times r_3 = 4.$

$\therefore \bar{N} = S_0 - S_1 + S_2 - S_3$

$= 24 - 30 + 16 - 4 = 6.$

$\therefore$ '6' ways of distribution, can be made, so that all of them are

8c) $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$, given $a_0 = 2$

Given: $a_n = 3a_{n-1} + (5 \times 3^n)$ ——— (1)

is a non-homogeneous relation with $C = 3$.

$f(n) = 5 \times 3^n$.

General Solution is given by,

$$a_n = 3^n a_0 + \sum_{k=1}^{n} 3^{n-k} \cdot f(k).$$

$$a_n = 3^n a_0 + 3^{n-1} f(1) + 3^{n-2} f(2) + \dots + 3^0 f(n).$$

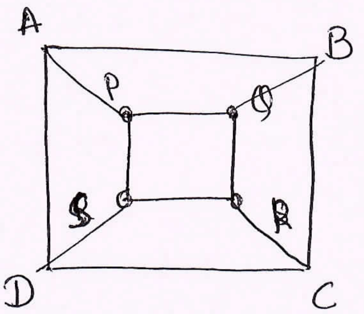
$$\Rightarrow a_n = 3^n \times 2 + 3^{n-1}(5 \times 3^1) + 3^{n-2}(5 \times 3^2) + \dots$$
$$+ 3^0 \times (5 \times 3^n)$$

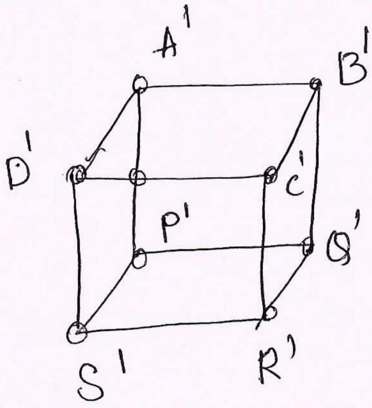
$$= 2 \times 3^n + 5 [3^n + 3^n + \dots + 3^n] \text{ n times}.$$

$$= 2 \times 3^n + 5 \times n \times 3^n$$

$\therefore a_n = 3^n (2 + 5n)$ is the required soln.

9a)





Let us consider the one-to-one correspondence between the vertices of the two graphs under which the vertices A, B, C, D, P, Q, R, S of the first graph correspond to the vertices A', B', C', D', P', Q', R', S' respectively of the second graph, and vice-versa.

$A \Leftrightarrow A'$
$B \leftrightarrow B'$
$C \leftrightarrow C'$
$D \leftrightarrow D'$

$P \leftrightarrow P'$
$Q \leftrightarrow Q'$
$R \leftrightarrow R'$
$S \leftrightarrow S'$

The edges determined by corresponding vertices

AB ⟷ A'B'   BQ ⟷ B'Q'

AP ⟷ A'P'   BC ⟷ B'C'

AD ⟷ A'D'   CD ⟷ C'D' ..... and so on

Edges determined by corresponding vertices correspond so that the adjacency of vertices is retained.

Both graphs have 8 - vertices & 12 - edges and are cubic graphs.

∴ The two graphs are isomorphic.

9b)

i) <u>Complement of a graph:</u>

If 'G' is a simple graph of order 'n', then the Complement of G in K_n is called the complement of G. It is denoted by $\bar{G}$

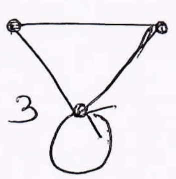
Ex:

'G' $\bar{G}$

ii) <u>vertex degree:</u>

Let $G = (V, E)$ be a graph and 'v' be a vertex of G, then the no of edges of 'G', that are incident on v with loops counted twice is called "vertex degree".

Ex:



$\deg(v_1) = 2$

$(v_2) = 2$

$(v_3) = 3.$

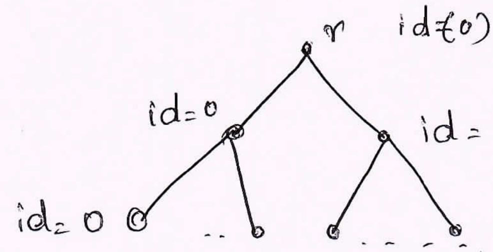
iii) <u>Rooted tree:</u>

A directed tree 'T' is called a rooted tree if

(i) T contains a unique vertex called the root whose indegree is equal to '0'

(ii) The in-degree of all other vertices of 'T' are equal to '1'.

