INFORMATION THEORY AND CODING

18EC54

(Time: 3 hrs) Feb/Mar. 2022 (Max. Marks: 100)

Solutions and Scheme Prepared by

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Module-1

1. a). Define self information. Why logarithmic expression is chosen for measuring information. — 4M

Ans Let S_k be a symbol chosen for transmission at any time with a probability equal to P_k. Then the self-information of message S_k is given by. $I_k = \log \frac{1}{P_k}$

If base of log is 2, then units is "Bits".
If base of log is 10; then units is "Hartleys".
If base of log is e, then units is "NATS". — 1M.

Logarithmic expression is chosen because of following reasons

1) Self information of any message cannot be negative.
2) lowest possible self-information is zero
3) More information is carried by a less likely message.
4) when independent symbols are transmitted, the total self information must be equal to the sum of individual self-information — 3M.

1. b). (i) Find relationship between Hartleys, Nats and bits
(ii) A discrete source emits one of the four symbols S_0, S_1, S_2 and S_3 with probabilities 1/3, 1/6, 1/4 and 1/4 respectively. The successive symbols emitted by the source are statistically independent. Calculate the entropy of the source. — 8M

Ans (i) $I = \log_{10} \frac{1}{P}$ Hartleys

$I = \log_e \frac{1}{P}$ nats

$I = \log_2 \frac{1}{P}$ bits.

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$$1\ \text{Hartley} = \frac{I}{\log_{10} 1/p} = \frac{\log_2 1/p}{\log_{10} 1/p} \text{ nats}$$

$$= \frac{-\log_e p}{-\log_{10} p} \text{ nats}$$

$$= \log_e 10 \text{ nats}$$

$$1\ \text{Hartley} = 2.303 \text{ nats} \quad \text{— 2M.}$$

Similarly; $1\ \text{Hartley} = \log_2 10$ bits

$$= \frac{1}{\log_{10} 2} \text{ bits}$$

$$1\ \text{Hartley} = 3.32 \text{ bits} \quad \text{— 2M}$$

and $1\ \text{nat} = \log_2 e$ bits

$$1\ \text{nat} = \frac{1}{\log_e 2} = 1.443 \text{ bits} \quad \text{— 2M.}$$

(ii) $$H(S) = \sum_{i=0}^{3} P_i \log \frac{1}{P_i} = P_0 \log \frac{1}{P_0} + P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + P_3 \log \frac{1}{P_3}$$

$$= \frac{1}{3}\log 3 + \frac{1}{6}\log 6 + \frac{1}{4}\log 4 + \frac{1}{4}\log 4$$

$$H(S) = 1.95914 \text{ bits/msg symbol} \quad \text{— 4M}$$

1.c (i) State the properties of entropy.

(ii) A source transmits two independent messages with probabilities of p and 1-p respectively. Prove that the entropy is maximum when both the messages are equally likely. Plot the variation of entropy (H) as a function of probability 'p' of the messages.

— 8M

1.C. Ans.

(I) Properties of entropy

1) The entropy function is continuous for every independent variable P_k in the interval $(0,1)$

2) The entropy function is a symmetrical function of its arguments. $H[P_k,(1-P_k)] = H[(1-P_k), P_k]$

3) The entropy attains a maximum value when all the source symbols become equiprobable.

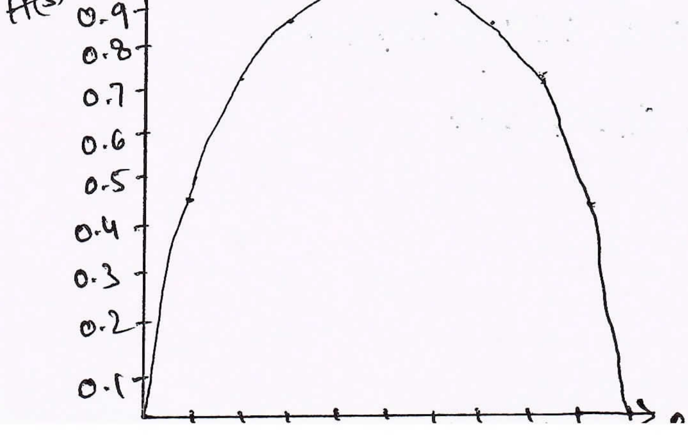
4) Partitioning of symbols into sub-symbols cannot decrease the entropy.

5) Source efficiency is given by $\eta_s = \dfrac{H(S)}{H(S)_{max}}$

and source redundancy is given by $R_{\eta_s} = 1 - \eta_s$. — 4M.

(II) $$H(S) = \sum_{i=1}^{2} P_i \log \frac{1}{P_i} = p \log \frac{1}{p} + (1-p) \log \frac{1}{(1-p)}$$

Let $p = 0.1$ $H(S) = 0.469$ bits/sym.
$p = 0.2$ $H(S) = 0.722$ bits/sym
$p = 0.3$ $H(S) = 0.881$ bits/sym
$p = 0.4$ $H(S) = 0.971$ bits/sym
$p = 0.5$ $H(S) = 1$ bits/sym
$p = 0.6$ $H(S) = 0.971$ bits/sym
$p = 0.7$ $H(S) = 0.881$ bits/sym
$p = 0.8$ $H(S) = 0.722$ bits/sym
$p = 0.9$ $H(S) = 0.469$ bits/sym. — 2M



— 2M.

