CBCS SCHEME

USN | | | | | | | | | | 18EE32

Third Semester B.E. Degree Examination, Aug./Sept.2020
Electric Circuit Analysis

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Determine voltage V_3 for the circuit shown in Fig.Q1(a), using Mesh analysis method.

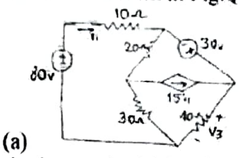


Fig. Q1(a) **(08 Marks)**

b. Apply node analysis method to find node voltages V1, V2, V3 for the circuit shown in Fig.Q1(b).

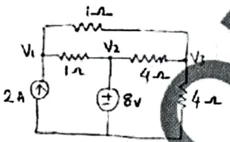


Fig.Q1(b) **(07 Marks)**

c. Determine the equivalent resistance between the terminals AB for circuit shown in Fig.Q1(c).

Fig.Q1(c) **(05 Marks)**

OR

2 a. Apply loop analysis method to find voltage V, such that current through (2 + j3) Ω resistor is zero. For the circuit shown in Fig.Q2(a).

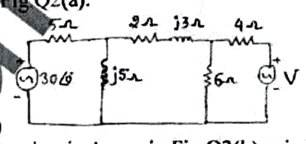


Fig.Q2(a) **(07 Marks)**

b. Determine the voltage V_x in the circuit shown in Fig.Q2(b) using Nodal analysis method.

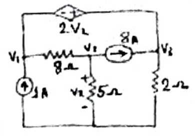


Fig.Q2(b) **(07 Marks)**

c. Apply source transformation and shifting method to reduce the circuit shown in Fig.Q2(c) to a single voltage source in series with resistance.

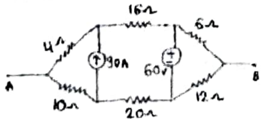


Fig.Q2(c) **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-2

3 a. In the circuit shown in Fig.Q3(a), determine current I_X using super position theorem.

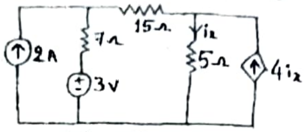


Fig.Q3(a)

(07 Marks)

b. Determine Thevenin's equivalent of the circuit in Fig.Q3(b).

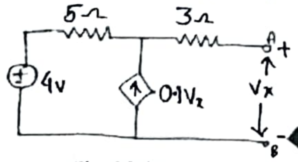


Fig.Q3(b)

(07 Marks)

c. Use Millman's theorem to find current I, for the circuit shown in Fig.Q3(c).

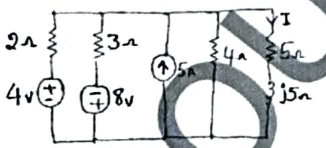


Fig.Q3(c)

(06 Marks)

OR

4 a. Determine current thorough 1Ω resistor. Using Nortan's theorem for the circuit shown in Fig.Q4(a).

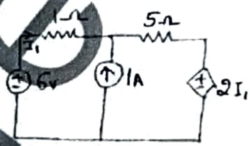


Fig.Q4(a)

(07 Marks)

b. Determine the fixed resistance R_L to receive maximum power from the source. Also find maximum power delivered to the load in the circuit shown in Fig.Q4(b).

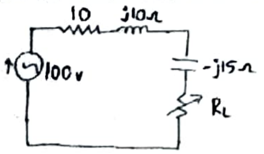


Fig.Q4(b)

(07 Marks)

c. State and verify reciprocity theorem for the circuit shown in Fig.Q4(c).

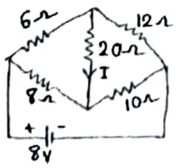


Fig.Q4(c)

(06 Marks)

Module-3

5 a. Derive an expression for the resonance frequency of a resonant circuit consisting of R_L, XL in parallel with RC, XC. (07 Marks)

b. An impedance coil having a resistance of 4Ω and an inductance of 1mH connected in series with 10μF capacitor. Determine resonant frequency, impedance at resonance, half power frequencies, Q of the circuit and bandwidth. (08 Marks)

c. For the circuit shown in Fig.Q5(C), the switch is moved from position 1 to 2 at t = 0. The steady state has been reached before switching. Determine : i, $\frac{di}{dt}, \frac{d^2i}{dt^2}$ at t = 0.

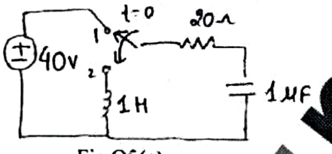


Fig.Q5(c) (05 Marks)

OR

6 a. In circuit shown in Fig.Q6(a), the switch K is closed at t = 0. Calculate $\frac{di_1(0^+)}{dt}$ and $\frac{di_2(0^+)}{dt}$

Assume that the circuit was not activated before t = 0.

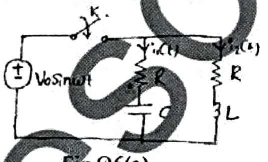


Fig.Q6(a) (10 Marks)

b. Determine R_L and R_C for which the circuit shown in Fig.Q6(b), resonances at all frequencies.

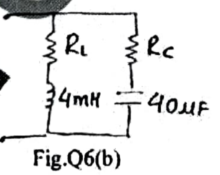


Fig.Q6(b) (04 Marks)

c Show that in series RLC circuit, the resonant frequency $f_0 = \sqrt{f_1 f_2}$. (06 Marks)

Module-4

7 a. State and prove initial and final value theorem in Lapalce transformation. (08 Marks)

b. Find Lapalce transform of the signal shown in Fig.Q7(b).

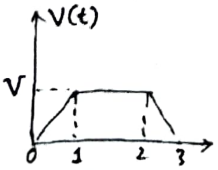


Fig.Q7(b) (08 Marks)

c. Find Laplace transform of unit step function. (04 Marks)

OR

8 a. State and prove shifting theorem.

b. Verify initial value theorem, given $f(t) = 10e^{...}$ (05 Marks)

c. Find Laplace transform of the signal in Fig.Q8(c). (05 Marks)

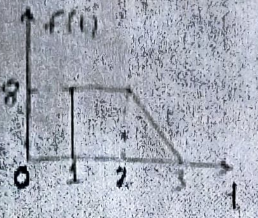


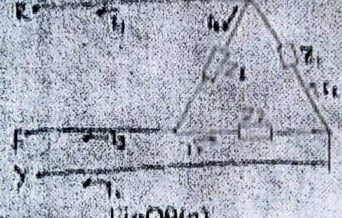
Fig.Q8(c)

(10 Marks)

Module-5

9 a. Three impedances $Z_1 = 20\angle 30^\circ\Omega$, $Z_2 = 40\angle 60^\circ\Omega$ and $Z_3 = 10\angle -90^\circ\Omega$ are delta connected to a 400V, 3 – phase system as show in Fig.Q9(a). Determine the .

i) Phase currents ii) Line currents.



FigQ9(a) (06 Marks)

b. Determine Y-parameters for the circuit shown in Fig.Q9(b).

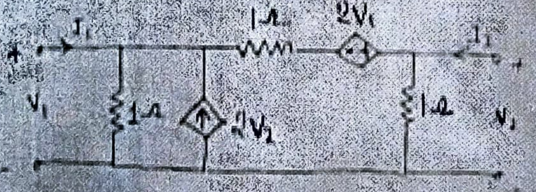


Fig.9(b) (08 Marks)

c. Express Y-parameters interms of Z-parameters. (06 Marks)

OR

10 a. An unbalanced four-wire, star connected load has a balanced voltage of 400V, the loads are $Z_1 = (4 + j8)\Omega$, $Z_2 = (3 + j4)\Omega$, $Z_3 = (15 + j20)\Omega$.

Calculate the : i) line currents ii) current in the neutral wire. (06 Marks)

b. Find Z-parameters and T-parameters for the circuit shown in Fig.Q10(b).

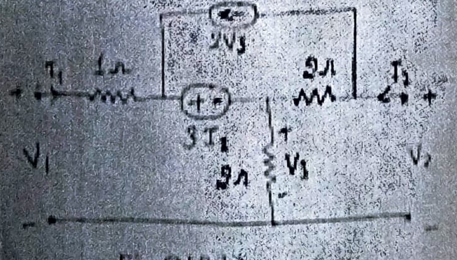


Fig.Q10(b) (10 Marks)

c. Define H – parameters with necessary equations. (04 Marks)

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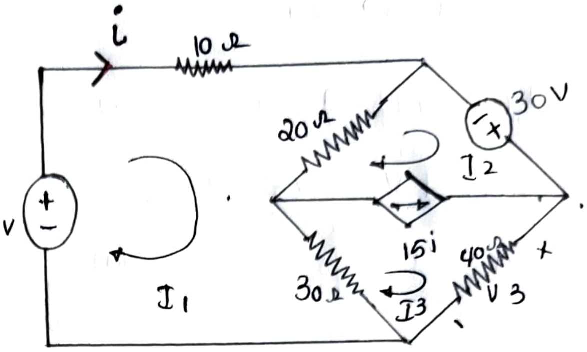
Aug / Sep - 2020

MODULE - 1

1 a. Determine Voltage V_3 for the circuits shown in fig using Mesh analysis methode.



