

Model Question Paper-II with effect from 2021 (CBCS Scheme)

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FIRST/SECOND Semester BE Degree Examination ENGINEERING PHYSICS - 21PHY12/22

TIME: 03 Hours					Max. Marks: 100		
Note:	1. Answer any FIVE full questions, choosing at least ONE question from each MODULE . 2. Draw neat sketches where ever necessary. 3. Constants : Speed of Light “c” = $3 \times 10^8 \text{ ms}^{-1}$, Boltzmann Constant “k” = $1.38 \times 10^{-23} \text{ JK}^{-1}$, Planck’s Constant “h” = $6.625 \times 10^{-34} \text{ Js}$, Acceleration due to gravity “g” = 9.8 ms^{-2} , Permittivity of free space “ ϵ_0 ” = $8.854 \times 10^{-12} \text{ F m}^{-1}$.						
Module -1					Marks		
Q.01	a	Discuss the theory of forced oscillations and hence classify the conditions of variation of amplitude and phase with angular frequency.				9	
	b	Illustrate the generation of shock waves using the Reddy shock tube.				6	
	c	Given the damping constant of the medium 0.1 kg s^{-1} calculate the amplitude of the oscillations at resonance given the mass attached to the spring-mass oscillator $50 \times 10^{-3} \text{ kg}$, the amplitude of the applied periodic force 1N and the period of oscillations 1 second.				5	
OR							
Q.02	a	Applying Hooke’s law arrive at the equations for the effective spring constants of Series and Parallel combinations of springs.				8	
	b	Enumerate the properties and applications of shock waves.				7	
	c	Compare the Mach number of a Jet fighter traveling with 2000 km hr^{-1} with that of a bullet traveling with a velocity of 400 ms^{-1} in the same medium given the speed of sound in the medium 330 ms^{-1} .				5	
Module-2							
Q. 03	a	Discuss the spectral distribution energy in the black body radiation spectrum and hence explain Wien’s displacement law.				8	
	b	State and Explain Heisenberg’s Uncertainty principle and infer on the classical and quantum mechanical measurements.				7	
	c	The kinetic energy of an electron is equal to the energy of a photon with a wavelength of 560 nm. Calculate the de Broglie wavelength of the electron.				5	
OR							
Q.04	a	Discuss the motion of a quantum particle in a one-dimensional potential well of the infinite height and of width ‘a’ and also examine the quantization of energy.				10	
	b	Deduce Rayleigh-Jeans law from Planck’s Law of radiation.				5	
	c	The speed of electron is measured to within an uncertainty of $2 \times 10^4 \text{ ms}^{-1}$ in one dimension. What is the minimum width required by the electron to be confined in an atom?				5	
Module-3							
Q. 05	a	Obtain the expression for energy density using Einstein’s A and B Coefficients and hence draw infer on the relation $B_{12}=B_{21}$.				8	
	b	Discuss the attenuation and various losses in optical fibers.				7	
	c	Calculate the number of photons emitted per pulse of duration 1 microsecond given the power output of LASER 3 mW and the wavelength of laser 632.8 nm.				5	
OR							
Q. 06	a	Define Modes of Propagation and RI Profile and Distinguish between the types of optical fibers.				6	

	b	Identify the requisites of the CO2 LASER and Explain its construction and working with the help of a neat sketch and band diagram.	9		
	c	Compare the acceptance angle of an optical fiber placed in air and water given the RI of water 1.33 and the RI of core and clad 1.5 and 1.45 respectively.	5		
Module-4					
Q. 07	a	Explain the Quantum Mechanical modifications to the classical free electron theory of metals to explain the electrical conductivity in solids and its success.	7		
	b	What is Hall effect and illustrate on the determination of the type of charge carriers in semiconductors.	8		
	c	An elemental solid dielectric material has polarizability $7 \times 10^{-40} \text{ Fm}^{-2}$. Assuming the internal field to be Lorentz, calculate the dielectric constant for the material if the material has 3×10^{28} atoms/m ³ .	5		
OR					
Q. 08	a	Deduce the expression for electrical conductivity of a conductor using the quantum free electron theory of metals.	8		
	b	Describe in brief the various types of polarization mechanisms.	7		
	c	Calculate the probability that an energy level at 0.2eV below Fermi level is occupied at temperature 500K.	5		
Module-5					
Q. 09	a	Define nano-material and classify the nano-materials based on the dimensional constraints.	5		
	b	Describe the construction and working of Scanning Electron Microscope with the help of a neat diagram.	10		
	c	X-rays are diffracted in the first order from a crystal with d spacing $2.8 \times 10^{-10} \text{ m}$ at a glancing angle 60° . Calculate the wavelength of X-rays.	5		
OR					
Q. 10	a	Mention the principle and applications of X-ray photoelectron spectroscope.	5		
	b	Illustrate the working of Transmission Electron Microscope.	10		
	c	Determine the crystallite size given the Wavelength of X-Rays 10 nm , the Peak Width 0.5° and peak position 25° for a cubic crystal given $K = 0.94$.	5		

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L2	1	1,2,12
	(b)	L2	1	1,2,12
	(c)	L3	1	1,2
Q.2	(a)	L3	1	1,2,12
	(b)	L1	1	1,2
	(c)	L3	1	1,2
Q.3	(a)	L1	2	1,2,12
	(b)	L3	2	1,2,12
	(c)	L3	2	1,2
Q.4	(a)	L3	2	1,2,12
	(b)	L2	2	1,2,12
	(c)	L3	2	1,2
Q.5	(a)	L4	3	1,2
	(b)	L2	3	1,2
	(c)	L3	3	1,2
Q.6	(a)	L4	3	1,2
	(b)	L2	3	1,2
	(c)	L3	3	1,2
Q.7	(a)	L2	4	1,2

	(b)	L4	4	1,2
	(c)	L3	4	1,2
	Q.8	(a)	L2	4
	(b)	L2	4	1,2
	(c)	L3	4	1,2
	Q.9	(a)	L1	5
	(b)	L2	5	1,2,12
	(c)	L3	5	1,2
	Q.10	(a)	L2	5
	(b)	L2	5	1,2,12
	(c)	L3	5	1,2
	Lower order thinking skills			
Bloom's Taxonomy Levels	Remembering (knowledge): L_1		Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher order thinking skills			
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6	



Module - 1

Q.1) a) Discuss the theory of forced oscillations and hence 01
classify the conditions of variations of amplitude and
phase with angular frequency.

Solⁿ :- considers a body of mass 'm' executing vibrations in
a damping medium acted upon by an external periodic force
 $F \sin(pt)$

where p is the angular frequency of the external force

If x is the displacement of the body at any instant of time 't'

Damping force which acts in a direction opposite to the
movement of the body is equated to the term $-r \left(\frac{dx}{dt} \right)$

where r is the damping constant. and the restoring force is
equated to the term $-kx$ where k is the force constant.

The net force acting on the body is the resultant
of all the three forces

$$\therefore \text{Resultant force} = -r \frac{dx}{dt} - kx + F \sin pt \rightarrow \textcircled{1}$$

The body's motion due to the resultant force obeys
Newton's second law of motion on the basis of which we
can write.

$$\text{Resultant force} = m \frac{d^2x}{dt^2} \rightarrow \textcircled{2}$$

\therefore From Equⁿ (1) & (2)

$$m \frac{d^2x}{dt^2} = -r \frac{dx}{dt} - kx + F \sin(pt)$$

$$\text{OR} \quad m \frac{d^2x}{dt^2} + r \frac{dx}{dt} + kx = F \sin pt$$

This is the Equⁿ of motion for forced vibration.

02

Dividing Through by m , we get

$$\frac{d^2x}{dt^2} + \frac{r}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F}{m} \sin pt \rightarrow (3)$$

$$\text{Let } \frac{r}{m} = 2b$$

The natural frequency of vibration of the body ω is given by

$$\omega = \sqrt{\frac{k}{m}}$$

$$\text{Squaring } \omega^2 = \frac{k}{m}$$

Eqn (3) can be written as.

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = \frac{F}{m} \sin pt \rightarrow (4)$$

As per the procedures followed to solve differential Eqn the above Eqn has a solution of the form.

$$x = a \sin(pt - \alpha) \rightarrow (5)$$

where a and α are the unknowns to be found

However since Eqn (5) represents a simple harmonic motion a & α must represent respectively, the amplitude and phase of the vibrating body.

Differentiating x with respect to t we get

$$\frac{dx}{dt} = ap \cos(pt - \alpha) \rightarrow (6)$$

Differentiating again

$$\frac{d^2x}{dt^2} = -ap^2 \sin(pt - \alpha) \rightarrow (7)$$

Substituting in Eqn (4), we get

$$-ap^2 \sin(pt - \alpha) + 2bap \cos(pt - \alpha) + \omega^2 a \sin(pt - \alpha) = \frac{F}{m} \sin(pt) \rightarrow (8)$$

The right side of the above Eqn can be written as

$$\frac{F}{m} \sin[(pt - \alpha) + \alpha] \quad \because \sin(A+B) = \sin A \cos B + \cos A \sin B$$

Substituting in Eqn (8) and simplifying, we get

$$[-ap^2 \sin(pt - \alpha) + \omega^2 a \sin(pt - \alpha) + 2bap \cos(pt - \alpha)] = \frac{F}{m} \sin(pt - \alpha) \cos \alpha + \left(\frac{F}{m}\right) \cos(pt - \alpha) \sin \alpha$$

By Equating the coefficient of $\sin(pt - \alpha)$ from both sides, we get

$$-ap^2 + \omega^2 a = \frac{F}{m} \cos \alpha \rightarrow (9)$$

Similarly by Equating the coefficients of $\cos(pt - \alpha)$ from both sides we get.

$$2bap = \frac{F}{m} \sin \alpha \rightarrow (10)$$

Squaring and adding Eqn (9) and Eqn (10) we get

$$[a(\omega^2 - p^2)]^2 + (2bap)^2 = \left[\frac{F}{m}\right]^2 [\cos^2 \alpha + \sin^2 \alpha]$$

$$a^2 [(\omega^2 - p^2)^2 + 4b^2 p^2] = \left[\frac{F}{m}\right]^2$$

$$\text{or } a = \frac{\left[\frac{F}{m}\right]}{\sqrt{4b^2 p^2 + (\omega^2 - p^2)^2}} \rightarrow (11)$$

The above Eqn represents the amplitude of the forced vibrations substituting Eqn (11) in Eqn (5), the solution of the Eqn for forced vibration can be written as

$$x = \frac{F/m}{\sqrt{4b^2 p^2 + (\omega^2 - p^2)^2}} \sin(pt - \alpha) \rightarrow (12)$$

04

Phase of forced vibrations:

