CBCS SCHEME

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18EC55

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022
Electromagnetic Waves

Time: 3 hrs.

Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. State and explain Coulomb's Law. Also express in Vector form. **(06 Marks)**

b. Derive the expression for electric field intensity due to infinite line charges. **(08 Marks)**

c. Find the electric field at a point P(2, 15, 13)m due to the uniform line charge density ρ_L = 25nc. Given that a perpendicular to drawn from A meets the line charge at a point B(3, 0, 4)m. **(06 Marks)**

OR

2 a. A charge $Q_2 = 121 \times 10^{-9}$ C is located in free space at P_2(-0.03, 0.01, 0.04)m. Find the force on Q_2 due to Q_1 where $Q_1 = 110 \times 10^{-6}$C at P_1 (0.03, 0.08, -0.02)m. **(06 Marks)**

b. Define Electric Field Intensity. Derive the expression for Electric field at a point due to may charges. **(08 Marks)**

c. Derive the expression for field due to continuous volume charge distribution. **(06 Marks)**

Module-2

3 a. State and explain Gauss Law. **(06 Marks)**

b. Evaluate both sides of divergence theorem for the field $D = 2xy\overline{ax} + x^2\overline{ay}$ c/m^2 and the rectangular parallel piped formed by the planes x = 0 and y = 1, y = 0 and y = 2, z = 0 and 3. **(10 Marks)**

c. Show that electric field intensity is negative potential gradient. **(04 Marks)**

OR

4 a. Obtain the expression for the work done in moving a point charge in an electric field. **(06 Marks)**

b. Derive the expression for equation of continuity. **(08 Marks)**

c. Give $V = 2x^2y - 5z$ at point P(-4, 3, 6). Find the potential, electric field intensity and volume charge density. **(06 Marks)**

Module-3

5 a. Solve the Laplace's equation to find the potential field in the homogeneous region between the two concentric conducting sphere with radii a and b such that b > a. If potential V = 0 at r = b and $V = V_o$ at r = a. Also find Electric field intensity. **(10 Marks)**

b. If the magnetic field intensity in a region is H = (3y – 2)az + 2 ×ay. Find the current density at the origin. **(04 Marks)**

c. State and explain Biot – Savart's law. **(06 Marks)**

OR

6 a. State and prove Uniqueness theory. **(08 Marks)**

b. Determine whether or not the following potential fields satisfy the Laplace's equation.
i) $V = x^2 - y^2 + z^2$ ii) $V = r\cos\phi + z$. **(08 Marks)**

c. Explain the concepts of Scalar Potential. **(04 Marks)**

Module-4

7 a. Derive an expression for force between differential current elements. **(06 Marks)**

b. Obtain the boundary conditions at the interface between two magnetic materials. **(10 Marks)**

c. Find the magnetization in a magnetic material, where

i) $\mu = 1.8 \times 10^{-5}$ H/m and H = 120 A/m ii) B = 300μT and suspectibility = 15. **(04 Marks)**

OR

8 a. State and explain Faraday's law of Electromagnetic Induction. Show its equation in differential form and integral form. **(10 Marks)**

b. A point charge Q = 18nc has a velocity of 5 × 106 m/s in the direction $a_v = 0.6\overline{a_x} + 0.75\overline{a_y} + 0.3\overline{a_z}$. Calculate the magnitude of force exerted on the charge by the field i) $\overline{E} = -3\overline{a_x} + 4\overline{a_y} + 6\overline{a_z}$ Kv/m ii) $\overline{B} = -3a_x + 4\overline{a_y} + 6\overline{a_z}$ MT

iii) $\overline{B}$ and $\overline{E}$ acting together. **(06 Marks)**

c. A conductor of length 4m long lies along the Y – axis with a current of 10 Amp in the $\overline{a_y}$ direction. Find the force on the conductor if the field in the region is B = 0.005 $\overline{a_x}$ tesla. **(04 Marks)**

Module-5

9 a. What is meant by Uniform Plane Wave? Derive the expression for Uniform Plane Wave in the free space. **(10 Marks)**

b. Let $\mu = 10^{-5}$H/m, $\epsilon = 4 \times 10^{-9}$F/m, $\sigma = 0$ and $\rho_v = 0$. Determine 'K' so that each of the following pair of fields satisfies Maxwell's equation :

i) $\vec{D} = 2x\hat{a}_x - 3y\hat{a}_y + 4z\hat{a}_z \, nC/m^2$, $\vec{H} = Kx\hat{a}_x + 10y\hat{a}_y - 25z\hat{a}_z \, A/m$

ii) $\vec{E} = (20y - kt)\hat{a}_x \, V/m$, $\vec{H} = (y + 2\times10^6 t)\hat{a}_z \, A/m$. **(10 Marks)**

OR

10 a. State and explain Poynting's theorem. **(10 Marks)**

b. Discuss Wave propagation in good conducting medium. **(06 Marks)**

c. Find the frequency at which conduction current density and displacement current density are equal in a medium with $\sigma = 2 \times 10^{-4}$ ℧/m and $\varepsilon_r = 81$. **(04 Marks)**

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Subject: Electromagnetic Waves

Subject code: 18EC55

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31.03.2022

Electromagnetic waves - 18E55

Solution & Scheme of QP Feb/Mar-2022

Module-1

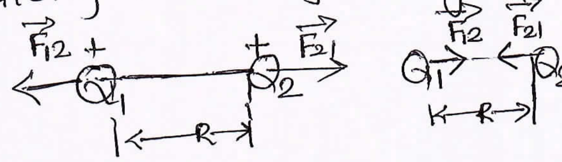
Q1a. State and Explain Coulomb's Law. Also express in vector form.

Coulomb's Law states that the force between two point-like objects with charge Q_1 & Q_2, seperated by a distance 'R' in vaccum or free space is

(i) Directly proportional to the product of magnitude of charge on each

(ii) Inversly proportional to the square of the distance between them.

(iii) The force acts along the line joining them. —02—



Mathematically

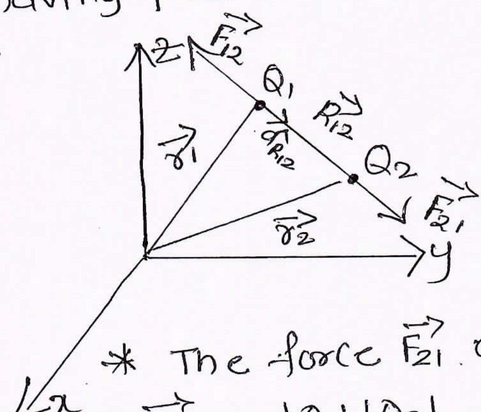
$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{|Q_1||Q_2|}{R^2}}$$

'ϵ_0' is permittivity of free space, $\epsilon_0 = 8.854\times10^{-12}$ F/m —01—

$k = \frac{1}{4\pi\epsilon_0} \cong 9\times10^{9}$ m/F

Vector form: let Q_1 & Q_2 are located at points having position vectors $\vec{r}_1$ & $\vec{r}_2$ respectively

$* \; \hat{a}_{R_{12}} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2-\vec{r}_1|}$



$* \; \vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

$* \; R_{12} = |\vec{r}_2 - \vec{r}_1|$

$* \; \hat{a}_{R_{12}}$ = unit vector along $\vec{R}_{12}$

* The force $\vec{F}_{21}$ on Q_2 due to Q_1 is given by

$$\vec{F}_{21} = \frac{|Q_1||Q_2|}{4\pi\epsilon_0 R_{12}^2}\hat{a}_{R_{12}}$$

$$= \frac{|Q_1||Q_2|}{4\pi\epsilon_0|\vec{r}_2-\vec{r}_1|^2}\cdot\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|}$$

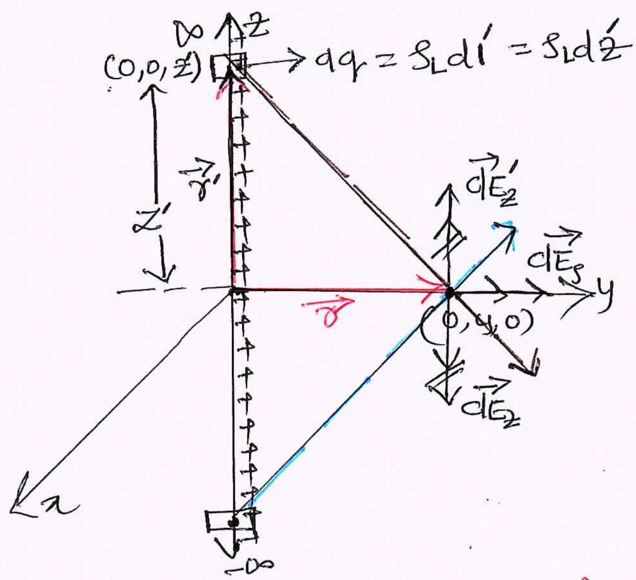
$$\boxed{\vec{F}_{21} = \frac{|Q_1||Q_2|(\vec{r}_2-\vec{r}_1)}{4\pi\epsilon_0|\vec{r}_2-\vec{r}_1|^3}}$$ —02—

* lly $\vec{F}_{12} = -\vec{F}_{21}$

According to Newton third Law. —01—

Q1b. Derive the Expression for electric field intensity due to infinite line charges.