applying KVL to loop 1

$-10(I_1) - 20(I_1 - I_2) - 30(I_1 - I_3) = -30V$

$60I_1 + 20I_2 + 30I_3 = -30$ — (1)

applying KVL to loop 2 & 3

$-20(I_2 - I_1) + 30 - 40I_3 - 30(I_3 - I_1) = 0$

$50I_1 - 20I_2 - 70I_3 = -30$ — (2)

eqn for 15i

$I_3 - I_2 = 15i$

but $i = I_1$

$I_3 - I_2 = +15I_1$

$-15I_1 - I_2 + I_3 = 0$ — (3)

solving (1) (2) & (3)

$I_1 = 1.024A$

$I_2 = -11.0A$

$I_3 = 4.3A$

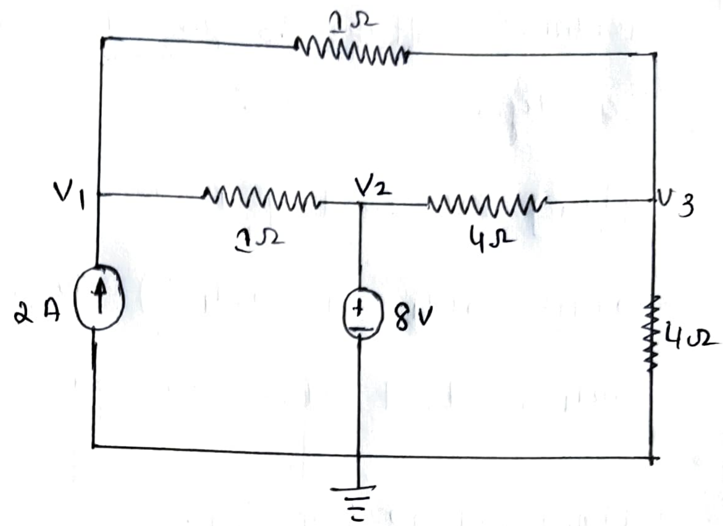
Voltage for $V_3 = I_3 40$

$= 40 \times 4.34$

$V_3 = 173.6V$

B. Apply node analysis method to find node voltages V_1, V_2, V_3 for circuit shown in fig.

Soln



Apply KCL to V_1

$2 - 1(V_1 - V_2) - 1(V_1 - V_3) = 0$

$2 - 2V_1 + V_2 + V_3 = 0$

$2V_1 - V_2 - V_3 = 2$ —— (1)

Apply KCL to V_3

$\frac{1}{4}(V_3 - 0) - \frac{1}{4}(V_3 - V_2) - 1(V_3 - V_1) = 0$

$0.25V_3 - 0.25V_3 + 0.25V_2 - 1V_3 + V_1 = 0$

$V_1 - V_3 + 0.25V_2 = 0$

$V_1 + 0.25V_2 - V_3 = 0$ —— (2)

eqn for 8V Source

$V_B - 0 = 8$

$V_B = 8.$ —— (3)

Solving eqn (1) (2) & (3)

$V_1 = 12V$

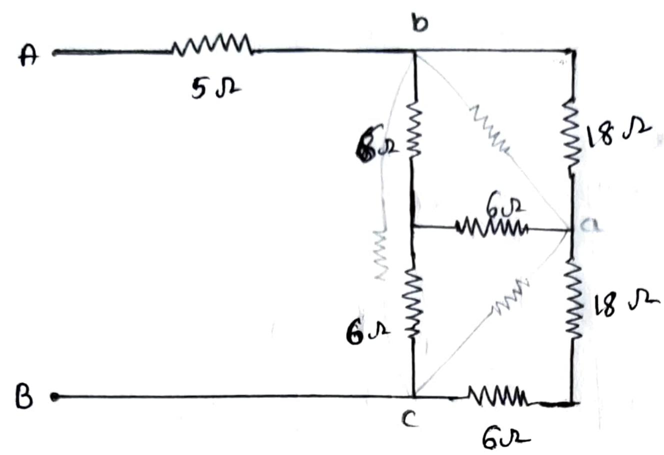
7, 8, 8.5

$V_2 = 8V$

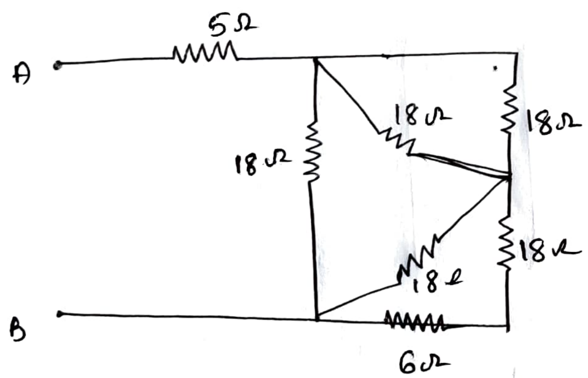
$V_3 = 14V.$ //

(2)

C. Determine the equivalent resistance between the terminals AB for circuit shown.



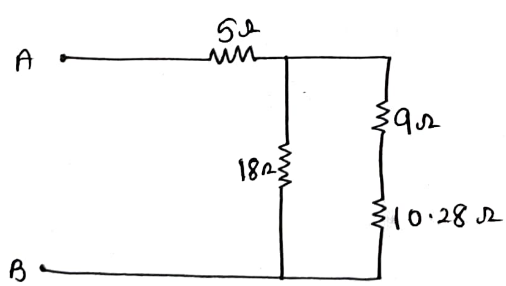
Soln Converting star to delta :- $R_{ab} = \frac{6\times6 + 6\times6 + 6\times6}{6} = 18\Omega$



$18 \| 18\Omega$, $18 \| (18 + 6\Omega)$

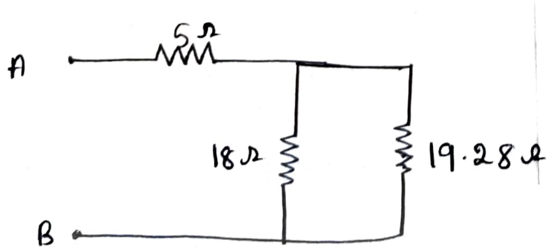
~~9Ω ‖ 10.28~~.

$9\Omega \| 10.28$

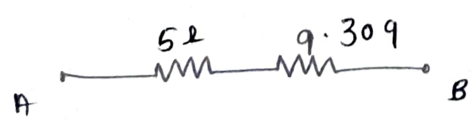


9Ω is series with 10.28

hence $R_{eq} = R_1 + R_2 = 19.28$.



$19.28 \| 18\Omega = 9.309\Omega$

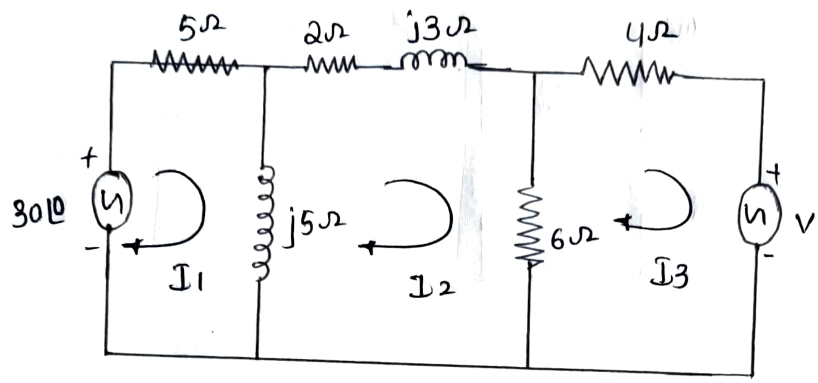


$\simeq$

$= 14.309\Omega$ //

3

2.a. Apply loop a mesh analysis method to find voltage V. Such that current through $(2+j3)\Omega$ resistor is zero for circuit shown in fig.



$$[\bar{Z}][\bar{I}] = [\bar{V}]$$

$$\begin{bmatrix} (5+j5) & (-j5) & 0 \\ (-j5) & (8+j8) & (-6) \\ 0 & (-6) & (10) \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0 \\ 0 \\ -V_2 \end{bmatrix}$$

$$\bar{I}_2 = \frac{\begin{vmatrix} (5+j5) & (30\angle 0) & (0) \\ (-j5) & 0 & (-6) \\ 0 & (-V_2) & (10) \end{vmatrix}}{\begin{vmatrix} (5+j5) & (-j5) & 0 \\ (-j5) & (8+j8) & (-6) \\ 0 & (-6) & (10) \end{vmatrix}}$$

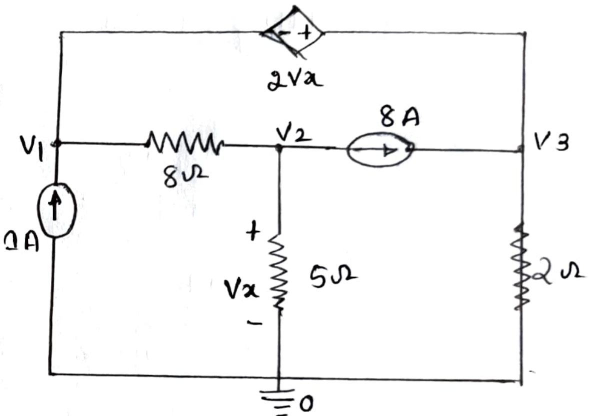
$$\frac{(5+j5)(6V_2) - (30\angle 0)[(10)(-j5)]}{(5+j5)[(8+j8)(10)-36] - (j5)[(-j5)(10)]} = 0$$

$$(5+j5)(6V_2) = (30\angle 0)[(-j5)(10)]$$

$$V_2 = \frac{(30\angle 0)(-j5)(10)}{6(5+j5)}$$

$$\boxed{V_2 = \frac{-j250}{5+j5}} = -25 - 25i$$

b. Determine the Voltage V_x in the Circuit shown in fig.