Ex:



iv) <u>prefix code:</u>

Let 'p' be a set of binary set of binary sequences that represent a set of symbols. Then 'p' is called a prefix code, if no sequence in 'p' is the prefix of any other sequence in P.

Ex. $P_1 = \{10, 0, 1101, 111, 1100\}$ is a prefix code.

$A_1 = \{01, 0, \underline{10}1, \underline{10}, 1\}$. is not a prefix code

coz 10 is sequence of other sequence.

9c) Apply merge sort.

−1, 7, 4, 11, 5, −8, 15, −3, −2, 6, 10, 3.

{−1, 7, 4, 11, 5, −8} {15, −3, −2, 6, 10, 3}.

{−1, 7, 4} {11, 5, −8} {15, −3, −2} {6, 10, 3}

{−1, 7} {4} {11, 5} {−8} {15, −3} {−2} {6, 10} {3}

{−1} {7} {4} {11} {5} {−8} {15} {−3} {−2} {6} {10} {3}

~~{−1, 7}~~

To merge.

−1, 7 4 11 5 −8 15 −3 −2 6 10 3

TO merge

$-1 \quad 7 \quad 4 \quad 11 \quad 5 \quad -8 \quad 15 \quad -3 \quad -2 \quad 6 \quad 10$

$\{-1, 7\} \quad 4 \quad \{5, 11\} \quad -8 \quad \{-3, 15\} \quad \{-2\} \quad \{6, 10\}$

$\{-1, 4, 7\} \quad \{-8, 5, 11\} \quad \{-3, -2, 15\} \quad \{3, 6, \ldots$

$\{-8, -1, 4, 5, 7, 11\} \quad \{-3, -2, 3, 6, 10, 15\}$

$\{-8, -3, -2, -1, 3, 4, 5, 6, 7, 10, 11, 15\}$

This is the Sorted version of the given list.

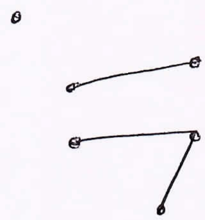
10a) Prove that a tree with 'n' vertices has n-1 edges.

Pf:→ (proof by M I)

$n = 1$

$n = 2$

$n = 3$



Result is true for $n = 1, 2, 3$.

consider a tree with k+1 vertices. Remove an edge 'e' from tree. Now there are two components. both of which are trees T_1 & T_2.

Let the no of edges in T_1 & T_2 be k_1 & k_2 resply.

$k_1, k_2 \leq k+1.$

no of edges in $T_1 = k_1 - 1$

" $T_2 = k_2 - 1$

Total no of edges in T_1 & T_2 $= k_1 + k_2 - 2$

$k_1 + k_2 - 2$
$= (k+1) - 2$
$= (k-1)$.

keeping the edge e, back in its 8 place
No of edges in $T = (k-1)+1 = k$.
So the result is true for $k+1$ also.
$\therefore$ Hence by Mathematical Induction, the result is true for all +ve in 'n'.

10 b) Determine the no of vertices in G.

i) G has 9 edges & all vertices have degree 3.
Let no of vertices be 'n'
Sum of degrees of all vertices $= 3n$.
Since 'G' has 9 edges, we have $3n = 2 \times E$

$3n = 2 \times 9$

$n = 6$.

$\therefore$ order of $G = 6$.

ii) G has 10 edges, with 2 vertices of degree 4 and all other have degree 3.
$\therefore$ the sum of deg of all vertices $= (2 \times 4) + (n-2) \times 3$