(2)

OR

2. a) Consider the following Markov source shown in fig Q2(a). Find the i) State probabilities ii) State entropies iii) Source entropy iv) G_1, G_2 v) Show that $G_1 > G_2 > H$

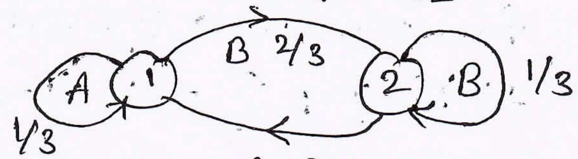


fig Q2(a)

10M

Ans

i) $P(1) = P(2) = 1/2$ — 2M.

ii) $$H_i = \sum_{j=1}^{n} P_{ij} \log \frac{1}{P_{ij}}$$

For 1st state ⇒ $H_1 = P_{11} \log \frac{1}{P_{11}} + P_{12} \log \frac{1}{P_{12}}$

$H_1 = 0.918$ bits/symbols

For 2nd state ⇒ $H_2 = P_{21} \log \frac{1}{P_{21}} + P_{22} \log \frac{1}{P_{22}}$

$H_2 = 0.918$ bits/symbols — 2M

iii). $$H = \sum_{i=1}^{n} P_i H_i = P(1) H_1 + P(2) H_2$$

$H = 0.918$ bits/symbols. — 2M

iv) $$G_N = H(\bar{S}) = \sum_{i=1}^{3} P(m_i) \log \frac{1}{P(m_i)}$$

$G_1 = \frac{1}{2} \log \frac{1}{1/2} + \frac{1}{2} \log \frac{1}{1/2} = 1$ bits/sym — 2M

$G_2 = 0.959$ bits/sym.

∴ Hence $G_1 > G_2 > H$ — 2M.

4

2. b) Consider a zero memory source emitting three symbols x, y and z with respective probabilities $\{0.6, 0.3, 0.1\}$. Calculate i) Entropy of the source. (ii) All symbols and the corresponding probabilities of the second order extension of the source. Find the memory entropy of 2nd order. (iii) Show that $H(S^2) = 2 * H(S)$.

10M

Ans i) The entropy of the source is given by

$$H(S) = \sum_{i=0}^{3} P_i \log \frac{1}{P_i}$$

$$= 0.6 \log \frac{1}{0.6} + 0.3 \log \frac{1}{0.3} + 0.1 \log \frac{1}{0.1}$$

$$H(S) = 1.2954 \text{ bits/sym}$$ — 3M

ii) $$H(S^2) = \sum_{k=1}^{9} P_k \log_2 \frac{1}{P_k} \text{ bits/sym}$$

$S_1S_1 = 0.36$
$S_1S_2 = 0.18$
$S_1S_3 = 0.06$
$S_2S_1 = 0.18$
$S_2S_2 = 0.09$
$S_2S_3 = 0.03$
$S_3S_1 = 0.06$
$S_3S_2 = 0.03$
$S_3S_3 = 0.01$

$\therefore H(S^2) = 2.5909$ bits/sym

— 6M

iii) $H(S^2) = 2 * H(S) = 2 \times 1.2954 = 2.5909$ bits/sym
— 1M

Module - 2

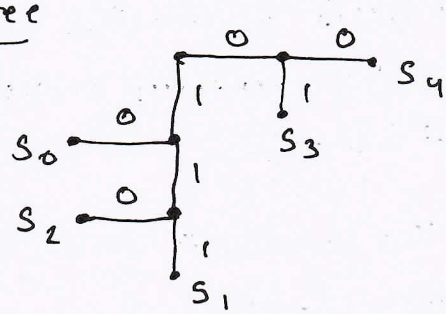
3. a) The table 3-1 below provides codes for five different symbols identify which of the following codes are prefix codes. Also draw the decision diagram for the prefix code.

Code - A	Code - B	Code - C	Code - D
0	1	00	10
10	01	110	111
110	111	1110	110
1110	10	001	01
1110	00	011	00

→ 4M

Ans code A, Code B, code-C are not prefix codes while code D is a prefix code as no codeword is a prefix of other codewords — 3M

Decision tree



— 1M.

3 b) Apply Shannon's encoding algorithm to the following set of messages and obtain code efficiency & redundancy.

M_1	M_2	M_3	M_4	M_5
$1/8$	$1/16$	$3/16$	$1/4$	$3/8$

→ 10M

Ans Step-1

M_5	M_4	M_3	M_1	M_2
$6/16$	$4/16$	$3/16$	$2/16$	$1/16$

Step-2 $a_1 = 0$, $a_2 = 0.375$, $a_3 = 0.625$, $a_4 = 0.8125$, $a_5 = 0.9375$, $a_6 = 1$

Step-3 $2^{l_i} \geq \frac{1}{P_i}$ $l_1 = 2$, $l_2 = 2$, $l_3 = 3$, $l_4 = 3$, $l_5 = 4$

Step-4 $a_1 = 0$, $a_2 = (0.375)_{10} = (0.011)_2$

$a_3 = (0.625)_{10} = (0.101)_2$

$a_4 = (0.8125)_{10} = (0.1101)_2$

$a_5 = (0.9375)_{10} = (0.1111)_2$

Step-5 →5M

Source	P_i	Code	l_i
M_5	$3/8$	00	2
M_4	$1/4$	01	2
M_3	$3/16$	101	3
M_1	$1/8$	110	3
M_2	$1/16$	1111	4

$\eta_c = \frac{H(S)}{L}$

$\eta_c = 0.865$

$\eta_c = 86.5\%$

$R\eta_c = 13.5\%$ — 3M.

$$L = \sum_{i=1}^{5} P_i l_i = 2.4375 \text{ bits/msg-sym.}$$ — 1M

$$H(S) = \sum P_i \log \frac{1}{P_i} = 2.1085 \text{ bits/sym}$$ — 1M

3(c) Construct a binary code by applying Huffman encoding procedure for the following messages with respective probabilities of 0.4, 0.2, 0.2, 0.1, 0.07 and 0.03. Also determine the code efficiency and redundancy of the code. → (6M)

Ans

Source	P_i	Code	Source Sa P_i	Source Sa code	Source Sb P_i	Source Sb Code	Source Sc P_i	Source Sc Code	Source Sd P_i	Source Sd Code
u_1	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
u_2	0.2	01	0.2	01	0.2	01	0.4	00	0.4	1
u_3	0.2	000	0.2	000	0.2	000	0.2	01		
u_4	0.1	0010	0.1	0010	0.2	001				
u_5	0.07	00110	0.1	0011						
u_6	0.03	00111								

— 4M.

$L = 2.3$ binits/msg-sym

$H(S) = 2.209$ bits/msg-sym

$\eta_c = \frac{H(S)}{L} = 96.04\%$

$R\eta_c = 2.96\%$ → 2M.