Soln KCL for V_1 & V_3

$$1 - 0.125(V_1 - V_2) - 0.5V_3 + 8 = 0$$

$$0.125V_1 - 0.125V_2 - 0.5_3 = -9 \quad \text{—(1)}$$

KCL for V_2

$$-8 - 0.2V_2 - 0.125(V_2 - V_1) = 0$$

$$0.125V_1 - 0.325V_2 = 8 \quad \text{—(2)}$$

eqⁿ for $2V_x$

$$V_1 - V_3 = 2V_x$$

but $V_x = V_2$

$$V_1 - 2V_2 - V_3 = 0 \quad \text{—(3)}$$

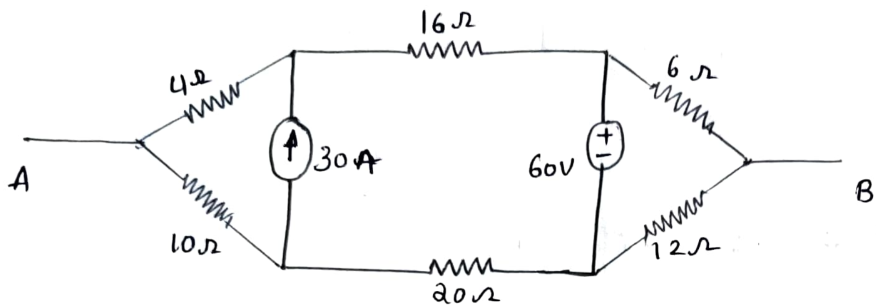
Solving (1) (2) & (3)

$V_1 = 42.9V$ $V_2 = 8.1V$

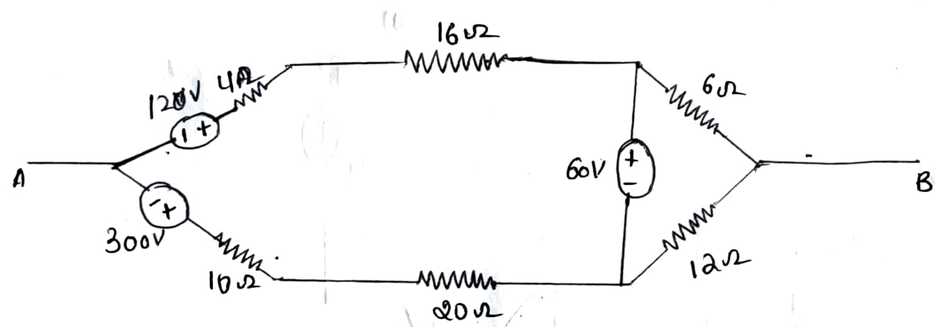
$V_3 = 26.7V$

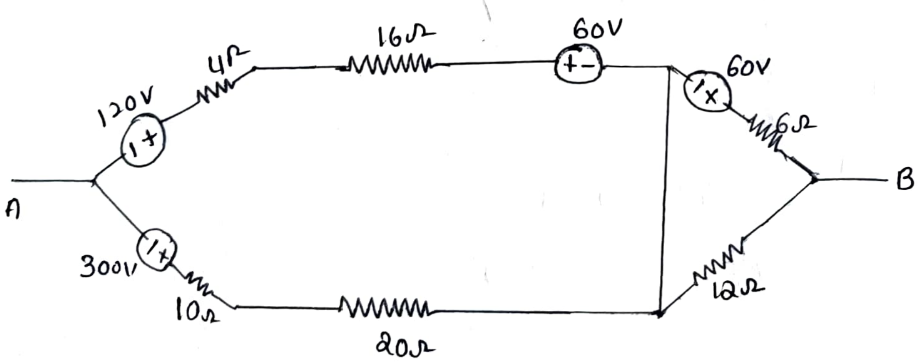
$$\boxed{V_x = V_2 = 8.1V}$$

c. Apply Source transformation and shifting method to reduce the circuit shown in fig to single voltage source in series with resistance.

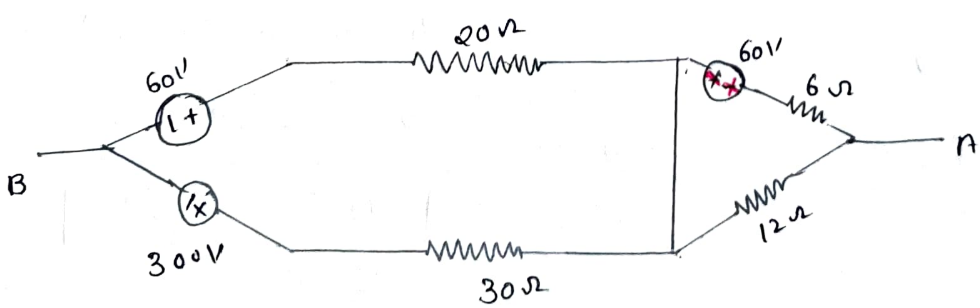


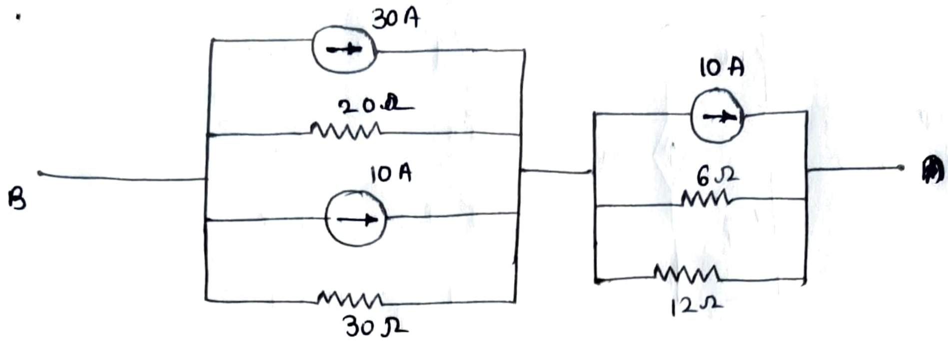
Sol^n Converting ideal current source to ideal voltage source.

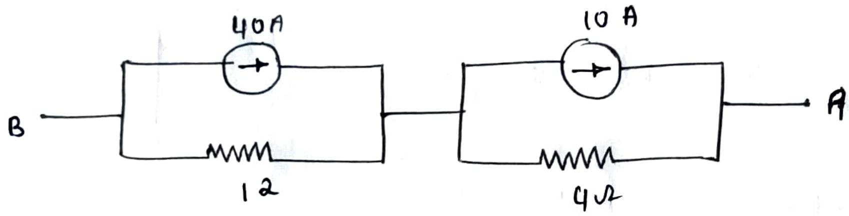


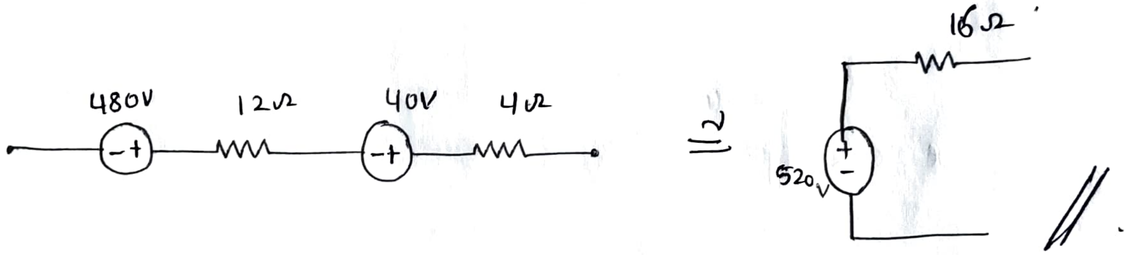


4Ω series with 16Ω & 20Ω series with 10Ω



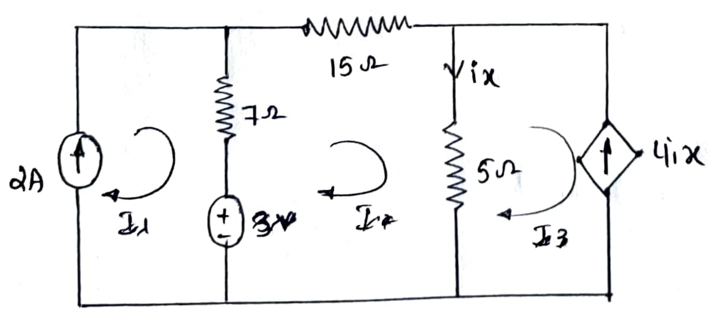






MODULE - 2

3.a. In the Circuit shown in fig determine current I_x using Super position theorem.

