Subject :- Signals and Systems.

Subject Code : 18EC45

Examination :- July/August 2022.

Scheme & Solution Prepared by :- Prof. SURAJ KADLI.

CBCS SCHEME

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18EC45

Fourth Semester B.E. Degree Examination, July/August 2022
Signals and Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Differentiate between Energy and Power signals. Identify whether u(t) is energy or power signals. Compute its energy / power. **(08 Marks)**

b. Given the signals x(t) & y(t) in the Fig. Q1(b), sketch
i) $x(t-2)+y(1-t)$ ii) $x(t)-y(t+2)$. **(08 Marks)**

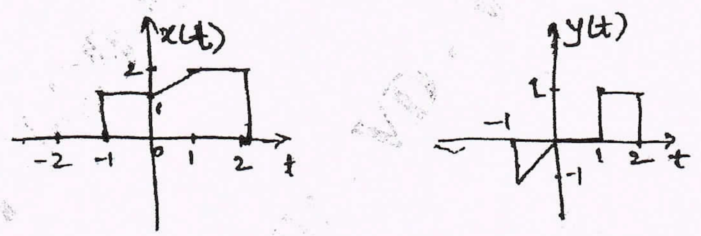

Fig. Q1(b)

c. Sketch the signal $Z(t) = r(t+2) - r(t+1) - 2u(t) + u(t-1)$. **(04 Marks)**

OR

2 a. For the signal shown in Fig. Q2(a), sketch its Even and Odd components. **(06 Marks)**

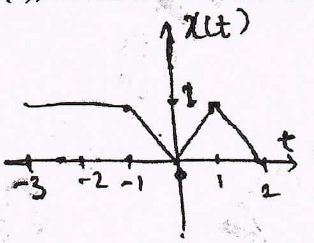

Fig. Q2(a)

b. Identify whether the following signals are periodic of not? If Periodic what is the period of it? i) $x(t) = \cos\sqrt{2}\,t + \sin 2\pi t$ ii) $x(t) = \cos 8\pi t$ iii) $x(n) = \sin\frac{\pi}{6}n + \sin\frac{\pi}{3}n$. **(08 Marks)**

c. Sketch the signals : i) $u(t-2) - 2u(t) + u(t+2)$ ii) $e^{-2t}\{u(t)-u(t-2)\}$. **(06 Marks)**

Module-2

3 a. Check whether the following system is linear, time variant , causal , static and stable.
$Y[n] = 2x[1-n] + 2$. **(08 Marks)**

b. Compute the following convolutions :
i) $y(t) = x(t) * h(t)$, where $x(t) = u(t+2)$ and $h(t) = e^{-2t}u(t)$.
ii) $y(t) = x(t) * h(t)$, where $x(t) = e^{-1+1}$ and $h(t) = u(t)$. **(12 Marks)**

OR

4 a. The system is described by the differential equation

$$\frac{dy(t)}{dt} = 2x(t) + \frac{d}{dt}x(t).$$

State whether this system is linear , time variant , causal and static. **(08 Marks)**

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b. i) Evaluate $y(n) = x(n) * h(n)$, if $x(n) = \alpha^n u(n)$ $\alpha < 1$ & $h(n) = u(n)$.
   ii) Evaluate $y(t) = x(t) * h(t)$, if $x(t)$ & $h(t)$ are as shown in Fig. Q4(b(ii)). **(12 Marks)**

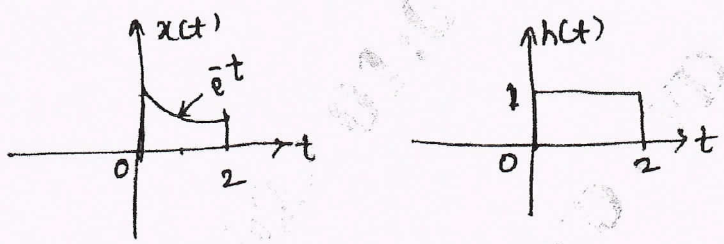


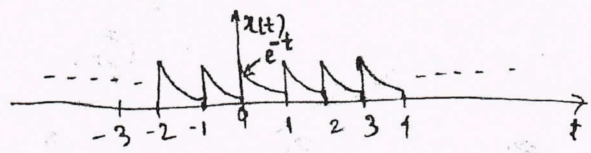
Fig. Q4(b(ii))

Module-3

5 a. Impulse responses of the various systems are described below. Identify whether these systems are memoryless, causal and stable.
   i) $h(n) = 2\delta(n)$ ii) $h(t) = e^{-2t} u(t + 2)$ iii) $h(t) = 2\{u(t) - u(t - 2)\}$. **(10 Marks)**

b. Obtain the Fourier representations of the signals :
   i) $x(n) = \cos 2\pi n + \sin 4\pi n$ with $\Omega_0 = 2\pi$ ii) $x(t)$ shown in Fig. Q5(b(ii)). **(10 Marks)**

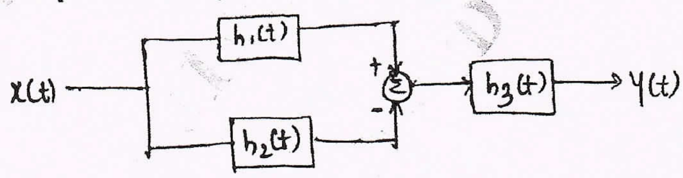
Fig. Q5(b(ii))



OR

6 a. Find the overall impulse response of the system shown in Fig. Q6(a). **(08 Marks)**

Fig. Q6(a)



where $h_1(t) = u(t + 1)$, $h_2(t) = u(t - 2)$, $h_3(t) = e^{-3t} u(t)$.

b. State and prove time shift property of Fourier Series. **(06 Marks)**

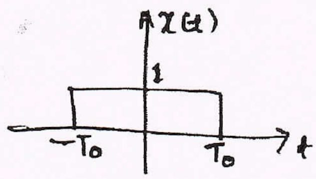
c. Obtain DTFS coefficients of $x(n)$ if $\Omega_0 = 3\pi$.
   i) $x(n) = \sin 6\pi n$ ii) $x(n) = \cos 3\pi n + \sin 9\pi n$. **(06 Marks)**

Module-4

7 a. State and prove Convolution property of DTFT. **(06 Marks)**

b. Find F.T. of the signal shown in Fig. Q7(b). **(06 Marks)**

Fig. Q7(b)



c. Find the time domain signal $x(t)$ if its F.T. $X(jw)$ given below :
   i) $X(jw) = \dfrac{jw}{(jw)^2 + 5jw + 6jw}$ ii) $X(jw) = \dfrac{1 - jw}{1 + w^2}$ **(08 Marks)**

OR

8 a. State and prove Parseval's theorem for Fourier transform. **(06 Marks)**

b. Using properties, find the DTFT of the signals.

i) $x(n) = (\frac{1}{2})^n u(n+2)$ ii) $x(n) = n \cdot a^n u(n)$. **(06 Marks)**

c. Obtain the signal x(t), if its Fourier transform is

i) $X(jw) = \dfrac{1}{2 + j(w-3)}$ ii) $X(jw) = e^{-j3w} \dfrac{1}{jw+2}$ **(08 Marks)**

Module-5

9 a. Find the Z – transform of the signals.

i) $x(n) = (\frac{1}{2})^n u(n) - (\frac{3}{2})^n u(-n-1)$ ii) $x(n) = (-\frac{1}{3})^n u(n)$. **(07 Marks)**

b. State and prove differentiation in the Z – domain property of Z – transform. **(06 Marks)**

c. Use Partial fraction expansion to find the inverse Z – transform of

$$X(z) = \frac{z^2 - 3z}{z^2 - \frac{3}{2}z - 1} \quad |\tfrac{1}{2}| < |z| < |2|$$ **(07 Marks)**

OR

10 a. Use properties to find Z – transform of the following signals :

i) $x(n) = 3^n u(n-2)$ ii) $x(n) = n \sin\left(\frac{\pi}{2} n\right) u(n)$. **(08 Marks)**

b. Find the Inverse Z - transform.

i) $X(z) = \dfrac{1}{1 - \frac{1}{2} z^{-1}} + \dfrac{2}{1 - 2z^{-1}} \quad |z| > |2|$.

ii) $X(z) = \dfrac{2 + z^{-1}}{1 - \frac{1}{2} z^{-1}} \; |z| < |\tfrac{1}{2}|$, Use Power Series Expansion method. **(12 Marks)**

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Signals and Systems - 18EC45

July/August 2022 - Scheme and Solution

01

Module - 1

1.a) Differentiate between Energy and Power signals. Identify whether u(t) is energy or power signal. Compute its energy/power. [Total - 8M]

→

Parameter	Energy Signal	Power Signal.
Definition	Total energy content of a signal will be finite quantity & all energy signals will have zero average power. $0 < E < \infty$ & $P = 0$	Average power of the signal will be finite quantity & all power signals will have infinite energy. $0 < P < \infty$ & $E = \infty$.
Examples	Both deterministic & aperiodic signals.	Periodic & power signals.
Equations.	$E = \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt$ $E = \sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2$	$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \lvert x(t) \rvert^2 dt$ $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x[n] \rvert^2$ for periodic signal $P = \frac{1}{T}\int_0^T \lvert x(t) \rvert^2 dt$.

— 2M + 2M

Example:- $x(t) = u(t)$

i) Total Energy: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$x(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

— 2M

$$E = \int_0^{\infty} 1\, dt = t\Big]_0^{\infty} = \infty.$$

$\therefore E = \infty$.

