

# GBCS SCHEME

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18EC63

## Sixth Semester B.E. Degree Examination, July/August 2022 Microwave and Antennas

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Making use of functional block diagram explain the working of reflex Klystron oscillator. Also discuss modes of oscillation. (10 Marks)
- b. A transmission line has the following parameters,  $R = 2\Omega$ ,  $G = 0.5\text{mho/m}$ ,  $f = 1\text{GHz}$ ,  $L = 8\text{nH/m}$ ,  $C = 0.23\text{PF}$ . Calculate :
  - i) Characteristic impedance
  - ii) Propagation constant. (04 Marks)
- c. List the characteristics of smith chart. (06 Marks)

### OR

- 2 a. A reflex Klystron is to be operated at frequency of  $10\text{GHz}$ , with DC beam voltage  $300\text{V}$ , repeller space  $0.1\text{cm}$  for 1 mode, calculate  $P_{\text{REMax}}$  and corresponding repeller voltage for a beam current of  $20\text{mA}$ . (04 Marks)
- b. Derive the equation of transmission line with possible solution. (10 Marks)
- c. A certain transmission line has the characteristics impedance of  $75 + j0.01\Omega$  and is terminated in a load impedance of  $70 + j50\Omega$ . Compute :
  - i) The reflection coefficient
  - ii) Transmission coefficient
  - iii) Standing wave ratio. (06 Marks)

### Module-2

- 3 a. Prove that impedance and admittance matrices are symmetrical for a reciprocal junction. (05 Marks)
- b. List the characteristics of magic - T when all the ports are terminated with matched load. Also derive the expression of S-matrix for magic T. (10 Marks)
- c. In a H-plane T junction compute power delivered to the loads of  $40\Omega$  and  $60\Omega$  connected to arms 1 and 2 when a  $10\text{mW}$  power is delivered to the matched port 3. (05 Marks)

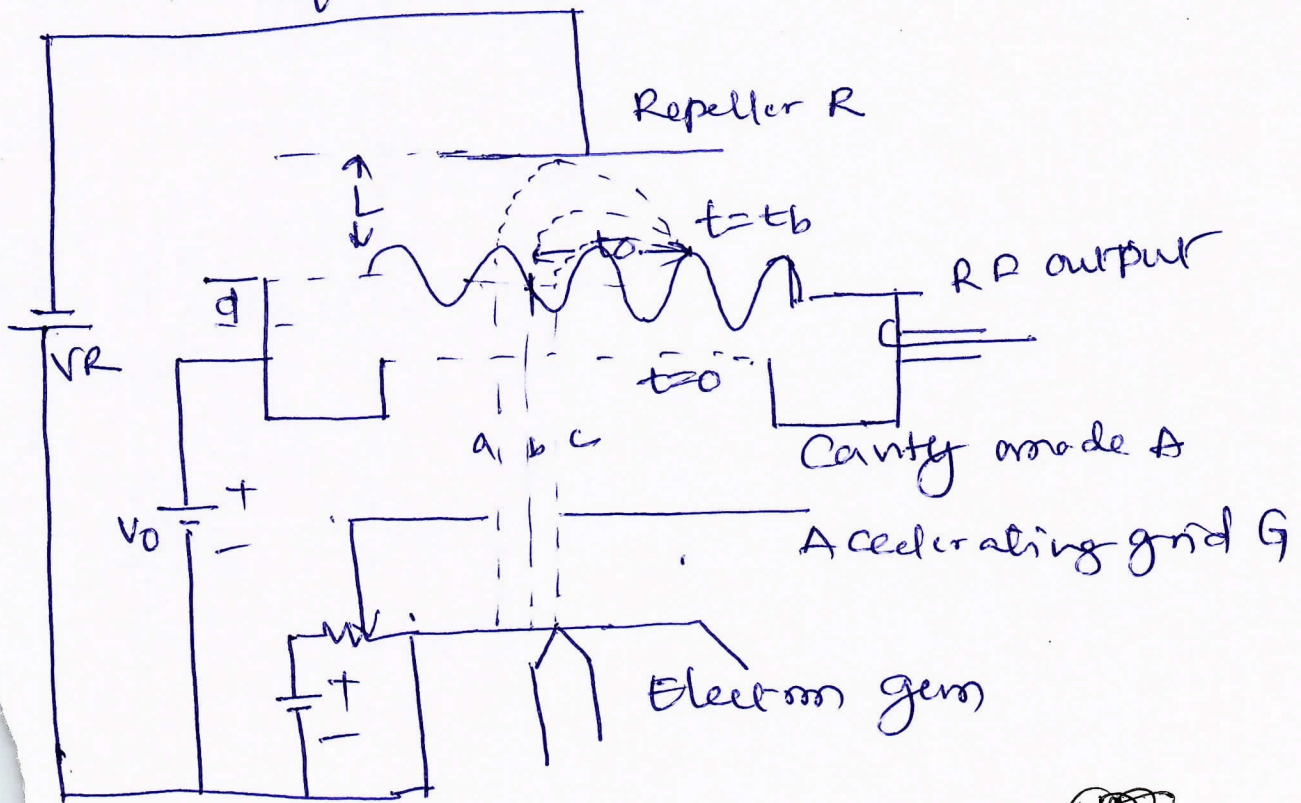
### OR

- 4 a. Derive the S-matrix representation for multiport network. Also define the losses in terms of S-parameters. (08 Marks)
- b. Explain briefly precision type variable attenuator. (05 Marks)
- c. What are waveguide tees? Explain its basic types with neat diagram. (07 Marks)

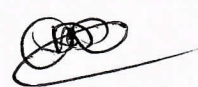
Scheme of solution July/August-2022  
 Sub: Microwave & Antennas (18EE63)

19 Mechanism of Oscillation in Reflex klystron  
 Reflex klystron tube uses only a single resonant microwave cavity as a resonator, which is shown below figure. The electron emitted from the cathode  $K$  is accelerated by grid  $G$  and pass through the cavity anode to the repeller space between the cavity anode and the repeller electrode  $R$ . The feedback required to maintain the oscillation within the cavity is obtained by reversing the electron beam emitted from the  $K$  towards  $R$  and sending back through the cavity.

The electron on the beam are velocity modulated before beam passed through the cavity.



4M



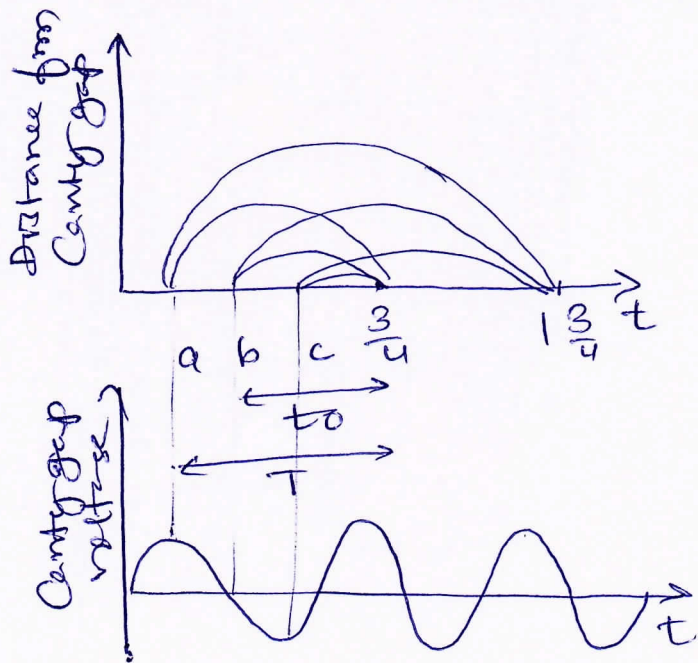
### Mechanism of oscillation.

Due to the DC voltage on the Cavity Circuit RF noise is generated on the Cavity. Electromagnetic noise pronounced at the Cavity resonant frequency.

The electrons a shown in the figure which enter during positive half cycle of free RF field on the Cavity gap will be accelerated. b electron which encountered zero RF field will pass with original unchanged velocity and free electron c which encountered free negative half cycle will be retarded on entering the repeller space.

6M

All velocity modulated electrons will be repelled back to the Cavity by the repeller due to negative potential. Repeller Space is adjusted to receive all the modulated electrons at the same time on the positive peak of the cavity RF velocity cycle and these bunched electrons lose their kinetic energy when they encounter the positive cycle of the Cavity RF field.



The power delivered by the bunched electron to the Cavity is greater than the power loss on the Cavity it produces the oscillations and is coupled to the o/p. A steady microwave oscillation is generated at resonant frequency of the Cavity.

