18

Subject Code-

Model Question Paper-I with effect from 2021 (CBCS Scheme)

USN | | | | | | | | | | |

First Semester Engineering Degree Examination

Subject Title 21PHY12/22

TIME: 03 Hours **Max. Marks: 100**

Note:
01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02. Draw neat sketches where ever necessary.
03. **Constants :** Speed of Light "c" = 3 $\times 10^{8}$ ms^{-1}, Boltzmann Constant "k" = 1.38 $\times 10^{-23}$ JK^{-1}, Planck's Constant "h" = 6.625 $\times 10^{-34}$ Js, Acceleration due to gravity "g" = 9.8 ms^{-2}, Permittivity of free space "ε_0"=8.854 $\times 10^{-12}$ F m^{-1}.

		Module -1	Marks
Q.01	a	Define SHM and mention any two examples. Derive the differential equation using Hooke's law.	07
	b	With a neat diagram, explain the construction and working of Reddy's shock tube. Mention the any three applications of shock waves.	09
	c	A free particle is executing S.H.M in straight line with a period of 5 seconds after it has crossed the equilibrium point, the velocity is found to be0.7m/s. Find the displacement at the end of10 seconds ,and also the amplitude of oscillation.	04
		OR	
Q.02	a	What are damped oscillations. Discuss the theory of damped oscillations. Represent overdamping, critical damping and under damping by graph.	10
	b	Define Mach number. Distinguish between Ultrasonic, subsonic, supersonic, and hypersonic waves.	06
	c	The distance between two pressure sensors in a shock tube is 200 mm. The time taken by a shock wave to travel this distance is 0.4ms. If the velocity of sound under the same condition is 340 m/s. Find the Mach number of the shock wave,	04
		Module-2	
Q. 03	a	State Wein's law and Rayleigh-Jeans law and mention their draw backs.	06
	b	Assuming the time independent Schrodinger's wave equation discuss the solution for a particle in one dimensional potential well of infinite height and hence obtain the normalized wave equation.	10
	c	A particle having mass of 0.5MeV/c^2 has a kinetic energy of 100 eV. Calculate the deBroglie wavelength, where c is the velocity of light.	04
		OR	
Q.04	a	Starting from Planck's quantum theory of radiation arrive at Wein's law and Rayleigh-Jean's law.	08
	b	State Heisenberg uncertainty Principle. Show that electron does not exists inside the nucleus by this Principle.	07
	c	A quantum particle confined to one dimensional box of width 'a' is in its first excited state. What is the probability of finding the particle over an interval of 'a/2' marked symmetrically at the center of the box.	05
		Module-3	
Q. 05	a	Define the terms population inversion and Meta stable state. Explain the construction and working of semiconductor laser.	09

	b	With neat diagram explain the working of Intensity based displacement sensor using optical fiber.	07
	c	Estimate the attenuation in an optical fiber of length 500m when a light signal of power 100mW emerges out of fiber with a power 90Mw.	04
		OR	
Q. 06	a	Derive the expression for numerical aperture of an optical fiber. Mention any two merits and demerits of optical communication.	10
	b	Explain how laser find application in eye surgery	05
	c	The ratio of population of two energy levels out of which upper one corresponds to a metastable state is 1.059×10^{-30}. Find the wavelength of light emitted at 330 K.	05
		Module-4	
Q. 07	a	Mention any four assumptions of Drude-Lorentz model and discuss the success of Quantum free electron theory.	10
	b	Derive Clausius-Mossotti equation.	05
	c	Show that occupation probability at an energy $E_F+\Delta E$ is equal to non-occupation probability at the energy $E_F-\Delta E$	05
		OR	
Q. 08	a	What is Hall effect. Obtain the expression for the Hall coefficient	08
	b	Obtain expression for electrical conductivity in metals on quantum model	08
	c	Find the temperature at which there is 1% probability that a state with an energy 0.5 eV above the fermi energy is occupied.	04
		Module-5	
Q. 09	a	With neat diagram, explain the principle, construction and working of Atomic Force Microscope.	10
	b	Explain in brief how crystal size is determined by Scherrer's equation.	05
	c	Determine the wave length of X-rays for crystal size of 1.188×10^{-6} m, peak width is 0.5^0 and peak position 30^0, for a cubic crystal. Given Scherrer's constant k=0.92.	05
		OR	
Q. 1 0	a	Explain the construction and working of X-Ray diffractometer.	07
	b	With neat diagram, explain the principle, construction and working of X-ray photoelectron spectroscope.	08
	c	The first order Bragg reflection occurs when a monochromatic beam of X-rays of wavelength 0.675A° is incident on a crystal at a glancing angle of 4°. What is the glancing angle for third order Bragg's reflection to occur?	05

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO.1	PO-1,2,12
	(b)	L2	CO.1	PO-1.2,12
	(c)	L3	CO.1	PO-1
Q.2	(a)	L3	CO.1	PO-1,2,12
	(b)	L2	CO.1	PO1,2.12
	(c)	L3	CO.I	PO-1
Q.3	(a)	L2	CO.2	PO-1,2,12
	(b)	L3	CO.2	PO-1,2,12
	(c)	L3	CO.2	PO-1
Q.4	(a)	L2	CO.2	PO-1,2,12
	(b)	L3	CO.2	PO1,2,12
	(c)	L3	CO.2	PO-1

KLS VDIT, HALIYAL

ENGINEERING PHYSICS 21PHY12

I/II SEMESTER B.E. MODEL QP SET-1

QUESTION PAPER SOLUTIONS:

Prepared by
Dr. Mahesh S. Rannu
Asst. Prof.
Dept. of Physics
KLS VDIT, Haliyal
9900111638

MODULE - I

1 a. Define SHM & mention any two examples. Derive the differential equation using Hooke's law.

Simple Harmonic Motion (SHM):

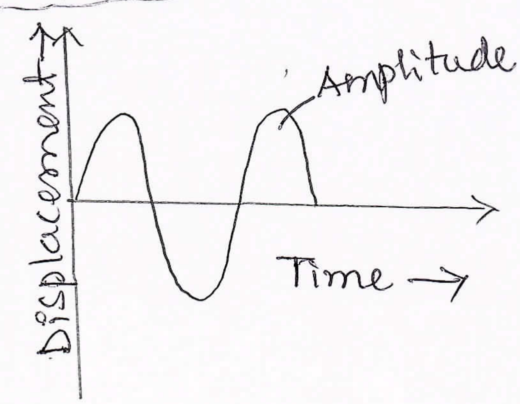
Definition: A motion in which the displacement varies simultaneously with time is known as SHM.

SHM is the oscillatory motion of a body whose the restoring force is proportional to the -ve of the displacement.

Examples of SHM:

1. Simple pendulum
2. Vibrations of mass held by a two stretched strings.
3. Vibrations of a stretched string.

Differential Equation of motion for SHM:



Let a body be initiated to an oscillatory motion after being displaced from its equilibrium position & left free.

For such oscillations, the lonely force acting on the body will be restoring force "F".
W.K.T. for a vibrating body,

$$\boxed{F = -Kx}$$

where K → is the force constant &
x → is the displacement
If 'm' is the mass of the body, then, as per Newton's 2nd law of motion.

$$F = ma \quad \text{—(1)}$$

$$m \cdot \frac{d^2x}{dt^2} = -Kx \quad \text{—(2)}$$

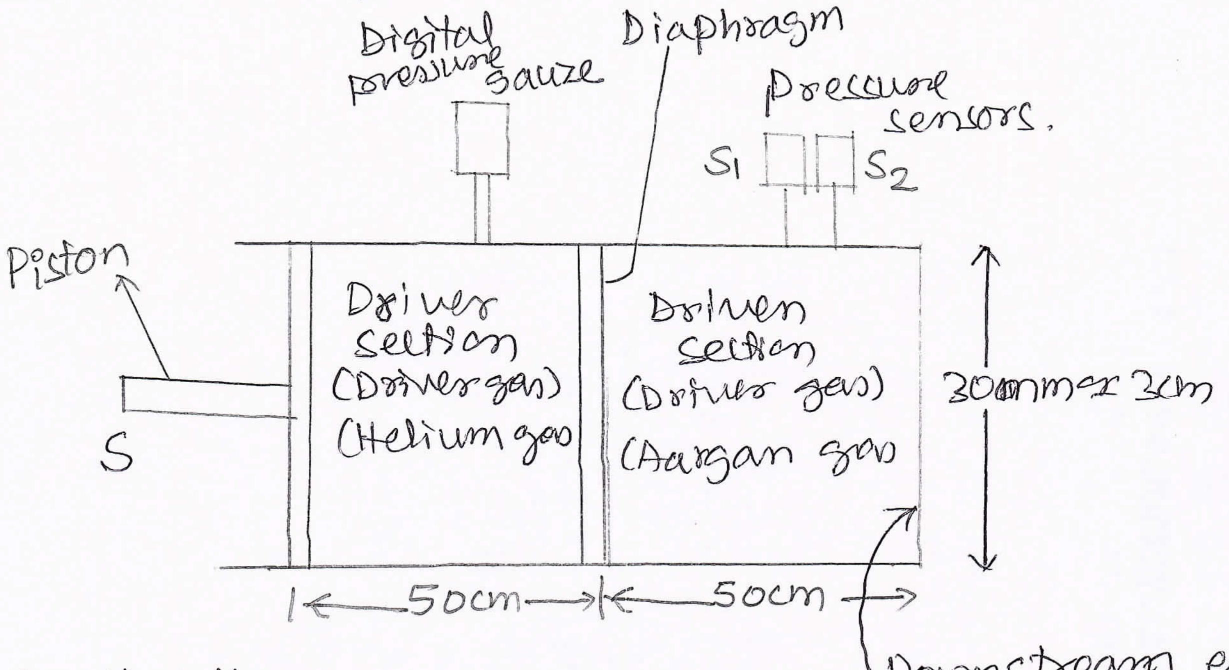
$$\frac{d^2x}{dt^2} = -\frac{Kx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0 \quad \text{—(3)}$$

The above eqn represents the equation of motion for a body executing free vibrations.

03

1 b. With a neat diagram, explain the construction & working of Reddy's shock tube. Mention the any three applications of shock tube.



Construction:

1. Reddy shock tube (RST) consists of a steel tube of length 100 cm.
2. A diaphragm of thickness 0.1m divides the tube into two compartments of length 50cm fitted with piston called Driver section filled with driver gas (Helium). The other compartment of length 50cm is called Driven section filled with driven gas (Aargan gas).
3. Sensor "S" is fitted to driver section measures the rupture pressure P_2, temperature T_2.
4. Two sensors S_1 & S_2 separated by a distance Δx fitted to driven section measures the pressures P_4, P_5 & temp T_4 & T_5 respectively.