* let us consider infinite line charge with uniform line charge density ρ_L C/m along z-axis in cylindrical coordinate system.



* let us consider symmetry of the problem
* No field component vary with ϕ & z', due to azimuthal & axial symmetry
* Field vary only with ρ'
* NO Element of charge produce ϕ component of field $E_\phi = 0$
* Each element produces component of field ~~Produce~~ E_ρ & E_ϕ component but contribution of E_z by symmetrically located charges will cancel.

* By symmetry argument

$$\boxed{\vec{E} = E_\rho(\rho)\hat{a}_\rho}$$ —02—

* let us choose point P(0,y,0) on the y-axis. This is perfect general point in view of the lack of variation of the field with ϕ & 'z'

* The differential Electrical field due to dq at 'P' is

$$d\vec{E} = \frac{\rho_L dz'}{4\pi\epsilon_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \quad ; \quad \vec{r} = y\hat{a}_y = \rho\hat{a}_y = \rho\hat{a}_\rho \quad ; \quad \vec{r}' = z'\hat{a}_z \; ; \; \vec{r}-\vec{r}' = \rho\hat{a}_\rho - z'\hat{a}_z$$ —02—

* $d\vec{E} = \dfrac{\rho_L dz'}{4\pi\epsilon_0} \dfrac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{(\rho^2+z'^2)^{3/2}}$ | lly $d\vec{E}' = \dfrac{\rho_L dz'}{4\pi\epsilon_0} \dfrac{(\rho\hat{a}_\rho + z'\hat{a}_z)}{(\rho^2+z'^2)^{3/2}}$

* By symmetry 'z' components will cancel, ρ components will add up

$$d\vec{E} = \frac{\rho_L dz' \, \rho\hat{a}_\rho}{2\pi\epsilon_0(\rho^2+z'^2)^{3/2}} \quad \text{—02—} \quad \vec{E} = \int_0^\infty \frac{\rho_L dz' \, \rho}{2\pi\epsilon_0(\rho^2+z'^2)^{3/2}} \hat{a}_\rho$$

let substitute $z' = \rho\tan\theta$, $dz' = \rho\sec^2\theta \, d\theta$, ; $z=0 \Rightarrow \theta = 0$
$z = \infty \Rightarrow \theta = \pi/2$

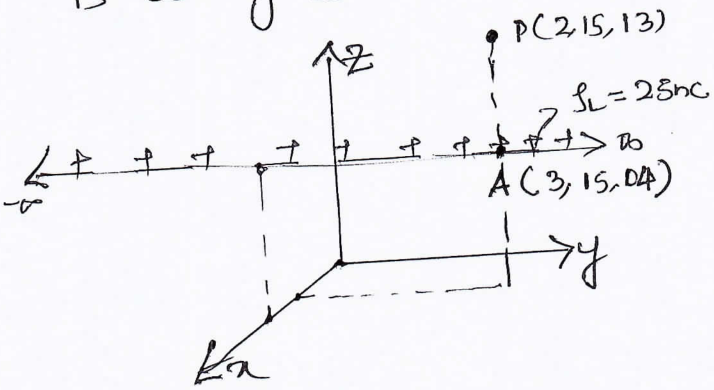
$$\vec{E} = \frac{\rho_L \hat{a}_\rho}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{\rho^2\sec^2\theta \, d\theta}{(\rho^2(1+\tan^2\theta))^{3/2}} \quad ; \quad \vec{E} = \frac{\rho_L \hat{a}_\rho}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{1}{\rho\sec\theta} d\theta$$

$$= \frac{\rho_L \hat{a}_\rho}{2\pi\epsilon_0 \rho} \left[\sin\theta\right]_0^{\pi/2}$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho}$$ —02—

Q1C. Find the electric field at a point P(2, 15, 13) m due to uniform line charge density $\rho_L = 25nC$. Given that Perpendicular to drawn from A meets the line charge at a point B(3,0,4)

* In given question point A is missing
* let us assume point A to be (2, 15, 13)
* It is given that distance from A→B is perpendicular distance
* from derivation of E-field due to infinite line charge we know that, there is no field component along the axis of line charge. From given parameters (data) it is along y-axis.



—02—

* E-field due to line charge is $\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho}$ —01—
* 'ρ' → perpendicular distance from line charge to given point, $\hat{a_\rho}$ → unit vector along 'ρ'
* $\vec{\rho} = \vec{R} = \langle 2, 15, 13\rangle - \langle 3, 15, 04\rangle = \langle -1, 0, 9\rangle = -\hat{a_x} + 9\hat{a_z}$

$\rho = \sqrt{1+81} = \sqrt{82}$, | $\hat{a_\rho} = \frac{-\hat{a_x} + 9\hat{a_z}}{\sqrt{82}}$ | $\hat{a_\rho} = -0.11\hat{a_x} + 0.993\hat{a_z}$

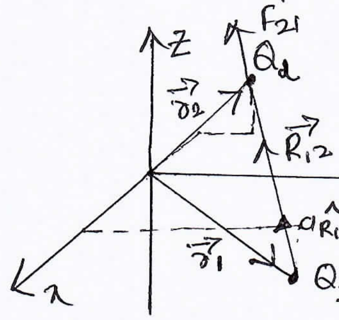
$\boxed{\rho = 9.055}$ —02—

* $\vec{E} = \frac{25\times10^{-9}}{\sqrt{82}\times2\pi\times8.854\times10^{-12}}\left[-0.11\hat{a_x} + 0.993\hat{a_z}\right]$

$\vec{E} = \frac{25\times10^{-9}}{2\pi\times8.854\times10^{-12}\times\sqrt{82}}\left[-0.11\hat{a_x} + 0.993\hat{a_z}\right]$

$\boxed{\vec{E} = -5.48\hat{a_x} + 49.3\hat{a_z}}$ | $\boxed{|\vec{E}| = 49.62\ V/m}$ —01—

Q2a. A charge $Q_2 = 121\times10^{-9}$ C is located in free space at $P(-0.03, 0.01, 0.04)$ m. Find the force on Q_2 due to Q_1 where $Q_1 = 110\times10^{-6}$ C at $P_1(0.03, 0.08, -0.02)$ m.



* $\vec{r}_2 = \langle -0.03, 0.01, 0.04\rangle$

$\vec{r}_1 = \langle 0.03, 0.008, -0.202\rangle$

$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

$= \langle -0.06, -0.07, 0.06\rangle$ -03-

$R_{12} = \sqrt{(0.06)^2 + (0.07)^2 + (0.06)^2}$

$= 0.11$

$$\vec{F}_{21} = \frac{Q_1 Q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^3} \quad \text{-01-}$$

$$= \frac{121\times110\times10^{-15}\times9\times10^{9}}{(0.11)^3}(-0.06\hat{a}_x - 0.07\hat{a}_y + 0.06\hat{a}_z)$$

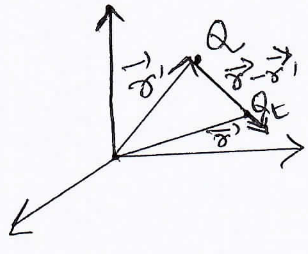
$$= 9\times10^{7}\times10^{-6}(-0.06\hat{a}_x - 0.07\hat{a}_y + 0.06\hat{a}_z)$$

$$\boxed{\vec{F}_{21} = -5.4\hat{a}_x - 6.3\hat{a}_y + 5.4\hat{a}_z}$$

$|\vec{F}_{21}| = \sqrt{(5.4)^2 + (6.3)^2 + (5.4)^2} = 9.9\text{N}$; $\boxed{|\vec{F}_{21}| = 9.9\text{ N}}$ -02-

Q2b. Define Electric field intensity. Derive the Expression for electric field at a point due to many charges.