~~6M~~



1b  $R=2\Omega$ ,  $G=0.5\text{S}$   $f=1\text{GHz}$   $L=8\text{nH}$   $\epsilon=2.23$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{2+j2\pi \times 10^9 \times 8 \times 10^{-9}}{0.5 \times 10^{-3} + j2\pi \times 10^9 \times 0.23 \times 10^{-12}}}$$

$$Z_0 = \sqrt{\frac{50.3 \angle 87.72^\circ}{15.29 \times 10^{-9} \angle 70.9^\circ}} = 181.39 \angle 8.40^\circ$$

2M

propagation constant

$$\begin{aligned} \gamma &= \sqrt{(R+j\omega L)(G+j\omega C)} \\ &= \sqrt{(50.31 \angle 87.72^\circ)(15.29 \times 10^{-9} \angle 70.9^\circ)} \\ &= \sqrt{769.24 \times 10^{-4} \angle 158.63^\circ} \\ &= 0.2774 \angle 79.31^\circ \\ \gamma &= 0.051 + j0.273 \end{aligned}$$

2M

1c. characteristics of Smith Chart

1. Constant  $r$  circle & constant  $x$  loci form two families of orthogonal circles
2. constant  $r$  & a circle pass through  $r=1$  and  $x=0$
3. upper half represents  $+jx$
4. Lower half represents  $-jx$
5. For admittance  $r$  circle becomes  $g$  circle and  $x$  circle becomes constant susceptance  $b$  circle
6. Distance around the Smith chart  $\lambda/2$  wavelength
7.  $Z_{min} = 1/\rho$  there is  $V_{min}$  on the line
8.  $Z_{max} = \rho$ , there is  $V_{max}$  on the line
9. Right of the chart center to  $V_{max}$   $Z_{min}$ ,  $Z_{max}$  and  $\rho$  (SWR)

$\frac{1 \times 10^9}{2}$   
 $= 6$





10. Left of the Chart center corresponds to  $V_{min}$ ,  $I_{max}$ ,  $Z_{min}$  and  $Y_p$
11. The normalized admittance  $Y$  is a reciprocal of normalized impedance  $Z$  corresponding quantities are  $180^\circ$  out of phase with those in the impedance chart
12. Normalized impedance or admittance is repeated for every half wavelength of distance
13. The distance are given in wavelengths towards the generator and also towards the load.

2a  $f = 10 \text{ GHz}$ ,  $V_0 = 300 \text{ V}$ ,  $L = 10^{-3} \text{ m}$ ,  $N = 1 \frac{3}{4}$   
 $I_0 = 20 \text{ mA}$

$$PRF_{\max} = \frac{0.398 V_0 I_0}{N} = \frac{0.398 \times 300 \times 20 \times 10^{-3}}{1 \frac{3}{4}}$$

$$= 1.365 \text{ Watts}$$

$$|VR| = 6.74 \times 10^6 f L \sqrt{V_0} / N - V_0$$

$$L = 10^{-3}, N = 1.75$$

$$|VR| = 6.74 \times 10^6 \times 10 \times 10^9 \times 10^{-3} \times \sqrt{\frac{300}{1.75}} - 300$$

$$VR = -367.08 \text{ V}$$

2 P2

4M

2b. The one possible sol<sup>n</sup> for transmission line

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}, \quad V_+ \text{ and } V_- \text{ represent the complex quantities, } \gamma = \alpha + j\beta$$

$$\text{and } I = Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z}) = Y_0 (V_+ e^{-\alpha z} e^{-j\beta z} - V_- e^{\alpha z} e^{j\beta z})$$

characteristic impedance

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

At microwave frequencies  $R \ll \omega L$  &  $G \ll \omega C$

$$\therefore \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(j\omega)^2 LC \left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)}$$

$$\approx j\omega \sqrt{LC} \left[1 + \frac{1}{2} \left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

$$\approx j\omega \sqrt{LC} \left[1 + \frac{1}{2} \left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

④



$$\gamma = \frac{1}{2} \left( R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right) + j\omega \sqrt{LC}$$

The attenuation constant and phase constant are

$$\alpha = \frac{1}{2} \left( R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right), \quad \beta = \omega \sqrt{LC}$$

The characteristic impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \left( 1 + \frac{R}{j\omega L} \right)^{1/2} \left( 1 + \frac{G}{j\omega C} \right)^{-1/2}$$

$$= \sqrt{\frac{L}{C}}$$

Phase velocity,  $V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

GM

$$\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10000}} = c = 3 \times 10^8 \text{ m/s}$$

For lossy microwave transmission line

$$V_e = \frac{1}{\sqrt{L_e C_e}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

The relative phase velocity factor

Velocity factor =  $\frac{\text{Actual phase velocity}}{\text{velocity of light in vacuum}}$

$$V_r = \frac{V_e}{c} = \frac{1}{\sqrt{\mu_r \epsilon_r}}$$



$$2c \quad Z_0 = 75 + j0.01 \quad Z_L = 70 + j50 \Omega$$

Reflection Coefficient

$$\begin{aligned} \Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{70 + j50 - (75 + j0.01)}{70 + j50 + (75 + j0.01)} \\ &= \frac{50.24 \angle 95.71}{153.38 \angle 19.03} = 0.33 \angle 76.68 \\ &= 0.08 + j0.32 \end{aligned}$$

Transmission Coefficient

$$\begin{aligned} T &= \frac{2Z_L}{Z_L + Z_0} = \frac{2(70 + j50)}{(70 + j50) + (75 + j0.01)} \\ &= \frac{172.05 \angle 35.54}{153.38 \angle 19.03} = 1.12 \angle 16.51 \quad \begin{array}{l} 2 \times 3 \\ = 6M \end{array} \\ &= 1.08 + j0.32 \end{aligned}$$

Standing wave ratio (SWR)

$$\begin{aligned} S &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \\ &= \frac{1 + 0.08}{1 - 0.08} \\ &= 1.17 \end{aligned}$$



39 Z and Y for reciprocal network

For multiport network, incident wave amplitude is  $V_0^+$ . Total voltage  $V_n = V_0^+ + V_n^-$   
= at all ports  $n=1, 2, \dots, N$

The  $i$ th ports the fields are  $\vec{E}_i, \vec{H}_i$   
for  $j$ th ports  $\vec{E}_j, \vec{H}_j$ . From Lorentz reciprocity theorem.

$$\int_S (\vec{E}_i \times \vec{H}_j - \vec{E}_j \times \vec{H}_i) \cdot d\vec{s} = 0$$

where  $S$  is the closed surface area of the conducting walls

$$\therefore \sum_{n=1}^N \int (\vec{E}_i \times \vec{H}_j - \vec{E}_j \times \vec{H}_i) \cdot d\vec{s} = 0$$

$V_n$  except  $V_i$  and  $V_j$  are zero.  $\vec{E}_i = \alpha \vec{e}_i$   
and  $\vec{E}_j = \alpha \times \vec{e}_j$  are zero on all reference planes at the corresponding ports except  $i$  and  $j$

$$\therefore \int_{t_i} (\vec{E}_i \times \vec{H}_j) \cdot d\vec{s} = \int_{t_j} (\vec{E}_j \times \vec{H}_i) \cdot d\vec{s}$$

$$P_{ij} = P_{ji}$$

where  $P_{ij}$  represents the power at reference plane  $i$  due to input voltage at plane  $j$

$$P = [Y][V] \text{ and } P = [V][Z]$$

$$\therefore V_i V_j Y_{ij} = V_j V_i Y_{ji} \text{ or } Y_{ij} = Y_{ji}$$

$$Z_{ij} = Z_{ji}$$

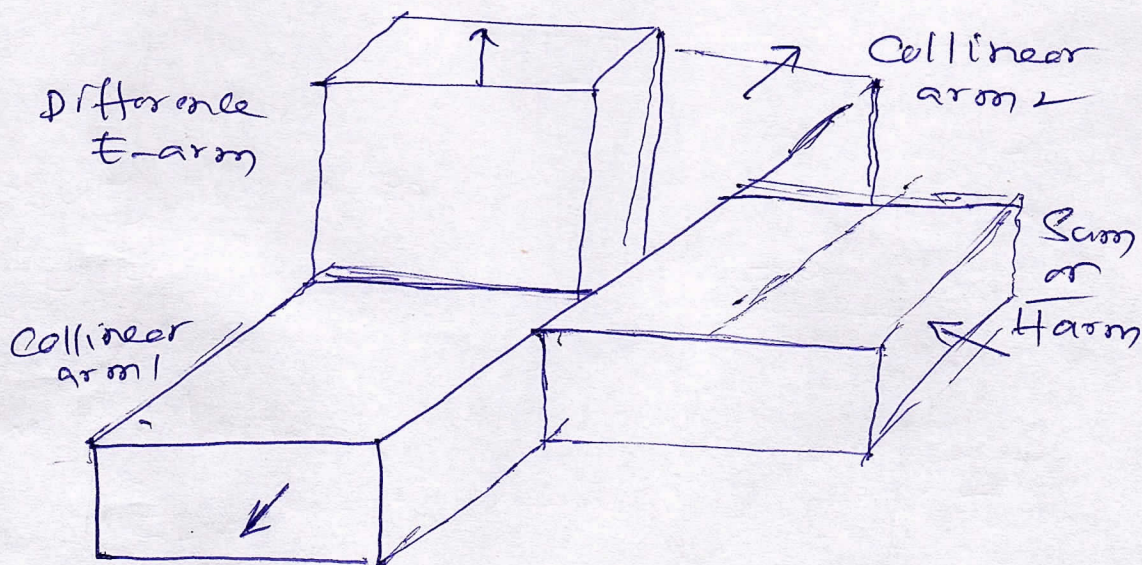
3 M

2 M



36 Hybrid or magic T

A combination of E plane and H-plane tees forms a hybrid tee, called magic tee (T) having 4 ports



Magic T has the following characteristics when all the ports are terminated with matched load.