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ii> Average power: $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} 1 \, dt = \lim_{T\to\infty} \frac{1}{2T} [t]_0^T$$

— 2M

$$P = \lim_{T\to\infty} \frac{1}{2\cancel{T}} [\cancel{T}]$$

∴ we can drop the limit function.

$$\therefore P = \frac{1}{2}.$$

Total energy is infinite & average power is finite & non-zero hence unit step function is a power signal.

1.b> Given the signals x(t) & y(t) in the Fig. Q1(b), sketch
i> $x(t-2) + y(1-t)$ ii> $x(t) - y(t+2)$. [Total – 8M]

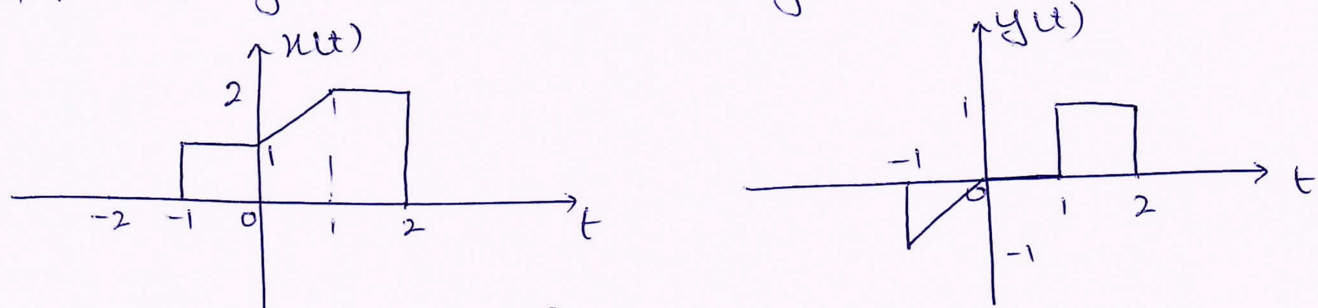
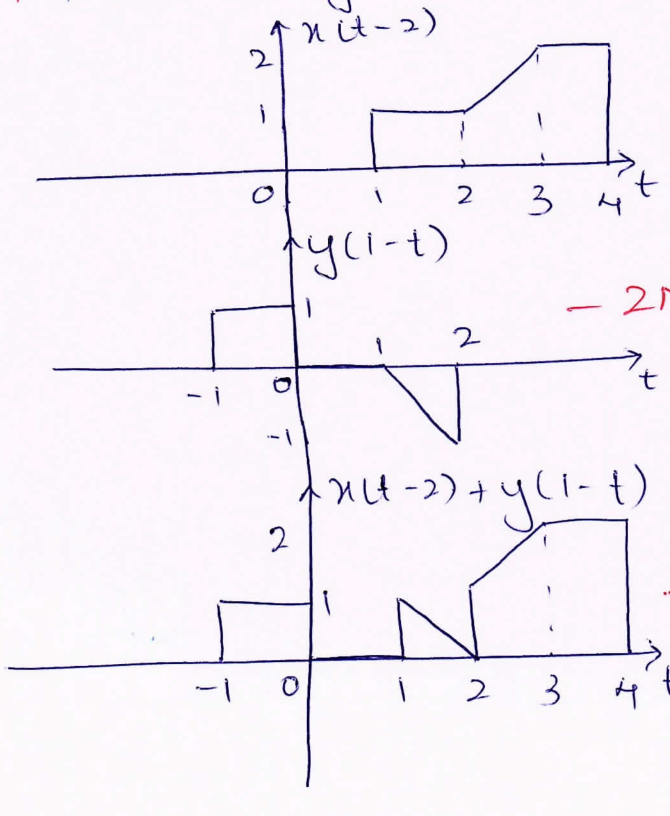


Fig. Q1(b)

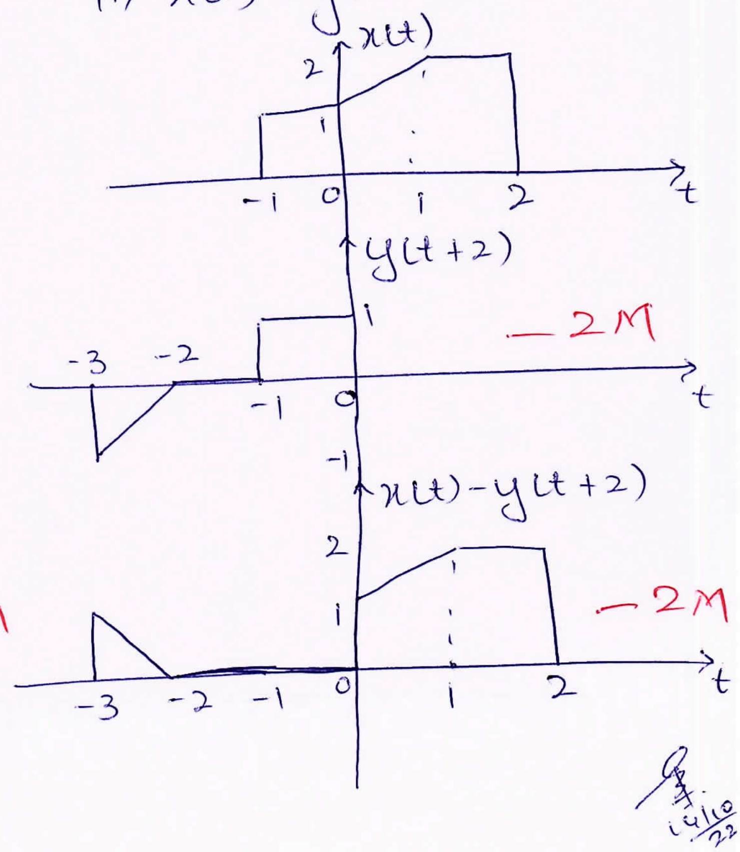
→ i> $x(t-2) + y(1-t)$



— 2M

— 2M

ii> $x(t) - y(t+2)$



— 2M

— 2M

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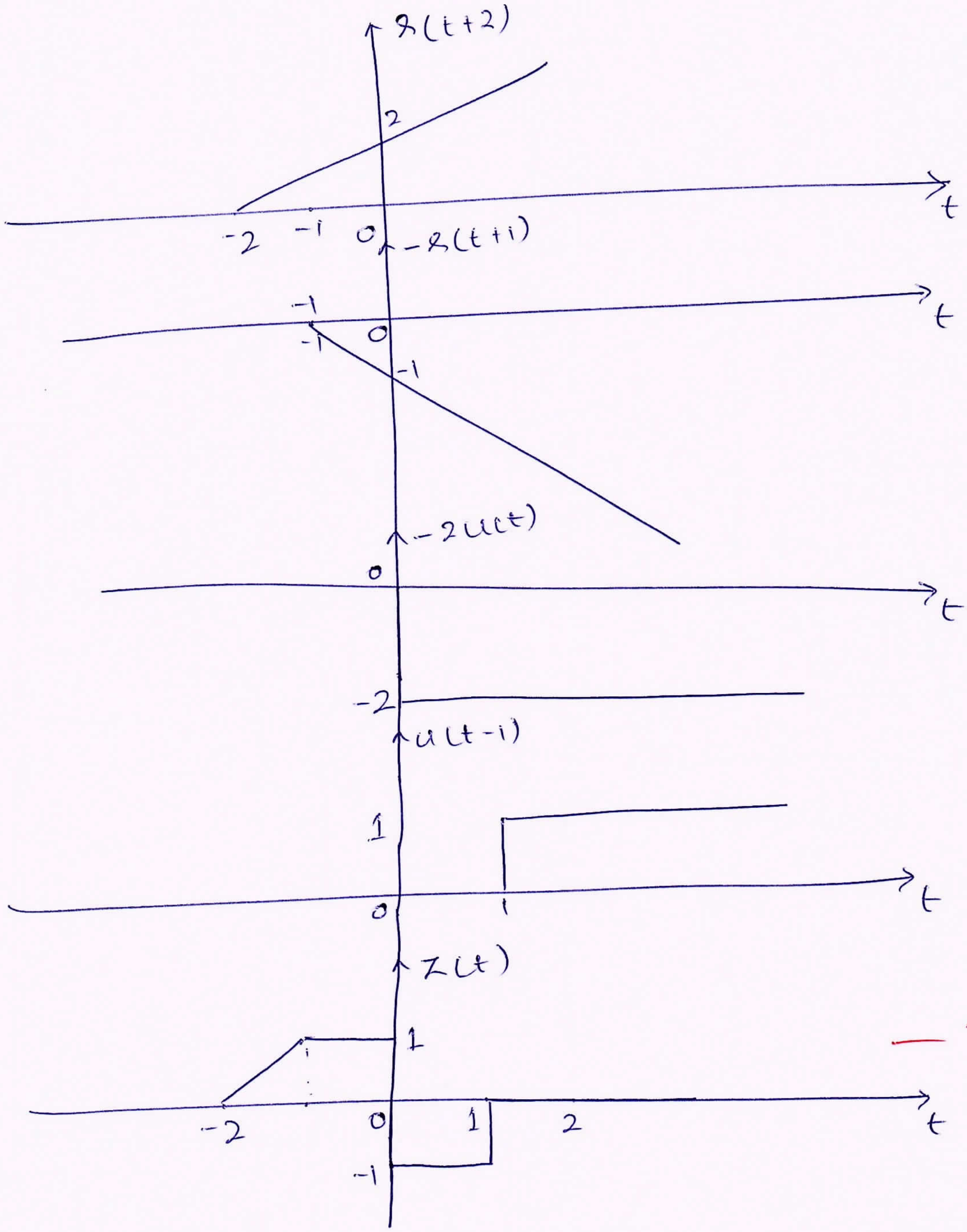
1. c) Sketch the signal $x(t) = r(t+2) - r(t+1) - 2u(t) + u(t-1)$. 05

[Total - 4M]

→ $x(t) = r(t+2) - r(t+1) - 2u(t) + u(t-1)$.



