CBCS SCHEME

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18CV32

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Strength of Materials

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Explain longitudinal strain and lateral strain. **(04 Marks)**

b. State and illustrate Saint Venant's principle. **(06 Marks)**

c. A tension test was conducted on mild steel bar and the following data was obtained from the test:
Diameter of the bar = 18mm
Gauge length of the bar = 82mm
Load at proportional limit = 75KN
Extension at a load of 62KN = 0.113mm
Load at failure = 82KN
Final gauge length of the bar = 106mm
Diameter of the bar at failure = 14mm
Determine the Young's modulus, proportional limit, true breaking stress, %elongation and percentage reduction in cross sectional area. **(10 Marks)**

OR

2 a. What are the elastic constants and explain them briefly. **(06 Marks)**

b. Obtain expression for temperature stress in a bar of uniform cross section when expansion or contraction is prevented partially. **(04 Marks)**

c. A weight of 390KN is supported by a short column of 250mm square in section. The column is reinforced with 8 steel bars of cross sectional area 2500mm^2. Find the stresses in steel and concrete if $E_s = 15E_c$.
If stress in concrete must not exceed 4.5MN/m^2, what area of steel is required in order that column may support a load of 480KN. **(10 Marks)**

Module-2

3 a. Derive Lame's equation for the radial and hoop stress for thick cylinder subjected to internal and external fluid pressure. **(08 Marks)**

b. A 2-dimensional element has the tensile stresses of 600MN/m^2 and compressive stress of 400MN/m^2 acting on two mutually perpendicular planes and two equal shear stresses of 200MN/m^2 on their planes. Determine
i) Resultant stress on a plane inclined at 30° wrt x-axis.
ii) The magnitude and direction of principal stresses.
iii) Magnitude and direction of maximum shear stress. **(12 Marks)**

OR

4 a. Obtain expression for volumetric strain in thin cylinder subjected to internal pressure in the form of $e_v = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$. **(08 Marks)**

b. A cast iron pipe has 200mm internal diameter and 50mm metal thickness and carries water under a pressure of 5N/mm^2. Calculate the maximum and minimum intensities of circumferential stresses and sketch the distribution of circumferential stress intensity and the intensity of radial pressure across the section. **(12 Marks)**

Module-3

5 a. Define shear force, bending moment and point of contraflexure. Explain how to calculate them? **(06 Marks)**

b. Develop shear force diagram and bending moment diagrams for the beam loaded shown in Fig. Q5(b) marking the values at salient points. Determine the position and magnitude of maximum bending moment.

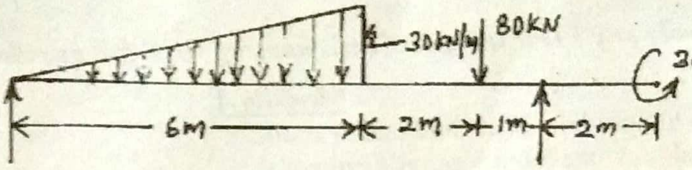

Fig. Q5(b) **(14 Marks)**

OR

6 a. Obtain the relationship between udl, shear force and bending moment. **(06 Marks)**

b. Construct SFD and BMD for the beam loaded shown in Fig. Q6(b). Also locate the point of contraflexure.

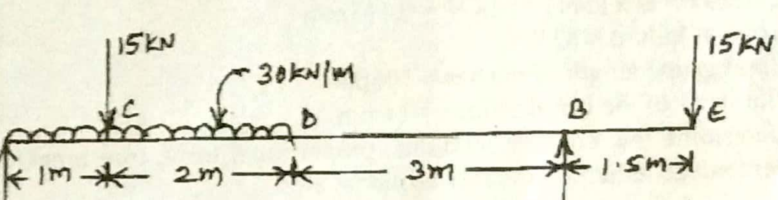

Fig.Q6(b) **(14 Marks)**

Module-4

7 a. Derive torsional equation with usual notations. **(06 Marks)**

b. A T-section of flange 120mm×12mm and overall depth 200mm with 12mm web thickness is loaded such that at a section it has a bending moment of 20KN.m and shear force of 120KN. Sketch the bending and shear stress distribution diagram marking the salient values. **(14 Marks)**

OR

8 a. Derive Bernoulli-Euler bending equation with usual notations. **(08 Marks)**

b. A solid circular shaft has to transmit power of 1000KW at 120rpm. Find the diameter of the shaft if the shear stress of the material is not to exceed $80N/mm^2$ The maximum torque is 1.25 times the mean torque. What percentage saving in material could be obtained if the shaft is replaced by a hollow one whose internal diameter is 0.6 times the external diameter? The length of the shaft, material and maximum shear stress being same. **(12 Marks)**

Module-5

9 a. Define slope, deflection and elastic curve. Explain Macaulay's method of determining slope and deflection. **(10 Marks)**

b. Compare the crippling loads given by Euler's and Rankine's formula for a tubular steel column 2.5m long having outer and inner diameter as 40mm and 30mm respectively. The column is loaded through pin joints at the ends. Take permissible compressive stress as $320N/mm^2$, Rankine constant as $\frac{1}{7500}$ and E=210GPa. For what length of the column of their cross section, does the Euler's formula cease to apply? **(10 Marks)**

OR

10 a. Differentiate between short and long column and what are the limitations of Euler's theory. **(06 Marks)**

b. Calculate slope at A and deflection at D for the overhanging beam shown in Fig. Q10(b). Take $E = 200GPa$ and $I = 50\times10^6 mm^4$. **(14 Marks)**

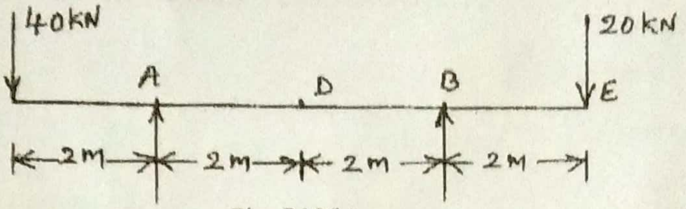

Fig. Q10(b).