1. If two in phase waves of equal magnitude are fed into port 1 and 2, the output at port 3 is subtractive and hence zero and total output appears additive at port 4.
2. Wave incident at port 3 divides equally between port 1 and port 2 but opposite in phase with no coupling to port 4  
 $S_{13} = S_{31} = 1/\sqrt{2} = S_{24} = S_{42}$  and  $S_{34} = 0$
3. Wave incident at port 4 divides equally between port 1 and port 2 in phase with no coupling to port 3  
 $S_{41} = S_{42} = 1/\sqrt{2} = S_{24} = S_{42}$  and  $S_{34} = 0$



4. A wave fed into one collinear port, or port 2 will not appear in the other collinear port 2 or P. collinear port 1 and 2 are isolated from each other

$$S_{12} = S_{21} = 0$$

For lossless magnet T matched at port 3 & 4

$$S_{33} = S_{44} = 0$$

The S-matrix consider the symmetry property of the junction

$$S_{14} = S_{41} = S_{24} = S_{42} \quad S_{31} = S_{13} = -S_{23} = -S_{32}$$

6M

$$S_{34} = S_{43} = 0 \quad S_{12} = S_{21} = 0$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{23} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

From the unitary property to rows 1 & 2

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

Subtracting two eqn  $|S_{11}|^2 - |S_{22}|^2 = 0$

or  $S_{11} = S_{22}$ ,

$$2|S_{13}|^2 = 1$$

or  $S_{13} = 1/\sqrt{2}$ ,  $2|S_{14}|^2 = 1$  or  $S_{14} = -1/\sqrt{2}$

$S_{11} = S_{12} = 0$  and  $S_{22} = 0$

$$\therefore [S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$



3c

with port 3 matched, the S matrix

$$[S] = \begin{bmatrix} -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{bmatrix}$$

8/p power at port 3 equally divided among 1 and 2, i/p at port 3 = 100mW = 0.01W  
power towards port 1 and port 2 = 0.005W  
=  $\frac{1}{2}|b_1|^2 = \frac{1}{2}|b_2|^2$

Reflected power from port 1 & port 2  
 $\frac{1}{2}|\Gamma_1 b_1|^2$  and  $\frac{1}{2}|\Gamma_2 b_2|^2$

power delivered to load  $Z_1 = 40 \Omega$  &  
 $Z_2 = 60 \Omega$

$$P_1 = \frac{1}{2}|b_1|^2 = \frac{1}{2}|\Gamma_1 b_1|^2 = \frac{1}{2}|b_1|^2(1 + |\Gamma_1|^2)$$

$$2 + 2 + 2 = 6W$$

$$P_2 = \frac{1}{2}|b_2|^2 = \frac{1}{2}|\Gamma_2 b_2|^2 = \frac{1}{2}|b_2|^2(1 - |\Gamma_2|^2)$$

Taking the characteristic impedance

$$|\Gamma_1| = \frac{|40 - 50|}{|40 + 50|} = \frac{1}{9}|\Gamma_1|^2 = 8.2694 \times 10^{-3}$$

$$\therefore P_1 = 0.005(1 - 0.01234) = 4.938 \times 10^{-3} = 4.9383 \text{ mW}$$

$$P_2 = 0.005(1 - 8.2694 \times 10^{-3}) = 4.9586 \times 10^{-3} \text{ W} = 4.9586 \text{ mW}$$





### 4a S-matrix representation of multi-port Network

The amplitudes are normalized and the average power in the wave

Input power at the  $n$ th port  $P_{in} = \frac{1}{2} |a_n|^2$

Reflected power at the  $n$ th port  $P_{rn} = \frac{1}{2} |b_n|^2$

$a_n$  and  $b_n$  represent incident and reflected wave peak amplitude at the  $n$ th port

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} = \frac{V_1 - V_1^-}{\sqrt{Z_0}}, \quad a_2 = \frac{V_2^+}{\sqrt{Z_0}} = \frac{V_2 - V_2^-}{\sqrt{Z_0}}$$

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = \frac{V_1 - V_1^+}{\sqrt{Z_0}}, \quad b_2 = \frac{V_2^-}{\sqrt{Z_0}} = \frac{V_2 - V_2^+}{\sqrt{Z_0}}$$

Total voltage is the sum of the incident and reflected voltage  $V^+$  and  $V^-$

$$\therefore V_1 = V_1^+ + V_1^- \quad \& \quad V_2 = V_2^+ + V_2^-$$

The total or net power flow into the port

$$P = P_i - P_r = \frac{1}{2} (|a|^2 - |b|^2)$$

For the two port network relation between incident and reflected waves expressed in terms of S parameter

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

The physical significance of S-parameters

$S_{11} = (b_1/a_1)_{a_2=0}$  = Reflection coefficient  $\Gamma_1$  at port 1

$S_{22} = (b_2/a_2)_{a_1=0}$  = Reflection coefficient  $\Gamma_2$  at port 2

4M

$S_{12} = (b_1/a_2) a_1=0 = \text{att\# of wave travelling from port 2 to port 1 when } a_1=0$

$S_{21} = (b_2/a_1) a_2=0 = \text{att\# of wave travelling port 1 to port 2 when } a_2=0$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

For multi-port network, the S-parameter

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1N} \\ S_{21} & S_{22} & \dots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \dots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

4N

In two port network if power fed at port 1 is  $P_i$  and reflected at the same port is  $P_r$  and output port 2 is  $P_o$ . The losses defined as

$$\begin{aligned} \text{Insertion loss (dB)} &= 10 \log \frac{P_i}{P_o} = 10 \log \frac{(a_1)^2}{(b_2)^2} \\ &= 20 \log \frac{1}{|S_{21}|} = 20 \log \frac{1}{S_{12}} \end{aligned}$$

$$\text{Transmission loss (dB)} = 10 \log \left( \frac{P_i - P_r}{P_o} \right)$$

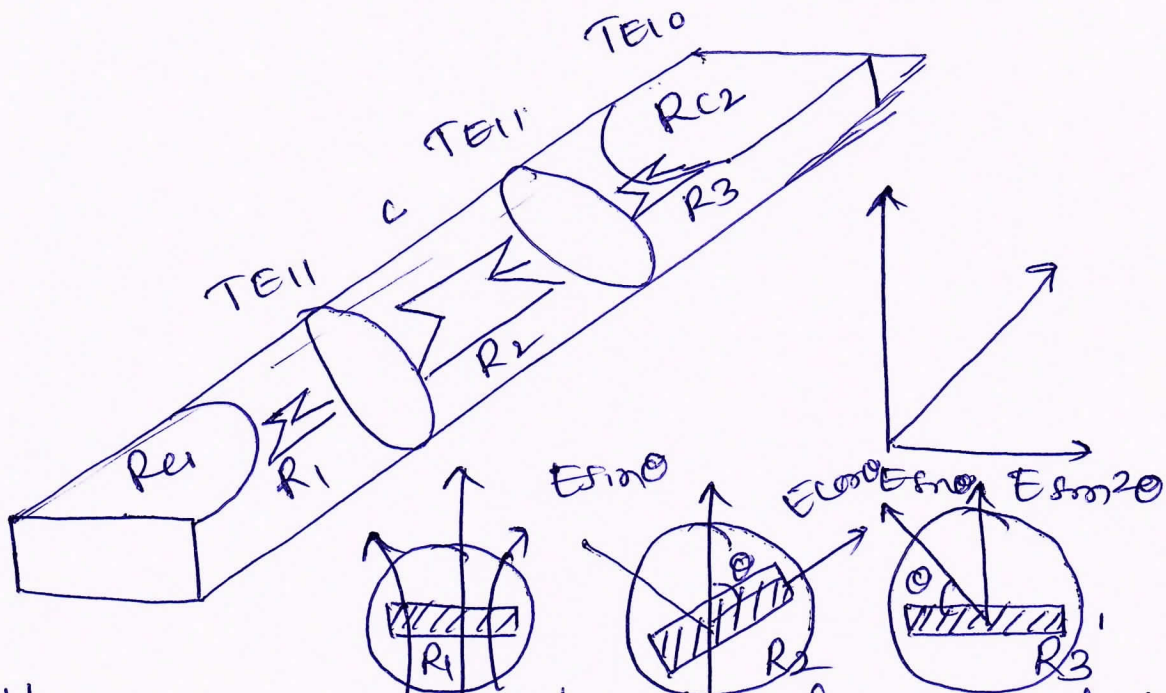
$$\text{Reflection loss (dB)} = 10 \log \frac{P_i}{P_i - P_r}$$

$$\text{Return loss (dB)} = 10 \log \frac{P_i}{P_r}$$

$$= 20 \log \frac{1}{|\Gamma|} = 20 \log \frac{1}{|S_{11}|} \quad \text{Ⓢ}$$



4b. precision type variable attenuator.