OR

4. a) Design a Trinary source code for the source shown using Huffman's coding procedure $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$

$P = \{1/3, 1/4, 1/8, 1/8, 1/12, 1/12\}$ → (10M)

Ans

$q = r + (r-1)\alpha$ $\quad \alpha = \frac{q-r}{r-1} = \frac{6-3}{3-1} = \frac{3}{2}$

Source	P_i	Code	Source Sa P_i	Source Sa Code	Source Sb P_i	Source Sb Code
S_1	1/3	1	1/3	1	5/12	0
S_2	1/4	2	1/4	2	1/3	1
S_3	1/8	01	1/6	00	1/4	2
S_4	1/8	02	1/8	01		
S_5	1/12	000	1/8	02		
S_6	1/12	001				
S_7	0	002 → Discard.				

→ 10M.

(4) (67)

4. b) Consider a source $S = \{S_1, S_2\}$ with probabilities 3/4 & 1/4 respectively. Obtain Shannon-Fano code for source S and its 2^{nd} & 3^{rd} extension. Calculate efficiencies for each case and justify the results. → 10M

Ans

	P_i	Code	l_i
S_1	3/4	1	1
S_2	1/4	0	1

$$L_1 = \sum_{i=1}^{2} P_i l_i = 1 \text{ bnits/msg-sym}$$

$$H(S) = \sum_{i=1}^{2} P_i \log \frac{1}{P_i} = 0.8113 \text{ bits/msg-sym}$$

$$\eta_c = 81.13\%$$

→ 3M.

For 2^{nd} extension

	P_i						Code	l_i
S_1S_1	9/16	1					1	1
S_1S_2	3/16	0	3/16	1			01	2
S_2S_1	3/16	0	3/16	0	3/16	1	001	3
S_2S_2	1/16	0	1/16	0	1/16	0	000	3

$$L_2 = 1.6875 \text{ bits/msg-sym}$$

$$H(S^2) = 2H(S) = 1.6226 \text{ bits/msg-sym}$$

$$\eta_c^{(2)} = 96.15\%$$

→ 3M.

For 3^{rd} extension

	P_i												Code	l_i
$S_1S_1S_1$	27/64	1											1	1
$S_1S_1S_2$	9/64	0	9/64	1	9/64	1							011	3
$S_1S_2S_1$	9/64	0	9/64	1	9/64	0							010	3
$S_2S_1S_1$	9/64	0	9/64	0	9/64	1							001	3
$S_1S_2S_2$	3/64	0	3/64	0	3/64	0	3/64	1					0001	4
$S_2S_1S_2$	3/64	0	3/64	0	3/64	0	3/64	0	3/64	1			00001	5
$S_2S_2S_1$	3/64	0	3/64	0	3/64	0	3/64	0	3/64	0	3/64	1	00001	6
$S_2S_2S_2$	1/64	0	1/64	0	1/64	0	1/64	0	1/64	0	1/64	0	000000	6

$$L_3 = 2.484375 \text{ bnits/msg-sym}$$

$$H(S^3) = 3 \times H(S) = 2.4339 \text{ bits/msg-sym}$$

$$\eta_c^{(3)} = 97.97\%$$

→ 3M.

Proper grouping of symbols will increase the efficiency with increase in extension. → 1M.

8

Module - 3

5. a) What is mutual information? Mention its properties. → 6M

Ans When an average information of $H(A)$ is transmitted over the channel, an average amount of info. equal to equivocation $H(A/B)$ is lost in the channel due to intersymbol conversion due to noise. The balance of information received at the receiver is the mutual information.

$$I(A,B) = H(A) - H(A/B).$$ → 2M.

Properties

1) Mutual information of a channel is symmetric
2) Mutual information is always non-negative
3) Mutual information of a channel may be expressed in terms of the entropy of the channel output
4) Mutual information is related to the joint entropy of the channel. → 4M.

5. b) A transmitter has an alphabet consisting of 5 letters $\{a_1, a_2, a_3, a_4, a_5\}$ and the receiver has an alphabet of 4 letters $\{b_1, b_2, b_3, b_4\}$. The JPM are given below. → 8M

$$P(A,B) = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.10 & 0.3 & 0 & 0 \\ 0 & 0.05 & 0.1 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix}$$

compute different entropies of the channel.

Ans

$P(b_1) = 0.35$ $\quad P(b_2) = 0.35$ $\quad P(b_3) = 0.2$ $\quad P(b_4) = 0.1$

$P(a_1) = 0.25$ $\quad P(a_2) = 0.4$ $\quad P(a_3) = 0.15$ $\quad P(a_4) = 0.15$ → 2M.

$P(a_5) = 0.05$

$H(A) = 2.066$ bits/msg-sym → 1M.

$$H(B) = \sum_{j=1}^{4} P(b_j) \log \frac{1}{P(b_j)} = 1.857 \text{ bits/msg-sym}$$ → 1M

$$H(A,B) = \sum_{i=1}^{5} \sum_{j=1}^{4} P(a_i, b_j) \log \frac{1}{P(a_i, b_j)} = 2.66 \text{ bits/msg.sym}$$ → 1M

$$H(B/A) = H(A,B) - H(B)$$
$$= 2.666 - 2.066 = 0.6 \text{ bits/msgg.sym}$$ → 1M.

$H(A/B) = H(A,B) - H(B)$

$= 2.66 - 1.857 = 0.809$ bits/msg-sym → 1M.

$I(A,B) = H(A) - H(A/B)$

$= 2066 - 0.809 = 1.257$ bits/msg-sym → 1M

5.c) For the channel matrix shown, find the channel capacity.

$$P(b_j/a_i) = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 1/3 & 1/6 & 1/2 \\ 1/6 & 1/2 & 1/3 \end{bmatrix}$$ → 6M

Ans

$$H(B/A) = h = \sum_{j=1}^{3} P_j \log \frac{1}{P_j}$$

$h = 1.4591$ bits/msg-sym →3M

$C = \log s - h$

$C. = \log 3 - 1.4591 = 0.1258$ bits/sec

→ 3M.