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III SEMESTER B.E DEGREE EXAMINATION, JAN/FEB.2021
STRENGTH OF MATERIALS (18CV32)
QUESTION PAPER SOLUTION

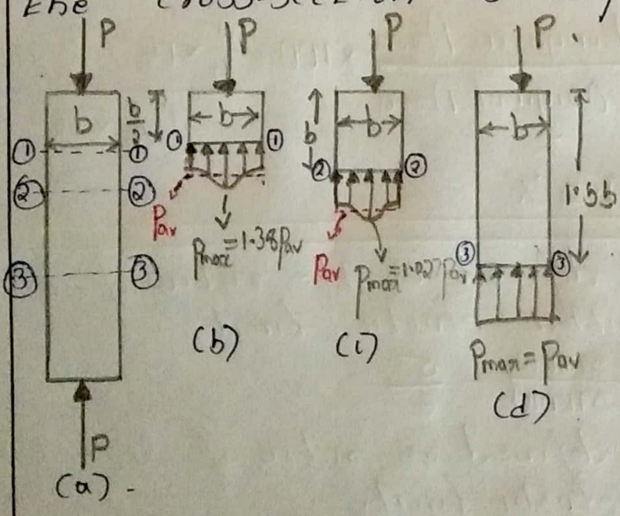
Q1.a **LONGITUDINAL STRAIN**: The change in length in the direction of force per unit length is known as LONGITUDINAL STRAIN.

$$\epsilon = \frac{\Delta L}{L} = \frac{\text{CHANGE IN LENGTH}}{\text{ORIGINAL LENGTH}}$$

LATERAL STRAIN: The change in lateral length (i.e ⊥ to direction of force) per unit lateral dimension is known as LATERAL STRAIN.

$$\text{Lateral strain} = \frac{b-b'}{b} = \frac{\Delta b}{b}.$$

b. St. Venant's principle states that "except in the region of the extreme ends of a bar carrying direct loading, the stress distribution over the cross-section is uniform."



- The stress distribution at section 1-1, at a distance of $b/2$ is shown in fig (b), where $P_{max} = 1.38 P_{av}$
- At S/c 2-2, distance 'b' from the end, max stress is very near to avg stress.
- At S/c 3-3, at a distance > b $P_{max} = P_{avg}$.

This illustrates the famous St. Venant's principle of rapid dissipation of localised stress.

Q1.C. Given, $d_0 = 18mm$ $L_0 = 82mm$ $P_E = 75kN$. $P_F = 82kN$

$L_F = 106mm$ $d_F = 14mm$

Original Area, $A = \frac{\pi d^2}{4} = \frac{\pi \times 18^2}{4} = 254.5mm^2$.

For a load of 62kN (i.e within elastic limit), $\Delta L = 0.113mm$

$$\therefore E = \frac{Stress}{Strain} = \frac{(P/A)}{(\Delta L/L)} = \frac{62\times10^3/254.5}{0.113/82} = 176.78\times10^3 N/mm^2.$$

Stress at proportionality Limit, $\sigma = \frac{P}{A} = \frac{75\times10^3}{\pi/4 \times 18^2} = 294.7N/mm^2$

$\therefore$ proportionality limit $= 294.7 N/mm^2$.

$$\text{True Breaking Stress} = \frac{\text{Breaking Load}}{\text{True Area}} = \frac{82\times10^3}{\pi/4\times14^2} = 532.68 N/mm^2.$$

$$\%\ \text{elongation} = \frac{\text{change in length}}{\text{Original length}} \times 100 = \frac{106-82}{82}\times100$$

$$= 29.27\%$$

$$\%\ \text{Reduction in area} = \frac{\text{Change in Area}}{\text{Original Area}}\times100 = \frac{\frac{\pi}{4}\times18^2 - \frac{\pi}{4}\times14^2}{\pi/4\times18^2}\times100$$

$$= 39.5\%$$

Q2.a. MODULUS OF ELASTICITY (E):- is defined as the ratio of LINEAR STRESS to LINEAR STRAIN within elastic limit.

$$E = \frac{\text{LINEAR STRESS}}{\text{LINEAR STRAIN}} = \frac{\sigma}{\epsilon}$$

MODULUS OF RIGIDITY (G):- is defined as ratio of SHEARING STRESS to SHEARING STRAN within elastic limit.

$$G = \frac{\text{SHEAR STRESS}}{\text{SHEAR STRAIN}} = \frac{\tau}{\phi}$$

BULK MODULUS (K):- is defined as ratio of direct stress to the volumetric strain when a body is subjected to identical stress in 3 mutually ⊥ directions & it undergoes changes in 3 directions w/o undergoing distortion of shape.

$$K = \frac{\text{DIRECT STRESS } (\sigma)}{\text{VOLUMETRIC STRAIN } (e_v)} = \frac{\sigma}{e_v}$$

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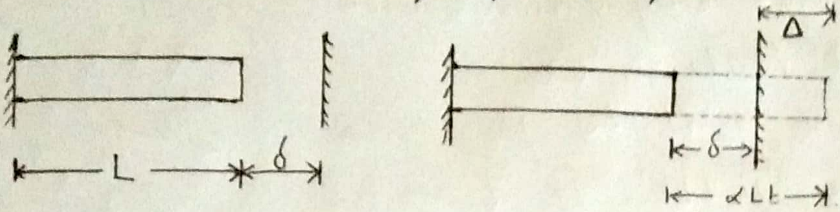
2.b. If the free expansion is prevented partly as shown in figure below, the force developed corresponds to the amount of free expansion (Δ)



If the bar is free to extend when temperature is increased by 't' degree its extension would be $\Delta = \alpha L t$
But this extension is prevented partially by forces at support

$\therefore \Delta = \alpha L t - \delta \Rightarrow \alpha L t = \Delta + \delta \Rightarrow \alpha E t \dfrac{L}{E} = \Delta + \delta$.