2M

Attenuators are passive device used to control power level in microwave system by partially absorbing transmitted signal wave

$R_1, R_2, R_3$  tapered resistive cards

Precision type variable attenuator makes use of a circular section (C) connected axisymmetric section of circular to Rectangular ( $R_{C1}$  &  $R_{C2}$ ). The center circular section with resistive card can be precisely rotated by  $360^\circ$  with respect to the two fixed sections of circular to Rectangular waveguide transition.

1M

Ⓢ

The induced current on the resistive card  $R_2$  due to the incident signal is dissipated as a heat-producing attenuator of transmitted signal. A very thin resistive card placed perpendicular to the E field at the circular end of each transmitted section so that the field perpendicular to it is negligible effect but absorbs the parallel component to it.

The center section is kept at an angle  $\theta$  relative to the E field direction of TE<sub>11</sub>. The component  $E \cos \theta$  parallel to the card gets absorbed. Finally the electric field component is  $E \sin^2 \theta$ .

$$\Rightarrow \alpha = \frac{E}{E \sin^2 \theta} = \frac{1}{\sin^2 \theta} = \frac{1}{|S_{21}|}$$

$$\text{or } \alpha(\text{dB}) = -40 \log(\sin \theta) = -20 \log |S_{21}|$$

$\therefore$  The S-matrix of precision attenuator

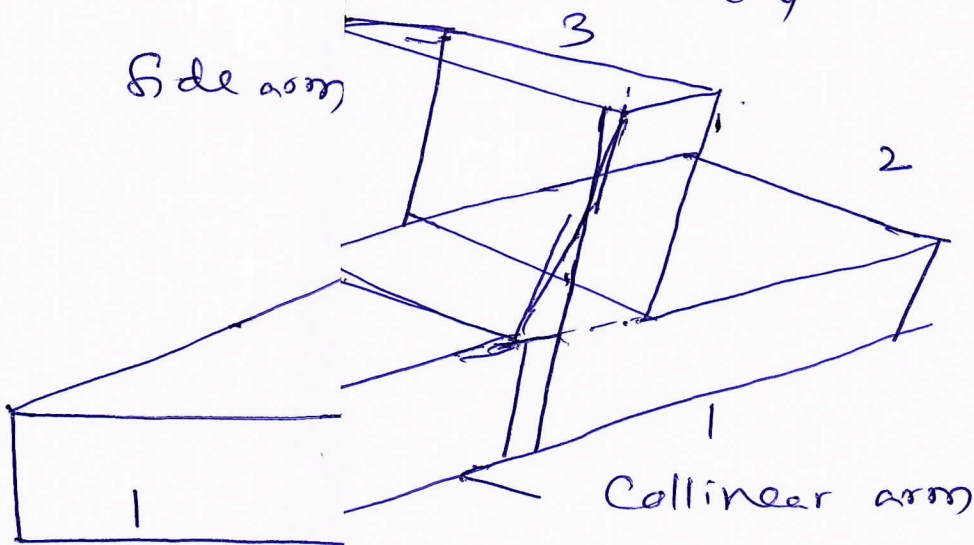
$$[S] = \begin{bmatrix} 0 & \sin^2 \theta \\ \sin^2 \theta & 0 \end{bmatrix}$$



Wave guide tees: Three port component used to connect the waveguide or section. It provides splitting and combining power in waveguide system.

- E-plane Tee (Series)
- H-plane Tee (Shunt)

These are named according to the arm which is parallel to the E field or H field respectively.

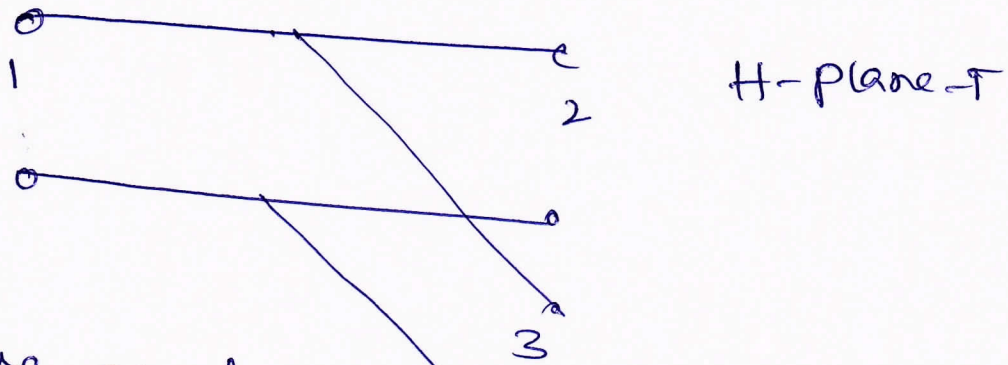
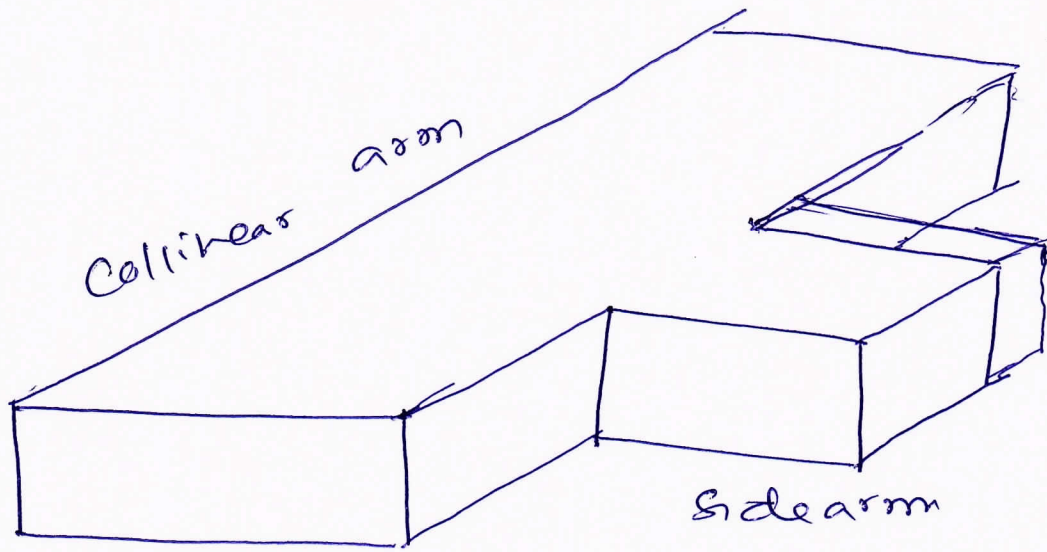


$$\frac{3}{2} \times 2 = 7m$$



E-plane-T





Wave guide tees are poorly matched devices. Adjust the matching scale once by means of turning the screw at the center. The S matrix is symmetric

$$S_{ij} = S_{ji} = 1, 2 \quad i, j = 1, 2, 3$$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$



59. The width of the conducting strip is

$$W = \frac{377 \cdot d}{\sqrt{\epsilon_0 \mu_0} Z_0} = \frac{377 \cdot 4 \times 10^{-3}}{\sqrt{6} \cdot 50} = 12.31 \times 10^{-3} \text{ m}$$

Strip line capacitance

$$C = \frac{\epsilon d W}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 12.31 \times 10^{-3}}{4 \times 10^{-3}}$$

$$= 163.57 \text{ pF/m}$$

The strip line inductance

$$L = \frac{\mu_0 d}{W} = \frac{4\pi \times 10^{-7} \times 4 \times 10^{-2}}{12.31 \times 10^{-3}}$$

$$= 0.41 \text{ uH/m}$$

2x4

8M

The phase velocity

$$v_p = \frac{c}{\sqrt{\epsilon_0 \mu_0}}$$

$$= \frac{3 \times 10^8}{\sqrt{6}}$$

$$= 1.22 \times 10^8 \text{ m/s}$$

5b) Directivity: The number of point sources can resolve is numerically equal to directivity of an antenna. or The ratio of the maximum power density to the average power density

$$\therefore D = N = \frac{P(\theta, \phi)_{\max}}{P(\theta, \phi)_{\text{avg}}}$$

(ii) Beam area: In the two dimensional co-ordinates an incremental area  $dA$  on the surface of the sphere is the product of the length  $r d\theta$  in the  $\theta$  direction &  $r \sin\theta d\phi$  in the  $\phi$  direction (longitude)

$$\therefore dA = (r \cdot d\theta) (r \sin\theta \cdot d\phi) = r^2 d\Omega$$

$d\Omega$  is solid angle

~~2x2~~

= 6

2x3

= 6M

(iii) Radiation pattern: Radiation pattern with respect to the field intensity. The field pattern can be presented by a plane cuts through the main lobe axis. Two such cuts at right angle principal plane pattern.