OR

6 a) In a communication system a transmitter has 3 input symbols $A = \{a_1, a_2, a_3\}$ and receiver also has 3 O/p symbols $B = \{b_1, b_2, b_3\}$. The matrix given below shows joint JPM with some marginal probabilities

i) Find the missing probabilities

ii) Find $P(b_3/a_1)$ and $P(a_1/b_3)$

iii) Are the events a_1 and b_1 statistically independent? why? → 6M

Ans

	b_1	b_2	b_3
a_1	1/12	*	5/36
a_2	5/36	1/9	5/36
a_3	*	1/6	*
$P(b_j)$	1/3	14/36	*

i) $$\sum_{j=1}^{3} P(b_j) = P(b_1) + P(b_2) + P(b_3) = 1$$

$P(b_3) = 5/18$

$P(a_3, b_1) = 1/3 - 1/12 - 5/36 = 1/9$

$P(a_1, b_2) = 14/36 - 1/9 - 1/6 = 1/9$

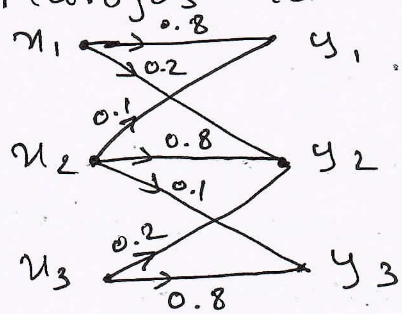
$P(a_3, b_3) = 5/18 - 5/36 - 5/36 = 0$ → 2M ii). $P(b_3/a_1) = \dfrac{P(a_1,b_3)}{P(a_1)} = \dfrac{5/36}{1/3} = 5/12$

$P(a_1/b_3) = \dfrac{P(a_1,b_3)}{P(b_3)} = \dfrac{5/36}{5/18} = 1/2$ → 2M

iii) $P(a_1,b_1) \neq P(a_1)\,P(b_1)$ → 2M.

∴ a_1 and b_1 are not statistically independent.

6 b) Find the capacity of the channel shown in fig. Q6 b) below using Murogo's Method. → 8M.



Ans

$$P(y_j/x_i) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0. & 0.2 & 0.8 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.8\log 0.8 + 0.2\log 0.2 \\ 0.8\log 0.8 + 2\times 0.1\log 0.1 \\ 0.8\log 0.8 + 0.2\log 0.2 \end{bmatrix} = \begin{bmatrix} -0.722 \\ -0.922 \\ -0.722 \end{bmatrix}$$ → 3M.

∴ $0.8Q_1 + 0.2Q_2 = -0.722$

$0.1Q_1 + 0.8Q_2 + 0.1Q_3 = -0.922$

$0.2Q_2 + 0.8Q_3 = -0.722$

$Q_1 = -0.6553$

$Q_2 = -0.9887$ → 3M.

$Q_3 = -0.6553$

$C = \log\left[2^{Q_1} + 2^{Q_2} + 2^{Q_3}\right]$

$C = 0.8269$ bits/msg-sym.

$C = (0.8269)\,r_s = 8269$ bits/sec. → 2M

6 c) Discuss Binary Ensure channel & derive channel capacity equation. → 6M

Ans The channel diagram of a binary erasure channel is shown below.

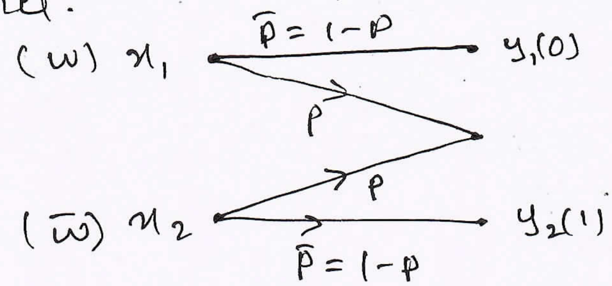
(11)

(6)

whenever an error occurs, the symbol will be received as 'y' and no decision will be made about the information, but an immediate request will be made through a reverse channel, for retransmission. till a correct symbol is received at the o/p.

→ 2M.

The disadvantage of this is the requirement of reverse channel.



Channel diagram of BEC.

BEC is also a symmetric channel whose channel matrix is constructed from the channel as

$$P(Y/X) = P(y_j/x_i) = \begin{bmatrix} \bar{p} & p & 0 \\ 0 & p & \bar{p} \end{bmatrix}$$

$$H(Y/X) = h = \sum_{j=1}^{s} P_j \log \frac{1}{P_j} = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p}$$

$$H(X) = w \log \frac{1}{w} + \bar{w} \log \frac{1}{\bar{w}}$$

$$P(X, Y) = \begin{bmatrix} \bar{p}w & pw & 0 \\ 0 & p\bar{w} & \bar{p}\bar{w} \end{bmatrix}$$

$$P(y_1) = \bar{p} w$$

$$P(y) = p$$

$$P(y_2) = \bar{p}\bar{w}$$

$$P(X/Y) = \begin{bmatrix} (\bar{p}w)/(\bar{p}w) & pw/p & 0 \\ 0 & (p\bar{w})/p & (\bar{p}\bar{w})/(\bar{p}\bar{w}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & w & 0 \\ 0 & \bar{w} & 1 \end{bmatrix}$$

$$H(X/Y) = p\left[w \log 1/w + \bar{w} \log 1/\bar{w}\right]$$ → 2M.

(ii) D = 1011

No. of shifts	Input	Shift registers R_0	R_1	R_2
		0	0	0
1	1	1	1	0
2	1	1	0	1
3	0	1	0	0
4	1	1	0	0

→ 3½M.