$\Rightarrow \alpha E t = \dfrac{E}{L}(\Delta + \delta)$ $\therefore$ Temp stress, $\sigma = \dfrac{E}{L}(\Delta + \delta)$.

C. Given $P = 390\,kN$ $B = 250\,mm$ $E_s = 15 E_c$ $A_s = 2500\,mm^2$

$A_c = 250 \times 250 = 250^2\,mm^2$

i) $P_c + P_s = 390 \times 10^3\,N$

$\Delta_c = \Delta_s \Rightarrow \dfrac{P_c \cancel{L_c}}{A_c E_c} = \dfrac{P_s \cancel{L_s}}{A_s E_s} \Rightarrow \dfrac{P_c}{250^2 \times \cancel{E_c}} = \dfrac{P_s}{2500 \times 15 \cancel{E_c}}$

$\Rightarrow P_c = 1.67 P_s$

$\therefore 1.67 P_s + P_s = 390 \times 10^3\,N$ $\therefore P_s = 146.07\,kN$ $\therefore P_c = 243.93\,kN$.

$\sigma_s = \dfrac{P_s}{A_s} = \dfrac{146.07 \times 10^3}{2500} = 58.43\,N/mm^2$ $\sigma_c = \dfrac{P_c}{A_c} = \dfrac{243.93 \times 10^3}{250^2} = 3.9\,N/mm^2$.

ii) $\sigma_c = 4.5\,N/mm^2$ (Given) $P = 480\,kN$ $A_s = ?$

$P_c + P_s = 480\,kN$.

$\Delta_c = \Delta_s \Rightarrow \dfrac{P_c}{250^2 E_c} = \dfrac{P_s}{A_s \times 15 E_c} \Rightarrow P_c = 4166.67 \dfrac{P_s}{A_s} \Rightarrow P_s = \dfrac{P_c \times A_s}{4166.67}$.

$\Rightarrow P_c + \dfrac{P_c \times A_s}{4166.67} = 480\,kN$. & $\sigma_c = \dfrac{P_c}{A_c} = \dfrac{480 \times 10^3\,N}{1 + \dfrac{A_s}{4166.67}} \times \dfrac{1}{250^2} = 4.5\,N/mm^2$.

$\Rightarrow P_c \left[1 + \dfrac{A_s}{4166.67}\right] = 480\,kN$

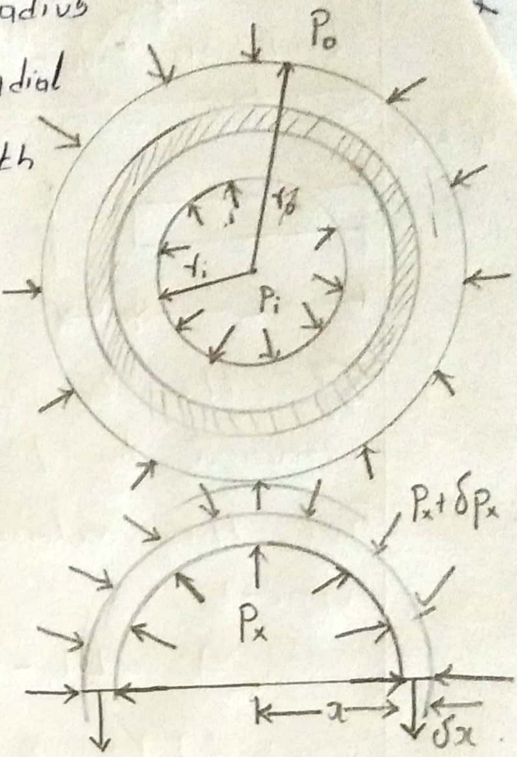
$\Rightarrow A_s = 2944.4\,mm^2$

Q3.a. Let $r_i \to$ inner radius, $r_o \to$ outer radius

$p_i \to$ internal radial pressure, $p_o \to$ external radial pressure

Consider a semicircular ring element with internal radius 'x' & thickness 'δx'.

Let internal pressure on it be p_x & external pressure '$p_x + \delta x$'. If 'L' is length then



Bursting Force $= p_x(2xL) - (p_x + \delta p_x) \times [2(x+\delta x)L] \to (1)$

Hoop stress $= f_1$

Then Resisting force $= f_1(2\delta x L) \to (2)$

$(1) = (2)$

$\Rightarrow f_1(\cancel{2}\delta x \cancel{L}) = p_x(\cancel{2}x\cancel{L}) - (p_x + \delta p_x)[\cancel{2}(x+\delta x)\cancel{L}]$.

$\Rightarrow f_1(\delta x) = p_x x - (p_x + \delta p_x)(x + \delta x). = \cancel{p_x x} - [\cancel{p_x x} + p_x \delta x + \delta p_x x + \delta p_x \delta x]$.

$\Rightarrow f_1(\delta x) = -p_x \delta x - \delta p\, x$ (Higher order of small quant neglected).

$\Rightarrow f_1 = -p_x - \delta p \dfrac{x}{\delta x}$ $\Rightarrow f_1 + x\dfrac{\delta p}{\delta x} + p_x = 0 \to (3)$

Let f_2 be the longitudinal stress. Then longitudinal strain is given by

$$e_2 = \frac{f_2}{E} - \mu\frac{f_1}{E} + \mu\frac{p_x}{E} = \frac{f_2}{E} - \frac{\mu}{E}[f_1 - p_x].$$

Acc to Lame theory 'e_2' is constant. $\therefore$ f_2 is constant $f_1 - p_x$ should be constant. $\therefore$ (3) becomes. [Let $f_1 - p_x = 2a$. $\Rightarrow f_1 = p_x + 2a$].

