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CBCS SCHEME

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18EC53

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022
Principles of Communication Systems

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Write an AM wave expression in time domain and in frequency domain. Draw AM waveform. **(07 Marks)**

b. With neat diagram, explain the demodulation of AM wave using envelope detector. **(08 Marks)**

c. An audio frequency signal $M(t) = 5 \sin 2\pi (10^3)t$ is used to amplitude modulate a carrier of $C(t) = 100 \sin 2\pi (10^6)t$. Assume modulation index $\mu = 0.4$. Find: i) Sideband frequencies ii) Amplitude of each sideband 8 iii) Bandwidth iv) Total power delivered to a load of 100μ v) Find efficiency of AM wave, assume $R = 1\Omega$. **(05 Marks)**

8.4k

OR

2 a. Explain the generation of DSBSC wave using a Ring modulator. **(10 Marks)**

b. Explain with a neat diagram, the working of Quadrature Carrier Multiplexing (QAM). **(08 Marks)**

c. An AM signal with a carrier of 1kW has 200W in each sideband. What is the percentage of modulation? **(02 Marks)**

Module-2

3 a. Define angle modulation. Derive the FM wave expression in time domain. **(08 Marks)**

b. Define the following terms:
   i) Modulation index
   ii) Frequency deviation
   iii) Bandwidth **(07 Marks)**

c. A FM wave is represented by the equation $V = 10 \sin [5 \times 10^8 t + 4 \sin 1250t]$. Find: i) Carrier frequency and modulating frequency ii) Modulation index and frequency deviation iii) Bandwidth using Carson's rule. **(05 Marks)**

OR

4 a. Write the basic block diagram of PLL. Derive the expression for non-linear model of PLL. **(10 Marks)**

b. Explain the direct method of generating FM wave using Hartley oscillator with relevant equations and diagram. **(06 Marks)**

c. Write the Narrowband FM and wideband FM expression. **(04 Marks)**

Module-3

5 a. Derive the expression for figure of merit of an AM receivers using envelope detection. **(10 Marks)**

b. Explain the noisy receiver model with neat diagram. Explain briefly the figure of merit. **(06 Marks)**

c. Explain the noise equivalent bandwidth with relevant equation. **(04 Marks)**

OR

6 a. Derive the expression for Figure Of Merit (FOM) for DSBSC receiver. (10 Marks)
b. Explain the use of pre-emphasis and de-emphasis circuit in an FM system. (06 Marks)
c. Define the white noise. Briefly explain the power spectral density and autocorrelation function of white noise. (04 Marks)

Module-4

7 a. State sampling theorem. Write the mathematical form of sampled signal and explain the steps to reconstruct the signal g(t) from the sequence of sample value. (10 Marks)
b. Explain the concept of TDM with a neat block diagram. (06 Marks)
c. What is aperture effect? Briefly explain how to overcome this effect. (04 Marks)

OR

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8 a. Briefly explain the following pulse modulation with waveform:
i) PAM ii) PWM iii) PPM. (09 Marks)
b. With neat block diagram, explain the generation of PPM wave. (05 Marks)
c. Explain the following terms:
i) Under sampling
ii) Over sampling
iii) Nyquist rate. (06 Marks)

Module-5

9 a. Derive the expression of output signal to noise ratio of a uniform quantizer. (08 Marks)
b. With neat block diagram, explain the transmitter, transmission path and receiver of a PCM system. (08 Marks)
c. An audio signal digitalized using PCM. Assume the audio signal bandwidth to be 20kHz.
i) What is the Nyquist rate and Nyquist period of the audio signal?
ii) If the samples are quantized to L = 4096 levels and then binary coded, determine the number of bits required to encode a sample. (04 Marks)

OR

10 a. Draw the line codes for given binary representation 01101001
i) Unipolar NRZ signaling
ii) Polar NRZ signaling
iii) Unipolar RZ signaling
iv) Bipolar RZ signaling
v) Manchester code. (10 Marks)
b. Explain granular noise and slope overload distortion in delta modulation. (04 Marks)
c. With neat diagram explain delta modulation system. (06 Marks)

* * * * *

Scheme of Evaluation & Answer sheet.

Name of the Faculty: Basavaraj. D. Goudar

Name of the Institution: VDIT Haliyal

Dept: E&CE.

Sem: 05

Subject: Principles of Commn systems (18EC53.)

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: Principles of Communication Systems :

Feb/march - 2022.

Scheme of Evaluation

Q 1a.
AM-wave eqn in TDD → } (03)
" waveforms " → }
AM-wave eqn in FDD } (04)
" " spectrum }
} (07)

1b.
Diagram & waveform — 02
Explanation / working — 04
Eqn's for charging & discharging - 02
} (08)

1c.
Five terms × 1 = (05)

Q 2a.
Diagram & waveform — (04)
Explanation — (04)
eqn's for C(t) & S(t) ~ (02)
} (10)

b.
Tr & Rx diagrams → (04)
Working + eqns — (04)
} (08)

c
% modn → (02)

Q 3 a
Definition — (02)
Derivation — (06)
} (08)

b.
Modln index — (03)
Freq - deviation — (02)
Bandwidth — 02
} 07

c
4 - terms — 1+1+1+2 = (05)

Q 4 a. Diagram — (02)
Assumptions — (02)
Initial 4 eqns — (02)
Non-linear model analysis — (03)
" diagram — (01)
} (10)

b CDF - diagram — (01)
Assumption — (01)
Derivation — (04)
} (06)

c Two eqns × 02 = (04)

Q 5 a. Diagram — (02)
$(SNR)_c$ derivation — (02)
Assumption — (01)
finding $y(t)$ — (02)
$(SNR)_D$ eqn — (02)
eqn for F — (01)
} (10)

b. Diagram — (01)
AWGN assumption — (01)
$x(t)$ eqn — (01)
$(SNR)_i$ definition — (01)
$(SNR)_o$ " — 01
F — 01
} (06)

c Ideal LPF — (02)
Practical — (02)
} (04)

Q 6 a

Diagram – (02)
Assumptions – (02)
P — (01)
$(SNR)_C$ – 02
$y(t)$ eqn – (01)
$(SNR)_D$ – (01)
F – (01)

(10)

b) Diagram – 02
(of $S_N(f)$ & PSD of $m(t)$)
Improving techniques – 02
Details of 2nd method with diagram – (02)

(06)

c) Diagrams – (01)
Definitions of $S_N(f)$, & N_0 – (02)
$R_N(T)$ – (01)

(04)

Q 7 a). statement – (02)
mathematical eqn – (02)
Steps to Reconstruct – (04)
eqn's – 02

(10)

b) TDM Block diagram – 02
Working – 04

(06)

c) Diagram – 02
Explanation – 02

(04)

Q 8 a) PAM – 03
PWM – 03
PPM – 03 } (09)

b) Diagram – 01
waveforms – 02
Explanation – 02 } (05)

c) under sampling – 02
over sampling – 02
Nyquist rate – 02 } (06)

Q 9 a) 8 eqns × 01 = (08)

b) Block diagram – 04
Explanation – 04 } (08)

c) f_s & T_s – 02
R – 02 } (04)

Q 10 a 5 waveforms × 2 = (10)

b) granular noise – 02
slope overload – 02 } (04)

c) Diagram – 02
Explanation – 03
eqn – 01 } (06)

"Principles of Communication Systems"

Q1a. Write an AM-wave expression in TDD & FDD. Draw AM-waveform. (07M)

Ans: Let $c(t) = A_c \cos(2\pi f_c t)$, $\rightarrow (1)$ then

$$s(t)_{AM} = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \rightarrow (2)$$

where $A_c \rightarrow$ Carrier Amplitude
$f_c \rightarrow$ " frequency
$k_a \rightarrow$ a constant called the "amplitude sensitivity of the modulator.

using eqn (2)

$$s(t)_{AM} = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t)$$

$$= \frac{A_c}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right] + \frac{A_c k_a m(t)}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

$$= \frac{A_c}{2}\left[\delta(t) \times e^{j2\pi f_c t} + \delta(t)\, e^{-j2\pi f_c t}\right]$$

$$+ \frac{A_c k_a}{2}\left[m(t)\, e^{j2\pi f_c t} + m(t)\, e^{j2\pi f_c t}\right] \rightarrow (3)$$

using frequency-shifting-property of the FT, and applying FT on both sides of eqn (3),

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

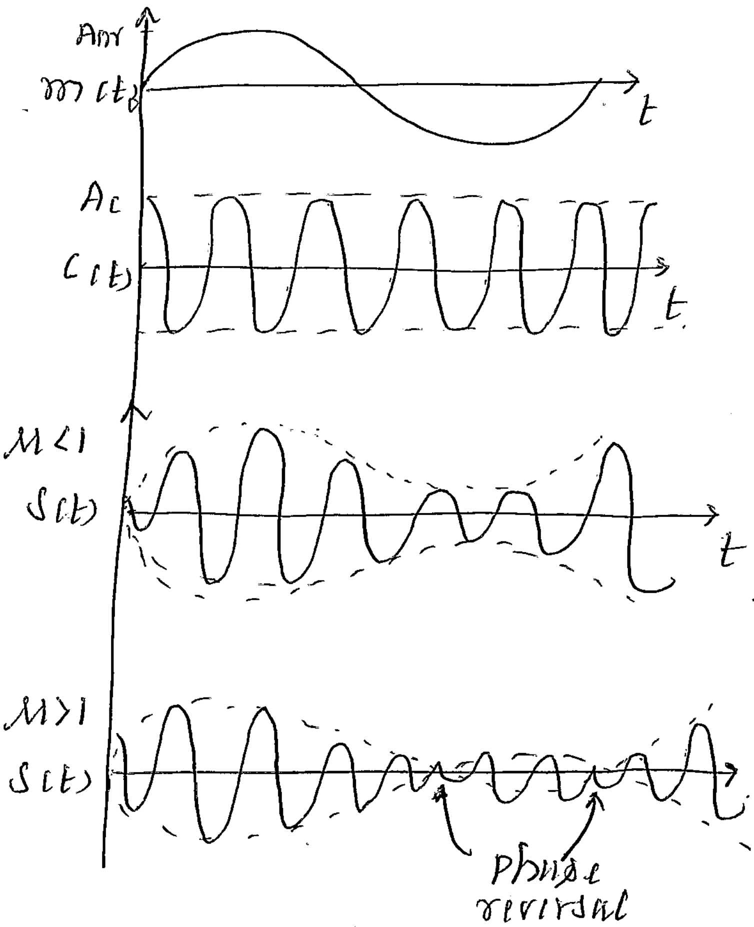
$$+ \frac{A_c k_a}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

$$\therefore S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c k_a}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

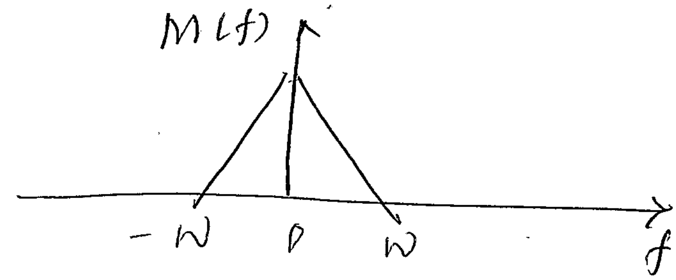
(4) $\longrightarrow$ Ans.

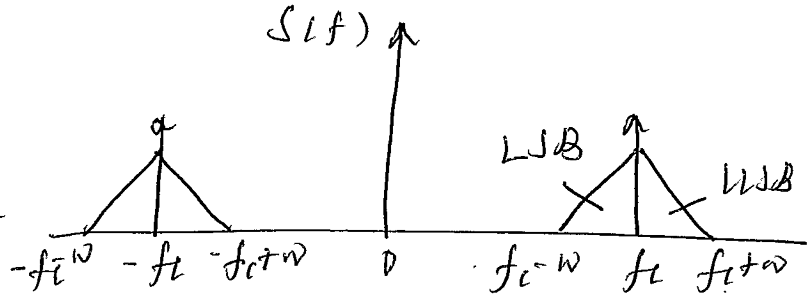
Thus, the eqn's (2) and (4) represents the expressions for an AM-wave in TDD and FDD.

TDD waveforms:



FDD Spectrum.

