CBCS SCHEME

USN | | | | | | | | | | **18EE71**

Seventh Semester B.E. Degree Examination, Feb./Mar. 2022

Power System Analysis - II

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Explain the following terms in network topology with an example. **(06 Marks)**
i) Tree ii) Basic loops iii) Basic cut-sets.

b. Consider an oriented graph of the power system network shown below Fig Q1(b). Choose branches 1, 3 and 5 as twigs. Build a bus incidence matrix A and basic cut-set matrix B for the oriented graph. Select node 2 as reference.

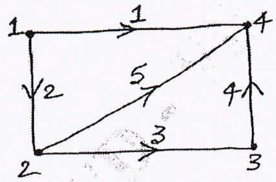

Fig Q1(b) **(08 Marks)**

c. A power system consists of four buses. The generators are connected at buses 1 and 3. The transmission lines are connected between buses 1-2, 1-4, 2-3 and 3-4 which have reactances of j0.25, j0.5, j0.4 and j0.1 respectively. Develop a bus admittance matrix by direct inspection method. Choose bus 1 as reference. **(06 Marks)**

OR

2 a. Build bus incidence matrix A and then bus admittance matrix Y_{bus} using singular transformation method for the power system network shown below in Fig Q2(a). Choose bus 1 as reference. The linedata of the power system are given in Table Q2(a) below.

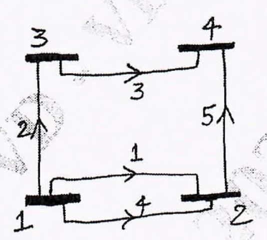

Fig Q2(a)

line No	Bus code (p-q)	Z(pu)	Mutual temperature Z_m(pu)
1	1 – 2	0.6	0.2 (line 2)
2	1 – 3	0.5	–
3	3 – 4	0.5	–
4	1 – 2	0.4	0.1 (line 1)
5	2 – 4	0.2	–

Table Q2(a)

(08 Marks)

b. Define primitive network and explain its two forms with neat representation circuit. Also derive their respective performance equations. **(06 Marks)**

c. Consider an oriented graph of the power system shown below in Fig Q2(c). Choose branches 1, 3 and 5 as twigs to form a tree. Build a basic loop incidence matrix C for the given oriented graph. Select node 2 as reference.

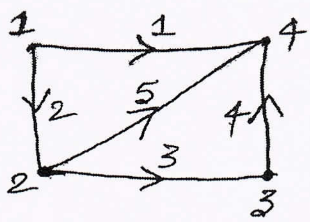

Fig Q2(c) **(06 Marks)**

Module-2

3 a. State the need of load flow study. Derive the static load flow equations or power flow equating to conduct load flow study in usual notations. **(06 Marks)**

b. For a 4 bus power system network shown below in Fig Q3(b), the generators are connected at all four buses, while loads are at buses 2 and 3. The real and reactive powers are listed below in table 3(b). Assuming a flat voltage start compute the unknown variables in all the buses other than the slack at the end of first GS iteration. Take acceleration factor as 1.4

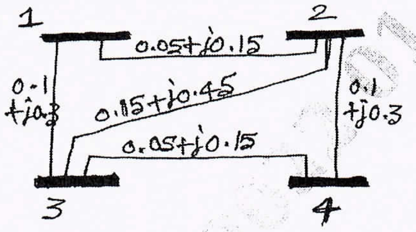

Fig Q3(b)

Bus No.	P_i(pu)	Q_i (pu)	V_i(pu)
1	–		$1.04\angle 0°$
2	0.5	–0.2	–
3	–1	0.5	–
4	0.3	–0.1	–

Table Q3(b)

(14 Marks)

OR

4 a. Explain the algorithm for Gauss – Seidel method to obtain load flow solution of a power system network with i) Absence of PV buses ii) Presence of PV buses. **(10 Marks)**

b. For the power system network shown below in Fig Q4(b), the line impedance are marked in pu. The bus data of the power system are shown below Table Q4(b). Compute the voltage in all buses other than slack at the end of first iteration using Gauss – Seidel method. Take $0 < Q_2 < 0.35$pu.

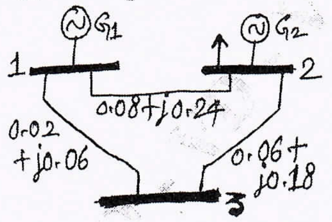

Fig Q4(b)

Bus No.	Voltage (pu)	Generation		Load	
		P_G	Q_G	P_D	Q_D
1	$0.05\angle 0°$	–	–	–	–
2	1.03	0.2	–	0.5	0.2
3	–	0	0	0.6	0.25

Table Q4(b)

(10 Marks)

Module-3

5 a. Derive the general expression for Jacobian elements in polar form with usual notations in NR method to obtain load flow solution. **(10 Marks)**

b. Explain the algorithm of Fast Decoupled Load Flow method with a neat flow chart for the load flow solution of a power system network. **(10 Marks)**

OR

6 a. In a two bus power system network shown below in Fig Q6(a), the bus – 1 is a slack bus with $V_1 = 1\angle 0°$pu and bus 2 is a load bus with P_2 = 100MW, Q_2 = 50MVAR. The line impedance is (0.12 + j0.16)pu on a base of 100MVA. Using NR method of load flow solution, compute the voltage at bus 2 at the end of first iteration.

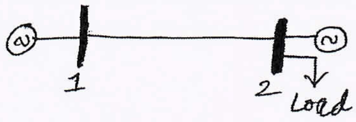

Fig Q6(a) **(10 Marks)**

b. Compare Gauss – Seidal, Newton Raphson and Fast decoupled load flow method of load flow solution with respect to various parameters. **(10 Marks)**

Module-4

7 a. A constant load of 300mW is supplied by two 200MW generators 1 and 2 for which the respective incremental fuel costs are, $\frac{dC_1}{dP_{G1}} = 0.1P_{G1} + 20$ and $\frac{dC_2}{dP_{G2}} = 0.12\,P_{G2} + 15$, where P_G's in MW and costs C_1 and C_2 are in Rs/hr. Determine : i) the most economical division of load between the generators and ii) the saving in Rs./day there by obtained compared to equal load sharing between generators. **(10 Marks)**

b. Explain various constraints involved in unit commitment solution. **(10 Marks)**

OR

8 a. Two units are connected at two buses through a transmission line. If 100MW is transmitted from unit 1 at bus 1 to the load at bus 2, a line loss of 10MW is incurred. The incremental cost curve of the two units are,
$IC_1 = 16 + 0.02\,P_1$ Rs./MWhr and
$IC_2 = 20 + 0.04\,P_2$ Rs./MWhr
If the system incremental cost is Rs.26/MWhrs no load fuel costs are Rs. 250 and Rs. 350 per hour for units 1 and 2 respectively, then determine the following :
i) Power generations from both units and the power received by the load if the losses are included and also coordinated
ii) Power generating from both units for the power received by the load as calculated above, if the losses are included but not coordinated
iii) Net saving in fuel cost by coordinating the losses. **(12 Marks)**

b. Explain the Dynamic program algorithm with the recursive relation and also explain forward DP approach with a neat flow chart. **(08 Marks)**

Module-5

9 a. Explain the algorithm for short circuit studies to be carried out in large power systems. **(08 Marks)**

b. A 20MVA, 50Hz generator delivers 18MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52MJ/MVA at rated speed. The generator has a transient reactance of 0.35pu. Each transmission line has a reactance of 0.2pu on a 20MVA base. The generator excitation voltage $|E'| = 1.1$pu and infinite bus voltage $V = 1\angle 0°$ pu. A three phase short circuit occurs at the midpoint of one of the lines. Plot the swing curve with the fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles after the occurrence of fault. Take a step size of time as 0.05sec. Also, calculate the critical clearing angle. Use point by point method. **(12 Marks)**

OR

10 a. For a three bus power system network show below in Fig Q10(a), the pu impedances are shown therein. Build bus impendence matrix Z_{bus} using step by step building algorithm. Add the elements in the order specified.

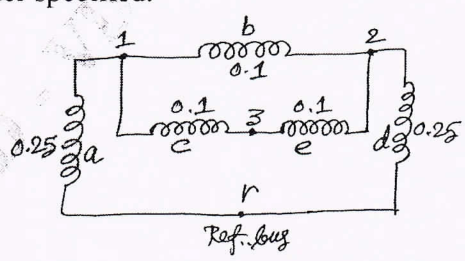

Fig Q10(a) **(10 Marks)**

b. Build an algorithm for numerical solution of swing equation by Runge - Kutta method. **(10 Marks)**

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Solution of VTU Question Paper [Feb/Mar. 2022]

Power System Analysis-2 [18EE71]

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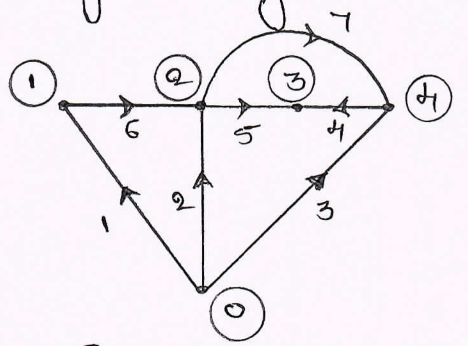
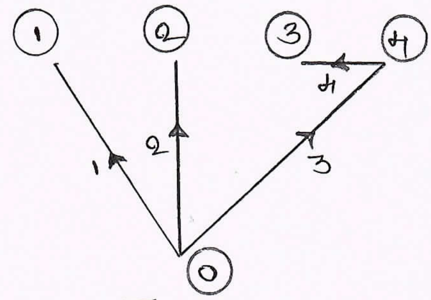
Q1a. Explain the following terms in network topology with an example. (i) tree (ii) basic loops. (iii) basic cut-sets. [6 marks]

Soln:- (i) Tree is a connected sub graph containing all the nodes of the graph G. but without any closed loops. In a graph with n nodes. the number of tree-branches is given by $t = n-1$

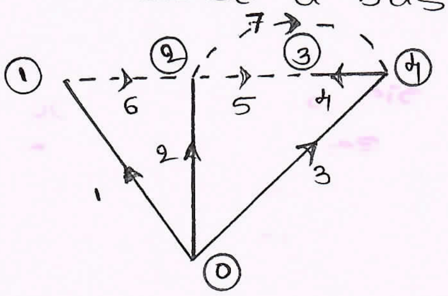


Graph



Tree.