WORKING:

1. Driver section is filled with a gas at high pressure (P_2) and driven section is filled with gas of low pressure (P_1).
2. Diaphragm is ruptured to produce shock waves by pushing the piston & the rupture pressure P_2 & temperature is measured using sensors S_1 & S_2 resp.
3. The time "t" taken by the shockwave to travel the distance "x" is measured using sensors S_1 & S_2 & CRO.

   The speed of the shock waves is calculated using $V = \frac{x}{t}$

4. Then, if "a" is the speed of sound at laboratory temperature, the Mach number of shock waves is calculated using $\boxed{M = \frac{V}{a}}$
5. The mach number increases with increase of the thickness of the diaphragm.

1 c. A free particle is executing SHM, in straight line with a ~~speed~~ period of ~~5~~ 25 seconds. 5 seconds after it has crossed the equilibrium point, the velocity is found to be 0.7 m/s. Find the displacement at the end of 10 seconds & also the amplitude of oscillation. 05

Solution:

Given that: Period of oscillations: T = 5 sec

T = 5 seconds. $v_1 = 0.7$ m/s. ~~t~~ at time $= t_1 = 5$ sec

after crossing to equilibrium position.

<u>To find</u>: Amplitude of oscillation, a = ?

Displacement x, at time $t_2 = 10$ se

Solution:

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{2\pi}{25}$

we have the eqn for displacement,

$x = a \cdot \sin\omega t$

∴ velocity, $v = \frac{dx}{dt} = a\omega \cos\omega t$

At time, $t_1 = 5$ seconds $v = 0.7$ m/s.

$\therefore 0.7 = a \cdot \frac{2\pi}{25} \cdot \cos\left[\frac{2\pi}{25} \times 5\right]$

$0.7 = 0.25 \times a \times 0.309$

$\boxed{a = 9.06\ m}$

∴ Displacement at the end of 10 seconds. is ~~5.3m & the amplitude is 9.06m~~. is obtained by the formula

$x = a \sin\omega t$

$x = 9.06 \times \sin\left[\frac{2\pi}{25} \times 10\right] = 9.06 \times 0.588 = 5.3\,m$ $\boxed{x = 5.3m}$

∴ The displacement at the end of 10 seconds is 5.3m & the amplitude is 9.06 m.

06

04

2a. What are damped oscillations. Discuss the theory of damped oscillations, Represent overdamping, critical damping & under damping by graph.

Damped oscillations:

Damped oscillations are the oscillations, whose amplitude goes on decreasing due to the frictional forces of the medium acting on the body.

Theory of damped oscillations:

Consider a body of mass "m" executing vibrations in a resistive medium. Then the resistive force acting on the body due to medium $= -r \cdot \frac{dx}{dt}$

where r is damping constant and $\frac{dx}{dt}$ is the velocity of the body.

Also restoring force acting on the body $= -Kx$

where K is force constant & x is the displacement.

$\therefore$ The net resultant restoring force acting on the body $= -r\frac{dx}{dt} - Kx$ —— (1)

From Newton's 2nd law of motion, the resultant force acting on the body

$$= m \cdot \frac{d^2x}{dt^2} \text{ —— (2)}$$

From equⁿ ① & ②, we get

$$m\frac{d^2x}{dt^2} = -r\cdot\frac{dx}{dt} - Kx$$

$$\therefore \boxed{m\frac{d^2x}{dt^2} + r\cdot\frac{dx}{dt} + Kx = 0}$$

This is equⁿ of damped motion.

On re-arranging, we get

$$\frac{d^2x}{dt^2} + \frac{r}{m}\cdot\frac{dx}{dt} + \frac{K}{m}x = 0 \quad \text{—— ②}$$

Let $\frac{r}{m} = 2b$ & w.k.t $\omega^2 = \frac{K}{m}$ $\left[\because \omega = \sqrt{\frac{K}{m}}\right]$

$$\therefore \frac{d^2x}{dt^2} + \frac{r}{m}\cdot\frac{dx}{dt} + \omega^2 x = 0 \quad \text{—— ③}$$

Let the solution of equⁿ ③ be

$$x = A e^{\alpha t} \quad \text{—— ④}$$

where A & α are constants

Differentiating equⁿ ④ twice w.r.t 't' we get

$$\frac{dx}{dt} = A\alpha e^{\alpha t} \quad \text{—— ⑤}$$

& $$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t} \quad \text{—— ⑥}$$

Substituting equⁿ ④, ⑤ & ⑥ in equⁿ ③, we get

$$A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$$

$$A e^{\alpha t}[\alpha^2 + 2\alpha b + \omega^2] = 0$$

$$x[\alpha^2 + 2\alpha b + \omega^2] = 0$$

Thuse $(\alpha^2 + 2b + \omega^2) = 0$ —— ⑦ as $x \neq 0$.

The solution of equ^n (7) is

$$\alpha = -b \pm \sqrt{b^2-\omega^2} \quad \text{—(8)}$$

From equ^n (4) & (8), the general sol^n can be written as

$$x = Ce^{(-b+\sqrt{b^2-\omega^2})t} + De^{(-b-\sqrt{b^2-\omega^2})t} \quad \text{—(9)}$$

where C & D are constants.

Let the time is counted from maximum displacement $x = x_0$, then $t=0$

From equ^n (9), we get

$$x_0 = C + D \quad \text{—(10)}$$

At maximum displacement the velocity

$$\frac{dx}{dt} = 0,$$

Differentiating equ^n (9) & equating to zero, we get

$$(-b+\sqrt{b^2-\omega^2})\,C\,e^{(-b+\sqrt{b^2-\omega^2})t} + (-b-\sqrt{b^2-\omega^2})\,D\,e^{(-b-\sqrt{b^2-\omega^2})t} = 0$$

when $t=0$, $(-b+\sqrt{b^2-\omega^2})C + (-b-\sqrt{b^2-\omega^2})D = 0$

On rearranging,

$$-b(C+D) + \sqrt{b^2-\omega^2}\,(C-D) = 0$$

$$-b\,x_0 + \sqrt{b^2-\omega^2}\,(C-D) = 0$$

$$\frac{b\,x_0}{\sqrt{b^2-\omega^2}} = (C-D) \quad \text{—(11)}$$

Adding equ^n (10) & (11), we get

$$C = \frac{x_0}{2}\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right]$$

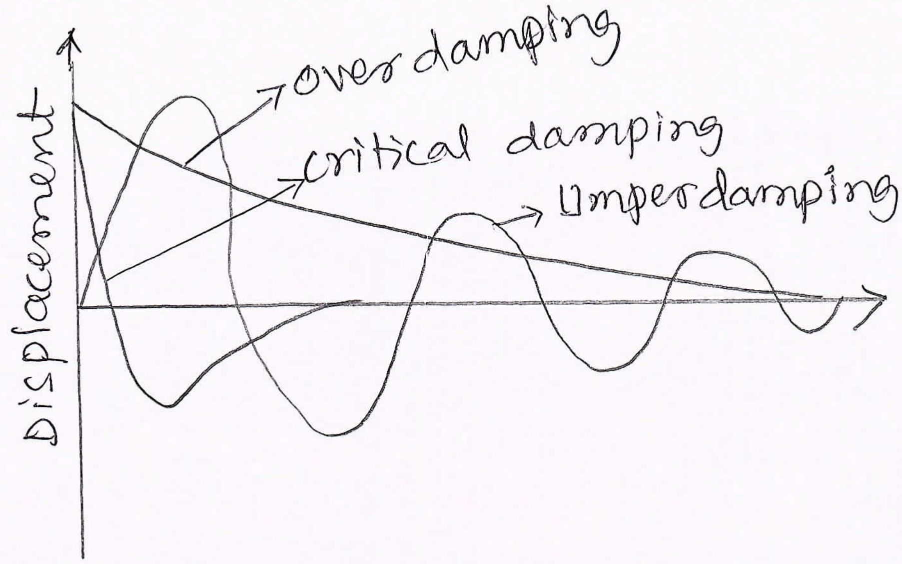
Also subtracting equ^n (11) & (10), we get

$$D = \frac{x_0}{2}\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right]$$

Substituting for C & D in equ^n (9), we get

$$x = \frac{x_0}{2}\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left(-b+\sqrt{b^2-\omega^2}\right)t} + \frac{x_0}{2}\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left(-b-\sqrt{b^2-\omega^2}\right)t} \quad \text{--- (12)}$$

This is the general solution for damped vibrations.



10 08

2 b. Define Mach number. Distinguish between Ultrasonic, Subsonic, Supersonic and Hypersonic waves.

Mach Number:

It is the ratio of the speed of the object to the speed of sound in the given medium

$$\text{Mach number} = \frac{\text{object speed}}{\text{Speed of sound in the medium.}}$$

$$\boxed{M = \frac{V}{a}}$$

Ultrasonic waves:

Ultrasonic waves are pressure waves having frequencies more than 20KHz

Subsonic waves:

The speed of mechanical wave or body moving in the fluid is lesser than that of sound, then such a speed is referred to as subsonic & the wave is a subsonic wave. and its mach number is less than 1.

Supersonic waves:

Supersonic waves are mechanical waves which travel with speeds greater than that of sound. i.e. with speeds for which, Mach number >1. (greater than 1).

Hypersonic waves:

It's mach number is more than 5.

2c. The distance between two pressure sensors in a shock tube is 200 mm. The time taken by a shock wave to travel this distance is 0.4 ~~meters~~ m-s. If the velocity of sound under the same condition is 340 m/s. Find the Mach number of the shock wave.

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Given data:

Distance betn the two pressure sensors, $d = 200\,mm = 200 \times 10^{-3} = 0.2\,m$,

Time taken to travel d is, $t = 0.4 \times 10^{-3}$ sec,

velocity of sound, $a = 340\,m/s$.

To find:

Mach number of the shock wave, $M = ?$

Solution:

Shock speed $= U_s = \frac{d}{t} = \frac{200 \times 10^{-3}}{0.4 \times 10^{-3}} =$

or $U_s = 500\,ms^{-1}$

Mach number, $M = \frac{U_s}{a} = \frac{500}{340} = 1.47$

$\therefore$ Mach number of the shock wave is 1.47

10

12

3 a. State Klein's law & Rayleigh-Jean law & mention their draw backs.