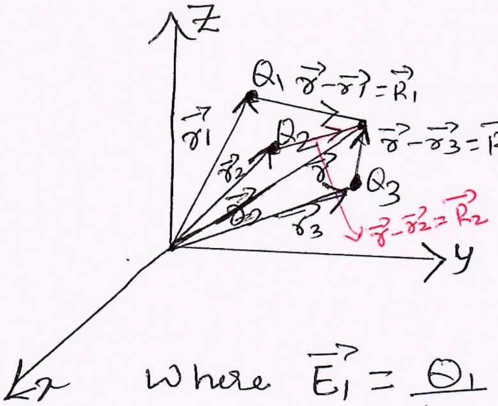
* The Electric field at any point is force experienced by unit postive test charge at that point.



$$\vec{E} = \frac{\vec{F}_t}{Q_t} \quad \text{N/c or V/m}$$

$$\boxed{\vec{E} = \frac{Q\,(\vec{r}-\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3}} \quad \text{-02-}$$

* Electric field due to 'N' point charge
* Principle of superposition: The Principle of superposition states that if there are 'N' charges $Q_1, Q_2, \ldots Q_N$ located respectively at points with position vector $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N$ the resultant Electric field at point $\vec{r}$ is vector sum of electric field due to 'N' individual charges acting alone. -02-



let

$\vec{R_1}=\vec{r}-\vec{r_1}$; $\vec{R_2}=\vec{r}-\vec{r_2}$

$\vec{R_3}=\vec{r}-\vec{r_3}$

* According to principle of superposition

$$\vec{E}_{net}=\vec{E_1}+\vec{E_2}+\vec{E_3}$$

-02-

where $\vec{E_1}=\dfrac{Q_1}{4\pi\epsilon_0}\dfrac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|^3}$; $\vec{E_2}=\dfrac{Q_2}{4\pi\epsilon_0}\dfrac{\vec{r}-\vec{r_2}}{|\vec{r}-\vec{r_2}|^3}$

$$\vec{E_3}=\frac{Q_3}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r_3}}{|\vec{r}-\vec{r_3}|^3}$$

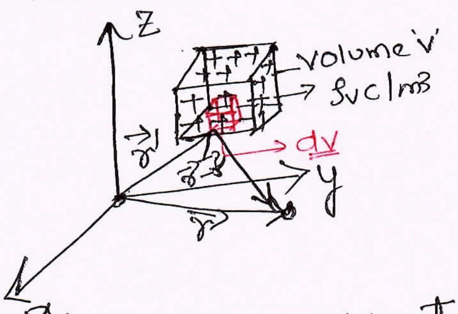
$$\therefore \vec{E}_{net}=\frac{Q_1}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|^3}+\frac{Q_2}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r_2}}{|\vec{r}-\vec{r_2}|^3}+\frac{Q_3}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r_3}}{|\vec{r}-\vec{r_3}|^3}$$

In general $\boxed{\vec{E}=\sum_{K=1}^{N}\frac{Q_K}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r_K}}{|\vec{r}-\vec{r_K}|^3}}$

-02-

Q2c. Derive the Expression for field due to continous volume charge distribution.

* let us consider volume charge with $\rho_v\, C/m^3$



let us consider differential volume 'dv' the charge within dv is differential charge dq

$$\therefore \boxed{dq=\rho_v dv}$$

* we know that differential electrical field due to differential charge is $d\vec{E}=\dfrac{dq}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')$

$$* \quad d\vec{E}=\frac{\rho_v dv\,(\vec{r}-\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3}$$

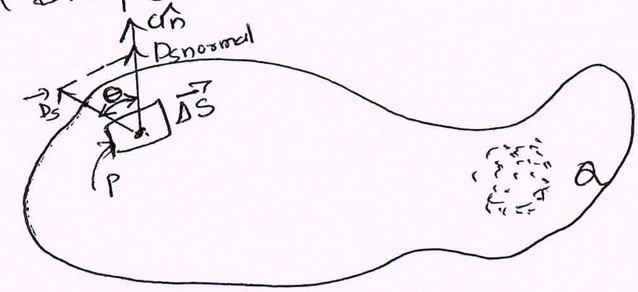
* The total electric field due to continous volume charge distribution is $\boxed{\vec{E}=\iiint_V \frac{\rho_v dv}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$

Module-2

Q3a. State and Explain Gauss Law

Gauss's Law : The Electric flux passing through any closed surface is equal to the total charge enclosed by that surface. —02—

* let us imagine a distribution of charge as a cloud of point charges surrounded by a closed surface of a shape as shown in below fig.



* At point 'p' consider incremental Element of surface ΔS & let $\vec{D_s}$ makes an angle 'θ' with $\vec{\Delta S}$, as shown in fig. The flux crossing ΔS is the product of the normal of $\vec{D_s}$ and area ΔS

$$\Delta S = D_{s,normal}\,\Delta S = D_s \cos\theta\,\Delta S = \vec{D_s}\cdot\vec{\Delta S}$$

* $\Delta\psi$ = flux crossing ΔS = $D_{s,normal}\,\Delta S = D_s\cos\theta\,\Delta S = \vec{D_s}\cdot\vec{\Delta S}$

* The total flux passing through the closed surface is obtained by adding the differential contributions crossing each surface element $\vec{\Delta S}$ —02—

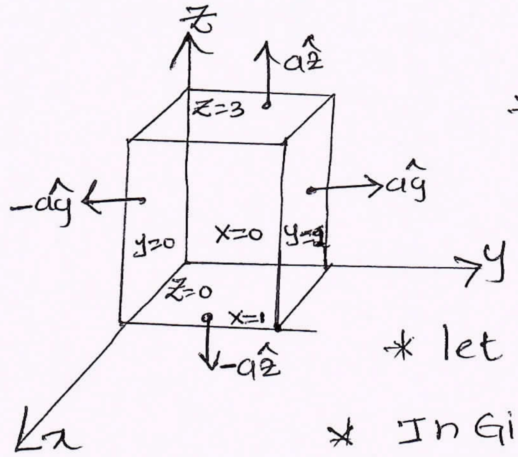
$$\psi = \int d\psi = \oint \vec{D_s}\cdot\vec{ds}$$

* Mathematical form of Gauss's Law —01—

$$\boxed{\psi = \oint \vec{D_s}\cdot\vec{ds} = Q_{enc}}$$

Q3b. Evaluate both sides of divergence theorem for the field $\vec{D} = 2xy\,a\hat{x} + x^2 a\hat{y}\ c/m^2$ and rectangular parallel piped formed by the planes $x=0$ & $x=1$, $y=0$ and $y=2$, $z=0$ and $z=3$.

* 7



* Divergence theorem

$$\oint_S \vec{D}\cdot\vec{ds} = \int_V \nabla\cdot\vec{D}\, dV$$

* let us evaluate surface integral

* In Given $\vec{D}$, the z-component is zero $\vec{D}\cdot\vec{ds}$ on top & bottom surface is zero.

* Surface integral for remaning four surface -02-

$$\int_{front}\vec{D}\cdot\vec{ds} = \int(\vec{D})_{x=1}\cdot dy\,dz\,\hat{a_x} = \int_0^3\int_0^2(\vec{D})_{x=1}\cdot dy\,dz\,\hat{a_x}$$

$$= \int_0^3\int_0^2 (D_x)_{x=1}\, dy\,dz = \int_0^3\int_0^2 2y\,dy\,dz = \int_0^3 4\,dz$$

$$\boxed{\int_{front}\vec{D}\cdot\vec{ds} = \int_0^3 4\,dz = 12\,C}$$

$$\int_{back}\vec{D}\cdot\vec{ds} = \int(\vec{D})_{x=0}\cdot dy\,dz\,(-\hat{a_x}) = \int(-D_x)_{x=0}\,dy\,dz$$

$$\boxed{\int_{back}\vec{D}\cdot\vec{ds} = 0} \quad \left[\,D_x|_{x=0} = 2xy|_{x=0} = 0\,\right]$$

$$\int_{left}\vec{D}\cdot\vec{ds} = \int(\vec{D})_{y=0}\cdot dx\,dz\,(-\hat{a_y}) = \int -(D_y)_{y=0}\,dx\,dz$$

$$= -\int_0^3\int_0^1 x^2\,dx\,dz = -\int_0^3\left[\frac{x^3}{3}\right]_0^1 dz = -3\times\frac{1}{3} = \boxed{-1 = \int_{left}\vec{D}\cdot\vec{ds}}$$

$$\int_{right}\vec{D}\cdot\vec{ds} = \int(\vec{D}\cdot)_{y=2}\cdot dx\,dz\,(\hat{a_y}) = \int(D_y)_{y=2}\,dx\,dz$$

$$= \int_0^3\int_0^1 x^2\,dx\,dz = \int_0^3\left(\frac{x^3}{3}\right)_0^1 dz = 3\times\frac{1}{3} = \boxed{1 = \int_{right}\vec{D}\cdot\vec{ds}}$$

$$\oint\vec{D}\cdot\vec{ds} = 12+0-1+1$$

$$\boxed{\oint\vec{D}\cdot\vec{ds} = 12\,C}$$

-04-

* $\nabla\cdot\vec{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$

$\frac{\partial}{\partial x}D_x = \frac{\partial}{\partial x}(2xy) = 2y$ | $\frac{\partial}{\partial y}(D_y) = \frac{\partial}{\partial y}(x^2) = 0$

$\frac{\partial}{\partial z}(D_z) = 0$ | $\boxed{\nabla\cdot\vec{D} = 2y}$ —02—

$\iiint \nabla\cdot\vec{D}\,dv = 12C$

$\int_V \nabla\cdot\vec{D}\,dv = \int_0^3\int_0^2\int_0^1 \nabla\cdot\vec{D}\,dv$

$= \int_0^3\int_0^2\int_0^1 2y\,dx\,dy\,dz$

$= \int_0^3\int_0^2 2y\,dy\,dz$

$= \int_0^3 \left[\frac{2y^2}{2}\right]_0^2$

$\boxed{\int_V \nabla\cdot\vec{D}\,dv = \int_0^3 4\,dz = 12}$ —02—

$\boxed{\oint \vec{D}\cdot d\vec{s} = \iiint \nabla\cdot\vec{D}\,dV = 12C}$

Divergence theorem verified.

Q3C. Show that electric field intensity is negative potential gradient.