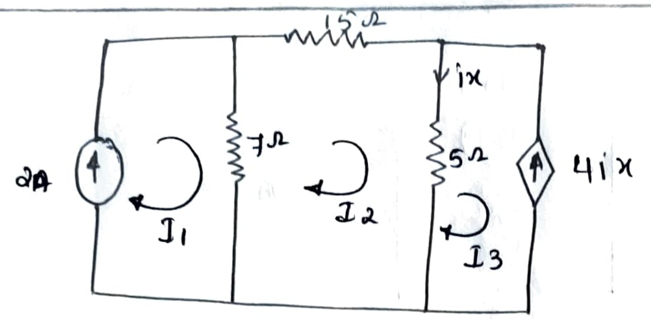


Sol^n I_x through 2A Source

$I_1 = 2A$

KVL to I_2 : $-15I_2 - 5(I_2 - I_3) - 7(I_2 - I_1) = 0$

$7I_1 - 27I_2 + 5I_3 = 0$



eqn for 4ix

$i_x = I_2 - I_3$

$I_3 = -4i_x$

$I_3 = -4(I_2 - I_3)$

$I_3 = -4I_2 + 4I_3$

$-4I_2 + 3I_3 = 0$

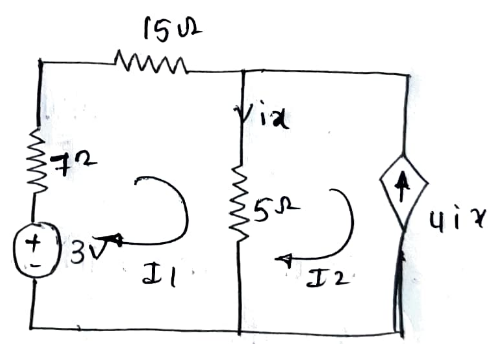
Solving ① ② & ③

$I_1 = 2A \qquad I_2 = 0.59A \qquad I_3 = 0.39A.$

$I_x = I_2 - I_3$

$\boxed{I_x = 0.2A}$

(b) Ix through 3V sourse



KVL to I1

$3 - 7I_1 - 15I_1 - 5(I_1 - I_2) = 0$

$-27I_1 + 5I_2 + 3 = 0$

$3 = 27I_1 - 5I_2$ —— ①

eqn for 4ix

$I_2 = -4i_x$ but $i_x = I_1 - I_2$

$I_2 = -4I_1 + 4I_2$

$4I_1 - 4I_2 + I_2 = 0$

$4I_1 - 3I_2 = 0$ —— ②

$I_1 = 0.147A \qquad I_2 = 0.196A.$

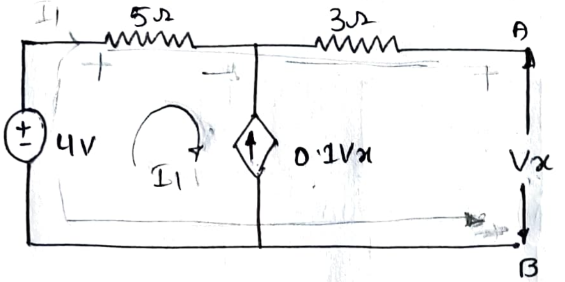
$$I_x = -0.049A$$

$$I_x\big|_{2A+3V} = I_x\big|_{2A} + I_x\big|_{3V}$$

$$0.2 - 0.049$$

$$\boxed{I_x = 0.151A}$$

B. Determine Thevenin's equivalent of the circuit



Sol^n a). finding V_{TH}.

$$I_1 = -0.1V_x \quad — ①$$

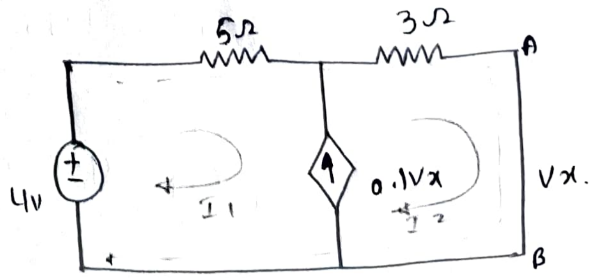
$$V_x + 5I_1 - 4 = 0$$

$$V_x + 5(-0.1V_x) - 4 = 0$$

$$0.5V_x = 4$$

$$\boxed{V_x = V_{TH} = 8V}$$

finding R_{TH}



KVL to super mesh

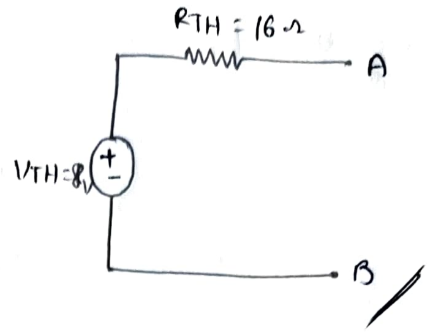
$$-5I_1 - 3I_2 + 4 = 0$$

$$5I_1 + 3I_2 = 4 \quad — ①$$

eq^n for current source

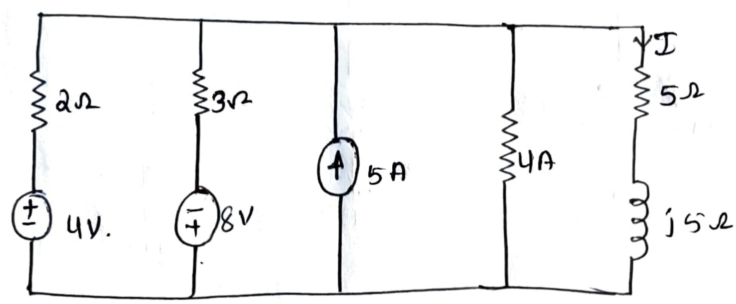
$I_1 = 0.5\,A$ $\qquad I_2 = 0.5\,A$

$$\boxed{I_{NOR} = 0.5\,A}$$

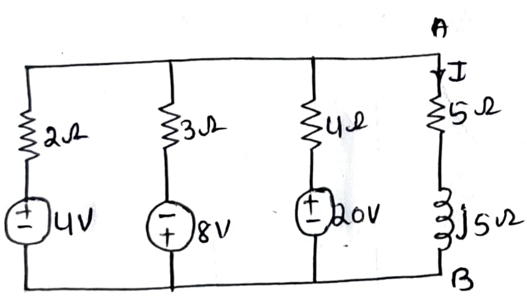


$$R_{TH} = \frac{V_{TH}}{I_{NOR}} = \frac{8}{0.5} = 16\,\Omega$$

c. Use Milliman's theorem to find current I. for the circuit shown in fig



Sol^n:- Convert PCS to P.VS

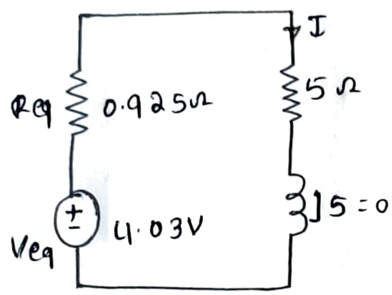


$$V_{eq} = \frac{V_1 G_1 - V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3} = \frac{(4 \times 0.5) - (8 \times 0.33) + (20 \times 0.25)}{0.5 + 0.33 + 0.25}$$

$$\boxed{V_{eq} = 4.03\ \text{Volts}}$$

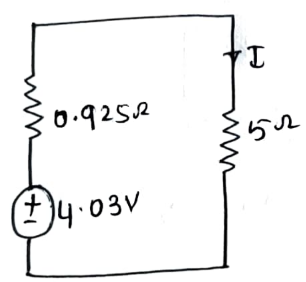
$$Req = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{0.5 + 0.33 + 0.25}$$

$$\boxed{Req = 0.925\,\Omega}$$



It is DC circuit hence effect of inductance is zero

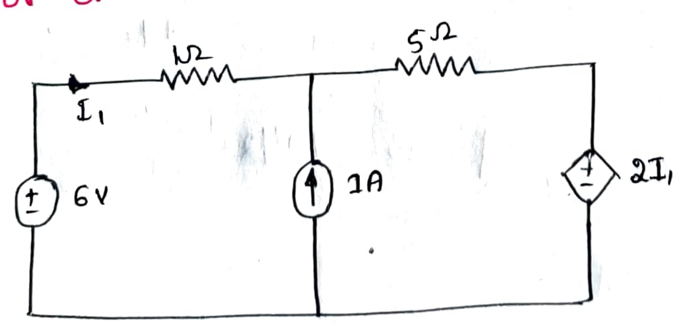
Hence $j5 = 0$

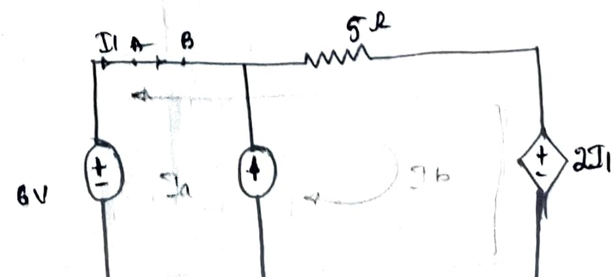


$$I = \frac{4.03}{(0.925 + 5)}$$

$$I = 0.68 \text{ Amps}$$

4a) Determine current through 1Ω resistor. Using Norton's theorem for circuit shown.