(ii) When a link is added to a tree, it formes a loop. A loop containing only one link and remaining twigs is called a basic loop or fundamental loop.



Basic loops

{1, 2, 6}

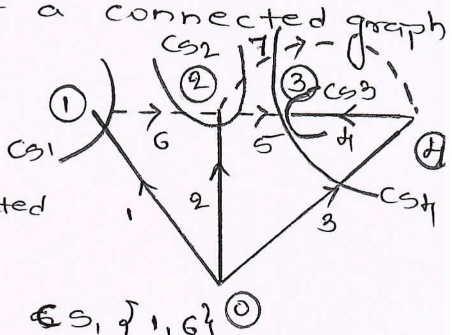
{2, 3, 4, 5}

{2, 3, 7}.

(iii) Cut set is a set of branches of a connected graph which satisfies the following

* The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graph.

* The removal of all but one of the branches of the set leaves the remaining graph connected.



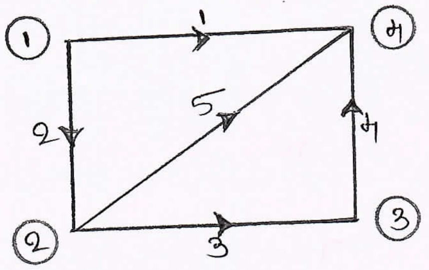
CS_1 {1, 6}

CS_2 {2, 5, 6}

CS_3 {4, 5}.

CS_4 {3, 5, 7}.

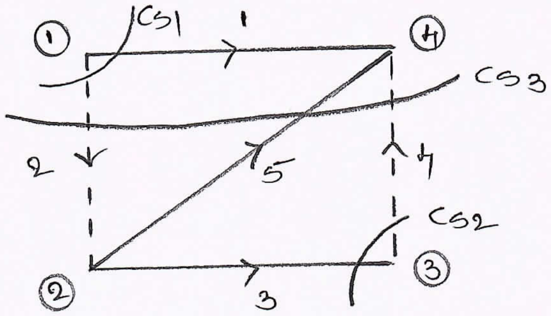
01.b Consider an oriented graph of the power system network shown below. Choose branches 1, 3 and 5 as twigs. Build a bus incidence matrix A and basic cut-set matrix B for the oriented graph. Select node 2 as reference. [8 marks]



Soln:- Bus incidence matrix A.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

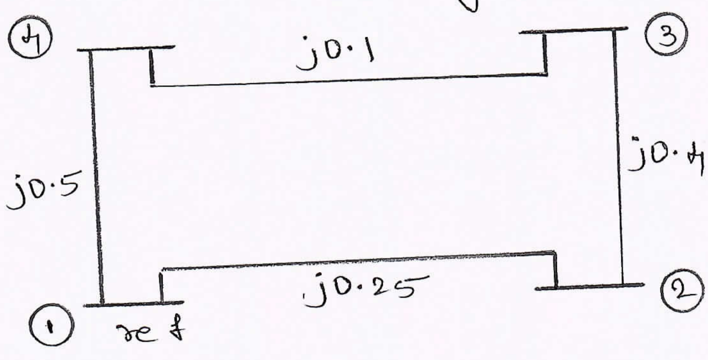
Basic cutset matrix B.



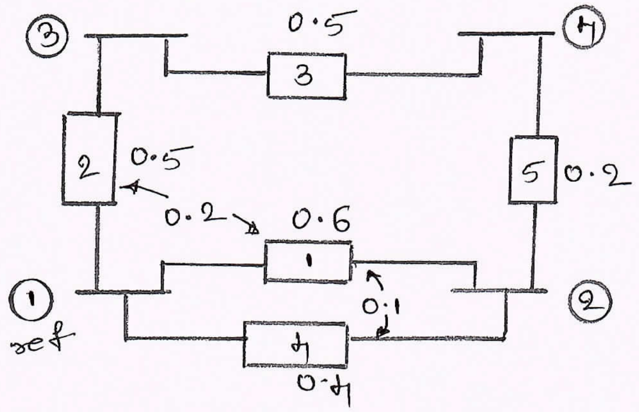
$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} CS_1 & CS_2 & CS_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

01.c A power system consists of four buses. The generators are connected at buses 1 and 3. The transmission lines are connected between buses 1-2, 1-4, 2-3 and 3-4 which have reactances of j0.25, j0.5, j0.4 and j0.1 respectively. Develop a bus admittance matrix by direct inspection method. Choose bus 1 as reference. [6 marks]

Soln:- PU impedance diagram.



PU impedance diagram.



$$Z_{prim} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 0.6 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \end{array}$$

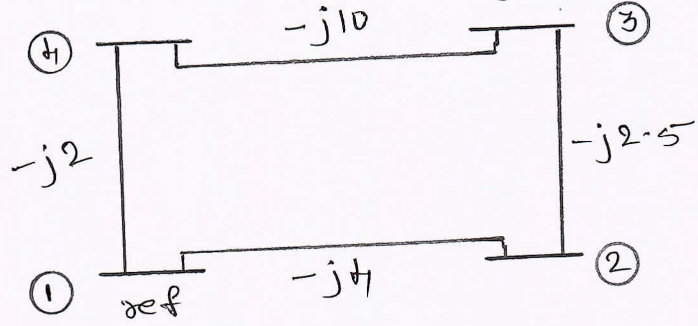
$$Y_{prim} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 2.02 & -0.8 & 0 & -0.5 & 0 \\ -0.8 & 2.32 & 0 & 0.2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -0.5 & 0.2 & 0 & 2.6 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \end{array}$$

$$Y_{bus} = A^T Y_{prim} A$$

$$= \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2.02 & -0.8 & 0 & -0.5 & 0 \\ -0.8 & 2.32 & 0 & 0.2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -0.5 & 0.2 & 0 & 2.6 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

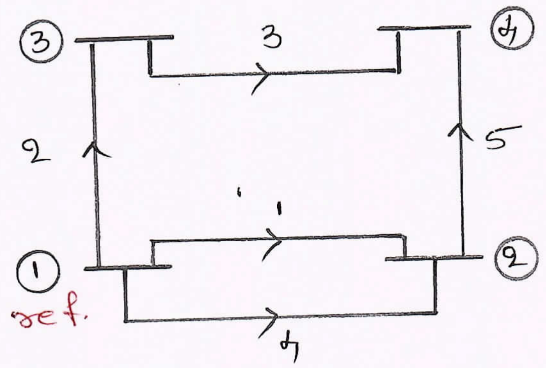
$$Y_{bus} = \begin{bmatrix} 8.63 & -0.60 & -5 \\ -0.60 & 4.32 & -2 \\ -5 & -2 & 7 \end{bmatrix}$$

PU admittance diagram.



$$Y_{bus} = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -j6.5 & j2.5 & 0 \\ j2.5 & -j12.5 & j10 \\ 0 & j10 & -j12 \end{bmatrix} \end{array}$$

Q2.a. Build bus incidence matrix A and then bus admittance matrix Y_{bus} using singular transformation method. for the power system network shown below. Choose bus 1 as reference. The line data of the power system are given in table below. [08 marks]



Line no.	Bus code	Zpu	Mutual impedance Zm PU
1	1-2	0.6	0.2 (Line 2)
2	1-3	0.5	–
3	3-4	0.5	–
4	1-2	0.4	0.1 (line 1)
5	2-4	0.2	–

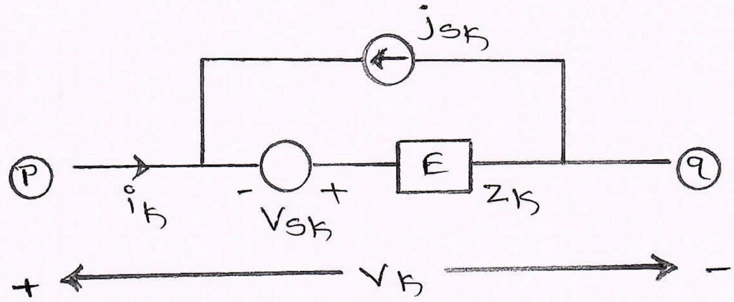
Soln:- Bus incidence matrix A

$$A = \begin{array}{c c} & \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

Q2.b. Define primitive network and explain its two forms with neat representation circuit. Also derive their respective performance equations. [6 marks]

Soln:- A primitive element is a fundamental element which is not connected to any other element. A set of such unconnected elements is defined as a primitive network.

General primitive element



V_k = Voltage across branch k.
i_k = current through branch k.
V_{sk} = independent voltage source in branch k.
j_{sk} = independent current source in branch k.
E = Passive element.

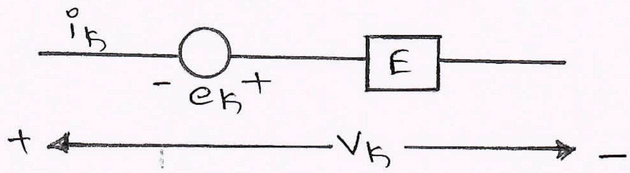
* Impedance form.

$$V_k + V_{sk} = (i_k + j_{sk}) Z_k$$

$$V_k = -V_{sk} + [i_k + j_{sk}] Z_k$$

$$= (j_{sk} Z_k - V_{sk}) + Z_k i_k$$

$V_k + e_k = i_k Z_k$ where $e_k = V_{sk} - j_{sk} Z_k$.



$$\bar{V} + \bar{e} = [Z]\,\bar{i}$$

where $\bar{V}$ = Vector of voltages across b branches.
$\bar{i}$ = Vector of currents through b branches.
$\bar{e}$ = vector of equivalent voltage sources.
$[Z]$ = Primitive impedance matrix.