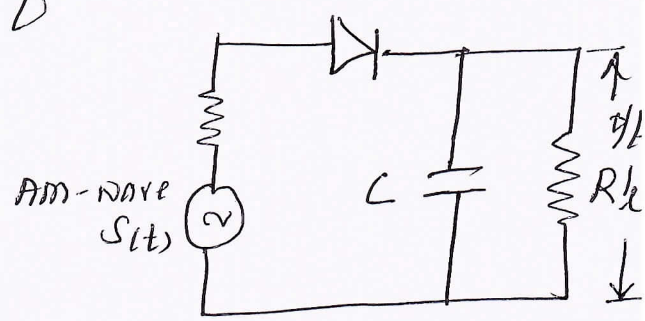




Q 1b With neat diagram, explain the demodulation of AM-wave using envelope-detector. (08M)

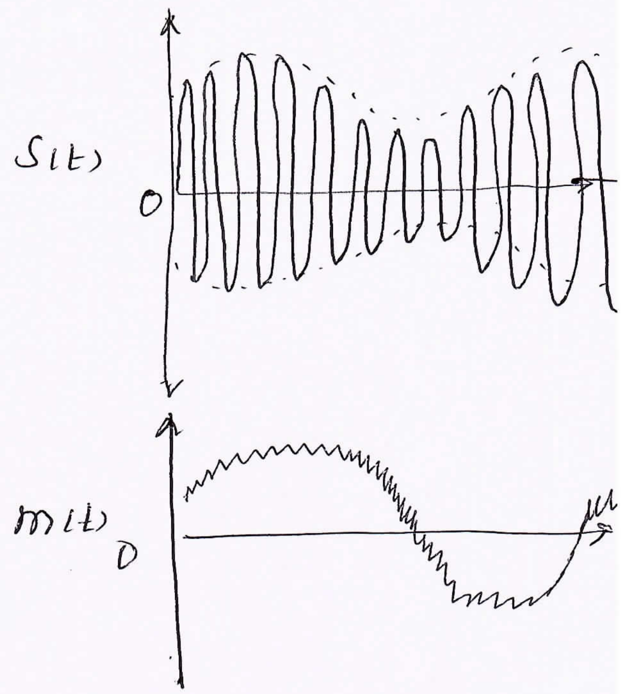
Ans: Simple & yet highly effective AM-demodulation device is known as the "Envelope Detector". Some versions of this demodulator is used in almost all commercial AM-radio receivers.

An envelope-detector of the series type is shown in fig (a), which consists of a diode and a RC LPF.



Working: On the positive half-cycle of the input signal, the diode is forward-biased & C charges up rapidly to the peak of the input. When the input falls below this value, the diode becomes reversed-biased and the C discharges slowly through the Rl.

The discharges-process continues until the next-positive half-cycle.



When the input-signal becomes greater-than the voltage across the C, the diode becomes forward biased & starts to conduct again, and the process is repeated.

(PTO)

we assume that the diode is ideal ie $r_f = 0$ and $r_r = \infty$. We further assume that the AM-wave applied to the envelope detector is supplied by a voltage source of internal-impedence R_s. Then the charging time-constant $(R_f + R_s)C$ must be short compared with the carrier period ie $1/f_c$ that is

$$(r_f + R_s)\, C \ll \frac{1}{f_c} \longrightarrow (1)$$

so that C charges rapidly & thereby follows the applied voltage upto the +ve peak when the diode is conducting.

On the other hand, the discharging-time constant $R_L C$ must be long enough to ensure that the C discharges slowly through the R_L between +ve peaks of the carrier-wave but not so long that the capacitance-voltage will not discharge at the maximum rate of change of the MLE ie

$$\frac{1}{f_c} \ll R_L C \ll 1/W \longrightarrow (2)$$

where $W \rightarrow$ message bandwidth.

The result is that the capacitor-voltage detector output is nearly the same as the envelope of the AM-wave.

Q1c. An audio frequency signal $m(t) = 5 \sin 2\pi(10^3)t$ is used to amplitude modulate a carrier wave of $c(t) = 100 \sin 2\pi(10^6)t$. Assuming $\mu = 0.4$, find

i) side band freq's
ii) Amplitude of each sidebands
iii) Transmission bandwidth B_T
iv) Total power delivered to a load of $100\,\Omega$
v) η of AM-wave assume $R = 1\,\Omega$!

Soln: → Given i) $m(t) = 5 \sin 2\pi(10^3)t \longrightarrow (1)$

in general $m(t) = A_m \sin 2\pi f_m t \longrightarrow (2)$

Comparing eqn (1) & (2), $A_m = 5$ volts
$f_m = 1\,KHz$

ii) $c(t) = 100 \sin 2\pi(10^6)t \longrightarrow (3)$

in general $c(t) = A_c \sin 2\pi f_c t \longrightarrow (4)$

Comparing (2) and (3) $A_c = 100$ volts
$f_c = 1000\,KHz$

$$S(t)_{AM} = A_c[1 + K_a A_m \sin(2\pi f_m t)] \sin 2\pi f_c t$$

$$= 100[1 + \mu \sin 2\pi(1KHz)t] \sin 2\pi(1000\,KHz)t$$

$$= 100 \sin 2\pi(10^6)t + 100 \times 0.4 \sin 2\pi(10^3)t \cdot \sin 2\pi(10^6)t$$

$$S(t)_{AM} = 100 \sin 2\pi(10^6)t + 40 \sin 2\pi(10^3)t \sin 2\pi(10^6)t$$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{A_c K_a}{2}[M(f - f_c) + M(f + f_c)] \longrightarrow (5)$$

For single tone

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{1}{4}\mu A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$
$$+ \frac{1}{4}\mu A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \longrightarrow (6)$$

Referring to eqn (6)

i) Sideband freq's are $f_c \pm f_m$

$$1000\,KH \pm 1\,KHz$$

$$= 1000\,KHz + 1\,KHz = \underline{1001\,KHz}$$

and $= 1000\,KH - 1\,KHz = \underline{999\,KHz}$

∴ side-freq's are ⇒ 1001 KHz
999 KHz //

ii) Amplitude of each sideband is

$$\frac{1}{4}\mu A_c = \frac{1}{4} \times 0.4 \times 100$$

$$= \underline{10\ volts}$$

iii) Bandwidth $B_T = f_c + f_m - (f_c - f_m)$

$$= 1001\,KHz - 999\,KHz$$

$$= 2\,KHz\ //$$

iv) Total power delivered to a load of $100\,\Omega$ is

$$= \left(\frac{100}{\sqrt{2}}\right)^2 \times \frac{1}{100} + \left[\frac{5 \times 0.4}{2\sqrt{2}}\right]^2 \times \frac{1}{100} \times 2$$

$$= \frac{100 \times \cancel{100}}{2} \times \frac{1}{\cancel{100}} + \frac{5^2 \times 0.4^2}{4 \times \cancel{2}} \times \frac{1}{100} \times \cancel{2}$$

$$= 50 + \frac{\overset{1}{\cancel{25}} \times 0.16}{4 \times \cancel{100}\,4} = 50 + \frac{0.16}{16} = 50 + 0.01$$

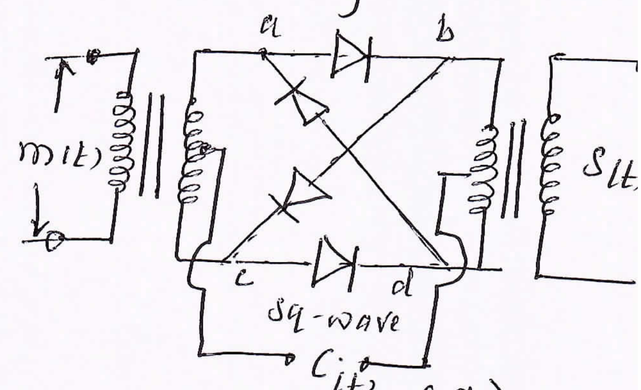
$$= 50.01\ Watts.$$

v) $$\eta = \frac{P_{LSB} + P_{USB}}{P_T} = \frac{0.01}{50.01} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{(0.4)^2}{2 + (0.4)^2} = \frac{0.16}{2 + 0.16} = \frac{0.16}{2.16} = 7.4\,\%$$

Q 2a. Explain the generation of DSBSCM-wave using a Ring-modulator. (10M)

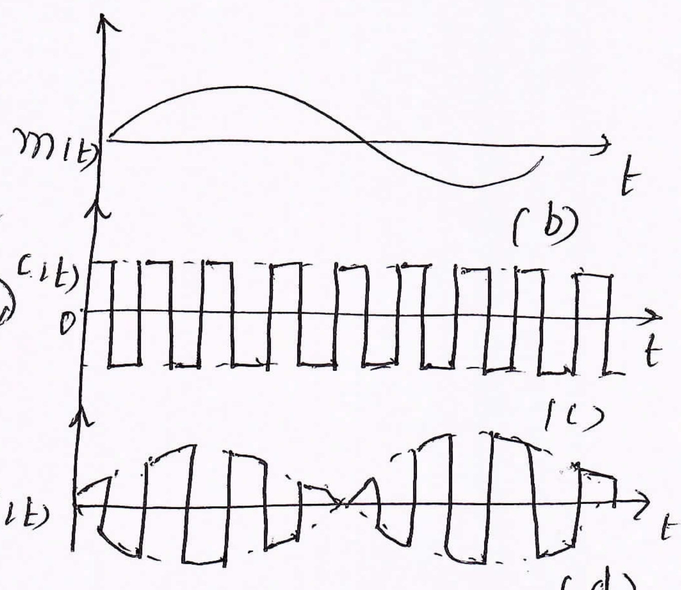
Ans: One of the most useful pm, well suited for generating a DSBSCM-wave is the ring modulator shown in fig (a)



(a)

Four diodes form a ring in which they all point in the same way - hence the name.

Diodes are controlled by a sq-wave carrier C(t) of frequency fc which is applied longitudinally by means of two centre-tapped transformers.



If the transformers are perfectly balanced and the diodes are identical. Assuming diodes are having a constant R_f & R_r, on one of half cycle of the C(t), outer diodes are forward (ON) biased inner-diodes are OFF. On the other ~~hand~~, half-cycle diodes operate in the opposite condition. In effect, the ring-modulator acts as a "commutator"

Fig (d) shows the idealized waveforms of the modu lated signal s(t) produced by the ring-modulator, assuming sinusoidal-m(t).

Now, the square-wave c(t) can be represented by a Fourier-series as follows

$$c(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t\,(2n-1)\right] \rightarrow (1)$$

∴ The ring-modulator output is

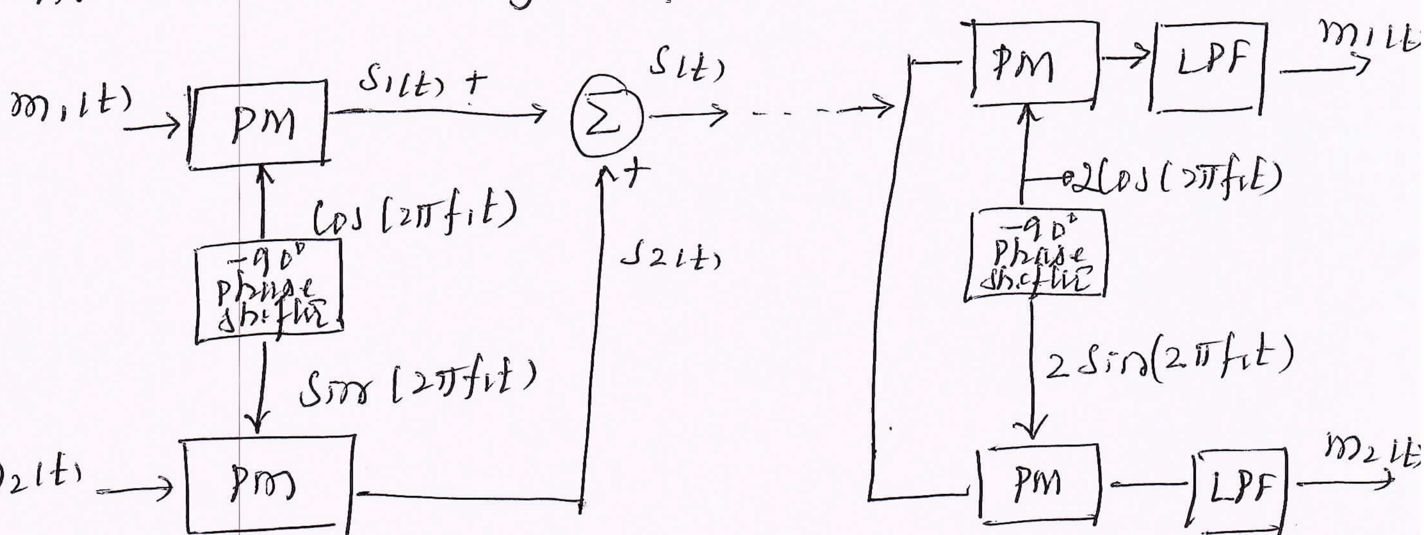
$$s(t) = c(t)\, m(t)$$

$$= \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t\,(2n-1)\right] m(t) \rightarrow (2)$$

Q 2b. Explain with a neat diagram the working of QAM / QCM. (08M)

Ans. QCM/QAM scheme enables two DSBSCM-waves resulting from two physically independent $m(t)$'s to occupy the same channel bandwidth, & yet it allows for the separation of the two $m(t)$'s at the rcvr-output. ∴ It is a bandwidth conservation scheme.

A block-diagram of the QCM system is shown in the below fig ( ).



(a) QCM Transmitter b) QCM Receiver

Transmitter part as in fig (a), involves the use of two separate PM's that are supplied with two carrier-waves of the same frequency and change in 90° phase. Thus the transmitted-signal $s(t)$ consists of sum of these two PM's o/p ie

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

Thus $s(t)$ occupies the same channel B_T of $2W$

&

(PTO)

According to above eqn, we may view $A_c m_1(t)$ as in-phase component of the multiplexed band pass signal $s(t)$ and $A_c m_2(t)$ as its quadrature component.