Finding I_{NOR}

KVL to supermesh

$2I_1 + 5I_b - 6 = 0$

but $I_1 = I_a$

$2I_a + 5I_b - 6 = 0$

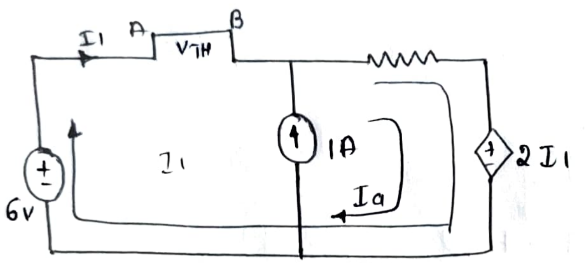
$5I_b = 6$ — (1)

$I_b - I_a = 1$

$I_a = 0.142$ $I_b = 1.143A$

$\boxed{I_{NOR} = 0.142}$

(b) finding R_{NOR}



$I_a = 1A$

KVL to supermesh

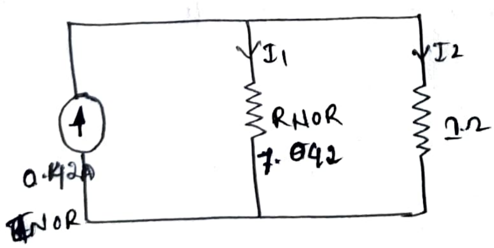
$-5I_a - 2I_1 + 6 - V_{TH} = 0$

$V_{TH} = 5I_a - 2I_1 - 6$

$\boxed{V_{TH} = 1V}$

I_1 becomes zero because of o.c

Current through 1Ω resistor is



$R_{NOR} = \frac{V_{TH}}{I_{NOR}} = \frac{1}{0.142}$

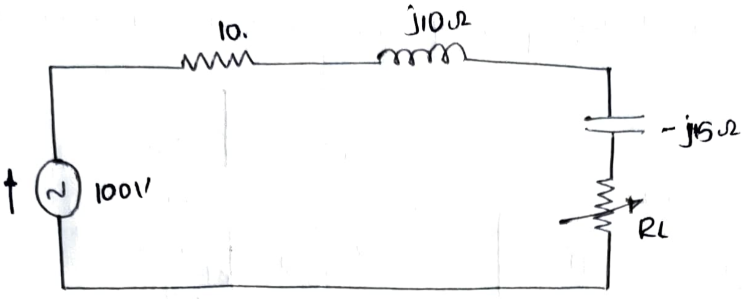
$\boxed{R_{NOR} = 7.042\Omega}$

by current dividing rule

$I_2 = \frac{R_{NOR}}{1 + R_{NOR}} \times I_{NOR} = \frac{7.042}{8.042} \times 0.142$

current through 1Ω resistor is 0.12A

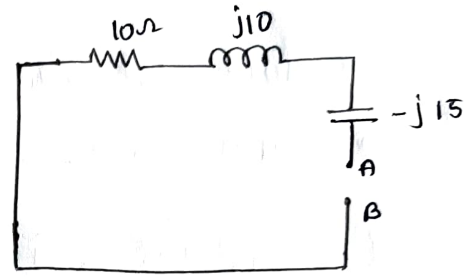
b. Determine the load resistance R_L to receive maximum power from Source. Also find maximum power delivered to the load in the circuit



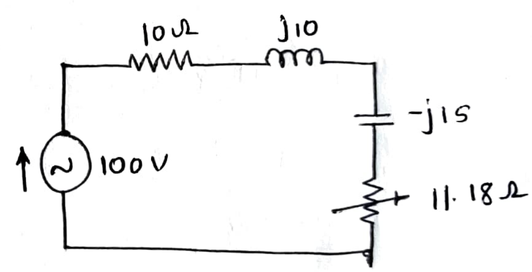
i) $Z_{TH} = 10 - j5$

$|Z_{TH}| = \sqrt{10^2 + 5^2} = \sqrt{125}$

$\boxed{R_L = 11.18\Omega}$



ii) $I = \dfrac{V}{Z} = \dfrac{100\angle 0}{21.18 - j5} = 4.47 + 1.05j$

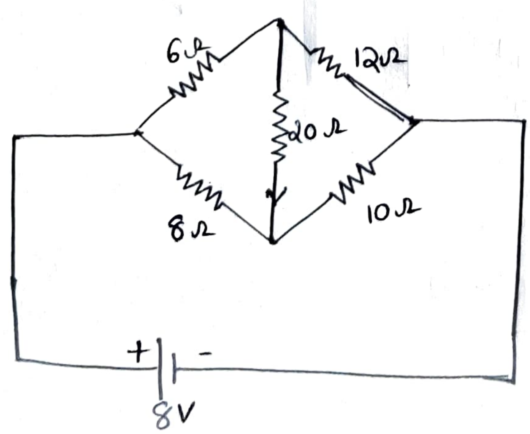


$|I| = \sqrt{4.47^2 + 1.05^2}$

$\boxed{|I| = 4.598A}$

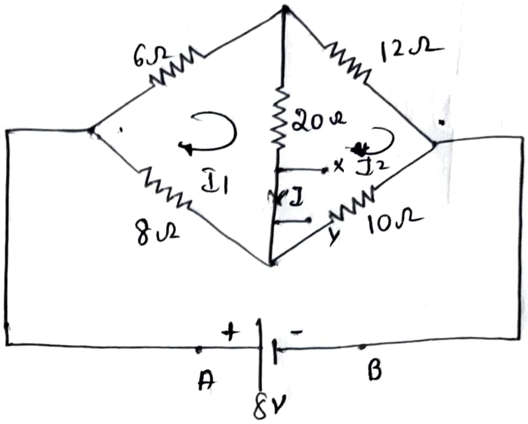
$P = |I|^2 R_L =$ 236.36W.

c) state and Verify reciprocity theorem for the Circuit shown fig.



Solⁿ:- **Statement :-** The theorem states that In any linear bilateral n/w the ratio of Excitation to response remains analter if the position of excitation & response are interchange

KVL to I_1, I_2 & I_3



$$-6I_1 - 20(I_1 - I_2) - 8(I_1 - I_3) = 0$$

$$-34I_1 + 20I_2 + 8I_3 = 0 \quad \text{--- (1)}$$

$$-12I_2 - 10(I_2 - I_3) - 20(I_2 - I_1) = 0$$

$$20I_1 - 40I_2 + 10I_3 = 0 \quad \text{--- (2)}$$

$$-8(I_3 - I_1) - 10(I_3 - I_2) + 8 = 0$$

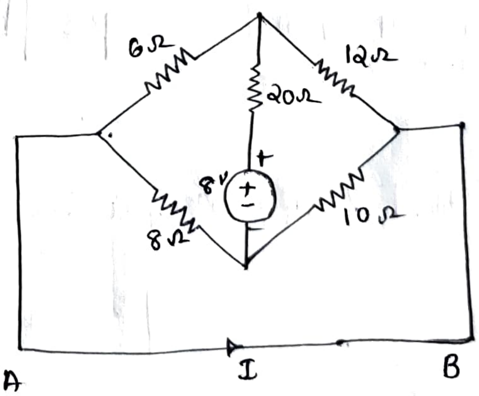
$$8I_1 + 10I_2 - 18I_3 = -8$$

Solving I_1 & I_2 & I_3

$$I = I_1 - I_2$$

$$\boxed{I = 0.03A}$$

By interchanging the excitation



$$-6I_1 - 20(I_1 - I_2) + 8 - 8(I_1 - I_3) = 0$$

$$-34I_1 + 20I_2 + 8I_3 = -8 \quad \text{--- (1)}$$

$$-12I_2 - 10(I_2 - I_3) - 8 - 20(I_2 - I_1) = 0$$

$$20I_1 - 42I_2 + 10I_3 = 8 \quad \text{--- (2)}$$

$$-8(I_3 - I_1) - 10(I_3 - I_2) = 0$$

$$8I_1 + 10I_2 - 18I_3 = 0 \quad \text{--- (3)}$$

$I_1 = 0.1875A$ $I_2 = -0.09375A$ $I_3 = 0.03125A$.

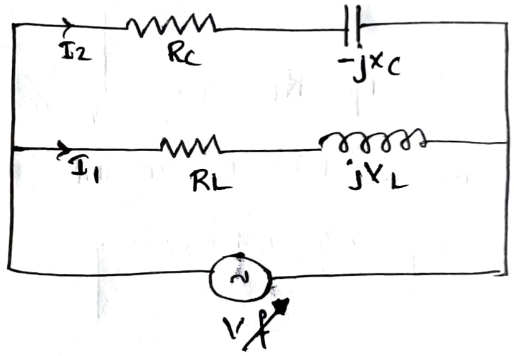
$$\boxed{I = 0.031A}$$

Hence reciprocity theorem is proved.

MODULE - 3

5a. Derive an expression for the resonance frequency of a resonant Circuit consisting of R_L, X_L in parallel with R_C, X_C.

→ Input impedence $Y = Y_1 + Y_2$



$$Y_1 = \frac{1}{R_L + jX_L} \times \frac{R_L - jX_L}{R_L - jX_L} =$$

$$\frac{R_L}{R_L^2 + X_L^2} - \frac{jX_L}{R_L^2 + X_L^2}.$$

$$Y_2 = \frac{1}{R_C - jX_C} \times \frac{R_C + jX_C}{R_C + jX_C} = \frac{R_C}{R_C^2 + X_C^2} + \frac{jX_C}{R_C^2 + X_C^2}$$

$$Y = \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right) + j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right)$$

equating imaginary Part to zero.