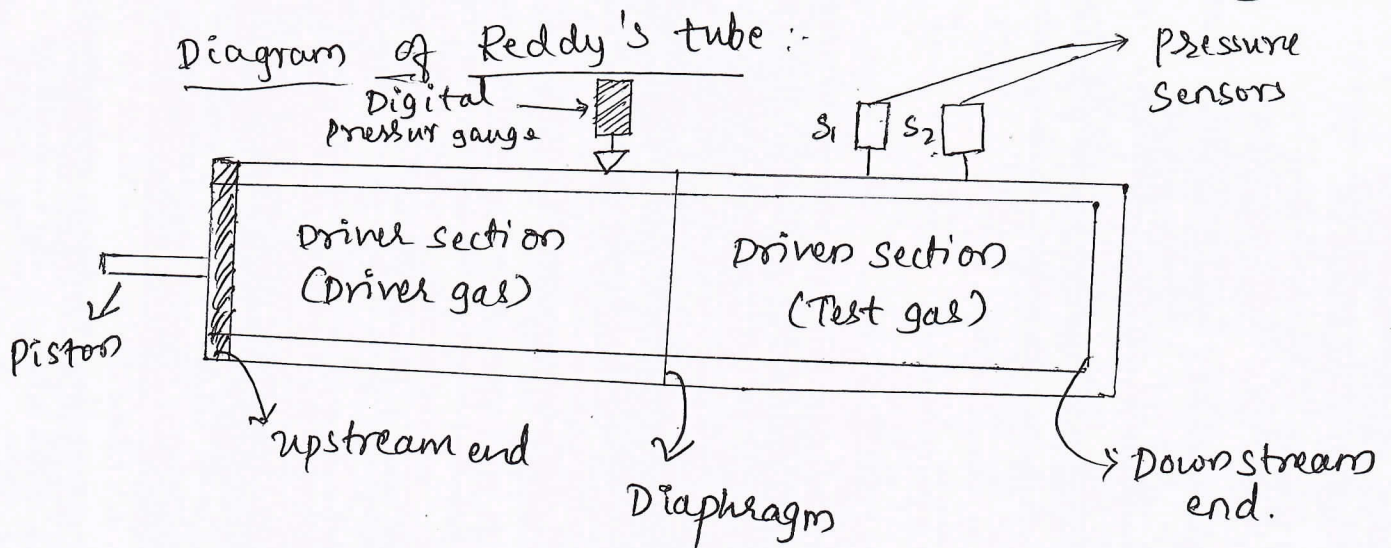
Dividing Eqn (10) by Eqn (9), we get.

$$\tan \alpha = \frac{2b\omega p}{p(\omega^2 - p^2)} = \frac{2b\omega}{(\omega^2 - p^2)}$$

∴ The phase α of the forced vibration is given by

$$\alpha = \tan^{-1} \left[\frac{2b\omega}{\omega^2 - p^2} \right] \rightarrow (13)$$

1 b) Illustrate the generation of shock waves using Reddy shock tube

Working:-

The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures.

- * Following the rupture, the driver gas rushes into the driven section and pushes the driver gas towards the far downstream end. This generates a moving shock wave that traverses the length of the driven section.
- * The shock wave instantaneously raises the temperature & pressure of the driven (test) gas as the shock moves over it.

* The propagating primary shock wave is reflected from the downstream end. After the reflection, the test gas undergoes further compression which boosts its temperature and pressure to still higher values of pressure and temperature. is sustained at the downstream end until an expansion wave reflected from the upstream end of the driver tube arrives there and neutralises the compression partially.

05

* Expansion waves are created at the instant the diaphragm is ruptured and they travel in a direction opposite to that of the shock wave.

* The period over which the extreme temperature and pressure conditions at the downstream end, is sustained is typically in the order of milliseconds.

* However the actual duration depends on the properties of the driver and test gases and the dimensions of the shock tube

c) Given the damping constant of the medium 0.1 kg s^{-1} calculate the amplitude of the oscillations at resonance given the mass attached to the spring-mass oscillator $50 \times 10^{-3} \text{ kg}$, the amplitude of the applied periodic force 1 N and the period of oscillations 1 second .

Solⁿ :- Given $b = \frac{\tau}{2m} = 0.1 \text{ kg s}^{-1}$, mass = $50 \times 10^{-3} \text{ kg}$. force (F) = 1 N

$$P = 2\pi f = 2\pi \frac{1}{T}$$

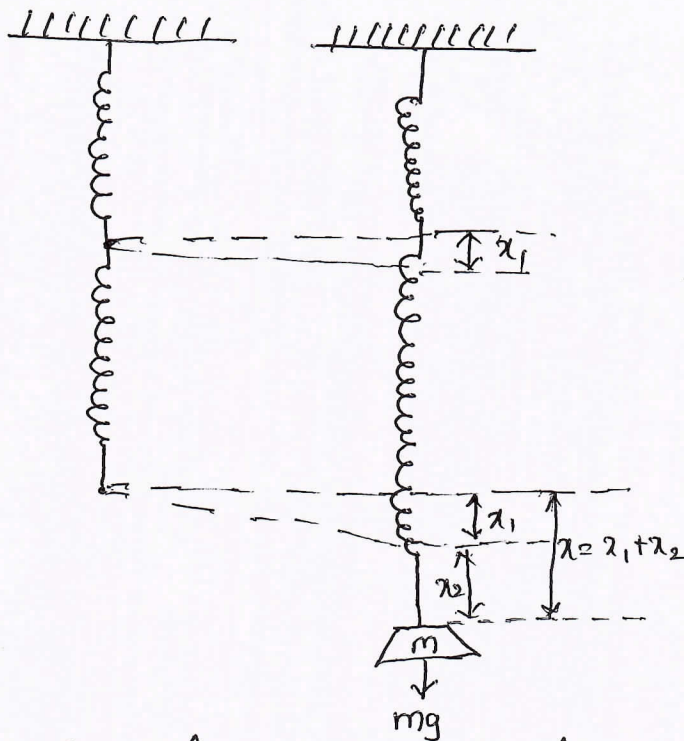
$$P = 2\pi$$

$$a = \frac{F/m}{2bp} = \frac{1/50 \times 10^{-3}}{2 \times 0.1 \times 2 \times 3.142} = \frac{20}{1.2568}$$

$$a = \frac{20}{1.2568} \quad \boxed{a = 15.91 \text{ m}}$$

06
Q. 2) a) Applying Hooke's law arrive at the Equations for the effective spring constant of Series and parallel combinations of springs

Solⁿ :-



Consider two idealized springs S_1 & S_2 with spring constants k_1 and k_2 respectively. x_1 be the Extension (within Elastic limit) in S_1 when a mass m is attached at its lower end.

Following Hooke's law we have $F = -k_1 x_1$

But $F = mg$. Hence $mg = -k_1 x_1$

$$\text{Or } x_1 = \frac{-mg}{k_1} \rightarrow \textcircled{1}$$

Similarly let x_2 be the Extension (within elastic limit) in S_2 when the same mass m is attached to it.

In similarly to Eqn (1), we can write

$$x_2 = \frac{-mg}{k_2} \rightarrow \textcircled{2}$$

Now let S_1 & S_2 be the suspended in series as shown in fig 3. Let the load m be suspended now at the bottom of this series combination.

Since each of the springs S_1 & S_2 experience the same pull by the mass m , S_1 extends by x_1 & S_2 extends by x_2

Thus the mass m comes down showing a total extension

$$x = x_1 + x_2$$

Let the force constant for this series combination as a whole be k_s

\therefore we can write

$$mg = -k_s x = -k_s (x_1 + x_2)$$

$$x_1 + x_2 = -\frac{mg}{k_s} \rightarrow (3)$$

Using Eqn (1) & (2), Eqn (3) can be written as

$$\frac{-mg}{k_1} - \frac{mg}{k_2} = -\frac{mg}{k_s}$$

Removing the common factor $-mg$ and rearranging, we have

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

If there are n no. of springs in series then

$$\frac{1}{k_s} = \sum_{i=1}^n \frac{1}{k_i}$$

If a mass is attached to the bottom of such a series

combination of springs & set for oscillations, its period of oscillation will be

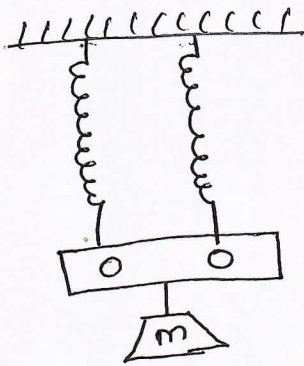
$$T = 2\pi \sqrt{\frac{m}{k_s}}$$

08 Equivalent force constant for springs in parallel combination

Consider two idealized springs S_1 & S_2 with spring constants k_1 & k_2 respectively. Let x_1 & x_2 be the respective extensions that the springs S_1 & S_2 would undergo individually under the pulling action of a suspended mass m . Hence we have

$$mg = -k_1 x_1 \quad \text{or} \quad x_1 = -\frac{mg}{k_1} \quad \rightarrow (1)$$

$$mg = -k_2 x_2 \quad \text{or} \quad x_2 = -\frac{mg}{k_2} \quad \rightarrow (2)$$



Let the restoring force acting on the support be F_p and the force constant for this combination be k_p .

$$\therefore F_p = -k_p x \quad \rightarrow (3)$$

The restoring force F_p is actually shared by the two springs. Let the restoring force in S_1 be F_1 & that in S_2 be F_2 .

$$F_p = F_1 + F_2 = -k_1 x_1 - k_2 x_2$$

But, since both springs undergo same extension x , $x_1 = x_2 = x$

$$F_p = -k_1 x - k_2 x$$

$$F_p = -(k_1 + k_2)x \quad \rightarrow (4)$$

Comparing Eqn (1) & (2), we have

$$k_p = k_1 + k_2$$

k_p is the Equivalent force constant for the parallel combination. If there are n no. of springs connected in parallel then

$$k_p = k_1 + k_2 + \dots + k_n$$

For this combination of mass-spring system, the period of oscillation will be.

$$T = 2\pi \sqrt{\frac{m}{K_p}}$$

09

Q27 b) Enumerate the properties and applications of shock waves.

Solⁿ: - (1) They always travel in the medium with mach number Exceeding 1.

(2) Shock waves obey the laws of fluid dynamics

(3) The effects caused by shockwaves result in increase of entropy.

(4) Across the shock wave. Supersonic flow is decelerated into subsonic flow. This process occurs adiabatically but with a change in internal energy.

(5) They are produced in very thin space of thickness not exceeding 1 μm so when the medium is subjected to an increase in pressure, temperature & density.

(6) They are not actually wave-like conventional ~~to~~ sense. However shock wave energy has similar physics as sound waves.

(7) on Impact, they physically travel through any medium (even in solid medium) through the energy is dissipated fast.

Application of shock waves

(1) cell information

(2) wood preservation

(3) use in pencil Industry

(4) kidney stone treatment.

(5) Gas dynamics Studies

(6) shock waves assisted needless drug delivery

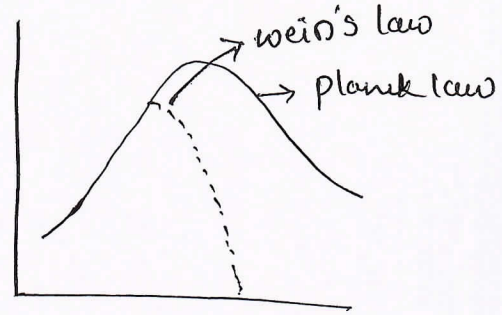
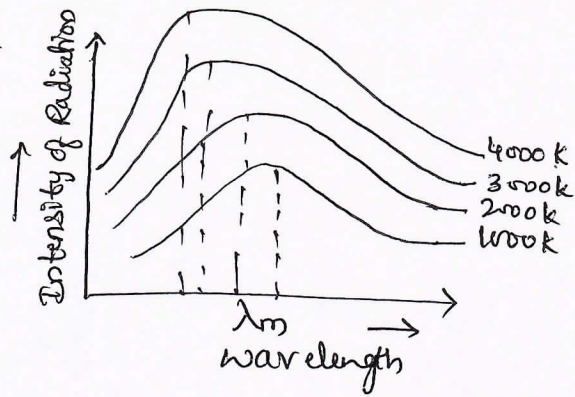
(7) Treatment of dry borewells.