Receiver part is shown in fig (b). The multiplexed signal $s(t)$ is applied simultaneously to two separate coherent-detectors that are supplied with two local carriers of the same frequency & phase-difference by $-90°$. The output of the top detector is $\frac{1}{2} A_c' m_1(t)$ & " " " bottom " " " $\frac{1}{2} A_c' m_2(t)$.

Note: Maintain coherence in phase & frequency betn LO and C(t) signal of the system. i.e use costas loop. or send a pilot-signal outside the passband of the modulated signal.

Q 2 C. An AM-signal with a carrier of 1 KW has 200 W in each side band. What is the % of modulation. (02M)

Ans: Given $P_c = 1\,KW = 1000\,Watts$

$P_{USB} = P_{LSB} = 200\,Watts.$

$$\%\ modulation = \frac{Side\ band\ power}{Carrier\ power}$$

$$= \frac{200+200}{200+200+1000} = \frac{400}{1400} \times 100$$

$$= 28\%$$

OR $\frac{400}{1000} \times 100 = 40\%$

Q 3 a. Define angle modulation. Derive the FM-wave expression in time domain. (08 M)

Ans: It is form of modulation, in which the angle of the carrier-wave is varied in accordance with the base band signal.

Let $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier, assumed to be function of the $m(t)$. Then we express the resulting angle-modulated wave as

$$s(t) = A_c \cos[\theta_i(t)] \longrightarrow (1)$$

A complete oscillation occurs whenever $\theta_i(t)$ changes by 2π radians.

If $\theta_i(t)$ increases monotonically with time, then the average-frequency in Hz over the interval from t to $t+\Delta t$ is given by

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \longrightarrow (2)$$

$\therefore$ We may define the instantaneous-frequency of the angle-modulated signal $s(t)$ as

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right]$$

$$\boxed{f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}} \longrightarrow (3)$$

FM: → Form of angle-modulation in which the instantaneous-frequency $f_i(t)$ is varied linearly with the $m(t)$ as given by

$$f_i(t) = f_c + k_f m(t) \longrightarrow (6)$$

where $f_c \rightarrow$ unmodulated carrier frequency

$k_f \rightarrow$ frequency sensitivity of the modulator Hz/volts

Then from eqn (3) which relates $f_i(t)$ in terms of $\theta_i(t)$,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$\Rightarrow \quad \theta_i(t) = 2\pi \int_0^t f_i(t)\,dt$$

using eqn (6)

$$= 2\pi \int_0^t [f_c + k_f m(t)]\,dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau \longrightarrow (7)$$

using eqn (7) in (1) we get

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]}$$

Q3b. Define the following terms
i> Modulation index
ii> Frequency - deviation
iii> Bandwidth.

(07)

Ans: Assuming single-tone $m(t)$, ie

$$m(t) = A_m \cos(2\pi f_m t)$$

then $f_i(t) = f_c + K_f A_m \cos(2\pi f_c t)$

$$= f_c + \Delta f \cos(2\pi f_m t) \longrightarrow (1)$$

where $\Delta f \rightarrow K_f A_m \rightarrow$ is called the "frequency - deviation" representing the max departure of the $f_i(t)$ of the FM - signal from the f_c

using eqn (1), the angle $\theta_i(t)$ of the FM - signal is obtain as

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \longrightarrow (2)$$

ii> The ratio of the frequency - deviation Δf to the modulation - frequency f_m is commonly called the "modulation - index" of the FM - signal, denoted it by β ie

$$\beta = \frac{\Delta f}{f_m} \longrightarrow$$

iii> Bandwidth : $\rightarrow$ It is the difference between the highest frequency component to lowest frequency - component of the modulated signal spectrum.

Q3C. A FM-wave is represented by the eqn
$V = 10 \sin[5\times10^8 t + 4 \sin 1250t]$
find i> f_c ii> f_m iii> β and Δf iv> B_T
using Carson's rule. (05)

Soln: w.k.t

$$S(t)_{FM} = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \rightarrow (1)$$

given FM $S(t) = V = 10 \sin[5\times10^8 t + 4 \sin 1250t] \rightarrow (2)$

by comparing eqn (1) and (2)

$$2\pi f_c t = 5\times10^8 t$$

i> $\therefore f_c = \dfrac{5\times10^8 \cancel{t}}{2\pi \cancel{t}} = \dfrac{5}{6.283}\times10^8 = 0.7957\times10^8$

$$= 79.5 \text{ MHz}$$

~~and~~ ii> $\beta = 4$

$$2\pi f_m t = 1250t$$

iii> $f_m = \dfrac{1250\cancel{t}}{2\pi\cancel{t}} = \dfrac{1250}{6.283} = 198.94$

iv> $\Delta f \Rightarrow \beta = \dfrac{\Delta f}{f_m} \Rightarrow \Delta f = \beta\times f_m = 4\times199$

$$\therefore \Delta f = 796 \text{ Hz}$$

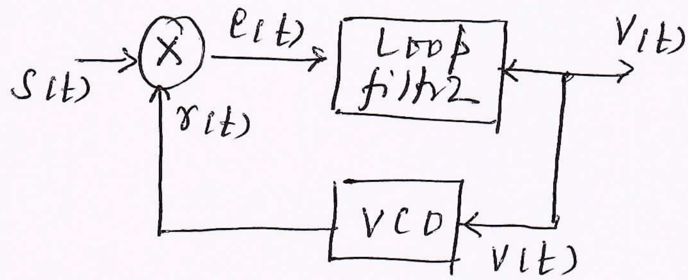
$$B_T \simeq 2\Delta f(1 + 1/\beta)$$

Q 4 a. Write the basic block diagram of PLL. Derive the expression for non-linear model. (10M)

Ans: Basically, the PLL consists of 3 major components

- a multiplier
- a loop filter
- & a VCO

connected together in the form of a feedback-loop as shown in below fig ( ).



Then, suppose that the input signal applied to the PLL is an FM-wave defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \rightarrow (1)$$

where $\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau \rightarrow (2)$

We assume that initially we have adjusted the VCO so that when the control-voltage is zero, two conditions are satisfied.

1. The frequency of the VCO is precisely set at unmodulated carrier frequency f_c.
2. VCO output has a 90° phase-shift w.r.t the unmodulated carrier-wave

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \rightarrow (3)$$

where $\phi_2(t) = 2\pi k_v \int_0^t v(\tau)\,d\tau \rightarrow (4)$

where $k_v \rightarrow$ frequency-sensitivity of the VCO measured in Hz/volt

(PTO)

To develop an understanding of the PLL, it is desirable to have a model of the loop. So now will develop a non-linear model.

Non-linear Model: → According to the above fig 1) the incoming FM-signal $s(t)$ and the VCO output $r(t)$ are applied to the multiplier producing two components.

1. HF component, represented by the double-frequency term $K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$

2. a LF-component, represented by the difference-frequency term
$$K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$
where K_m → multiplier-gain measured in volt

The Loop-filter in the PLL is a LPF and its response to HF-component will be negligible, so
$$e(t) = K_m A_c A_v \sin[\phi_e(t)] \longrightarrow (5)$$
where
$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$
$$= \phi_1(t) - 2\pi k_v \int_0^t v(\tau)\, d\tau \longrightarrow (6)$$

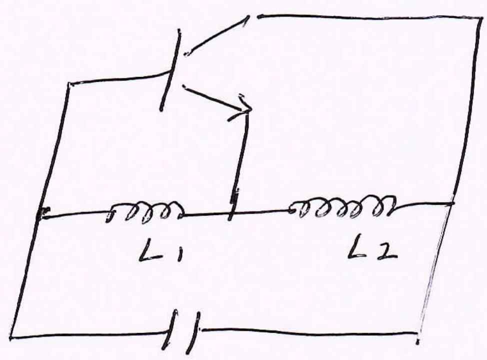
* Loop-filter operates on the input $e(t)$ to produce an output $v(t)$ defined by the convolution-integral ie
$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \rightarrow (7)$$
where $h(t)$ → impulse-response of the loop-filter.
using eqn (5) and (6) to relate $\phi_e(t)$ and $\phi_1(t)$,
ie from eqn (5)
$$e(\tau) = K_m A_c A_v \sin[\phi_e(\tau)] \rightarrow (9)$$
∴ using (9) in (7)

Q4b. Explain the direct-method of generating FM-wave using Hartly-oscillator with relevant equations & diagrams. (06)

Ans: In a direct-FM system a device known as a "VCO" is used. One way of implementing such a device is to use a sinusoidal oscillator having ① a highly selective frequency-determining resonant-network ② and to control the oscillator by symmetrical incremental variation of the reactive components of this network.

An example of such a scheme is shown in the below fig 1), depicting a Hartly-oscillator.

Assuming that capacitive component of the frequency determining-network consists of a fixed capacitor



C(t)

shunted by a voltage-variable capacitor

The resultant capacitance is represented by C(t) in the fig 1), and is the sum of the fixed-capacitor and the variable-voltage capacitor ie

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t)$$

assuming variable-voltage is a sinusoidal signal.

(PTO)

$$v(t) = \int_{-\infty}^{\infty} k_m A_c A_v \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau \rightarrow (10)$$

using (10) in (6)

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} k_m A_c A_v \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_0 \int_0^t \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau\, d\tau \rightarrow (11)$$

differentiating eqn (11) w.r.t t,

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau \rightarrow (12)$$

where $K_0 \rightarrow K_m K_v A_c A_v$ is the loop-gain parameter

The A_c, A_v both measured in volts

$K_m \rightarrow$ is " " 1/volt

$K_v \rightarrow$ " " " Hz/volt

$\therefore\ K_0 \Rightarrow \cancel{volt}\ \cancel{volt}\ \frac{1}{\cancel{volt}} \times \frac{Hz}{\cancel{volt}} \Rightarrow Hz.$

The dimension of K_0 is the dimension of frequency

Eqn (12) suggests that the model shown in below fig 1) for the PLL.

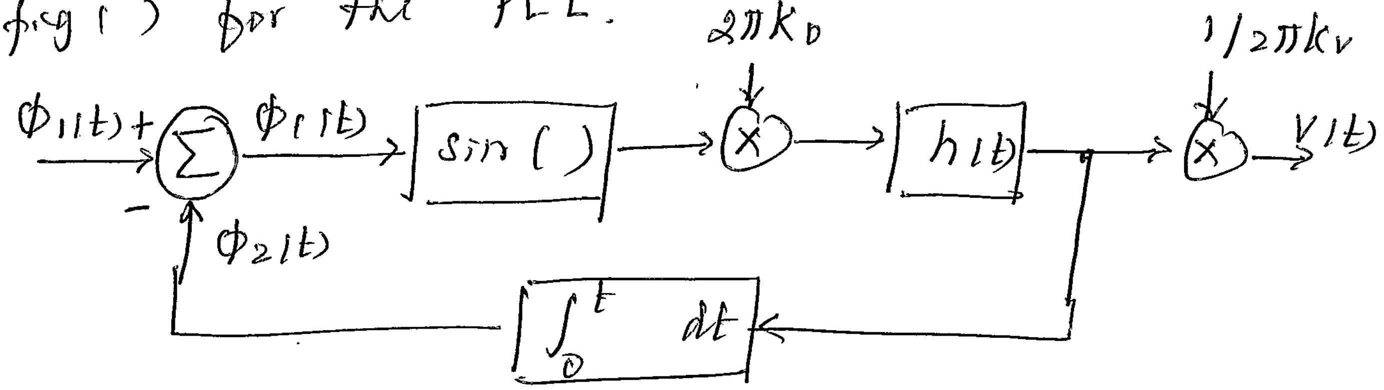


Fig 1) Non linear model of PLL.

We see that the

1) multiplier replaced by a subtractor and a sinusoidal non linearity

& 2). VCO by an integrator

(PTO)

Then, the frequency of oscillation of the Hartly oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}} \longrightarrow (2)$$

using eqn (1) in (2)

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\left(C_0 + \frac{C_0}{C_0}\Delta C \cos(2\pi f_m t)\right)}}$$

by mul & Div by C_0 with ΔC

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left(1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)}}$$

$$f_i(t) = f_0\left[1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{-1/2} \longrightarrow (3)$$

where $f_0 = \dfrac{1}{2\pi\sqrt{(L_1+L_2)\,C_0}}$

Provided that the maximum-change in ΔC is small compared with C_0, we may approximate eqn (3) as

$$f_i(t) \simeq f_0\left[1 - \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right] \longrightarrow (4)$$

Let $\dfrac{\Delta C}{2C_0} = -\dfrac{\Delta f}{f_0} \longrightarrow (5)$

using (5) in (4)

$$f_i(t) \cong f_0 + \Delta f \cos(2\pi f_m t) \longrightarrow (6)$$

Eqn (6) is the desired relation for $f_i(t)$ of the FM-wave assuming sinusoidal modulation.