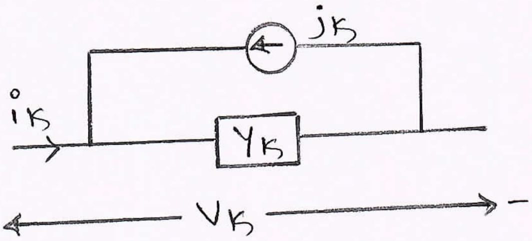
* Admittance form.

$$i_K + j_{sK} = (V_K + V_{sK})\, Y_K$$

$$i_K = (Y_K V_{sK} - j_{sK}) + V_K Y_K$$

$$i_K + j_K = V_K Y_K$$

where $j_K = j_{sK} - Y_K V_{sK}$.

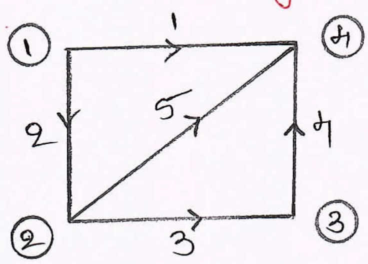


$$\bar{i} + \bar{j} = [Y]\,\bar{v}$$

where $\bar{j}$ = Vector of injected currents.

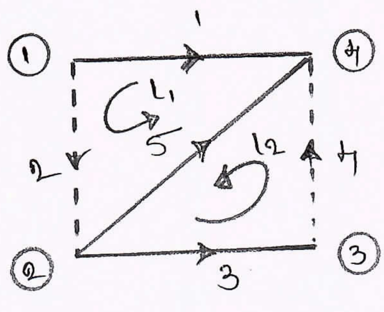
$[Y]$ = Primitive admittance matrix.

02.c Consider an oriented graph of the power system below. Choose branches 1, 3 and 5 as twigs to form a tree. Build a basic loop incidence matrix C for the given oriented graph. Select node 2 as reference. [06 marks]



Solⁿ:-

Tree



Basic loop incidence matrix.

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cc} L_1 & L_2 \end{array} \\ \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \end{array}$$

Q3. a State the need of load flow study. Derive the static load flow equations or power flow equations to conduct load flow study in usual notations. [6 marks]

Soln:- Load flow study or power flow study gives steady state solutions of the voltages at all the buses for particular load condition. Load flow studies are important in planning and designing future expansion. Load flow studies throw light on

- Violation of voltage magnitudes at the buses.
- Over loading of lines.
- Over loading of generators.
- Stability margin reduction.
- Effect of contingencies.

Power flow equations.

At any bus i, the complex power injected is given by

$$S_i = S_{Gi} - S_{Di}$$

where S_i = net complex power injected into bus i

S_{Gi} = Complex power injected by the generator at bus i.

S_{Di} = Complex power drawn by the load at bus i.

According to conservation of complex power at any ith bus the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines.

Hence $\quad S_i = \Sigma S_{ik} \quad i = 1, 2 \ldots n.$

where S_{ik} is sum over all the lines connected to the bus

Bus current injected

$$I_i = I_{Gi} - I_{Di} \quad i = 1, 2 \ldots n.$$

In bus frame of reference

$$I = Y_{bus} V$$

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k . \quad i = 1, 2 \ldots n$$

Complex power $\quad S_i^* = V_i^* I_i$

$$= V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right]$$

$$S_i = V_i \left[\sum_{k=1}^{n} Y_{ik}^* V_k^* \right] \longrightarrow (1)$$

Let $V_i = |V_i| \angle \delta_i = |V_i| (\cos\delta_i + j\sin\delta_i)$

$$\delta_{ik} = \delta_i - \delta_k$$

$$Y_{ik} = G_{ik} + jB_{ik}$$

Substituting in equation (1)

$$S_i = \sum_{k=1}^{n} |V_i||V_k| (\cos\delta_{ik} + j\sin\delta_{ik}) [G_{ik} - jB_{ik}]$$

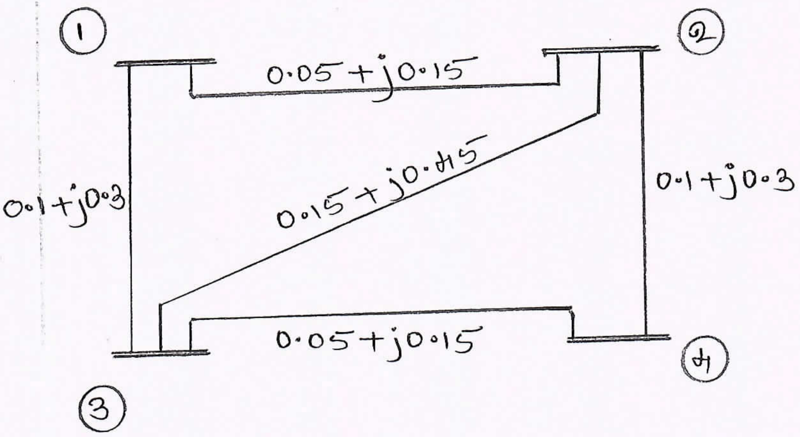
Separating real and imaginary parts.

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

and

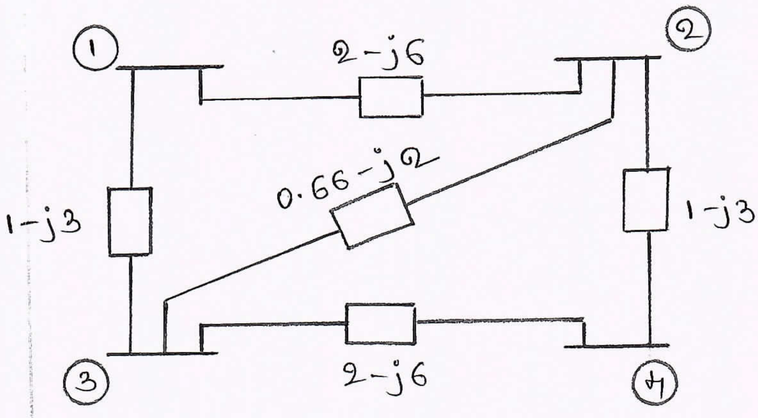
$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

03. b For a 4 bus power system network shown below, the generators are connected at all four buses, while loads are at buses 2 and 3. The real and reactive powers are listed below. Assuming a flat start compute the unknown variables in all the buses other than the slack at the end of first GS iteration. Take acceleration factor as 1.4. [14 marks]



Bus no.	P_i (PU)	Q_i (PU)	V_i (PU)
1	-	-	$1.04 \angle 0$
2	0.5	-0.2	-
3	-1	0.5	-
4	0.3	-0.1	-

Soln:- PU admittance diagram.



$$Y_{bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.66-j11 & -0.66+j2 & -1+j3 \\ -1+j3 & -0.66+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Assume initial voltages as

$V_1^0 = 1.04\angle 0$, $V_2^0 = 1\angle 0$ $V_3^0 = 1\angle 0$ $V_4^0 = 1\angle 0$

we have

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ \neq i}}^{n} Y_{ik}V_k\right]$$

$P_2 = 0.5$, $P_3 = -1$ $P_4 = 0.3$

$Q_2 = -0.2$ $Q_3 = 0.5$ $Q_4 = -0.1$

$$\therefore V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left[Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right]\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{0.5+j0.2}{1} - \left[(-2+j6)\times 1.04 + (-0.66+j2) + (-1+j3)\right]\right]$$

$V_2^1 = 1.019 + j0.046$ pu

$V_{2\,acc}^1 = V_2^0 + \alpha\left[V_2^1 - V_2^0\right]$

$= 1 + 1.4\left[1.019 + j0.046 - 1\right]$

$V_{2\,acc}^1 = 1.026 + j0.065$ pu

$= 1.028\angle 3.618^\circ$ pu

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \left[Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right]\right]$$

$$= \frac{1}{3.66-j11}\left[\frac{-1-j0.5}{1} - \left[(-1+j3)\times 1.04 + \left[(-0.66+j2) \times (1.026+j0.065)\right] + (-2+j6)\right]\right]$$

$V_3^1 = 1.029 - j0.083$ pu.

$V_{3\,acc}^1 = 1 + 1.4\left(1.029 - j0.083 - 1\right)$

$= 1.04 - j0.1162$ pu

$V_{3\,acc}^1 = 1.047\angle -6.37^\circ$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - [Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3]\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1} - [0 + (-1+j3)(1.026+j0.065) + (-2+j6)(1.04-j0.1162)]\right]$$

$$V_4^1 = 1.035 - j0.022 \text{ pu}$$

$$V_{4acc}^1 = 1 + 1.4[1.035 - j0.022 - 1]$$

$$= 1.049 - j0.031 \text{ pu}$$

$$V_{4acc}^1 = 1.049 \angle{-1.71^\circ} \text{ pu}$$

At the end of first iteration

$V_1^1 = 1.04 \angle 0^\circ$ pu

$V_{2acc}^1 = 1.028 \angle 3.618^\circ$ pu

$V_{3acc}^1 = 1.047 \angle -6.37^\circ$ pu

$V_{4acc}^1 = 1.049 \angle -1.71^\circ$ pu

04.a Explain the algorithm for Gauss-Seidel method to obtain load flow solution of a power system network with (i) Absence of PV buses. (ii) Presence of PV buses. [10 marks]

Soln:- Algorithm for GS method in absence of PV buses.

01. Read the given data.

02. Formulate Ybus.

03. Assume initial voltages for all the buses except slack bus.

04. Update the voltages.

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{r*}} - \sum_{\substack{k=1 \\ \neq i}}^{n} Y_{ik}V_k\right] \quad i = 1, 2 \ldots n.$$

05. Continue iteration till.

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)} - V_i^{(r)}| < \epsilon \quad i = 1, 2 \ldots n.$$

ϵ = tolerance value

06. Compute slack bus power.

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{k=1}^{n} Y_{1k}V_k\right)$$

07. Compute all line flows.

Algorithm for GS method in presence of PV buses.