(i) Field pattern (ii) power pattern

~~(iii)~~



SC  $U = U_m \sin\theta \cdot \sin^2\phi$        $0 \leq \theta \leq \pi$  &  $0 \leq \phi \leq \pi$

$$D = \frac{4\pi}{\int_0^\pi \int_0^\pi \sin^3\theta \cdot \sin^2\phi \, d\theta \, d\phi}$$

$$= \frac{4\pi}{2\pi/3} = 6 \quad \text{Exact directivity } 20\text{dB}$$

$$D \approx \frac{41,253^\square}{90^\circ \times 90^\circ} = 5.1 \quad \text{Approximate directivity}$$

$$10 \log \frac{6.0}{5.1} = 0.7\text{dB} = \text{Difference}$$

6a.  $d = 30 \text{ miles}$        $D = 45\text{dB}$        $f = 3\text{GHz}$

$P_t = 9$        $P_r = 1\text{mW}$

$$P_r = \frac{P_t A_{et} \cdot A_{er}}{r^2 \lambda^2} \quad \left[ G_t = \frac{4\pi A_{et}}{\lambda^2} \right] \text{---3dB}$$

$$= \frac{P_t G_t G_r}{4\pi r^2} =$$

$$\Rightarrow P_t \approx \frac{P_r \cdot 4\pi r^2}{G_t G_r} = \frac{1 \times 10^{-3} \times 4\pi \times (48)^2}{(45\text{dB}) \times (45\text{dB})}$$

6b- Losses in microstrip line.

Two types of losses occur in microstrip

① dielectric loss in the substrate and  
ohmic loss in the strip conductor

$$\alpha = -\frac{d\beta/dz}{2P(z)} = \alpha_d + \alpha_c,$$

$\alpha_d \rightarrow$  dielectric attenuation,  $\alpha_c =$  ohmic attenuation

$$\alpha_d = \frac{\sigma'}{2\sqrt{\epsilon}} \quad \sigma' \rightarrow \text{Conductivity of dielectric substrate}$$

$$\alpha_d = 27.3 \frac{\epsilon_r \sigma'}{\epsilon_r \epsilon_0} \frac{\tan \delta}{\lambda_g} \text{ dB}/\lambda_g$$

2x3  
= 6M

Ohmic losses: loss due to non-perfect conductor, Conduction attenuation constant

$$\alpha_c = \frac{8.686 R_s}{Z_0 W} \text{ dB/cm for } \frac{W}{h} \gg 1$$

Radiation losses: The radiation loss depends on the substrate thickness and dielectric constant. Radiation loss is calculated using the assumption,

1. TEM Transmission
2. Uniform dielectric in the neighbourhood of the strip
3. Neglect the radiation from TE

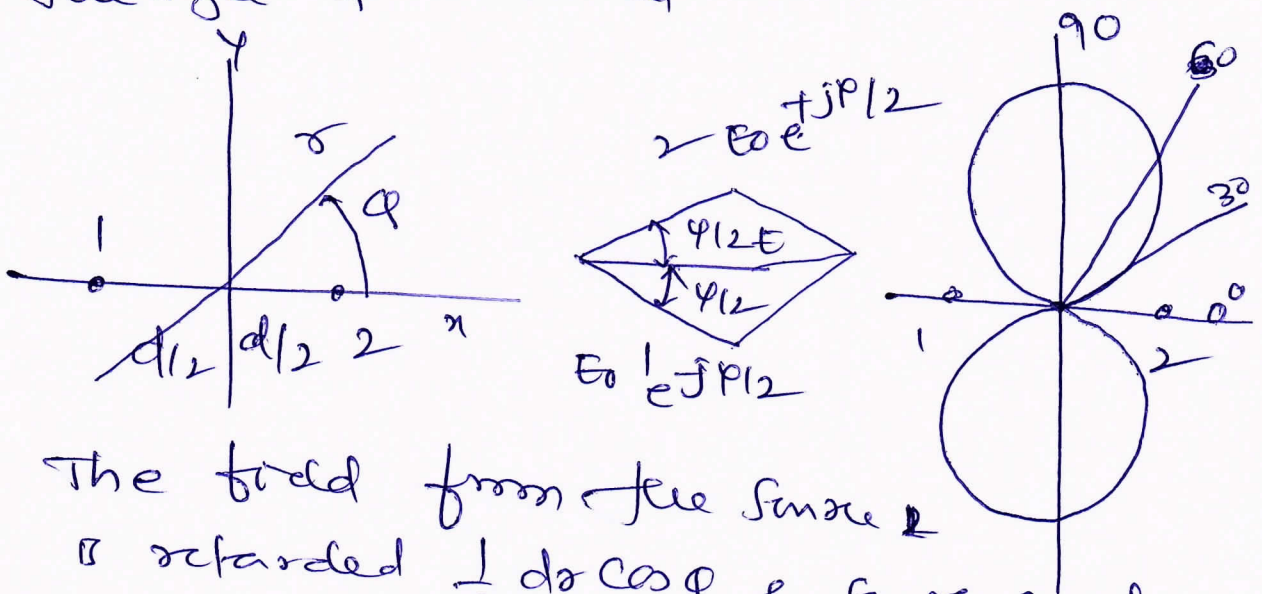
$$R_r = 240 \pi^2 \left(\frac{h}{\lambda}\right)^2 R(\epsilon_r)$$

①



19. Two isotropic point sources of same amplitude and same phase

Let two point sources 1 and 2 separated by distance  $d$  and located symmetrically into respect to the origin of the co-ordinates. The angle  $\phi$  measured counter clockwise



The field from the source 1

is retarded  $\frac{1}{2} d \cos \phi$  & source 2 advanced by  $\frac{1}{2} d \cos \phi$ .  $\therefore d \cos \phi = \frac{2\pi d}{\lambda} = \beta d$

The total field at large distance  $r$  in the direction  $\phi$   $E = E_0 e^{-j\beta r/2} + E_0 e^{+j\beta r/2}$

$$\phi = d \cos \phi \quad \therefore E = 2 E_0 \frac{e^{+j\beta r/2} + e^{-j\beta r/2}}{2}$$

$$\therefore E = 2 E_0 \cos \beta r/2 = 2 E_0 \cos \left( \frac{d}{\lambda} \cos \phi \right)$$

$$\text{Set } 2 E_0 = 1, \quad d = \lambda/2 \quad d \cos \phi = \pi$$

$$\therefore E = \cos \left( \frac{\pi}{2} \cos \phi \right)$$

The field pattern  $E$  versus  $\phi$ . The pattern is bi-directional figure of 8 with maxima along the  $y$  axis. The space pattern is doughnut-shaped being a figure of 8 when you look at this pattern around the  $z$  axis.

$$E = E_0 + E_0 e^{j\phi} \quad \phi = d r \cos \theta$$

$$\therefore E = 2E_0 \cos \phi / 2 = 2E_0 \cos \frac{d r \cos \theta}{2}$$

The phase of the total field  $E$  is

$$E = E_0 (1 + e^{j\phi}) = 2E_0 e^{j\phi/2} \left( \frac{e^{j\phi/2} + e^{-j\phi/2}}{2} \right)$$

$$= 2E_0 e^{j\phi/2} \cos \phi/2$$

$$\therefore E = e^{j\phi/2} \cos \phi/2 = \cos \phi/2 \angle \phi/2$$

$$[\because 2E_0 = 1]$$



7b Radiation resistance of short dipole

The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiated. The power then equated to  $I^2 R$ , where  $I$  is the rms current of the dipole and  $R$  resistance called as radiation resistance.

$$P = \frac{1}{2} \operatorname{Re}(E \times H^*)$$

The far field components are  $E_\theta$  and  $H_\phi$  & the radial component

$$S_r = \frac{1}{2} \operatorname{Re} E_\theta \cdot H_\phi^*$$

-4M

The far field components related by the intrinsic impedance of the medium

$$E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\epsilon}}$$

$$S_r = \frac{1}{2} \operatorname{Re} Z H_\phi H_\phi^* = \frac{1}{2} |H_\phi|^2 \operatorname{Re} Z$$

$$= \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\epsilon}}$$

The total power

$$P = \iint S_r \cdot dS$$

$$= \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin \theta \, d\theta \, d\phi$$

-4M

$H_\phi$  is absolute value of the magnitude

$$|H_{\theta}| = \frac{W \mathcal{P}_0 L \sin \theta}{4\pi Cr}$$

$$\therefore P = \frac{1}{32} \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 \mathcal{P}_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^{\pi} \sin^3 \theta \sin \theta d\theta d\phi$$

Double integral equals  $8\pi/3$

$$\therefore P = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 \mathcal{P}_0^2 L^2}{12\pi}$$

The average power which is streaming out of the sphere surrounding the dipole is equal to the power radiated

$$P = \mathcal{P}^2 R$$

$$\therefore \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 \mathcal{P}_0^2 L^2}{12\pi} = \left(\frac{\mathcal{P}_0}{\sqrt{2}}\right)^2 R$$

Solving for  $R$

$$R = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 \cdot L^2}{6\pi} = \sqrt{\frac{\mu}{\epsilon}} = 377 = 120\pi \Omega$$

2M

$\therefore$  Radiation resistance of free short dipole

$$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 = 80\pi^2 L_{\lambda}^2 = 790 L_{\lambda}^2 \Omega$$

$$R_r = 790 L_{\lambda}^2 \Omega$$



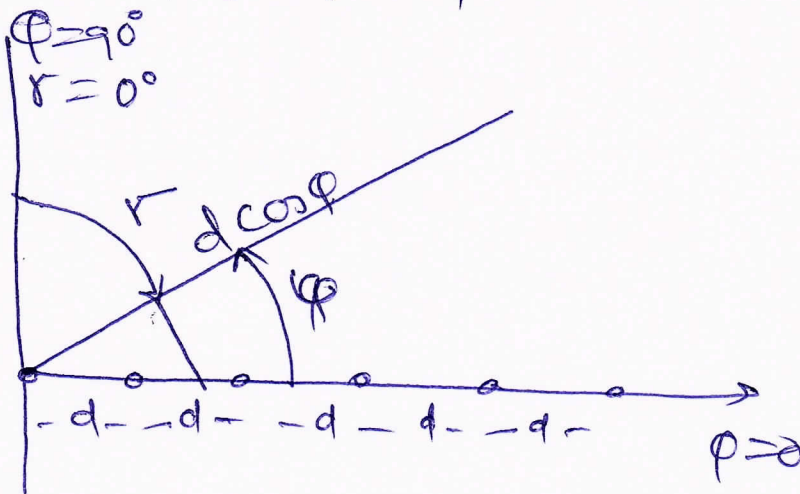


89 Linear array of  $n$  isotropic point sources of equal amplitude and spacing

Let the case of the  $n$  isotropic point sources, where  $n$  is a positive integer.