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{1/w_0 C}{R_C^2 + \left(\frac{1}{w_0 C}\right)^2} = \frac{w_0 L}{R_L^2 + (w_0 L)^2}$$

$\cancel{R_L^2 + w_0^2 L^2}$

$$\frac{R_L^2}{w_0 C} + \frac{w_0 L^2}{C} = R_C^2 w_0 L + \frac{L}{w_0 C^2}$$

$$\frac{R_L^2 + w_0^2 L^2}{w_0 C} = \frac{R_C^2 w_0^2 L C^2 + L}{w_0 C^2}$$

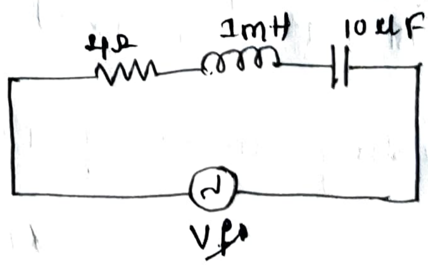
$$R_L^2 C + w_0^2 L^2 C = R_C^2 w_0^2 L C^2 + L$$

$$w_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L - \frac{L}{C}}{R_C - L/C}}$$

If $R_L = R_C = \sqrt{L/C}$ then resonant frequency of the circuit is resonant for all the frequency at resonance I become minimum.

---

b. An impedence coil having a resistance of 4Ω & an inductance of 1 mH connected in series with $10\,\mu F$ capacitor. Determine resonant frequency, impedence at resonance. half power frequency. Q of the circuit and bandwidth.



Resonant frequency $w_0 = \frac{1}{\sqrt{LC}}$

$$w_0 = \frac{1}{\sqrt{1 \times 10\mu}} = 316.2 \text{ rad/sec.}$$

$$w_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \frac{-4}{2 \times 1} + \sqrt{\left(\frac{4}{2 \times 1}\right)^2 + \frac{1}{1 \times 10\mu}}$$

$$= -2 + 316.2$$

$\omega_1 = 314.2$ rad/sec.

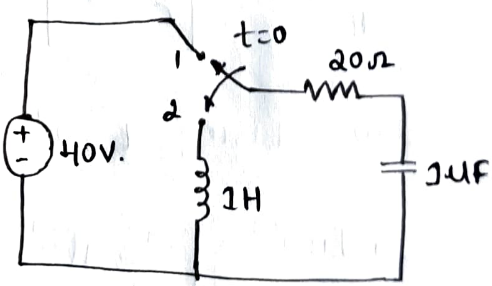
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \frac{4}{2\times 1} + \sqrt{\left(\frac{4}{2\times 1}\right)^2 + \frac{1}{1\times 10\mu}} = 318.2 \text{ rad/sec}$$

$$\text{Bandwidth} = \frac{R}{L} = \frac{4}{1} = 4$$

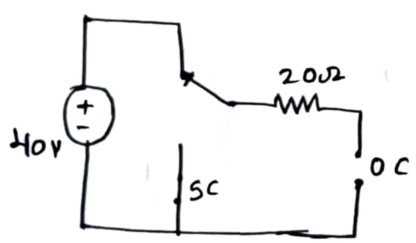
$$Q.F = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{4}\sqrt{\frac{1}{10\mu}} = 316.4 \text{ rad/sec.}$$

c. For circuit shown in fig. the switch is moved from position 1 to 2 at $t=0$. The steady state has been reached before switching. Determine; i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0$.

→ When $t<0$

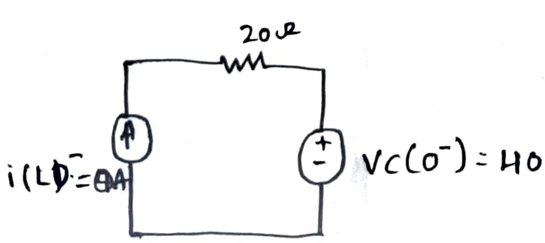


at $t(0^-)$

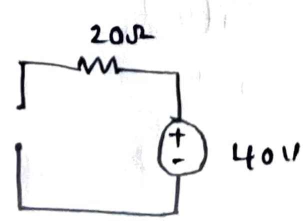


$i_L(0^-) = 0$ Amps.

at $t(0^+)$

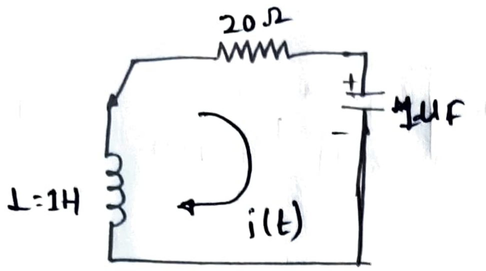


$\cong$



$i(0^+) = 0$ Amps.

at $t > 0$



KVL eq^n 1.

$$-20i(t) - \left[\frac{1}{C}\int_{0^-}^{t} i(t)\,dt + V_C(0^-)\right] + L\frac{d\,i(t)}{dt} = 0 \longrightarrow (1)$$

Substitute $t = 0^+$

$$-20\,i(0^+) - [\,40\,] + 1\frac{di}{dt}(0^+) = 0$$

$$0 - 40 + 1\frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = 40 \text{ Amp/sec}$$

Differentiate eq^n 1 again.

$$-20\frac{d\,i(t)}{dt} - \left[\frac{1}{C}\,i(t)\right] + L\frac{d^2 i(t)}{dt^2} = 0$$

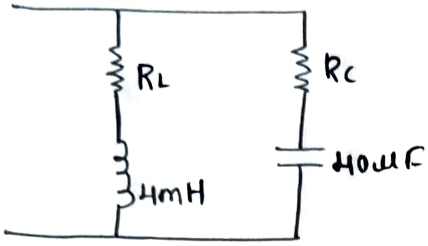
Substituting $t = 0^+$

$$-20\frac{di(0^+)}{dt} - \left[\frac{1}{4}\,i(0^+)\right] + 1\frac{d^2 i(0^+)}{dt^2} = 0$$

$$-20\,(40) + 1\frac{d^2 i(0^+)}{dt^2} = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = 800 \text{ Amp/sec}$$

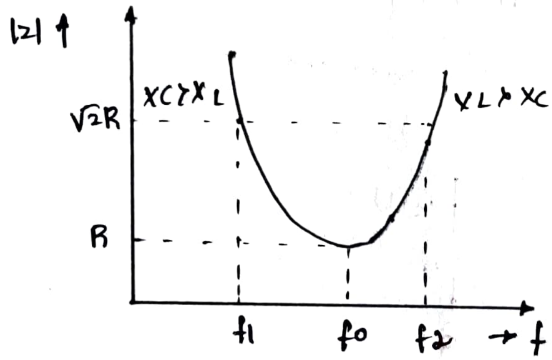
6. Determine R_L and R_C for which the circuit shown in fig resonantes at all frequencies.



$$R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{4 \times 10^{-3}}{40 \times 10^{-6}}}$$

$$\boxed{R_L = R_C = 10}$$

b. Show that in series RLC circuit, the resonant frequency $f_0 = \sqrt{f_1 f_2}$.



at Half power frequencies $|Z_1| = |Z_2|$

$$\sqrt{R^2 + (X_{C1} - X_{L1})^2} = \sqrt{R^2 + (X_{L2} - X_{C2})^2}$$

$$X_{C1} - X_{L1} = X_{L2} - X_{C2}$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$\frac{1}{C}\left[\frac{\omega_1 + \omega_2}{\omega_1 + \omega_2}\right] = L(\omega_1 + \omega_2)$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\omega_1 \omega_2 = \omega_0^2$$

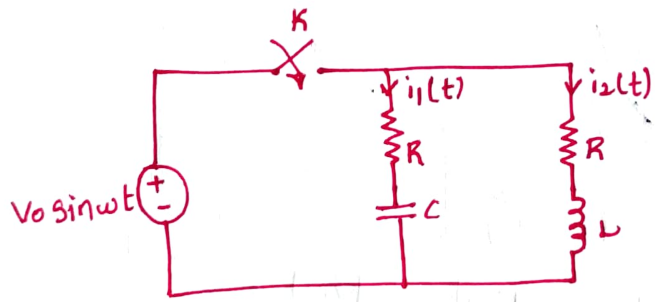
~~do~~

$$2\pi f_1 \times 2\pi f_2 = 2\pi f_0^2$$

$$f_0^2 = f_1 f_2$$

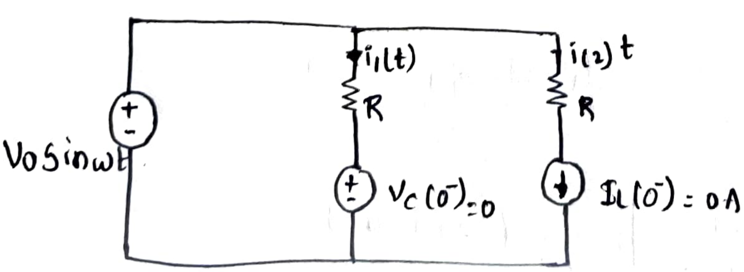
$$f_0 = \sqrt{f_1 \times f_2} \quad //$$

6 a In circuit shown in fig the switch K is closed at $t=0$ Calculate $\frac{di_1(0^+)}{dt}$ and $\frac{di_2(0^+)}{dt}$



→ draw circuit at $t=0^-$ $V_C(0^-) = 0V$ $I_L(0^-) = 0A$

circuit at $t=0^+$



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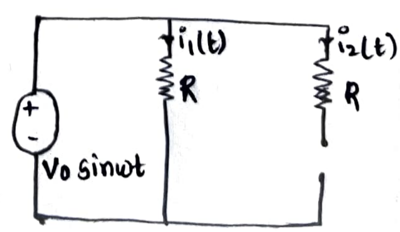
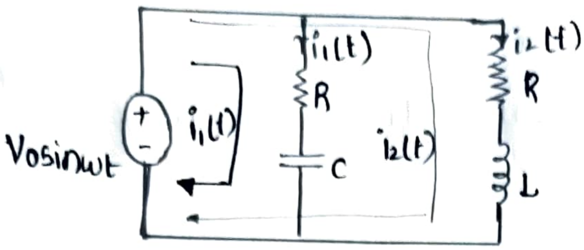


20

$i_2(0^+) = 0A$. $i_1(0^+) = \frac{V_0 \sin\omega t}{R}$



KVL to $i_1 t)$

$$R i_1(t) + \frac{1}{C}\int_{0^-}^{t} i_1(t)\,dt + V(0^-) = V_0 \sin\omega t$$

Differentiating.

$$R\frac{di_1(t)}{dt} + \frac{1}{C} i_1(t) = V_0 \cos\omega t$$

KVL to $i_2(t)$

$$-R i_2(t) - L\frac{di_2}{dt} + V_0 \sin\omega t = 0$$

Substitute 0^+

$$L\frac{di_2(t)}{dt} = V_0 \sin\omega t$$

$$\boxed{\frac{di_2(0)}{dt} = \frac{V_0 \sin\omega t}{L}}$$

7) a. State and Prove initial and final Value theorem in Laplace Transform.