Q 27 ¹⁰cs Compare the Mach number of a jet fighter travelling with 2000 km h^{-1} with that of a bullet travelling with a velocity of 400 m s^{-1} in the same medium given the speed of sound in the medium 330 m s^{-1}

Q 3) as Discuss the Spectral distribution energy in the black body radiation spectrum and hence Explain Wien's displacement law.

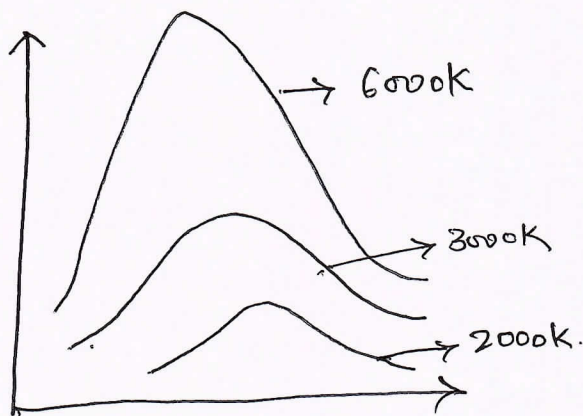
11

Solⁿ:



The curves in the figure represent the variation of Intensity of the radiation with wavelength for different temp's of the black body ($T_1 < T_2 < T_3 < T_4$)

- ① At a given temperature of the black body emits continuous range of wavelengths.
- ② Energy at a definite wavelength increases with increase in temperature of the black body.
- ③ The energy distribution is not uniform. There is a particular wavelength λ_m at which the energy emitted is maximum
- ④ The wavelength λ_m at which maximum emission of energy takes place, decreases with increase in temp
- ⑤ The area under the curve represents the total energy emitted and is proportional to the fourth power of the absolute temp.



12 The plot had following features:

(1) There are different curves for different temps.

(2) There is a peak for each of the curves which indicates

that the e.m. wave of that wavelength corresponding to the peak is emitted to the large extent at that temp to which the curve corresponds.

(3) As the temp increases, the peak shifts to the lower wavelength

(4) An increase in temp results in the increase of the energy emitted.

Wein's displacement law:

Statement: "The wavelength of maximum intensity λ_m is inversely proportional to the absolute temp of the emitting body because of which the peaks of the energy curves for different temps to get to get displaced towards the lower wavelengths side"

$$\lambda_m \downarrow \text{ or } \lambda_m \uparrow T \downarrow$$

$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m T = \text{constant} = 2.898 \times 10^{-3} \text{ mK}$$

Further Wein showed that the maximum energy E_m at the peak maximum peak emission is directly proportional to the λ_m power of absolute temp

$$E_m \propto T^5$$

$$E_m = \text{const } T^5$$

$$E_m T^{-5} = \text{const}$$

where $E_m \rightarrow$ maximum energy radiated, $T \rightarrow$ absolute temp
 $E \lambda \rightarrow$ energy radiated (unit area)

Wein's Law

Wein also deduced the relation between the wavelength of emission and the temp of the source as

$$E_{\lambda} d\lambda = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}} d\lambda$$

where $E_{\lambda} d\lambda \rightarrow$ is the energy emitted (unit volume) for wavelength in the range λ and $\lambda + d\lambda$

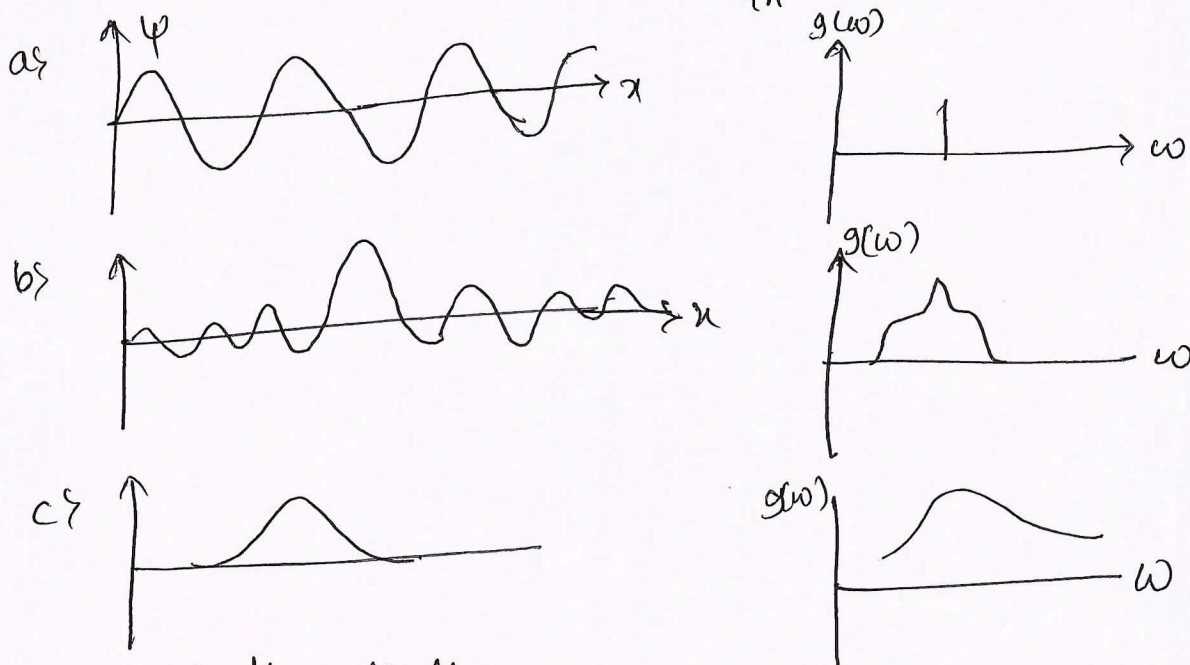
C_1 & $C_2 \rightarrow$ are constant

This law is called Wein's law of energy distribution in the black body radiation spectrum.

13

Q 3 b) State and Explain Heisenberg uncertainty principle and infer on the classical & mechanical ~~res~~ measurements.

Solⁿ:- Statement:- It is impossible to determine both the exact position and exact momentum of a particle at the same time. The product of uncertainty in these quantities is always greater than or equal to $\frac{h}{4\pi}$.



Mathematically The uncertainty principle in x -direction.

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

14 The uncertainty principle in y and z-direction, can be written as

$$\Delta y \Delta p_y \geq \frac{h}{4\pi}$$

$$\Delta z \Delta p_z \geq \frac{h}{4\pi}$$

Infer on the Classical & Mechanical Measurements:-

① Non-Existence of electron in the nucleus.

If an electron is confined to the nucleus which has a radius of the order 10^{-14} m, the maximum uncertainty in position of the electron will be of the order of the radius

$$(\Delta x)_{\max} = 10^{-14} \text{ m}$$

By Heisenberg uncertainty principle,

$$(\Delta x)_{\max} (\Delta p)_{\min} = \frac{h}{4\pi}$$

$$(\Delta p)_{\min} = \frac{h}{4\pi (\Delta x)_{\max}}$$

$$= \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$(\Delta p)_{\min} = 5.276 \times 10^{-21} \text{ kg m/s}$$

The momentum of electron ~~is~~ has to be at least comparable in magnitude to this uncertainty.

$$\therefore p_{\min} \approx (\Delta p)_{\min} = 5.276 \times 10^{-21} \text{ kg m/s}$$

The equation for energy from theory of relativity is

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

$$\text{Here } m_0^2 c^4 \ll p^2 c^2$$

$$E = pc$$

$$E_{\min} = 5.276 \times 10^{-21} \times 3 \times 10^8 = 1.583 \times 10^{-12} \text{ J} = 9.9 \text{ MeV}$$

2) minimum kinetic energy of an electron in an atom.

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consider an electron in a hydrogen atom of radius $5.3 \times 10^{-11} \text{ m}$

$$\Delta x = 5.3 \times 10^{-11} \text{ m}$$

using uncertainty principle.

$$\Delta p \geq \frac{h}{4\pi\Delta x} \Rightarrow \Delta p \geq \frac{6.63 \times 10^{-34}}{4\pi \times 5.3 \times 10^{-11}}$$

$$\Delta p \geq 9.955 \times 10^{-25} \text{ kg m/s}$$

The momentum of electron must be atleast of the same order as Δp

$$p \geq 9.955 \times 10^{-25} \text{ kg m/s}$$

The kinetic energy will be

$$K \geq \frac{p^2}{2m} \geq \frac{(9.955 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$$\boxed{K \geq 3.4 \text{ eV}}$$

c) The kinetic energy of an electron is equal to the energy of a photon with a wavelength of 560 nm. Calculate the de Broglie wavelength of the electron.

Solⁿ :- Energy of photon is given by

$$E_p = \frac{hc}{\lambda_p} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{560 \times 10^{-9}}$$

$$E_p = \frac{0.0354 \times 10^{-17} \text{ J}}{1.6 \times 10^{-19}}$$

$$E_p = 0.021 \times 10^2 \text{ eV}$$

$$E_p = 2.18 \text{ eV}$$

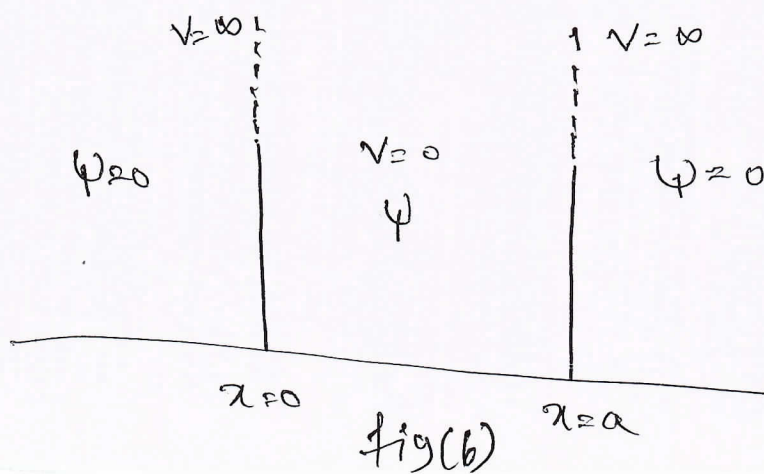
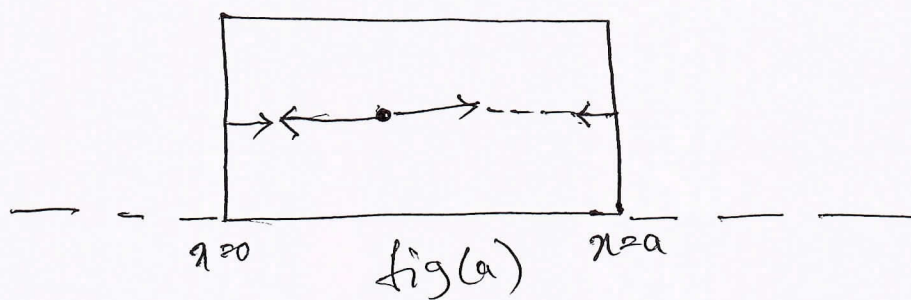
16 The wavelength λ_e of an electron in terms of its kinetic energy 'E' is given by

$$\begin{aligned}\lambda_e &= \frac{h}{\sqrt{2m_e E_p}} \\ &= \frac{6.623 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2.18}} \\ &= \frac{6.623 \times 10^{-34}}{1.991 \times 10^{-15}}\end{aligned}$$

$$\lambda_e = 3.32 \times 10^{-19} \text{ m}$$

Q4a) Discuss the motion of a quantum particle in one-dimensional potential well of the infinite height and of width a and also examine the quantization of energy.

