

VSSN



V33

CBGS SCHEME**Third Semester B.E. Degree Examination, Jan./Feb. 2021**
Fluid Mechanics

Time: 3 hrs

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Assume missing data (if any) suitably.

Module-1

1. a. Define the following and mention their units
(i) Capillarity (ii) Surface tension (iii) Viscosity (06 Marks)
b. Derive an expression for capillary rise/fall of liquid in a tube of small diameter with sketches. (06 Marks)
c. A 100 mm diameter cylinder rotates concentrically inside a 105 mm diameter fixed cylinder. The length of both the cylinders is 250 mm. Find the viscosity of the liquid that fills the space between the cylinders, if a torque of 10 N-m is required to maintain a rotating speed of 120 rpm. (06 Marks)

OR

2. a. State and prove Pascal's law - the incompressibility of pressure at a point in a static fluid. (06 Marks)
b. Derive an expression for difference in pressure between two points using a U-tube differential manometer. (06 Marks)
c. Determine the pressure intensity at the bottom of a tank filled with an oil of specific gravity 0.7 to a height of 10 m. (06 Marks)

Module-2

3. a. Define: (i) Total pressure (ii) Center of pressure (06 Marks)
b. Derive an expression for total pressure and center of pressure for an inclined plane surface submerged in a liquid. (06 Marks)
c. A 1200 mm x 1800 mm size rectangular plate is immersed in water with an inclination of 30° to the horizontal. The 1200 mm side of the plate is kept horizontal at a depth of 30 m below the water surface. Compute the total pressure on the surface and the position of center of pressure. (06 Marks)

OR

4. a. Differentiate between:
(i) Uniform and non-uniform flow
(ii) Steady and unsteady flow (06 Marks)
b. Derive continuity equation for a three dimensional flow in Cartesian coordinates. (06 Marks)
c. Evaluate stream function ψ and compute velocity of flow, V , for a two-dimensional flow field given by, $u = 4x^3$ and $v = -12x^2y$ at point (1, 2). Assume $\psi = 0$ at point (0, 0). (06 Marks)

Module-3

- a. State Impulse Momentum principle. Give fields where it is applied. (06 Marks)
b. Derive an expression for force exerted by a fluid on a pipe bend. (06 Marks)
c. A pipe of 300 mm diameter, carrying 15000 litres per minute of water is bent by 135°. Find the magnitude and direction of resultant force exerted by the flowing fluid on the bend if the pressure of the flowing water is 39.24 N/cm². (06 Marks)

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OR

6. a. What is ventur effect? Derive an expression for discharge through a venturimeter. (06 Marks)
 b. A pitot tube fixed in a pipe of 300 mm diameter is used to measure the velocity and rate of flow. If the stagnation and static pressure heads are 6.0 m and 5.0 m respectively, compute the velocity and rate of flow. Assume $C_v = 0.98$ for the pitot tube. (06 Marks)
 c. A 20 cm \times 10 cm venturimeter is used to measure the flow of water in a horizontal pipe. The pressure at the inlet of venturimeter is 17.658 N/cm² and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through the venturimeter assuming $C_d = 0.98$. (06 Marks)

Module-4

7. a. Define hydraulic coefficients for an orifice and give the relation between them. (06 Marks)
 b. Give classification of mouth pieces with suitable sketches. (06 Marks)
 c. A jet of water issuing from an orifice 25 mm diameter under a constant head of 1.50 m, falls 0.915 m vertically before it strikes the ground at a horizontal distance of 2.288 m from vena-contracta. The discharge is found to be 102 litres per minute. Calculate the hydraulic coefficients of the orifice. (08 Marks)

OR

8. a. Enumerate advantages of triangular notches over rectangular notches. (04 Marks)
 b. Derive the expression for discharge through a triangular notch. (08 Marks)
 c. A river 60 m wide has vertical banks and 1.50 m depth of flow. The velocity of flow is 1.20 m/s. A broad crested weir 2.40 m high is constructed across the river. Find the head on the weir crest considering the velocity of approach. Assume $C_d = 0.90$. (08 Marks)

Module-5

9. a. Derive Darcy-Weisbach equation for head loss due to friction in a pipe. (08 Marks)
 b. List major and minor losses in pipe flow. (04 Marks)
 c. Water is required to be supplied to a colony of 4000 residents at a rate of 180 litres per person from a source 3 km away. If half the daily requirement needs to be pumped in 8 hours against a friction head of 1.1 m, find the size of the main pipe supplying water. Assume friction factor as 0.028. (08 Marks)

OR

10. a. What is an equivalent pipe? Derive an expression for diameter of an equivalent pipe. (08 Marks)
 b. Explain phenomena of water hammer in pipes. (04 Marks)
 c. Water is flowing in a pipe of 150 mm diameter with a velocity of 2.5 m/s, when it is suddenly brought to rest by closing the valve. Find the pressure rise in the pipe assuming it to be elastic with $E = 206 \text{ GN/m}^2$ and Poisson's ratio = 0.25. The bulk modulus of water, $K = 206 \text{ GN/m}^3$. Thickness of pipe wall is 5 mm. (08 Marks)

1a

Module-1

Ques. **i)** **Capillarity :-** Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.

06

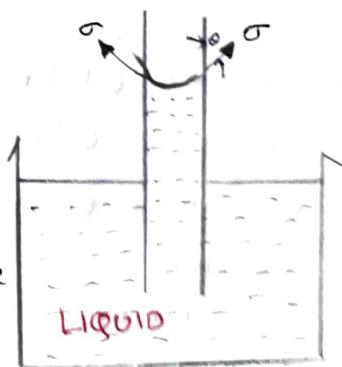
ii) Surface tension :- Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension, it is expressed as kgf/m.

iii) Viscosity :- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

1b. Expression for capillary rise :-

Consider a glass tube of small diameter 'd' opened at both ends & is inserted in a liquid, say water.

The liquid will rise in the tube above the level of the liquid.



Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

06

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

the weight of liquid of height h in the tube:

$$= (\text{Area of tube}) \times h \times \rho \times g$$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad \text{--- (i)}$$

where ρ = Density of liquid

Vertical Component of the Surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos \theta$$

$$= \sigma \times \pi d \times \cos \theta. \quad \text{--- (ii)}$$

for equilibrium equating (i) & (ii) we get

$$\frac{\pi}{4} (d^2) \times h \times \rho \times g = \sigma \times \pi d \times \cos \theta$$

$$h = \frac{\sigma \times \pi d \times \cos \theta}{\pi/4 d^2 \times \rho \times g} = \frac{4 \sigma \cos \theta}{\rho \times g \times d}$$

the value of θ between water and clean glass tube is approximately equal to zero and hence $\cos \theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

1C.

Given:-

$$d_1 = 105 \text{ mm} = 0.105 \text{ m}$$

$$d_2 = 100 \text{ mm} = 0.1 \text{ m}, A_1 = \pi/4 (0.1)^2 = 0.00785 \text{ m}^2$$

$$L = 250 \text{ mm} = 0.25 \text{ m}$$

$$T = 1.0 \text{ N-m}$$

$$N = 120 \text{ r.p.m}$$

$$t = 5 \text{ mm} = 0.005 \text{ m}$$

$$u = \frac{\pi D N}{60} = \frac{\pi (0.1) (120)}{60} = 0.62 \text{ m/s}, T = \text{Force} \times \frac{D}{2}$$

$$\text{Force} = \frac{T \times 2}{D} = 20 \text{ N.}$$

08

force = Shear Stress \times Area. $\Rightarrow 20 = \text{Shear stress} \times 0.00785$

$$\therefore \text{Shear stress} = \tau = 2547.7 \text{ N/m}^2$$

$$\therefore \tau = y \cdot \frac{du}{dy} \quad \text{--- (iii)} \quad y = \frac{\tau \times dy}{du} = \boxed{y = 3.159 \text{ N-1/m}^2}$$

Qo. **PASCAL'S LAW** :- It States that the pressure or Intensity of pressure at a point in a Static fluid is equal in all directions this is proved as.

The fluid element is of very small dimensions i.e. $dm \cdot dy$ and ds .

Consider an arbitrary fluid element of wedge shape in a fluid, mass at rest

The force acting on the element are :-

1. pressure forces normal to the surface
2. weight of element in the Vertical direction

The forces on the faces are

$$\text{Force on the face AB} = P_n \times \text{Area of face AB}$$

$$= P_n \times dy \times 1$$

Similarly force on the face AC = $P_y \times dm \times 1$

Force on the face BC = $P_z \times ds \times 1$

$$\text{Weight of element} = (\text{Mass of element}) \times g$$

$$= (\text{Volume} \times \rho)g = \left(\frac{AB \times AC}{2} \times 1 \right) \times \rho \times g$$

where ρ = density of fluid

Resolving the forces in n -direction, we have

$$P_n \times dy \times 1 - P_z \times ds \times 1 \sin(90^\circ - \theta) = 0$$

$$\text{or } P_n \times dy \times 1 - P_z \times ds \times 1 \cos\theta = 0.$$

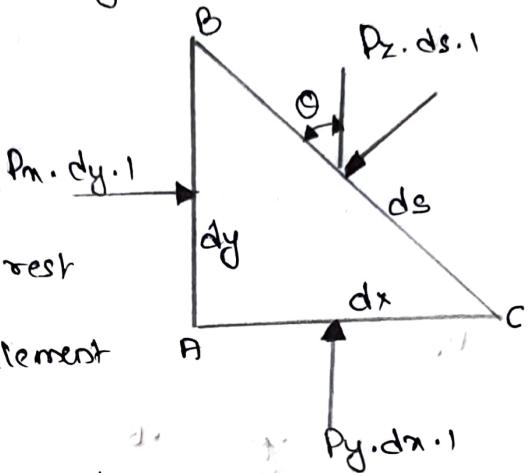
We know that $ds \cos\theta = AB = dy$

$$\therefore P_n \times dy \times 1 - P_z \times dy \times 1 = 0 \quad \text{or } P_n = P_z \quad \text{--- (1)}$$

Similarly, resolving the forces in y -direction we get

$$P_y \times dm \times 1 - P_z \times ds \times 1 \cos(90^\circ - \theta) - \frac{dy \times dy \times 1 \times \rho \times g}{2} = 0$$

$$\textcircled{1} \quad P_y \times dm - P_z \times ds \sin\theta - \frac{dm \cdot dy}{2} \times \rho \times g = 0$$



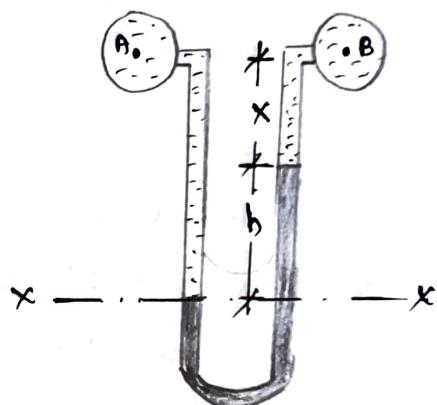
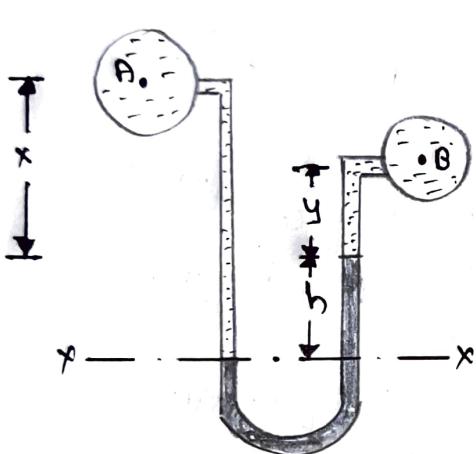
But $dS \cdot g \sin \theta = dm$ and also the element is very small and hence weight is negligible

$$\therefore P_y \cdot dm - P_z \cdot dm = 0 \quad \text{or} \quad P_y = P_z \quad \text{--- (2)}$$

from equation (1) & (2) we have

$$P_m = P_y = P_z$$

2b. U-tube differential Manometer :-



Let h = Difference of mercury level in the U-tube

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_g = Density of heavy liquid or mercury