* We know that $V = -\int \vec{E}\cdot d\vec{l}$

* $\Delta V \doteq -\vec{E}\cdot\Delta\vec{L}$ | or $\boxed{\frac{dV}{dL} = -E\cos\theta}$

* $\Delta V \cong -E\cos\theta\,\Delta L$

* $\boxed{-\frac{dV}{dL}\Big|_{max}\hat{a}_N = \vec{E}}$

Let $\hat{a}_N$ be unit vector normal to equipotential surface directed towards higher potentials

* Since $\frac{dV}{dL}\Big|_{max}$ occurs when $\Delta\vec{L}$ is in the direction of $\hat{a}_N$

$\therefore \boxed{\vec{E} = -\frac{dV}{dN}\hat{a}_N}$ $\therefore \boxed{\vec{E} = -\text{grad}\,V}$ —02—

* We know that total differentiation is defined as

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz \quad -(1)-$$

* we also know that

$$dv = -\vec{E} \cdot \vec{dl}$$
$$= -E_x dx - E_y dy - E_z dz \quad -(2)-$$

* Eqn (1) & (2) are true for any dx, dy, & dz

$$E_x = -\frac{\partial v}{\partial x} \; ; \; E_y = -\frac{\partial v}{\partial y} \; ; \; E_z = -\frac{\partial v}{\partial z}$$

Combining above result & writting vectorially as

$$\boxed{\vec{E} = -\left[\frac{\partial v}{\partial x} a\hat{x} + \frac{\partial v}{\partial y} a\hat{y} + \frac{\partial v}{\partial z} a\hat{z}\right]}$$

—02—

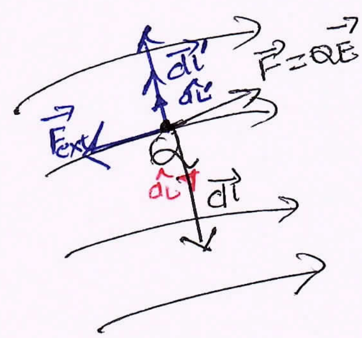
-OR-

Q4a. Obtain the Expression for the workdone in Moving a point Charge in an Electric field.

* let us consider a region of Electric field

* The force arising from E-field is

$$\vec{F_E} = Q\vec{E}$$



* Under this force, the charge moves a differential distance $\vec{dl}$. The incremental Energy expended by the electric field, or simply amount of workdone by E-field is $dW_e = \vec{F} \cdot \vec{dl} = Q\vec{E} \cdot \vec{dl}$

$$\boxed{dW_e = (Q\vec{E} \cdot a\hat{L})dl}$$

* where $Q\vec{E} \cdot a\hat{L}$ is component of force in the direction of $\vec{dl}$.

—02—

* If we attempt to move the charge against the Electric field, we have to exert a force equal & opposite to that exerted by the field & this require us to expend energy or do work.

* The force we must apply

$$F_{Apply} = -Q\,\vec{E}\cdot\hat{a}_L$$

* The Differential work done by External Source moving charge Q is $dW = (-Q\vec{E}\cdot\hat{a}_L)\,dl$

$$\boxed{dW = -Q\vec{E}\cdot\vec{dL}}$$ -03-

* Total work done in moving point charge from initial point to final point is given by

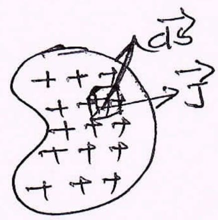
$$W = -\int_{initial}^{final} Q\vec{E}\cdot\vec{dl} \qquad \boxed{W = -Q\int_{initial}^{final} \vec{E}\cdot\vec{dl}}$$ -01-

Q4b. Derive the Expression for equation of continuity

* let us Consider the region bounded by closed surface, if we assume ρ is volume charge density in that region

* The total current crossing the closed surface

$$\boxed{I = \oint \vec{J}\cdot\vec{ds}} \quad \text{--(1)}$$



* The outward flow of postive charge must be balanced by a decrease of postive charge within the closed surface. If the charge inside the closed surface is Q_i, then rate of decrease is $-\frac{dQ_i}{dt}$

* The principle of conservation of charge requires

$$\boxed{I = \oint \vec{J}\cdot\vec{ds} = -\frac{dQ_i}{dt}} \quad \text{--(2)}$$ -03-

* The Eqn (2) is Integral form of continuity Eqn or mathematical Expression for conservation of Charge } —01—

* The point form or differential form of continuity Eqn is obtained by using divergence theorem.

$$\oint_{surface} \vec{J}\cdot\vec{ds} = \int_{vol} \nabla\cdot\vec{J}\,dv \quad --(3)$$ } —01—

But $\oint \vec{J}\cdot\vec{ds} = -\frac{dQ_i}{dt} = -\frac{d}{dt}\int_{vol} \rho_v\,dv = -\int_{vol}\frac{\partial \rho_v}{\partial t}dv$

$\therefore \oint \vec{J}\cdot\vec{ds} = -\int_{vol}\frac{\partial \rho_v}{\partial t}dv = \int_{vol}\nabla\cdot\vec{J}\,dv$

$\therefore \int_{vol}\nabla\cdot\vec{J}\,dv = -\int_{vol}\frac{\partial \rho_v}{\partial t}dv \quad --(4)$ —02—

* The above Eqn (4) is true for any volume, however small, it is true for an incremental volume

$$\nabla\cdot\vec{J}\,\Delta v = -\frac{\partial \rho_v}{\partial t}\Delta v$$

$$\boxed{\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ → Point form of Equation of continuity —01—

Q4 c. Given $V = 2x^2y - 5z$ at point P(-4, 3, 6). Find the potential, electric field intensity & volume charge density

*(a) V at 'p' is $V|_p = 2(-4)^2(3) - 5(6)$

$= 2(16)(3) - 30$

$$\boxed{V_p = 66V}$$ —01—

*(b) $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right]$

$\frac{\partial V}{\partial x} = 4xy$ | $\frac{\partial V}{\partial y} = 2x^2$ | $\frac{\partial V}{\partial z} = -5$

$$\vec{E} = -\left[+4xy\,a\hat{x} + 2x^2 a\hat{y} - 5a\hat{z}\right] \text{V/m}$$

$$\boxed{\vec{E}_p = 48a\hat{x} - 32a\hat{y} + 5a\hat{z} \text{ V/m}}$$

$$E_p = \sqrt{48^2 + (-32)^2 + 5^2} = 57.9 \text{V/m}$$

$$\boxed{E_p = 57.9 \text{V/m}}$$

-02-

* (C) $\vec{D} = \epsilon_0 \vec{E} = \epsilon_0\left[-4xy\,a\hat{x} - 2x^2 a\hat{y} + 5a\hat{z}\right] \text{C/m}^2$

$$\nabla \cdot \vec{D} = \rho_v$$

$$\therefore \rho_v = \nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \epsilon_0\left[-4y - 0 + 0\right]$$

$$\boxed{\rho_v = -\epsilon_0 4y} \quad \}\text{-02-}$$

$$\therefore \boxed{\rho_{vp} = -\epsilon_0 4(3) = -12\epsilon_0}$$

$$\rho_{vp} = -12 \times 8.854 \times 10^{-12}$$

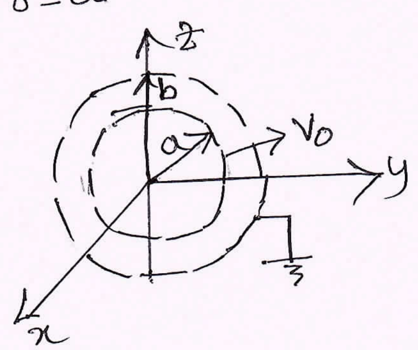
$$\boxed{\rho_{vp} = -106.2p \text{ C/m}^3}$$

-01-

Module-3

Q5a. Solve the Laplace's Equation to find the potential field in the homogeneous region between the two concentric conducting sphere with radii a & b such that b>a. If potential V=0 at r=b and V=V₀ at r=a. Also find Electric field intensity

* let us consider concentric conducting sphere with radius a & b (b>a), with potential $V=0$ @ $r=b$ & $V=V_0$ @ $r=a$



* Step 1 : For Given problem, the boundary Conditions Show that V is function of 'r' alone — 01 -

* Laplace Eqn $\nabla^2 V = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

or $$\frac{d}{dr}\left[r^2\frac{dV}{dr}\right]. \quad [r \neq 0]$$ — 01 —

* Integrating twice, we get

$$r^2\frac{dV}{dr} = A$$

or $$\boxed{\frac{dV}{dr} = \frac{A}{r^2}} \quad -(1)-$$

$$\boxed{V(r) = -\frac{A}{r} + B} \quad (2)$$ —02—

* Step 2 : Apply Boundary conditions

$$V(b) = 0 = -\frac{A}{b} + B$$

$$V(a) = V_0 = -\frac{A}{a} + B$$

$$\therefore -\frac{A}{b} + B = 0$$

$$\therefore \boxed{B = \frac{A}{b}}$$

$$V_0 = -\frac{A}{a} + \frac{A}{b} = A\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$\boxed{V_0 = A\left[\frac{1}{b} - \frac{1}{a}\right]} \quad \text{or} \quad \boxed{A = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]}}$$ —02

Sub'g A & B in Eqn (2)

$$\therefore V(r) = \frac{-V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} \cdot \frac{1}{r} + \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} \cdot \frac{1}{b}$$

$$\boxed{V(r) = \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)}\left[-\frac{1}{r} + \frac{1}{b}\right]}$$ Potential field —01—

* Step 3 $\vec{E} = -\nabla V$

$$= -\left[\frac{dV}{dr}\hat{a_r}\right]$$

$$\vec{E} = -\left[\frac{V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} \cdot \frac{1}{r^2}\hat{a_r}\right]$$

$$\left[\because \frac{dV}{dr} = \frac{A}{r^2} \text{ from Eqn (1)}\right]$$ —02—

$$\boxed{\vec{E} = \frac{V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \cdot \frac{1}{r^2}\hat{a_r}}$$ —01— Electric field

#

Q5b. If the magnetic field intensity in a region is $\vec{H} = (3y-2)\hat{a_z} + 2x\hat{a_y}$. Find the current density at the origin.