Klein's law states that the energy density in the wavelength interval λ & $\lambda+d\lambda$ is

$$\boxed{E_\nu\, d\lambda = C_1 \lambda^{-5} e^{-C_2/\lambda T} d\lambda}$$

$C_1 = 8\pi h\nu$

$C_2 = \frac{hc}{K}$

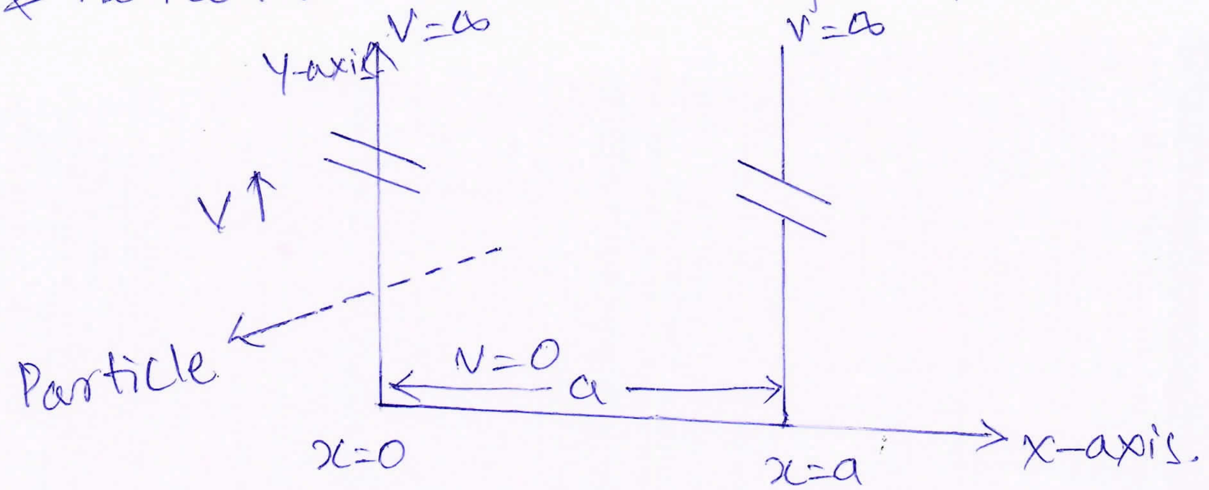
where C_1 & C_2 are constants

Klein's law explained only for shorter λ region of BBR spectra below λ_m & it failed to explain the longer wavelength region of the BBR spectra beyond λ_m.

Rayleigh-Jean's law: It states that the energy density in the wavelength interval λ & $\lambda+d\lambda$ is $\boxed{E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4}}$ where $K \to$ is Boltzman's constant

It explain's for longer wavelength region of BBR spectra beyond λ_m & it failed to explain for shorter wavelength region of BBR spectra below λ_m.

3 b. Assuming the time-independent Schrodinger's wave equation discuss the solution for a particle in one dimensional potential well of infinite height & hence the obtained wave ~~fun~~ equation.



Considera particle of a mass "m" free to move in one dimension along positive X-direction between $x=0$ to $x=a$. the potential energy outside this region is infinite & within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box.

The Schrodinger's equation outside the well is

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0.}$$

This is Schrodinger's time independent equation for a particle

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \longrightarrow (1) \quad (\because V=\infty)$$

For outside, the equation holds good if $\psi=0$ & $|\psi|^2=0$. That is particle cannot be found outside the well & also at the walls.

The Schrodinger's equation inside the well is

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E)}{h^2}\psi = 0} \longrightarrow (2) \quad (\because V=0).$$

This is Eigen-value equation.

Put $\frac{8\pi^2 m}{h^2} = K^2 \longrightarrow (3)$ ~~in equn (2)~~

$\therefore$ Equn (2) becomes.

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \longrightarrow (4)$$

The general solution of the quadratic equation (4) is of the form.
the solution of this equn is

$$\psi = C \cos Kx + D.\sin Kx \longrightarrow (5)$$

where C & D are constants & determined by using boundary conditions, as follows.

$\psi(x)=0$ at $x=0$ from equn (5)

case (i) $x=0$ at $\psi=0$.

$$0 = C.\cos K(0) + D.\sin K(0)$$
$$= C.\cancel{\cos}1 + D\times(0).$$

$$\therefore \boxed{C=0} \longrightarrow (6)$$

Case (ii) $x = a$ at $\psi = 0$.

Equ^n (5) becomes

$$0 = C \cdot \cos Ka + D \cdot \sin Ka$$

But $\boxed{C = 0}$

$$\therefore D \cdot \sin Ka = 0 \quad \text{---} (7)$$

If $D \neq 0$ (Because the concept vanishes).

$$Ka = \sin^{-1}(0)$$

ie $Ka = n\pi$ where $n = 0, 1, 2, 3, 4 \ldots$, quantum number

$$\boxed{K = \frac{n\pi}{a}} \quad \text{---} (8)$$

Using this equ^n (5), we have.

$$\psi_n = D \cdot \sin\left[\frac{n\pi}{a}\right] x \quad \text{---} (9)$$

which gives permitted wave functions.

To find out the value of D, normalization of the wave function is to be done.

$$\int_0^a |\psi_n|^2 dx = 1 \quad \text{---} (10)$$

using the value of ψ_n from equ^n (9)

$$\int_0^a D^2 \sin^2\left(\frac{n\pi}{a}\right) x \, dx = 1$$

$$D^2 \int_0^a \left[\frac{1 - \cos(2n\pi/a)x}{2}\right] dx = 1 \qquad \left[\sin^2\theta = \frac{1 - \cos 2\theta}{2}\right]$$

$$\frac{D^2}{2}\left[\int_0^a dx - \int_0^a \cos \frac{2n\pi}{a} x \, dx\right] = 1$$

$$\frac{D^2}{2}[a - 0] = 1$$

$$\frac{D^2}{2} a = 1 \qquad D^2 = \frac{2}{a} \qquad \boxed{D = \sqrt{\frac{2}{a}}}$$

Hence, the normalized wave functions of a particle in one dimensional infinite potential well

$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}\right) x$$

14

16

3 c. A particle having mass of 0.5 MeV/c^2 has a kinetic energy of 100 eV. Calculate the de-Broglie wavelength, where c is the velocity of light.

Soln Mass of the particle $m = 0.5\ MeV/c^2$

$$= \frac{0.5 \times 10^{6} \times 1.6 \times 10^{-19}}{(3 \times 10^{8})^2}$$

$$= 8.9 \times 10^{-31}\ Kg.$$

Kinetic energy $E = 100\ eV = 100 \times 1.6 \times 10^{-19}\ J$

$$E = 1.6 \times 10^{-17}\ J$$

We have de-Broglie wavelength eqn

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 8.9 \times 10^{-31} \times 1.6 \times 10^{-17}}}$$

$$\boxed{\lambda = 1.24 \times 10^{-10}\ m}$$

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4.a) Starting from Planck's quantum theory of radiation arrive at Wein's law and Rayleigh-Jean's law.

Planck's derived the law which holds good for the entire spectrum of the black body radiation.

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{h\nu/KT}-1}\right]d\lambda} \quad \text{---} (1) \qquad \left(\because\ c=\nu\lambda \text{ or } \nu=\frac{c}{\lambda}\right)$$

This is Planck's law for radiation

Since Wein's law holds good in the shorter or lower wavelength. Rayleigh-Jeans law ~~for~~ the longer wavelength.

Reduction of Planck's law to Wein's law for Shorter wavelength:

1. For shorter wavelengths, $\nu = c/\lambda$ is large

when ν is large, $e^{h\nu/KT}$ is very large

$e^{h\nu/KT} >> 1$

$$(e^{h\nu/KT}-1) = e^{h\nu/KT} = e^{hc/\lambda KT}$$

Substituting in eqn (1), we have

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{h\nu/KT}}\right]d\lambda$$

$$\boxed{E_\lambda d\lambda = C_1 \lambda^{-5} \cdot e^{-C_2/\lambda T} d\lambda}$$

where $C_1 = 8\pi hc$ $\qquad C_2 = \frac{hc}{K}$

This is Wein's law for radiation.

(ii) Reduction of Planck's law to Rayleigh-Jeans for longer wavelength:

For longer wavelength, $\nu = \frac{c}{\lambda}$ is small.

When ν is small, $\frac{h\nu}{KT}$ will be very small.

Expanding $e^{h\nu/KT}$ as power series, we have

$$e^{h\nu/KT} = 1 + \frac{h\nu}{KT} + \left[\frac{h\nu}{KT}\right]^2 + \left[\frac{h\nu}{KT}\right]^3 + \ldots\ldots$$

$$e^{h\nu/KT} = 1 + \frac{h\nu}{KT}$$

Since $\frac{h\nu}{KT}$ is very small, its higher power terms could be neglected.

$$\left(e^{h\nu/KT} - 1\right) = \frac{hc}{\lambda KT}$$

Substituting in equn ①, we have

$$E\nu\, d\lambda = \left[\frac{8\pi hc}{\lambda^5\left(\frac{hc}{\lambda KT}\right)}\right] d\lambda$$

$$\boxed{E\nu\, d\lambda = \frac{8\pi KT}{\lambda^4}\, d\lambda}$$

This is Rayleigh-Jeans law of radiation.

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4 b. State Heisenberg's uncertainty principle. Show that electron does not exist inside the nucleus by this principle.

Statement of Heisenberg's uncertainty principle:

"In any simultaneous determination of the position & momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than $\frac{h}{4\pi}$.

$$\boxed{\Delta x \cdot \Delta P \geq \frac{h}{4\pi}}$$

$\Delta x \rightarrow$ is uncertainty in position.
$\Delta P \rightarrow$ is " " momentum.

Electron cannot exist inside the nucleus using HU principle:

Electron to be present in the nucleus, maximum uncertainty in position $\Delta x = 10^{-14} m$
ie. Diameter of nucleus $\Delta x = 10^{-14} m$.

According HUP
The minimum uncertainty in momentum,

$$\Delta x \cdot \Delta P \geq \frac{h}{4\pi}$$

$$\Delta P \geq \frac{h}{4\pi \cdot \Delta x}$$

$$\Delta P_x \geqslant \frac{h}{4\pi \cdot \Delta x}$$

$$\Delta P \geqslant \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}}$$

$$\Delta P \geqslant 5.275 \times 10^{-21}\ \text{Kg.m/s} = P\ (\text{say}).$$

The minimum energy of the electron in the nucleus is given by

$$\therefore\ \cancel{P} = E \geqslant \frac{P^2}{2m}$$

$$\geqslant \frac{(5.275 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}}$$

$$\geqslant 1.527 \times 10^{-11}\ \text{J}$$

$$\geqslant \frac{1.527 \times 10^{-11}}{1.6 \times 10^{-19}}\ \text{MeV}$$

$$\geqslant 95.45\ \text{MeV}$$

$$\boxed{E \geqslant 95.45\ \text{MeV}}$$

But the maximum K.E. of the electrons (β-particle) emitted from the nucleus does not exceed 4 MeV.

Hence electrons do not present in the nucleus.

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4 C. A quantum particle confined to one dimensional box of width "a" is in ~~the~~ its first excited state. What is the probability of finding the particle over an interval of a/2 marked symmetrically at the center of the box.

Solution:

We have the wave function for particle in one-dimensional box of width "a" as

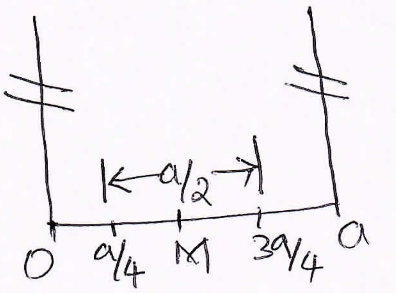
$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}\right)x$$

Since, the particle is in the first excited state, $n=2$.