The generated code-vector = 1001011

OR

8 a) Define G and H matrix and show that $GH^T = 0$. → 5M

Ans
[G] is called as generator matrix of order $(k \times n)$ given by

$$[G] = [I_k \vdots P]_{(k \times n)}$$

[G] can also be expressed as $[G] = [P \vdots I_k]$

Associated with [G] is another matrix called "parity check matrix - H" given by

$$[H] = [P^T \vdots I_{n-k}]$$

[H] matrix is a $(n-k) \times n$ matrix and this matrix is used in error correction.

$$[G] = \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & \cdots & 0 & P_{11} & P_{12} & \cdots & P_{1,n-k} \\ 0 & 1 & 0 & \cdots & 0 & P_{21} & P_{22} & \cdots & P_{2,n-k} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & P_{k1} & P_{k2} & \cdots & P_{k,n-k} \end{array}\right]$$

$$[H] = \left[\begin{array}{cccc|ccccc} P_{11} & P_{21} & \cdots & P_{k1} & 1 & 0 & 0 & \cdots & 0 \\ P_{12} & P_{22} & \cdots & P_{k2} & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \\ P_{1,n-k} & P_{2,n-k} & \cdots & P_{k,n-k} & 0 & 0 & 0 & \cdots & 1 \end{array}\right]$$

→ 2M.

i^{th} row of [G] is given by

$$g_i = [0\ 0 \cdots 1 \cdots 0\ P_{i1}\ P_{i2} \cdots P_{ij} \cdots P_{i,n-k}]$$

j^{th} row of [H] is given by

$$h_j = [P_{1j}\ P_{2j} \cdots P_{ij} \cdots P_{kj}\ 0\ 0 \cdots 1 \cdots 0]$$

Consider $g_i h_j^T$

(8)

$$[0\ 0 \cdots 1 \cdots 0\ P_{i1}\ P_{i2} \cdots P_{ij} \cdots P_{i,n-k}] \begin{bmatrix} P_{1j} \\ P_{2j} \\ \vdots \\ P_{ij} \\ \vdots \\ P_{kj} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$g_i h_j^T = P_{ij} + P_{ij} = 0.$$

This equation is true for all values of i and j, hence

$$[G][H]^T = 0$$

Multiplying both sides by $[D]$

$$[D][G][H]^T = [D]\,0 = 0$$

$$[C][H^T] = 0 \quad \rightarrow 3M.$$

8\. b) The parity check bits of a (8, 4) block code are generated by
$C_5 = d_1 + d_2 + d_4$ $C_6 = d_1 + d_2 + d_3$ $C_7 = d_1 + d_3 + d_4$
$C_8 = d_2 + d_3 + d_4$ i) Find the $[G]$ & $[H]$ for this code
ii) Find minimum weight iii) show that its capable of correcting all single errors and capable of detecting all double errors by preparing syndrome table for them. → 10M.

Ans i) $[G] = \begin{bmatrix} 1&0&0&0&1&1&1&0 \\ 0&1&0&0&1&1&0&1 \\ 0&0&1&0&0&1&1&1 \\ 0&0&0&1&1&0&1&1 \end{bmatrix}$ →2M $H = \left[\begin{array}{cccc|cccc} 1&1&0&1&1&0&0&0 \\ 1&1&1&0&0&1&0&0 \\ 1&0&1&1&0&0&1&0 \\ 0&1&1&1&0&0&0&1 \end{array}\right]$ → 2M.

ii) By inspecting the H-matrix, we observe that not less than 4 columns of H will addup to zero.
Therefore, we can conclude that $d_{min} = 4$. Hence minimum weight $= d_{min} = 4$.

→ 3M.

$$I(X;Y) = H(X) - H(X/Y)$$

$$I(X,Y) = \bar{p}\,H(X)$$

$\therefore$ The channel capacity of a BEC is given by

$$C = \mathrm{Max}\left[I(X,Y)\right]$$

$$= \max\left[\bar{p}\left(H(X)\right)\right]$$

$$= \bar{p}\,\max\left[H(X)\right]$$

$$C = \bar{p}\log 2 = \bar{p}$$ → 2M

Module - 4

7 a). For a systematic (7,4) linear block code, the parity matrix P is given by. $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

i) Find all possible code vectors

ii) Draw the encoding circuit

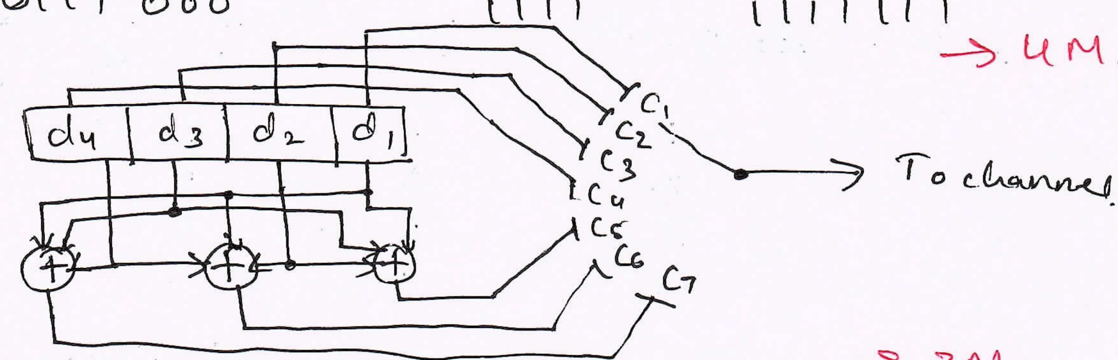
iii) Draw the syndrome calculation circuit. → 10M

Ans i) $G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$ $\quad [C] = [D][G]$

Message	Codevector	Message	Codevector
0000	0000000	1000	1000111
0001	0001011	1001	1001100
0010	0010101	1010	1010 010
0011	0011 110	1011	1011 001
0100	0100 110	1100	1100 001
0101	0101 101	1101	1101 010
0110	0110 011	1110	1110 100
0111	0111 000	1111	1111111

→ 4M

(ii)



→ 3M

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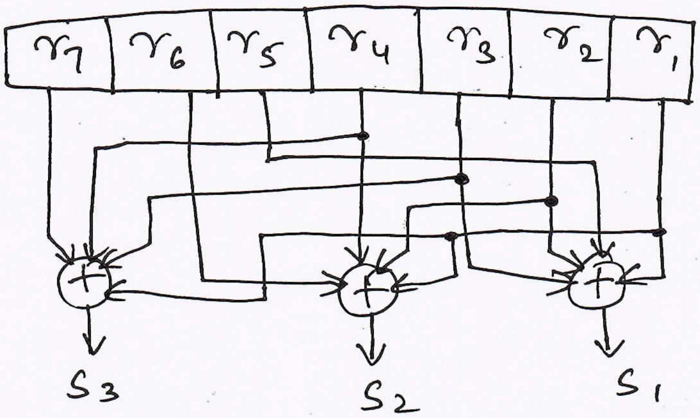
$$(C) = [D][G] = [d_1, d_2, d_3, d_4, (d_1+d_2+d_3), (d_1+d_2+d_3), (d_1+d_3+d_4)]$$