* The current density for Given magnetic field is, given by

$$\vec{J} = \nabla \times \vec{H} \quad \}\text{-01-}$$

* $$\vec{J} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 2x & 3y-2 \end{vmatrix} = \hat{a_x}\left(\frac{\partial}{\partial y}(3y-2) - \frac{\partial 2x}{\partial z}\right) - \hat{a_y}\left(\frac{\partial}{\partial x}(3y-2)\right.$$
$$\hat{a_z}\left(\frac{\partial}{\partial x}(2x)\right) \quad \}\text{-02-}$$

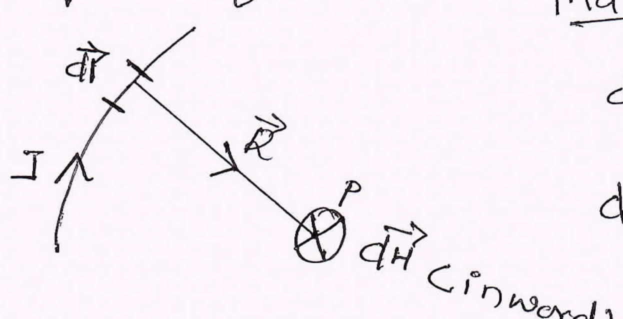
$$\vec{J} = 3\hat{a_x} - \hat{a_y}(0) + \hat{a_z}(2)$$

$$\boxed{\vec{J} = 3\hat{a_x} + 2\hat{a_z}} \qquad \therefore \vec{J}\ @\text{origin} \quad \boxed{\vec{J}_{(0,0,0)} = 3\hat{a_x} + 2\hat{a_z}} \quad \}\text{-01-}$$

Q5c. State and Explain Biot-savart's Law

* Biot-savart's Law States that the differential magnetic field Intensity dH produced at a point 'P', by the differential current element Idl is proportional to the product Idl and the sine of the angle 'α' between the element and the line joining 'P' to the Element and is inversly proportional to the square of the distance R between 'P' & the Element }-02-

Mathematically



$$dH \propto \frac{Idl\sin\alpha}{R^2} \quad \text{--(1)}$$

$$dH = k\frac{Idl\sin\alpha}{R^2} \quad \text{--(2)}$$

* where 'k' is constant of proportionality. In SI units $k = \frac{1}{4\pi}$ ∴ * Eqn ② can be written as

$$\boxed{dH = \frac{Idl\sin\alpha}{4\pi R^2}} \quad \text{--(3)} \quad \}\text{-02-}$$

* from the defn of crossproduct Eqn (3) can be writtenas in vector form as,

$$\vec{dH} = \frac{I\vec{dl} \times a_R^{\wedge}}{4\pi R^2}$$

or $$\boxed{\vec{dH} = \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}}\ A/m$$

—02—

* The direction of $\vec{dH}$ can be determined by the right-hand rule. —01—

—OR—

Q6a. State and Prove Uniqueness theorem

* Uniqueness Theorem : If a solution to Laplace's Equation can be found that satisfies the boundary Conditions, then the Solution is unique. —01—

* The theorem is proved by contradiction. we assume that there are two solutions. V_1 and V_2 of Laplace's equation. both of which satisfy the Prescribed boundary conditions.

$\nabla^2 V_1 = 0$ ---(1) | $V_1 = V_2$ on the boundary

$\nabla^2 V_2 = 0$ ---(2) |

* let us consider their difference

$V_d = V_1 - V_2$ --(3) | $V_d = 0$ .. on the boundary

$\nabla^2 V_d = \nabla^2 V_1 - \nabla^2 V_2$ --(4) |

—02—

* we have vector identity

$$\nabla \cdot (V\vec{D}) \equiv V(\nabla \cdot \vec{D}) + \vec{D} \cdot (\nabla V) \text{ --(5)--}$$

which holds for any scalar V & any vector $\vec{D}$

* for present derivation let us choose V_d is scalar & ∇V_d is vector

* $\nabla\cdot(V_d \nabla V_d) = V_d \underbrace{(\nabla\cdot\nabla V_d)}_{(\nabla^2 V_d)} + \nabla V_d\cdot(\nabla V_d)$ --(6)

But $\nabla^2 V_d = 0$

$\therefore\ \nabla\cdot(V_d\nabla V_d) = \nabla V_d\cdot(\nabla V_d)$ --(7)

-02-

* let us integrate eqn (7) throught volume enclosed by boundary surface

$$\int_V \nabla\cdot(V_d\nabla V_d)\,dV = \int_V \nabla V_d\cdot\nabla V_d\,dV \quad \text{--(8)}$$

* By divergence theorem, the left side of eqn (8) can be written as

$$\int_V \nabla\cdot(V_d\nabla V_d)\,dV = \oint_S (V_d\nabla V_d)\cdot\vec{ds}$$

* we know that $V_d = V_1 - V_2$, the closed surface consist of boundaries on which $V_{db} = V_{1b} - V_{2b} = 0$

$$\oint_S (V_d\nabla V_d)\cdot\vec{ds} = 0 \quad \text{--(9)}$$

$$\therefore\ \int_V \nabla\cdot(V_d\nabla V_d)\,dV = 0 \quad \text{--(10)}$$

from eqn (8) $\int_V (\nabla V_d\cdot\nabla V_d)\,dV = 0$ --(11)

$\vec{A}\cdot\vec{A} = |\vec{A}|^2$

* $\int_V |\nabla V_d|^2 dV =$ --(12)

* The above integration is zero. because, the Integrand is zero every where. $\nabla(V_1 - V_2) = 0$

$|\nabla V_d|^2 = 0$

-03-

* If the gradient of $V_1 - V_2$ is zero everywhere, implies (17)

$V_1 - V_2 = \text{constant}$

* The constant is easily evaluated by considering point on the boundary. Here $V_1 - V_2 = V_{1b} - V_{2b} = 0$ from above we conclude that constant is zero

$\therefore \boxed{V_1 = V_2}$ } -01-

Q6b. Determine whether or not the following potential fields satisfy the Laplaces equation.

(i) $V = x^2 - y^2 + z^2$ (ii) $V = r\cos\phi + z$

* Laplace's eqn in cartesian coordinate is given by

(i) $\nabla^2 V = 0$; $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ -01-

$\frac{\partial^2 V}{\partial x^2} = 2$; $\frac{\partial^2 V}{\partial y^2} = -2$; $\frac{\partial^2 V}{\partial z^2} = 2$ } -01-

$\therefore \nabla^2 V = 2 - 2 + 2 = 2$; $\boxed{\nabla^2 V = 2 \neq 0}$ → doesnot satisfy Laplaces eqn } -01-

(ii) Laplace's eqn in cylindrical coordinate system is given by

Note = $\rho = r$

-01- { $\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$

$\rho\frac{\partial V}{\partial \rho} = \cos\phi\,\rho$; $\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) = \boxed{\cos\phi \cdot \frac{1}{\rho} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right)}$

$\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = -\rho\cos\phi \cdot \frac{1}{\rho^2} = \boxed{\frac{-\cos\phi}{\rho} = \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}}$ -03-

$\frac{\partial^2 V}{\partial z^2} = 0$

$\nabla^2 V = \frac{\cos\phi}{\rho} - \frac{\cos\phi}{\rho} + 0 = 0$

$\boxed{\nabla^2 V = 0}$ --- Satisfy Laplace's eqn } -01-

Q6c. Explain the concepts of Scalar potential.

* Scalar magnetic potential is defined as

$$\boxed{\vec{H} = -\nabla V_m} \quad (\vec{J} = 0) \quad \underline{A'} \text{ (Ampere unit of } V_m)$$

* V_m is scalar magnetic potential
* We can define $\vec{H}$ as the gradient of a scalar magnetic potential, then the current density must be zero throughout the region in which the scalar magnetic potential is so defined

—02—

* V_m is not single-valued function of position
* The reason for Multivaluedness is

$\nabla \times \vec{H} = 0$ (wherever $\vec{J} = 0$)

but, $\oint \vec{H} \cdot \vec{dl} = I$ -- even if $\vec{J} = 0$ along the path of Integration

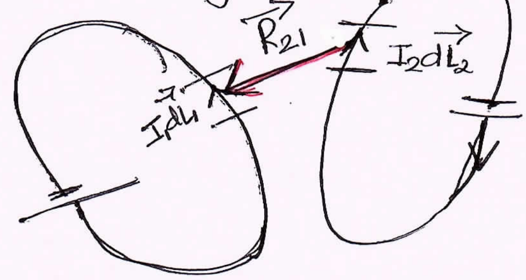
* In general, $V_{m\,a,b} = -\int_b^a \vec{H} \cdot \vec{dl}$ (specified path)

—02—

Module-4

Q7a. Derive an expression for force between differential Current elements.