— 2M

— 2M

"OR"

2. a) For the signal shown in Fig. Q2(a), sketch its Even and Odd components.

[Total - 6M]

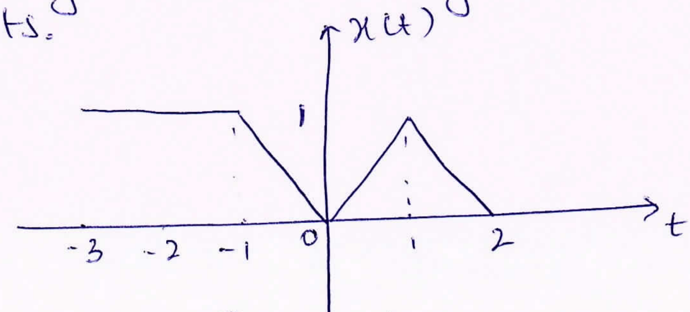


Fig Q2(a)

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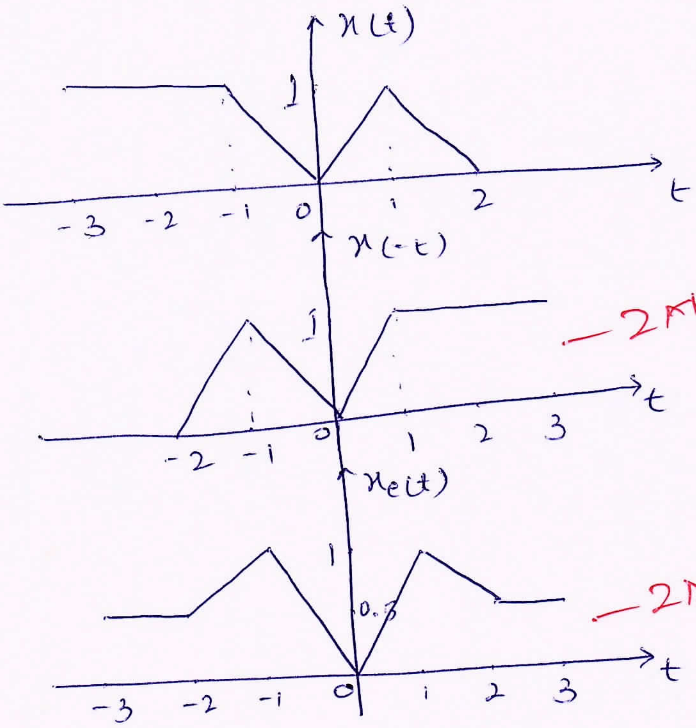
→ Even component

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$.

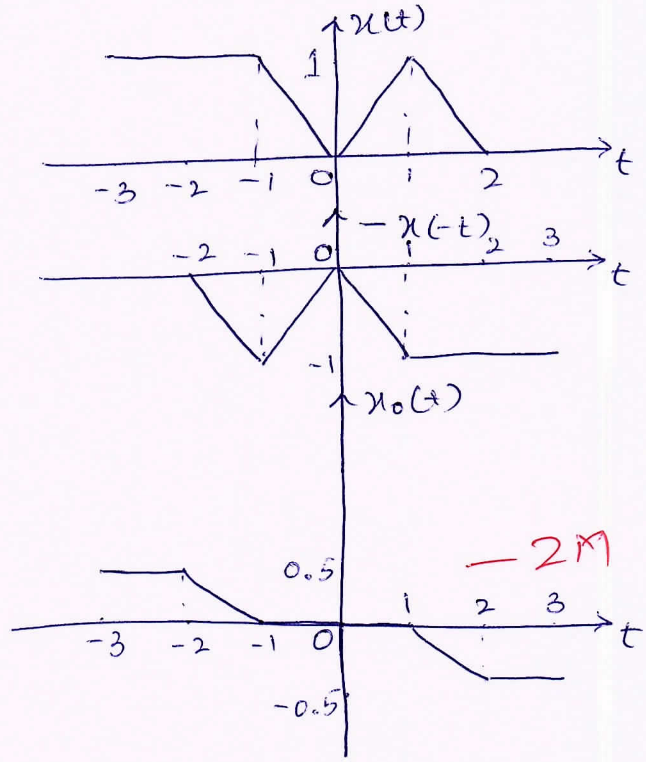
Odd component

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$



— 2M

— 2M



— 2M

2.b) Identify whether the following signals are periodic or not? If periodic what is the period of it?

i) $x(t) = \cos\sqrt{2}t + \sin 2\pi t$ ii) $x(t) = \cos 8\pi t$ iii) $x[n] = \sin\frac{\pi}{6}n + \sin\frac{\pi}{3}n$

[Total - 8M]

→ i) $x(t) = \cos\sqrt{2}t + \sin 2\pi t$

$x_1(t) = \cos\sqrt{2}t$ & $x_2(t) = \sin 2\pi t$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{2}}$ $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{2\pi}$ — 1M

$T_1 = 1.414\pi = \sqrt{2}\pi$ $T_2 = 1$ — 1M

$\frac{T_1}{T_2} = \frac{\sqrt{2}\pi}{1}$ is an irrational number. — 1M

$\therefore x(t) = \cos\sqrt{2}t + \sin 2\pi t$ is Aperiodic signal.

ii) $x(t) = \cos 8\pi t$

$\omega_0 = 8\pi$ $\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$ rational number

— 2M

$\therefore x(t)$ is periodic signal with fundamental period $T = \frac{1}{4}$.

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iii) $x[n] = \sin\frac{\pi}{6}n + \sin\frac{\pi}{3}n$

$x_1[n] = \sin\frac{\pi}{6}n$ & $x_2[n] = \sin\frac{\pi}{3}n$

$\Omega_1 = \frac{\pi}{6}$ $\Omega_2 = \frac{\pi}{3}$

$\Omega_1 = \frac{2\pi}{12}$ $\Omega_2 = \frac{2\pi}{6}$ — 1M

$\therefore N_1 = 12.$ $N_2 = 6.$ — 1M

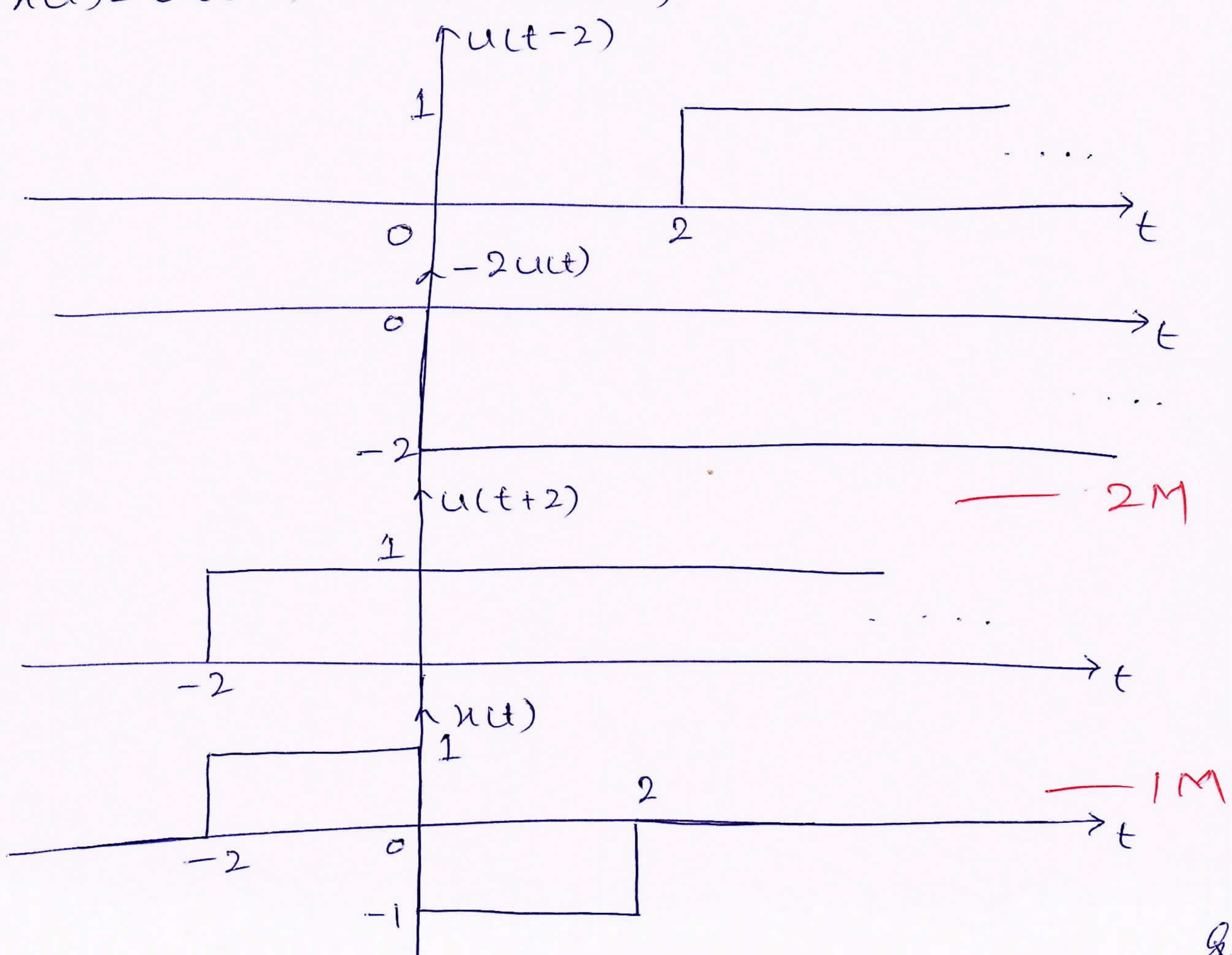
$\therefore \frac{N_1}{N_2} = \frac{12}{6} = 2$ is a rational number. — 1M.