Soln:-



Consider a one dimensional problem in which a particle of mass m moving with speed ' v ' along x -axis is confined to box of length a with perfectly rigid walls at $x=0$ and $x=a$ as shown in fig 17

The particle does not lose energy when it collides with the walls so that its total energy remains constant

This physical problem of a particle confined between two rigid walls can be converted into a problem of potential distribution by specifying the potential energy of the particle to be infinite at and beyond the walls i.e

$$V = \infty \quad \text{for } x \leq 0 \quad \text{and } x \geq a$$

The potential energy of the particle is constant within the box which can be taken to be zero for convenience i.e

$$V = 0 \quad \text{for } 0 < x < L$$

The potential energy distribution is shown in fig (6) It is as if the particle is inside an infinite potential well as the particle does not exist at the walls and beyond them

$$\psi = 0 \quad \text{for } x \leq 0 \quad \text{and } x \geq a$$

The wave function ψ exists only for $0 < x < a$

Schrodinger's time independent wave Eqnⁿ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E-V)\psi = 0 \rightarrow (1)$$

Substituting $V=0$ for $0 < x < a$ in Eqnⁿ (1)

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0$$

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$$\text{Let } k^2 = \frac{8\pi^2 m E}{h^2} \rightarrow \textcircled{2}$$

$$\therefore \frac{d^2\psi}{dx^2} + k^2\psi = 0 \rightarrow \textcircled{3}$$

The general solution of Equation (3) is

$$\psi = A \sin kx + B \cos kx \rightarrow \textcircled{4}$$

where A & B are arbitrary constants which are to be determined using boundary conditions.

The first boundary condition is

$$\psi = 0 \text{ at } x = 0$$

\therefore From Eqnⁿ (4)

$$0 = A \sin(0) + B \cos(0) \\ = Ax_0 + Bx_1$$

$$\Rightarrow \boxed{B=0}$$

From Eqnⁿ (4)

$$\psi = A \sin kx \rightarrow \textcircled{5}$$

The second boundary condition is

$$\psi = 0 \text{ at } x = a$$

\therefore From Eqn (5)

$$0 = A \sin ka$$

$A \neq 0$ as for $A = 0$, $\psi = 0$ for all values of x which will mean that the particle does not exist inside the box

$$\therefore \sin kL = 0$$

$$\therefore ka = n\pi \quad ; \quad n = 1, 2, 3, \dots$$

$n \neq 0$ as $n=0 \Rightarrow \psi=0$ for all values of x which is not possible

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$$k = \frac{n\pi}{a} \rightarrow (6)$$

From Eqn (2) & (5)

$$\frac{8\pi^2 m E}{h^2} = \frac{n^2 \pi^2}{a^2}$$

As energy 'E' depends on 'n' we use suffix 'n' to 'E'

$$E_n = \frac{n^2 h^2}{8ma^2} \rightarrow (7)$$

where $n = 1, 2, 3, \dots$

From the above Eqn the smallest value of energy that the particle can have is

$$E_1 = \frac{h^2}{8ma^2}$$

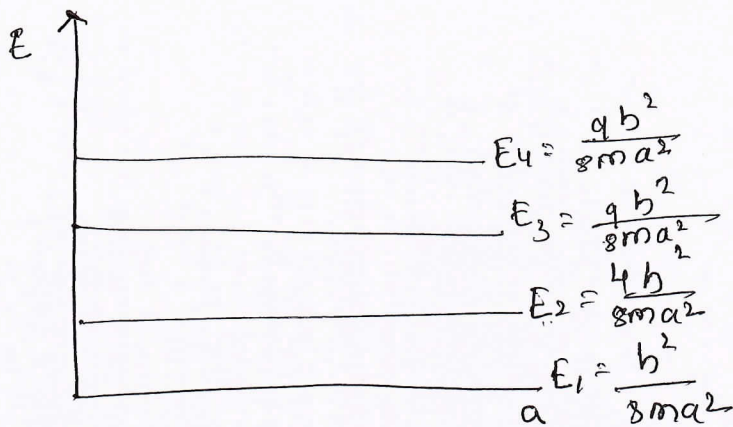
which is non-zero. This contradicts classical mechanics as to which the particle can have zero energy.

The other possible values of energy are

$$E_2 = \frac{4h^2}{8ma^2}, \quad E_3 = \frac{9h^2}{8ma^2} \text{ etc}$$

These energy values are discrete. They are not continuous as expected from classical mechanics.

Thus a/c to quantum mechanics the particle inside a rigid box cannot have all values of energy but only these discrete energy values are known as energy Eigen values



wave function substituting Eqn (6) in Eqn (5) we get

$$\psi = A \sin\left(\frac{n\pi x}{a}\right)$$

The complex conjugate of ψ is

$$\psi^* = A \sin\left(\frac{n\pi x}{a}\right)$$

To normalize the wave function we find $\int_0^a \psi \psi^* dx$

$$\int_0^a \psi \psi^* dx = \int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= A^2 \int_0^a \left[\frac{1 - \cos\left(\frac{2n\pi x}{a}\right)}{2} \right] dx$$

$$= \frac{A^2}{2} \left[x - \frac{\sin\left(\frac{2n\pi x}{a}\right)}{\frac{2n\pi}{a}} \right]_0^a$$

$$= \frac{A^2}{2} [a - 0]$$

$$\therefore \int_0^a \psi \psi^* dx = \frac{A^2 a}{2}$$

Let the RHS be N^2 i.e.

$$N^2 = \frac{A^2 a}{2}$$

$$N = A \sqrt{\frac{a}{2}}$$

The normalized wave function ψ_n is obtained using

$$\psi_n = \frac{\psi}{N} = \frac{A \sin\left(\frac{n\pi x}{a}\right)}{A \sqrt{\frac{a}{2}}}$$

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$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

These normalized wave functions are called Eigenfunctions

$$\psi_n^* = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

The probability function is

$$p(x) = |\psi_n|^2 = \psi \psi_n^* = \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right)$$

The wave function and probability functions for $n=1, 2, 3$ are shown in below fig

A particle having the lowest energy E_1 , has wave function ψ_1 , for which the probability of finding the particle is maximum, at the center of the box.

Q4(c) Deduce Rayleigh-Jeans law from Planck's law of radiation.

Solⁿ. For longer wavelengths

$$v = \frac{c}{\lambda} \text{ is small } \lambda \uparrow v \downarrow$$

Since v is small, $\frac{h\nu}{kT}$ will be very small.

Expanding $e^{\frac{h\nu}{kT}}$ as power series, we have

$$e^{\frac{h\nu}{kT}} = 1 + \frac{h\nu}{kT} + \left(\frac{h\nu}{kT}\right)^2 + \dots$$

$$\boxed{e^{\frac{h\nu}{kT}} = 1 + \frac{h\nu}{kT}} \quad \because v \text{ is very small.}$$

[Since $\frac{h\nu}{kT}$ is very small, its higher power terms could be neglected]

$$\left(e^{\frac{h\nu}{kT}} - 1 \right) = \frac{h\nu}{kT} = \frac{hc}{\lambda kT} \quad \left(v = \frac{c}{\lambda} \right)$$

Substituting in Eqn (1)

$$E_{\lambda} d\lambda = \left[\frac{8\pi h c}{\lambda^5 \left(\frac{hc}{\lambda kT} \right)} \right] d\lambda$$

$$\boxed{E_{\lambda} d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda} \quad \text{This Eqn is Rayleigh-Jeans Law of radiation}$$

Thus Wein's law & Rayleigh-Jeans law come out as special cases showing the general form of Planck's law of radiation.

4 c) The speed of electron is measured to within an uncertainty $2 \times 10^4 \text{ ms}^{-1}$ in one dimension. what is the ~~maximum~~ minimum width required by the electron to be confined in an atom.

Solⁿ :- By Heisenberg's uncertainty principle,

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$p = mv$$

$$\Delta p = m \Delta v$$

$$\Delta x \geq \frac{h}{4\pi m \Delta v}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\Delta v = 2 \times 10^4 \text{ ms}^{-1}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4\pi \times 9.1 \times 10^{-31} \times 2 \times 10^4} = \frac{6.63}{228.73}$$

$$\Delta x = 0.028 \times 10^{-7} \text{ m}$$

$$\Delta x = 2.8 \times 10^{-9} \text{ m} = 0.28 \times 10^{-10} \text{ m}$$

$$\boxed{\Delta x = 0.28 \text{ \AA m}}$$

Q7 or Q8 Discuss the motion of quantum particle in one-dimensional

5(a) ~~Discuss~~ Obtain the Expression for energy density using Einstein's A and B Co-efficients and hence draw infer on the Relation $B_{12} = B_{21}$

Solⁿ:- consider a system of atoms having a ground state energy E_1 and Excited State energy E_2 with number densities of atoms in these states N_1 and N_2 respectively.

If a photons of frequency is given by

$$\nu = \frac{E_2 - E_1}{h} \rightarrow (1)$$

The rate of absorption of photons will be proportional to the number density N_1 of the atoms in ground state, and the energy density E_ν in the frequency range ν to $\nu + d\nu$ incident radiation,

\therefore Rate of absorption $\propto N_1 E_\nu$

\therefore Rate of absorption $= B_{12} N_1 E_\nu \rightarrow (2)$

where B_{12} is a constant known as Einstein's Co-efficient of induced absorption.