∴ We can write the above equation as

$$\psi = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{2\pi}{a}\right)x \quad \text{——} (1)$$

In the range "0" to "a" of the box width, Consider an interval $(a/2)$ symmetric about the center M.



Since the interval is bound between $x = a/4$ to $x = 3a/4$, the above eqn is written as

$$P = \int_{a/4}^{3a/4} \psi^2 dx$$

Eqn (1) becomes

$$P = \int_{a/4}^{3a/4} \left(\frac{2}{a}\right)\left[\sin^2\left(\frac{2\pi}{a}\right)x\right]dx$$

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22 Since $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$,

the above equation can be written as

$$P = \frac{2}{a}\int_{a/4}^{3a/4} \frac{1}{2}\left[1-\cos 2\left(\frac{2\pi}{a}\right)x\right]dx$$

$$= \frac{1}{a}\left[\int_{a/4}^{3a/4} dx - \int_{a/4}^{3a/4} \cos\left(\frac{4\pi}{a}\right)x\,dx\right]$$

$$= \frac{1}{a}\left[(x)_{a/4}^{3a/4} - \frac{a}{4\pi}\left[\sin\left(\frac{4\pi}{a}\right)x\right]_{a/4}^{3a/4}\right]$$

$$= \frac{1}{a}\left[\left(\frac{3a}{4}-\frac{a}{4}\right) - \frac{a}{4\pi}\left[\sin\left(\frac{4\pi}{a}\right)\left(\frac{3a}{4}\right) - \sin\left(\frac{4\pi}{a}\right)\left(\frac{a}{4}\right)\right]\right]$$

$$= \frac{1}{a}\left[\frac{a}{2} - \frac{a}{4\pi}(\sin 3\pi - \sin\pi)\right]$$

$$= \frac{1}{a}\left[\frac{a}{2} - \frac{a}{4\pi}(0-0)\right]$$

$$\boxed{P = 0.5}$$

$\therefore$ The probability of finding the particle somewhere in the given interval is 50%

~~20~~1

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5a. Define the terms population inversion & Meta stable state. Explain the construction and working of semiconductor laser.

Population inversion:

It is a state in which the number of atoms/molecules populated in higher energy (N_2) state (excited state) is more than lower energy level (N_1) $E_2 > E_1 \Rightarrow N_2 > N_1$

$N_1 \rightarrow$ no. of atoms in lower energy level.

$N_2 \rightarrow$ " " " higher energy level.

Meta stable state:

It is a state in which the atoms/molecules stay for a long duration in the order 10^{-2} to 10^{-3} second or in terms of milli second. Meta stable state exist between lower & higher energy level.

It is an intermediate state in which the average life of the atoms is of the order of 10^{-2} sec to 10^{-3} sec or milli second.

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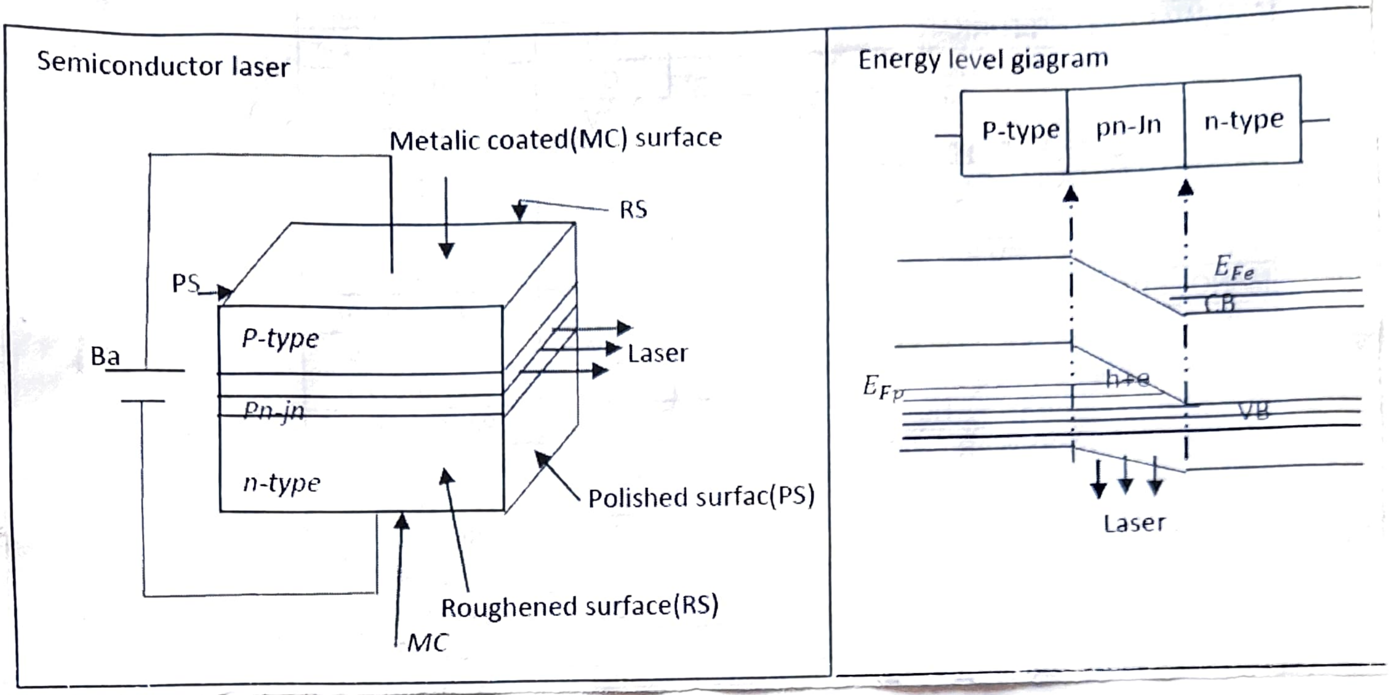
Semiconductor Diode Laser:

Construction:



Metallic contacts are provided to the P & N-types in a heavily doped P-N junction diode. Two opposite faces which are $\perp^{lar}$ to the plane of the junction are polished & made parallel to each other. These parallel faces constitute the resonant cavity & laser is obtained through these faces as shown in figure. The remaining two faces are roughened to prevent lasing action in that direction.

1. The schematic diagram of Ga-As semi-conductor diode is as shown in fig.
2. It consists of heavily doped n-region of Ga-As doped with tellurium & p-region of Ga-As doped with zinc.

3. The dopants are added in the concentration
of the order 10^{17} to 10^{19} no. of dopants/cm^3.
4. The upper & lower surfaces are metalized, so that pn junction is forward biased.
5. Two surfaces $\perp^{lar}$ to the junction are polished, so that they act as optical resonators & the other two surfaces roughened to prevent lasing in that direction.

Principle: semiconductor diode laser works on the principle of stimulated emission.

Working:

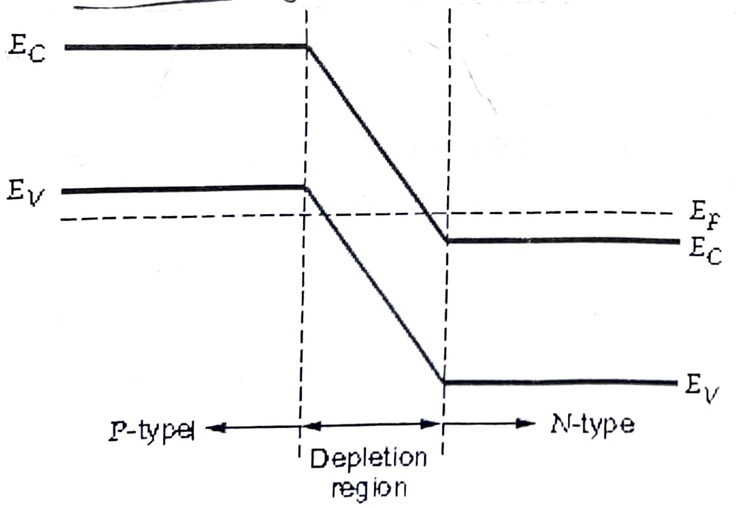
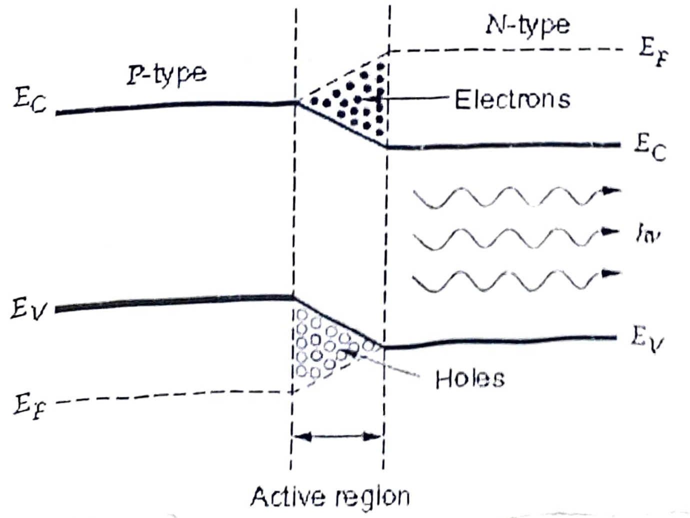


Fig. 5.2 (a)



1. Semiconductor laser are made up of highly degenerate semiconductors having direct band gap like GaAs.
2. When Ga-As diode is forward biased with voltage nearly equal to the energy gap voltage, electrons from n-region & holes from p-region flow across the junction creating population inversion in the active pn junction.
3. As the voltage is gradually increased due to forward biasing population inversion is achieved befo

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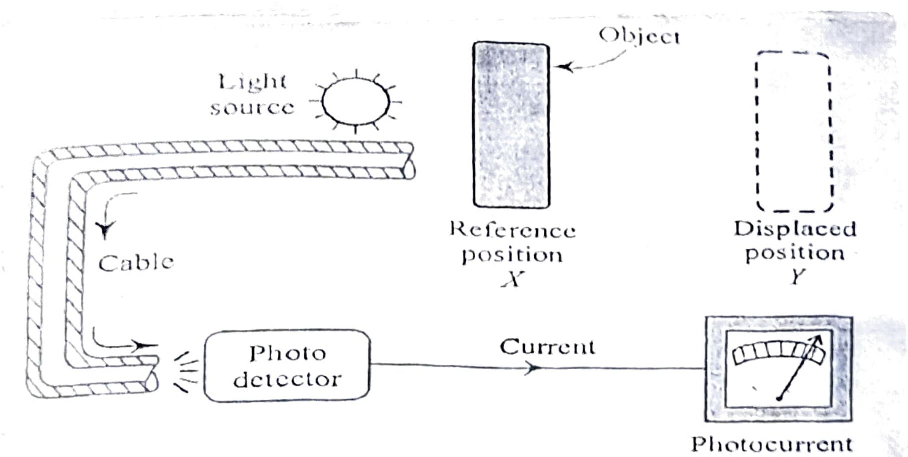
between the valence band & conduction band which is turn result in stimulated emission.