01. Read the given data.
02. Formulate Y_{bus}
03. Assume initial voltages at PQ buses and voltage angles at PV buses.
04. At PV bus calculate Q_i

$$Q_i = -\text{Imaginary}\left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k\right]$$

05. Check for Q limit violations.

    if $Q_i < Q_{i\,min}$ then $Q_i = Q_{i\,min}$

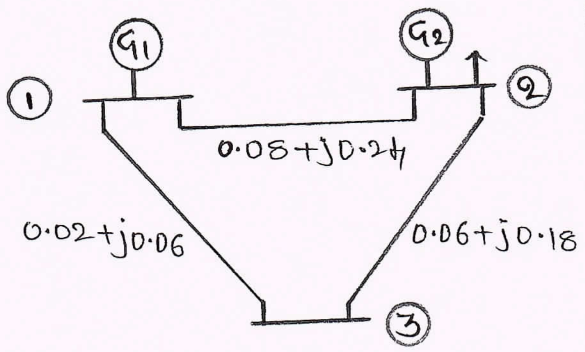
    if $Q_i > Q_{i\,max}$ then $Q_i = Q_{i\,max}$

    and treat bus as PQ bus. and calculate V_i and δ_i.

    if Q limit is not violated only update δ_i.

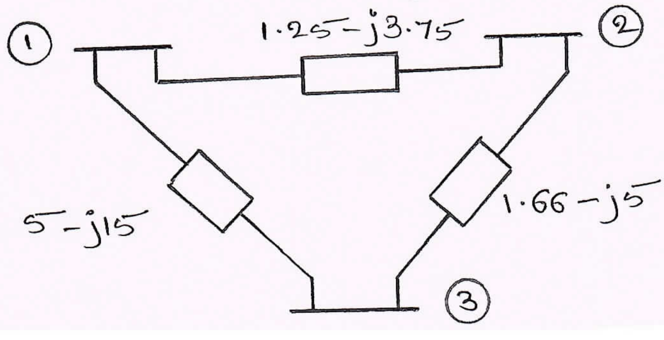
06. At PQ buses calculate V_i and δ_i.
07. Continue the above till convergence.
08. Compute slack bus power.
09. Compute all line flows.

04.b. For the power system network shown below, the line impedance are marked in pu. The bus data of the power system are shown below. Compute the voltage in all buses other than slack bus at the end of first iteration using Gauss-Seidel method. Take $0 < Q_2 < 0.3$ pu [10 marks]



Bus no.	Voltage (PU)	Generation		Load	
		P_G	Q_G	P_D	Q_D
1	$1.05\angle 0°$	–	–	–	–
2	1.03	0.2	–	0.5	0.2
3	–	0	0	0.6	0.25

Soln:- PU admittance diagram.



$$Y_{bus} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.91 - j8.75 & -1.66 + j5 \\ -5 + j15 & -1.66 + j5 & 6.66 - j20 \end{bmatrix}$$

Assume initial voltages as

$V_1^0 = 1.05 \angle 0, \quad V_2^0 = 1.03 \angle 0 \quad V_3^0 = 1 \angle 0$

$P_2 = P_{G2} - P_{D2} = 0.2 - 0.5 = -0.3 \text{ pu.}$

$Q_{D2} = 0.2$

$P_3 = -0.6 \quad , \quad Q_3 = -0.25$

Calculate Q_2 at bus 2.

$$Q_{2cal} = -\text{Img}\left[V_2^* \sum_{k=1}^{n} Y_{2k} V_k\right]$$

$$= -\text{Img}\left[V_2^* \left[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3\right]\right]$$

$$= -\text{Img}\left[1.03\left[(-1.25 + j3.75)(1.05) + (2.91 - j8.75)(1.03) + (-1.66 + j5)\right]\right]$$

$$= 0.077 \text{ pu.}$$

$0 < 0.077 < 0.35 \quad \therefore Q_2$ is within the limit

$Q_2 = 0.077 - 0.2 = -0.123 \text{ pu.}$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left[Y_{21}V_1 + Y_{23}V_3\right]\right]$$

$$= \frac{1}{2.91 - j8.75}\left[\frac{-0.3 + j0.123}{1.03} - \left[(-1.25 + j3.75)(1.05) + (-1.66 + j5)\right]\right]$$

$$= 0.99 - j0.025 \text{ pu}$$

$$V_2^1 = 0.99 \angle -1.48^\circ \text{ pu}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \left[Y_{31}V_1 + Y_{32}V_2\right]\right]$$

$$= \frac{1}{6.66 - j20}\left[\frac{-0.6 + j0.25}{1} - \left[(-5 + j15)(1.05) + (-1.66 + j5)(0.99 - j0.025)\right]\right]$$

$$= 1.014 - j0.029 \text{ pu}$$

$$V_3^1 = 1.015 \angle -1.66^\circ \text{ pu.}$$

At the end of first itteration

$V_1' = 1.05 \angle 0^\circ$ pu

$V_2' = 0.99 \angle -1.48^\circ$ pu

$V_3' = 1.015 \angle -1.66^\circ$ pu.

05.a Derive the general expression for Jacobian elements in polar form with usual notations in NR method to obtain load-flow solution. [10 marks]

Soln:- We have

$$P_{i\,cal} = G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i\,cal} = -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

(*) Elements of J_1

→ Diagonal elements

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(-G_{ik}\sin\delta_{ik} + B_{ik}\cos\delta_{ik})$$

$$= -\sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -Q_i - B_{ii}|V_i|^2$$

→ Off diagonal element

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

(*) Elements of J_3

→ Diagonal elements.

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= P_i - G_{ii}|V_i|^2$$

→ Off diagonal elements.

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

(*) Elements of J_2

→ Diagonal elements.

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i|G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = P_i + |V_i|^2 G_{ii}.$$

→ Off diagonal elements.

$$\frac{\partial P_i}{\partial |V_k|} = |V_i|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|\,(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

(*) Elements of J_4

→ Diagonal elements

$$\frac{\partial Q_i}{\partial |V_i|} = -2B_{ii}|V_i| + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = -2B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= Q_i - B_{ii}|V_i|^2$$

→ Off diagonal elements

$$\frac{\partial Q_i}{\partial |V_k|} = |V_i|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

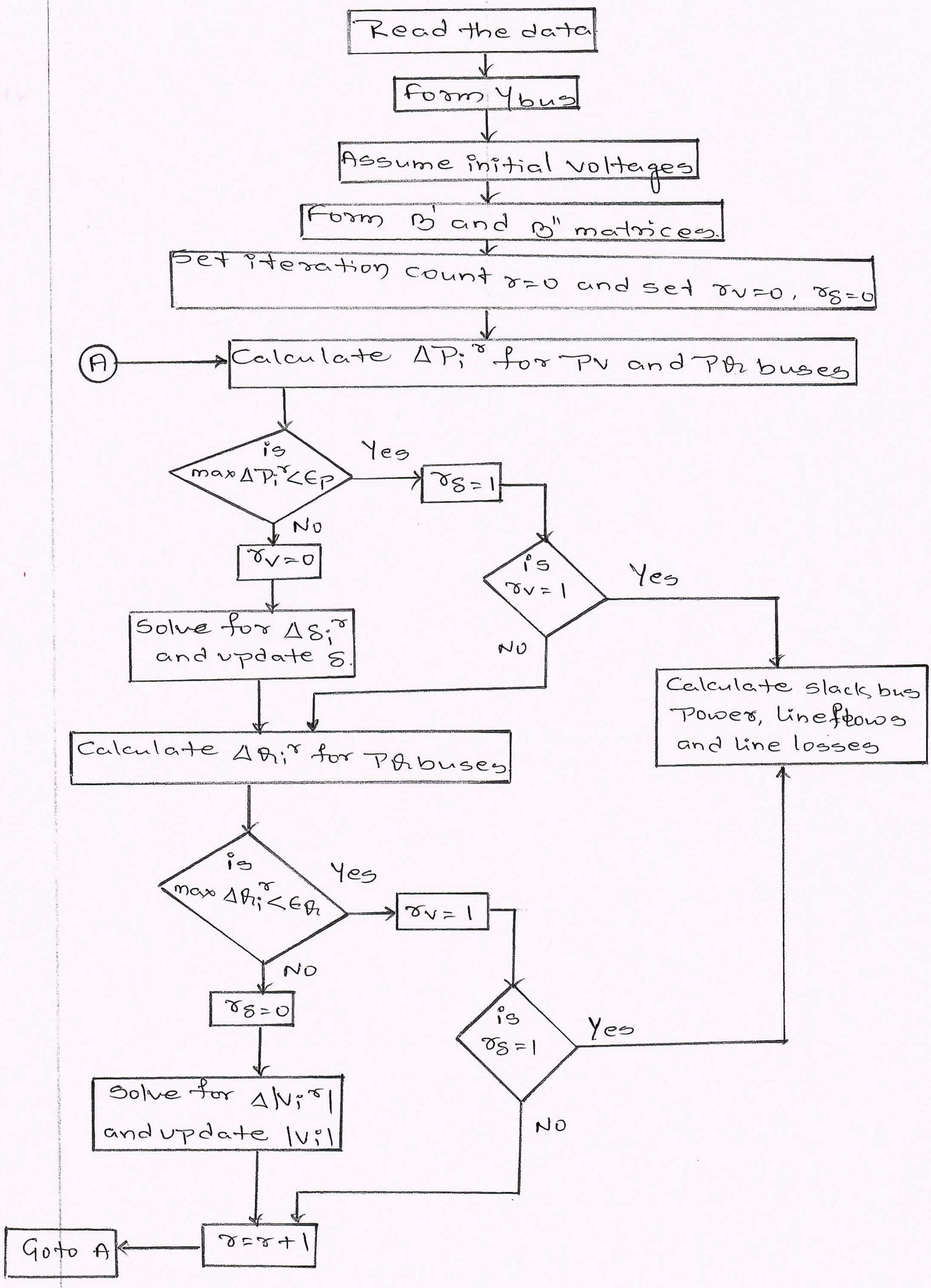
$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

05. b. Explain the algorithm of Fast Decoupled load flow meth-d with a neat flow chart for the load flow solution of a power system network. [10 marks]

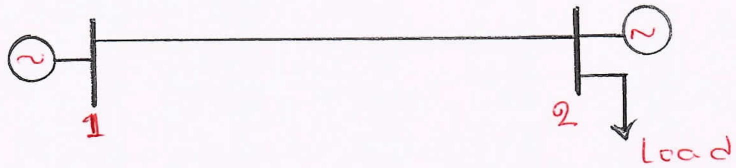
Soln:- Algorithm for Fast Decoupled Load Flow method.