$\therefore$ $x[n]$ is periodic signal with fundamental period

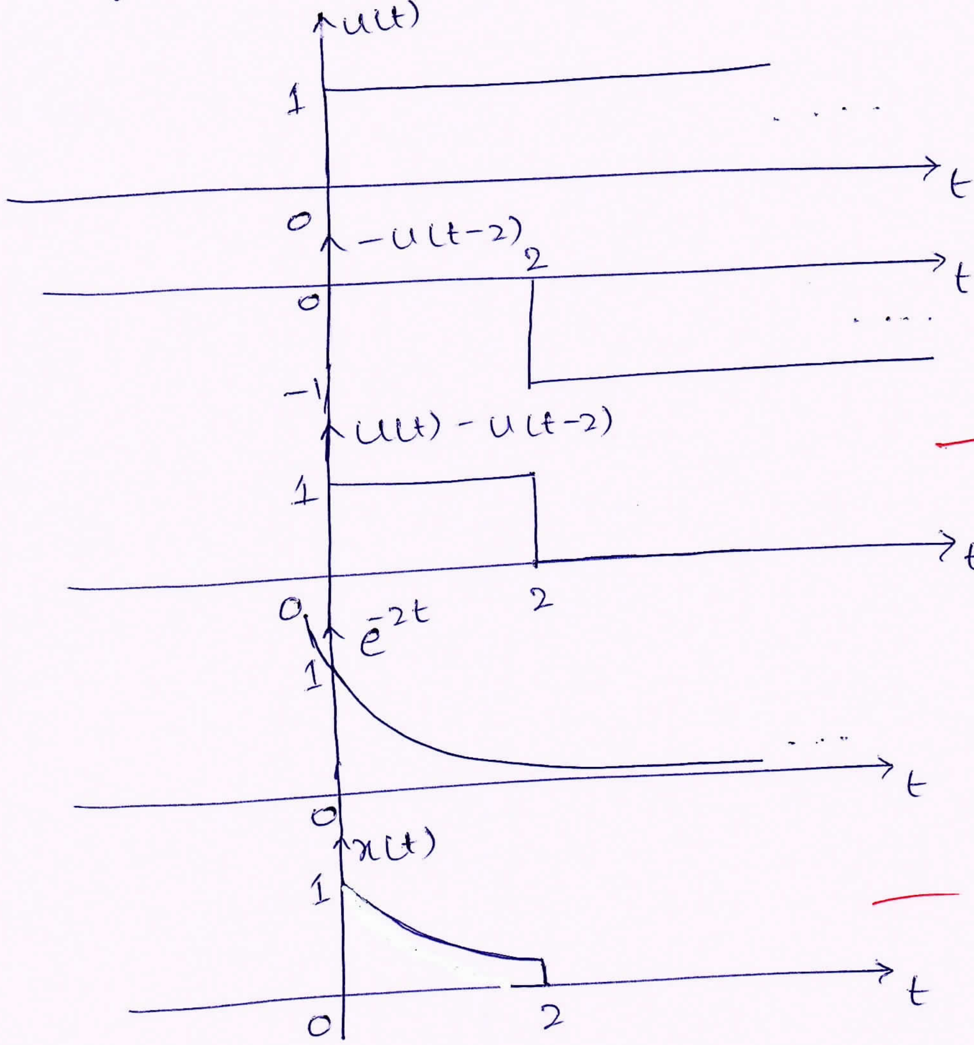
$N = LCM(N_1, N_2) = LCM(12, 6) = \underline{\underline{12}}$

2.c) Sketch the signals: i) $u(t-2) - 2u(t) + u(t+2)$
ii) $e^{-2t}\{u(t) - u(t-2)\}$. [Total - 6M]

→ i) $x(t) = u(t-2) - 2u(t) + u(t+2)$



ii) $x(t) = e^{-2t}\{u(t) - u(t-2)\}$.



— 2M

— 1M

Module-2.

3.a) Check whether the following system is linear, time variant, causal, static and stable. $y[n] = 2x[1-n] + 2$.

[Total - 8M]

→ $y[n] = 2x[1-n] + 2$

a) For two separate inputs the system produces the response of,

$y_1[n] = T\{x_1[n]\} = 2x_1[1-n] + 2$

$y_2[n] = T\{x_2[n]\} = 2x_2[1-n] + 2$.

The response of the system to linear combination of two inputs will be,

$y_3[n] = T\{a_1x_1[n] + a_2x_2[n]\} = 2\{a_1x_1[1-n] + a_2x_2[1-n]\} + 2$

$\therefore y_3[n] = 2\{a_1x_1[1-n] + a_2x_2[1-n]\} + 2$.

The linear combination of two outputs will be,

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$$y_3'[n] = a_1 y_1[n] + a_2 y_2[n] = a_1 2x_1[1-n] + 2 + a_2 2x_2[1-n] + 2$$

$$y_3'[n] = a_1 2x_1[1-n] + a_2 2x_2[1-n] + 4$$

$$y_3[n] \neq y_3'[n]$$ — 2M

$\therefore$ System is nonlinear.

b) The output $y[n]$ for delayed input will be,

$$y[n,k] = T\{x[n-k]\} = 2x[1-n-k] + 2$$

Now, delayed output will be obtained by replacing n by $n-k$, $\therefore y[n-k] = 2x[1-(n-k)] + 2 = 2x[1-n+k] + 2$

— 2M

$y[n,k] \neq y[n-k]$ $\therefore$ System is time variant.

c) for $n = 0$

$$y[0] = 2x[1] + 2$$

Output depends upon future inputs. Hence the system is non-causal. — 1M

d) n^{th} output is equal to $(-n+1)^{th}$ sample of input

$\therefore$ the system is dynamic. — 1M

e) As long as the input is bounded, output will be bounded.

$\therefore$ System is stable. $|y[n]| = 2|M_x| + 2 < \infty$.

— 2M

3.b) Compute the following convolutions:

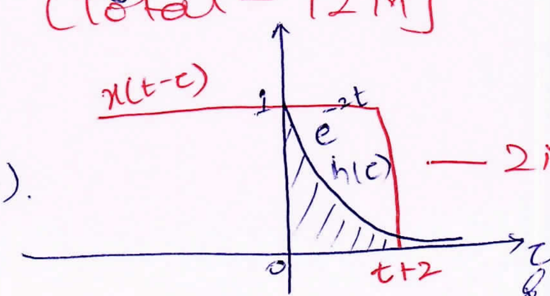
i) $y(t) = x(t) * h(t)$, where $x(t) = u(t+2)$ & $h(t) = e^{-2t}u(t)$

ii) $y(t) = x(t) * h(t)$, where $x(t) = e^{-|t|}$ & $h(t) = u(t)$.

[Total – 12 M]

$\rightarrow$ i) $y(t) = x(t) * h(t)$

$x(t) = u(t+2)$ & $h(t) = e^{-2t}u(t)$.



— 2M

Convolution integral,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau) \cdot u(t-\tau+2)\,d\tau$$

$u(\tau) = 1, \quad \tau \geq 0 \quad$ & $\quad u(t-\tau+2) = 1, \quad \tau \leq t+2.$

$\therefore$ the limits of integration will be modified as,

$$y(t) = \int_{0}^{t+2} e^{-2\tau}\,d\tau \quad — 2M$$

$$y(t) = -\frac{1}{2}\left[e^{-2\tau}\right]_0^{t+2} = -\frac{1}{2}\left[e^{-2t-4} - 1\right]$$

$$\boxed{\therefore y(t) = \frac{1}{2}\left[1 - e^{-2(t+2)}\right]} \quad t \geq -2. \quad — 2M$$

ii) $y(t) = x(t) * h(t)$

$x(t) = e^{-1+1} = e^{0} = 1 \quad$ & $\quad h(t) = u(t).$

Convolution integral,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \quad — 2M$$

$$y(t) = \int_{-\infty}^{\infty} 1 \cdot u(t-\tau)\,d\tau$$

$$y(t) = \int_{0}^{t} 1\,d\tau \qquad \because u(t-\tau) = 1, \quad \tau \leq t$$

— 2M

$$y(t) = \tau\Big]_0^{t} = t - 0$$

$$\boxed{\therefore y(t) = t} \quad — 2M$$

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"OR"

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4.a) The system is described by the differential equation

$$\frac{dy(t)}{dt} = 2x(t) + \frac{dx(t)}{dt}.$$

State whether this system is linear, time variant, causal and static. [Total – 8 M]

→

$$\frac{dy(t)}{dt} = 2x(t) + \frac{dx(t)}{dt}.$$

a) In response to $x_1(t)$ & $x_2(t)$

$$\frac{dy_1(t)}{dt} = 2x_1(t) + \frac{dx_1(t)}{dt} \quad , \quad \&$$

$$\frac{dy_2(t)}{dt} = 2x_2(t) + \frac{dx_2(t)}{dt}.$$

Linear combination of above equation

$$a_1\frac{dy_1(t)}{dt} + a_2\frac{dy_2(t)}{dt} = 2(a_1x_1(t)) + a_1\frac{dx_1(t)}{dt} + 2(a_2x_2(t)) + a_2\frac{dx_2(t)}{dt}.$$

rearranging

$$\frac{d}{dt}\left(a_1y_1(t) + a_2y_2(t)\right) = 2\left[a_1x_1(t) + a_2x_2(t)\right] + \frac{d}{dt}\left(a_1x_1(t) + a_2x_2(t)\right)$$

this equation is similar to original difference equation.