* Let us consider the force between two Elements $I_1\vec{dl_1}$ and $I_2\vec{dL_2}$. According to Biot-savart's Law, both current elements produce magnetic fields
* let us find the force $d(dF_1)$ on the Element $I_1\vec{dL_1}$ due to the field $\vec{dB_2}$ produced by $I_2\vec{dl_2}$ as shown in below fig



—02—

* $d(d\vec{F_1}) = I_1 d\vec{L_1} \times d\vec{B_2}$

* From Biot-savart's Law

$$d\vec{B_2} = \frac{\mu_0 I_2 d\vec{L_2} \times \hat{a}_{R21}}{4\pi R_{21}^2}$$

$$d(d\vec{F_1}) = \frac{\mu_0 I_1 d\vec{L_1} \times (I_2 d\vec{L_2} \times \hat{a}_{R21})}{4\pi R_{21}^2}$$

* The above Eqn is Law of force between two current elements and is analogous to the coulomb's Law

* $d(d\vec{F_2}) \neq -d(d\vec{F_1})$. This is because of non physical nature of current elements. —02—

* The total force $\vec{F_1}$ on current loop 1 due to current loop 2 is obtained by integrating twice

$$\vec{F_1} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \left[d\vec{L_1} \times \oint_{L_2} \left(\frac{d\vec{L_2} \times \hat{a}_{R21}}{R_{21}^2} \right) \right]$$ —01—

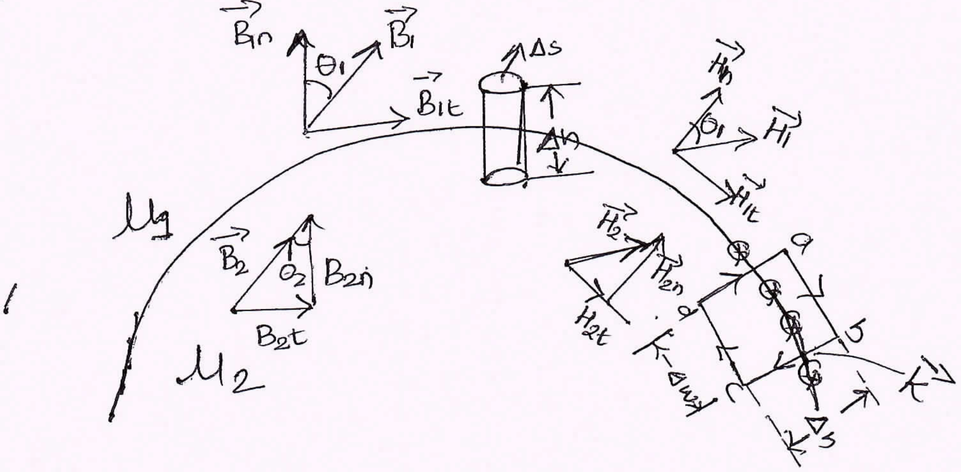
* The force on loop 2 due to magnetic field B_1 from loop 1 is obtained from above eqn by interchanging subscripts 1 & 2

* It can be shown that $\boxed{\vec{F_2} = -\vec{F_1}}$ -- obey Newton third Law —01—

Q7b. Obtain the boundary conditions at the interface between two magnetic materials

* If the field exist in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called Boundary conditions. —01—

* let us Consider Boundary between two linear linear isotropic Media 1 and 2, Characterized by μ_1 & μ_2 respectively as shown in abelow fig.



}—02—

* Normal components is determined by Applying Gauss's Law to the pill box & allowing $\Delta h \to 0$

$B_{1n}\Delta S - B_{2n}\Delta S = 0$ | $\oint \vec{B}\cdot d\vec{S} = 0$ --- Gauss's Law for magnetic field

$\oint \vec{B}\cdot d\vec{S} = \int_{top} \vec{B_1}\cdot d\vec{S} + \int_{bottom} \vec{B_2}\cdot d\vec{S} + \int_{side} \vec{B}\cdot d\vec{S} = 0$ | $\Delta h \to 0$

$\therefore \oint \vec{B}\cdot d\vec{S} = B_{1n}\Delta S - B_{2n}\Delta S = 0$

$\therefore \boxed{\vec{B_{1n}} = \vec{B_{2n}}}$ | but $\vec{B} = \mu\vec{H}$ | $\therefore \mu_1\vec{H_{1n}} = \mu_2\vec{H_{2n}}$ | $\therefore \boxed{\vec{H_{2n}} = \frac{\mu_1}{\mu_2}\vec{H_{1n}}}$

* from above we can say that the Normal component of $\vec{B}$ is continous @ the Boundary.

* Normal component of $\vec{H}$ is discontinous @ the boundary $\vec{H}$ undergoes some changes at the interface }—03—

* To Determine tangential component. let us apply Ampere's Circuit Law about small closed path abcda

$\oint \vec{H}\cdot d\vec{l} = I$

* let us assume boundary carry surface current $\vec{K}$ whose component normal to the plane of closed path is K.

* $\oint \vec{H}\cdot d\vec{l} = H_{1t}\Delta w + H_{1n}\cancel{\frac{\Delta h}{2}} - H_{1n}\cancel{\frac{\Delta h}{2}} - H_{2n}\cancel{\frac{\Delta h}{2}} - H_{2t}\Delta w + H_{2n}\cancel{\frac{\Delta h}{2}} = K\Delta w$

$(H_{1t} - H_{2t})\Delta w = K\Delta w$ | $\boxed{H_{1t} - H_{2t} = K}$ }—02—

* In vector form

$$\vec{H_{1t}} - \vec{H_{2t}} = \hat{a_{n12}} \times \vec{K}$$

* where $\hat{a_{n12}}$ is unit normal @ the Boundary directed from region 1 to region 2 —02—

Q7C. Find the magnetization in a magnetic material, where (i) $\mu = 1.8 \times 10^{-5}$ H/m & H = 120 A/m

(ii) B = 300 μT & suspectibility = 15

(i) $M = \chi_m H$ | $\chi_m = \mu_r - 1$ | $\mu = \mu_0 \mu_r$

$\mu_r = \dfrac{\mu}{\mu_0}$

$\chi_m = \dfrac{\mu}{\mu_0} - 1$

$\therefore M = \left(\dfrac{1.8 \times 10^{-5}}{4\pi \times 10^{-7}} - 1\right) 120$ | $\boxed{M = 1599 \text{ A/m}}$ —02—

(ii) $\cdot$ $B = 300 \mu T$, $\chi_m = 15$ | $\mu_r = 1 + \chi_m = 16$

$M = \chi_m \vec{H}$ | $\boxed{M = 224 \text{ A/m}}$ —02—

$= \chi_m \dfrac{\vec{B}}{\mu_0 \mu_r}$ | $\vec{B} = \mu \vec{H}$

$M = 15 \dfrac{300 \times 10^{-6}}{4\pi \times 10^{-7} \times 16}$

—OR—

Q8a. State and Explain Farday's Law of Electromagnetic Induction Show its equation in differential form & integral form.

* Faraday's Law :- Farday's Law states that the induced emf, in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit

Mathematically $\boxed{V_{emf} = \dfrac{-d\lambda}{dt} = -N\dfrac{d\phi}{dt}}$ --(1)

—01—

* where $\lambda = N\phi$ is the flux linkage, N is the no of turns in the circuit

* Negative sign shows that the induced Voltage acts in such way that it oppose the flux producing it. This behavior is described by Lenz's Law.

* The Eqn ① can be written in terms of $\vec{E}$ & $\vec{B}$ field

* we know that Emf is defined as

$$Emf = \oint \vec{E} \cdot \vec{dl}$$

$$\therefore \oint \vec{E} \cdot \vec{dl} = \frac{-d}{dt} \int_s \vec{B} \cdot \vec{ds} \quad --(2)$$

-02-

$\therefore$ As integration over space & differentiation with time are independent operations

$$\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}}$$ → faraday's Law integral form

-02-

* Applying stoke's theorem for above left side of Eqn

$$\int_s \nabla \times \vec{E} \cdot \vec{ds} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

Two integrals to be equall, their integrands must be equall

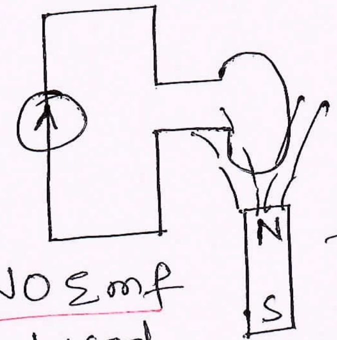
$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ → faraday's Law in point form.

-01-

* Lenz's Law :- Lenz's Law states that the current induced in closed loop by changing magnetic flux to the loop is in a direction such that flux generated by Induced current tends to counter balance Change in original magnetic flux.