4. Photons produced are amplified between polished optical resonator surfaces producing laser beam.

5. Ga-As laser produce laser beam of wavelength 8870 Å in IR region, GaAsP produce laser beam of 6500Å in visible region etc.

5b. With neat diagram, explain the working of intensity based displacement sensor using optical fiber.

Displacement sensor (Based on Intensity modulation):



DISPLACEMENT SENSOR

Here, the optical fiber is used to convert the the displacement of the object into a proportionate variation in the intensity of light. This is one of the ~~fiber~~ types of intensity modulated sensors.

Principle: Light reflected from a body undergoing displacement is detected by using an optical fiber in combination with a photo-detector. The displacement that the body undergoes results in a proportionate change in the intensity of light.

Construction: It consists of light source of high intensity fixed firmly to a support (Figure). A bundle of of optical fibers enclosed in a cable is taken. One end of the cable (front end) is fixed to an adjustable support and properly oriented to gather the light reflected from the ~~box~~ object. The position of the light source must be well behind the front end. The other end (back end) of the cable is also fixed to an adjustable support. The emergent light from this end is gathered by a photo detector and is measured as a corresponding electric current.

Working:

The object is placed next to the cable's front end. Its position then is treated as its reference position, say X. It is then illuminated by the light source. The orientation of the cable at its front end is adjusted till the photo current reads maximum at which inclination it's firmly clamped. The same procedure is repeated for its other end also. Let the corresponding photo current be I_x which now corresponds to the position X of the object. The object is moved to a different position Y. The intensity of the light entering the cable changes due to the change in distance. Let the corresponding photocurrents be I_y.

Then the difference between in photocurrent $I = I_x - I_y$ is a measure of the distance $\Delta L = X - Y$. The value of ΔI for different known distances ΔL are measured and a graph of calibration of ΔL versus ΔI is drawn.

Now the object is released to follow its intended movement. As the object moves to a different location, its distance from the front end of the cable changes resulting in different value for Iy. The actual displacement is then found out with the help of the calibration graph after evaluating ΔI.

Measurement of displacement following this procedure helps when the movement of the object is in difficult - to reach locations. Through this method, the amplitude & frequency of vibration of a body can also ~~be~~ measured.

~~29~~ 30

5c. Estimate the attenuation in an optical fiber of length 500 m, when a light signal of power 100 mW emerges out of fiber with a power 90 mW.

Soln Given data

$L = 500\,m = 0.5\,Km$

$P_{in} = 100\,mW = 100 \times 10^{-3}\,W$

$P_{out} = 90\,mW = 90 \times 10^{-3}\,W$

$\alpha = ?$

Attenuation constant (α) is given by

$\boxed{\alpha = \frac{P_L}{L}}$ and $P_L = 10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)$

$$P_L = 10 \log_{10}\left[\frac{100 \times 10^{-3}}{90 \times 10^{-3}}\right]$$

$$= 10 \log_{10}\left[\frac{10}{9}\right] = 10 \log_{10}[1.11]$$

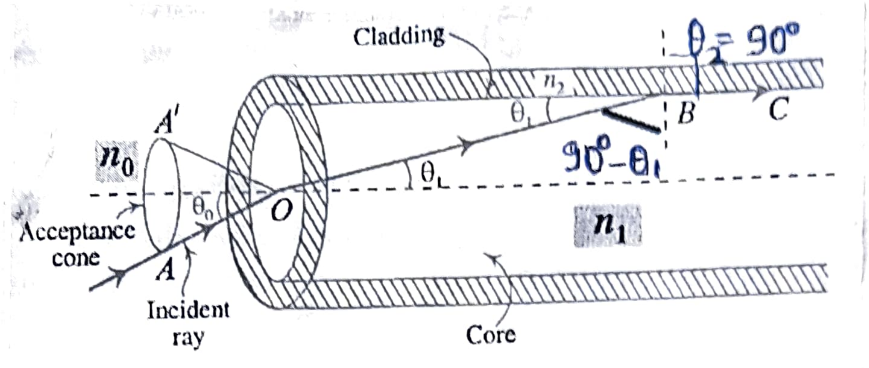
$$= 10 \times 0.04532$$

$$P_L = 0.4532\ dB.$$

$$\alpha = \frac{P_L}{L} = \frac{0.4532}{0.5}$$

$$\alpha = 0.9064\ dB/Km$$

6a. Derive the expression for numerical aperture of an optical fiber. Mention any two merits & demerits of optical communications.



ACCEPTANCE CONE

Consider a light ray AO incident at an angle "θ_0" enters into the fiber.

Let "θ_1" be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle $(90^\circ - \theta_1)$ at B grazes the interface between core & cladding along BC.

If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus θ_0 is called the waveguide acceptance angle & $\sin\theta_0$ is called the ~~acceptance an~~ numerical aperture.

Let n_1, n_2 & n_0 be the R.I. of core, cladding & surrounding medium respectively.

Also AO be the incident ray, AB → refracted ray, BC totally reflected ray, θ_i & θ_r be the angle of incidence & refraction at A, θ_0 & θ_r be the angle of incidence & angle of refraction at B resp.

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32 Using Snell's law

At position "O"

$n_0 \cdot \sin\theta_0 = n_1 \sin\theta_1$ —— ①

At position "B", angle of incidence $= 90^\circ - \theta_1$

angle of refraction $= 90^\circ$

Using Snell's law

$$n_1 \sin(90^\circ - \theta_1) = n_2 \sin 90^\circ$$

$$n_1 \sin(90^\circ - \theta_1) = n_2$$

$$n_1 \cos\theta_1 = n_2$$

$$\boxed{\cos\theta_1 = \frac{n_2}{n_1}} \text{ —— ②}$$

Re arranging the equ^n (1), we have

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0} \cdot \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0} \sqrt{1 - \cos^2\theta_1} \text{ —— ③} \quad \left[\sin\theta_1 = \sqrt{1 - \cos^2\theta_1}\right]$$

Using equ^n ②, equ^n ③ becomes.

Put the value of $\cos\theta_1$ in equ^n ③, we have

$$\sin\theta_0 = \frac{n_1}{n_0} \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0} \times \frac{1}{n_1} \sqrt{n_1^2 - n_2^2}$$

$$\boxed{\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}}$$

If the surrounding medium is air $n_0 = 1$, then

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

Merits of optical communications:

1. Used in networking systems.
2. It carries very large amount of information in either digital or analog form due to its large bandwidth
3. It is easily compatible with electronic system.
4. It can be operated in high temperature range.

Demerits:

1. Fiber may suffer line break, operations required to re-establish the connections are highly skilful & time consuming.
2. Fiber undergo expansion & contraction with temp that upset some critical alignments which leads to loss in signal power.

6 b. Explain how laser find application in eye surgery.

In the field of medicine, the success achieved in the use of lasers in eye surgery, especially in treating retinal detachment, is spectacular.

In the eye, under certain abnormal conditions, the retina may get detached from the choroid, which may result in the blindness at the detached part of the retina. The only way to overcome this problem is to reset the retina back to its original state, i.e. attaching it back to choroid by heating it over a number of spots almost of the size of tissues. The effect of heating is achieved by focussing a laser beam of predetermined intensity on to the retina. The laser beam enters into the front of the eye and then passes through transparent tissues and the liquid part (aqueous and vitreous humors), without being absorbed to reach the right spot where the welding of the retina to the choroid takes place.

The flash of the laser beam lasts for a such a short period (one millisecond) that, the patient does not react to the light so that, no involuntary contraction of the eye muscle would occur. The patient feels no pain which means no anaesthesia is required. The operation does not even require an operating theater and ~~a~~ essentially the cases are dealt with as outpatients.

6 c. The ratio of population of two energy levels out of which upper one corresponds to a metastable state is 1.059×10^{-30}. Find the wavelength of light emitted at 330 K

Soln Given data:
The ratio of population, $\frac{N_2}{N_1} = 1.059 \times 10^{-30}$

Ambient temperature $T = 330\,K$

To find:
The wavelength of light emitted, $\lambda = ?$

We have Boltzmann factor

$$\frac{N_2}{N_1} = e^{-\Delta E/KT} = e^{-\frac{hc}{\lambda KT}}$$

By taking natural log on both sides, we have

$$\ln\left[\frac{N_2}{N_1}\right] = -\frac{\Delta E}{KT} = -\frac{hc}{\lambda KT} = -\left(\frac{hc}{K}\right)\left(\frac{1}{\lambda T}\right)$$

$$= -\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23}}\right)\left(\frac{1}{\lambda T}\right)$$

$$\ln\left[1.059 \times 10^{-30}\right] = -\frac{0.014413}{\lambda T}$$

$$\therefore \lambda = -\frac{0.014413}{\ln\left[\frac{N_2}{N_1}\right] \cdot T} = \frac{-0.014413}{\ln[1.059 \times 10^{-30}] \times 330}$$

$$\boxed{\lambda = 632\ nm}$$

$\therefore$ The wavelength of light emitted by spontaneous emissions is 632 nm

7a. Mention any four assumptions of Drude-Lorentz model & discuss the success of Quantum free electron theory.

Assumptions of Drude - Lorentz model (CFET):

1. All the metals consists of free moving valence electrons called free electrons. when electric field is applied these $\bar{e}$ move opposite to the direction of applied field.
2. The free $\bar{e}$ are treated as equivalent to gas molecules and thus obey laws of kinetic theory of gases.

$$\frac{3}{2}KT = \frac{1}{2}mV_{th}^2$$

3. The electrical potential due to ionic cores is taken to be essentially constant throughout the metal.
4. The attraction between $\bar{e}$ & lattice ions and repulsion between themselves are considered insignificant.

Success of QFET:

The QFET is successful in explaining the drawbacks of CFET:

① Specific heat capacity:

As per CFET, all the conduction electrons are capable of absorbing the heat energy which results in a large value of specific heat ($C_V = 3/2 R$).
But according to QFET, only those electrons which are close to E_F are capable of absorbing the heat energy to get excited to higher energy

levels & hence the specific heat value becomes very small.

It was shown that

$$C_V = \left[\frac{2K}{E_F}\right] RT$$

Taking the typical value of $E_F = 5eV$, we get

$$\left[\frac{2K}{E_F}\right] \simeq 10^{-4} \qquad \therefore \boxed{C_V = 10^{-4} RT}$$

This is experimental value.

(2) Temperature dependence of electrical conductivity:

According to classical theory,

$$\sigma \propto \frac{1}{\sqrt{T}} \quad \text{—— (1)}$$

According to QFET, the electrical conductivity, is given by

$$\sigma = \frac{ne^2}{m^*}\left[\frac{\lambda}{V_F}\right] \quad \text{—— (2)}$$

We have $E_F \rightarrow$ temperature independent.