∴ System is linear — 2M

b) Output does not change with time, ∴ the System in time invariant. — 2M

c) Output depends only on present or past inputs.

∴ System is causal. — 2M

d) Since differentiation is involved, system is dynamic. — 2M

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4.b) i) Evaluate $y[n] = x[n] * h[n]$, if $x[n] = \alpha^n u[n]$ $\alpha < 1$ & $h[n] = u[n]$. 19

ii) Evaluate $y(t) = x(t) * h(t)$, if $x(t)$ & $h(t)$ are as shown in Fig. Q4(b(ii)).

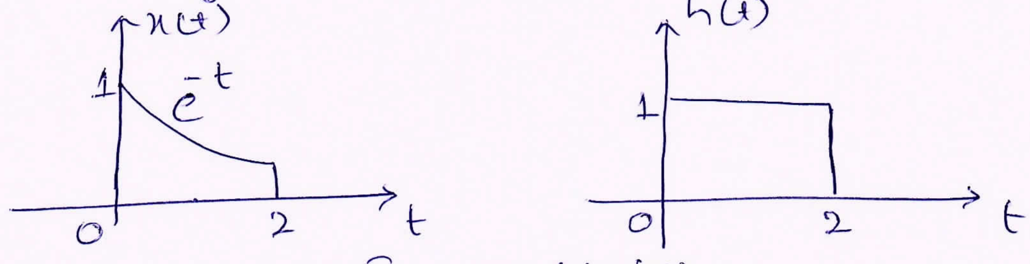

Fig. Q4(b(ii)). [Total - 12M]

→ i) $y[n] = x[n] * h[n]$

$x[n] = \alpha^n u[n]$, $\alpha < 1$ & $h[n] = u[n]$.

Convolution summation

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \quad \text{— 1M}$$

$$x[k] = \alpha^k u[k] \quad \& \quad h[n-k] = u[n-k]. \quad \text{— 1M}$$

$$\therefore y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] u[n-k] \quad \text{— 2M}$$

$u[k]u[n-k] = 1$ for $0 \le k \le n$.

$$\therefore y[n] = \sum_{k=0}^{n} \alpha^k, \quad \alpha < 1$$

$$\therefore \boxed{y[n] = \frac{\alpha^{n+1} - 1}{\alpha - 1}} \quad n \geq 0 \quad \text{— 2M.}$$

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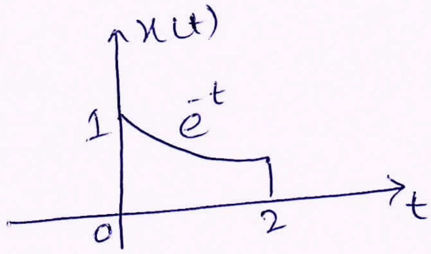
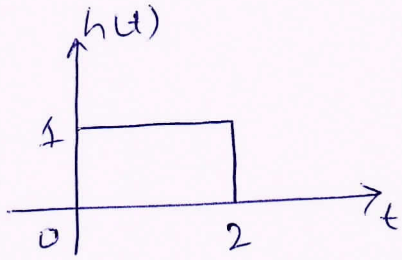
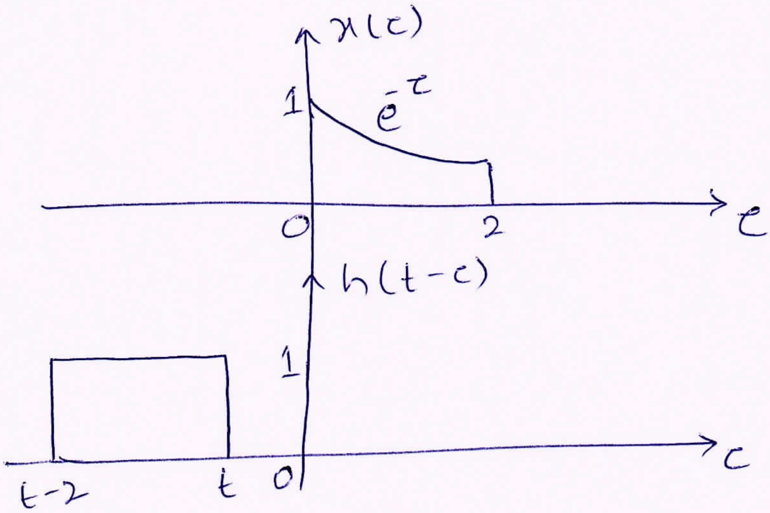
ii) $y(t) = x(t) * h(t)$





Graphical Method.

$$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau \quad , \text{ where } \quad w_t(\tau) = x(\tau)\, h(t-\tau).$$



— 1M

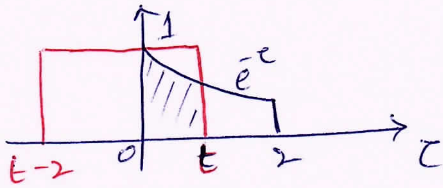
a) for, $t < 0$

$w_t(\tau) = 0 \quad \therefore \; y(t) = 0 \; , \; t < 0$

b) for, $t \geq 0$

$w_t(\tau) = e^{-\tau} , \quad 0 \leq \tau \leq t$

$$y(t) = \int_0^t e^{-\tau} d\tau = -e^{-\tau}\Big]_0^t$$

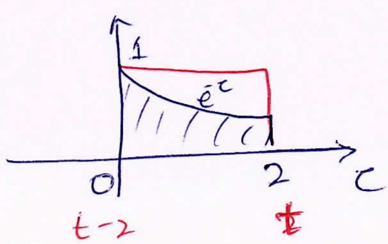


$\therefore \; y(t) = [1 - e^{-t}] , \quad 0 \leq t < 2$ — 2M

c) for, $t - 2 = 0$ or $t = 2$.

$w_t(\tau) = e^{-\tau} . \quad 0 \leq \tau \leq 2.$

$$y(t) = \int_0^2 e^{-\tau} d\tau = -e^{-\tau}\Big]_0^2 = 1 - e^{-2} , \; t = 2.$$

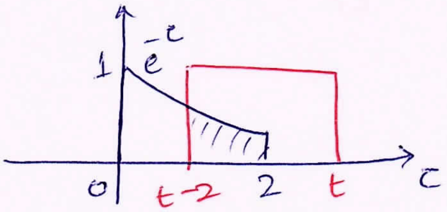


d) for, $t-2>0$ or $t>2$

$w_t(\tau) = e^{-\tau}, \; t-2 \le \tau \le 2$

$$y(t) = \int_{t-2}^{2} e^{-\tau} d\tau = -e^{-\tau}\Big]_{t-2}^{2}$$

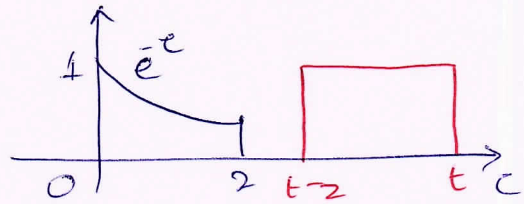
$$y(t) = e^{-t+2} - e^{-2}, \quad 2<t<4$$ — 2M



e) for, $t-2>2$ or $t>4$

$w_t(\tau) = 0$

$\therefore y(t) = 0, \quad t>4$



$$\therefore y(t) = \begin{cases} 0 & , \; t<0 \\ 1-e^{-t} & , \; 0 \le t < 2 \\ 1-e^{-2} & , \; t=2 \\ e^{-t+2}-e^{-2} & , \; 2<t \le 4 \\ 0 & , \; t>4 \end{cases}$$ — 1M.

Module-3

5.a) Impulse responses of the various systems are described below. Identify whether these systems are memoryless, causal and stable.

i) $h[n] = 2\delta[n]$ ii) $h(t) = e^{-2t}u(t+2)$ iii) $h(t) = 2\{u(t)-u(t-2)\}$.

[Total − 10M]

→ i) $h[n] = 2\delta[n]$

as $h[n]$, impulse response is of the form $c\,\delta[n]$

∴ System is memoryless. — 1M

b) Impulse response is defined at $n=0$, $h[n]=0, n<0$

∴ System is causal. — 1M

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c) let, $y = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} 2\delta[n]$ @ $n=0$

$y = 2$ ∴ System is stable. — 1M

ii) $h(t) = e^{-2t} u(t+2)$.

as Impulse response is not of the form, $h(t) = c\delta(t)$

∴ System is not memoryless. — 1M

b) $h(t) \neq 0$, $t < 0$ ∴ System is non-causal. — 1M

c) Let, $S = \int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{\infty} e^{-2t} u(t+2) dt = \int_{-2}^{\infty} e^{-2t} dt \cong 110$. Absolutely integrable

∴ System is stable. — 1.5M

iii) $h(t) = 2\{u(t) - u(t-2)\}$.

as Impulse response is not of the form, $h(t) = c\delta(t)$

∴ System is not memoryless. — 1M

b) $h(t) = 0$, $t < 0$ ∴ system is causal. — 1M

c) let, $S = \int_{-\infty}^{\infty} |h(t)| dt = \int_{0}^{2} 2\, dt = 2\, t\big]_0^2 = 2[2] = 4$. Absolutely integrable.

∴ System is stable. — 1.5M.

5.b) Obtain the Fourier representations of the signals:

i) $x[n] = \cos 2\pi n + \sin 4\pi n$ with $\Omega_0 = 2\pi$

ii) $x(t)$ shown in Fig. Q5(b(ii)).