Atoms in a Excited State E_2 can come down to ground state through spontaneous emission In this case emission does not depend on the energy density, in the incident radiation

So Rate of Spontaneous emission $\propto N_2$

\therefore Rate of Spontaneous emission $= A_{21} N_2 \rightarrow (3)$

Where A_{21} is a constant known as Einstein's coefficient of Spontaneous emission. 25

In case of Stimulated Emission a photon of frequency is required to stimulate the atoms

So

Rate of Stimulated emission $\propto N_2 E_\gamma$

$$\text{Rate of stimulated emission} = B_{21} N_2 E_\gamma \rightarrow (4)$$

Where B_{21} is a constant known as Einstein's coefficient of Stimulated emission

In a state of thermal Equilibrium, the rate of transition of atoms from E_1 to E_2 must Equal the total rate of transition from E_2 to E_1

\therefore Rate of absorption = Rate of Spontaneous emission + Rate of Stimulated emission.

$$\therefore B_{12} N_1 E_\gamma = A_{21} N_2 + B_{21} N_2 E_\gamma \rightarrow (5)$$

Dividing by N_1 we get.

$$B_{12} E_\gamma = A_{21} \frac{N_2}{N_1} + B_{21} \frac{N_2}{N_1} E_\gamma$$

$$E_\gamma \left[B_{12} - B_{21} \frac{N_2}{N_1} \right] = A_{21} \frac{N_2}{N_1}$$

$$E_\gamma = \frac{A_{21} \left[\frac{N_2}{N_1} \right]}{\left[B_{12} - B_{21} \left[\frac{N_2}{N_1} \right] \right]}$$

$$E_\gamma = \frac{A_{21} \left[\frac{N_2}{N_1} \right]}{B_{12} - B_{21} \left[\frac{N_2}{N_1} \right]}$$

$$26 \quad E_r = A_{21} \left[\frac{N_2}{N_1} \right]$$

$$\left[\frac{N_2}{N_1} \right] \left[B_{12} \left[\frac{N_1}{N_2} \right] - B_{21} \right]$$

$$E_r = \frac{A_{21}}{\left[B_{12} \left[\frac{N_1}{N_2} \right] - B_{21} \right]}$$

Taking B_{21} take it as common

$$E_r = \frac{A_{21}}{B_{21} \left[\frac{B_{12}}{B_{21}} \left[\frac{N_1}{N_2} \right] - 1 \right]}$$

$$E_r = \left[\frac{A_{21}}{B_{21}} \right] \left[\frac{1}{\frac{B_{12}}{B_{21}} \left[\frac{N_1}{N_2} \right] - 1} \right]$$

maxwell - Boltzman distribution

$$\frac{N_2}{N_1} = e^{\frac{-(E_2 - E_1)}{kT}} = e^{-\frac{h\nu}{kT}}$$

$$\therefore \frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} \rightarrow (6)$$

$$E_r = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right] \rightarrow (7)$$

Comparing with the energy density from planck law

$$E_n = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \rightarrow (8)$$

we get

$$\frac{B_{12}}{B_{21}} = 1$$

$$\therefore B_{12} = B_{21}$$

and

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \rightarrow (9)$$

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From the above Eqn we can write

$$\boxed{\frac{A_{21}}{B_{21}} \propto \nu^3}$$

Discussion:-

For large ν , $A_{21} \gg B_{21}$ as $\nu = \frac{E_2 - E_1}{h}$ for large Energy difference between the ground state and excited state.

The probability of spontaneous emission is much larger than the probability of stimulated emission.

* It means that to build lasers in the ultraviolet x-ray region would be much more difficult than to build lasers in visible or infrared regions.

* The process of stimulated emission becomes significant at lower frequencies

5⁷ 28
b) Discuss the attenuation and various losses in optical fibers.

Solⁿ:- The loss of light energy of the optical signal as it propagates through the fiber is called attenuation

The main reason for the loss of light intensity over the length of the cable is due to (1) light absorption
(2) Scattering (3) Radiation losses.

① Light Absorption:- The absorption in the fiber glass occurs due to the presence of impurities like copper, chromium, iron etc.

* During the light propagation the electrons of the impurity atoms absorb the photons and get excited to higher energy level.

* After a fraction of time they come back to ground state with the emission of photons.

* But the emitted photons will have different wavelength or different phase w.r.t light signal.

* Therefore they fail to undergo total internal reflection

* Even if the material has no impurities the material itself may absorb some light energy. This is called intrinsic absorption.

② Scattering losses:- Since the glass is a heterogeneous mixture of many oxides like SiO_2 , P_2O_5 , etc. The composition of the molecular distribution varies from point to point. Hence due to the non-homogeneity in the material there will be sharp variation in refractive index value inside the glass over distance.

* When light travels in the fiber, the photons may be scattered. This type of scattering is similar to Rayleigh scattering. 29

* Rayleigh scattering occurs when the dimensions of the object are smaller than the wavelength of the ~~object~~ light. Rayleigh scattering is inversely proportional to the 4th power of wavelength.

* Due to Rayleigh scattering photons move in random direction and fail to undergo TIR and (leaves) escapes from the fiber through cladding it becomes a loss.

* The scattering losses can be minimized by the use of light waves of longer wavelength.

3) Radiation losses :-

Radiation losses occur due to bending of fiber when the optical fiber is curved extensively such that incidence angle of the light ray falls below the critical angle then no total internal reflection takes place and some of the light rays leak through the cladding and leads to loss in the intensity of light.

The net attenuation can be determined by a factor called attenuation coefficient (α) expressed in dB/km that is

$$\alpha = \frac{-10}{L} \log_{10} \left[\frac{P_{out}}{P_{in}} \right]$$

where P_{out} is the power output and P_{in} is the power coupled into the fiber. L is the length of the fiber.

Q 57 c) ³⁰ Calculate the number of photons emitted per pulse of duration 1 microsecond given the power out of laser 3mW and wavelength of laser 632.8nm

Solⁿ :
$$E = \frac{Nhc}{\lambda} = P \times t$$

$$N = \frac{P t \lambda}{hc}$$

$$P = 3 \text{ mW} = 3 \times 10^{-3} \text{ W}, t = 1 \mu\text{s} = 1 \times 10^{-6} \text{ s}, \lambda = 632.8 \times 10^{-9} \text{ m}$$

$$N = \frac{3 \times 10^{-3} \times 10^{-6} \times 632.8 \times 10^{-9}}{6.624 \times 10^{-34} \times 3 \times 10^8}$$

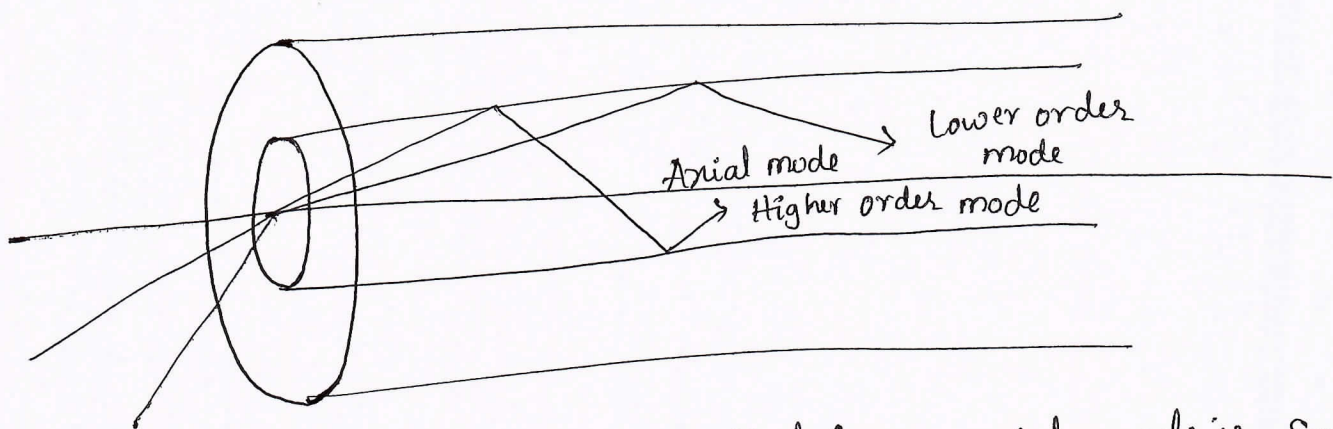
$$N = \frac{1898.4}{19.872} \times 10^{-18} \times 10^{+26}$$

$$N = 95.53 \times 10^8$$

Q 6) a) Define modes of propagation and RI Profile and Distinguish between the types of optical fibers.

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Solⁿ: When light is transmitted through optic fibre cores of diameter in the range 50 μm to 200 μm , it can travel along different ray paths known as modes of propagation. A ray travelling along the axis is known as axial mode and the higher order modes are the rays travelling at smaller angles of incidence on the Core - Cladding interface.



The distance travelled by the axial mode is smaller than the higher order modes due to which the different modes reach the other end of the fibre at different times.

The number of modes that can travel in a fibre is determined from V-Number which is given by

$$V = \frac{\pi d}{\lambda} \times \text{NA}$$

Where λ is the wavelength of light used and d is the diameter of core of the optic fibre.

The number of modes = $\frac{V^2}{2}$

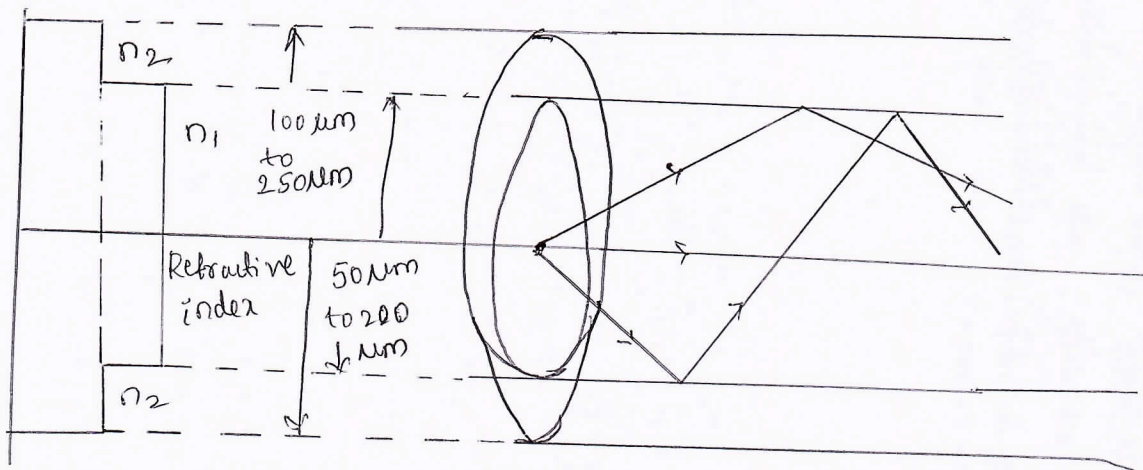
\therefore The number of modes $N = \frac{\pi^2 d^2}{2\lambda^2} \times (\text{N.A})^2$

32 Optical fibre types

① Step index fibres:-

Such fibres have a core of homogeneous transparent material of refractive index n_1 and cladding of another homogeneous transparent material of refractive index n_2 ($n_2 < n_1$). There is an abrupt change in refractive index at the core-cladding interface due to which they are known as step index fibres.

If the core diameter of the step index fibre is of the order of $50\mu\text{m}$ to $200\mu\text{m}$ it can transmit a large number of modes as shown in fig (1). The cladding thickness is typically 20 to $25\mu\text{m}$. These fibres are known as multimode step index fibres. These fibres are cheaper compared to other fibres but not suitable for long distance.



- ✦ Step index fibres which have core diameters in the range of $2\mu\text{m}$ to $10\mu\text{m}$ are known as single mode step index fibres.
- ✦ These fibres essentially transmit only the axial mode due to very small core diameters.
- ✦ The thickness of the core is typically $25\mu\text{m}$ to $30\mu\text{m}$.
- These fibres are used for long distance communications as intermodal dispersion is eliminated.

Graded index multimode fibres:

In graded index fibres, the refractive index is maximum at the axis of the core decreases gradually upto the cladding and then remains constant throughout the cladding as shown in fig. The rays travelling at an angle to the axis travel in curved paths due to gradually decreasing refractive index.

longer distances compared to the axial ray. They travel in region of small refractive index and hence travel faster. This reduces the intermodal dispersion. The core diameter is typically about 20 μm to 100 μm . With cladding thickness of about 25 μm . They are commonly used for medium distance communication.