Pressure above $x-x$ in the left limb = $\rho_g \cdot g \cdot (h+y) + P_A$

where P_A = Pressure at A

Pressure above $x-x$ in the right limb = $\rho_g \cdot g \cdot h + \rho_2 \cdot g \cdot y + P_B$

where P_B = Pressure at B

Equating the two pressure we get

$$\rho_g \cdot g \cdot (h+y) + P_A = \rho_g \cdot g \cdot h + \rho_2 \cdot g \cdot y + P_B$$

$$P_A - P_B = \rho g \times g \times h + \rho g y - \rho g (h+n) \\ = h \times g (\rho g - \rho_i) + \rho g y - \rho g n$$

from fig (b) the two points A & B are at the same level and contains the same liquid of density ρ_i . Then pressure above x-x in right limb = $\rho_i \times g \times h + P_B$
 Pressure above x-x in left limb = $\rho_i \times g \times (h+n) + P_A$

Equating the two pressure.

$$\rho_i \times g \times h + \rho_i g n + P_B = \rho_i \times g \times (h+n) + P_A$$

$$P_A - P_B = \rho_i \times g \times h + \rho_i g n - \rho_i \times g \times (h+n)$$

$$P_A - P_B = g \times h (\rho_i - \rho)$$

Q.C.

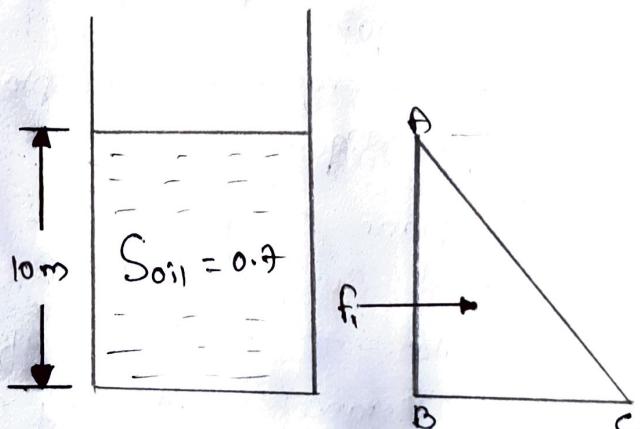
Given:-

$$\text{Depth of oil} = d = 10\text{m}$$

$$\text{Sp. gravity of oil} = S_{oil} = 0.7$$

$$\text{Density of oil} = \rho_{oil} =$$

$$\rho_{oil} = 0.7 \times 1000 = 700 \text{ kg/m}^3$$



Intensity of pressure at A = 0

Intensity of pressure at B = $P_{i,high}$

$$= 700 \times 9.81 \times 10$$

$$P_B = 68670 \text{ N/m}^2$$

Q.D.

Module - 2

3a. **if Total Pressure :-** Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surface. this force always acts normal to the surface.

ii. Centre of pressure :- It is defined as the point of application of the total pressure on the surface. there are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined.

3b. **Inclined plane Surface Submerged in Liquid :-**

Let A = Total area of inclined surface

h = Depth of C.G of inclined area from free surface

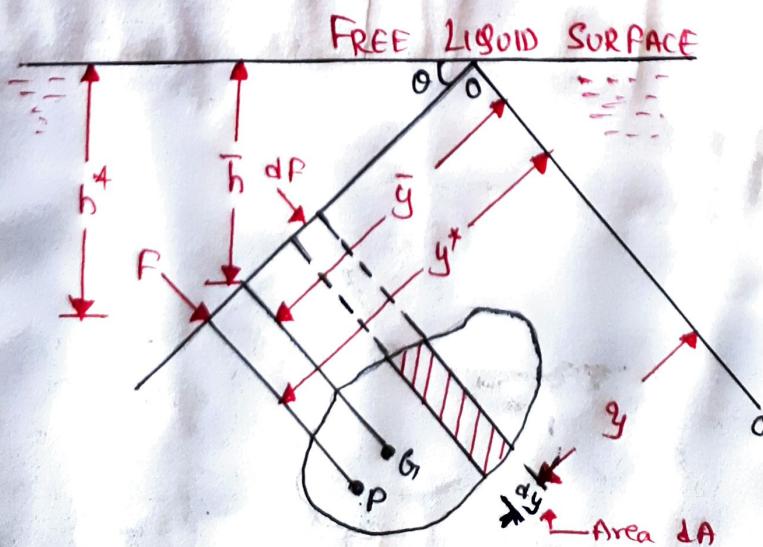
h^* = Distance of Centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. then O-O is the axis perpendicular to the plane of the surface.

Let \bar{y} = distance of the C.G of the inclined surface O-O

y^* = distance of the centre of pressure from O-O.



Consider a small strip of area dA at a depth 'h' from free surface and at a distance 'y' from the axis O-O

Pressure intensity on the strip $p = \rho gh$

$$\therefore \text{Pressure force, } df, \text{ on the strip } dA = p \times \text{area of strip}$$

$$= \rho gh \times dA$$

Total pressure force on the whole area, $F = \int df = \int \rho g h dA$

$$\text{But } \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$$

$$\therefore h = y \sin\theta$$

$$\therefore F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y \cdot dA$$

$$\text{But } \int y \cdot dA = A\bar{y}$$

where \bar{y} = Distance of Cg from axis O-O

$$F = \rho g \sin\theta \bar{y} \times A = \rho g A \bar{y}$$

Center of pressure (h^*) $df = \rho g h \cdot dA = \rho g y \cdot \sin\theta \cdot dA$

Moment of the force df about axis O-O

$$= df \times y = \rho g y \cdot \sin\theta \cdot dA \times y = \rho g \sin\theta y^2 \cdot dA$$