-01-

* Faraday's Experiment

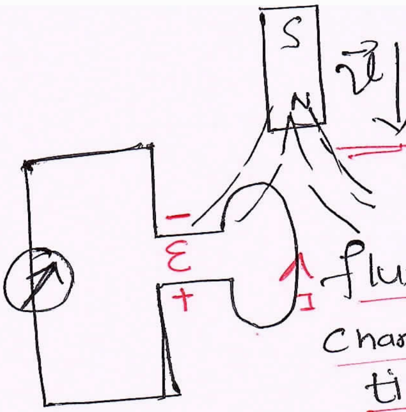


No Emf induced

flux linkage not changing with time

(Static)

−02−



Moving

flux linkage change with time

* Emf induced in loop

* I is induced current

* The variation of flux with time may be caused in three ways

① Stationary loop in time-varying B-field

② Moving loop in static B-field

③ Moving loop in time-varying B-field. } −01−

Q8 b. A point charge Q = 18nC has ~~a~~ velocity of 5×10^6 m/s in the direction $\hat{a}_v = 0.6\hat{a_x} + 0.75\hat{a_y} + 0.3\hat{a_z}$. Calculate the magnitude of force exerted on the charge by field i) $\vec{E} = -3\hat{a_x} + 4\hat{a_y} + 6\hat{a_z}$ kV/m

(ii) $\vec{B} = -3\hat{a_x} + 4\hat{a_y} + 6\hat{a_z}$ mT

(iii) $\vec{B}$ & $\vec{E}$ acting together.

* (i) $\vec{F}_e = Q\vec{E} = 18\times10^{-9}\langle -3, 4, 6\rangle \times10^3$

$\vec{F}_e = 18\langle -3, 4, 6\rangle \times10^{-6}$

$\vec{F}_e = -54\hat{a_x} + 72\hat{a_y} + 108\hat{a_z}$ μN

$|\vec{F}_e| = \left[\sqrt{(54)^2+(72)^2+(108)^2}\right]\times10^{-6}$

$\boxed{|\vec{F}_e| = 140.6\ \mu N}$ −01−

* (ii) $\vec{F}_m = Q(\vec{v} \times \vec{B})$

$\vec{v} = 5\times10^6 \langle 0.6, 0.7, 0.3 \rangle$

$$\vec{v}\times\vec{B} = 5\times10^6 \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ 0.6 & 0.75 & 0.3 \\ -3 & 4 & 6 \end{vmatrix} \times10^{-3}$$

$\vec{v}\times\vec{B} = (3.3\hat{a_x} - 4.5\hat{a_y} + 4.65\hat{a_z})\,5\times10^3$

$\vec{F}_m = Q\vec{v}\times\vec{B} = 18\times10^{-9}\times5\times10^3\,(3.3\hat{a_x} - 4.5\hat{a_y} + 4.65\hat{a_z})$

$\boxed{|\vec{F}_m| = 654\,\mu N}$ —03— $|\vec{F}_m| = \left[\sqrt{(3.3)^2+(4.5)^2+(4.65)^2}\right]\times 90\times10^{-6}$

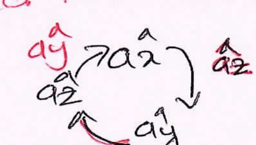
* (iii) $\vec{F} = Q\vec{E} + Q\vec{v}\times\vec{B}$

$= 18\times10^{-6}\langle -3, 4, 6\rangle + 18\times10^{-6}\times5\langle 3.3, -4.5, 4.65\rangle$

$= 18\times10^{-6}\langle 13.5, -18.5, 29.25\rangle$

$\boxed{|\vec{F}| = 668.7\,\mu N}$ —02— $|\vec{F}| = \left[\sqrt{(13.5)^2+(18.5)^2+(29.25)^2}\right]18\times10^{-6}$

Q8c. A conductor of length 4m long lies along the Y-axis with a current of 10 Amp in the $\hat{a_y}$ direction. Find the force on the conductor if the field in the region is $\vec{B} = 0.005\hat{a_x}$ T.



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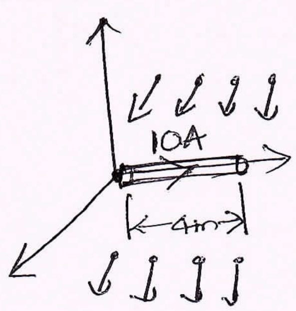


$\vec{B} = 0.005\hat{a_x}$ T

* Given B-field is uniform

* force on filament conductor in uniform B-field is given by —01—

−01− $\vec{F} = I\vec{L}\times\vec{B}$; $\vec{L} = 4\hat{a_y}$; $\vec{B} = 0.005\hat{a_x}$

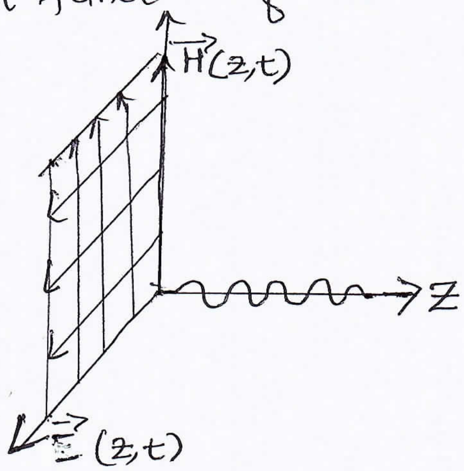
$\vec{F} = 10[4\hat{a_y}\times0.005\hat{a_x}] = 10[0.02(-\hat{a_z})]$ —02—

$\boxed{\vec{F} = -0.2\,\hat{a_z}\,N}$

Module-5

Q9a. What is meant by uniform plane wave? Derive the Expression for uniform plane wave in the free Space.

* Uniform plane waves :- The uniform plane wave implies that a field has same magnitude and direction in a plane containing it. For uniform plane wave propagating in the z-direction, $\vec{E}$ & $\vec{H}$ are not function of x & y, they are function of 'z' alone.



-02-

* Maxwell's Equation in free space

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{---(1)---} \qquad \nabla \cdot \vec{H} = 0 \quad \text{---(3)---}$$

$$\nabla \times \vec{E} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{---(2)---} \qquad \nabla \cdot \vec{E} = 0 \quad \text{---(4)---}$$

-01-

* lets us take curl of Eq^n (1)

$$\nabla \times \nabla \times \vec{E} = -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t} \quad \text{---(5)---}$$

* we have vector identity

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$\nabla \cdot \vec{E} = 0 \quad \text{-- from Eq}^n \text{ (4)}$$

$$\therefore \boxed{\nabla \times \nabla \times \vec{E} = -\nabla^2 E}$$

* Eqn (5) can be written as

$$\nabla^2 \vec{E} = \mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t}$$

* Changing the order of differentation w.r.t space & time, we can write Eqn (5) as

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial}{\partial t} \nabla \times \vec{H} \quad \text{--- (6)}$$

* subn of $\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ from Eqn (2) in Eqn (6) we get

$$\boxed{\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$ → wave Eqn in timedomain for E-field (3D wave Eqn)

* By following above similar steps, we can obtain wave eqn for H-field

$$\boxed{\nabla^2 H = \epsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2}}$$ → wave Eqn in timedomain for H-field (3D wave Eqn) } —04—

* let us assume uniform plane wave propagating in z-dir by defn of uniform plane wave $E_z, H_z = 0$, & further $\vec{E}, \vec{H}$ are not function of x & y —01—

* for uniform plane wave propagating in z-dir, Helmboltz Eqn can be Expressed in Scalar form as

$$\frac{\partial^2 E_x}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad \frac{\partial^2 H_x}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 H_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0 \qquad \frac{\partial^2 H_y}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2} = 0$$

* further let us assume $\vec{E}$ has only x-component & $\vec{H}$ has only y-component. The above Eqn reduces

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad \frac{\partial^2 H_y}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}}$$ } —02—

Uniform plane wave eqn for E-field (1D Eqn) — Uniform plane wave eqn for H-field (1D Eqn)

Q9b. let $\mu = 10^{-5}$ H/m, $\epsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ & $\rho_v = 0$. Determine 'k' so that each of the following pair of fields satisfies Maxwell's Equation:

(i) $\vec{D} = 6x\,\hat{a_x} - 2y\,\hat{a_y} + 2z\,\hat{a_z}$ nC/m^2; $\vec{H} = kx\,\hat{a_x} + 10y\,\hat{a_y} - 25z\,\hat{a_z}$ A/m.