Q.65 b) Identify the requisites of the CO₂ laser. and Explain its construction and working with the help of a neat sketch and band diagram

- Solⁿ: Explain in detail of
- (1) symmetric stretching mode
 - (2) asymmetric stretching mode
 - (3) Bending mode.

CO₂ Laser construction:

A typical CO₂ laser consists of a discharge tube of 2.5 cm diameter and 1.5 meter length.

* A tube is filled with a mixture of CO₂, N₂ and He gases in the ratio 1:2:3

* Sometimes, traces of Hydrogen or water vapour is added. This is because during discharge some CO₂ molecules break

into CO and O. The hydrogen or water vapour additives help to reoxidise CO to CO₂.

* The pressure inside tube is 6-17 torr.

The actual size pressure and proportion of gases

vary with particular application of the laser.

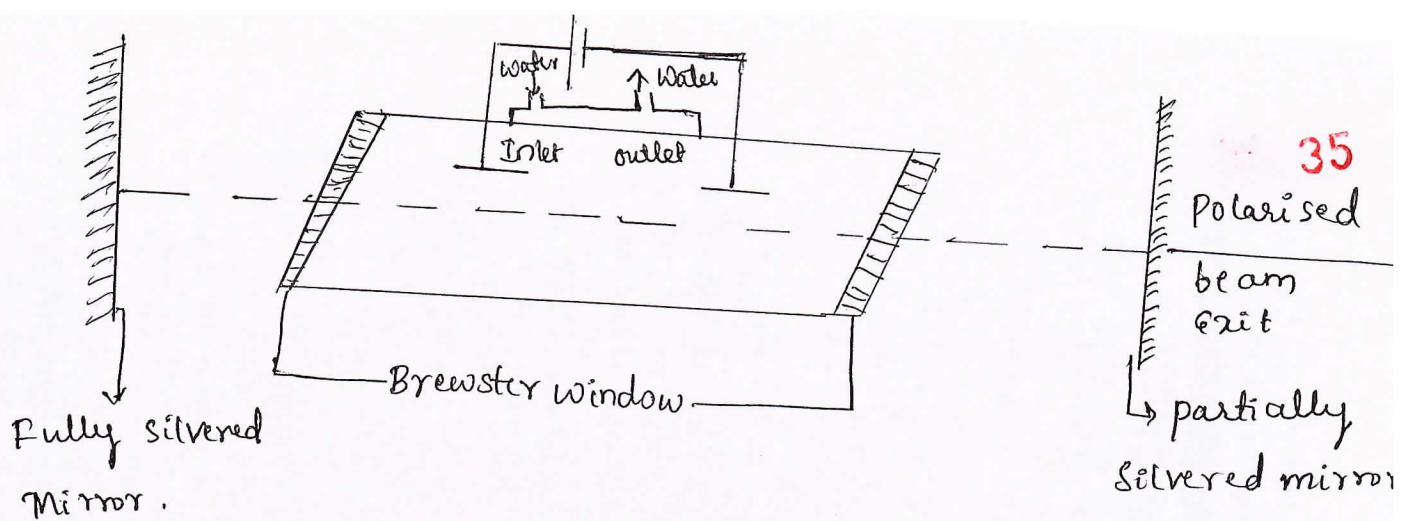
* In CO₂ laser. In a glass tube which has two electrodes

connected to a power supply

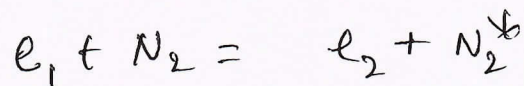
* One end of the tube has parabolically silvered mirror and

the other end has a Brewster's window. A completely

silvered mirror is kept beyond the Brewster's window.



Working:- The high voltage across the electrodes excited the gas molecule. The Nitrogen molecules in the gas are excited to higher levels. and transfer energy to CO_2 molecules by first kind collision like that many of the CO_2 molecules will also be raised to their 001 energy state (which is not metastable state) For N_2 molecules this process can be represented as



Here e_1 & e_2 are the energy values of the electrons before & after collisions respectively

N_2 & N_2^* are the energy values of the N_2 molecules in the ground state $v=1$ state respectively.

The CO_2 molecules are excited to the metastable state E_3 where population inversion takes place w.r.t the two lower lasing levels E_3 & E_4

In this state second collision takes place which can be represented as

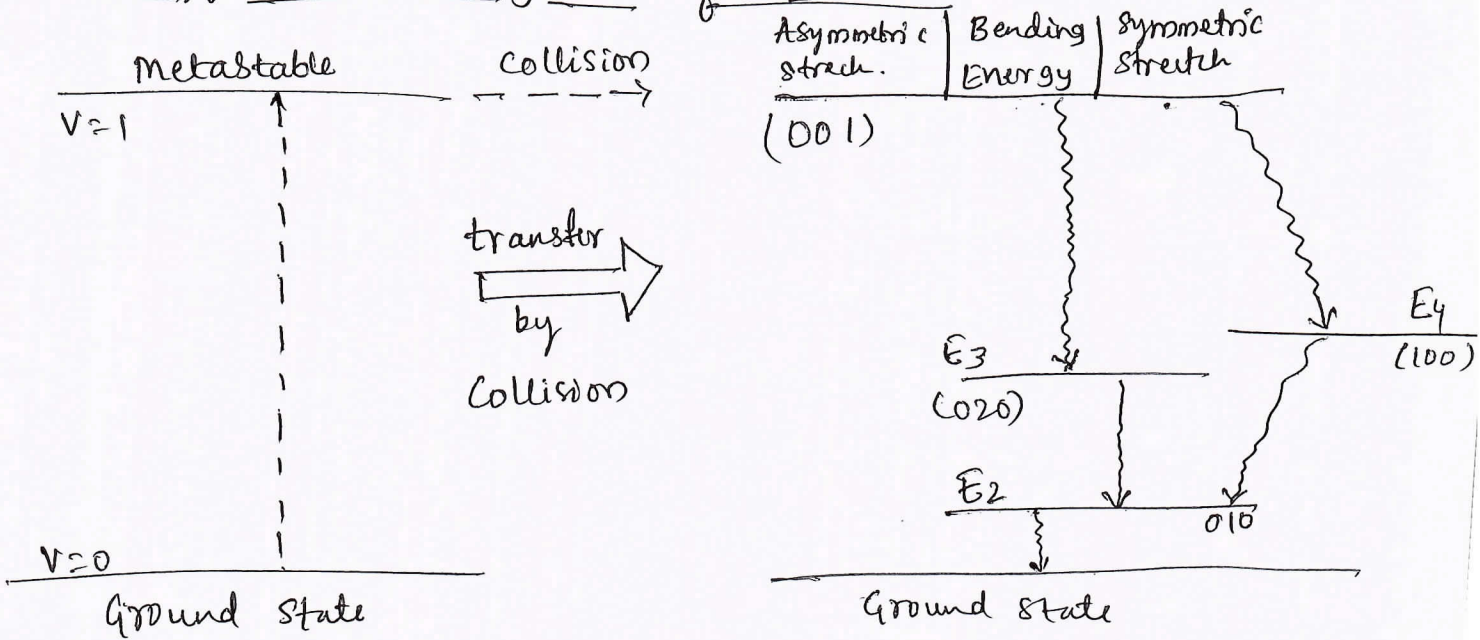


where CO_2 & CO_2^* refer to the energies of CO_2 molecules in the ground and excited state respectively

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- * In the CO_2 laser. transition E_3 level to E_4 level which gives rise to radiation of wavelength $10.6 \mu\text{m}$ which is in the far infrared region.
- * Transition from E_3 level to E_2 level which gives rise to radiation of wavelength $9.6 \mu\text{m}$ which is also in the far infrared region.
- * The procedure is repeated again and again we get ~~the~~ polarised beam on the exit side of the CO_2 laser.

Energy level diagram of CO_2 laser:-



Q 6) c) Compare the acceptance angle of an optical fiber placed in air and water given the RI of water 1.33 and RI of core & clad 1.5 & 1.45 respectively. 37

Solⁿ:- Given
 $n_{\text{air}} = 1$ $n_{\text{water}} = 1.33$ $n_{\text{core}} = 1.5$ $n_{\text{clad}} = 1.45$

$$\frac{\sin \theta_{\text{air}}}{\sin \theta_{\text{water}}} = \frac{n_{\text{water}}}{n_{\text{air}}}$$

$$\frac{\frac{\sqrt{n_1^2 - n_2^2}}{n_{\text{air}}}}{\frac{\sqrt{n_1^2 - n_2^2}}{n_{\text{water}}}} = \frac{n_{\text{water}}}{n_{\text{air}}}$$

$$\frac{\frac{\sqrt{(1.5)^2 - (1.45)^2}}{1}}{\frac{\sqrt{(1.5)^2 - (1.45)^2}}{1.33}} = \frac{1.33}{1} \Rightarrow \left(\frac{0.05}{1} \right) \frac{1}{1.33}$$

$$= \frac{1.33 \times 0.05}{1 \times 1.33}$$

$$\boxed{\frac{n_{\text{water}}}{n_{\text{air}}} = 1.33}$$

Q.7) Explain the Quantum mechanical modifications to the Classical free electron theory of metals to explain the electrical conductivity in solids and its success.

- Solⁿ:-
- ① The energy of free electrons are quantized
 - ② Free electrons obey Pauli's Exclusion Principle
 - ③ The distribution of free electrons in energy levels is governed by Fermi-Dirac Statistics.
 - ④ Free electrons move in uniform potential field due to ionic cores in a metal.
 - ⑤ The electrostatic electron-ion attractions and electron-electron repulsions are negligible.

Expression for conductivity using Quantum free electron theory:-

The energy of free electron can be written in terms of momentum 'p' is

$$E = \frac{p^2}{2m} \rightarrow \textcircled{1} \quad (\because p^2 = 2mE)$$

using de-Broglie wavelength ' λ '

$$p = \frac{h}{\lambda} \quad (\because \lambda = \frac{h}{p})$$

Eqn^t ① becomes

$$E = \frac{(h/\lambda)^2}{2m\lambda^2} \rightarrow \textcircled{2}$$

E can be expressed in terms of wavenumber k

$$\lambda = \frac{2\pi}{k}$$

\therefore using the value of λ , Eqn (2) becomes

$$E_k = \frac{h^2}{2m \left(\frac{2\pi}{k}\right)^2}$$

$$E_k = \frac{h^2 k^2}{8m \pi^2} \rightarrow (3)$$

In the ground state of the free electrons, the maximum energy of electrons is the fermi energy E_F

Eqn (3) can be written for this state is

$$E_F = \frac{h^2}{8\pi^2 m} k_F^2 \rightarrow (4)$$

We know that, the general expression for drift velocity V_d is

$$V_d = \frac{eE\tau}{m} \rightarrow (5)$$

where $\tau \rightarrow$ is the average time elapsed after the collision

\therefore The energy density is

$$J = nev_d \rightarrow (6) \quad (\because J = I/A)$$

using Eqn (5) Eqn (6) becomes

$$J = nev_d$$

$$J = ne \left(\frac{eE\tau}{m} \right)$$

$$J = \frac{ne^2 E \tau}{m}$$

$$\text{Also } J = \sigma E$$

$$\sigma E = \frac{ne^2 E \tau}{m} \Rightarrow \sigma = \frac{ne^2 \tau}{m} \rightarrow (7)$$

40 If λ is the mean free path & v_f is the speed of free electrons whose kinetic energy is equal to Fermi energy. Since only electrons near Fermi level ~~contributions~~ contribute to the conductivity.