$\therefore$ V_F also temp. independent.

But "λ" is temp. dependent. The nature of dependence of "λ" and "T", can be analysed as follows

During the conduction electrons scattered by the vibration of lattice ions & the displacement of ions takes place equally in all directions.

If "r" is the amplitude larger area of cross section should scatter more efficiently then the means free path of the electron will reduce.

$$\therefore \lambda \propto \frac{1}{\pi r^2}$$

But $\pi r^2 \propto T$

$$\therefore \boxed{\lambda \propto \frac{1}{T}}$$

Comparing ~~as~~ this with equⁿ ②, we have

$$\sigma \propto \lambda \propto \frac{1}{T}$$

$$\therefore \boxed{\sigma \propto \frac{1}{T}}$$

③ Electrical conductivity & electron concentration:

We have equⁿ for the electrical conductivity.

$$\boxed{\sigma = \frac{ne^2}{m^*}\left[\frac{\lambda}{V_F}\right]}$$

The value of σ depends on both "n" & λ/V_F.

If we compare the cases of copper & aluminium, the value of "n" for aluminium is 2.13 times higher than that of copper. But the value of λ/V_F for copper is about 3.73 times higher than that of "Al". Thus the conductivity of copper exceeds that of "Al". Hence the dependence of electrical conductivity on electron concentration is correctly explained by the Quantum free electron theory.

7b. Derive Clausius - Mossotti equation.

Consider an elemental solid dielectric material of dielectric constant ϵ_r.

If N is the number of atoms/unit volume of the material,

$\mu \rightarrow$ is the atomic dipole moment, then we have

Dipole moment/unit volume $= N\mu$ —— (1)

Here the field experienced by the atoms is the internal field E_i.

Hence, if α_e is the electronic polarizability of the atoms, we can write the equation for μ as

$$\boxed{\mu = \alpha_e E_i} \quad \text{—— (2)}$$

$\therefore$ Equn (1) becomes

Dipole moment/unit volume $= N\alpha_e E_i$ —— (3)

In equn (3), its left side is same as polarization P,

$$P = N\alpha_e E_i \quad \text{—— (4)}$$

or

$$E_i = \frac{P}{N\alpha_e} \quad \text{—— (5)}$$

But, we have the relation for P as,

$$P = \epsilon_0(\epsilon_r - 1)E,$$

where E is the applied field.

$$E = \frac{P}{\epsilon_0(\epsilon_r - 1)} \quad \text{---} (6).$$

Also, we have the equation for internal field as,

$$E_i = E + \gamma \frac{P}{\epsilon_0} \quad \text{---} (7)$$

where, γ is the internal field constant.

Substituting for E_i and E from equn (5), & (6) in equn (7), we have

$$\frac{P}{N\alpha_e} = \frac{P}{\epsilon_0(\epsilon_r - 1)} + \gamma \frac{P}{\epsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0}\left[\frac{1}{(\epsilon_r - 1)} + \gamma\right]$$

Considering the internal field in the material to be Lorentz field, we have $\gamma = 1/3$. Substituting the same in the above equn, we get

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0}\left[\frac{1}{(\epsilon_r - 1)} + \frac{1}{3}\right]$$

$$= \frac{1}{\epsilon_0}\left[\frac{3 + \epsilon_r - 1}{3(\epsilon_r - 1)}\right]$$

$$\therefore \frac{\epsilon_0}{N\alpha_e} = \left[\frac{(\epsilon_r + 2)}{3(\epsilon_r - 1)}\right]$$

By rearranging the above we have

$$\boxed{\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}} \quad \text{---} (8)$$

This is equn for Clausius-Mossotti equn & it holds good for crystals of high degree of symmetry

7C. Show that occupation probability at an energy $E_F + \Delta E$ is equal to non-occupation probability at the energy $E_F - \Delta E$.

Soln/: Let $E = E_F + \Delta E$ & $E' = E_F - \Delta E$

To prove:
Occupation probability at $E = E_F + \Delta E$ is equal to non-occupation probability at $E' = (E_F - \Delta E)$.

Proof:
W.K.T occupation probability is given by the fermi factor.

$$f(E) = \frac{1}{e^{(E-E_F)/KT} + 1} \quad \text{—} (1)$$

∴ The occupation probability for the energy state $E = E_F + \Delta E$ is obtained by substituting $(E_F + \Delta E)$ in place of E in eqn (1).

$$\therefore \left[\text{Occupation probability}\right]_{E=(E_F+\Delta E)} = \frac{1}{e^{\frac{\Delta E}{KT}} + 1} \quad \text{—} (2)$$

Similarly, occupation probability for the energy state $E' = (E_F - \Delta E)$ is,

$$f(E') = \frac{1}{e^{(E'-E_F)/KT} + 1}$$

By substituting $E' = (E_F - \Delta E)$, we get

$$f(E') = \frac{1}{e^{\frac{-\Delta E}{KT}} + 1} \quad \text{—} (3)$$

Now, we ~~have~~ shall consider the non-occupation probability for the energy state, $E' = (E_F - \Delta E)$.

~~Wk~~

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We know from the theory of probability, that,

probability of absence = 1 − probability of presence

∴ Non-occupation probability = 1 − occupation probability

or $[\text{Non-occupation probability}]_{E'} = 1 - f(E')$

∴ $[\text{Non-occupation probability}]_{E' = (E_F - \Delta E)}$

$$= 1 - \frac{1}{e^{-\Delta E/KT} + 1} \quad \text{—— (6)}$$

For the sake of handling the terms let us put,

$$e^{(\Delta E/KT)} = x \quad \text{—— (5)}.$$

$$e^{-(\Delta E/KT)} = \frac{1}{x}$$

∴ Equn (4) becomes,

$$\text{Non-occupation probability} = 1 - \frac{1}{\left(\frac{1}{x}+1\right)} = 1 - \frac{1}{\left(\frac{1+x}{x}\right)}$$

$$= 1 - \frac{x}{1+x} = \frac{(1+x) - x}{(1+x)} = \frac{1}{1+x}$$

Equn (5), the above equation becomes.

$[\text{Non-occupation probability}]_{E' = E_F - \Delta E} = \cancel{}$

$$= \frac{1}{1 + e^{\left(\frac{\Delta E}{KT}\right)}} \quad \text{—— (6)}$$

comparing Equn (2) & (6) we see that, occupation probability for $E = (E_F + \Delta E)$ is equal to non-occupation probability for $E' = (E_F - \Delta E)$.

Hence, it is ~~the~~ proved..

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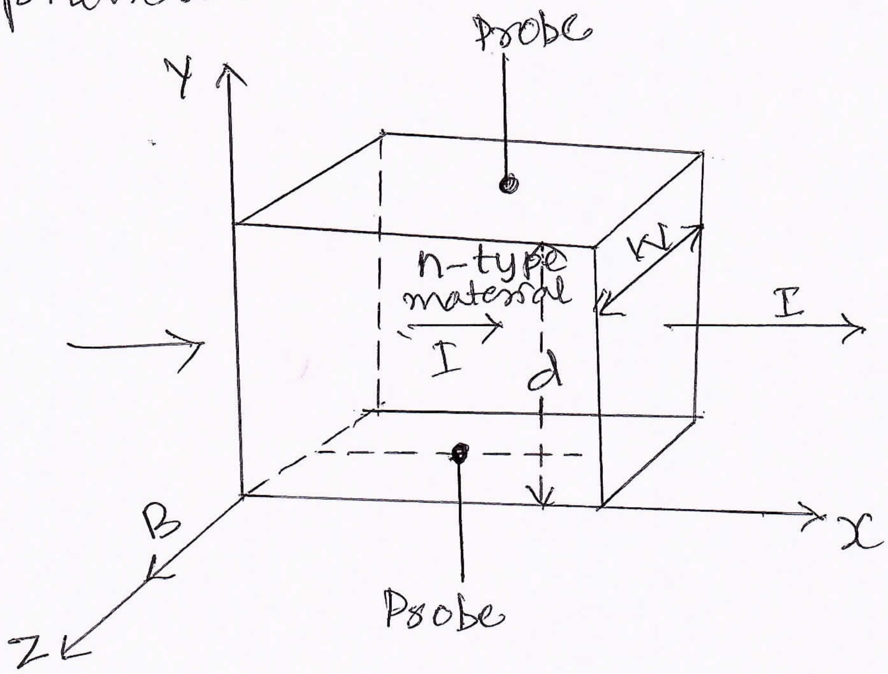
8a. What is Hall effect. Obtain the expression for the Hall coefficient.

Statement: If a material carrying current is placed in a transverse magnetic field, an electric field is produced in the material in a direction $\perp^{lar}$ to both the current & magnetic field. This phenomenon is called Hall effect.



Consider a rectangle slab of a semiconductor material in which a current "I" is flowing in +ve x-direction.

Let the semiconductor material be of n-type which means that the charge carriers are electrons.

Let a magnetic field "B" is applied along z-direction as shown in the figure.

Under the influence of a magnetic field, the electrons experience the Lorentz-force, F_L is given by

$$\boxed{F_L = -BeV} \quad (1)$$

-ve sign is due to electron charge.

e → magnitude of charge on the $\bar{e}$ &

v - is drift velocity of the $\bar{e}$.

Applying the Flemings Left-hand rule, the force F_L is acting on the electron along the -ve Y-direction. The electrons are therefore deflected downwards. As a results the density of electrons increases in lower end of the material, due to which its bottom edge becomes -vely charged. On the other hand the loss of $\bar{e}$ from the upper end causes the top edge of the material to be +vely charged. Hence a potential difference V_H called Hall voltage appears between the upper and lower surface of the semiconductor material which establishes an electric field E_H, called Hall field across the conductor in the -ve Y-direction.

The field E_H, exert an upward force F_H on the electrons given by

$$F_H = -e E_H \quad — ②$$

Now as the deflection of $\bar{e}$ continues in the downward direction due to Lorentz force F_L, the Hall field increases.

As a result the force F_H which acts on the $\bar{e}$ in upward direction also increases till it becomes equal to F_L.

Thus at equilibrium,

$$F_L = F_H$$

$$-BeV = -e E_H$$

$$\boxed{E_H = BV} \quad — ③$$

The current density (J),

$$J = \frac{I}{A} = \frac{neVA}{A}$$

$$J = neV \qquad \boxed{v = \frac{J}{ne}}$$

Substituting for v, in equⁿ (3) we get

$$E_H = B \cdot \frac{J}{ne} \quad \text{--- (4)}$$

The Hall voltage, $V_H = E_H \cdot d = \frac{BJd}{ne}$ --- (5)

where n is charge carrier concentration.