01. Read the given data.
02. Formulate Y_{Bus}.
03. Assume initial voltages.
04. Form B' and B'' matrices.
05. Calculate ΔP_i for all PV and PQ buses
06. Solve for $\Delta\delta_i$ and update δ_i
07. Calculate ΔQ_i for all PQ buses
08. Solve for $\Delta|V_i|$ and update V_i
09. Goto step 05 and repeate till ΔP_i and ΔQ_i is within tolerance value

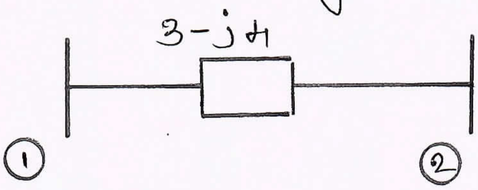
Flowchart for Fast Decoupled load flow method



06.a In a two bus power system network shown below, the bus 1 is a slack bus with $V_1 = 1\angle 0$ pu and bus 2 is a load bus with $P_2 = 100$ MW, $Q_2 = 50$ MVAR. The line impedance is $(0.12 + j0.16)$ pu on a base of 100 MVA. Using NR method of load flow solution, compute the voltage at bus 2 at the end of first iteration. [10 marks]



Soln:- PU admittance diagram.



$$Y_{bus} = \begin{bmatrix} 3-j4 & -3+j4 \\ -3+j4 & 3-j4 \end{bmatrix}$$

Assume initial voltages as

$$V_1^0 = 1\angle 0^\circ, \quad V_2^0 = 1\angle 0^\circ$$

$$P_{2sp} = \frac{100}{100} = 1\,\text{pu} \qquad Q_{2sp} = \frac{50}{100} = 0.5\,\text{pu}.$$

$$S_2^* = V_2^* \sum_{k=1}^{2} Y_{2k} V_k = V_2^* \left[Y_{21}V_1 + Y_{22}V_2 \right]$$

$$= 1\left[(-3+j4) + (3-j4) \right]$$

$$= 0$$

$$P_{2cal} = 0 \qquad Q_{2cal} = 0$$

$$\Delta P_2 = P_{2sp} - P_{2cal} = 1$$

$$\Delta Q_2 = Q_{2sp} - Q_{2cal} = 0.5.$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} H_{22} & N_{22} \\ M_{22} & L_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \frac{\Delta |V_2|}{|V_2|} \end{bmatrix}$$

$$H_{ii} = -Q_i - B_{ii}|V_i|^2$$

$$H_{22} = -Q_2 - B_{22}|V_2|^2$$

$$= 0 - (-4)(1)^2$$

$$= 4$$

$$N_{22} = P_2 + G_{22}|V_2|^2$$

$$= 0 + 0 = 0$$

$$M_{22} = P_2 - G_{22}|V_2|^2 = 0$$

$$L_{22} = Q_2 - B_{22}|V_2^2|$$

$$= 4$$

$$\therefore \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \frac{\Delta|V_2|}{|V_2|} \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \frac{\Delta|V_2|}{|V_2|} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.125 \end{bmatrix}$$

$$\therefore \Delta\delta_2 = 0.25^r = 14.32^\circ$$

$$\frac{\Delta|V_2|}{|V_2|} = 0.125 \quad \therefore \Delta|V_2| = 0.125 \text{ pu.}$$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2^0 = 0 + 14.32 = 14.32^\circ$$

$$|V_2^1| = |V_2^0| + \Delta|V_2^0| = 1 + 0.125 = 1.125 \text{ pu.}$$

at the end of first iteration

$$V_1^1 = 1 \angle 0^\circ \text{ pu}$$

$$V_2^1 = 1.125 \angle 14.32^\circ \text{ pu}$$

Q6. b Compare Gauss-Seidal, Newton Raphson and fast decoupled load flow method of load flow solution with respect to various parameters. [10 marks]

Soln.:-

Sl no.	Parameter of comparison	G-S method.	N-R method	FDLF method
01.	Coordinates	works well with rectangular co-ordinates	Polar coordinates are prefered	Polar coordinates.
02.	Arithmetic operations	Least in no. to complete one iteration	Elements of Jacobian to be calculated in each iteration	Less than NR method.
03.	Time	Requires less time per iteration, but increases with no. of buses	Time/iteration is 7 times of G-S method	less compared to N-R or G-S method
04	Convergence	Linear convergence	Quadratic convergence	Geometric convergence
05	Accuracy	less accurate	more accurate	Moderate
06	Memory required	less	Large	around 60% when compared with N-R

Q7. a A constant load of 300 MW is supplied by two 200 MW generators 1 and 2 for which the respective incremental fuel costs are

$\frac{dC_1}{dP_{G1}} = 0.1 P_{G1} + 20$ and $\frac{dC_2}{dP_{G2}} = 0.12 P_{G2} + 15$, where

PG's in MW and costs C_1 and C_2 are in Rs/hr.

Determine : (i) the most economical division of load between the generators and (ii) the saving in Rs/day there by obtained compared to equal load sharing between generators. [10 marks]

Soln.- $P_D = 300\,MW$

For economic operation $\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$

i.e $0.1P_{G1} + 20 = 0.12P_{G2} + 15 \rightarrow (1)$

$P_{G1} + P_{G2} = 300 \rightarrow (2)$

$\therefore \quad P_{G2} = 300 - P_{G1}$

So $0.1P_{G1} + 20 = 0.12(300 - P_{G1}) + 15$

or $P_{G1} = 140.9\,MW$

and $P_{G2} = 300 - 140.9 = 159.1\,MW.$

Cost function $C = \int \frac{dc_1}{dP_G}$

$\therefore C_1 = 0.05P_{G1}^2 + 20P_{G1} + X$

$C_2 = 0.06\,P_{G2}^2 + 15P_{G2} + Y$

$C_1 = (0.05 \times 140.9^2) + (20 \times 140.9) + X = 3810.64 + X$ Rs/hr

$C_2 = (0.06 + 159.1^2) + (15 \times 159.1) + Y = 3905.26 + Y$ Rs/hr

total cost with economic generation scheduling

$C_T = C_1 + C_2 = 7715.9 + X + Y$ Rs/hr

With equal load sharing

$P_{G1} = P_{G2} = P_D/2 = 150\,MW.$

$C_1 = (0.05 + 150^2) + (20 * 150) + X = 4125 + X$ Rs/hr

$C_2 = (0.06 + 150^2) + (15 \times 150) + Y = 3600 + Y$ Rs/hr

$C_T = C_1 + C_2 = 7725 + X + Y$ Rs/hr

Saving $= 7725 + X + Y - (7715.9 + X + Y)$

$= 9.1$ Rs/hr

or $9.1 \times 24 = 218.4$ Rs/day.

07.b. Explain various constraints involved in unit Commitment solution. [10 marks]

Soln.- Constraints in unit commitment

01. Spinning reserve

In any power system, some amount of active power generation capacity has to be kept in reserve to restablish the balance between load and generation at all times, even under the eventuality of a unit failing

Spinning reserve = (sum of the capacities of all units synchronized at a time) − (load + losses in the system at that time)

Spinning reserve is necessary so that the loss of a generating unit does not lead to a drop in system frequency.

02. <u>Thermal unit constraints.</u>

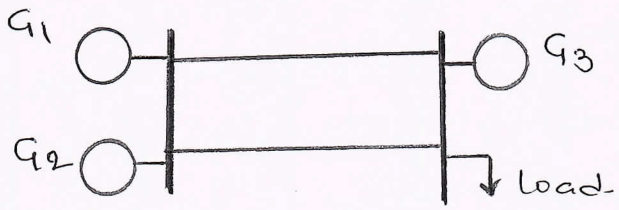
a. Minimum Uptime – This is the minimum time for which a unit once committed should run. It should not be turned off immediately

b. Minimum down time - A unit which has been shut down cannot be started up before a minimum time has elapsed.

c. Start up cost - Start up costs are the costs incurred in starting a thermal unit.

03. <u>Network constraints.</u>

Transmission network may have an effect on the commitment of the units.



Generation of G_3 may be more than G_1 and G_2. The transfer of power from G_1 and G_2 to load is limited by the transmission network. Hence G_3 even it is more expensive is committed.

04. <u>Emission constraints.</u>

As per the emission norms there is a limit on Green house gas emission. It sets a limit on Plant usage

05. <u>Capacity of generator</u>

The limits of the generators may vary over the period of the day. This has to be considered while commiting the unit.

06. Fule constraints.

Some units may have a limit on the fule consumption.

07. Hydel plant constraints.

Hydel plant donot have operating cost, but we need to maintain water level in the dams.

08.a. Two units are connected at two buses through a transmission line. If 100MW is transmitted from unit 1 at bus 1 to the load at bus 2, a line loss of 10MW is incurred. The incremental cost curve of the two units are

$IC_1 = 16 + 0.02P_1$ Rs/Mwh and

$IC_2 = 20 + 0.04P_2$ Rs/Mwh.

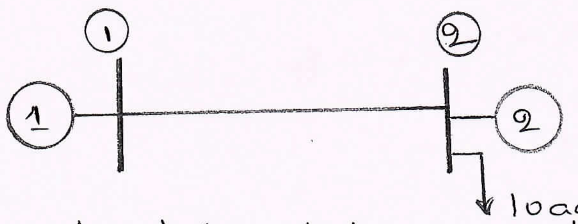
If the system incremental cost is Rs. 25/Mwh, no load fuel costs are Rs. 250 and Rs. 350 per hour for unit 1 and 2 respectively, then determine the following.

(i) Power generations from both units and the power received by the load if the losses are included and also coordinated

(ii) Power generating form both units for the power received by the load as calculated above, if the losses are included but not coordinated.