Sum of moments $= \int \rho g \sin\theta y^2 \cdot dA = \rho g \sin\theta \int y^2 \cdot dA$

$$\text{But } \int y^2 \cdot dA = I_o$$

Moment of the force $= F \times y^*$

$$\therefore F \times y^* = \rho g \sin\theta I_o$$

$$y^* = \frac{\rho g \sin\theta I_o}{F}$$

$$y^* = \frac{h^*}{\sin\theta}; \quad F = \rho g A \bar{y}$$

$$I_o = \bar{y} + A\bar{y}^2$$

Substituting these values we get

$$\frac{h^*}{\sin \theta} = \frac{\rho g \sin \theta}{\rho g A h} [Ig + A \bar{y}^2]$$

$$\therefore h^* = \frac{\sin^2 \theta}{A h} [Ig + A \bar{y}^2]$$

But $\frac{h^*}{\bar{y}} = \sin \theta$ or $\bar{y} = \frac{h^*}{\sin \theta}$

$$\therefore h^* = \frac{\sin^2 \theta}{A h} \left[Ig + A \times \frac{\bar{y}^2}{\sin^2 \theta} \right]$$

$$h^* = \frac{Ig \sin^2 \theta}{A h} + \bar{y}$$

Q.C.

Given:-

$$b = 1200 \text{ mm} = 1.2 \text{ m}$$

$$d = 1800 \text{ mm} = 1.8 \text{ m}$$

$$A = b \times d = 1.2 \times 1.8 = 2.16 \text{ m}^2$$

$$\bar{h} = 30 + 0.9 \sin 30^\circ = 30.45 \text{ m}$$

$$\theta = 30^\circ$$

$$\therefore F = \rho g A \bar{h} = 1000 \times 9.81 \times 30.45 \times 2.16 = 1042.42 \text{ N}$$

$$\therefore h^* = \frac{Ig \sin^2 \theta}{A h} + \bar{y}$$

$$Ig = \frac{bd^3}{12} = \frac{1.2(1.8)^3}{12} = 0.58 \text{ mt}$$

$$\therefore h^* = \frac{0.58 \sin^2(30^\circ)}{2.16(30.45)} + 30.45 = 30.45 \text{ m}$$

$$h^* = 30.45 \text{ m}$$

08

4a. Uniform and Non-uniform flow :-

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space.

Non-Uniform flow is that type of flow in which flow velocity at any given time changes with respect to space.

4b. Steady and Unsteady flow :-

Steady flow is defined as that type of flow in which the fluid characteristics like Velocity, pressure density etc., at a point do not change with time.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time.

4b. Continuity Equation in three-dimensions :-

Consider a fluid element of length $dm \cdot dy \cdot dz$ in the direction of x, y, z . Let u, v, w are the two inlet velocity components in x, y, z directions respectively.

Mass of fluid entering the face ABCD per second.

$$= \rho \times \text{velocity in } x\text{-direction} \times \text{Area of ABCD}$$

$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face EFGH per second.

$$= \rho \times u \times dy \times dz + \frac{\partial}{\partial n} (\rho u dy dz) dz$$

08

\therefore gain of mass in x -direction = Mass through ABCD

- Mass through EFGH per second

$$= \rho u dy dz - \rho u dy dz - \frac{\partial}{\partial n} (\rho u dy dz) dz$$

$$= - \frac{\partial}{\partial n} (\rho u dy dz) dz$$

$$= - \frac{\partial}{\partial n} (\rho u) dm \cdot dy \cdot dz$$

Similarly, the net gain of mass in y -direction ..

$$= -\frac{\partial}{\partial y} (\rho u) dx \cdot dy \cdot dz$$

$$= -\frac{\partial}{\partial z} (\rho w) dx \cdot dy \cdot dz$$

\therefore Net gain of Masses

$$= -\left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx \cdot dy \cdot dz$$

Equating the two expressions

$$-\left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx \cdot dy \cdot dz = \frac{\partial P}{\partial t} dx \cdot dy \cdot dz$$

(or)

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0.$$

The above equation is the continuity equation.

4C.

Given:-

$$u = 4x^3, v = -12x^2y \text{ at } P(1, 2)$$

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x} [4x^3] = 12x^2$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y} [-12x^2y] = -24xy$$

$$\therefore u = 12x^2 = 12(1)^2 = 12 \text{ Unit/sec}, v = -24xy = -24(1)(2) = -48 \text{ Unit/sec}$$

$$\therefore P = \sqrt{(12)^2 + (-48)^2} = 49.43 \text{ Units/sec.}$$

$$\frac{\partial \psi}{\partial y} = -u = -4x^3 \quad \text{--- (1)}, \quad \frac{\partial \psi}{\partial x} = v = -24xy \quad \text{--- (2)}$$

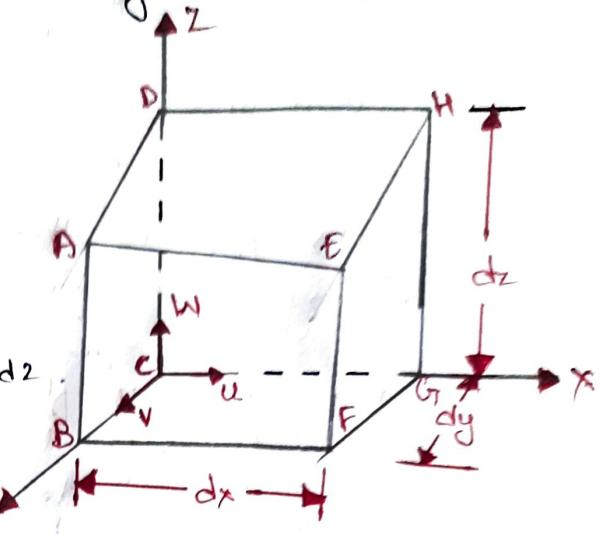
Integrating (1) we get .

$$\int d\psi = f(x^3) = \psi = -4x^3 + K.$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x} \quad \therefore \frac{\partial \psi}{\partial x} = -24xy. \quad \text{i.e. } \frac{\partial K}{\partial x} = -24xy.$$

$$K = \int -24xy = -24 \cdot \frac{x^2}{2} \cdot y = -12x^2y.$$

$$\therefore \boxed{\psi = -4x^3 - 12x^2y}$$



08

MODULE - 3

5a. Impulse Momentum Principle is "The net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction."