$(p_x + 2a) + p_x + x(\delta p_x/\delta x) = 0$

$\Rightarrow \dfrac{\delta p_x}{p_x + a} = -2\dfrac{\delta x}{x}$. Integrating on both sides

$\Rightarrow \log(p_x + a) = -2\log x + c$. $\Rightarrow \log(p_x + a) = -2\log x + \log b$ [$c = \log b$].

$\Rightarrow \log(p_x + a) = -\log x^2 + \log b = \log(b/x^2)$ $\Rightarrow p_x + a = b/x^2$

$\Rightarrow p_x + 2a = b/x^2 + a$ $\therefore f_1 = \dfrac{b}{x^2} + a$ & $p_x = \dfrac{b}{x^2} - a$.

Q3.b. Given, $\sigma_x = 600\ MN/m^2$ $\sigma_y = -400\ MN/m^2$ $\tau_{xy} = 200\ MN/m^2$.

$\theta = 90 - 30 = 60°$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = \frac{600-400}{2} + \frac{600+400}{2}\cos 120 + 200\sin 120 = 23.2\ MN/m^2$$

$$\sigma_t = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta = \frac{600-(-400)}{2}\sin 120 - 200\cos 120$$
$$= 533\ MN/m^2.$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \frac{600+(-400)}{2} + \frac{1}{2}\sqrt{[600-(-400)]^2 + 4\times 200^2}$$
$$= 500 + 538.5 = 1038.5\ MN/m^2.$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2} = 500 - 538.5 = -38.5\ MN/m^2.$$

$$2\theta = \tan^{-1}\left[\frac{2\tau}{\sigma_x - \sigma_y}\right] = \tan^{-1}\left[\frac{2\times 200}{600-(-400)}\right] = 21.8° \quad \therefore \theta = 10.9°$$

$$\theta' = \theta + 90 = 10.9 + 90 = 100.9\ MN/m^2.$$

$$\sigma_{t.max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2} = 538.5\ MN/m^2.$$

Direction of plane of Max shear stress $= \theta + 45° = 10.9° + 45°$
$= 55.9°$

c. At any point in a cylinder, 2 principal stresses acting are

$$f_1 = \frac{pd}{2t} \qquad f_2 = \frac{pd}{4t}.$$

Circumferential strain, $e_1 = \frac{f_1}{E} - \mu\frac{f_2}{E} = \frac{1}{E}\left[\frac{pd}{2t} - \mu\frac{pd}{4t}\right] = \frac{pd}{4tE}[2-\mu]$

$$e_1 = \frac{\text{Final circum} - \text{Original circum}}{\text{Original circum}} = \frac{\pi(d+\delta d) - \pi d}{\pi d} = \frac{\delta d}{d}.$$

$e_2 = \frac{f_2}{E} - \frac{\mu f_1}{E} = \frac{pd}{4tE}[1-2\mu]$. Also $e_2 = \frac{\delta L}{L}$.

$V = \frac{\pi}{4}d^2.L$ $\quad \delta V = \frac{\pi}{4}d^2.\delta L + 2\frac{\pi}{4}(Ld)\delta d$

$$e_v = \frac{\delta V}{V} = \frac{\delta l}{L} + 2\frac{\delta d}{d} \Rightarrow e_v = \frac{pd}{4tE} + \frac{2pd}{4tE}(2-\mu).$$

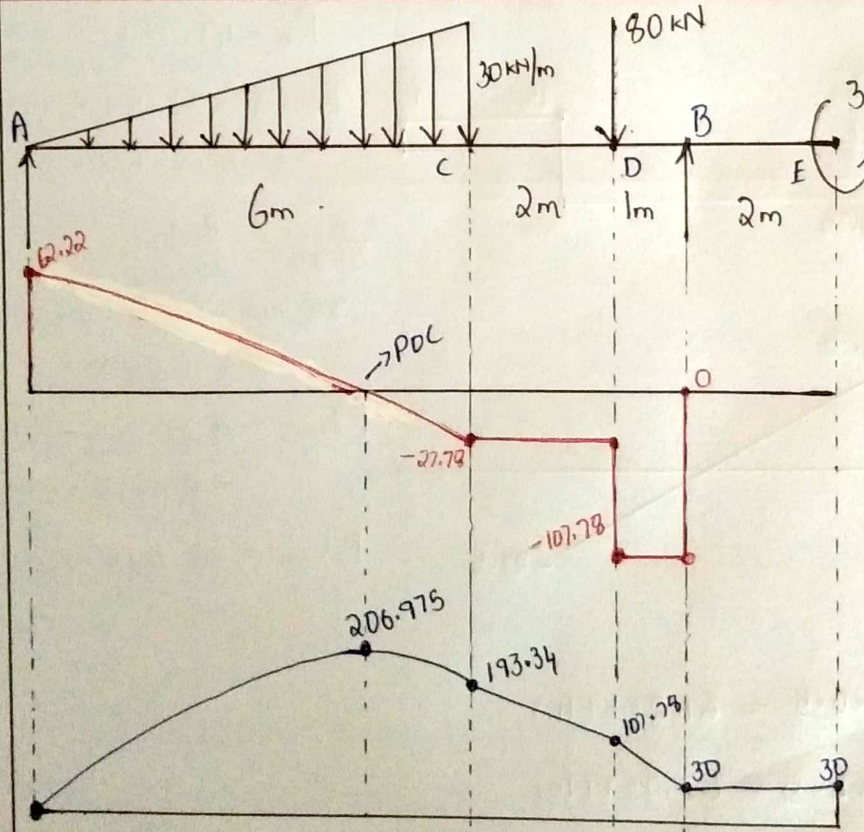
$$e_v = \frac{pd}{4tE}[4-2\mu+1-2\mu] \quad \therefore e_v = \frac{pd}{4tE}[5-4\mu].$$

Q5.a. SHEAR FORCE: at a section in a beam is the force that tries to shear off the section & is obtained as algebraic sum of all the forces including the reaction acting normal to the axis of the beam either to the left or right of the section.