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Q4C Write the narrowband-FM and wideband FM expression. (04M)

Ans: $S(t)_{NB} \cong A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \rightarrow (1)$

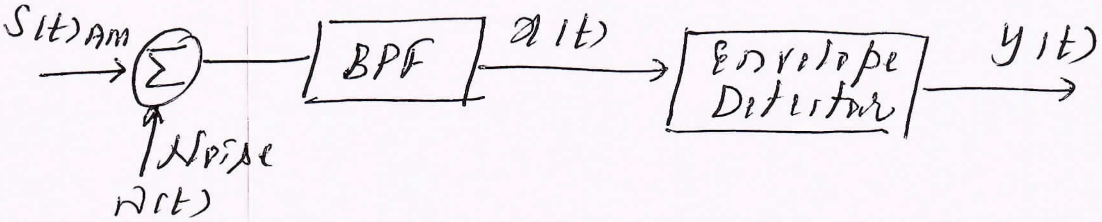
$$S(t)_{WB} = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] \rightarrow (2)$$

//.

: module - 3 :

Q 5a. Derive the expression for figure of merit of an AM-receiver using envelope-detection (10M)

Ans: → The receiver-model for AM-system using envelope-detection is as shown in fig (1)



Analysis: → $S(t) = A_c[1 + K_a m(t)]\cos(2\pi f_c t) \rightarrow (1)$

where $A_c\cos(2\pi f_c t) \rightarrow$ Carrier-wave

$m(t) \rightarrow$ message-signal

$K_a \rightarrow$ Constant determines % modⁿ.

It is reasonable to assume that A_c has same unit as the additive-noise. The factor K_a is then assumed to have the units necessary to make the remainder of the expression dimensionless.

$S(t) = A_c\cos(2\pi f_c t) + A_c K_a m(t)\cos(2\pi f_c t) \rightarrow (2)$

∴ The average-power of the carrier component is $\frac{1}{2}A_c^2$, and the average power of the information bearing component $A_c K_a m(t)\cos(2\pi f_c t)$ is $\frac{1}{2}A_c^2 K_a^2 P$. where $P \rightarrow$ Avg. power of the m(t).

∴ Avg-power of the full AM-wave is given by

$$\frac{1}{2}A_c^2 + \frac{1}{2}A_c^2 K_a^2 P$$

$$= A_c^2(1 + K_a^2 P)/2 \longrightarrow (3)$$

Similar to DSBSC-sys, the average-power of the noise in the message bandwidth is WN_0.

$$\therefore (SNR)_{C,AM} = \frac{A_c^2(1 + K_a^2 P)}{2WN_0} \longrightarrow (4)$$

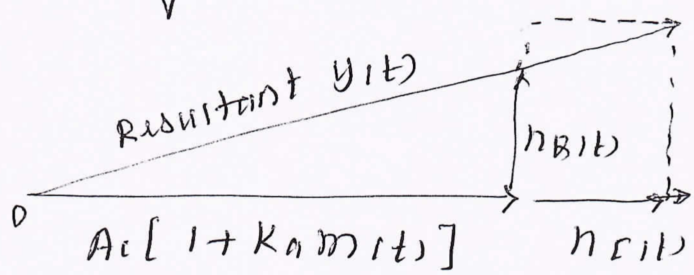
To evaluate $(SNR)_{O,AM}$, we first represent the filtered noise $n(t)$ in terms of in-phase and quadrature components. Therefore we may define filtered signal $x(t)$ applied to the envelope detector as

$$x(t) = s(t) + n(t)$$

$$= A_c[1+K_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$= [A_c + A_c K_a m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \rightarrow (5)$$

It is informative to represent the components that comprise the signal $x(t)$ by means of phasors as in below fig ( ).



From this phasor diagram the receiver output is readily obtained as

$$y(t) = \text{Envelope of } x(t)$$

$$= \left\{[A_c + A_c K_a m(t) + n_I(t)]^2 + n_Q^2(t)\right\}^{1/2} \longrightarrow (6)$$

The signal $y(t)$ defines the output of an ideal envelope-detector. The phase of $x(t)$ is of no interest because an ideal envelope-detector is totally insensitive to variations in the phase of $x(t)$.

The expression defining $y(t)$ is somewhat complex so needs to be simplified to permit the derivation of insightful results. ie ** or specifically we would like to approximate the output $y(t)$ as the sum of message term + a term due to noise.

So, when the average power of the carrier is large compared with the average power of noise so that receiver is operating satisfactorily then, the signal term $A_c[1+k_a m(t)]$ will be large compared to $n_I(t)$ & $n_Q(t)$ at least most of the time.

Then we may approximate $y(t)$ as

$$y(t) \cong A_c + A_c k_a m(t) + n_I(t) \rightarrow (7)$$

Presence of DC/constant term A_c can be removed by means of a blocking capacitor.

Accordingly, the $(SNR)_{o,AM}$ using envelope-detector is approximately

$$(SNR)_{o,AM} = \frac{A_c^2 k_a^2 P}{2WN_0} \rightarrow (8)$$

Thus by using eqn (4) and (8)

$$\left.\frac{(SNR)_o}{(SNR)_c}\right|_{AM} \cong \frac{k_a^2 P}{1+k_a^2 P} \rightarrow (9) \text{ Ans.}$$

~~[illegible]~~

Q 5b. Explain the noisy receiver-model with neat diagram. Explain briefly the figure of merit. (06)

Ans: For the situation at hand, we propose to use the receiver-model of below fig(1), in its basic form.

(PTO)

$$s(t) \rightarrow \underset{\uparrow\, w(t)}{\Sigma} \rightarrow \boxed{\text{BPF}} \xrightarrow{x(t)} \boxed{\text{Demodulator}} \rightarrow \text{output signal.}$$

Fig 1) Noisy receiver-model.

$s(t) \rightarrow$ incoming modulated signal
$w(t) \rightarrow$ front-end receiver noise

$\therefore$ Received signal $= s(t) + w(t)$, is the signal that receiver has to work on.

BPF $\rightarrow$ represents the combined filtering action of the tuned-amplifiers used in the actual rx for the purpose of signal amplification, whose BW enough to pass the $s(t)$ without distortion

Type of demodulator naturally depends on type of modulation used.

In noise-analysis, the customary practice is to <u>assume</u> that $w(t)$ is AWGN type (for many reasons).

Thus, we let the psd of the noise $w(t)$ be denoted by $\frac{N_0}{2}$, defined for both positive & -ve frequencies. <u>ie</u> N_0 is the avg-power of noise per unit bandwidth measured at the front end of the receiver.

Assume noise as a narrowband-noise, representing in the canonical form

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \rightarrow (1)$$

Then the filtered-signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \longrightarrow (2)$$

The details of the $s(t)$ depends on the type of modulation used at transmitter, but our

5b continued...

However, average noise-power at the demodulator input is equal to the total area under the curve of the PSD $S_N(f)$ ie

$$\text{Avg-power of noise} = \int_{-\infty}^{\infty} \frac{N_0}{2}\,df = 2\int_{0}^{\infty} \frac{N_0}{2}\,df$$

$$= 2\int_{-W}^{W} \frac{N_0}{2}\,df = N_0 W \text{ // or } \underline{N_0 B_T}.$$

Then, given the format of $s(t)$, we may also determine, the avg-power at the demodulator input. ∴ we may define $(SNR)_i$ as the ratio of the average-power of the modulated signal $s(t)$ to the avg-power of the filtered noise $n(t)$.

$$(SNR)_i = \frac{\text{Avg-power of } s(t)}{\text{Avg-power of filtered noise } n(t)}$$

** $(SNR)_o$: → A more useful measure of 'noise-performance' is $(SNR)_o$ defined as the ratio of avg-power of the demodulated $m(t)$ to the avg-power of the o/p noise, both measured at the receiver end. $(SNR)_o$ provides an <u>intuitive-measure</u> for describing <u>fidelity</u> with which the demodulation process recovers $m(t)$ from the $s(t)$ in the presence of <u>additive noise</u>.

<u>Note</u>: For such a criteria to be well defined, the recovered $m(t)$ & the corruptive noise component must appear additively at the demodulator output. It is perfectly valid for coherent detector.

For envelope-detector, we have to assume avg-power of $n(t)$ is <u>relatively low</u> to justify the use of $(SNR)_o$ as a measure of receiver-performance.

$(SNR)_o$ depends on other factors like the type of modulation, demodulation. Thus it is informative to compare the $(SNR)_o$'s for different mod-demod'n systems.

However, for this comparison to be of meaningful value it must be made on an equal-basis as explained below.

Accordingly, as a frame of reference we define channel-SNR ie $(SNR)_c$ as the ratio of the avg-power of the sig to the avg-power of the noise in message bandwidth, both measured at rxr input.

For the purpose of comparing different comm-systems, we normalize the receiver-performance $(SNR)_o$ w.r.t $(SNR)_c$. Thus ~~ref~~ we define a "figure of merit F as

$$F = \frac{(SNR)_o}{(SNR)_c} \longrightarrow \text{Ans.}$$

Q 5c. Explain the noise-equivalent bandwidth with relevant equation (04)

Ans: → w.k.t when a source of white-noise of zero mean and psd $\frac{N_0}{2}$ is connected across an ideal-LPF of bandwidth B, and passband amplitude response one, average output noise power denoted as $R_N(0)$ is equal to $N_0 B$.

Similarly, when such a noise-source is connected to the input of the simple RC-LPF, then the corresponding avg-power of output noise is equal to $\frac{N_0}{4RC}$ (ref eq 5.14 & 5.15 of text)

& For this filter 3-dB bandwidth is $= \frac{1}{2\pi RC}$

Then Avg-power of o/p noise can be expressed interms of 3-dB bandwidth

$$\frac{1}{2}\frac{\pi N_0}{2\pi RC} \times = \frac{\pi N_0}{2} BW$$

$$= \frac{\pi}{2} \times N_0 \times \text{3-dB BW of the LPF } (B)$$

$$= \frac{\pi}{2} N_0 B \Rightarrow \frac{\pi}{2} N_0 B.$$

Thus, we may generalize this statement to include all kinds of LPF's by definition a noise equivalent-bandwidth as follows.

⇒ Suppose, we have a source of white-noise of zero-mean and PSD $N_0/2$ connected to the input of an arbitrary LPF of TF $H(f)$

Then, the resulting avg-power of output noise is given by

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$$N_{out} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$$

$$N_{out} = N_0 \int_0^{\infty} |H(f)|^2 df \rightarrow (1)$$

out = in × TF

$= \frac{N_0}{2} \times H(f)$

avg. power of o/p = Area under psd

$= \int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 df$

$= \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$

$= N_0 \int_0^{\infty} |H(f)|^2 df$

Consider, next the same source of white-noise connected to the input of an ideal LPF of zero frequency response $H(0)$ and bandwidth B, Then the average power of output noise is

$$N_{out} = N_0 \int_0^{B} |H(0)|^2 df$$

$$= N_0 B H^2(0) \longrightarrow (2)$$

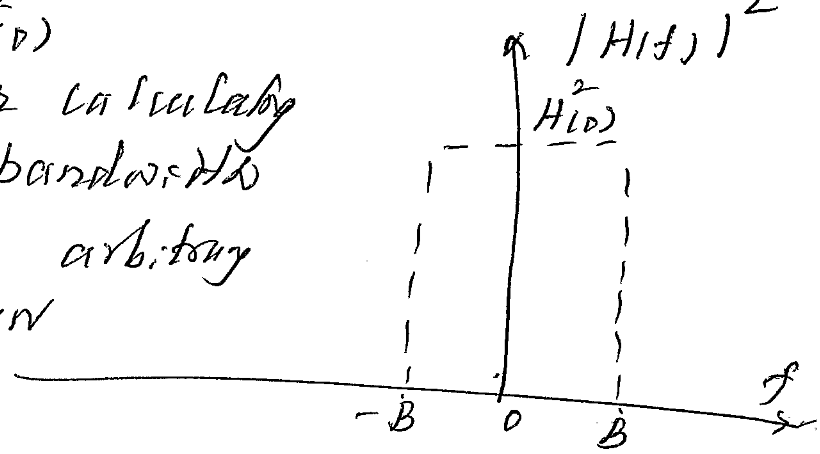
∴ Equating eqn (1) & (2)

$$N_0 B H^2(0) = N_0 \int_0^{\infty} |H(f)|^2 df$$

$$\Rightarrow B = \frac{\cancel{N_0}\int_0^{\infty} |H(f)|^2 df}{\cancel{N_0} H^2(0)}$$

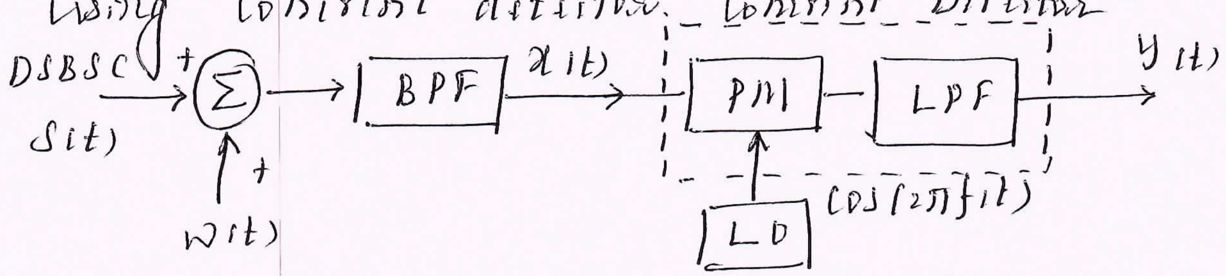
$$\therefore B = \frac{\int_0^{\infty} |H(f)|^2 df}{H^2(0)} \longrightarrow (3)$$

Thus, the procedure for calculating the noise equivalent bandwidth consists of replacing the arbitrary LPF of TF $H(f)$ by an equivalent ideal-LPF of TF $H(0)$, BW = B.