Application:-

04

1. Applicable in pipe network Systems
2. To identify the force exerted by a fluid flowing in a pipe bend.

5b. Force Exerted by a fluid flowing in a pipe bend:-

Consider two sections (1) & (2), as shown in fig

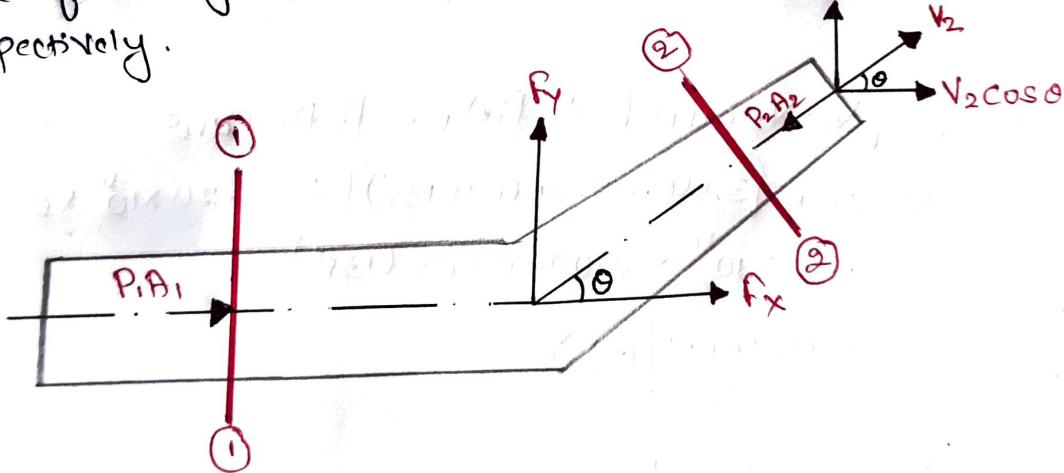
v_1 = Velocity of flow at section (1)

p_1 = Pressure intensity at section (1)

A_1 = Area of cross-section of pipe at section (1) &

v_2 , p_2 & A_2 = Velocity, Pressure & Area at Section (2).

Let F_x & F_y be the components of the forces exerted by the flowing fluid on the bend in x - and y -direction respectively.



Net force acting on fluid in the direction of x = Rate of change of momentum in x -direction.

$$\begin{aligned} \therefore p_1A_1 - p_2A_2 \cos\theta - F_x &= (\text{Mass per sec}) (\text{change of velocity}) \\ &= \rho Q (\text{final velocity} - \text{initial velocity}) \\ &= \rho Q (v_2 \cos\theta - v_1) \end{aligned}$$

$$\therefore F_x = \rho Q (v_1 - v_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta$$

08

Similarly the momentum equation in y-direction is

$$0 - P_2 A_2 \sin \theta - f_y = P \varphi (V_2 \sin \theta - 0)$$

$$f_y = P \varphi (-V_2 \sin \theta) - P_2 A_2 \sin \theta$$

Now the resultant force (F_R) acting on the bend.

$$F_R = \sqrt{F_x^2 + F_y^2}$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan \theta = \frac{F_y}{F_x}$$

SC.

Given:-

$$g = 15000 \text{ lit/min} = \frac{15000}{1000 \times 60} = 0.25 \text{ m}^3/\text{sec}$$

$$\theta = 135^\circ$$

$$P = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$D_1 = 300 \text{ mm} = 0.3 \text{ m}, A_1 = \pi/4 (D)^2 = 0.0706 \text{ m}^2$$

$$D_2 = 300 \text{ mm} = 0.3 \text{ m}, A_2 = 0.0706 \text{ m}^2$$

$$\therefore V_1 = V_2 = 3.54 \text{ m/s}, P_1 = P_2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$\begin{aligned} \therefore F_R &= P \varphi [V_1 - V_2 \cos \theta] + P A_1 - P_2 A_2 \cos \theta \\ &= 1000 \times 0.25 [3.54 - 3.54 (\cos 135^\circ)] + 39.24 \times 10^4 \times 0.0706 \\ &\quad - 39.24 \times 10^4 \times 0.0706 \cos (135^\circ) \end{aligned}$$

$$\therefore F_R = 29213.44 \text{ N} \quad (1)$$

$$F_y = P \varphi [-V_2 \sin \theta] - P_2 A_2 \sin \theta$$

$$= 1000 \times 0.25 [-3.54 \sin (135^\circ)] - 39.24 (0.0706 \sin 135^\circ)$$

$$F_y = 245.5 \text{ N.} \quad (1)$$

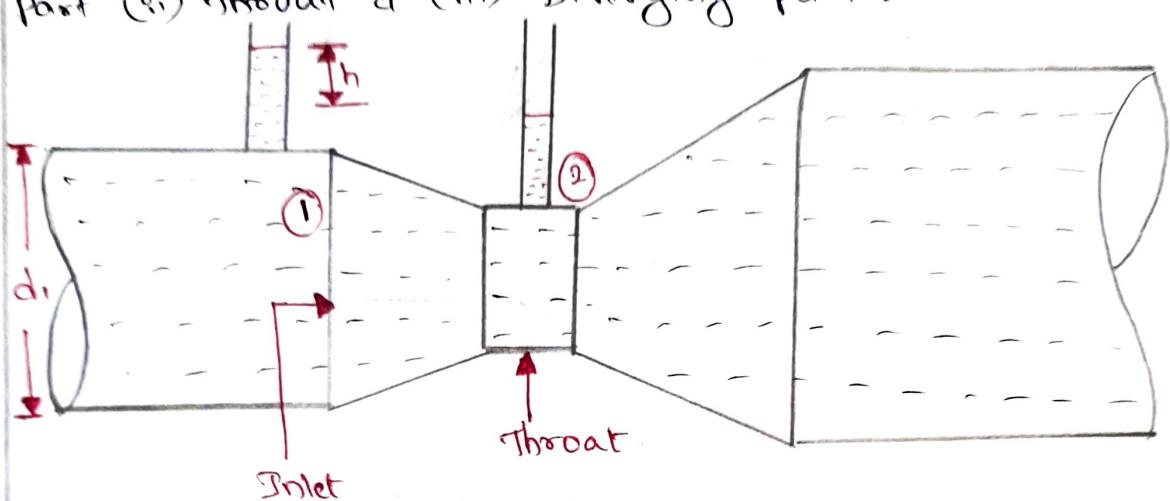
$$\therefore \text{Resultant force } F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(29213.44)^2 + (245.5)^2}$$

$$F_R = 29214.03 \text{ N}$$

08

6a. **Venturiometer**: A Venturiometer is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of (i) A short constricting part (ii) throat & (iii) Diverging part.