The average time τ between two collisions of free electrons with core ions is given by

$$\tau = \frac{\lambda}{v_f}$$

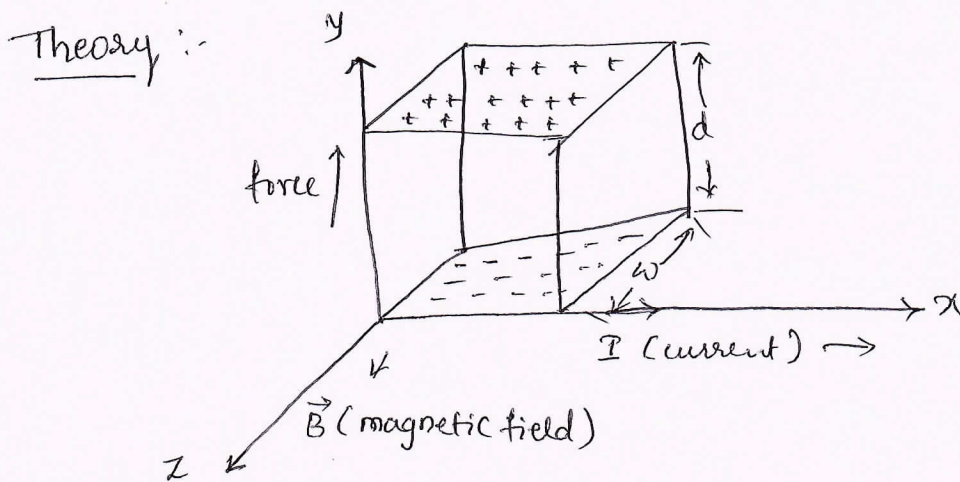
\therefore Eqn (7) becomes.

$$\sigma = \frac{ne^2}{m} \left(\frac{\lambda}{v_f} \right)$$

Q 7) b) what is Hall effect and illustrate on the determination of the type of charge carriers in Semiconductors. 01

Solⁿ :- Statement :- "When magnetic field is applied \perp to the direction of current in a conductor a potential difference develops along an axis \perp to both current and magnetic field. This effect is known as Hall effect."

Hall effect finds important application in studying the electron properties of Semiconductor such as determination of carrier concentration and carrier mobility. It is also used to determine whether a Semiconductor is n-type or p-type.



Consider a rectangular slab of a Semiconductor material in which a current I is flowing in the positive x -direction.

Let the semiconducting material be of n-type, which means that the charge carriers are electrons.

Let a magnetic field B be applied along the z -direction under the influence of the magnetic field. The electrons experience a Lorentz force F_L is given by

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$$F_L = -Bev \rightarrow (1)$$

Where e is the magnitude of charge on the electron
 $v \rightarrow$ is the drift velocity

Applying the Flemings left hand rule, we see that the force is exerted on the electrons in negative y-direction

* The electrons are therefore deflected downwards as a result the density of the electrons increases in the lower end of the material due to which its bottom edge becomes negatively charged.

* on the other hand the loss of electrons from the upper end causes the top edge of the material to become positively charged Hence a potential V_H called the Hall voltage, appears between upper and lower surfaces of the semiconductor material which establishes an electrical field E_H , called the "Hall field" across the conductor in the negative y direction. The field E_H exerts an upward force F_H on the electrons given by

$$F_H = -eE_H \rightarrow (2)$$

By defⁿ of electric field

$$E_H = \frac{F}{q} = \frac{F}{e}$$

Now as the deflection of electrons continues in the downward direction due to the Lorentz force F_L It also contributes to the growth of Hall field.

As a result the force F_H which acts on the electron in the upward direction also increases.

These two opposing forces ~~sub~~ reach an equilibrium at which stage

$$F_L = F_H$$

∴ using Eqn (1) & (2) the above equation becomes

$$-Bev = -eEH$$

$$E_H = Bv \rightarrow (3)$$

If d is the distance between the upper end and lower end surfaces of the slab then

$$E_H = \frac{V_H}{d} \quad \left\{ \begin{array}{l} \text{Electric} \\ \text{Potential} \end{array} \right. = \frac{\text{potential drop}}{\text{unit length}}$$

$$\therefore V_H = E_H \times d$$

using Eqn (3) we have

$$V_H = Bvd \rightarrow (4)$$

Let w be the thickness of the material in the z -direction
∴ Its area of cross-section normal to the direction

$$A = w \cdot d$$

The current density J

$$J = \frac{I}{wd}$$

04 If n is the concentration of electrons

$$I = neAV$$

$$J = \frac{I}{A} = \frac{neAV}{A}$$

$$J = neV \Rightarrow J = \rho V \rightarrow (5)$$

where n is charge carrier concentration & ρ is the charge density.

$$J = \frac{I}{A} = \frac{I}{wd}$$

$$\rho V = \frac{I}{wd}$$

$$V = \frac{I}{\rho wd} \rightarrow (6)$$

Substituting for V_d from Eqn (6), Eqn (4) we get

$$V_H = BV_d$$

$$V_H = B \cdot \frac{I}{\rho wd} \cdot d$$

$$V_H = \frac{BI}{\rho w} \Rightarrow \rho = \frac{BI}{V_H w}$$

7) c) An elemental solid dielectric material has polarizability $7 \times 10^{-40} \text{ Fm}^2$. Assuming the internal field to be Lorentz calculate the dielectric constant for the material if the material has $3 \times 10^{28} \text{ atoms/m}^3$.

05

Solⁿ:- Polarizability $\alpha_e = 7 \times 10^{-40} \text{ Fm}^2$

No of atoms/m³ $\cdot N = 3 \times 10^{28}$

The internal field is Lorentz field

Dielectric constant of the material $\epsilon_r = ?$

$$\left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) = \frac{N \alpha_e}{3 \epsilon_0}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{3 \times 10^{28} \times 7 \times 10^{-40}}{3 \times 8.854 \times 10^{-12}}$$

$$= 0.7906$$

$$(\epsilon_r - 1) = (\epsilon_r + 2) \times 0.7906$$

$$\epsilon_r - 1 = 0.7906 \epsilon_r + 1.5812$$

$$\epsilon_r (1 - 0.7906) = 2.5812$$

$$\epsilon_r = \frac{2.5812}{0.2094} = 12.33$$

\therefore The dielectric constant of the material is 12.33

Q. 87⁰⁶ as Deduce the expression for electrical conductivity of a conductor using the quantum free electron theory of metals.

Solⁿ:- The energy of free electron can be written in terms of momentum 'P' is

$$\boxed{E = \frac{P^2}{2m}} \rightarrow \textcircled{1} \quad (\because p^2 = 2mE)$$

using de-Broglie wavelength λ

$$P = \frac{h}{\lambda} = \quad (\because \lambda = \frac{h}{P})$$

Eqnⁿ ① becomes

$$E = \frac{(h/\lambda)^2}{2m} \rightarrow \textcircled{2}$$

E can be expressed in terms of wavenumber K

$$\lambda = \frac{2\pi}{K}$$

\therefore using the value of λ Eqnⁿ (2) becomes

$$E_K = \frac{h^2}{2m \cdot \left(\frac{2\pi}{K}\right)^2}$$

$$\boxed{E_K = \frac{h^2 K^2}{8m \pi^2}} \rightarrow \textcircled{3}$$

In the ground state of the free electrons. The maximum energy of electrons is the Fermi energy E_F . Eqnⁿ (3) can be written for this state is

$$E_F = \frac{h^2}{8\pi^2 m} K_F^2 \rightarrow \textcircled{4}$$

we know that the general expression for drift velocity 07

v_d is

$$v_d = \frac{e E \tau}{m} \rightarrow (5)$$

where $\tau \rightarrow$ is the average time elapsed after the collision

\therefore The energy density is

$$J = n e v_d \rightarrow (6) \quad (\because J = I/A)$$

using Eqn (5) & Eqn (6) becomes.

$$J = n e v_d$$

$$J = n e \left(\frac{e E \tau}{m} \right)$$

$$J = \frac{n e^2 E \tau}{m}$$

$$\text{Also } J = \sigma E$$

$$\sigma E = \frac{n e^2 E \tau}{m}$$

$$\Rightarrow \boxed{\sigma = \frac{n e^2 \tau}{m}} \rightarrow (7)$$

If λ is the mean free path & v_f is the speed of free electron whose k.E is equal to Fermi energy since only electrons near Fermi level contribute to the conductivity.

$$\tau = \frac{\lambda}{v_f} \Rightarrow \boxed{\sigma = \frac{n e^2}{m} \left(\frac{\lambda}{v_f} \right)}$$

Q 8) b) Describe in brief the various types of polarization mechanisms.

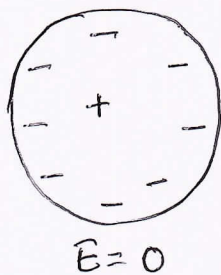
08

Solⁿ: There are mainly 3 different mechanisms through which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. All three different types of polarization are identified.

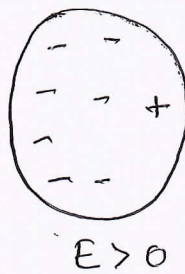
They are

- ① Electric polarization
- ② Ionic polarization
- ③ Orientational polarization.

① Electronic polarization



Charge distribution in the absence of the field.



Charge displacement due to the applied field.

The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material owing to the application of an external electric field.

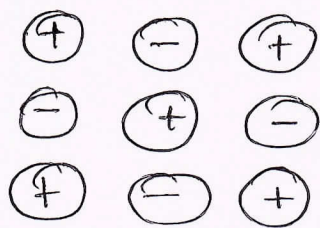
- * The separation created between the charges, leads to development of a dipole moment.
- * This process occurs throughout the material. Thus the material as a whole will be polarized.

The electronic polarizability α_e for a rare gas atom is given by $\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$ where N is the number of atoms/unit volume.

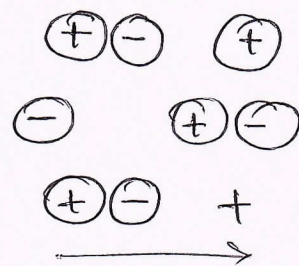
2) Ionic Polarization :-

Ionic polarization occurs only in those dielectric materials which possess ionic bonds such as in NaCl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement.

* The displacement causes an increase or decrease in the distance of separation between the atoms depending upon the location of the ion pair in the lattice



Ion placement in
the absence of the
field



Ion displacement due
to the applied field.

③ Orientalional Polarization :-

This polarization occurs in those dielectric material whether liquid or solids - which possess molecules with permanent dipole moment. (i.e. polar dielectrics)

* The orientation of these molecules will be random normally due to thermal agitation. Because of randomness in orientation the material has net zero dipole moment.

* The orientational polarization is strongly temperature dependent and decreases with increase of temperature.