Q 6 a. Derive the expression for F for DSBSC-receiver. (10M)

Ans: Fig 1) shows the model of a DSBSC-receiver, using coherent detector.



Assuming perfect synchronization btwn LO output & the carrier, the DSBSC-component of the x(t) is expressed as

$$s(t) = C\, A_c \cos(2\pi f_c t)\, m(t) \rightarrow (1)$$

where $C \rightarrow$ system-dependent scaling-factor purpose of which is to ensure that the signal-component s(t) is measured in the same unit as the additive noise component n(t).

Assuming m(t) is the sample-function of a stationary-process of zero-mean whose PSD $S_M(f)$ is limited to a max-freq W. Then the average-power P of the m(t) is the total area under the curve of PSD ie

$$P = \int_{-W}^{W} S_M(f)\, df \longrightarrow (2)$$

* The carrier-wave is statistically independent of the m(t). Then we may express the average power of the DSBSCM-signal component s(t) as

$$\frac{1}{2} C^2 A_c^2 P \longrightarrow (3)$$

With a noise spectral density of $N_0/2$, the avg power of noise in the message bandwidth W

$$\int_{-W}^{W} \frac{N_0}{2}\,df = \frac{N_0}{2}\,[W-(-W)] = \frac{N_0}{\cancel{2}}\times \cancel{2}W$$

$$= N_0 W \longrightarrow (4)$$

$$\therefore (SNR)_{C,DSB} = \frac{C^2 A_c^2 P}{2WN_0} \longrightarrow (5)$$

where the constant C^2 in the numerator ensures that this ratio is dimensionless.

To find $(SNR)_{O,DSBSC}$ we write the signal $x(t)$ as

$$x(t) = s(t) + n(t)$$

$$x(t) = CA_c\cos(2\pi f_c t)\,m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \longrightarrow (6)$$

$\therefore$ The output of the PM is given by

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= \{CA_c\cos(2\pi f_c t)\,m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)\} \times \cos(2\pi f_c t)$$

$$= \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t) + \frac{1}{2}\{CA_c m(t) + n_I(t)\}\cos(4\pi f_c t) - \frac{1}{2}A_c n_Q(t)\sin(4\pi f_c t) \longrightarrow (7)$$

Then the LPF output $y(t)$ is given by

$$y(t) = \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t) \longrightarrow (8)$$

The message signal component at the receiver output is $CA_c m(t)/2$. Therefore, avg-power may be expressed as $\frac{1}{2}\left(\frac{1}{2}C^2A_c^2P\right) = \frac{1}{4}C^2A_c^2P \longrightarrow (9)$

Similarly the avg-power of the noise at the receiver output is $\left(\frac{1}{2}\right)^2 2WN_0 = \frac{1}{2}WN_0$

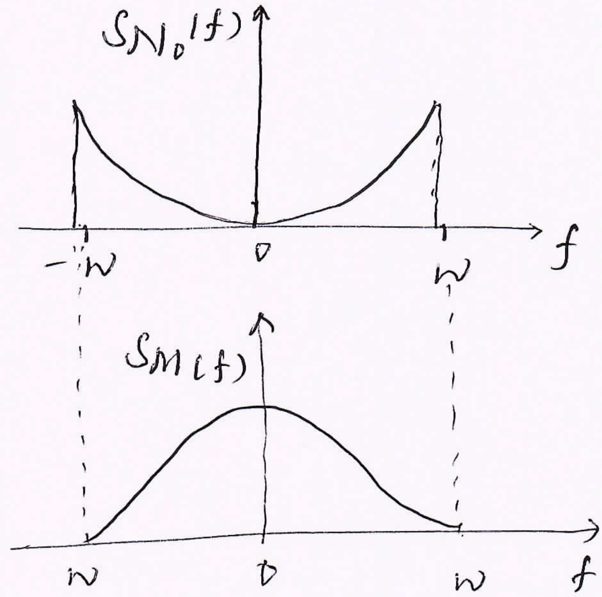
$$\therefore (SNR)_O = \frac{C^2A_c^2P/4}{WN_0/2} = \frac{C^2A_c^2P}{2WN_0} \longrightarrow (10)$$

$$\therefore F = \left.\frac{(SNR)_O}{(SNR)_C}\right|_{DSBSC} = 1 \longrightarrow (11)$$

Q6 b. Explain the use of pre-emphasis and de-emphasis circuit for an FM-system. (06 M)

Ans: w.r.t the PSD of the noise at the output of an FM-receiver has a square-law dependence on the operating frequency as illustrated in the below fig(a). In fig(b) we have included the PSD of a typical m(t) source (typically audio & video) have spectra of this form.

In particular we see that the PSD of the m(t) usually falls-off appreciably at higher freq's.



On the other hand, the PSD of the output noise increases rapidly with frequency.

* Thus, around $f = \pm W$, the relative PSD of the m(t) is quite low, whereas that of the output noise is quite high compared (to m(t)'s PSD.

∴ m(t) is not utilizing the frequency band allotted to it in an efficient manner.

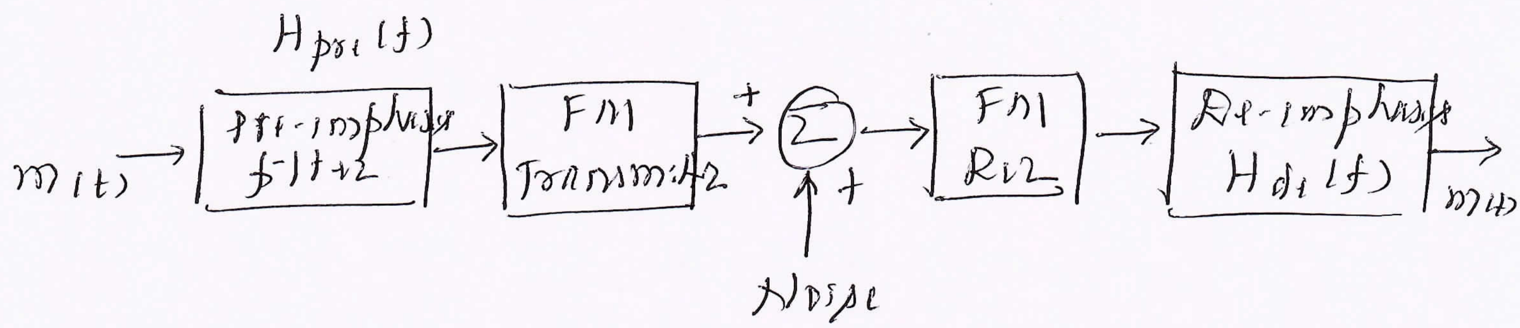
One way of improving the noise-performance of the system is to slightly reduce the bandwidth of the post-detection LPF so as to reject a large amount of noise-power while losing only a small amount of message-power.

(PTO)

But, such an approach is usually not satisfactory because the distortion of m(t) caused by the reduced bandwidth, even though slight, may not be tolerable.

eg: In case of music, we find that HF note contribute only a very small fraction of the total power, they contribute a great deal of from esthetic viewpoint (esthetic = appreciation of ~~but~~ beauty or feel)

2> A more satisfactory approach to the efficient utilization of the allowed freq-band is based on the use of <u>pre-emphasis</u> in the transmitter & " " <u>De-emphasis</u> in the receiver as illustrated in fig( ).



ie we artificially emphasize the HF components of the m(t) prior to modulation in the transmitter, ie before noise is introduced in the receiver. Then at the discriminator output in the receiver, we perform the inverse operation by de-emphasizing the HF

Q 6C. Define white noise. Briefly explain the psd and ACF of the white-noise. (04)

Ans: It is an idealized form of noise whose psd is independent of the operating frequency. The adjective white is used in the sense that the white light contains equal amounts of all freq's within the visible-band of em-spectrum.

The psd of the white-noise with a sample function denoted by $W(t)$, is defined as

$$S_W(f) = \frac{N_0}{2} \rightarrow (1)$$

where $N_0 \rightarrow$ watts/Hz. N_0 is usually referenced to the input stage of the receiver of a comm'n s/m.

It may be expressed as

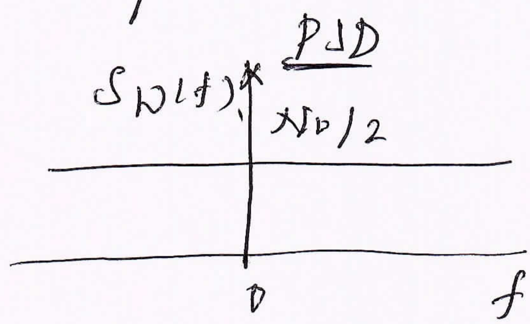
$$N_0 = KT_e \longrightarrow (2)$$

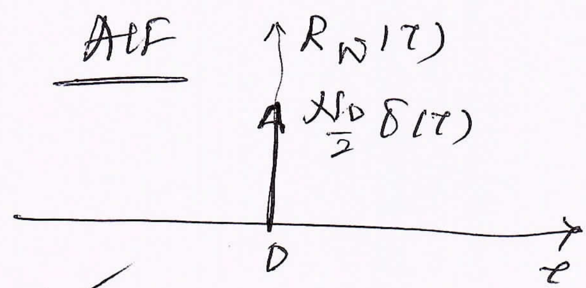
$K \longrightarrow$ Boltzmann's constant
$T_e \rightarrow$ Noise equivalent temperature.

Since the ACF is the IFT of the PSD, it follows that for white noise

$$R_W(\tau) = \frac{N_0}{2}\delta(\tau) \longrightarrow (3)$$

as shown in fig (b) //





Q7a. State sampling theorem. Write the mathematical form of sampled signal and explain the steps to reconstruct the $g(t)$ from the sequence of sample values. (10M)

Ans: → We may state the sampling-theorem for strictly band-limited signal of finite energy in two equivalent parts.

1. A band-limited signal of finite-energy is completely described by specifying the values of the signal at instants of time separated by $1/2W$ seconds or less.

2. A band-limited signal of finite-energy with $f < W$ Hz, may be completely recovered from a knowledge of its samples taken at the rate of $2W$ samples/sec.

Mathematical form of sampled signal are

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t-nT_s) \rightarrow (1)$$

in time-domain and

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f-mf_s) \longrightarrow (2)$$

and

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi n f T_s) \rightarrow (3)$$

eqn (2) and (3) are in frequency-domain.

To reconstruct $g(t)$ from $[g(n/2W)]$ samples use the eqn (1) ie

$$G(f) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} g(n/2W)\exp\left(\frac{j\pi n f}{W}\right) \quad -W<f<W \rightarrow (1)$$

in the formula for the IFT defining $g(t)$ in terms of $G(f)$ ie

$$g(t) = [\text{IFT of } G(f)] \rightarrow (2)$$

$$= \int_{-\infty}^{\infty} G(f)\exp(j2\pi f t)\,df$$

$$= \int_{-W}^{W} \frac{1}{2W}\sum_{n=-\infty}^{\infty} g(n/2W)\exp\left(-\frac{j\pi n f}{W}\right)\exp(j2\pi f t)\,df$$

Interchanging the order of summation & integration, we get

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\frac{1}{2W}\int_{-W}^{W}\exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]df$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\frac{\sin(2\pi W t - n\pi)}{(2\pi W t - n\pi)} \rightarrow (3)$$

$$\therefore g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\,\text{sinc}(2Wt-n) \rightarrow \text{Ans} \rightarrow (4)$$

eqn (4) provides an interpolation formula for reconstructing the original-signal $g(t)$ from the sequence of sample-values $\{g(n/2W)\}$, with $\text{sinc}(2Wt)$ playing the role of interpolation function.

(PTO)

Q 7b. Explain the concept of TDM with a neat block diagram. (08M)

Ans: Sampling-theorem provides the basis for transmitting band-limited m(t) as a sequence of samples m(nTs) taken uniformly at the rate $f_s \geq 2W$ Hz (Nyquist-rate)

Thus transmission of m(nTs) engages the comm'n-channel for only a fraction of the sampling-interval Ts. on a periodic basis. & time-interval betn adjacent samples is cleared (or free) for use by other independent m(t)'s on time shared basis.

Thereby we obtain a TDM-system, which enables the joint utilization of channel by a plurality of independent m(t)'s without mutual interference among them.