BENDING MOMENT: at a section in a beam is the moment that tries to bend it & is obtained as the algebraic sum of the moments of all the forces about the section including the reaction acting on the beam either to the left or to the right of the section.

POINT OF CURVATURE: is a point on the beam where SF = 0 and BM is maximum.

b.



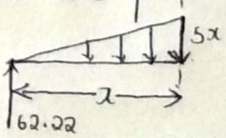
$\Sigma M_A = 0$

$\Rightarrow \left(\frac{1}{2}\times30\times6\right)\left(\frac{2}{3}\times6\right) + 80\times8 - R_B\times9 - 30 = 0$

$\Rightarrow R_B = 107.78\,kN$.

$R_A = \frac{1}{2}\times30\times6 + 80 - 107.78$
$= 62.22\,kN$.

For AC portion $? \rightarrow x$
$30 \rightarrow 6$



$F_x = 62.22 - 0.5\times(5x)(x)$
$= 62.22 - 2.5x^2$

$M_x = 62.22x - [0.5(5x)(x)](x/3)$
$= 62.22x - 0.833x^3$

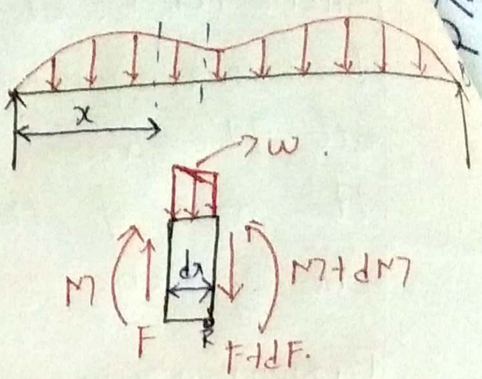
At POC, $F_x = 0 \Rightarrow 62.22 - 2.5x^2 = 0 \Rightarrow x = 5m$.

$\therefore M_{max} = 62.22\times5 - 0.833\times5^3 = 206.975\,kNm$

$M_c = -80\times2 + 107.78\times3 + 30 = 193.34\,kNm$.

$M_D = 107.78\times1 = 107.78\,kNm$. $M_B = 30\,kNm$.

Q6.a. Consider a beam AB subjected to a general loading as shown in figure. Consider an element of length 'dx' at distance 'x' from left support & draw its FBD.



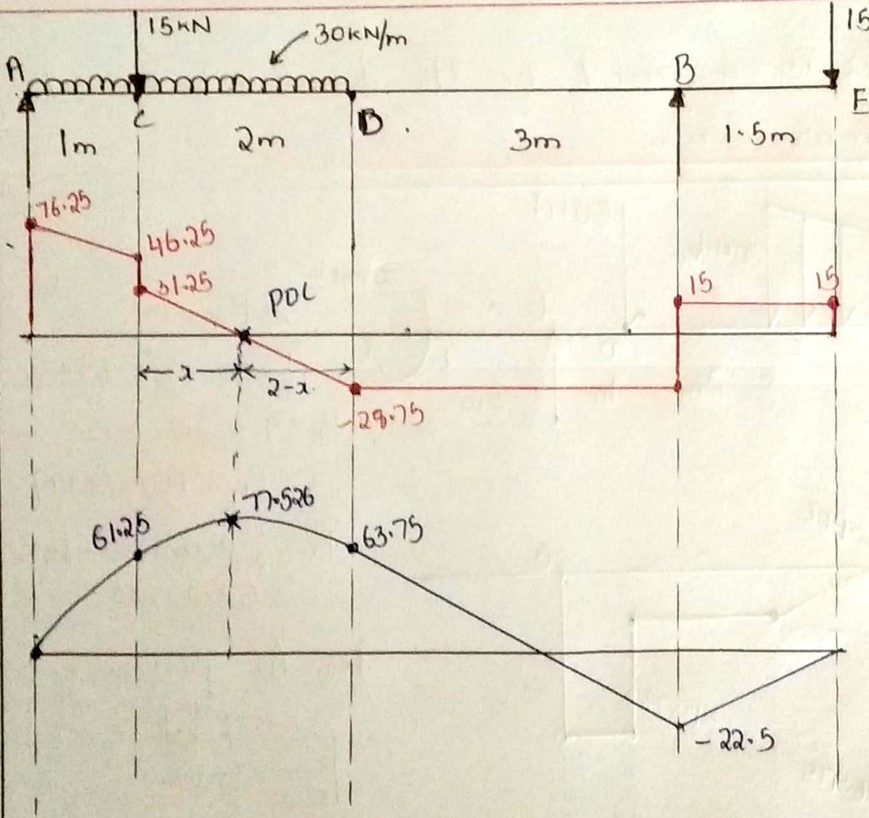
$$\Sigma F_v = 0 \Rightarrow F - (F + dF) - w(dx) = 0$$

$$\Rightarrow -dF = w(dx) \quad \therefore \frac{dF}{dx} = -w$$

$$\Sigma M_R = 0 \Rightarrow M + F(dx) - [w(dx)][dx/2] - (M + dM) = 0$$

$$\Rightarrow F(dx) - w\cancel{\left(\frac{dx^2}{2}\right)}^{\to 0} - dM = 0 \quad \Rightarrow F(dx) = dM \quad \therefore \frac{dM}{dx} = F$$

b.



$$\Sigma M_A = 0$$

$$(30\times 3)\times 1.5 + 15\times 1 - R_B\times 6 + 15\times 7.5 = 0$$

$$R_B = 43.75\ kN$$

$$R_A = 30\times 3 + 15 + 15 - 43.75 = 76.25\ kN$$

$$\frac{x}{31.25} = \frac{2-x}{28.75}$$

$$28.75x = 62.5 - 31.25x$$

$$x = 1.04\ m$$

$$M_{max} = 76.25\times 2.04 - 15\times 1.04 - 30\times 2.04\times \frac{2.04}{2}$$

$$M_{max} = 77.526\ kNm$$

$$M_C = 76.25\times 1 - 30\times 1\times 0.5 = 61.25\ kNm$$

$$M_D = 43.75\times 3 - 15\times 4.5 = 63.75\ kNm$$

$$M_B = -15\times 1.5 = -22.5\ kNm$$

$$M_E = 0$$

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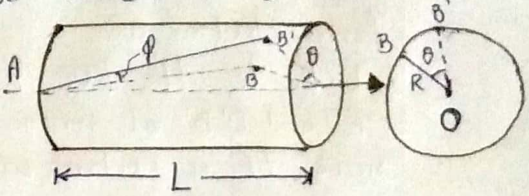
Q7.a. Consider a shaft of length 'L' & radius 'R' fixed at one end & subjected to torque 'T'. Let 'O' be the center & 'B' be a point on surface.