Let d_1 = diameter at inlet or at Section (1)

P_1 = Pressure at Section (1)

v_1 = Velocity of fluid at section (1)

a = area at section (1) = $\pi/4 d_1^2$

and d_2, P_2, v_2, a_2 are corresponding values at section (2)

08

Applying Bernoulli's equation at sections (1) & (2) we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads

$$\text{i.e. } h = \frac{P_1 - P_2}{\rho g} \quad \text{or} \quad h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Now applying continuity equation at sections 1 & 2

$$a_1 V_1 = a_2 V_2 \quad \text{or} \quad V_1 = \frac{a_2 V_2}{a_1}$$

Substituting this value of V_1 in equation,

$$h = \frac{V_2^2}{2g} - \frac{(a_2 V_2 / a_1)^2}{2g} + \frac{V_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2} \right] = \frac{V_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2} \right]$$

$$V_2^2 = agh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \sqrt{agh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

\therefore Discharge $Q = a_2 V_2$

$$= a_2 \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Actual discharge will be less than theoretical discharge.

$$\therefore Q_{\text{act}} = cd \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}.$$

6b.

Given :-

$$d = 300 \text{ mm} = 0.3 \text{ m}, A = \pi/4 (0.3)^2 = 0.0706 \text{ m}^2$$

h_s = Stagnation pressure = 6.0 m

h_t = Static pressure = 5.0 m

$$\therefore h = h_s - h_t = 6 - 5 = 1 \text{ m}$$

$$C_v = 0.98$$

$$V = C_v \sqrt{2gh} = 0.98 \times \sqrt{2(9.81)(1)} = 4.34 \text{ m/s.}$$

$$V = 4.34 \text{ m/s}$$

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$$Q = A \times V = 0.0706 \times 4.34 = 0.306 \text{ m}^3/\text{s}$$

$$Q = 0.306 \text{ m}^3/\text{s}$$

6C. Given :-

$$d_1 = 20 \text{ cm}$$

$$a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$$

$$P_1 = 12.658 \text{ N/cm}^2 = 12.658 \times 10^4 \text{ N/m}^2$$

$$= 1000 \text{ kg/m}^3 \text{ & } \therefore \frac{P_1}{\text{sg}} = \frac{12.658 \times 10^4}{9.81 \times 1000} = 1.286 \text{ m of water}$$

$$\frac{P_2}{\text{sg}} = -30 \text{ cm of mercury}$$

$$= -0.30 \text{ m of mercury} = -0.30 \times 13.6 = -4.08 \text{ m of H}_2\text{O}$$

$$\therefore \text{Differential head} = h = \frac{P_1}{\text{sg}} - \frac{P_2}{\text{sg}} = 1.286 - (-4.08) = 5.37 \text{ m of H}_2\text{O}$$

The discharge Q is given by

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = 0.98 \times 314.16 \times$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 9.81 \times 5.37}$$

$$= \frac{5032883.21}{304} \times 165555 \text{ cm}^3/\text{s} = 165.55 \text{ lit/sec}$$

$$Q = 165.55 \text{ lit/sec}$$

7a. Hydraulic Co-efficients :-

1. Co-efficient of Velocity C_v
2. Co-efficient of Contraction C_c
3. Co-efficient of discharge C_d

MODULE - 4

1. Co-efficient of Velocity (C_v): It is defined as the ratio between the actual velocity of a jet of liquid at Vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, C_v is

$$C_v = \frac{\text{Actual Velocity of jet at Vena-contracta}}{\text{Theoretical Velocity}}$$

$$C_v = \frac{V}{\sqrt{gH}} \quad (C_v \text{ varies from 0.95 to 0.99})$$

2. Co-efficient of contraction (C_c): It is defined as the ratio of the area of the jet at Vena-contracta to the area of the orifice. It is denoted by C_c .

Let a = area of Orifice and

a_c = area of jet at Vena-contracta.

$$\therefore C_c = \frac{\text{Area of jet at Vena-contracta}}{\text{Area of Orifice}} = \frac{a_c}{a}$$

(C_c varies from 0.61 to 0.69)

3. Co-efficient of discharge (C_d): It is denoted as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice.

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual Velocity} \times \text{Actual Area}}{\text{Theoretical Velocity} \times \text{Theoretical Area}}$$

$$C_d = C_v \times C_c \quad (C_d \text{ varies from 0.61 - 0.65})$$

7b. Classification of Mouthpiece :-

1. The mouthpiece are classified as
a) Internal mouthpiece

b) External mouthpiece

2. The mouthpiece are classified as
a) Cylindrical (b) Convergent (c) Divergent mouthpiece

3. the mouthpiece are classified

a. Mouthpiece running full (b) Mouthpiece running free
depending upon the nature of discharge at the outlet of the mouthpiece. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the side of the mouthpiece. But if the jet after contraction expands & fills the whole mouthpiece it is known as running full.

06

7C.