(iii) $S = RH^T = (r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S = [(r_1+r_2+r_3+r_5), (r_1+r_2+r_4+r_6), (r_1+r_3+r_4+r_7)]$$

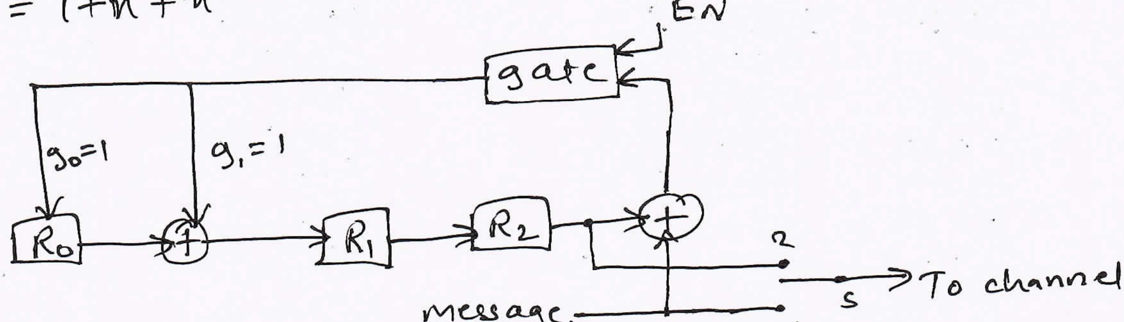


→ 3M.

7.b) Design an encoder for a (7,4) binary cyclic code generated by $g(x) = 1+x+x^3$ and verify its operation using the message vectors (1001) & (1011) → (10M)

Ans

$$g(x) = 1+x+x^3$$



→ 3M.

(i) 1001

No. of shifts	Input		R_0	R_1	R_2
			0	0	0
1	1	—	1	1	0
2	0	—	0	1	1
3	0	—	1	1	1
4	1	—	0	1	1

(shift registers)

→ 3½M

The code vector is 0111001

(19)

iii)

Single error pattern		Syndrome
10000000	–	1110
01000000	–	1101
00100000	–	0111
00010000	–	1011
00001000	–	1000
00000100	–	0100
00000010	–	0010
00000001	–	0001

The eight syndromes listed for eight single error patterns are all distinct. These syndromes are unique to single-error patterns. Hence the code can detect and correct single errors.

The syndrome for double-error patterns are not unique For example: the double error patterns (11000000) and (00011000) have the same syndrome 0011. Hence the code can detect the double errors but cannot correct them.

→ 3M.

8. c) Design a linear block code with minimum distance $d_{min} = 3$ and message length of 4 bits. → 5M

Ans. $k = 4$ code length $n \leq 2^{n-k} - 1$

$$n \leq 2^{n-4} - 1$$

By trial and error $n = 7$ $\therefore (n, k) = (7, 4)$

$$H^T = \begin{bmatrix} [P] \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [P] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow 1M$$

$$H^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \rightarrow 2M$$

$$[G] = [I_k \vdots P] = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

$$[C] = [D][G]$$

Message	Code	Message	Code
0000	0000000	1000	1000011
0001	0001111	1001	1001100
0010	0010110	1010	1010101
0011	0011001	1011	1011010
0100	0100101	1100	1100110
0101	0101010	1101	1101001
0110	0110011	1110	1110000
0111	0111100	1111	1111111

→ 3M.

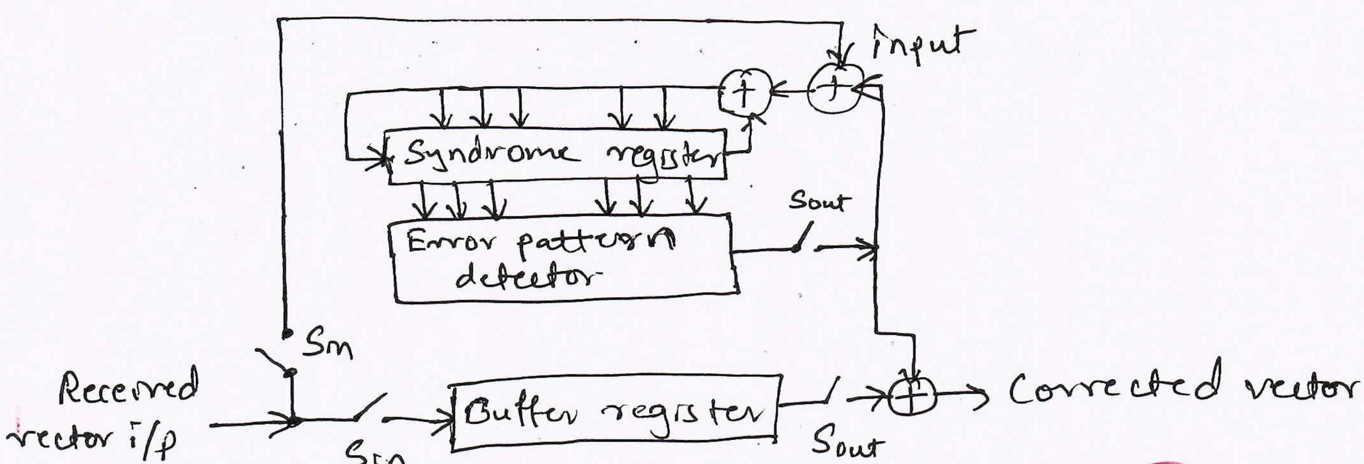
Module - 5

9. a) With a neat block diagram, draw a general decoding circuit for a linear block code. Also draw the complete error correcting circuit for a (7, 4) linear block code if the error bits are given in terms of the syndrome bits as given in equation below.