(iii) Net saving in fuel cost by coordinating the losses. [12 marks]

Soln:-



Since load is at bus 2 alone, P_{G2} will not have any effect on P_L

$\therefore B_{22} = 0 \qquad B_{12} = 0 = B_{21}$

Hence $P_L = B_{11} P_{G_1}^2$

for $P_{G_1} = 100\,MW$ $\quad P_L = 10\,MW$

$\therefore \quad 10 = B_{11} (100)^2$

$B_{11} = 0.001\,MW^{-1}$

We have co-ordination equation for plant 1

$0.02 P_{G_1} + 2\lambda B_{11} P_{G_1} + 2\lambda B_{12} P_{G_2} = \lambda - 16$

for plant 2

$0.04 P_{G_2} + 2\lambda B_{22} P_{G_2} + 2\lambda B_{21} P_{G_1} = \lambda - 20$

Given $\lambda = 25$ Rs/MWh and $B_{11} = 0.001\,MW^{-1}$

$0.02 P_{G_1} + 2 \times 25 \times 0.001 P_{G_1} = 25 - 16.$

$\therefore P_{G_1} = 128.57\,MW$

$0.04 P_{G_2} = 25 - 20$

$P_{G_2} = 125\,MW.$

transmission loss $= P_L = 0.001 \times 128.57^2$

$= 16.53\,MW$

load $P_D = P_{G_1} + P_{G_2} - P_L = 237.04\,MW.$

$\therefore$ Power generation with losses co-ordinated

$P_{G_1} = 128.57\,MW \quad P_{G_2} = 125\,MW.$

With losses included but not coordinated.

$0.02 P_{G_1} + 16 = 0.04 P_{G_2} + 20. \longrightarrow (1)$

Power delivered to load is

$P_{G_1} + P_{G_2} = 0.001 P_{G_1}^2 + 269.6. \longrightarrow (2)$

Solving equations (1) and (2)

$P_{G_1} = 275.18\,MW$ and $P_{G_2} = 37.59\,MW.$

Loss coordination causes the load on plant 1 to reduce from 275.18 MW to 128.57 MW.

$\therefore$ Saving in plant 1 due to loss co-ordination is

$$\int_{128.51}^{275.18} (0.02 P_{G_1} + 16)\, dP_{G_1} = 0.01 P_{G_1}^2 + 16 P_{G_1} \Big|_{128.51}^{275.18}$$

$= 2937.69$ Rs/hr.

at plant 2 the load incresed from 37.59 MW to 125 MW due to loss co-ordination, the saving at plant 2 is

$$\int_{125}^{37.59} (0.04 P_{G2} + 20)\, dP_{G2} = 0.02 P_{G2}^2 + 20 P_{G2} \Big|_{125}^{37.59}$$

$= -2032.43$ Rs/hr

$\therefore$ Saving $= 2937.69 - 2032.43 = 905.26$ Rs/hr.

08. b. Explain the Dynamic program algorithm with the recursive relation and also explain forward DP approach with a neat flow chart. (08 marks)

Soln:- The recursive algorithm to compute the minimum cost in hour K with combination C is

$$F_{cost}(K, C) = \min_{\{L\}} \left[P_{cost}(K, C) + S_{cost}(K-1, L : K, C) + F_{cost}(K-1, L) \right]$$

where

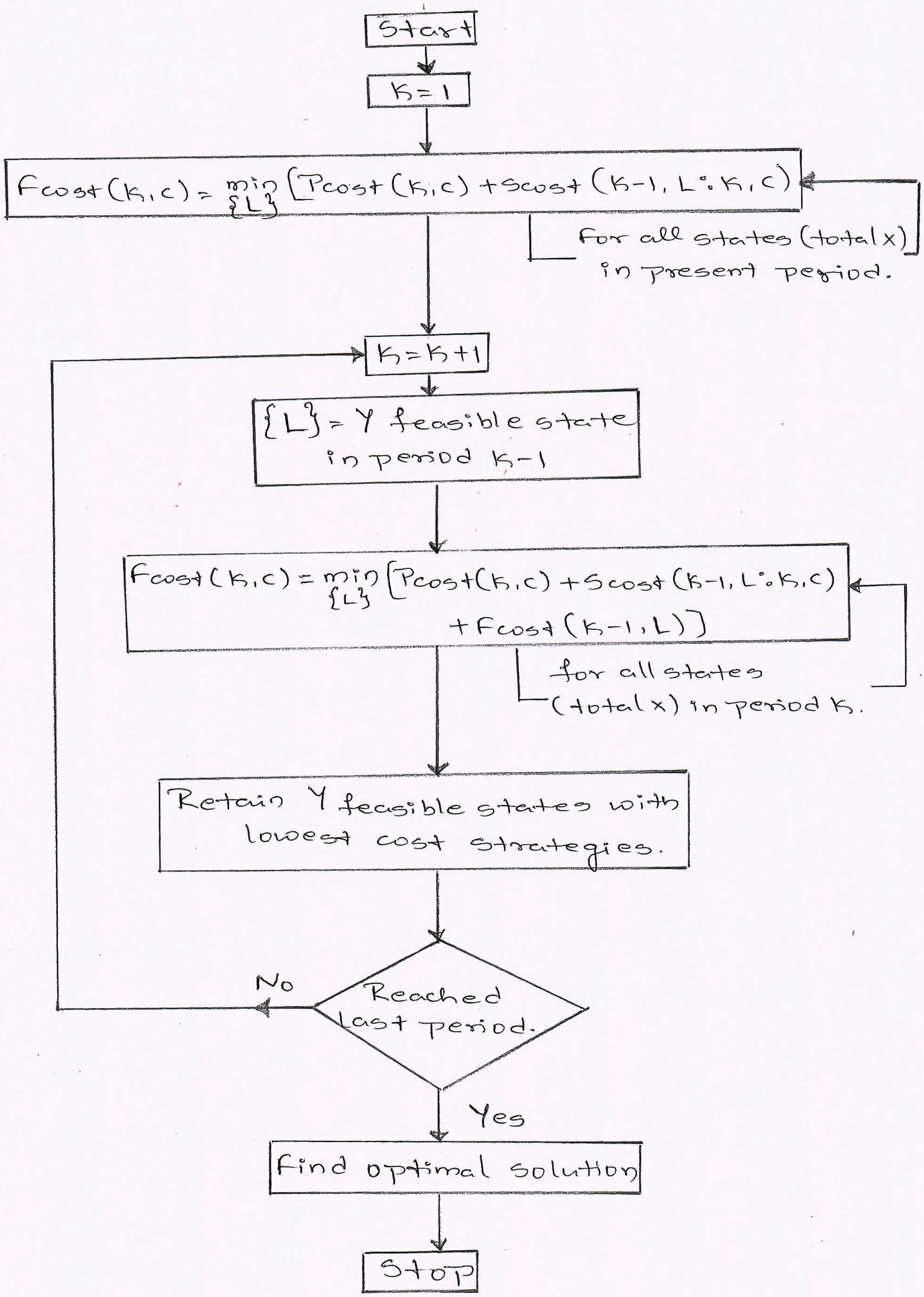
$F_{cost}(K, C)$ = least total cost to arrive at state (K, C)

$P_{cost}(K, C)$ = production cost for state (K, C)

$S_{cost}(K-1, L : K, C)$ = transition cost from state $(K-1, L)$ to state (K, C)

State (K, C) is the C^{th} combination in hour K. For the forward DP approach, we define a strategy as the transition or path, from one state at a given hour to a state at the next hour.

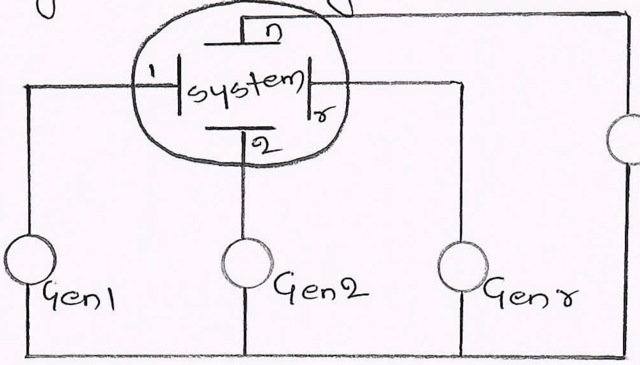
Forward DP approach flow chart



Q9.a. Explain the algorithm for short circuit studies to be carried out in large power systems. (08 marks)

Soln:- Algorithm for short circuit studies.

Consider an n-bus system shown in fig below. operating at steady load.



Step 01: Obtain prefault voltages at all buses and currentes in all lines through a load flow study. Let prefault bus voltage vector be

$$V^0_{bus} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_r^0 \\ \vdots \\ V_n^0 \end{bmatrix}$$

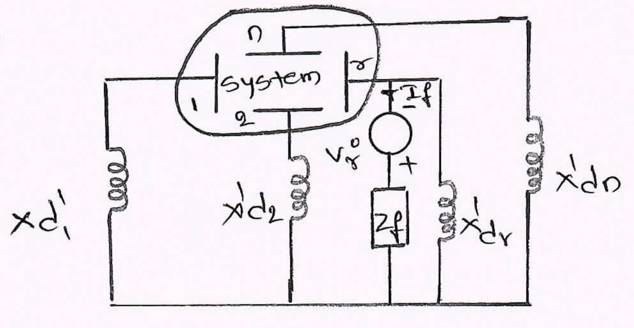
Assume that rth bus is faulted through a fault impedance Z^f. The post-fault bus voltage vector will be given by

$$V^f_{bus} = V^0_{bus} + \Delta V$$

where ΔV is the vector of changes in bus voltage caused by the fault.

Step 02: Draw the passive thevenin network of the system with generators replaced by transient or Subtransient reactances with their emf's shorted.

Step 03: Excite the passive Thevenin network with $-V_r^0$ in series with Z_f as shown below.