Due to torque 'T' let 'B' move to B' & 'φ' be the shear strain.

$BB' = R\theta$ and $\tan\phi = \frac{BB'}{AB}$ => $\phi = \frac{BB'}{AB}$ (for small angles) => $BB' = AB(\phi)$

$\Rightarrow AB(\phi) = R\theta$ => $L\phi = R\theta$.

WKT, $G = \tau/\phi$ => $\phi = \tau_s/G$.

$\therefore L(\tau_s/G) = R\theta$ => $\frac{\tau}{r} = \frac{G\theta}{L}$ (if any pnt B at a dist 'r' is considered) $\left(\tau_s/R = \frac{\tau}{r}\right)$

Consider an elemental area δa at a distance 'r' from centre. ∴ Resisting Torsional moment, $dT = dF \times r$

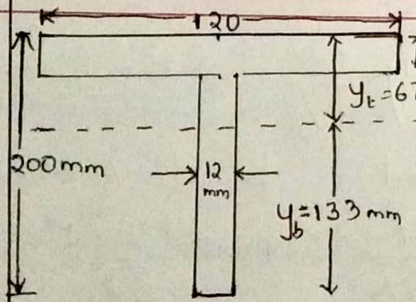
$\Rightarrow dT = (\tau\, da)r = \frac{\tau_s r}{R} da\, r = \frac{\tau_s r^2 . da}{R}$

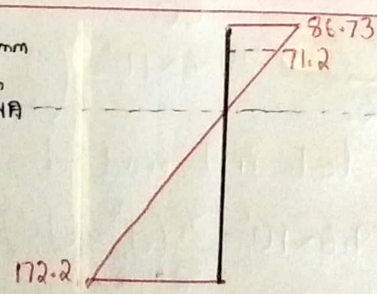
Total Resisting moment, $T = \Sigma(dT) = \frac{\tau_s}{R}\Sigma r^2.da = \frac{\tau_s}{R}(J)$

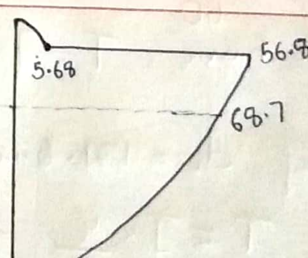
where J → Polar moment of inertia

$\therefore T = \frac{\tau_s}{R}(J)$ => $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$.

Q7.b.







$A = 120\times12 + 188\times12 = 3696\,mm^2$, $\bar{y}_t = (120\times12)\times6 + (188\times12)\times(12+188/2) / [120\times12 + 188\times12] = 67\,mm$

$I = \frac{120\times12^3}{12} + (120\times12)(67-6)^2 + \frac{12\times188^3}{12} + (188\times12)(106-67)^2 = 15.45\times10^6\,mm^4$

$\sigma_{top} = \frac{M}{I}y = \frac{20\times10^6}{15.45\times10^6}\times67 = 86.73$, $\sigma_{flange} = \frac{20\times10^6}{15.45\times10^6}\times(67-12) = 71.2$ $\sigma_b = \frac{20}{15.45}\times(200-67) = 172.2\,N/mm^2$

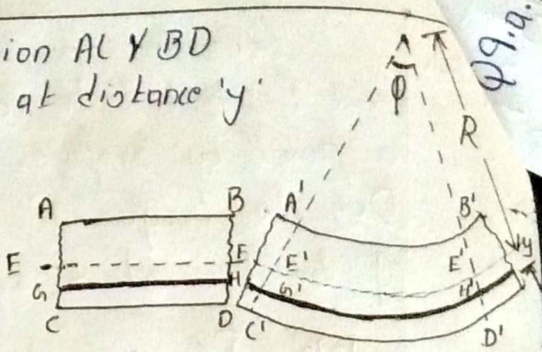
$\tau_{flange\ above} = \frac{F}{bI}(a\bar{y}) = \frac{120\times10^3}{120\times15.45\times10^6}(120\times12)(67-6) = 5.68\,N/mm^2$. $\tau_{NA} = \frac{120\times10^3}{12\times15.45\times10^6}(133\times12)(133/2)$

$\tau_{flange\ below} = \frac{120\times10^3}{12\times15.45\times10^6}(120\times12)(67-6) = 56.85\,N/mm^2$. $= 68.7\,N/mm^2$.

Q8.a. Consider a portion of beam b/w section AC & BD

Let EF be the neutral axis & GH an element at distance 'y' from Neutral axis

Let 'R' be the ROC & 'ϕ' the angle subtended by C'A' and D'B' at centre.



Since EF is neutral axis, there is no change in length

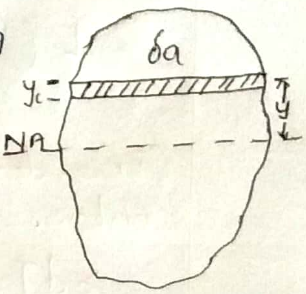
$$EF = E'F' = R\phi$$

$$\epsilon_{GH} = \frac{G'H' - GH}{GH} \quad \text{where } GH = EF = R\phi \ \& \ G'H' = (R+y)\phi = R\phi + y\phi.$$

$$\therefore \epsilon_0 = \frac{R\phi + y\phi - R\phi}{R\phi} = \frac{y\phi}{R\phi} \Rightarrow \epsilon = \frac{y}{R} \Rightarrow \frac{\sigma}{E} = \frac{y}{R} \quad \left(\because \epsilon = \frac{\sigma}{E}\right).$$

Consider an element of area δa for a distance 'y' from NA

$\sigma = \frac{E}{R} y$. is stress on element.