$$S = [S_1\ S_2\ S_3] = [(r_1 + r_2 + r_3 + r_5),\ (r_1 + r_2 + r_4 + r_6),\ (r_1 + r_3 + r_4 + r_7)]$$

→ 6M

Ans



→ 6M

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9.b) Consider a (7,4) cyclic code with $g(x) = 1+x+x^3$. obtain the code polynomial in non systematic and systematic form for the input sequence. → 10M.

i) 1010 ii) 1100

Ans Non systematic form

i) 1010

$$V(x) = D(x)\,g(x) = (1+x^2)\,g(x)$$

$$V(x) = 1110010$$ → $2\frac{1}{2}$M

ii) 1100

$$V(x) = D(x)\,g(x) = (1+x)(1+x+x^3)$$

$$V = 1011100$$ → $2\frac{1}{2}$M.

Systematic form

i) 1010

$$R(x) = \frac{x^{n-k}D(x)}{g(x)} + Q(x)$$

$$[V] = [0011010]$$ → $2\frac{1}{2}$M

ii) 1100 $(V) = [1011100]$ → $2\frac{1}{2}$M.

9.c) write a short note on BCH codes. → 4M.

Ans BCH codes form a large class of powerful random error-correcting codes.

For any positive integers $m (\geq 3)$ and $t (< 2^{m-1})$, there exists a binary BCH code with the following parameters → 2M.

Block length : $n = 2^m - 1$

no. of parity-check digits : $n-k \leq mt$

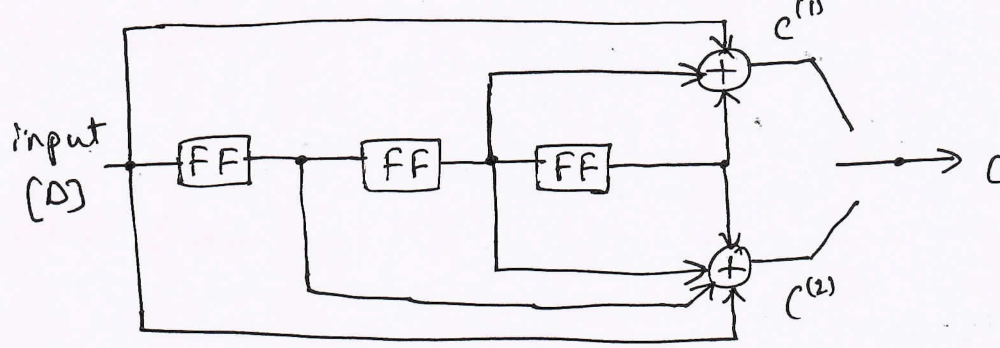
minimum distance : $d_{min} \geq 2t+1$

This code is capable of correcting 't' or fewer errors in a block of 'n' digits. This code is hence called "t-error correcting BCH code". → 2M or

10 a) For a (2, 1, 3) convolutional encoder with $g^{(1)} = 1011$ and $g^{(2)} = 1111$. Find the output sequence using the following two approaches. i) Time domain approach ii) Transform domain approach. Also draw the encoder diagram. → (10M)

Ans

(i)



→ 2M.

Taking $D = 10111$ $\qquad n(L+M) = 2(5+3) = 16$

$$G = \begin{bmatrix} 11 & 01 & 11 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 01 & 11 & 11 \end{bmatrix}$$

$C = DG$

$C = [11, 01, 00, 01, 01, 01, 00, 11]$ → 4M

(ii) $g^{(1)}(x) = 1 + x^2 + x^3 \qquad g^{(2)}(x) = 1 + x + x^2 + x^3$

$c^{(1)}(x) = d(x)\, g^{(1)}(x)$

$c^{(1)}(x) = 1 + x^7$

$c^{(2)}(x) = d(x)\, g^{(2)}(x)$

$c^{(2)}(x) = 1 + x + x^3 + x^4 + x^5 + x^7$

$c(x) = c^{(1)}(x^2) + x\, c^{(2)}(x^2)$

$c(x) = 1 + x + x^3 + x^7 + x^9 + x^{11} + x^{14} + x^{15}$

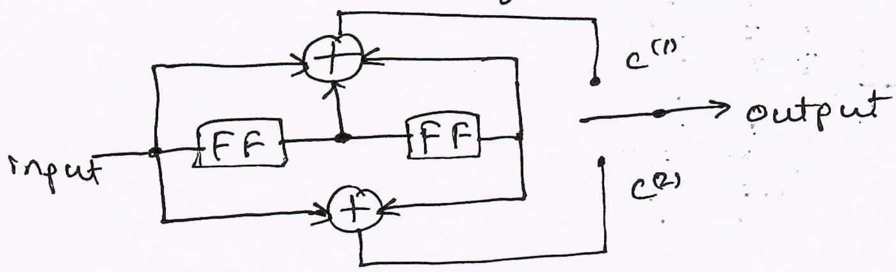
$C = [11, 01, 00, 01, 01, 01, 00, 11]$

→ 4M.

(20)

10 b) For a (2, 1, 2) convolutional encoder with $g^{(1)} = [1\,1\,1]$, $g^{(2)} = [1\,0\,1]$ i) Draw transition table ii) state diagram iii) Draw code tree iv) Using code tree, find the encoded sequence for the message 10111 v) Draw Trellis diagram