→ **Initial Value Theorem :-** States that the initial value of the function [f(t)] can be found as $f(0) = \lim_{S \to \infty} SF(S)$

Proof :- W.K.T $LT\left[\frac{df(t)}{dt}\right] = SF(S) - f(0)$

$$SF(S) = \int_0^\infty \frac{df(t)}{dt} e^{-st} dt + f(0)$$

$$\lim_{S \to \infty} SF(S) = \int_0^\infty \frac{df(t)}{dt}$$

$$\boxed{f(0) = \lim_{S \to \infty} SF(S)}$$

Final Value Theorem :- States that the final Value of the function f(t) can be found as $f(\infty) = \lim_{S \to 0} SF(S)$

Proof! $SF(S) = \int_0^\infty \frac{df(t)}{dt} dt + f(0)$

$$= f(t) \Big|_0^\infty + f(0)$$

$$= f(\infty) - f(0) + f(0)$$

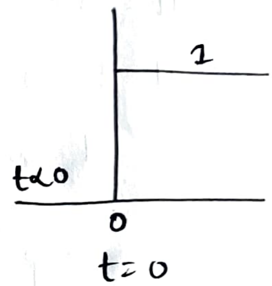
$$= f(\infty)$$

$$\boxed{f(\infty) = \lim_{s \to 0} sF(s)}$$

b] Find the Laplace Transform of Unit step function.

→ for $t < 0$ $u(t) = 0$

for $t > 0$ $u(t) = 1$



$$F(s) = \int_0^\infty f(t) e^{-st} \, dt$$

$$= \int_0^\infty u(t) e^{-st} \, dt$$

WKT $u(t) = 1$ for $t = 0$ ~~to ∞~~

$$= \int_0^\infty 1 e^{-st} \, dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^\infty$$

$$= \frac{-1}{s} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{-1}{s} \left[0 - 1 \right]$$

$$= 1/s$$

Hence Laplace Transform of unit step function is $\frac{1}{s}$.

8) a. State and Prove shifting theorem.

Statement:- If $F(S)$ is LT of $f(t)$ then $LT[f(t-t_0)] = e^{-st_0} F(S)$

Chang

Proof:- $LT[f(t-t_0)] = \int_0^\infty f(t-t_0) e^{-st} dt$

Change the Variable

let $t - t_0 = z$ Then $t = z + t_0$

$dt = dz$

at $t = 0$ $z = -t_0$

$t = \infty$ $z = \infty$

$$= \int_{-t_0}^{\infty} f(z)\, e^{-s(z+t_0)} dz$$

$$= \int_{-t_0}^{0} f(z)\, e^{-s(z+t_0)} dz + \int_0^\infty f(z)\, e^{-s(z+t_0)} dz$$

$$LT[f(t-t_0)] = e^{-st_0} \int_0^\infty f(z) e^{-zs} dz$$

$$LT[f(t-t_0)] = e^{-st_0} F(s)$$

Proved.

b) Verify initial Value theorem, given $f(t) = 10\,e^{5t}$

$f(t) = 10e^{5t}$

$f(0) = 10e^{5\times0} = 10e^{0} = 10$

Verifying $f(0) = \lim_{s\to\infty} sF(s)$

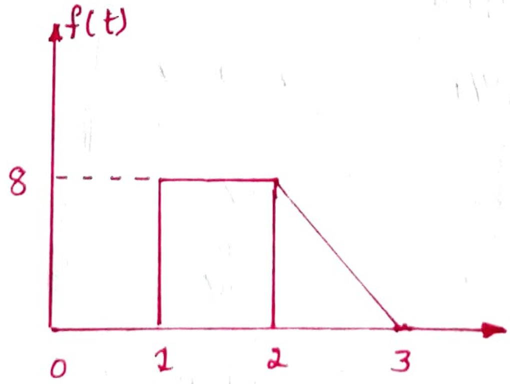
$F(s) = 10\dfrac{1}{s-5}$

$sF(s) = \dfrac{10s}{s-5} = \dfrac{10\cancel{s}}{\cancel{s}(1-5/s)} = \dfrac{10}{1-5/s}$ $\quad s\to\infty$

$f(0) = sF(s)$

$\boxed{f(0) = 10}$ Verified.

c] Find the Laplace transform of the fig shown.



$f_1(t) = +8u(t-1)$

$f_2(t) = -16r(t-2)$

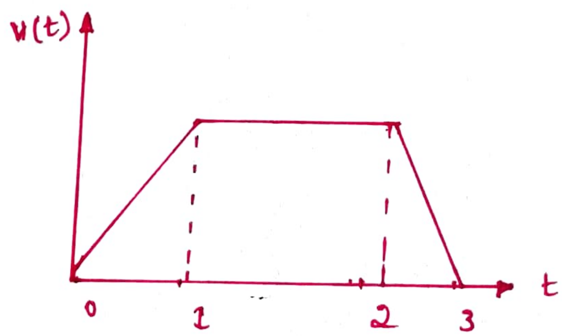
$f_3(t) = 8r(t+3)$

$f(t) = f_1(t) + f_2(t) + f_3(t) = 8u(t-1) - 16r(t-2) + 8r(t+3)$

taking Laplace Transform of f(t)

$$LT[f(t)] = \frac{8e^{-s}}{s} - \frac{16e^{-2s}}{s^2} + \frac{8e^{2s}}{s^2}.$$

7c) Find the Laplace Transform of fig show.



	Slope Before	slope after	Change	
0	0	V/1	V − 0 = V	$Vr(t)$
1	V/1	0	0 − V = −V	$-Vr(t-1)$
2	0	−V	−V	$-Vr(t-2)$
3	−V	0	0 − (−V) = V	$Vr(t-3)$

$V(t) = Vr(t) - Vr(t-1) - Vr(t-2) + Vr(t-3)$

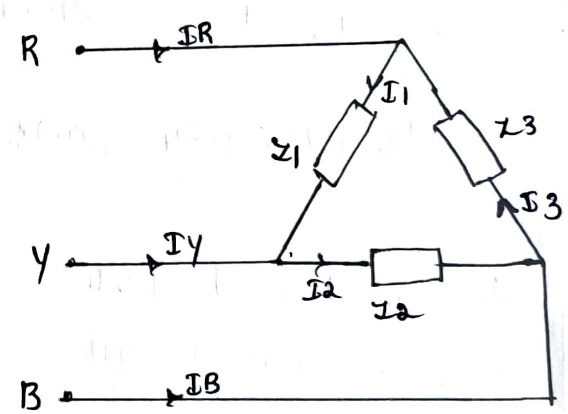
Taking Laplace transform.

$$LT[V(t)] = V\frac{1}{s^2} - \frac{Ve^{-s}}{s^2} - \frac{Ve^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

MODULE - 5

9]a] Three impedence $Z_1 = 20\angle 30\ \Omega$, $Z_2 = 40\angle 60^\circ\ \Omega$ and $Z_3 = 10\angle -90$ ohm are delta connected to 400V, 3 phase systems as shown in fig. Determine the i) Phase current ii) Line Current.