⇒ The concept of TDM is illustrated by the block-diagram shown in the below fig1)

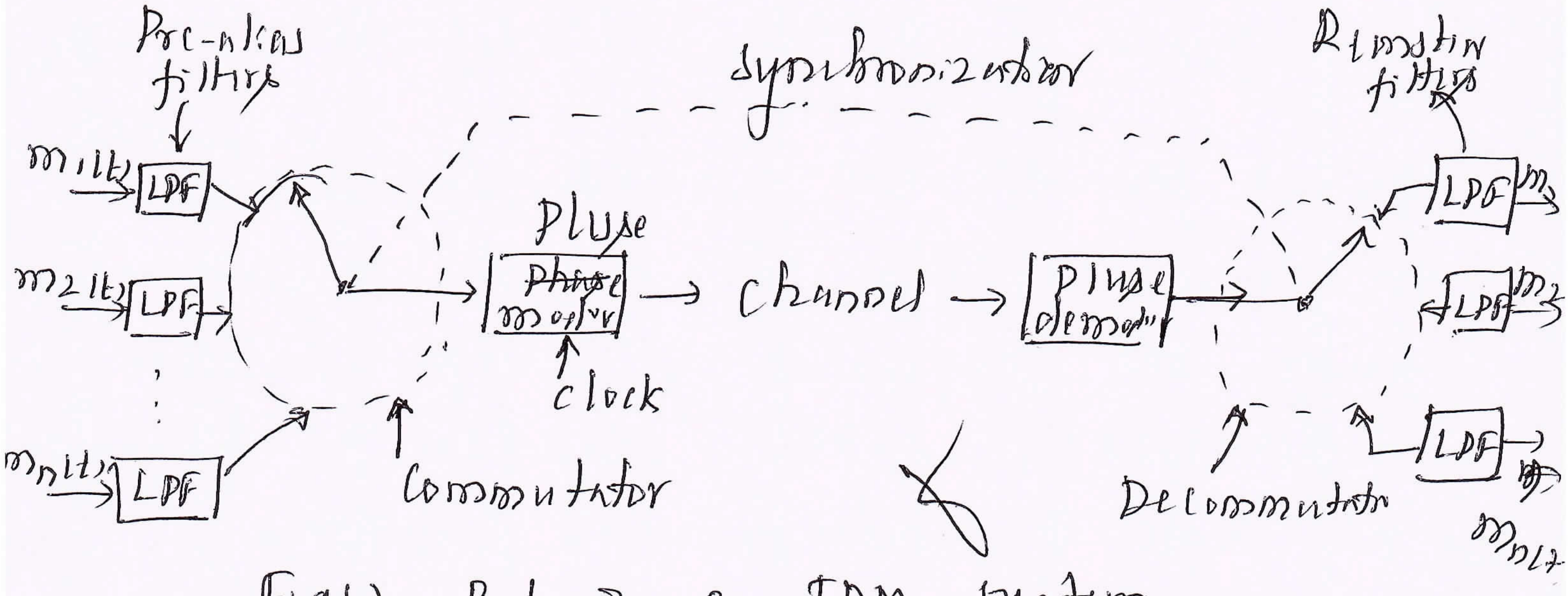


Fig1) B.d of a TDM system.

Each inputs is converted into a strictly-bandlimited signal by a pre-alias LPF. Then the output of this LPF applied to a commutator which is basically electronic-switching-circuity. The function of the commutator is

1) to take narrow samples of each N input at f_s rate $\geq 2W$ Hz

2) to sequentially interlay these N-samples inside the T_s interval.

Then these multiplexed-signal is applied to a pulse-modulator to transform into a form for transmission over the common channel.

At the receiving end, received signal is applied to a pulse-demodulator to recover narrow pulses, then they passed through a appropriate reconstruction-LPF by means of a decommutator, which operates synchronously with the commutator //.

Q 7C. What is aperture-effect? Briefly explain how to overcome this effect. (04M)

<u>Ans</u>: The spectrum of the received-PAM wave is given by

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)\, H(f) \rightarrow (1)$$

&

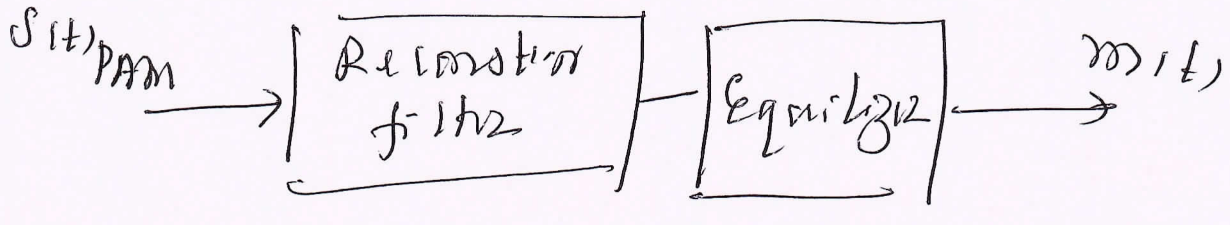
(PTO)

W.k.that, to recover $m(t)$, we may pass $s(t)$ through a LPF, assuming $f_s > 2W$ Hz. Then this output is equivalent to passing the original $m(t)$ through another LPF of TF $H(f)$. $H(f)$ is defined by the eqn

$$H(f) = T\,\mathrm{sinc}(fT)\exp(-j\pi fT)$$

∴ we see that by using flat-top samples to generate PAM-signal, we have introduced amplitude-distortion as well as delay of $T/2$. This effect is similar to scanning aperture effect in TV & so is known as the aperture effect.

This distortion may be corrected by connecting an equilizer in cascade with the reconstn-LPF as shown in below fig ( )



The amplitude response of this equalizer is given by

$$\frac{1}{|H(f)|} = \frac{1}{T\,\mathrm{sinc}(fT)} = \frac{\pi f}{\sin(\pi f T)} \quad //.$$

Q 8 a. Briefly explain the following modulation with waveforms (04)
i) PAM ii) PWM iii) PPM.

Ans: Simplest & most basic form of analog-pulse modulation is PAM, defined as → amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample-values of a continuous $m(t)$.

$m(t) \times c(t) \rightarrow$ Natural sampling

$m(nT_s) \times c(t) \rightarrow$ Flat-top sampling

In PAM top maintained flat, as shown in fig (c) and is mathematically defined as

$$s(t)_{PAM} = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s) \rightarrow (1)$$

where $h(t) \rightarrow$ standard unit-pulse defined as

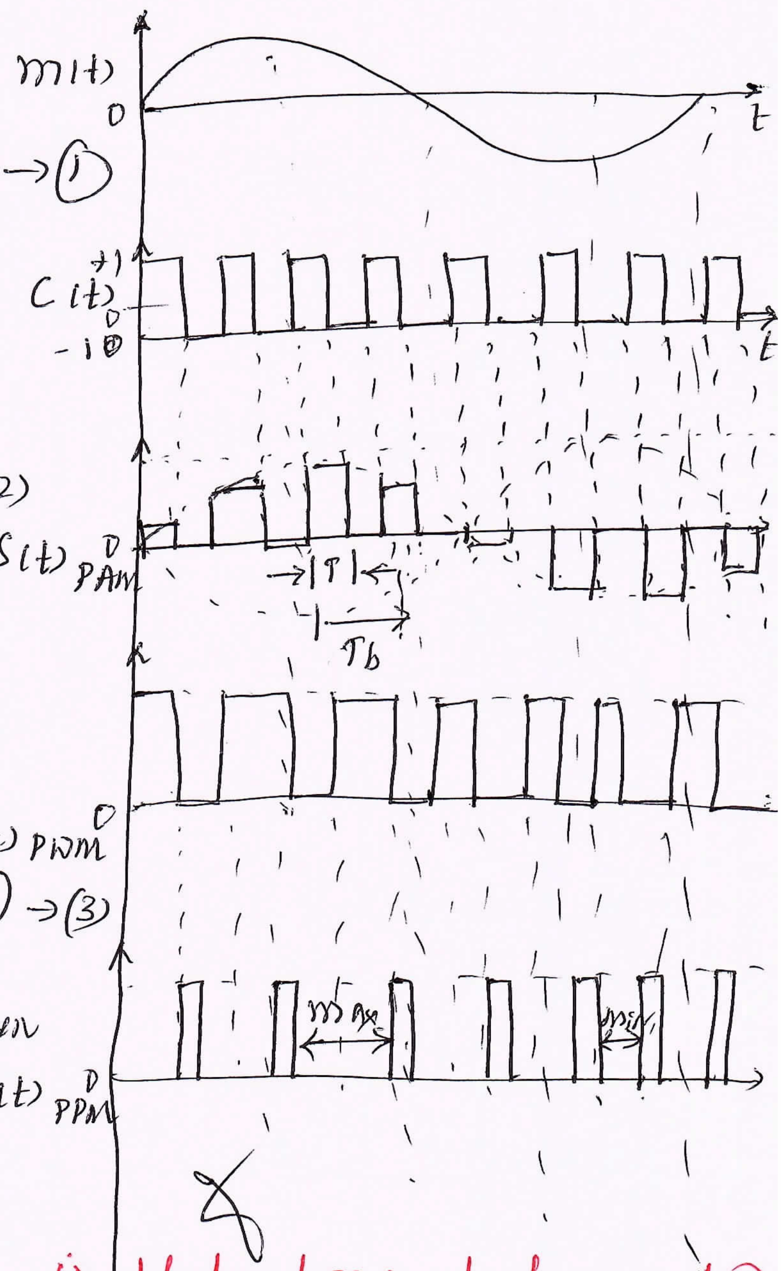
$$h(t) = \begin{cases} 1 & 0 < t < T \rightarrow (2) \\ 1/2 & t = 0,\ t = T \\ 0 & \text{otherwise.} \end{cases}$$

$m(nT_s) \rightarrow$ sample values of $m(t)$

$$s(t)_{PPM} = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s)) \rightarrow (3)$$

and the sufficient condition for non-overlapping pulses is

$$k_p |m(nT_s)| < \frac{T_s}{2} \rightarrow (4).$$



Limitation of PWM / PDM is that long-pulses expend considerable power during the pulse while bearing no additional information. In order in PPM ...

Q 8b With neat block diagram, explain the generation of PPM-wave. (05).

Ans: The PPM signal defined by the eqn (1) i.e

$$s(t)_{PPM} = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s)) \rightarrow (1)$$

may be generated using the system defined in the below fig (1).

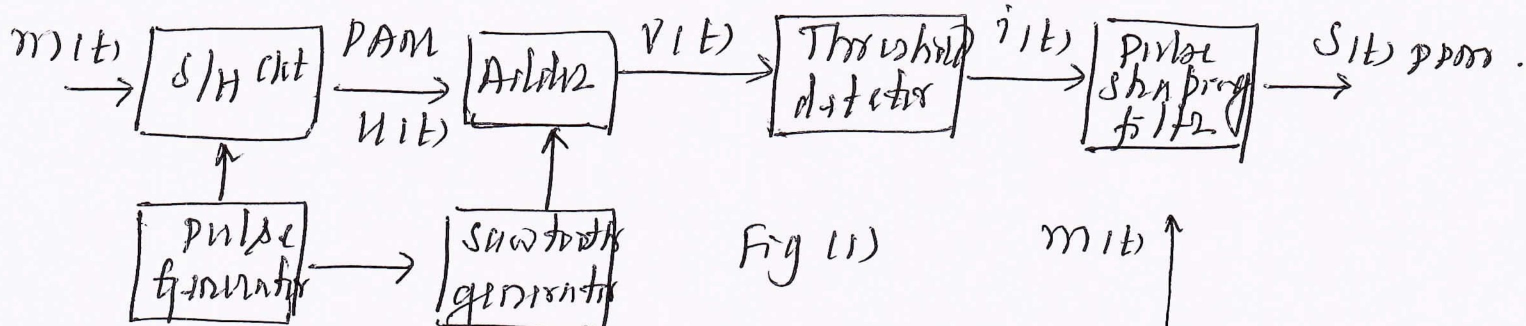
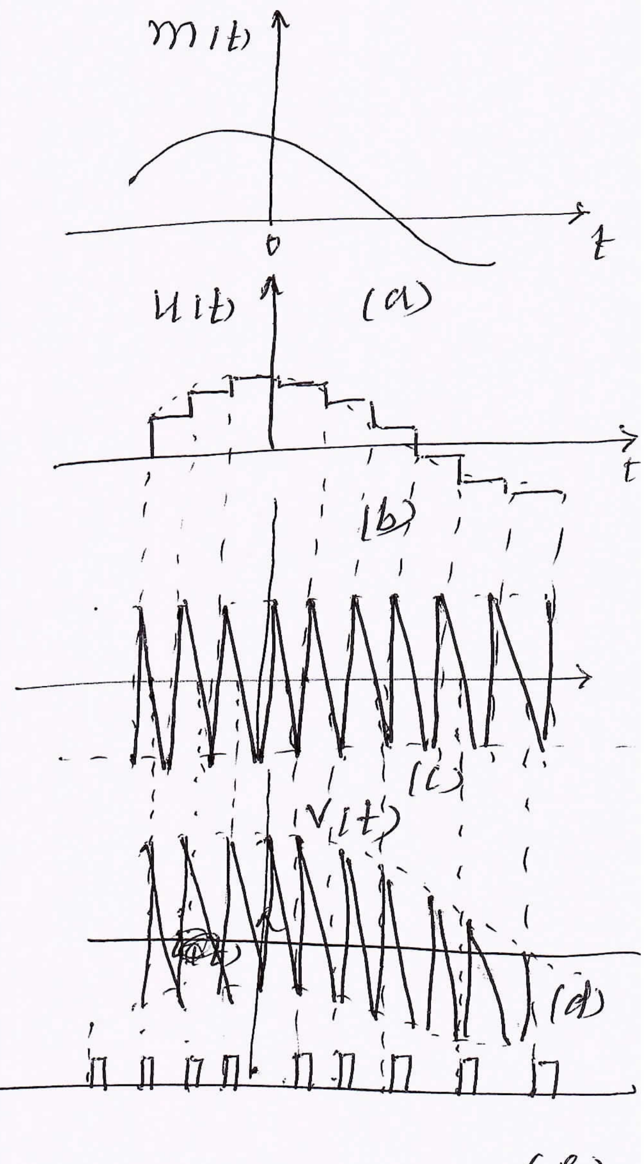


Fig (1)



S/H circuit generating a staircase-waveform u(t)

<u>Note:</u> Pulse-duration T of the S/H ckt is same as the sampling duration T_s.