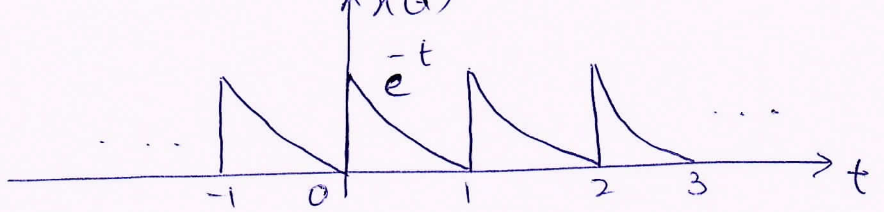

Fig. Q5(b(ii)).

[Total - 10M]

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→ i> $x[n] = \cos 2\pi n + \sin 4\pi n$, for $\Omega_0 = 2\pi$

$x[n] = \cos \Omega_0 n + \sin 2\Omega_0 n$.

using Euler's identity.

$$x[n] = \frac{e^{j\Omega_0 n} + e^{-j\Omega_0 n}}{2} + \frac{e^{j2\Omega_0 n} - e^{-j2\Omega_0 n}}{2j} \quad \text{— 2M}$$

rearranging the terms.

$$x[n] = -\frac{1}{2j} e^{-j2\Omega_0 n} + \frac{1}{2} e^{-j\Omega_0 n} + \frac{1}{2} e^{j\Omega_0 n} + \frac{1}{2j} e^{j2\Omega_0 n} \quad \text{— 1M}$$

Discrete time Fourier Series is given by,

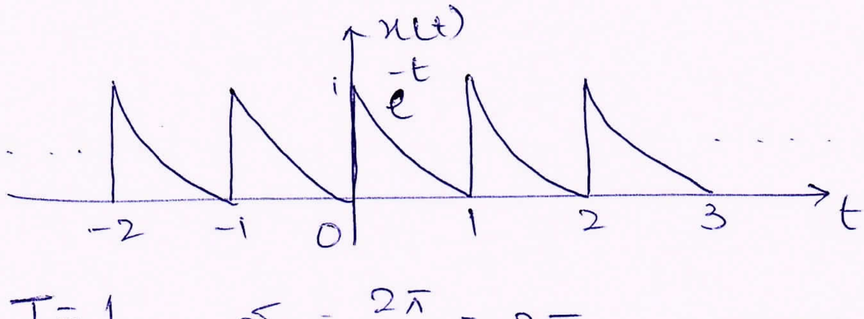
$$x[n] = \sum_{k=\langle N\rangle} X[k] e^{jk\Omega_0 n}, \quad k = -2 \text{ to } 2.$$

$$x[n] = X(-2) e^{-j2\Omega_0 n} + X(-1) e^{-j\Omega_0 n} + X(0) + X(1) e^{j\Omega_0 n} + X(2) e^{j2\Omega_0 n} \quad \text{— 1M}$$

Comparing both the equations., DTFS coefficients are given by.

$X(-2) = -\frac{1}{2j}$, $X(-1) = \frac{1}{2}$, $X(1) = \frac{1}{2}$ & $X(2) = \frac{1}{2j}$. — 1M

ii>



$T = 1$, $\omega_0 = \frac{2\pi}{T} = 2\pi$ — 1M

$x(t) = e^{-t}$, $0 \le t \le 1$.

$$X[k] = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt$$

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$$X(k) = \frac{1}{1}\int_0^1 e^{-t} e^{-jk2\pi t}\, dt$$

$$X(k) = \int_0^1 e^{-(1+jk2\pi)t}\, dt = \left[\frac{e^{-(1+jk2\pi)t}}{-(1+jk2\pi)}\right]_0^1 \text{ — 1M}$$

$$X(k) = \left[\frac{e^{-(1+jk2\pi)} - 1}{-(1+jk2\pi)}\right] = \frac{1}{1+jk2\pi}\left[1 - e^{-(1+jk2\pi)}\right] \text{ — 1M}$$

$$X(k) = \frac{1}{1+jk2\pi}\left[1 - e^{-1} e^{-jk2\pi}\right]$$

$$e^{-jk2\pi} = 1.$$

$$\therefore \boxed{X(k) = \frac{1 - e^{-1}}{1+jk2\pi}} \text{ — 1M}$$

$k=0$, $X(0) = 0.632$. , $|X(0)| = 0.632$, $\angle X(0) = 0$

$k=1$, $X(1) = 0.015 - 0.098j$, $|X(1)| = 0.099$, $\angle X(1) = -1.418$

$k=-1$, $X(-1) = 0.015 + 0.098j$, $|X(-1)| = 0.099$, $\angle X(-1) = 1.418$

$k=+2$, $X(2) = 0.003 - 0.049j$, $|X(2)| = 0.049$, $\angle X(2) = -1.509$

$k=-2$, $X(-2) = 0.003 + 0.049j$, $|X(-2)| = 0.049$, $\angle X(-2) = 1.509$

— 1M

"OR"

6.a) Find the overall impulse response of the system shown in Fig. Q6(a).

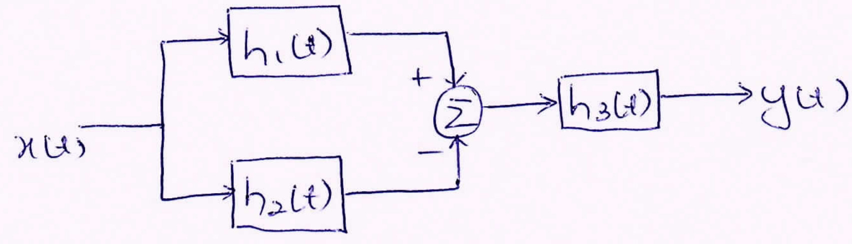

Fig. Q6(a).

where, $h_1(t) = u(t+1)$, $h_2(t) = u(t-2)$, $h_3(t) = e^{-3t}u(t)$.

[Total — 8M]

→ a)

$x(t) \longrightarrow \boxed{h_{12}(t) = h_1(t) - h_2(t)} \longrightarrow \boxed{h_3(t)} \longrightarrow y(t)$

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$$h_{12}(t) = h_1(t) - h_2(t)$$

$$h_{12}(t) = \{u(t+1) - u(t-2)\}.$$

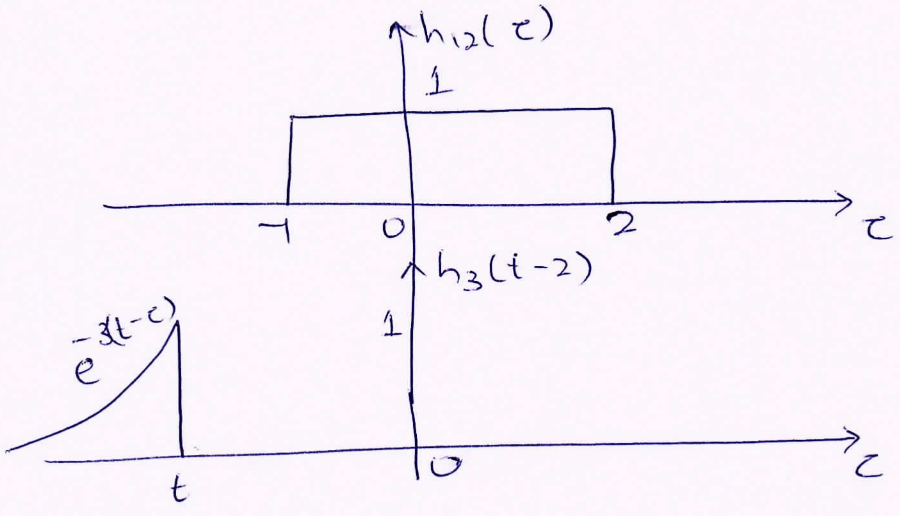
b) $x(t) \longrightarrow \boxed{h_{123}(t) = [h_1(t) - h_2(t)] * h_3(t)} \longrightarrow y(t).$

$$h_{123}(t) = [h_1(t) - h_2(t)] * h_3(t).$$

$$h_{123}(t) = [u(t+1) - u(t-2)] * e^{-3t}u(t). \quad \text{— 2M}$$

$$h_{123}(t) = h_{12}(t) * h_3(t).$$

$$h_{123}(t) = \int_{-\infty}^{\infty} h_{12}(\tau)\, h_3(t-\tau)\, d\tau. = \int_{-\infty}^{\infty} h_3(\tau)\, h_{12}(t-\tau)\, d\tau.$$



— 2M

$t < -1, \quad h_{123}(t) = 0.$

$-1 \leq t < 2, \quad \omega_t(\tau) = e^{-3\tau}, \quad -1 \leq \tau \leq t$

$$\therefore h_{123}(t) = \int_{-1}^{t} e^{-3\tau} d\tau = -\frac{1}{3}\left[e^{-3\tau}\right]_{-1}^{t}$$

$$= -\frac{1}{3}\left[e^{-3t} - e^{3}\right]$$

$$\therefore h_{123}(t) = \frac{e^{3} - e^{-3t}}{3}, \quad -1 \leq t < 2. \quad \text{— 2M}$$

$t \geq 2, \quad \omega_t(\tau) = e^{-3\tau}, \quad -1 \leq \tau \leq 2.$

$$h_{123}(t) = \int_{-1}^{2} e^{-3\tau} d\tau = -\frac{1}{3}\left[e^{-3\tau}\right]_{-1}^{2} = \frac{e^{3} - e^{-6}}{3}. \quad t \geq 2.$$

— 1M

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$$\therefore h_{123}(t) = \begin{cases} 0, & t < -1 \\ \dfrac{e^{3} - e^{-3t}}{3}, & -1 \le t < 2 \\ \dfrac{e^{3} - e^{-6}}{3}, & t \geq 2 \end{cases}$$ — 1M

6.b> State and prove time shift property of Fourier Series. [Total - 6M]

→ Time shift property of Fourier Series.