Let "w" be the thickness of the material in Z-direction, then the area of cross section normal to direction of current "I" is $A = w \cdot d$.

$$\therefore \quad V_H = \frac{BId}{neA} = \frac{BId}{ne(w \cdot d)}$$

$$V_H = \frac{BI}{new}$$

$$V_H = \frac{BI}{\rho_{ch} w} \qquad (\rho_{ch} = ne)$$

where ρ_{ch} is the charge density which is a constant for a given material

Hence, $\boxed{V_H = \frac{R_H BI}{w}}$

where R_H = is a constant called Hall coefficient.

$$\boxed{R_H = \frac{1}{\rho_{ch}} = \frac{1}{ne}}$$

Thus by measuring V_H, I & w & by knowing B, the charge density ρ_{ch} can be determined using which Hall coefficient can be evaluated.

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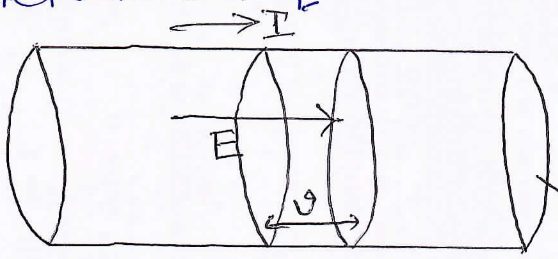
8b. Obtain expression for electrical conductivity in ~~metals on quantum model~~. semiconductor.



When an electric field is applied to a semiconductor, there will be movement of $\bar{e}$ in conduction band (C.B) & holes in valence band (V.B). Hence there will be two currents, an electron current in C.B & holes current in V.B. As these currents are constituted by oppositely charged carriers, the net current will be sum of the two currents.

Consider a semiconductor of area of cross section "A".

Let an electric field of "E" is applied to the conductor.

Let Ve and Vh be the drift velocities of electrons & holes resp. due to applied field "E".

~~The current~~

We know that the expression for current in a conductor is

$$\boxed{I = neAVd} \quad \text{—} (1)$$

current due to electrons can be written as

$$\boxed{I_e = n_e e A V_e} \longrightarrow (2)$$

where $I_e \rightarrow$ current in a semiconductor due to $\bar{e}$.

$e \rightarrow$ charge of $\bar{e}$

$A \rightarrow$ Surface area of semiconductor

$V_e \rightarrow$ drift velocity of electrons.

$n_e \rightarrow$ number ~~of~~ density of electrons.

Similarly, current due to holes is

$$\boxed{I_h = n_h e A V_h} \longrightarrow (3)$$

where $V_h \rightarrow$ drift velocity of holes. in SC's

$n_h \rightarrow$ number density of holes in SC's.

The total current is

$$I = I_e + I_h \longrightarrow (4)$$

Using equn (2) & (3), Equn (4) becomes.

$$I = I_e + I_h$$

$$= n_e e A V_e + n_h e A V_h$$

$$I = eA[n_e V_e + n_h V_h]$$

$\therefore$ The current density, $\boxed{J = \frac{I}{A}}$, then

$$J = e[n_e V_e + n_h V_h] \longrightarrow (5)$$

Also, w.k.t. Relation betn I, J & E then

$$\boxed{J = \sigma E} \longrightarrow (6)$$

then.

Equn (5) can be written as

$$\sigma E = e[n_e v_e + n_h v_h]$$

Dividing by "E" on both sides

$$\sigma = e\left[n_e \frac{v_e}{E} + n_h \cdot \frac{v_h}{E}\right]$$

$\therefore$ $\boxed{\mu_e = \frac{v_e}{E}}$, Mobility of electrons.
= drift velocity of $\bar{e}$ / applied field

and $\boxed{\mu_h = \frac{v_h}{E}}$, Mobility of holes

$$\boxed{\sigma = e[n_e \mu_e + n_h \mu_h]} \quad —(6)$$

This is the equn for conductivity in Semiconductor.

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8 b. Obtain expression for electrical conductivity in metals on quantum model.

The energy of free electron can be written in terms of momentum "P" is

$$\boxed{E = \frac{P^2}{2m}} \quad \text{—} (1) \qquad (\because P^2 = 2mE)$$

Using de-Broglie wavelength "λ"

$$\boxed{P = \frac{h}{\lambda}} \qquad (\because \lambda = \frac{h}{P}).$$

Equn (1) becomes

$$E = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m}$$

$$\boxed{E = \frac{h^2}{2m\lambda^2}} \quad \text{—} (2)$$

E can be expressed in terms of wave number "K".

$$\lambda = \frac{2\pi}{K}$$

$\therefore$ Using the value of "λ"
Equn (2) becomes

$$E_K = \frac{h^2}{2m(2\pi/K)^2}$$

$$E_K = \frac{h^2 \cdot K^2}{2m\pi^2} \quad \text{—} (3).$$

In the ground state of the free $\bar{e}$, the maximum energy of electrons is the fermi energy E_F.

∴ Equ^n 3 can be written for this state is

$$\boxed{E_F = \frac{h^2}{8\pi^2 m} \cdot K_F^2} \quad \text{— (4)}$$

We know that, the general expression for drift velocity V_d is,

$$\boxed{V_d = \frac{eE\tau}{m}} \quad \text{— (5)}$$

where $\tau \rightarrow$ is the average time elapsed after the collision.

∴ the energy density is

~~J = ne~~

$$\boxed{J = neV_d} \quad \text{— (6)} \qquad \left(\because I = neAV_d,\ J = \frac{I}{A}\right)$$

Using equ^n 5, the equ^n 6 becomes

$$J = neV_d$$

$$J = ne\left(\frac{eE\tau}{m}\right)$$

$$\boxed{J = \frac{ne^2 E\tau}{m}}$$

Also $J = \sigma E$

$$\sigma E = \frac{ne^2 E\tau}{m}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \quad \text{— (7)}.$$

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If "λ" is the ~~sum~~ mean free path & "V_F" is the speed of free electrons whose kinetic energy is equal to fermi energy.

Since, only electrons near fermi level contribute to the conductivity.

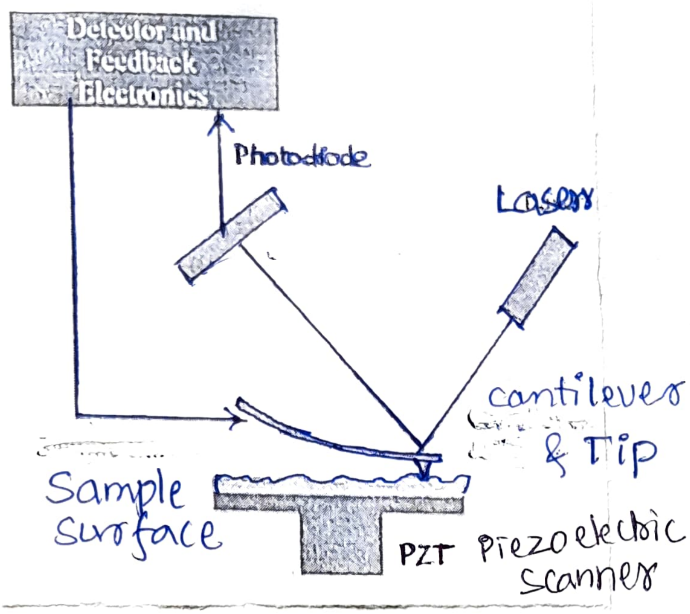
The average time τ between two collision of free electrons with core ions is given by

$$\boxed{\tau = \frac{\lambda}{V_F}}$$

$\therefore$ Equ^n (7) becomes,

$$\boxed{\sigma = \frac{ne^2}{m}\left(\frac{\lambda}{V_F}\right)}$$

9a. With neat diagram, explain the principle, and construction and working of Atomic Force Microscope (AFM). 119



The atomic Force microscope (AFM) is a type of scanning probe microscope whose primary roles include measuring properties such as magnetism, height, friction. The resolution is measured in a nanometer, which is more accurate & effective than the optical diffraction limit. It uses a probe for measuring & collection of data involves touching the surface that has the probe.

Principle: The AFM works on the principle ~~sees~~ sees measuring intermolecular forces & sees surfaces of the specimen atoms by using probed surfaces in nanoscale.

An Atomic Force Microscope (AFM) consists of following components:

1) LASER
2) photodiode
3. cantilever with sharp tip
4. Detector & feedback circuit
5. Piezoelectric sensor.

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Working of AFM:

1. AFM consists of microscope cantilever with a sharp tip (probe) at its end used to scan the specimen surface.
2. The cantilever is typically silicon or silicon nitride with the tip radius of curvature of the orders of nm. Basically, AFM is modified TEM in which limitations of TEM is overcomed. When the tip is bought close to the sample, force between the tip and sample leads to the deflection of the cantilever according to the Hooke's law. Instead of using an electrical signal, the AFM relies on forces between the atom on the tip & in the sample.
3. The force present in the tip is kept constant and ~~is~~ the scanning is done. As the scanning continues, the tip will have vertical movements depending upon the topography of the sample. The force present in the tip is kept constant & the scanning is done. As the scanning continue the tip will have vertical movement depending upon the topography of the sample.

4. A LASER beam is used to have a record of vertical movement of the needle. This information is later converted into visible from ~~the~~ using photo diode. Depending upon the situation, AFM measures different types of forces like a ~~Vaand~~ Vander waal's forces, capillary force, mechanical contact force etc.

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9 b. Explain in brief how crystal size is determined by scherrer's equation.

The Scherrer equation, in X-ray diffraction & crystallography, is a formula that relates the size of sub micrometer crystallites in a solid to the broadening of a peak in a diffraction pattern. It is used in the determation of size of crystals in the form of powder.

The Scherrer eqn can be written as

$$\boxed{D = \frac{K\lambda}{\beta \cos\theta}}$$

where "D" is the mean size of the ordered (crystalline) domains, which may be smaller or equal to the grain size, which may be smaller or equal to the particle size.

"K" is a dimensionaless shape factor, with a value close to unity. The shape factor has a typical value of about 0.9, but varies with the actual shape of the crystallite.

λ is the X-ray wavelength,

$\beta \rightarrow$ is the line broadening at half the maximum intensity [Full width at half maximum - FWHM] value in radians], after subtracting the instrumental line broadening in radians.

$\theta \rightarrow$ is the peak position Bragg's angle.

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The scherrer equation is a widely used to determine the crystallite size of polycrystalline samples. However, it is not clear if one can apply it to large crystallite sizes because its derivation is based on the kinematical theory of x-ray diffraction.

The FWHM is then extracted & crystallite size is computed using the scherrer equation. It is shown that for the crystals with linear absorption coeffs below 2117.3 cm^{-1}, ~~to~~ the Scherrer equ^n is valid for crystallites with sizes up to 600nm.

It is also shown that the as the size increases only the peaks at higher 2θ angles give good results, and if one uses peaks with $2\theta > 60^\circ$ the limit for use of the scherrer equation would go up to 1 um.