The vector ΔV comprises the bus voltage of this network.

now $\Delta V = Z_{bus} J^f$

Z_{bus} = bus impedance matrix of Thevenin network

J^f = bus current injection vector.

$$J^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_r^f = -I^f \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

So $\Delta V_r = -Z_{rr} I^f$

Step 04 : Voltage at the r^{th} bus under fault is

$$V_r^f = V_r^o + \Delta V_r^o = V_r^o - Z_{rr} I^f$$

also $V_r^f = Z^f I^f$

So $Z^f I^f = V_r^o - Z_{rr} I^f$

$\therefore I^f = V_r^o / Z^f + Z_{rr}$

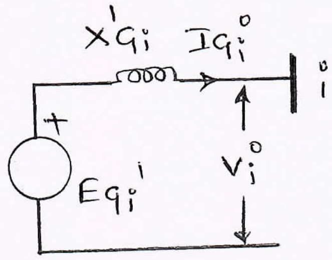
at i^{th} bus $\Delta V_i = [-Z_{ir} I_f]$

$$V_i^f = V_i^o - Z_{ir} I^f$$

Substituting I^f

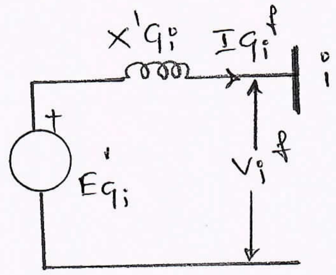
$$V_i^f = V_i^o - \frac{Z_{ir}}{Z^f + Z_{rr}} V_r^o$$

Post fault current in lines



$$I_{ij}^f = Y_{ij} (V_i^f - V_j^f)$$

$$I_{qi}^o = \frac{P_{qi} - jQ_{qi}}{V_i^o}$$

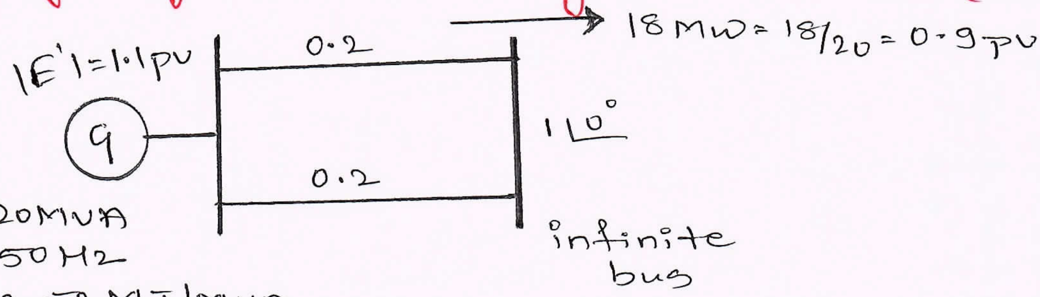


$$E_{qi}' = V_i + j x_{qi} I_{qi}$$

$$I_{qi}^f = \frac{E_{qi}' - V_i^f}{j x'_{qi}}$$

Q9.b. A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator has a transient reactance of 0.35 pu. Each transmission line has a reactance of 0.2 pu on a 20 MVA base. The generator excitation voltage |E'| = 1.1 pu and infinite bus voltage V = 1∠0 pu. A three phase short circuit occurs at the midpoint of one of the lines. Plot the swing curve with the fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles after the occurrence of fault. Take a step size of time as 0.05 sec. Also, calculate the critical clearing angle. Use point by point method. [12 marks]

Soln.:-



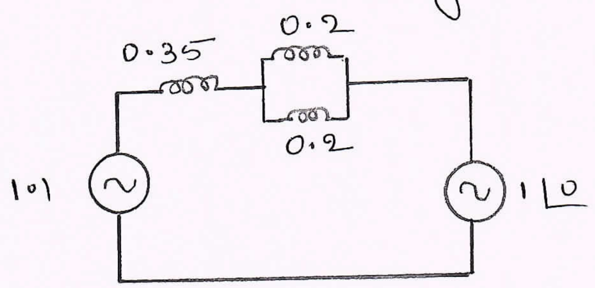
$H = 2.52\ MJ/MVA$

$X_d' = 0.35$

$$C.B\ time = \frac{2.5}{50} = 0.05\ sec$$

(*) Pre fault condition

PU impedance diagram.



$P_e = P_{max\,I} \sin \delta_0$

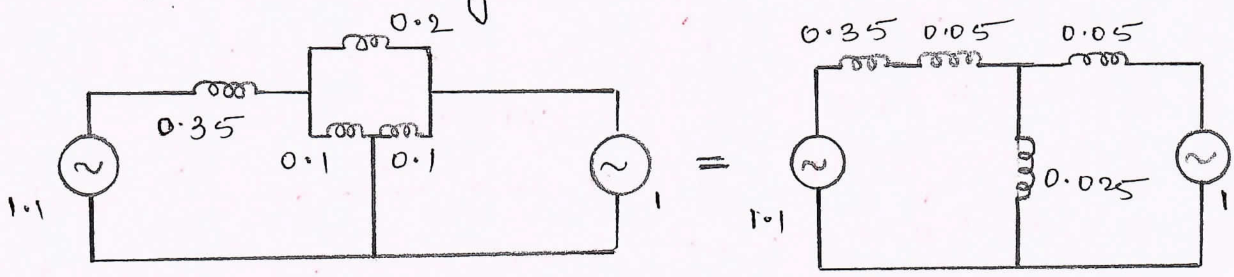
$$P_{max\,I} = \frac{|V||E|}{X_T} = \frac{1.1 \times 1}{0.35 + (0.2 \| 0.2)} = 2.44$$

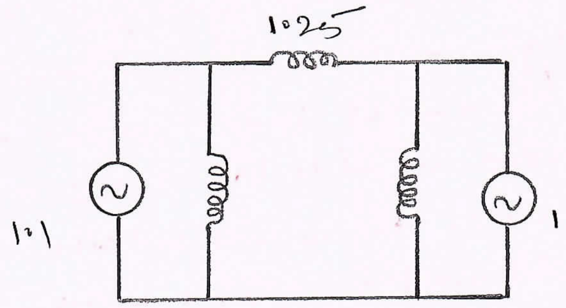
$\therefore \; 0.9 = 2.44 \sin \delta_0$

$$\delta_0 = 21.64^\circ = 0.377^{r}$$

(*) During fault

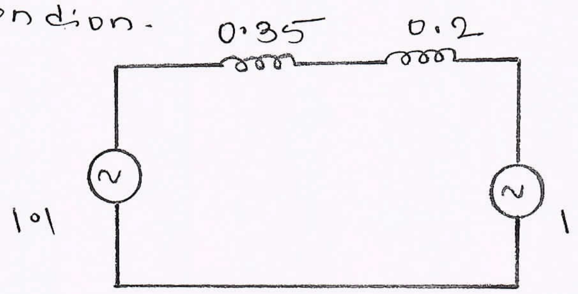
PU impedance diagram.





$$P_{max\,II} = \frac{1.1 \times 1}{1.25} = 0.88 \text{ pu.}$$

(*) Fault is cleared at 0.05 sec. So post fault condion.

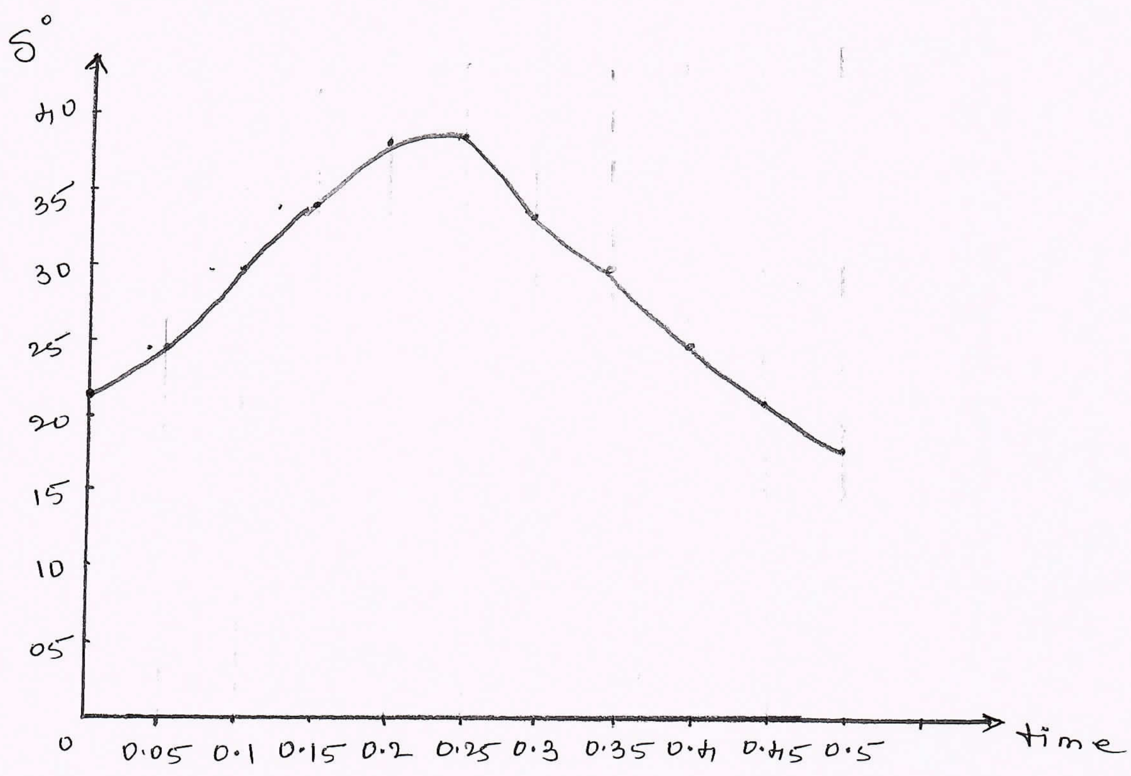


$$P_{max\,III} = \frac{1.1 \times 1}{0.55} = 2 \text{ pu.}$$

Given $\Delta t = 0.05$ sec

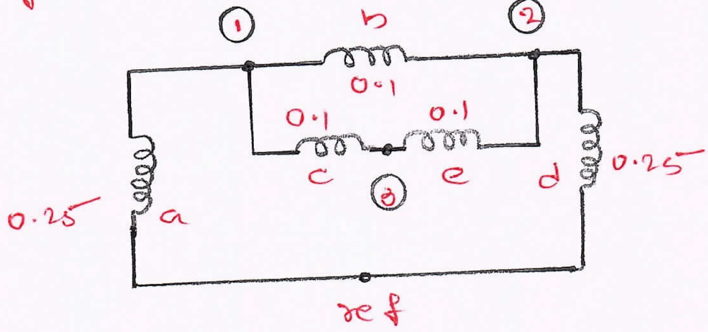
$$M = \frac{H}{180 f} = \frac{2.52}{180 \times 50} = 2.8 \times 10^{-4}$$