Force on element, $F = (\sigma) \times (\delta A) = \frac{E}{R} y (\delta a)$.

Moment of Resistance, $F \times y = \frac{E}{R} y^2 (\delta a)$.

MOR of whole section, $M = \Sigma \frac{E}{R} y^2 (\delta a) = \frac{E}{R} \Sigma y^2 (\delta a) = \frac{E}{R}(I) \Rightarrow \frac{M}{I} = \frac{E}{R}$

$$\therefore \frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}.$$

b. Given $N = 120\,rpm$ $P = 1000\,kW$ $\tau = 80\,N/mm^2$

$P = 1000 \times 10^3\,W = 1000 \times 10^3\,Nm/s = 1000 \times 10^6\,Nmm/s$

$$P = \frac{2\pi NT}{60} \Rightarrow 1000 \times 10^6 = \frac{2\pi \times 120 T}{60} \Rightarrow T = 79.6 \times 10^6\,Nmm.$$

i) We have, $T = J \times \frac{\tau}{R} \Rightarrow 79.6 \times 10^6 = \left(\frac{\pi}{32} d^4\right) \times \frac{80}{d/2} \Rightarrow \underline{d = 171.6\,mm}$

ii) If d let internal diameter be $d_i = 0.6 d_o$.

$$\therefore T = J \times \frac{\tau}{R} \Rightarrow 79.6 \times 10^6 = \frac{\pi}{32}(d_o^4 - d_i^4) \times \frac{\tau}{d_o/2} \Rightarrow 79.6 \times 10^6 = \frac{\pi}{32}(d_o^4 - 0.6^4 d_o^4) \times \frac{80 \times 2}{d_o}$$

$$\Rightarrow \frac{79.6 \times 10^6 \times 32}{\pi \times 80 \times 2} = \frac{d_o^{\cancel{4}\,3}(1 - 0.6^4)}{\cancel{d_o}}$$

$\Rightarrow d_o = 179.89\,mm$ $\therefore d_i = 0.6 d_o = 108\,mm$.

C/s of hollow shaft $= \pi/4 (d_o^2 - d_i^2) = 15975.9\,mm^2$

C/s of solid " $= (\pi/4) d^2 = (\pi/4) \times 171.6^2 = 23127.2\,mm^2$.

$$\%\ \text{saving} = \frac{23127.2 - 15975.9}{23127.2} \times 100 = 30.92\,\%.$$

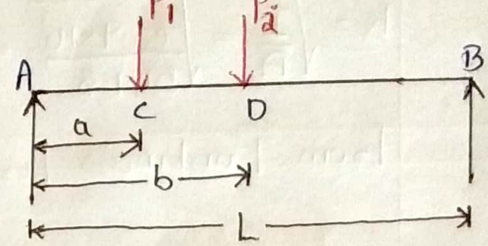
Scanned with CamScanner

Q9.a. SLOPE: of a beam at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of beam.

DEFLECTION: of a beam at any section on the axis of the beam is distance b/w its position before & after loading.

MACAULAY'S METHOD OF DETERMINING SLOPE & DEFLECTION

Consider a simply supported beam subjected to 2 concentrated loads.



Let V_A be the reaction at A. Then

for portion AC, $M_x = V_A x$

CD, $M_x = V_A x - P_1(x-a)$

DB, $M_x = V_A x - P_1(x-a) - P_2(x-b)$

In general $M_x = V_A x - P_1(x-a) - P_2(x-b)$.

(If term $(x-a)$ or $(x-b)$ becomes $-ve$ then it is not applicable for that point).

$$M_x = EI\frac{d^2y}{dx^2} = V_A x - P_1(x-a) - P_2(x-b).$$

$$\Rightarrow EI\frac{dy}{dx} = C_1 + \frac{V_A x^2}{2} - P_1\frac{(x-a)^2}{2} - P_2\frac{(x-b)^2}{2}.$$

$$EIy = C_2 + C_1 x + V_A\frac{x^3}{6} - P_1\frac{(x-a)^3}{6} - P_2\frac{(x-b)^3}{6}.$$

The constants 'C_1' and 'C_2' are found from boundary conditions.

Once they are found, slope & deflection at any point can be found.

Q9.b. Given, $L = 2.5m$ $d_o = 40mm$ $d_i = 30mm$ $\sigma_c = 320 N/mm^2 = f_c$

END CONDITION: BOTH ENDS PINNED, $\alpha = 1/7500$ $E = 210GPa = 210\times10^3\times10^6 Pa$
$= 210\times10^3 MPa = 210\times10^3 N/mm^2$.

$A = \pi/4 (d_o^2 - d_i^2) = \pi/4 (40^2 - 30^2) = 549.8 mm^2$.

$I = \pi/64 (d_o^4 - d_i^4) = \pi/64 (40^4 - 30^4) = 85.9\times10^3 mm^4$.

From Euler's Formula, $P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \times 210\times10^3 N/mm^2 \times 85.9\times10^3 mm^4}{2500^2 mm^2}$

$= 28486 kN$.

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{85.9\times10^3}{549.8}} = 12.5 mm$

From Rankine's Formula, $P_{cr} = \frac{f_c A}{1+\alpha(L/k)^2} = \frac{320\times549.8}{1+(1/7500)\left(\frac{2500}{12.5}\right)^2} =$

$= 27.79 kN$.