Given:-

$$d = 25 \text{ mm} = 0.025 \text{ m}$$

$$a = \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (0.025)^2 = 0.00049 \text{ m}^2$$

$$H = 1.50 \text{ m}$$

$$x = 2.288 \text{ m}$$

$$y = 0.915 \text{ m}$$

$$g = 10.2 \text{ lit/sec} = \frac{10.2}{1000} = 0.102 \text{ m}^3/\text{sec} = \frac{0.102}{60} = 0.0017 \text{ m}^3/\text{sec}$$

$$C_v = ?$$

$$C_d = ?$$

$$C_c = ?$$

$$V_{th} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.5} = 5.42 \text{ m/s}$$

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{2.288}{\sqrt{4 \times 0.915 \times 1.5}} = 0.97 \quad \therefore C_v = 0.97$$

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$$Q_{act} = \frac{0.102}{60} \text{ lit/sec} = 0.0017 \text{ m}^3/\text{sec}$$

$$Q_{th} = V_{th} \times \text{Area of orifice}$$

$$= 5.42 \times 0.00049 = 0.0026 \text{ m}^3/\text{sec}$$

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.0017}{0.0026} = 0.65 \quad \boxed{C_d = 0.65}$$

$$C_c = \frac{C_d}{C_v} = \frac{0.65}{0.97} = 0.67 \quad \boxed{C_c = 0.67}$$

8a. Advantages of triangular notch or weir over rectangular notch or weir :-

1. The expression for discharge for a right angled V-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.
3. In case of triangular notch, only one reading, i.e., h is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

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8b. Discharge over a triangular notch :-

Let H = head of water above the

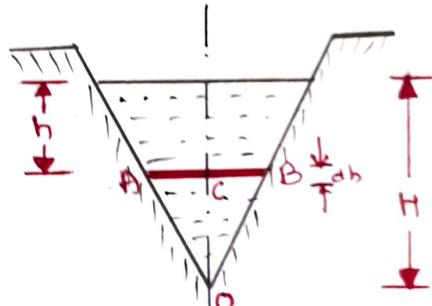
V-notch

θ = angle of notch

dh = thickness of strip.

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h) \tan \frac{\theta}{2}$$



$$\text{Width of strip} = AB = 2AC = 2(H-h) \tan \frac{\theta}{2}$$

$$\therefore \text{Area of strip} = 2(H-h) \tan \frac{\theta}{2} \times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

\therefore Discharge through the strip

$$\begin{aligned} dQ &= Cd \times \text{Area of strip} \times \text{Velocity} \\ &= Cd \times 2(H-h) \tan \frac{\theta}{2} \times dh \times \sqrt{2gh} \\ &= 2Cd (H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh \end{aligned}$$

$$\therefore \text{Total discharge } Q = \int_0^H 2Cd (H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2Cd \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h)^{1/2} dh$$

$$= 2 \times Cd \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{H^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

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8C

Given:-

$$b = 6 \text{ m}$$

$$d = 1.5 \text{ m}$$

$$V = 1.2 \text{ m/s}$$

$$H_1 = 2.4 \text{ m}$$

$$Cd = 0.90$$

$$Q = A \times V = (60 \times 1.5) \times 1.2$$

$$Q = 108 \text{ m}^3/\text{s}$$

$$Q = 2/3 \times Cd \times L \times \sqrt{2g} \times (H_1)^{3/2}$$

$$108 = 2/3 \times 0.90 \times L \times \sqrt{2 \times 9.81} \times (2.4)^{3/2}$$

$$L = 11 \text{ m}$$

$$\text{Velocity of approach} = V_a = Q/A = 108/90 = 1.2 \text{ m/s}$$

$$\text{Additional head} = h_a = V_a^2/2g = 0.0233 \text{ m}$$

Discharge with Velocity of approach

$$Q = 2/3 \times Cd \times L \times \sqrt{2g} [(H_{\text{itha}})^{3/2} - h_a^{3/2}]$$

$$= 2/3 \times 0.90 \times 11 \times \sqrt{2g} [3.88 - 0.0198]$$

$$Q = 111.47 \text{ m}^3/\text{s}$$

9a. Darcy - Weisbach Equation:-

Let P_1 = pressure intensity at Section 1-1

V_1 = velocity of flow @ 1-1

L = length of the pipe b/w 1-1 and 2-2

d = diameter of pipe

f' = frictional resistance per unit weight area.

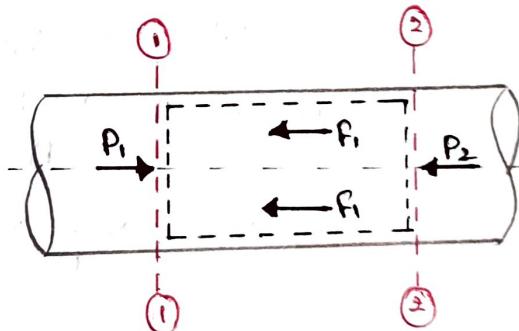
h_f = loss of head due to friction.

& P_2, V_2 = are values of pressure intensity & velocity @ 2-2

Applying Bernoulli's equation b/w Section 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$Z_1 = Z_2$ as pipe is horizontal



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MODULE - 5

$V_1 = V_2$ as dia. of pipe is same at 1-1 & 2-2

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \Rightarrow h_f = \frac{P_1 - P_2}{\rho g}$$

Frictional resistance = Frictional resistance per unit wetted area per unit velocity \times wetted area $\times V^2$

$$F_f = f' \times \pi d L \times V^2$$

$$= f' \times \rho \times L \times V^2$$

The forces acting on the fluid b/w section 1-1 & 2-2 are:

1. pressure force at section 1-1 = $P_1 \times A$

2. pressure force section 2-2 = $P_2 \times A$

3. frictional force F_f

Resolving all forces in the horizontal direction, we have

$$P_1 A - P_2 A - F_f = 0$$

$$(P_1 - P_2)A = F_f = f' \times \rho \times L \times V^2$$

$$P_1 - P_2 = \frac{f' \times \rho \times L \times V^2}{A} \quad (P_1 - P_2 = \rho g h_f)$$

Equating the value of $(P_1 - P_2)$, we get

$$\rho g h_f = \frac{f' \times \rho \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{\rho}{A} \times L \times V^2$$

$$\frac{\rho}{A} = \frac{\text{Wetted Parameter}}{\text{Area}} = \frac{\pi d}{\pi/4 d^2} = \frac{4}{d}$$

$$\therefore h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4L V^2}{d}$$

$$\text{Putting } \frac{f'}{\rho} = \frac{f}{2}$$

$$\therefore h_f = \frac{4f}{9} \cdot \frac{L V^2}{d} = \frac{4f L V^2}{2gd}$$

$$h_f = \frac{4f L V^2}{2gd}$$

9b. Major and Minor losses in a pipe flow :-

Energy losses.