t	P_{max}	δ°	$\sin\delta$	$P_e = P_{max}\sin\delta$	$P_a = 0.9 - P_e$	$\frac{\Delta t^2}{M}\cdot P_a = 8.93 P_a$	$\Delta\delta = \Delta\delta_{n-1} + \frac{\Delta t^2}{M} P_a$
0^-	2.44	21.64	0.368	0.9	0	–	–
0^+	0.88	21.64	0.368	0.323	0.575	–	–
0^{av}	–	21.64	–	–	0.2875	2.56	2.56
0.05^-	0.88	24.2	0.41	0.36	0.539	–	–
0.05^+	2.0	24.2	0.41	0.82	0.08	–	–
0.05	2	24.2	–	–	0.309	2.76	5.32
0.1	2	29.52	0.49	0.98	−0.085	−0.764	4.55
0.15	2	34.07	0.56	1.12	−0.22	−1.969	2.58
0.2	2	36.65	0.59	1.19	−0.293	−2.624	−0.044
0.25	2	36.60	0.59	1.19	−0.292	−2.613	−2.65
0.3	2	33.94	0.55	1.11	−0.21	−1.93	−4.58
0.35	2	29.35	0.49	0.98	−0.08	−0.718	−3.86
0.4	2	25.48	0.43	0.86	0.039	0.351	−3.508
0.45	2	21.9	0.37	0.74	0.151	1.35	−2.15
0.5	2	19.74	–	–	–	–	–



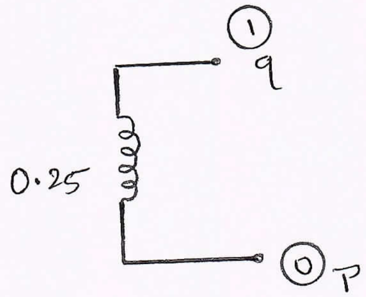
10.a. For a three bus power system network shown below, the pu impedances are shown therein. Build bus impedance matrix Zbus using step by step building algorithm. Add the elements in the order specified.

[10 marks]



Soln:- adding element between ref bus and bus 1.

Addition of a branch $p=0 \quad q=1$



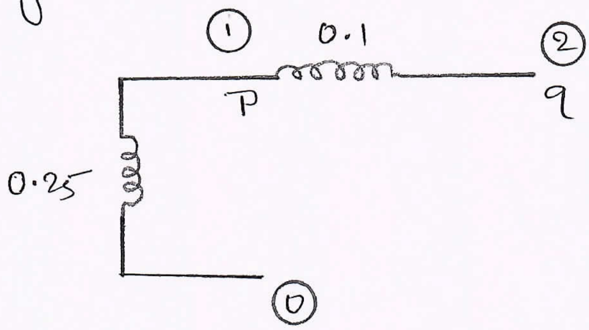
$Z_{bus} = [Z_{11}]$

$Z_{qq} = Z_{pqpq}$

$Z_{11} = Z_{0101} = 0.25$

$\therefore Z_{bus} = [0.25]$

adding element between bus 1 and 2. Addition of a branch



$$Z_{bus} = \begin{bmatrix} 0.25 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$Z_{qi} = Z_{pi}$

$Z_{21} = Z_{11} = 0.25 = Z_{12}$

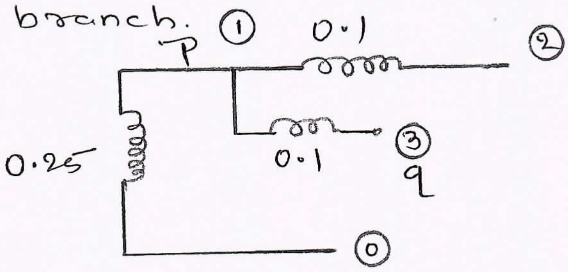
$Z_{qq} = Z_{pq} + Z_{pqpq}$

$Z_{22} = Z_{12} + Z_{1212}$

$= 0.25 + 0.1 = 0.35$

$$\therefore Z_{bus} = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix}$$

adding element between bus 1. and 3. Addition of a branch.



$$Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & Z_{13} \\ 0.25 & 0.35 & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$Z_{qi} = Z_{pi}$

$Z_{31} = Z_{11} = 0.25 = Z_{13}$

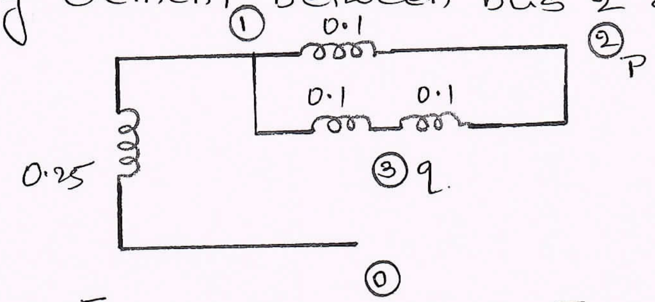
$Z_{32} = Z_{12} = 0.25 = Z_{23}$

$Z_{qq} = Z_{pq} + Z_{pqpq}$

$Z_{33} = Z_{13} + Z_{1313} = 0.25 + 0.1 = 0.35$

$$\therefore Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix}$$

adding element between bus 2 and 3. Addition of a link.



$$Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & Z_{1l} \\ 0.25 & 0.35 & 0.25 & Z_{2l} \\ 0.25 & 0.25 & 0.35 & Z_{3l} \\ Z_{l1} & Z_{l2} & Z_{l3} & Z_{ll} \end{bmatrix}$$

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{l1} = Z_{21} - Z_{31} = 0.25 - 0.25 = 0 = Z_{1l}$

$Z_{l2} = Z_{22} - Z_{32} = 0.35 - 0.25 = 0.1 = Z_{2l}$

$Z_{l3} = Z_{23} - Z_{33} = 0.25 - 0.35 = -0.1 = Z_{3l}$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$

$= Z_{2l} - Z_{3l} + Z_{2323} = 0.1 - (-0.1) + 0.1$

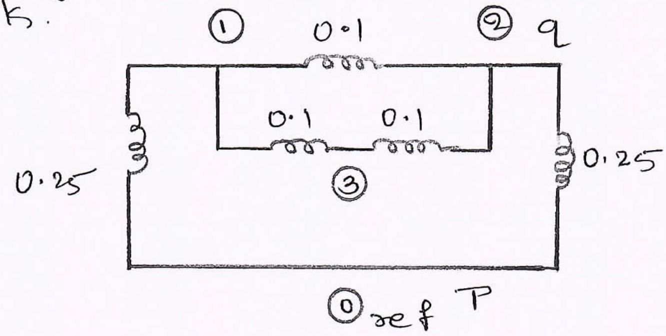
$Z_{ll} = 0.3$

$$Z_{bus\,new} = Z_{bus\,old} - \frac{\overline{Z_{il}}\,\overline{Z_{lj}}}{Z_{ll}}$$

$$Z_{bus\,new} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.3}\begin{bmatrix} 0 \\ 0.1 \\ -0.1 \end{bmatrix}\begin{bmatrix} 0 & 0.1 & -0.1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.316 & 0.283 \\ 0.25 & 0.283 & 0.316 \end{bmatrix}$$

adding element between bus 2 and ref. Addition of a link.



$Z_{li} = -Z_{qi}$

$Z_{l1} = -Z_{21} = -0.25 = Z_{1l}$

$Z_{l2} = -Z_{22} = -0.316 = Z_{2l}$

$Z_{l3} = -Z_{23} = -0.283 = Z_{3l}$

$$\begin{aligned} Z_{ll} = -Z_{ql} + Z_{pqpq} &= -Z_{2l} + Z_{0202} \\ &= -(-0.316) + 0.25 \\ &= 0.566. \end{aligned}$$

$$Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.316 & 0.283 \\ 0.25 & 0.283 & 0.316 \end{bmatrix} - \frac{1}{0.566}\begin{bmatrix} -0.25 \\ -0.316 \\ -0.283 \end{bmatrix}\begin{bmatrix} -0.25 & -0.316 & -0.28 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.139 & 0.11 & 0.125 \\ 0.11 & 0.139 & 0.125 \\ 0.125 & 0.125 & 0.174 \end{bmatrix}$$

10.b. Build an algorithm for numerical solution of swing equation by Runge-Kutta method. [10 marks]

Soln.:- In Runge-Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation to replace a truncated Taylor's series expansion. The formulae for Runge-Kutta fourth order approximation for solution of two simultaneous differential equations are given below.

Given $dx/dt = f_x(x, y, t)$

$dy/dt = f_y(x, y, t)$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where

$$k_1 = f_x(x_0, y_0, t_0)h$$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h]h$$

$$l_1 = f_y(x_0, y_0, t_0)h$$

$$l_2 = f_y\left(x_0 + k_1/2, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

The two first order differential equations to be solved to obtain solution for the swing equation are

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_m - P_{max}\sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and step size of Δt the formulae are as follows.

$$k_1 = \omega_0 \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max} \sin\delta_0}{M}\right]\Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right)\Delta t$$

$$l_2 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right]\Delta t$$

$$k_3 = \left[\omega_0 + \frac{l_2}{2}\right]\Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{3}\right)}{M}\right]\Delta t$$

$$k_4 = \left[\omega_0 + l_3\right]\Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + k_3\right)}{M}\right]\Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

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