Ans → 10M



state table

State	S_0	S_1	S_2	S_3
Binary Description	00	10	01	11

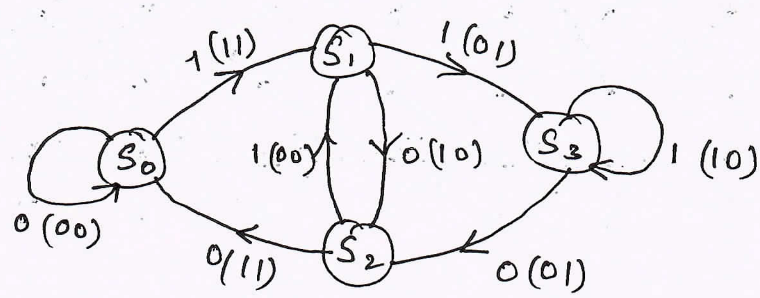
→ 2M

state transition table

Present State	Binary Description	Input	Next State	Binary Description	d_l	d_{l-1}	d_{l-2}	Output $c^{(1)}$	$c^{(2)}$
S_0	00	0	S_0	00	0	0	0	0	0
		1	S_1	10	1	0	0	1	1
S_1	10	0	S_2	01	0	1	0	1	0
		1	S_3	11	1	1	0	0	1
S_2	01	0	S_0	00	0	0	1	1	1
		1	S_1	10	1	0	1	0	0
S_3	11	0	S_2	01	0	1	1	0	1
		1	S_3	11	1	1	1	1	0

→ 2M

State diagram

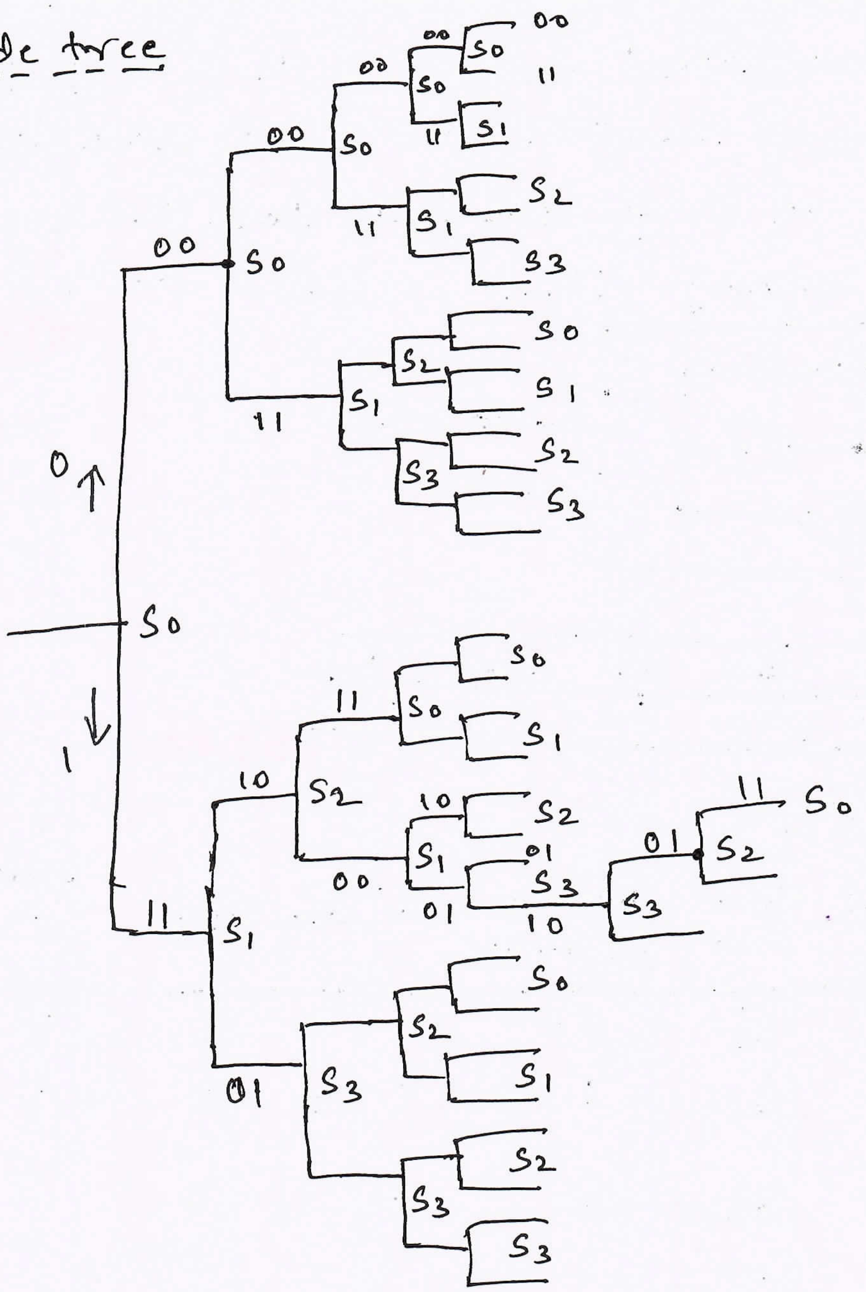


→ 2M

(24)

(11)

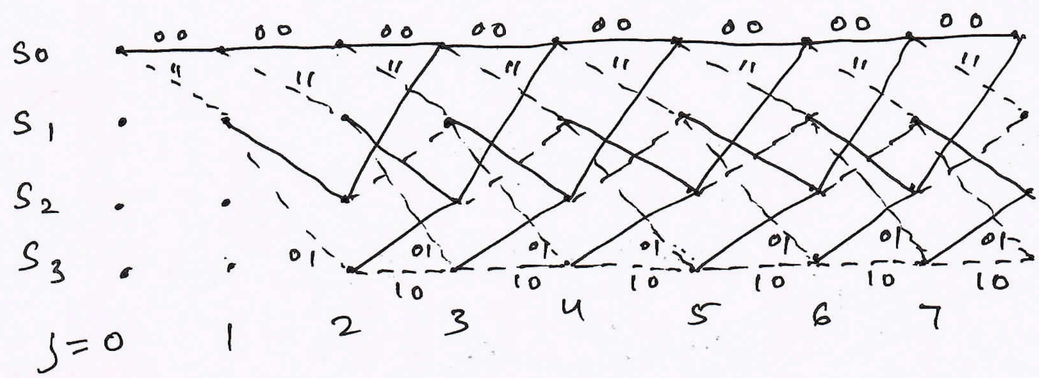
Code tree



IV) Encoded sequence => $S_0 \to S_1 \to S_2 \to S_1 \to S_3 \to S_3 \to S_2 \to S_0$

[11, 10, 00, 01, 10, 01, 11] → 2M

V)



→ 2M.

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