The total field at large distance in the direction of  $\phi$

$$E = 1 + e^{j\phi} + e^{j2\phi} + \dots + e^{j(n-1)\phi} \quad \text{--- (1)}$$

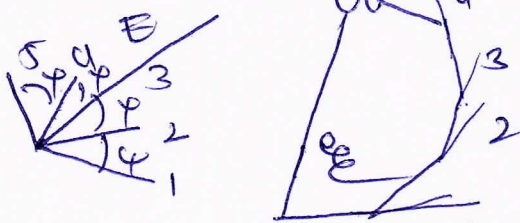


4M

$\phi$  is the total phase difference

$$\phi = \frac{2\pi d}{\lambda} \cos \phi + \delta = dr \cos \phi + \delta$$

$\delta \rightarrow$  Phase difference of adjacent sources.



Multiply eqn (1) by  $e^{j\phi}$

4M

$$\Rightarrow E \cdot e^{j\phi} = e^{j\phi} + e^{j2\phi} + \dots + e^{jn\phi} \quad \text{--- (3)}$$

Subtract (3) from (1) and divide by  $1 - e^{j\phi}$

~~4M~~

$$E = \frac{1 - e^{jN\varphi}}{1 - e^{j\varphi}} \quad \text{--- (1)}$$

$$\begin{aligned} \therefore E &= \frac{e^{jN\varphi/2} (e^{jN\varphi/2} - e^{-jN\varphi/2})}{e^{j\varphi/2} (e^{j\varphi/2} - e^{-j\varphi/2})} \\ &= e^{j\frac{N-1}{2}\varphi} \frac{\sin N\varphi/2}{\sin \varphi/2} = \frac{\sin N\varphi/2}{\sin \varphi/2} \underline{E_0} \end{aligned}$$

$E_0$  is referred to the field from source

$$\therefore E = \frac{\sin}{2} \varphi$$

The phase referred to the center part of the array

$$E = \frac{\sin N\varphi/2}{\sin \varphi/2}$$

$$\text{for } \varphi = 0 \quad E = N$$

2M

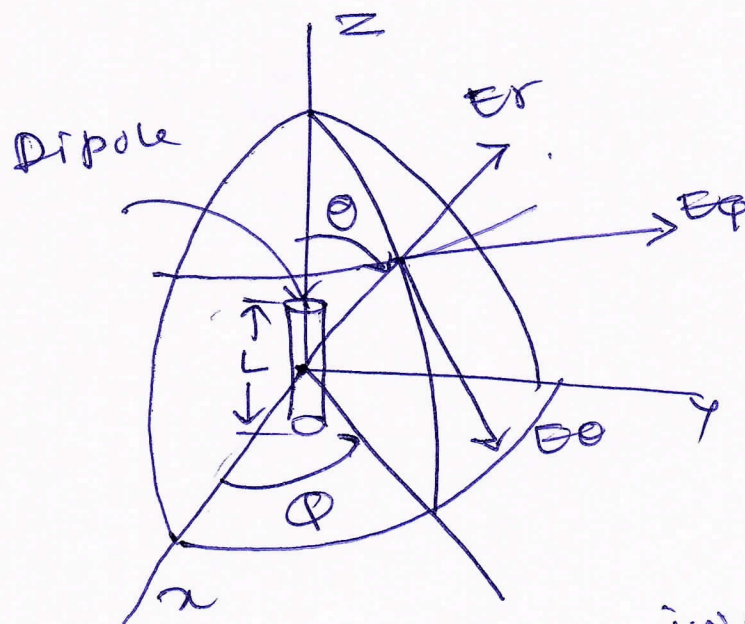
The maximum value of  $E$  can attain

$$E_{\max} = N \quad \therefore E = \frac{1}{N} \frac{\sin(N\varphi/2)}{\sin(\varphi/2)}$$



8b Field of Short dipole

Fields everywhere around the short dipole. Let the dipole length  $L$  placed on  $z$  axis with  $z$  axis and center at origin



The electric field components are  $E_r, E_\theta,$  and  $E_\phi$  medium from around the dipole or air

AM

The current  $I = I_0 e^{j\omega t}$  — (1)

$[I] = I_0 e^{j\omega[t - (r/c)]}$

$I \rightarrow$  Called retarded current

$r/c \rightarrow$  Results in phase retardation

$\omega r/c = 2\pi f r/c = 360^\circ t/T, T = 1/f$

The retarded vector potential of electric current  $A_z$

$$A_z = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{[I]}{r} dz$$

$z \rightarrow$  distance to the point on the conductor

$I_0 \rightarrow$  Peak value of the

(10)

Current

$\epsilon_0 \rightarrow$  Permittivity of free space  
 $= 4\pi \times 10^{-7} \text{ H/m}$

$$A_2 = \frac{\epsilon_0 L \rho_0 e^{j\omega(t - r/c)}}{4\pi r} \quad \begin{matrix} r \gg L \\ \lambda \gg L \end{matrix}$$

3M

The retarded scalar potential

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{[\rho]}{s} d\tau \quad \text{--- (5)}$$

$$\rho = \rho_0 \cdot e^{j\omega(t - r/c)}$$

$d\tau \rightarrow$  infinitesimal volume element

$\epsilon_0 \rightarrow$  Permittivity =  $8.85 \times 10^{-12} \text{ F/m}$

$$V = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{s_1} - \frac{q}{s_2} \right] L/2 \cos\theta$$

$$\begin{aligned} \Rightarrow q &= \int \rho \cdot dt = \rho_0 \int e^{j\omega(t - r/c)} \cdot dt \\ &= \rho / j\omega = \rho / j\omega \end{aligned}$$

$$\Rightarrow V = \frac{\rho_0}{4\pi\epsilon_0 j\omega} \left[ \frac{e^{j\omega(t - s_1/c)}}{s_1} - \frac{e^{j\omega(t - s_2/c)}}{s_2} \right]$$

3M

$$s_1 = r - \frac{L}{2} \cos\theta, \quad s_2 = r + \frac{L}{2} \cos\theta$$

Electric field of short dipole

$$E_r = \frac{\rho_0 L \cos\theta e^{j\omega(t - r/c)}}{2\pi\epsilon_0} \left[ \frac{1}{r^2} + \frac{1}{j\omega r^3} \right]$$

$$E_\theta = \frac{\rho_0 L \sin\theta e^{j\omega(t - r/c)}}{4\pi\epsilon_0} \left[ \frac{j\omega}{cr} + \frac{1}{r^2} + \frac{1}{j\omega r^3} \right]$$

$$H_\phi = \frac{\rho_0 L \sin\theta e^{j\omega(t - r/c)}}{4\pi} \left[ \frac{j\omega}{cr} + \frac{1}{r^2} \right]$$

(10)



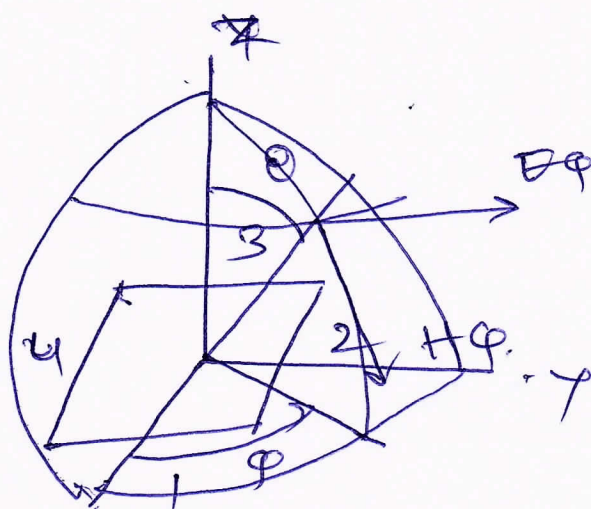
9a far field of small loop antenna

Small circular loop of radius  $a$  and considering square loop of same area

$$d^2 = \pi a^2, \quad d \rightarrow \text{side length of square loop}$$

The far field Electric field has only the  $E_\phi$  component. The far field pattern in the  $\phi$  plane