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9c. Determine the wavelength of x-rays for crystal size of 1.188×10^{-6} m, peak width is 0.5° & peak position 30° for a cubic crystal.
Given: Scherrer constant $K = 0.92$

Solution:

Given data:

$\lambda = ?$

$D = 1.188 \times 10^{-6}$ m $\beta = 0.5$ $\theta = 30^\circ$

$K = 0.92$

Using Scherrer's equation, we have

$$D = \frac{K\lambda}{\beta \cos\theta}$$

$$\lambda = \frac{D.\beta.\cos\theta}{K}$$

$$= \frac{1.188 \times 10^{-6} \times 0.5 \times \cos 30^\circ}{0.92}$$

$$= \frac{1.188 \times 10^{-6} \times 0.5 \times 0.8660}{0.92}$$

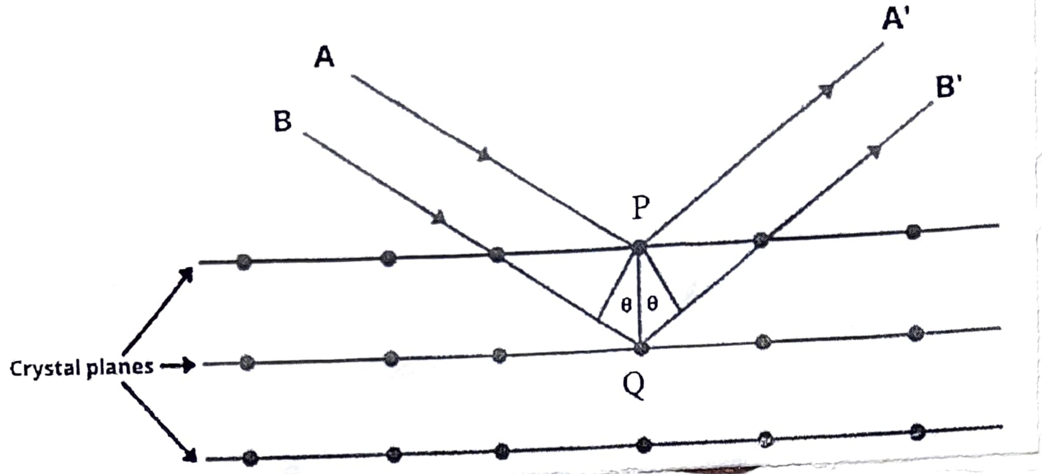
$$= \frac{0.5144}{0.92}$$

$$\lambda = 0.55913 \times 10^{-6}$$

$$\lambda = 5591.34 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 5591.34 \text{ A}^\circ}$$

10.a. Explain the construction & working of X-ray diffractometer.



Principle:

X-ray diffraction is based on constructive interference of monochromatic X-rays & a crystalline sample. This law relates the wavelength of EM ~~waves~~ radiation to the diffraction angle & the lattice spacing in a crystalline sample. These diffracted X-rays are then detected, processed & counted.

Construction:

Consider a set of parallel atomic planes of the crystal with Miller indices [h k l], such that the distance between the two successive planes is d. Let a parallel beam of monochromatic X-rays of wavelength 'λ' be incident on the plane at a glancing angle "θ", such that the incident rays lie in the plane of the paper.

Let AP & BQ be two parallel incident rays that are reflected from points P & Q on the crystal planes & travel along with PA' & QB' resp.

If the path difference between APA' & BQB' is an integral multiple of "λ", there will be constructive interference & a maximum will be observed.

Therefore, Bragg's law is given as

$$\boxed{n\lambda = 2d \cdot \sin\theta}$$

Working:

The technique of single crystal x-ray crystallography has three basic steps.

The crystal is placed in an intense beam of x-rays usually of a single wavelength of monochromatic x-rays producing the regular pattern of deflections. the crystal is gradually rotate at previous reflections disappear & new ones appear the intensity of every spot is recorded at every orientation of the crystal.

Multiple data sets may ~~also~~ have to be collected set of covering slightly more than half a full rotation of the crystal & type weekly containing of tens of thousands of reflections.

Third step this data source or combined computationally with complimentary chemical information to produce & refine your model of the arrangement of atom within the crystal the final refined model of the atomic arrangement now called as crystal structure.

The first tees to after in an adequate crystal of the material under study which is often most difficult. The crystal should be sufficiently large typically larger than 0.1mm in all dimensions. The material should be pure in composition in structure, no significant internal in perfections such as tracks of twinning.

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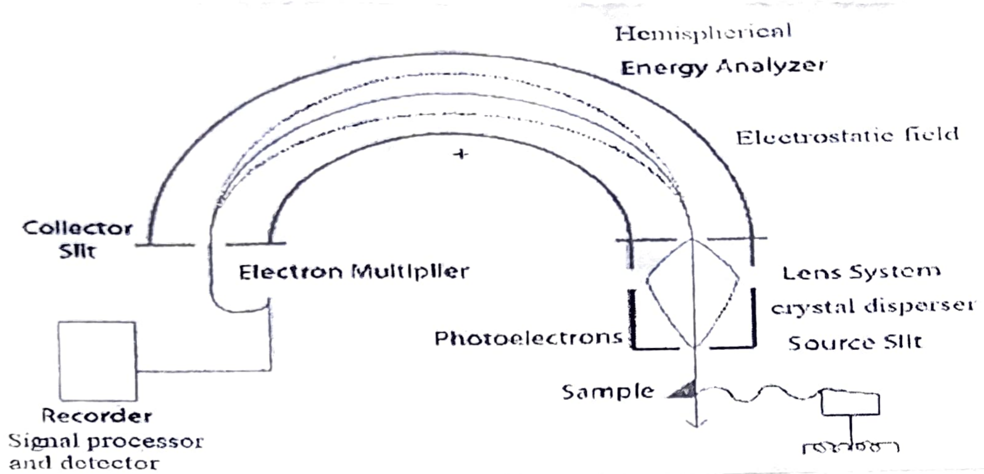
10.b. With neat diagram, explain the principle, construction & working of X-ray photoelectron spectroscope.



The technique of X-ray photoelectron spectroscopy is also known as Electron spectroscopy for chemical analysis. X-Ray photo electron spectroscopy is a type of electron spectroscopy, it is an analytical technique to study the electronic structure & it is dynamic in atoms & molecules.

Principle:

Due to the bombardment of X-ray photon on the sample surface K and L electron are ejected which are further analysed by the analyser.

Let us consider E_b, E_b' & E_b'' are binding energy of lower energy levels inner core orbitals. Where E_v, E_v' and E_v'' are the energies of the vehicles shell electron.

The monochromatic x-ray photon incident on the sample surface cell electron abstract the energ from this x-ray photon & get ejected in terms of electron. Kinetic energy of the ejected electron is recorded by spectrometer & is given by

$$\boxed{E_K = h\nu - E_b - \phi}$$

where $E_K \rightarrow$ is K.E of ejected electron
$h\nu \rightarrow$ energy associated with incident photon.
$E_b \rightarrow$ binding energy ejected electron.
$\phi \rightarrow$ work function of the instrument.

Construction:

The electron spectrometer made up of following components:

* source
* Sample holder
* Analyser
* Detector
* Processor & the Read out.

Source: The simple x-ray photon source for x-ray photoelectron spectra ~~M~~ is ~~X~~-ray tube equipped with magnetism or aluminium metal ~~too~~ target. Monochromator crystal can also provide having band width of 0.3 electron-volt. Much smaller spots on a surface to be examined.

Sample Holder:

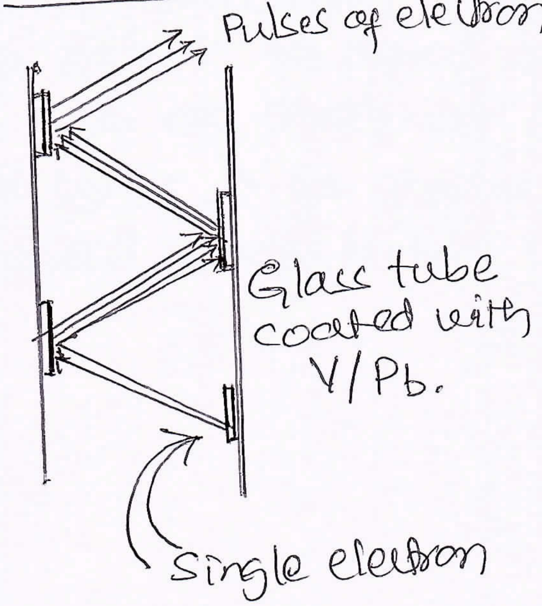
Sample holder is located in between the source & the entrance slit of spectrometer. Crystal dispenser selects the photon of known energy from the source & incident on the sample. The area inside the sample holder should be evacuated within 10^{-5} Torr. Pressure to avoid communication of the surface sample.

The gaseous sample for introduced into a sample compartment through a slit, to provide a pressure of 10^{-12} Torr. If the pressure is higher than attenuation of electron beam may take place, weaker signal may be ~~attenuated~~ obtained.

Analyser:

It is hemispherical in shape with very high electrostatic field is applied on analyser. Pressure maintained inside the analyser is 10^{-5} Torr. When the electron enters, into the hemispherical analyser, it travels in curved path & radius of curvature depends upon ~~magnitude~~ magnitude of field & kinetic energy of the electron.

Detector:



The electron channel multilayer tube or transducer are required of X-ray photo electron spectroscopy. When single electron pass through the electron multiplier tube, it is converted into number of electrons are pulses of electrons.

Signal processor & Read out:

The function of signal processor is to amplify the signal & read out device, converts the signal into spectrum.

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10.c. The first order Bragg's reflection occurs when a monochromatic beam of x-rays of wavelength 0.675 A° is incident on a crystal at a glancing angle of 4°. What is the glancing angle for third order Bragg's reflection to occur?

Solution:

Wavelength of X-rays, $\lambda = 0.675 \times 10^{-10}$ m

For first order reflection $n=1$

Glancing angle, $\theta_1 = 4°$.

To find:

To find third order reflection $(n=3)$, $\theta_3 = ?$

Soln: $2d\sin\theta = n\lambda$

For first order reflection

$$2d\sin\theta_1 = 1 \times \lambda$$

$$2 \times d \times \sin 4° = 1 \times 0.675 \times 10^{-10}$$

$$2 \times d \times 0.0697 = 0.675 \times 10^{-10}$$

$$d \times 0.1395 = 0.675 \times 10^{-10}$$

$$d = \frac{0.675 \times 10^{-10}}{0.1395} = 4.8387 \times 10^{-10}$$

$$d = 4.838 \times 10^{-10}\text{ m}$$

For third order reflection,

$$2d \cdot \sin\theta_3 = 3\lambda$$

$$2 \times 4.838 \times \sin\theta_3 = 3 \times 0.675 \times 10^{-10}$$

$$\sin\theta_3 = 0.253125$$

$\boxed{\theta_3 = 14.6625°}$ or $\boxed{\theta_3 = 14° 39' 45''}$

Dean, Academics
KLS VDIT, HALIYAL

(Dr. M.S. Bannur).