If, $x(t) \xleftrightarrow{FS} X(k)$. — 2M

then, $z(t) = x(t - t_0) \xleftrightarrow{FS} Z[k] = e^{-jk\Omega_0 t_0} X[k]$.

$$Z[k] = \frac{1}{T}\int_0^T z(t)\, e^{-jk\Omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T x(t - t_0)\, e^{-jk\Omega_0 t}\, dt \qquad \text{Put, } t - t_0 = \lambda.$$

$$\therefore Z[k] = \frac{1}{T}\int_0^T x(\lambda)\, e^{-jk\Omega_0 (t_0 + \lambda)}\, d\lambda$$ — 4M

$$= e^{-jk\Omega_0 t_0}\, \frac{1}{T}\int_0^T x(\lambda)\, e^{-jk\Omega_0 \lambda}\, d\lambda.$$

$$\boxed{\therefore Z[k] = e^{-jk\Omega_0 t_0}\, X[k]}$$

6.c> Obtain DTFS coefficients of $x[n]$ if $\Omega_0 = 3\pi$.

i> $x[n] = \sin 6\pi n$ ii> $x[n] = \cos 3\pi n + \sin 9\pi n$. [Total - 6M]

→ i> $x[n] = \sin 6\pi n = \sin 2\Omega_0 n$ for $\Omega_0 = 3\pi$.

$$x[n] = \frac{e^{j2\Omega_0 n} - e^{-j2\Omega_0 n}}{2j} = -\frac{1}{2j} e^{-j2\Omega_0 n} + \frac{1}{2j} e^{j2\Omega_0 n}$$ — 1M

comparing with DTFS $x[n] = \sum_{k=\langle N \rangle} X(k)\, e^{jk\Omega_0 n}$

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$\therefore X(-2) = -\frac{1}{2j}$ & $X(2) = \frac{1}{2j}$ — 1M

ii) $x[n] = \cos 3\pi n + \sin 9\pi n$

$x[n] = \cos \Omega_0 n + \sin 3\Omega_0 n$, for $\Omega_0 = 3\pi$

$$x[n] = \frac{e^{j\Omega_0 n} + e^{-j\Omega_0 n}}{2} + \frac{e^{j3\Omega_0 n} - e^{-j3\Omega_0 n}}{2j} \quad \text{— 1M}$$

Rearranging,

$$x[n] = -\frac{1}{2j} e^{-j3\Omega_0 n} + \frac{1}{2} e^{-j\Omega_0 n} + \frac{1}{2} e^{j\Omega_0 n} + \frac{1}{2j} e^{j3\Omega_0 n} \quad \text{— 2M}$$

On comparing with DTFS, $x[n] = \sum_{k=\langle N \rangle} X(k) e^{jk\Omega_0 n}$

$X(-3) = -\frac{1}{2j}$, $X(-1) = \frac{1}{2}$, $X(1) = \frac{1}{2}$, $X(3) = \frac{1}{2j}$ — 1M

Module-4

7.a) State and prove convolution property of DTFT. [Total — 6M]

→ Convolution property of DTFT.

If, $x[n] \xleftrightarrow{DTFT} X(\Omega)$

and, $y[n] \xleftrightarrow{DTFT} Y(\Omega)$ — 2M

then, $z[n] = x[n] * y[n] \xleftrightarrow{DTFT} Z(\Omega) = X(\Omega)Y(\Omega)$.

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} z[n] e^{-j\Omega n}$$

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} (x[n] * y[n]) e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] y[n-k] \right) e^{-j\Omega n} \quad \text{— 4M}$$

$m = n-k$. & interchanging the summation

$$Z(\Omega) = \sum_{k=-\infty}^{\infty} x[k] \sum_{m=-\infty}^{\infty} y[m] e^{-j\Omega(m+k)}$$

$$Z(\Omega) = \sum_{k=-\infty}^{\infty} x[k] e^{-j\Omega k} \sum_{m=-\infty}^{\infty} y[m] e^{-j\Omega m}$$

$$\boxed{Z(\Omega) = X(\Omega) Y(\Omega)}$$

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7.b) Find F.T. of the signal shown in Fig. Q7(b).

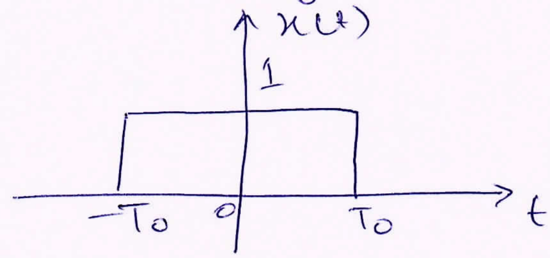
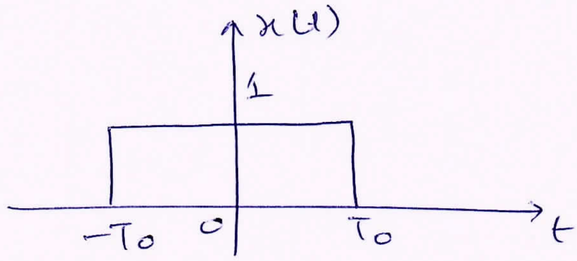


Fig. Q7(b)

[Total – 6M]

→



$$x(t) = \begin{cases} 1, & -T_0 \le t \le T_0 \\ 0, & \text{otherwise.} \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-T_0}^{T_0} e^{-j\omega t}\, dt$$

$$X(j\omega) = \frac{1}{-j\omega}\left[e^{-j\omega t}\right]_{-T_0}^{T_0}$$

$$= \frac{1}{-j\omega}\left[e^{-j\omega T_0} - e^{j\omega T_0}\right] \quad \text{— 2M}$$

$$X(j\omega) = \frac{2}{\omega}\left[\frac{e^{j\omega T_0} - e^{-j\omega T_0}}{2j}\right]$$

$$\therefore \boxed{X(j\omega) = \frac{2}{\omega}\sin\omega T_0} \quad \omega \neq 0. \quad \text{— 2M}$$

$\omega = 0$

$$X(j\omega) = \int_{-T_0}^{T_0} 1\, dt = t\Big]_{-T_0}^{T_0} = T_0 + T_0 = 2T_0.$$

— 1M

$$\therefore X(j\omega) = \begin{cases} \frac{2}{\omega}\sin\omega T_0, & \omega \neq 0 \\ 2T_0, & \omega = 0. \end{cases} \quad \text{— 1M}$$

7.c) Find the time domain signal $x(t)$ if its F.T. $X(j\omega)$ given below:

i) $X(j\omega) = \dfrac{j\omega}{(j\omega)^2 + 5j\omega + 6}$ ii) $X(j\omega) = \dfrac{1-j\omega}{(1+j\omega)^2}$.

[Total - 8M]

→ i) $X(j\omega) = \dfrac{j\omega}{(j\omega)^2 + 5j\omega + 6}$.

$$X(j\omega) = \frac{j\omega}{(j\omega+3)(j\omega+2)} = \frac{A}{j\omega+3} + \frac{B}{j\omega+2}. \quad - 1M$$

$$\therefore A = \frac{j\omega}{\cancel{(j\omega+3)}(j\omega+2)} \cancel{(j\omega+3)} \Big|_{\omega=-3} = 3.$$

— 2M

$$B = \frac{j\omega}{(j\omega+3)\cancel{(j\omega+2)}} \cancel{(j\omega+2)} \Big|_{\omega=-2} = -2.$$

$$\therefore X(j\omega) = \frac{3}{j\omega+3} - \frac{2}{j\omega+2}. \quad - 1M$$

$$\therefore \boxed{x(t) = \{3e^{-3t} - 2e^{-2t}\} u(t).} \quad - 1M$$

ii) $X(j\omega) = \dfrac{1-j\omega}{(1+j\omega)^2}$

$$X(j\omega) = \frac{1-j\omega}{(1+j\omega)(1-j\omega)} = \frac{1}{1+j\omega} = e^{-t} u(t). \quad - 2M$$

$$\therefore \boxed{X(j\omega) = e^{-t} u(t)} \quad - 1M$$

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"OR"

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8.a) State and prove Parseval's theorem of Fourier transform. [Total - 6M]

→ Parseval's theorem of Fourier transform.

If, $x[n] \xleftrightarrow{DTFT} X(\Omega)$

then, $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |X(\Omega)|^2 d\Omega.$ — 2M

$|X(\Omega)|^2$ is known as energy density spectrum of the signal $x[n]$ & E is the total energy content of the sequence $x[n]$.

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} x[n]\, x^*[n]$$

$$x[n] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\Omega)\, e^{j\Omega n} d\Omega.$$

taking conjugate on both sides.

$$x^*[n] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X^*(\Omega)\, e^{-j\Omega n} d\Omega.$$

$$\therefore E = \sum_{n=-\infty}^{\infty} x[n] \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X^*(\Omega)\, e^{-j\Omega n} d\Omega.$$ — 4M

$$= \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X^*(\Omega) \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} d\Omega.$$

$$= \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X^*(\Omega)\, X(\Omega)\, d\Omega.$$

$$\therefore E = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |X(\Omega)|^2 d\Omega.$$

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8.b) Using properties, find the DTFT of the signals.

i) $x[n] = \left(\frac{1}{2}\right)^n u[n+2]$ ii) $x[n] = n \cdot a^n u[n]$.