Next u(t) is added to a sawtooth waveform yielding the combined signal v(t) shown in fig (d)

The combined signal v(t) applied to a threshold detector that produces a very narrow pulse (≈ impulse) each time v(t) crosses zero in -ve going direction. as in fig (e).

Finally $s(t)_{PPM}$ is generated by using these sequence of impulses to excite a filter whose response is g(t)

Q 8C. Explain the following terms. i) under Sampling ii) over Sampling iii) Nyquist rate (06)

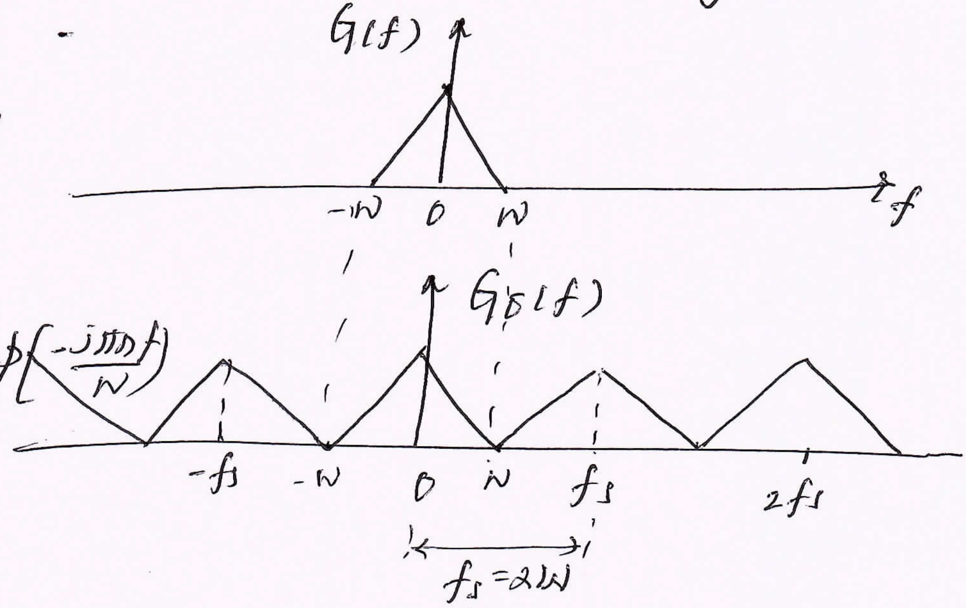
Ans: → Let us assume that the signal $g(t)$ of finite-energy and infinite duration. Suppose $g(t)$ is a strictly band limited signal with

$|G(f)| = 0$ for $|f| \geq W$, as illustrated in fig 1a;

1) Suppose we choose $f_s = 2W$ (or $T_s = 1/2W$), then the corresponding spectrum $G_\delta(f)$ of the sampled signal $g_\delta(t)$ is as shown in fig 1b.

Then, by solving the case using FDD of $g_\delta(t)$ we get

$$G(f) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} g(n/2W)\exp\left(\frac{-j\pi nf}{W}\right)$$



ie

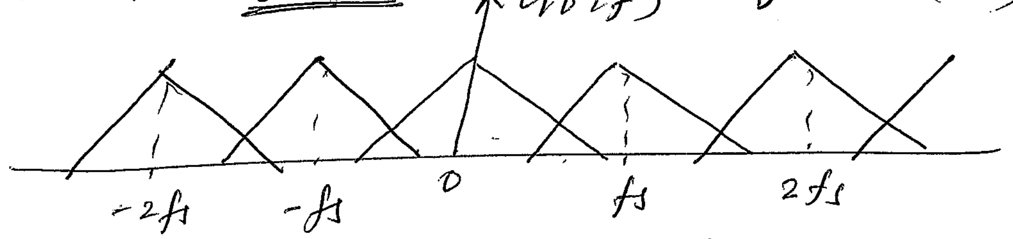
$$G(f) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} g(n/2W)\exp\left(-\frac{j\pi nf}{W}\right) \longrightarrow (2)$$

or $$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\,\mathrm{sinc}(2Wt-n) \longrightarrow (3)$$ Ans.

Case I: Under Sampling => $f_s < 2W$.

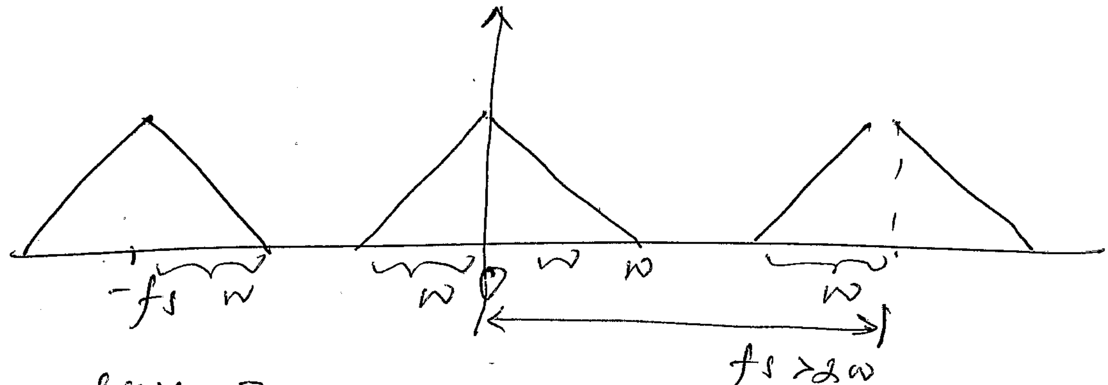
In this case the resultant spectrum $G_\delta(f)$ is as shown in fig 1c.

Case II : $f_s < 2W$



(c)

Case III : $f_s > 2W$



Thus in the case of I and III i.e. $f_s \geq 2W$ we can recover $g(t)$ by passing the $G_\delta(f)$ through appropriate reconstⁿ filter.

Case II: $f_s < 2W$, there is a ~~overla~~ overlapping of the periodic spectrum of $g(t)$, which is known as alias-effect, & hence the spectrum is distorted by the amount that it is less from 2W.

Nyquist Rate: → From the above discussion we can conclude that sampling rate $f_s \geq 2W$ i.e. over-sampling is practically preferred, under-sampling avoided.

Thus $f_s \geq 2W$ is known as "Nyquist rate"

Q 9 a. Derive the expression of $(SNR)_0$ of a uniform quantizer. (08M)

Ans: The use of quantization introduces an error defined as the difference between the input signal m and the output signal v. This error is called "quantization-noise".

Assuming the uniform-quantizer, the quantization error q is given by

$$q = m - v \longrightarrow (1)$$

or correspondingly $Q = M - V \rightarrow (2)$

With the input M having zero-mean, and the quantizer assumed to be symmetric, it follows that the quantizer output V and hence the quantization-error Q will also have zero-mean

Thus, for a practical statistical characterization of the quantizer in terms of $(SNR)_0$, we need to find only the mean-sq value of the quantization-error Q.

W.k.t the step-size Δ of the quantizer is given by

$$\Delta = \frac{2 m_{max}}{L} \longrightarrow (3)$$

where $L \rightarrow$ Total no. of representation levels

For a uniform quantizer, the quantization-error Q will have its sample values bounded by $-\frac{\Delta}{2} \leq q \leq \frac{\Delta}{2}$.

* If Δ is very small (& hence L is very large) it is reasonable to assume that Q is a RV of

uniformly-distributed type, hence the interfering effect of the quantization-noise on the quantizer input is similar to that of thermal-noise.

Thus we may express the pdf of the quantization-error Q as follows.

$$f_Q(q) = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases} \longrightarrow (4)$$

With mean-zero, its variance is same as the mean-sq value,

$$\therefore \underset{\text{variance}}{\sigma_Q^2} = \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq = \underset{\text{Mean sq value}}{E[Q^2]} \rightarrow (5)$$

using (4) in (5)

$$\sigma_Q^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq \longrightarrow (6)$$

$$\therefore \sigma_Q^2 = \frac{\Delta^2}{12}.$$

Let R denote the no. of bits/sample (codeword size)

then

$$\underset{\substack{\uparrow \\ \text{Representation Level}}}{L} = 2^R$$

OR $\quad R = \log_2 L$

w.k.t $\quad \Delta = \frac{2m_{max}}{L} = \frac{2m_{max}}{2^R} \longrightarrow (7)$

using (7) in (6)

$$\therefore \sigma_Q^2 = \frac{1}{3} m_{max}^2\, 2^{-2R} \longrightarrow (8)$$

Let $P \rightarrow$ Avg power of the m(t), then the

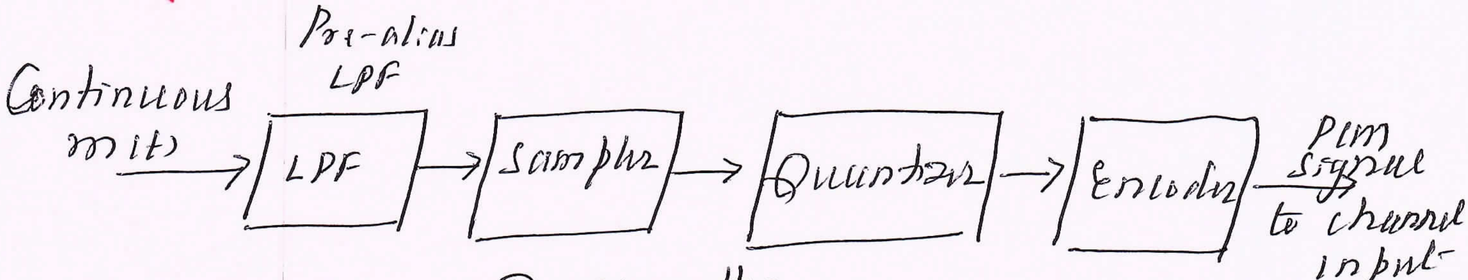
$$(SNR)_0 = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{max}^2}\right) 2^{2R} \longrightarrow (8)$$

$\therefore$ SNR of Quantizer increases exponentially with R //.

Q 9b. With neat block-diagram explain the transmitter, channel and receiver of a PCM system. (08M)

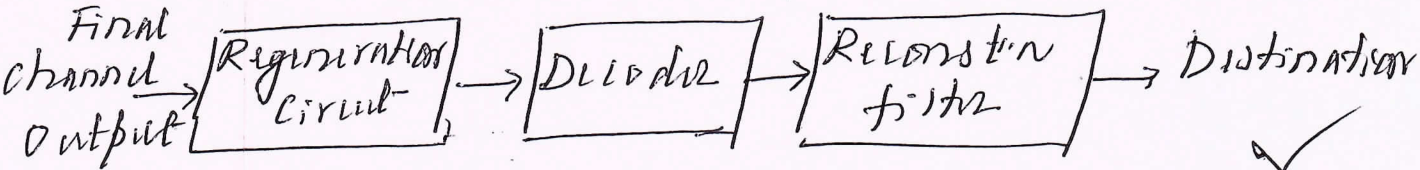
Ans: In PCM a m(t) is represented by a sequence of coded pulses which is accomplished by representing the signal in discrete form in both time and amplitude.

The basic operations performed in the transmitter of a PCM-system are sampling, quantizing and encoding as shown in fig 1a.



a) Transmitter

b) Transmission path



c) Receiver.

<u>Transmitter</u>: → The quantizing & encoding performed in the same circuit called an "ADC".

<u>Sampling</u>: Incoming m(t) is sampled with a train of narrow rectangular pulses so as to closely approximate instantaneous sampling process, and $\boxed{f_s \geq 2W}$. Pre-alias LPF converts m(t) into a strictly band limited signal.

Quantization: → Sampled signal then quantized that is its output is a discrete in both time and amplitude. Basically there are two types of quantizers

- Uniform
  - Symmetric
  - Asymmetric
- Non-uniform.
  - A-law Compander
  - M-law Compander

Encoding: Output of a quantizer becomes discrete set of values but not in the form best suited to transmission over the radio-path.

So to exploit the advantages of sampling & quantizing for making transmitted signal robust to noise, and other channel degradations, we use encoding process to translate the discrete set of sample values to a more appropriate form of signal. Binary coding is more popular.

Receiver: → The basic operations in the receiver are
- Regeneration
- Decoding
- Reconstruction

&

Note: Regeneration also occurs at intermediate points along the transmission path as necessary.

Regeneration may be 3R or 2R, or R type
where 3R → Retime
→ Reshape
→ Retransmit //.