In case of polar dielectrics, the orientational polarizability α_0 is given by

$$\alpha_0 = \frac{\mu^2}{3kT}$$

Q. 08) CS 10 calculate the probability that an energy level at 0.2 eV below Fermi level is occupied at temperature 500K.

Soln. - $E - E_F = 0.2 \text{ eV} = 0.2 \times 1.6 \times 10^{19} \text{ J}$

$f(E)$ at 500K = ?

$$f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1}$$

$$= \frac{1}{e^{\frac{0.2 \times 1.6 \times 10^{19}}{1.38 \times 10^{-23} \times 500}} + 1} = \frac{1}{e^{\frac{3.2 \times 10^{26}}{690 \times 10^{23}} + 1}}$$

$$= \frac{1}{e^{4.636 \times 10^3} + 1} = \frac{1}{e^{102.5} + 1}$$

$f(E) = 9.66 \times 10^{-3}$ at 500K.

Q 9) as Define nano-material and classify the nano-materials based on the dimensional constraints.

Solⁿ:- Nanoscale materials are defined as a set of substances where at least one dimension is less than approximately 100 nanometers. A nanometre is one millionth of a millimetre - approximately 100,000 times smaller than the diameter of a human hair.

* Nanomaterials are of interest because at this scale unique optical, magnetic, electrical and other properties emerge.

These emergent properties have the potential for great impacts in electronics, medicine and other fields.

Classification of Nanomaterials:-

Nanomaterials have extremely small size which having at least one dimension 100nm or less. Nanomaterials can be nanoscale in one dimension (Eg surface films), two dimension (Eg, strands or fibres) or three dimensions (Eg, particles). They can exist in single, fused, aggregated or agglomerated forms with spherical, tubular and irregular shapes. Common types of nanomaterials include nanotubes, dendrimers, quantum dots and fullerenes. Nanomaterials have applications in the field of nanotechnology. & displays different physical chemical characteristics from normal chemicals (i.e Silver nano, Carbon nano tube, fullerene, photo catalyst, Carbon Nano & silica)

Alc to Siegel Nanostructured materials are classified as zero dimensional, one dimensional, two dimensional, Three-dimensional nano structures.

Q. 9(b) Describe the construction and working of Scanning
12 Electron microscope with the help of a neat diagram.

Solⁿ:- Scanning electron microscope is an improved model of an electron microscope. SEM is used to study the three dimensional image of the specimen.

Principle:- When the accelerated primary electrons strike the sample, it produces secondary electrons. These secondary electrons are collected by a positively charged electron detector which in turn gives a three dimensional image of the sample.

Construction:-

It consists of an electron gun to produce high-energy electron beam. Magnetic condensing lens is used to condense the electron beam and scanning coil is arranged in between the magnetic condensing lens and the sample.

The electron detector scintillator is used to collect the secondary electrons and convert into electrical signal. These signals can be fed into CRO through video amplifier.

Working:-

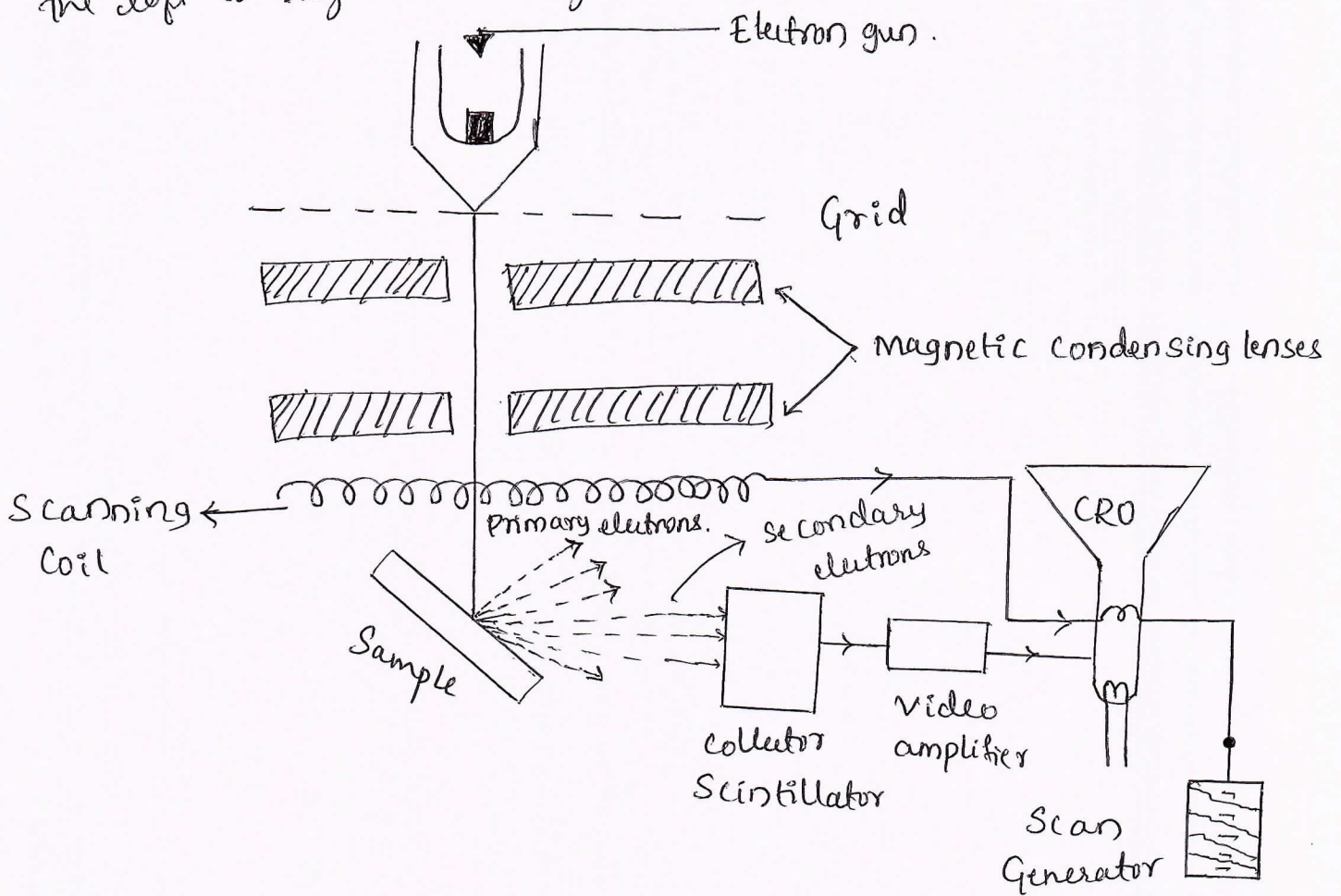
Stream of electrons are produced by the electron gun and the primary electrons are accelerated by the grid and anode. These accelerated primary electrons are made to fall on the sample through condensing lenses and scanning coil.

This high speed primary electron on falling over the sample produces low-energy secondary electrons. The collection

- of secondary electrons are very difficult because of their low-energy. Therefore to collect the secondary electrons a very high voltage is applied to the collector.

This is collected electrons produce skin t relations on photomultiplier tube or detector and are converted into electrical signals. These signals are amplified by the video amplifiers and is fed to the CRO.

in procedure the electron beam scan the sample from the left to right and again from the left to right etc.



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9 c) X rays are diffracted in the first order from a crystal with d spacing 2.8×10^{-10} m at a glancing angle 60° . Calculate the wavelength of X-rays.

Soln:- Given $n = 1$ $d = 2.8 \times 10^{-10}$ $\theta = 60^\circ$

$$\lambda = ?$$

$$n\lambda = d \sin \theta$$

$$\lambda = \frac{d \sin \theta}{n}$$

$$\lambda = \frac{2.8 \times 10^{-10} \cdot \sin 60^\circ}{1}$$

$$\lambda = \frac{2.8 \times 10^{-10} \times 0.866}{1}$$

$$\lambda = 2.42 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 2.42 \text{ \AA}}$$

Q 10) as mention the principle and application of X-Ray photoelectron Spectroscopy.

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Solⁿ:- Principle:-

Due to the bombardment of X-ray photon on the sample surface K and L electron are ejected which are further analysed by the analyser. Let us consider E_b , E_b' and E_b'' are binding energy of lower energy levels inner core orbitals, where E_v , E_v' and E_v'' are the energies of the valence shell electron

The monochromatic X-ray photon incident on the sample surface will electron abstract the energy from this X-ray photon and get ejected in terms of electron. Kinetic energy of the ejected electron is recorded by spectrometer and is given by

$$E_k = h\nu - E_b - \Phi$$

where E_k is K.E of the ejected electron

$h\nu$ → energy associated with incident photon

E_b → binding energy ejected electron

Φ → work function of the instrument.

The electron spectrometer made up of following components.

- (1) source, (2) sample holder (3) Analyser, (4) Detector
- (5) processor and The Read-out.

SOURCE:-

The simple X-Ray Photon source for X-Ray photoelectron spectra is X-ray tube equipped with magnesium or Aluminium metal target. monochromator crystal can also provide having bandwidth of 0.3 electron volt. much smaller spots on a surface to be examined.

16 Sample holder :-

Sample holder is located in between the source and the entrance slit of spectrometer. Crystal disperser selects the photon of known energy from the source and incident on the sample. The area inside the sample holder should be evacuated, within 10^{-5} Torr. pressure to avoid contamination of the surface sample.

The gaseous sample is introduced into a sample compartment through a slit, to provide a pressure of 10^{-10} torr. If the pressure is higher than attenuation of electron beam may take place, weaker signal may be obtained.

Analysers :- It is hemispherical in shape with very high electrostatic field is applied on analyser. Pressure maintained inside the analyser is 10^{-5} torr. when the electron enters, into the hemispherical analyser.

It travels in curved path and radius of curvature depends upon magnitude of field and kinetic energy of the electron.

Detector :- The electron channel multiplier tube or transducer are required of X-Ray photoelectron spectroscopy.

When single electron pass through the electron multiplier tube it is converted into number of electrons are pulses of electrons.

Q.104 b) Illustrate the working of transmission electron microscope.

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Applications of XPS :-

- ① Identification of active sites
- ② Determination of Surface Contamination on Semiconductors
- ③ Study of oxide layers on metals
- ④ Analysis of dust on the sample
- ⑤ Determination of oxidation state all the elements of periodic table can be determined

Q.105 b) Illustrate the working of TEM.

Solⁿ :- Stream of electrons are produced by the electron and is made to fall over the specimen using magnetic condensing lens.

Based on the angle of incidence the beam is partly transmitted and partly diffracted as shown in fig. Both the transmitted beam and the diffracted beams are recombined at the E-WALLED SPHERE of deflection, which encloses all possible reflections from the crystal or specimen satisfying the Bragg's law image as shown in figure. The combined image is called the phase contrast image.

In order to increase the intensity and the contrast of the image and amplitude contrast image has to be obtained for stop this can be achieved only by using the transmitting beam and does the diffracted beam has to be eliminated

18 Now in order to eliminate the diffracted beam that beam is passed through the magnetic objective lens and the aperture is shown in fig adjusted in such a way that the diffracted image is illuminated.

Thus the final image being alone is passed through the projector lens for further magnification.

Final image is recorded in the fluorescent screen or CCD this high contrast image is called Bright field image. In addition it has to be noted that the bright field image obtained is purely due to the elastic scattering ~~non~~ no energy change. that is due to the transmitted beam alone.