$\frac{P_{cr}, \text{Euler's}}{P_{cr}, \text{Rankine's}} = \frac{28486 kN}{27.79 kN} = 1025$.

Euler's formula ceases to apply when $f_{cr} = f_c$.

$\Rightarrow \frac{\pi^2 E}{(L/k)^2} = f_c \Rightarrow \frac{\pi^2 E}{f_c} = (L/k)^2 \Rightarrow \frac{\pi^2 E k^2}{f_c} = L^2$.

$\Rightarrow L^2 = \frac{\pi^2 \times 210\times10^3 \times 12.5^2}{320} = 1012019.9$

$\therefore L = 1006 mm$.

Q9.b. Given, $L = 2.5m$ $d_o = 40mm$ $d_i = 30mm$ $\sigma_c = 320 N/mm^2 = f_c$

END CONDITION: BOTH ENDS PINNED, $\alpha = 1/7500$ $E = 210 GPa = 210\times10^3\times10^6 Pa$
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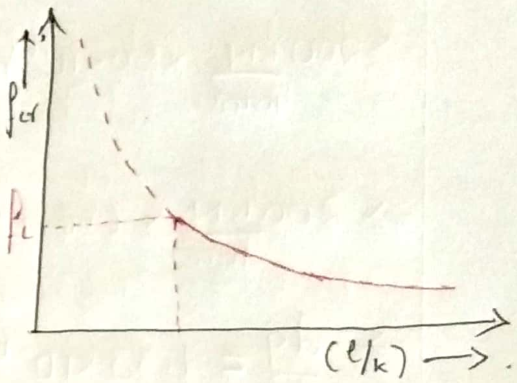
$\Rightarrow L^2 = \dfrac{\pi^2 \times 210\times10^3 \times 12.5^2}{320} = 1012019.4$

$\therefore L = 1006 mm$.

Q10.a.

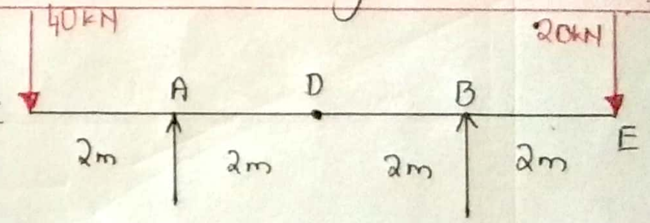
SHORT COLUMN	LONG COLUMN
→ Slenderness Ratio < 12	Slenderness ratio > 12
→ Tendency to Buckle is very low	Tendency to crushing is very low.
→ Usually fails by crushing	Usually fails by buckling.

LIMITATIONS OF EULER'S THEORY



- As (l/k) approaches zero, th crippling stress tends to infinity. But this cannot happen. Befor this, th material will get crushed. Hence it is valid only upto crushing stress.
- Euler's formula holds good for if column fails by buckling ie for long columns with high (l/k) values. where bending stresses > compressive stress.
  But there is a range of SR for which bending & compressive stress are comparable.
- For such cases Euler's formula does not hold good.

Q10.b. $\Sigma M_B = 0$.



$V_A \times 4 - 40\times 6 + 20\times 2 = 0 \Rightarrow V_A = 50\,kN$

$\therefore V_B = 40 + 20 - 50 = 10\,kN$.

From C, $M_x = -40x + 50(x-2) + 10(x-6)$ $\Rightarrow EI\frac{d^2y}{dx^2} = -40x + 50(x-2) + 10(x-6)$.

$\Rightarrow EI\frac{dy}{dx} = -20x^2 + 25(x-2)^2 + 5(x-6)^2 + C_1$ $\Rightarrow EIy = -\frac{20x^3}{3} + \frac{25}{3}(x-2)^3 + \frac{5}{3}(x-6)^3 + C_1x + C_2$

At $x = 2$, $y = 0 \Rightarrow 0 = 2C_1 + C_2 - \frac{20\times 2^3}{3} + \cancel{\frac{25}{3}(2-2)^3} + \cancel{\frac{5}{3}(2-6)^3} \Rightarrow 0 = 2C_1 + C_2 - 53.3 \rightarrow (1)$

At $x = 6$, $y = 0 \Rightarrow 0 = 6C_1 + C_2 - \frac{20\times 6^3}{3} + \frac{25}{3}(6-2)^3 \Rightarrow 0 = 6C_1 + C_2 - 906.67 = 0 \rightarrow (2)$

(2) − (1) $\Rightarrow 4C_1 - 853.34 = 0$ $\therefore C_1 = 213.335$ & $C_2 = -373.37$

$\therefore$ At D, $x = 4m$ $\therefore y_D EI = -\frac{20\times 4^3}{3} + \frac{25}{3}(4-2)^3 + \cancel{\frac{5}{3}(4-6)^3} + 213.335\times 4 + (-373.34) =$

$\Rightarrow y_D \times (200\frac{kN}{mm^2}\times 50\times 10^6 mm^4) = 120\,kNm^3$ $\Rightarrow y_D \times 200\frac{kN}{mm^2}\times 50\times 10^6 mm^4 = 120\times 10^9\,kNmm^3$

$y_D = 12mm$.

$\therefore y_D = 12mm$

At $x=2$ i.e A.

$$EI\frac{dy}{dx} = -20(2)^2 + 25\cancel{(2-2)}^2 + 5\cancel{(2-6)}^2 + C_1$$

$\rightarrow 0$, $\rightarrow -ve$

$$\Rightarrow EI\frac{dy}{dx} = -20\times 2^2 + 213.335$$

$$\Rightarrow 200\frac{kN}{mm^2}\times(50\times10^6)mm^4\frac{dy}{dx} = 133.335\ kNm^2.$$

$$\Rightarrow 200\frac{kN}{mm^2}\times(50\times10^6)mm^4\frac{dy}{dx} = 133.335\times10^4\ kN\,mm^2$$

$$\Rightarrow \frac{dy}{dx} = 1.33\times10^{-4}\ \text{radian}.$$

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