Q 9c. An audio-signal digitized using PCM. Assume audio signal bandwidth is 20 kHz
i) what is Nyquist rate & Nyquist period of the audio signal?
ii) If the samples are quantized to L = 4096 levels, & then binary coded, determine the no. of bits required to encode a sample. (04M)

Soln: Given $W = 20\,kHz$

Nyquist rate $f_s = 2W$

$= 2 \times 20\,kHz$

$\boxed{f_s = 40\,kHz}$

$\therefore T_s = \frac{1}{f_s} = \frac{1}{40} \times 10^{-3}$

$= 0.25 \times 10^{-4}$

$= 25.0 \times 10^{-6}$

$\boxed{T_s = 25\,\mu sec}$

$2^{10} = 1024$

$2^{11} = 2048$

$2^{12} = 4096$

$\therefore L = 4096 = 2^{12}$

$\therefore R = \log_2 L$

$= \log_2 2^{12}$

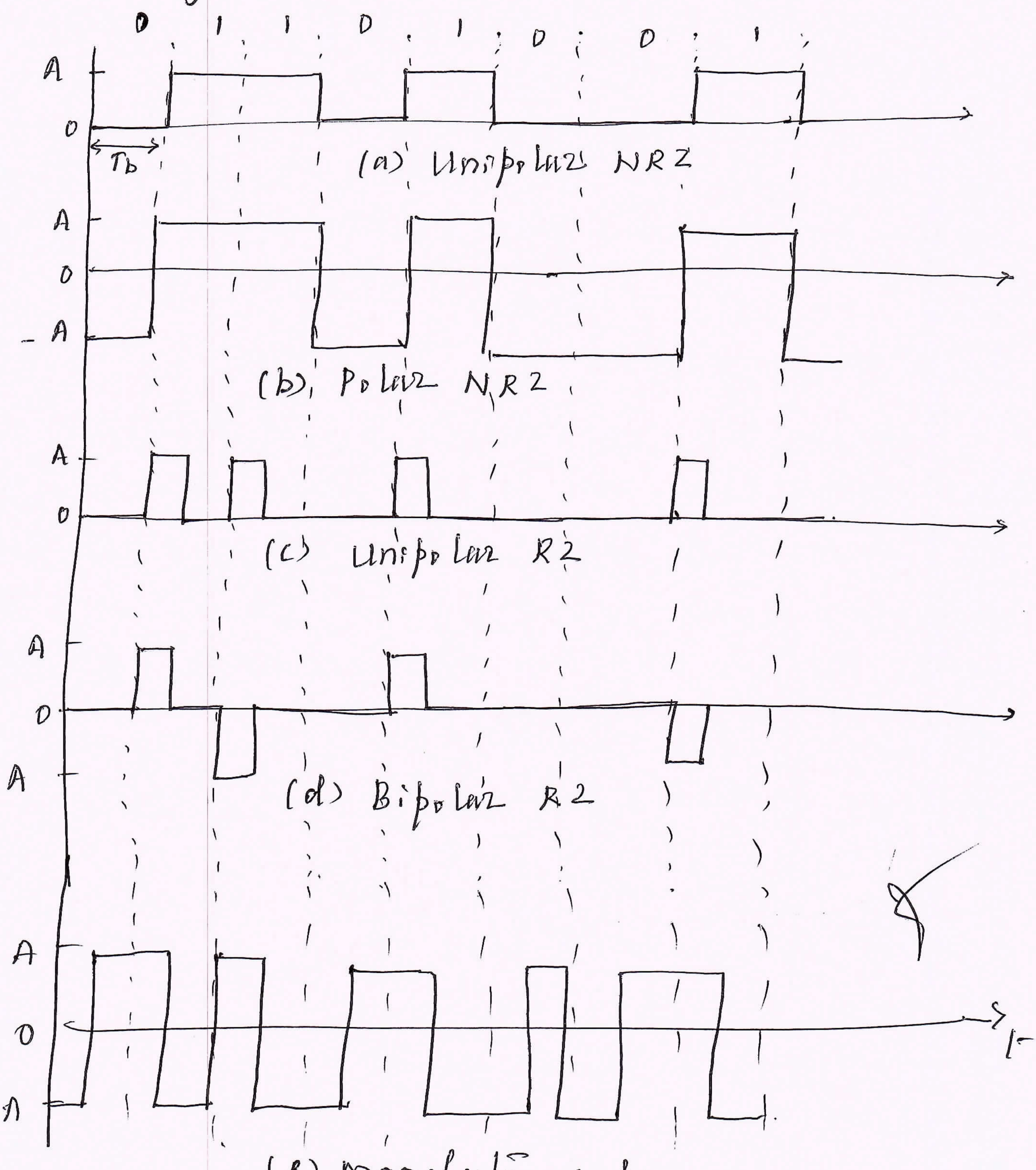
$= 12 \times 1$

$\boxed{R = 12 \text{ bits / codeword}}$

Q 10 a. Draw the line codes for given binary representation 01101001.
i) Unipolar NRZ
ii) Polar NRZ
iii) Unipolar RZ
iv) Bipolar RZ
v) manchester code

(10M)

Ans: Binary Data 01101001.



Q10 b. Explain granular noise & slope overload distortion in delta modulation. (04M)

Ans: Delta modulation is subject to two types of quantization error.

1> slope overload distortion
2> Granular noise.

Slope overload distortion: → We observe that eqn

$$m_q(nT_s) = m_q(nT_s - T_s) + e_q(nT_s) \rightarrow (1)$$

is that the digital equivalent of integration in the sense that it represents the accumulation of +ve and -ve increments of magnitude Δ. Also denoting the quantization error by $q(nT_s)$

$$m_q(nT_s) = m(nT_s) + q(nT_s) \rightarrow (2)$$

Thus the input to the quantizer is

$$e(nT_s) = m(nT_s) - m(nT_s - T_s) - q(nT_s - T_s) \rightarrow (3)$$

Thus except for the quantization error $q(nT_s - T_s)$ the quantizer input is a <u>first-order</u> backward difference of the input signal which may be viewed as a digital approximation to the derivative of the input signal or equivalently inverse of the digital integration process.

If we consider the max slope of the original input waveform $m(t)$, it is clear that in order for the sequence of samples $\{m_q(nT_s)\}$ to increase as fast as the input sequence of samples $\{m(nT_s)\}$ in a region of max

Q10C. With a neat diagram explain the delta modulation scheme. (06M)

Ans: Increased bandwidth requirement of PCM is a reason for concern, alternative method of digitally representing analog source is "DM".

In DM, m(t) is <u>oversampled purposely</u> to increase the correlation between adjacent samples of the signal. to permit the use of a simple quantizing strategy for constructing the encoded signal.

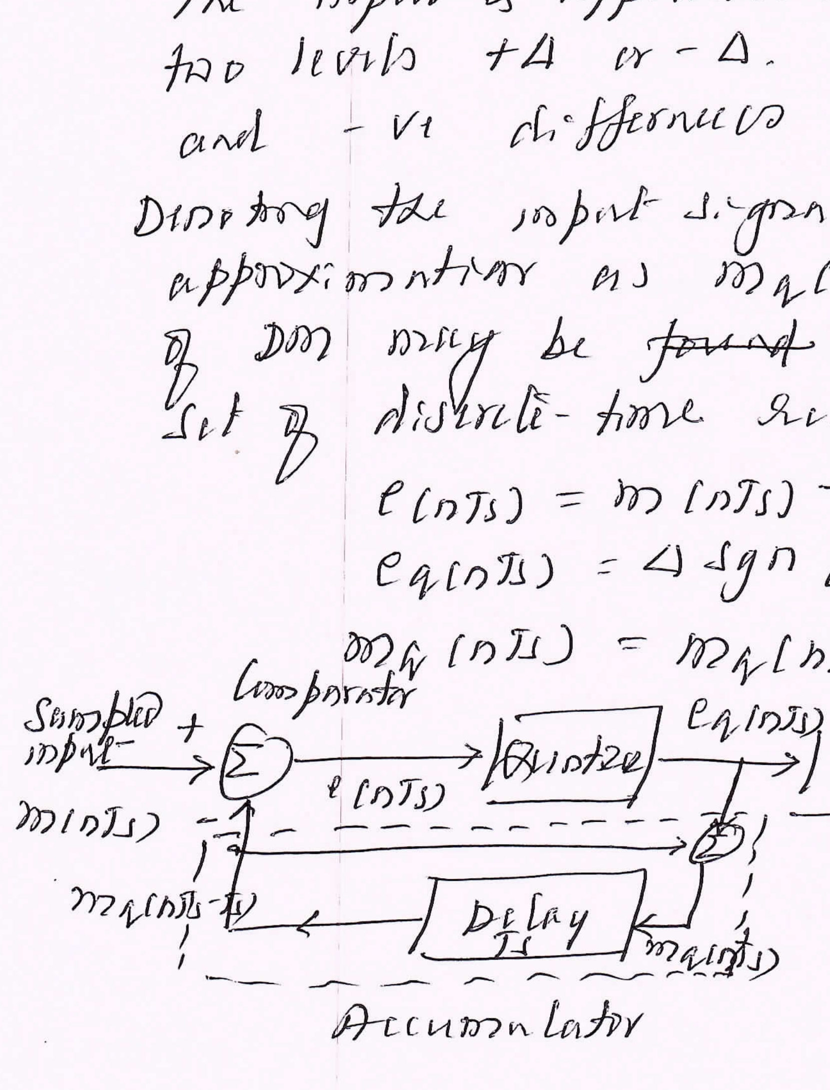
In its simplest basic form, DM provides a staircase approximation to the oversampled version of the m(t). Then the difference between the input & approximation is quantized into two levels $+\Delta$ or $-\Delta$. Corresponding to +ve and -ve differences respectively.

Denoting the input signal as m(t) & its staircase approximation as $m_q(t)$, the basic principle of DM may be ~~found~~ formalized in the following set of discrete-time relations

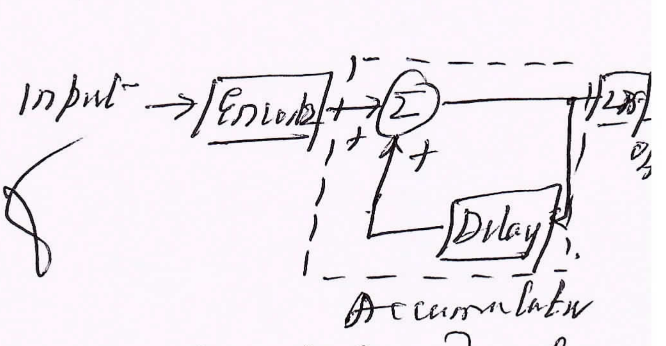
$$e(nT_s) = m(nT_s) - m_q(nT_s - T_s) \rightarrow (1)$$

$$e_q(nT_s) = \Delta \, sgn[e(nT_s)] \rightarrow (2)$$

$$m_q(nT_s) = m_q(nT_s - T_s) + e_q(nT_s) \rightarrow (3)$$



(a) Transmitter (ADC)



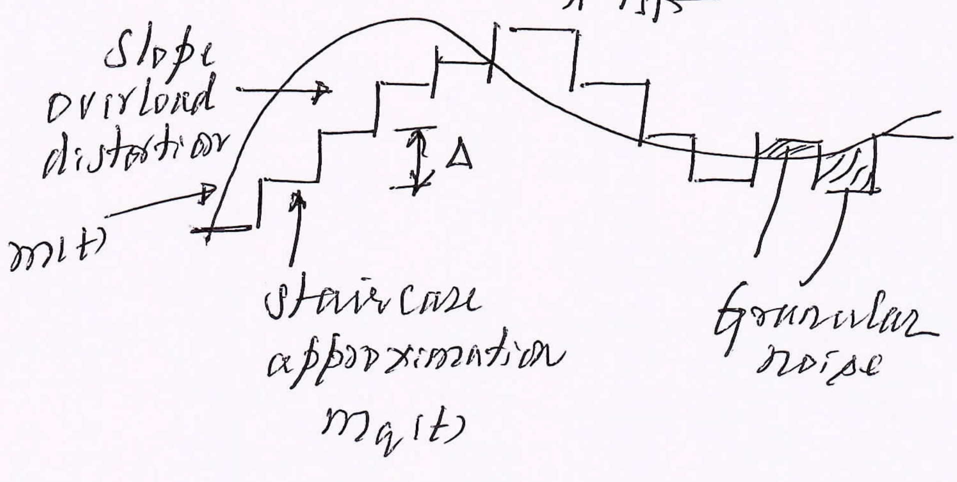
(b) DAC decoder

slope of $m(t)$, we require that the condition

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right| \longrightarrow (4)$$

be satisfied. Otherwise, we find that the step size Δ is too small for staircase approximation $m_q(t)$ to follow a steep segment of the $m(t)$ with the result that $m_q(t)$ falls behind $m(t)$ as in fig(). This condition is called slope overload and the resulting quantization error is called slope over load distortion (noise).

In contrast to slope over-load distortion, granular noise occurs when Δ is too large relative to the local slope characteristic of the $m(t)$ thereby causing $m_q(t)$ to hunt around a relatively flat segment of the input $m(t)$. Granular-noise is analogus to quantization in a PCM system.



Fig() Illustration of quantization error in the delta modulation.