Consider two of the four small dipoles 2 & 4



Since the individual small dipoles are 4M non-directional in the  $\phi$  plane

It is same as the two isotropic power sources.

$$\Rightarrow E_\phi = -E_{\phi 0} \cdot e^{j\phi/2} + E_{\phi 0} \cdot e^{-j\phi/2}$$

$E_{\phi 0} \rightarrow$  Electric field from the individual dipole  $\Rightarrow \psi = \frac{2\pi d}{\lambda} \sin\theta = dr \sin\theta$

$$\Rightarrow E_\phi = -2jE_{\phi 0} \sin\left(\frac{dr \sin\theta}{2}\right)$$

$\phi$  indicates the total field  $E_\phi$  in

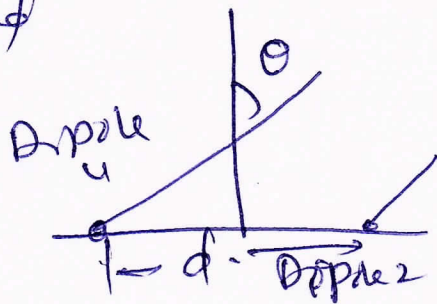
phase quadrature.  $E_{\theta 0}$  of the individual dipole if  $d \ll \lambda$ , the above  $E_{\theta}$  can be written as

$$E_{\theta} = -j E_{\theta 0} d r \sin \theta$$

The angle  $\theta$  is different with small dipole. Thus the far field of individual dipole

$$E_{\theta 0} = \frac{j 60 \pi [I] L}{r} \quad \text{--- (6)}$$

where  $I$  is the retarded current on the dipole and  $r$  is the distance from the dipole



$$\therefore E_{\theta} = \frac{60 \pi [I] L d r \sin \theta}{r^2}$$

Length  $L$  of short dipole is  $L = d$  as  $d \ll \lambda$  ie  $L = d$ ,  $d r = 2 \pi d / \lambda$

and the area  $A$  of the loop is  $d^2$

$$\therefore \text{Small loop } E_{\theta} = \frac{120 \pi^2 [I] \sin \theta A}{r^2}$$

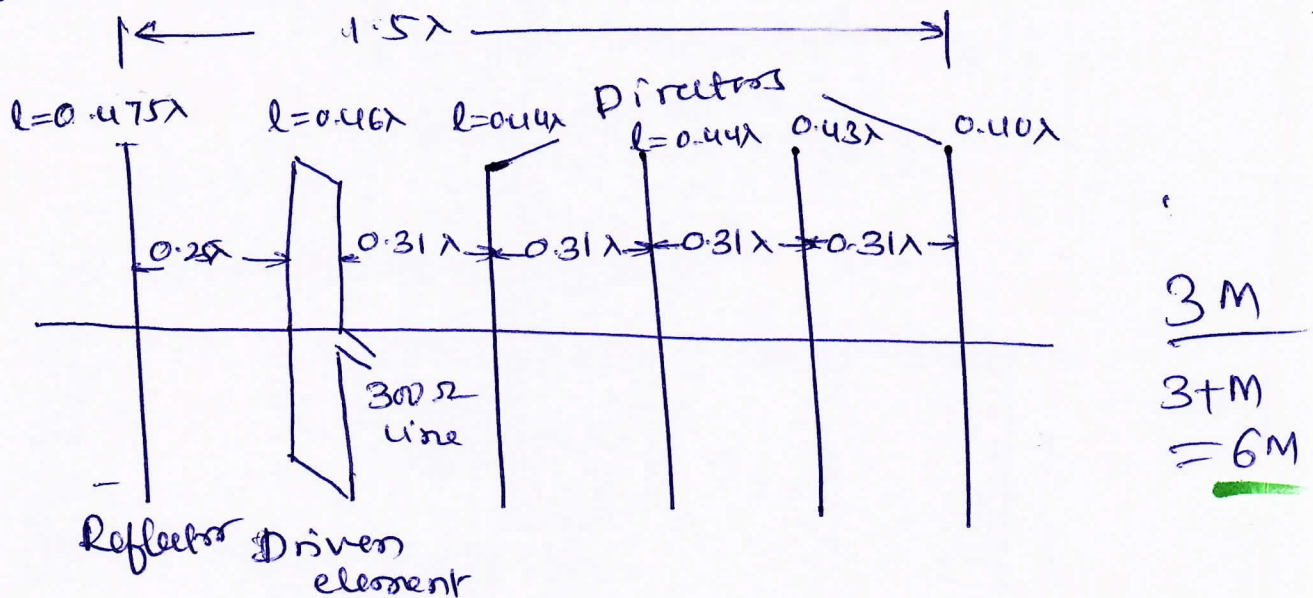
Far  $E_{\theta}$  field.

4M



Q.6 i) Yagi Uda. Array. :- Yagi - professor of Electrical Engineering and Uda senior presented paper in 1926. The narrow beam of short wave produced by guiding action of the multidirector periodic structure which is called wave canal.

A modern 6-element Yagi-Uda antenna. It consists of driven element  $\lambda/2$  folded dipole fed by  $300\ \Omega$  two wire transmission line (two wire), a reflector and four directors.



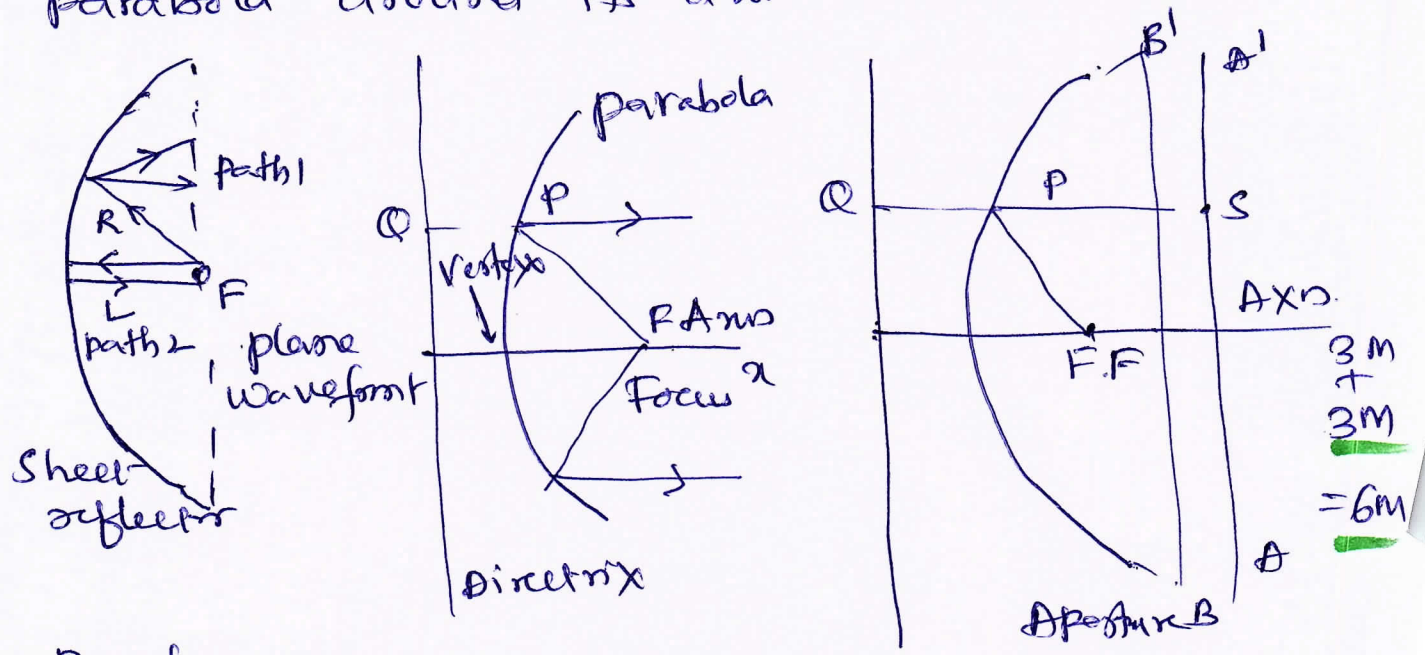
Dimensions and length & spacing are indicated in the figure. The antenna provides gain of 10dBi maximum with bandwidth of half power of about 10 percent.

Narrow bandwidth of Yagi Uda antenna can be broadened to 1.5 to 1 by lengthening the reflector or to improve the operation at low frequencies.

and shortening the directors to improve the high frequency operation this sacrifice the gain of as much as 5dB

Qb(11) Parabolic Reflector

The surface generated by the revolution of the parabola around its axis is called the paraboloid



Parabolic Curve - The distance from the point P on parabolic curve to fixed point F. Called the focus perpendicular distance to the fixed line called the directrix  $PF = P\phi$ . Let  $AA'$  be the line Normal to the axis at the arbitrary distance  $\phi$  from the directrix  $PS = \phi - P\phi$ .  $\therefore PF = P\phi$ ,  $PF + PS = PF + \phi - P\phi = \phi$ .