(ii) $\vec{E} = (20y - kt)\,\hat{a_x}$ V/m, $\vec{H} = (y + 2\times10^{6}t)\,\hat{a_z}$ A/m

(i) * Given fields are static fields -01-

* Medium is dielectric * Maxwell's eqn for Given medium

$$\left.\begin{array}{ll} \nabla\cdot\vec{D} = 0 & \nabla\times\vec{E} = 0 \\ \nabla\cdot\vec{B} = 0 & \nabla\times\vec{H} = 0 \end{array}\right\} -01-$$

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \quad - \quad + \quad = 0 \text{ -- satisfied}$$

$$\nabla\cdot\vec{B} = \mu\left[\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z}\right] = \mu\left[k + 10 - 25\right]$$

* inorder to statisf maxwell's eqn

$$\left.\begin{array}{l} \nabla\cdot\vec{B} = 0 \quad \therefore \quad \mu\left[k + 10 - 25\right] = 0 \\ \boxed{k = 15 \text{ A/m}^2} \end{array}\right\} -02-$$

(ii) Given fields are dynamic (time varying) fields

maxwell's eqn are

$$\left.\begin{array}{ll} \nabla\cdot\vec{D} = 0 & \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B} = 0 & \nabla\times\vec{H} = \dfrac{\partial\vec{D}}{\partial t} \end{array}\right\} -01-$$

* $\nabla \cdot \vec{D} = \epsilon \dfrac{\partial E_x}{\partial x} = 0$ [E_x component is function of 'y']

* $\nabla \cdot \vec{B} = \mu \dfrac{\partial H_z}{\partial z} = 0$ [H_z component is function of 'y']

—02—

* $\nabla \times \vec{H} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = \hat{a_x}\left(\dfrac{\partial H_z}{\partial y} - 0\right) - \hat{a_y}\left(\cancel{\dfrac{\partial H_z}{\partial x}}^{=0} - 0\right) + \hat{a_z}(0-0)$

* $\boxed{\nabla \times \vec{H} = 1\hat{a_x}}$ | * $\dfrac{\partial \vec{D}}{\partial t} = \epsilon \dfrac{\partial \vec{E}}{\partial t} = -\epsilon K \hat{a_x}$

—02—

* In order to satisfy maxwell's eqn

$\nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}$ | $1\hat{a_x} = -\epsilon K \hat{a_x}$

$\therefore K = \dfrac{1}{-\epsilon}$ | $K = \dfrac{-1}{4 \times 10^{-9}} = -2.5 \times 10^{8}$ V/m.s

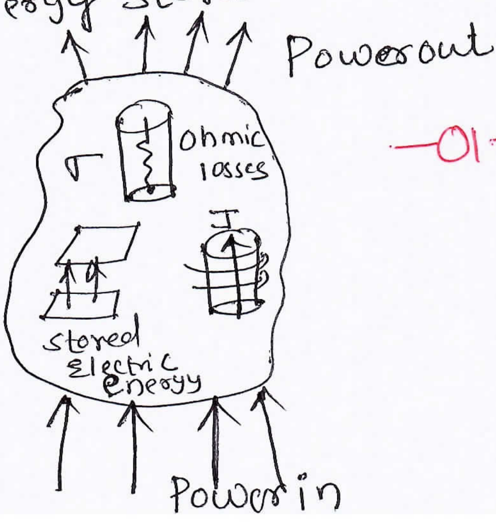
—01—

$\boxed{K = -2.5 \times 10^{8} \text{ V/m.s}}$

— OR —

Q10a. State and Explain Poynting's theorem.

* Poynting's Theorem :- States that net power flowing out of a given Volume V is equal to the time rate of decrease in the energy stored within V minus the ohmic losses.



—01—

* we have Maxwell's Equation

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad --- (1)$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad --(2)$$

* Taking Dot product bothside of Eqn (2) with E

$$\vec{E} \cdot \nabla \times \vec{H} = \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t} \quad --(3)$$

* we have vector identity

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

* Apply this vector identity to Eqn (3)
  let $\vec{A} = \vec{H}$, $\vec{B} = \vec{E}$

$$\nabla \cdot (\vec{H} \times \vec{E}) = \vec{E} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{E})$$

$$\vec{H} \cdot (\nabla \times \vec{E}) + \nabla \cdot (\vec{H} \times \vec{E}) = \vec{E} \cdot (\nabla \times \vec{H}) \quad -(4)$$

-03-

$$\therefore \vec{H} \cdot (\nabla \times \vec{E}) + \nabla \cdot (\vec{H} \times \vec{E}) = \sigma E^2 + \vec{E} \cdot \frac{\epsilon \partial \vec{E}}{\partial t} \quad --(5)$$

But $\vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \epsilon \frac{\partial E^2}{\partial t}$ - (5b) -01-

* Dot product both side of Eqn (1) with $\vec{H}$ we get

$$\therefore \vec{H} \cdot (\nabla \times \vec{E}) = H \cdot \left(-\mu \frac{\partial H}{\partial t}\right) = -\mu \frac{1}{2} \frac{\partial H^2}{\partial t} \quad --(6)$$

$$* \quad \boxed{\vec{H} \cdot (\nabla \times \vec{E}) = -\frac{1}{2} \mu \frac{\partial H^2}{\partial t}} \quad -(7)$$

-01-

Sub of (7) & (5b) in (5) we get

$$-\frac{1}{2} \mu \frac{\partial H^2}{\partial t} - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \frac{1}{2} \epsilon \frac{\partial E^2}{\partial t} \quad --(8)$$

* Rearranging terms & taking volume integral bothside of Eqn (8)

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv = -\frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\epsilon E^2+\frac{1}{2}\mu H^2\right]dv-\int_V \sigma E^2\,dv \quad -(9)-$$

* Apply divergence theorem on the left hand side of above Eqn (9)

$$\underbrace{\oint_S(\vec{E}\times\vec{H})\cdot\vec{ds}}_{\text{Total Power leaving volume}} = \underbrace{-\frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\epsilon E^2+\frac{1}{2}\mu H^2\right]dv}_{\text{rate of decrease in energy stored in E \& H fields}}-\underbrace{\int_V \sigma E^2\,dv}_{\text{Ohmic Power dissipated}}$$

Total Power leaving volume = rate of decrease in energy stored in E & H fields — Ohmic Power dissipated

* The above Eqn (9) is Integral form of poynting's theorem —02—

* Eqn (8) $-\nabla\cdot(\vec{E}\times\vec{H}) = \sigma E^2+\frac{1}{2}\epsilon\frac{\partial E^2}{\partial t}+\mu\frac{1}{2}\frac{\partial H^2}{\partial t}$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\left[\frac{1}{2}\epsilon\frac{\partial E^2}{\partial t}+\frac{1}{2}\mu\frac{\partial H^2}{\partial t}\right]-\sigma E^2 \quad -01-$$

Point form of poynting theorem.

* The vector product $\vec{E}\times\vec{H}$ has units of power density W/m^2 & it is known as poynting vector S

$$\vec{S} = \vec{E}\times\vec{H} \ W/m^2$$

* $\vec{S}$ represents the instantaneous power density vector associated with EM field @ a given point

* Integration of poynting vector over any closed surface gives net power flowing out of the surface

$$\oint_S \vec{E}\times\vec{H}\cdot\vec{ds} = \oint \vec{S}\cdot\vec{ds} \ \ 'W' \quad -01-$$

$\hat{a}_E\times\hat{a}_H=\hat{a}_k$

$\hat{a}_k$ is unit vector along the direction of wave propagation.

Q10b. Discuss wave propagation in good conducting medium.

* Wave propagation in Good conductor

* A medium is said to be good conductor if

$\frac{\sigma}{\omega\epsilon} >> 1.$ —01—

* Propagation constant of general medium is

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

$\frac{\sigma}{\omega\epsilon} >> 1$ Applying this condⁿ for above eqⁿ

$$\boxed{\gamma = (1+j)\sqrt{\pi f \mu \sigma}} = \alpha + j\beta \quad \boxed{\alpha = \beta = \sqrt{\pi f \mu \sigma}}$$

* ||ly $\eta \cong (1+j)\sqrt{\frac{\pi f \mu}{\sigma}}$

$$\boxed{\eta \cong \sqrt{\frac{\mu\omega}{\sigma}} \angle 45^\circ}$$

$$\boxed{v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\pi f \mu \sigma}}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\pi f \mu \sigma}}}$$

* $$\boxed{\vec{E} = E_{xo} e^{-\alpha z} \cos(\omega t - \beta z) \hat{a_x}}$$

$$\boxed{\vec{H} = \frac{E_{xo}}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ) \hat{a_y}}$$

$\vec{E}$ leads $\vec{H}$ by 45° —03—

* Skin depth: When $\vec{E}$ & $\vec{H}$ travels in conducting medium its amp is attenuated by the factor $e^{-\alpha z}$

* The distance 'δ' through which wave amp decreases to a factor of e^{-1} of original value (37% of original value) is called skin depth of medium

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} \text{ 'm'}}$$

—02—

Q10c Find the frequency at which conduction current density & displacement current density are equal in a medium $\sigma = 2\times10^{-4}$ S/m & $\epsilon_r = 81$

* we have $\frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{\sigma}{\omega \epsilon}$; According to given cond^n $\frac{\sigma}{\omega \epsilon} = 1$ —02

$\therefore \omega = \frac{\sigma}{\epsilon} = \frac{2 \times 10^{-4}}{8.854 \times 10^{-12} \times 81} = 0.2788 \times 10^{6}$ rad/s

$\therefore \boxed{\omega = 0.2788 \times 10^{6} \text{ rad/s}}$; $\omega = 2\pi f$ or $f = \frac{\omega}{2\pi}$

$\therefore \boxed{f = 44.372 \text{ KHZ}}$ —02