$\Rightarrow 2 \times 4 + (n-2) \times 3 = 2 \times 10$

$\Rightarrow 8 + (n-2) \times 3 = 20$

$(n-2) = \frac{12}{3} = 4$

$\underline{n = 6.}$

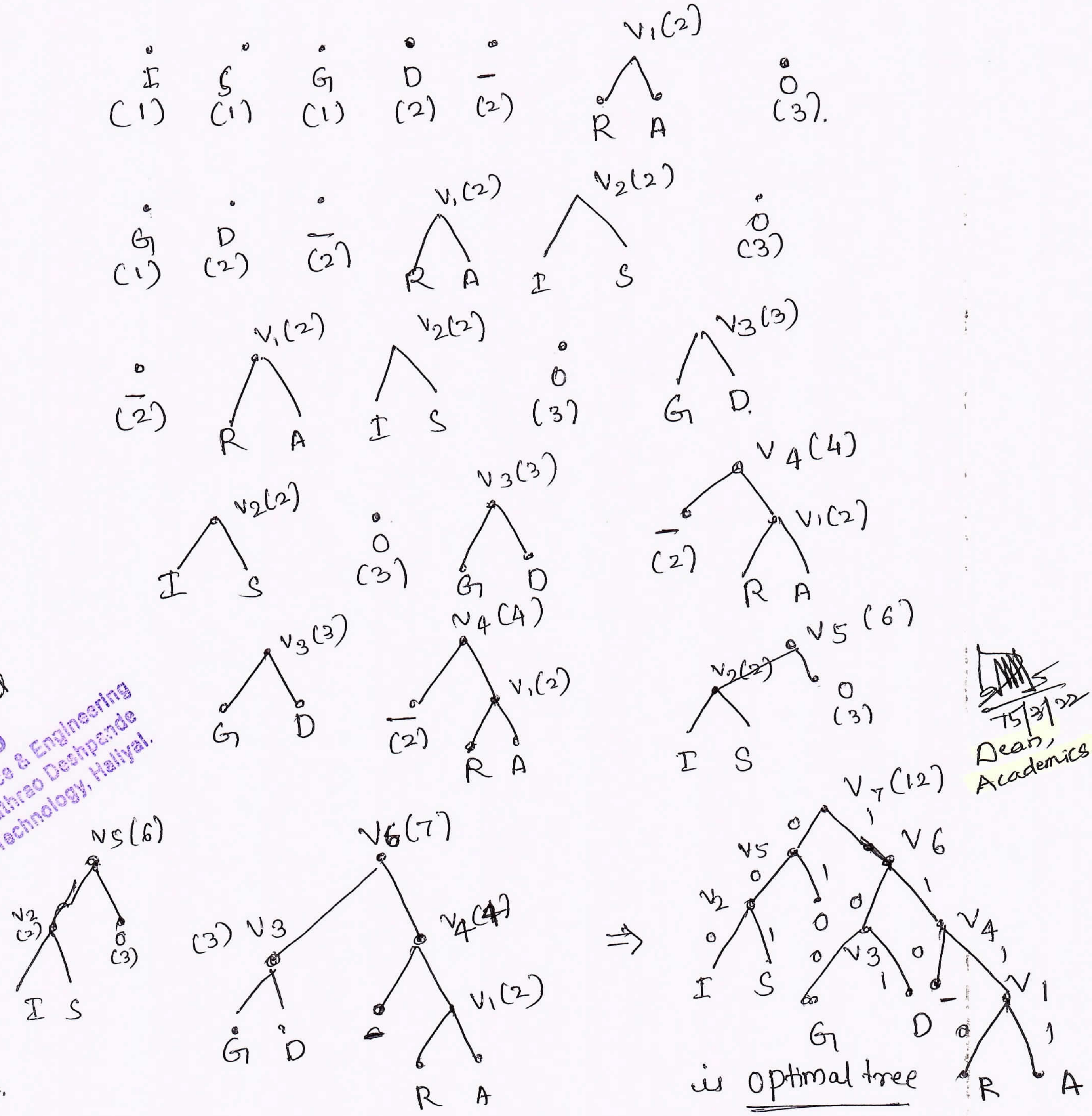
$\therefore$ order of $G = 6$.

10 c) ROAD IS GOOD.

The given message consists of the letters R, O, A, D, I, S, G with frequencies 1, 3, 1, 2, 1, 1, 1 resply. Also there are 2 blank spaces (_) betn two words.

Arranging all these in non-decreasing order.

R	A	I	S	G	D	_	O
(1)	(1)	(1)	(1)	(1)	(2)	(2)	(3).



R : 1111 O : 01 A : 1111 D : 101

_ : 110 I : 000 S : 001 G : 100

Code : 111101 1111 101 110 000 001 110 100 01 01 101