1. Major Energy losses

This is due to friction & it is calculated by the following formulae

- (a) Darcy - Weisbach formula
- (b) Chezy's formula.

2. Minor Energy losses

This is due to

- (a) Sudden Expansion of pipe
- (b) Sudden Contraction of pipe
- (c) Bend in pipe
- (d) Pipe fittings etc
- (e) An Obstruction in pipe

9c. Given :-

$$Q = 180 \text{ lit/person} = \frac{180}{60} = 3 \text{ lit/sec} = 0.003 \text{ m}^3/\text{sec}$$

$$d = 3 \text{ Km} = 3000 \text{ m}$$

$$t = 8 \text{ hr}$$

$$h_f = 18 \text{ m}$$

$$\text{friction factor } f = 0.028$$

$$V = \frac{3000}{8 \times 60 \times 60} = 0.10 \text{ m/s}$$

$$Q = A \times V, \quad A = \frac{Q}{V} = \frac{0.003}{0.10} = 0.03 \cdot$$

$$\frac{\pi}{4} (D)^2 = 0.03$$

$$D = 0.19 \text{ m, or } 195 \text{ mm}$$

$$h_f = \frac{4 f L V^2}{2 g \times d} =$$

$$18 = \frac{4(0.028) \times L \times 0.10^2}{2 \times 9.81 \times 0.19} = \frac{1.120 \times 10^{-3} L}{3.728}$$

$$L = 5.9 \times 10^{-2} \text{ m}$$

04

08

10a. **Equivalent Pipe** :- This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe.

Let L_1 = length of pipe 1 & d_1 = diameter of pipe 1

L_2 = length of pipe 2 & d_2 = diameter of pipe 2

L_3 = length of pipe 3 & d_3 = diameter of pipe 3

H = total head loss

L = Length of equivalent pipe

d = diameter of the equivalent pipe

$$\text{then } L = L_1 + L_2 + L_3$$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{4f_2 L_2 V_2^2}{2g d_2} + \frac{4f_3 L_3 V_3^2}{2g d_3} \quad (9)$$

Assuming $f_1 = f_2 = f_3 = f$

$$\text{Discharge } Q = A_1 V_1 = A_2 V_2 = A_3 V_3 \Rightarrow \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4} d_3^2 V_3$$

$$\therefore V_1 = \frac{4Q}{\pi d_1^2}, \quad V_2 = \frac{4Q}{\pi d_2^2} \quad \& \quad V_3 = \frac{4Q}{\pi d_3^2}$$

Substituting these values in equation (9)

$$H = \frac{4f L_1 \left(\frac{4Q}{\pi d_1^2} \right)^2}{d_1 \times 2g} + \frac{4f L_2 \left(\frac{4Q}{\pi d_2^2} \right)^2}{d_2 \times 2g} + \frac{4f L_3 \left(\frac{4Q}{\pi d_3^2} \right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 f Q^2}{\pi^2 \times g} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right]$$

Head loss on the equivalent pipe, $H = \frac{4f L V^2}{2g d}$.

$$\text{where } V = \frac{Q}{A} = \frac{Q}{\pi A(d)} = \frac{4Q}{\pi d^2}$$

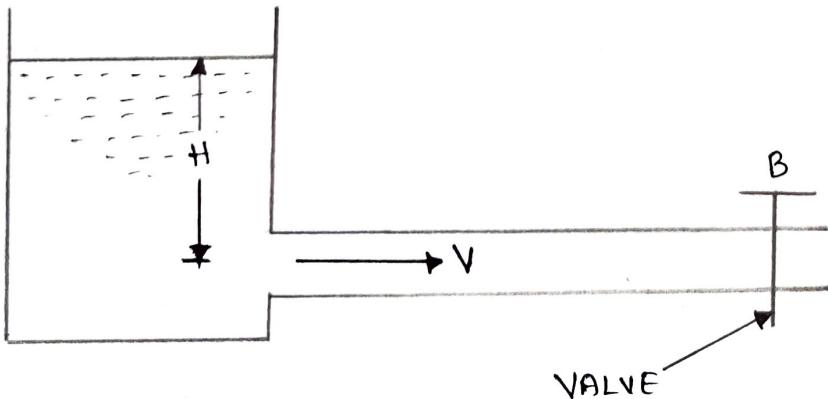
$$\therefore H = \frac{4fL(40/\pi d^2)^2}{dx \times 2g} = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g} \left[\frac{L}{d^5} \right]$$

$$\frac{4 \times 16 f g^2}{\pi^2 \times 2g} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g} \left[\frac{L}{d^5} \right]$$

(ii)
$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Qb. Water hammer in pipes :-

Consider a long pipe AB connected at one end to a tank containing water at a height of H from the centre of the pipe. At the other end of the pipe, a valve to regulate the flow of water is provided. When the valve is completely open, the water is flowing with a velocity, V. Consequently a wave of high pressure will be set up. This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking. Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is also known as water hammer.



10C.

Given :-

$$d = 150\text{mm} = 0.15\text{m}$$

$$V = 2.5 \text{ m/s}$$

$$E = 206 \text{ GPa} = 206 \times 10^9 \text{ N/m}^2$$

$$m = 0.25$$

$$K = 206 \text{ GPa} = 206 \times 10^9 \text{ N/m}^2$$

$$t = 5\text{mm} = 0.005\text{m}$$

$$P = ?$$

$$P = V \sqrt{\frac{1}{K} + \frac{P}{Et}}$$

$$= 2.5 \sqrt{\frac{1000}{206 \times 10^9} + \frac{0.15}{206 \times 10^9 \times 0.005}}$$

$$= 2.5 \sqrt{6.6 \times 10^{-12}} = 2.5 \times 2.57 \times 10^{-6} = \cancel{644 \text{ N/m}}$$

$$\boxed{P = 644 \times 10^4 \text{ N/m}^2}$$

08


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