The parabolic reflector that all the waves from the isotropic source at the focus that are reflected from the parabola arrive at line  $AA'$  with



Equal phase. The image of the focus is the directrix. The plane BB1 at which the reflector is cut off is called aperture plane.

The distance from the source to the plane wave front via path 1 and path 2 to be equal

$$2L = R(1 + \cos\theta) \quad \& \quad R = \frac{2L}{1 + \cos\theta}$$

~~to e~~ The helix is represented by end fire array of point sources, spaced  $\lambda/4$  which source for each turn

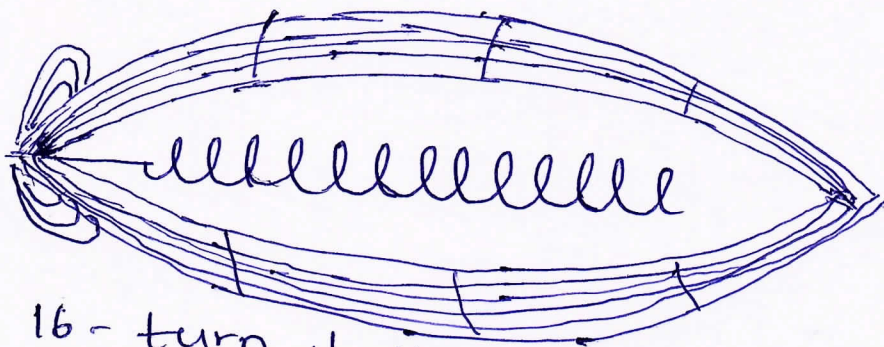
The end fire array phase between source is  $-90^\circ$

The axial mode helix  $\delta = 101.25^\circ$

$$\text{HPBW (Half power beam width)} = \frac{52}{C\lambda\sqrt{N}} = 26^\circ$$

$$\text{Axial ratio} = \frac{(2n+1)}{2n} = \frac{33}{32} = 1.03$$

Gain = 15.4 dBi from the pattern integration

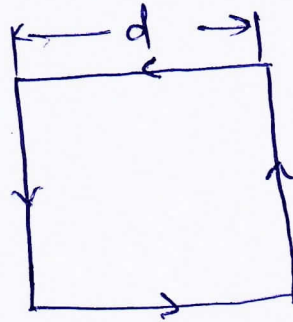
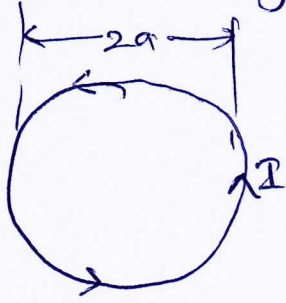


16 - turn helical beam antenna & its Power Pattern

109 Loop Antenna: The field pattern of small circular loop of radius  $a$  is determined by considering a square loop of the same area

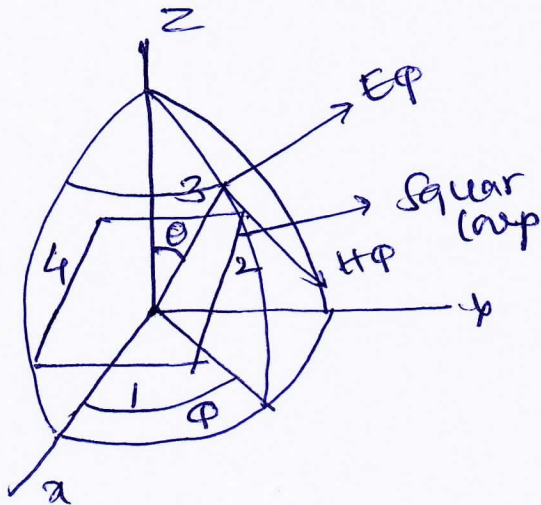
$$d^2 = \pi a^2$$

$d \rightarrow$  side length of the square loop



$d =$  side length of the square loop. The far field pattern of circular or square loop are of the same area are the same when loops are small, but different when they are large in terms of the wavelength.

The far field Electric field has only  $E_\theta$  component. Consider 4 dipoles of the four small linear dipole (2 & 4) through the loop in the  $yz$  plane



It is same as the two isotropic point sources

$$E_\theta = -E_{\theta 0} e^{j\psi/2} + E_{\theta 0} e^{-j\psi/2}$$

$E_{\theta 0} =$  Electric field from individual dipole

$$\psi = \frac{2\pi d}{\lambda} \sin\theta = dr \sin\theta$$

~~(scribble)~~



$$\therefore E_{\phi} = -j E_{\phi 0} \sin\left(\frac{dr}{2} \sin\theta\right)$$

$j$  indicates the total field  $E_{\phi}$  is in phase quadrature with the field  $E_{\phi 0}$  of individual dipole

$$\text{If } d \ll \lambda \quad \therefore E_{\phi} = -j E_{\phi 0} dr \sin\theta \quad \text{GM}$$

The angle  $\theta$  is measured from the dipole axis and is  $90^\circ$ . The far field of  $E_{\phi 0}$  of individual antenna

$$E_{\phi 0} = \frac{j 60 \pi [I] L}{r \lambda}$$

where  $[I] \rightarrow I$  the retarded current on the dipole

$$\therefore E_{\phi} = \frac{60 \pi [I] L dr \sin\theta}{r \lambda}$$

$L$  of short dipole is same as  $d$ ,  $dr = \frac{2\pi d}{\lambda}$  and area  $A$  of the loop is  $d^2$ . Then above eqn becomes

$$E_{\phi} = \frac{120 \pi^2 [I] \sin\theta}{r} \frac{A}{\lambda^2} \quad (\text{Small loop})$$

Far field  $E_{\phi}$

10b

Radiation resistance of the loop

The radiation resistance of the loop antenna. The Poynting vector integrated over the large sphere yielding total power  $P$  radiated. The power then equated to the effective current on the loop times the radiation resistance.

$$P = \frac{I_0^2}{2} \cdot R_r$$

So is the peak current in time on the loop  
The average Poynting vector of the far field

$$S_r = \frac{1}{2} |H|^2 \text{Re } Z$$

4M

$H \rightarrow$  absolute magnetic field

$Z \rightarrow$  Intrinsic impedance of the medium

Substitute the  $H_p$  value

$$S_r = \frac{15\pi (Ba\beta_0)^2}{r^2} J_1^2(\beta a \sin\theta)$$

The total power radiated

$$P = \iint S_r \cdot d\vec{s} = 15\pi (Ba\beta_0)^2 \int_0^{2\pi} \int_0^\pi J_1^2(\beta a \sin\theta) \times \sin\theta \cdot d\theta \cdot d\phi$$

$$P = 30\pi^2 (Ba\beta_0)^2 \int_0^\pi J_1^2(\beta a \sin\theta) \cdot \sin\theta \cdot d\theta$$

In case of the small loop in terms of wavelength

$$P = \frac{15}{2} \pi^2 (Ba)^4 \beta_0^2 \int_0^\pi \sin^3\theta \cdot d\theta$$

$$= 10\pi^2 B^4 a^4 \beta_0^2$$

Area  $A = \pi a^2$

$$\therefore P = 10 B^4 A^4 \beta_0^2$$

4M

Assume No antenna losses the power delivered to the loop

$$R_r \frac{\beta_0^2}{2} = 10 B^4 A^4 \beta_0^2$$

$$R_r = 31,171 \left(\frac{A}{\lambda^2}\right)^2 = 197 C_A^4$$

2M

$$\underline{R_r} \approx 31,200 \left(\frac{A}{\lambda}\right)^2$$

2M



106. Pyramidal horn antenna E-plane -  $10\lambda$

$$\delta = 0.2\lambda \text{ in E plane}$$

$$\text{H plane } - 0.375\lambda$$

$$L = \frac{a^2}{8\delta} = \frac{100\lambda}{8 \cdot 0.2} = 62.5\lambda$$

$$\theta_E = 2 \tan^{-1} \frac{a}{2L} = 2 \tan^{-1} \frac{10}{125} = 9.1^\circ$$

Taking  $\delta = 0.3\lambda/8$  in H-plane

$$\theta_H = 2L \tan \frac{\theta_H}{2} = 2 \times 6$$

$$= 2 \cos^{-1} \frac{L}{L+\delta} = 2 \cos^{-1} \frac{62.5}{62.5+0.375} = 12.52$$

H plane aperture.

$$a_H = 2L \tan \frac{\theta_H}{2} = 2 \times 62.5\lambda \tan 6.26$$

$$= 13.7\lambda$$

$$\text{HPBW (E plane)} = \frac{56^\circ}{a_E \lambda} = \frac{56^\circ}{10} = 5.6^\circ$$

$$\text{HPBW (H-plane)} = \frac{67^\circ}{a_H \lambda} = \frac{67^\circ}{13.7} = 4.9^\circ$$

$$\text{and } D \approx 10 \log \frac{7.5 A_p}{\lambda^2} = 10 \log (7.5 \times 10 \times 13.7)$$

$$\approx 30.1 \text{ dBi}$$