→ i) Phase current



$\bar{V}_{RY} = 400\angle 0^\circ$ V

$\bar{V}_{YB} = 400\angle -120$ V

$\bar{V}_{BR} = 400\angle -240$ V

$$\bar{I}_1 = \frac{\bar{V}_{RY}}{Z_1} = \left(\frac{400\angle 0}{20\angle 30}\right) = 20\angle -30 \text{ or } 17.32 - j10\text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}_{YB}}{Z_2} = \frac{(400\angle -120)}{(40\angle 60)} = -10 + j0\text{ A}$$

$$\bar{I}_3 = \frac{\bar{V}_{BR}}{Z_3} = \frac{(400\angle -240)}{(10\angle -90)} = -34.64 - j20\text{ A.}$$

Line current :-

KCL

$I_R - I_1 + I_3 = 0$

$I_R = I_1 - I_3 = (17.32 - j10) - (-34.64 - j20)$

$I_R = (51.96 - j10)\ A$

$I_y + I_1 - I_2 = 0$

$I_y = I_2 - I_1 = (-10 + j0) - (17.32 - j10)$

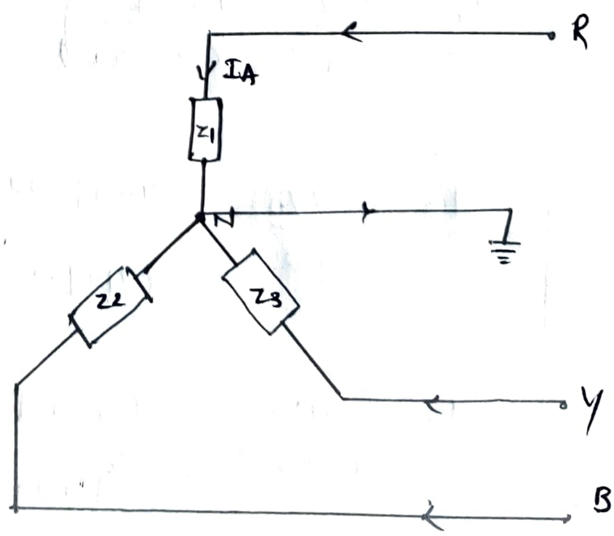
$I_y = (-117.32 + j10)\ A$

$I_B + I_2 - I_3 = 0$

$I_B = I_3 - I_2 = (-34.64 - j20) - (-10 + j0)$

$I_B = (-24.64 - j20)\ A$

10 a) An Unbalanced 4 wire star connected, Star connected load has a balanced Voltage of 400V the loads are $Z_1 = (4 + j8)\ \Omega$ $Z_2 = (3 + j4)$ $Z_3 = (15 + j20)\ \Omega$ Calculate the line & Neutral current.



$V_{RN} = \frac{400 \angle 0}{\sqrt{3}} = 230.9V$

$V_{YN} = 230.9 \angle -120\ V$

$V_{BN} = 230.9 \angle -240\ V$

$V_{RY} = 400 \angle 30$

$V_{YB} = 400 \angle -90$

$V_{BR} = 400 \angle -240$

Line current

$$I_A = \frac{V_{AN}}{Z_1} = \frac{(230.9 \angle 0)}{(4+j8)} = (11.5 - j\,23.09)A$$

$$I_B = \frac{V_{YN}}{Z_2} = \frac{(230.9 \angle -120)}{(3+j4)} = (-45.85 - j5.52)A.$$

$$I_C = \frac{V_{BN}}{Z_3} = \frac{(230.9 \angle -240)}{(15+j20)} = (3.63 + j8.49)A.$$

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10 c) Define H parameter with necessary equation:

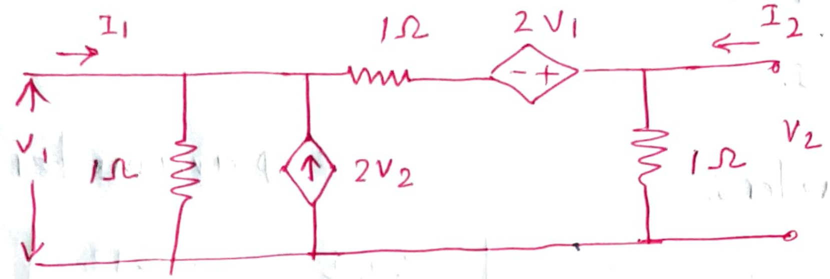
→ Parameter relating H Variable called H Parameter. the term Hybrid was choosen because of the mixture for Variable V & I in each equation.

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

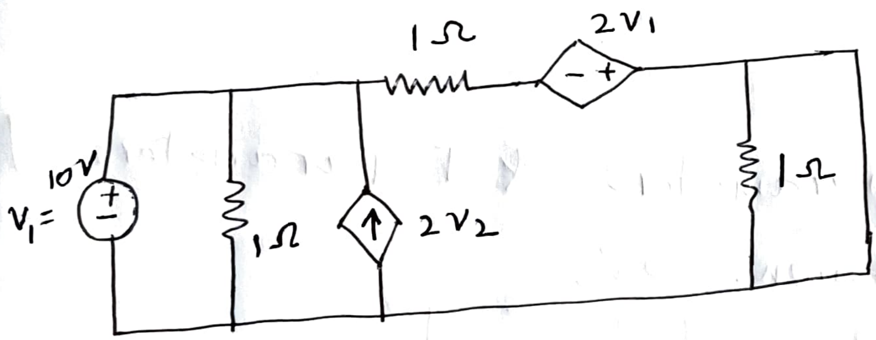
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

9b. Determine Y-Parameters for the circuit shown.



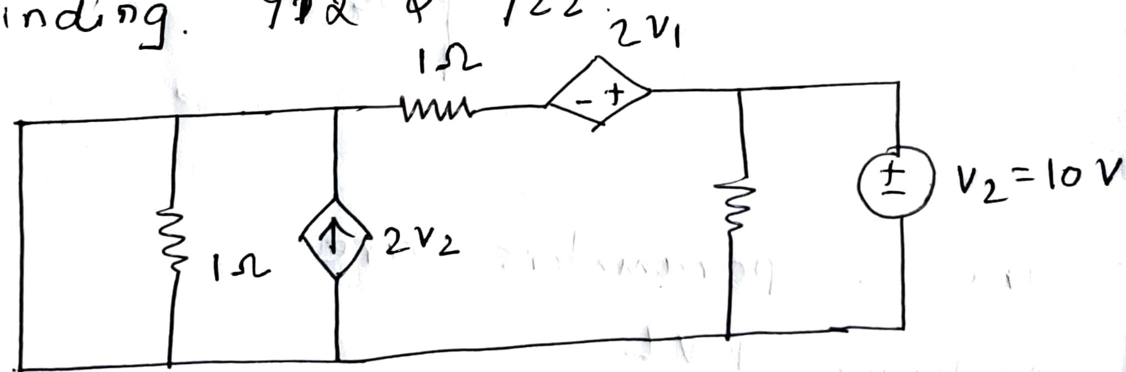
Sol^n: A. Finding y_{11} & y_{21}.

$$y_{11} = \frac{I_1}{V_1} \quad , \quad y_{21} = \frac{I_2}{V_1} \quad \text{with } V_2 = 0.$$



$$\therefore y_{11} = \frac{+40}{10} = +4\,℧ \qquad y_{12} = \frac{-30}{10} = -3\,℧$$

B. Finding y_{12} & y_{22}



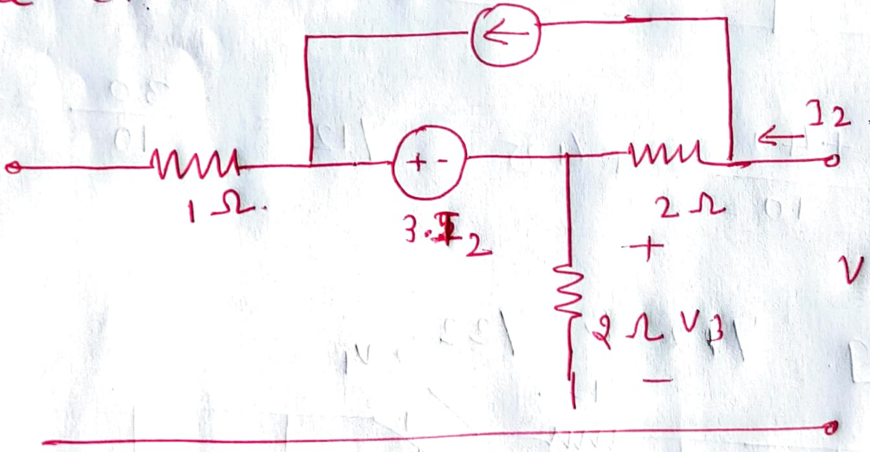
$$y_{12} = \frac{I_1}{V_2} = \frac{30}{10} = 3\,℧ \qquad y_{22} = \frac{20}{10} = 2\,℧$$

9c Express Y parameters in terms of Z parameters.

Sol^n: Y parameters. Z parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}.$$

10b. Find Z- parameters & T parameters for the ckt shown.



Sol^n: The Z- parameters are

Applying KVL

$$V_1 = (I_1 + 2V_3) \times 1 + 2(I_1 + I_2) \quad \text{—} (1)$$

$$V_2 = (I_2 - 2V_3) \times 3 + 2(I_1 + I_2) \quad \text{—} (2)$$

$$V_3 = 2(I_1 + I_2) \quad \text{—}$$

Putting values of V_3 in (1) & (2)

$V_1 = I_1 + 4I_1 + 4I_2 + 2I_1 + 2I_2$

$V_1 = 7I_1 + 6I_2$

$V_2 = (I_2 - 4I_1 - 9I_2) \times 3 + 2I_1 + 2I_2$

$V_2 = -12I_1 - 9I_2 + 2I_1 + 2I_2$

$V_2 = -10I_1 - 7I_2$

$\therefore$ $Z_{11} = 7, \quad Z_{12} = 6\,\Omega, \quad Z_{21} = -10, \quad Z_{22} = -7.$

$\therefore$ $A = \frac{Z_{11}}{Z_{21}} = \frac{7}{-10} = -0.7$

$B = \frac{|Z|}{Z_{21}} = 0.6$

$C = \frac{1}{Z_{21}} = \frac{1}{-10} = -0.1$

$D = \frac{Z_{22}}{Z_{11}} = \frac{-7}{7} = -1.$

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