[Total - 6M]

→ i) $x[n] = \left(\frac{1}{2}\right)^n u[n+2]$.

we can rewrite it as.

$$x[n] = \left(\frac{1}{2}\right)^{-2} \left(\frac{1}{2}\right)^{n+2} u[n+2].$$

we know that,

$$a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - a e^{-j\Omega}}$$

$$\therefore \left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - \frac{1}{2} e^{-j\Omega}} \quad \text{— 1M}$$

using time shift property.

$$\left(\frac{1}{2}\right)^{n+2} u[n+2] \xleftrightarrow{DTFT} e^{j2\Omega} \frac{1}{1 - \frac{1}{2} e^{-j\Omega}} \quad \text{— 1M}$$

using linearity property.

$$\left(\frac{1}{2}\right)^{-2} \left(\frac{1}{2}\right)^{n+2} u[n+2] \xleftrightarrow{DTFT} \left(\frac{1}{2}\right)^{-2} \frac{e^{j2\Omega}}{1 - \frac{1}{2} e^{-j\Omega}}.$$

$$\therefore \boxed{X(\Omega) = 4 \frac{e^{j2\Omega}}{1 - \frac{1}{2} e^{-j\Omega}}.} \quad \text{— 1M}$$

ii) $x[n] = n a^n u[n]$.

we know that

$$a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - a e^{-j\Omega}}$$

using frequency differentiation property.

$$n a^n u[n] \xleftrightarrow{DTFT} j \frac{d}{d\Omega} \frac{1}{1-ae^{-j\Omega}} = j \frac{-aj e^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$ — 2M

$$\therefore \boxed{X(\Omega) = \frac{a e^{-j\Omega}}{(1-a e^{-j\Omega})^2}}$$ — 1M

8.c) Obtain the signal $x(t)$, if its Fourier transform is,

i) $X(j\omega) = \frac{1}{2+j(\omega-3)}$ ii) $X(j\omega) = e^{-j3\omega} \frac{1}{j\omega+2}$.

[Total - 8M]

→ i) $X(j\omega) = \frac{1}{2+j(\omega-3)}$

we know that,

$$e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}$$ — 2M

using frequency shift property,

$$e^{j3t} e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2+j(\omega-3)}$$

$$\therefore \boxed{x(t) = e^{j3t} e^{-2t} u(t)}$$ — 2M

ii) $X(j\omega) = e^{-j3\omega} \frac{1}{j\omega+2}$.

we know that,

$$e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}$$ — 1M

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using time shift property,

$$e^{-2(t-3)} u(t-3) \xleftrightarrow{FT} e^{-j3\omega} \frac{1}{2+j\omega} \quad \text{— 2M.}$$

$$\therefore \boxed{x(t) = e^{-2(t-3)} u(t-3)} \quad \text{— 1M}$$

Module-5

9.a> Find the z-transform of the signals.

i> $x[n] = (1/2)^n u[n] - (3/2)^n u[-n-1]$ ii> $x[n] = (-\frac{1}{3})^n u[n]$

[Total - 7M]

→ i> $x[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{3}{2}\right)^n u[-n-1]$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n u[n] - \left(\frac{3}{2}\right)^n u[-n-1]\right] z^{-n} \quad \text{— 1M}$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2z}\right)^n - \sum_{n=-\infty}^{-1} \left(\frac{3}{2z}\right)^n$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2z}\right)^n + 1 - \sum_{k=0}^{\infty} \left(\frac{2}{3}z\right)^k \quad \text{— 2M}$$

$$\therefore X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + 1 - \frac{1}{1-\frac{3}{2}z^{-1}}, \quad \frac{1}{2} < |z| < \frac{3}{2}.$$

$$\therefore \boxed{X(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{3}{2}}}, \quad \frac{1}{2} < |z| < \frac{3}{2} \quad \text{— 1M}$$

ii> $x[n] = \left(-\frac{1}{3}\right)^n u[n]$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(-\frac{1}{3}\right)^n u[n] z^{-n} = \sum_{n=0}^{\infty} \left(-\frac{1}{3}z^{-1}\right)^n \quad \text{— 1M}$$

$\therefore X(z) = \frac{1}{1+\frac{1}{3}z^{-1}}$, $|-\frac{1}{3}z^{-1}| < 1$ — 1M

$$\boxed{\therefore X(z) = \frac{z}{z+\frac{1}{3}}}, \quad |z| > |\tfrac{1}{3}|.$$ — 1M.

9.b> State and prove differentiation in the Z-domain property of Z-transform. [Total - 6M]

→ Differentiation in the Z-domain property.

or

Multiplication by a Ramp.

If, $Z\{x[n]\} = X(z)$; $R_{x^-} < |z| < R_{x^+}$

then, $Z\{n\,x[n]\} = -z\frac{d}{dz}X(z)$, $R_{x^-} < |z| < R_{x^+}$ — 2M

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$

differentiating both sides with respect to z.

$$\frac{dX(z)}{dz} = -\sum_{n=-\infty}^{\infty} n\,x[n]\,z^{-n-1}$$

— 4M.

multiplying by $-z$.

$$-z\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty}(x[n]\,n]\,z^{-n}$$

$$\therefore Z\{n\,x[n]\} = -z\frac{d}{dz}X(z); \quad R_{x^-} < |z| < R_{x^+}$$

9.c> Use Partial fraction expansion to find the inverse Z-transform of,

$$X(z) = \frac{z^2-3z}{z^2-\frac{3}{2}z-1}, \quad |\tfrac{1}{2}| < |z| < |2|$$

[Total - 7M]

→ $X(z) = \frac{z^2-3z}{z^2-\frac{3}{2}z-1}$

$$\frac{X(z)}{z} = \frac{z-3}{(z+0.5)(z-2)} \quad \text{— 1M}$$

$$\therefore \frac{X(z)}{z} = \frac{z-3}{(z+0.5)(z-2)} = \frac{A}{(z+0.5)} + \frac{B}{(z-2)}.$$

$$A = \frac{z-3}{(z+0.5)(z-2)}(z+0.5)\Big|_{z=-0.5} = 1.4 = \frac{7}{5}$$

— 2M

$$B = \frac{z-3}{(z+0.5)\cancel{(z-2)}}\cancel{(z-2)}\Big|_{z=2} = -0.4$$

$$\therefore \frac{X(z)}{z} = \frac{1.4}{(z+0.5)} - \frac{0.4}{(z-2)} \quad \text{— 1M}$$

$$X(z) = 1.4\frac{z}{z+0.5} - 0.4\frac{z}{z-2}. \quad \text{— 1M}$$

$$X(z) = 1.4\frac{1}{1+0.5z^{-1}} - 0.4\frac{1}{1-2z^{-1}} \quad \text{— 1M}$$

taking inverse z-transform.

$$\boxed{x[n] = 1.4(-0.5)^n u[n] + 0.4(2)^n u[-n-1]} \quad \text{— 1M.}$$

"OR"

10.a) Use properties to find z-transform of the following signals:

i) $x[n] = 3^n u[n-2]$ ii) $x[n] = n\sin(\frac{\pi}{2}n)u[n]$.

[Total - 8M]

→ i) $x[n] = 3^n u[n-2]$.

$$x[n] = 3^2\, 3^{n-2}\, u[n-2]. \quad \text{— 1M}$$

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we know that,

$$a^n u[n] \xleftrightarrow{Z} \frac{1}{1-az^{-1}}$$

$$\therefore\ 3^n u[n] \xleftrightarrow{Z} \frac{1}{1-3z^{-1}}$$

using time shift property.

$$3^{n-2} u[n-2] \xleftrightarrow{Z} \frac{z^{-2}}{1-3z^{-1}} \quad \text{— 1 M}$$

using linearity property

$$3^2\, 3^{n-2} u[n-2] \xleftrightarrow{Z} 3^2 \frac{z^{-2}}{1-3z^{-1}}$$

$$\therefore\ X(z) = 9\, z^{\cancel{-2}} \frac{z}{\cancel{z}-3} = \frac{9\, z^{-1}}{z-3}$$

$$\therefore\ \boxed{X(z) = \frac{9z^{-1}}{z-3}} \qquad |z|>3. \quad \text{— 1 M}$$

ii) $x[n] = n \sin\left(\frac{\pi}{2}n\right) u[n]$.

we know that,

$$Z\{\sin(\tfrac{\pi}{2}n)\, u[n]\} = \frac{z \sin\frac{\pi}{2}}{z^2 - 2z\cos\frac{\pi}{2} + 1} = \frac{z}{z^2+1}. \quad \text{— 2 M}$$

using differentiation in z-domain property.

$$Z\{n \sin(\tfrac{\pi}{2}n)\, u[n]\} = -z \frac{d}{dz}\left[\frac{z}{z^2+1}\right] \quad \text{— 1 M}$$

$$\therefore\ \boxed{X(z) = \frac{z^3-z}{(z^2+1)^2}} \quad \text{— 2 M}$$

10.b) Find the inverse Z-transform.

i) $X(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} + \dfrac{2}{1-2z^{-1}}$; $|z| > |2|$.

ii) $X(z) = \dfrac{2+z^{-1}}{1-\frac{1}{2}z^{-1}}$; $|z| < |\frac{1}{2}|$, use power series expansion method.

[Total-12M].

→ i) $X(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} + \dfrac{2}{1-2z^{-1}}$; $|z| > |2|$. — 1M

$X(z) = \dfrac{z}{z-\frac{1}{2}} + 2\dfrac{z}{z-2}$; $|z| > |2|$. — 1M

taking inverse z-transform.

$$\boxed{x[n] = \left(\tfrac{1}{2}\right)^n u[n] + 2(2)^n u[n]}$$ — 2M

ii) $X(z) = \dfrac{2+z^{-1}}{1-\frac{1}{2}z^{-1}}$; $|z| < |\frac{1}{2}|$.

using power series Expansion.

$$\begin{array}{r|l} & -2-8z-16z^2-32z^3\cdots\cdots \\ \hline -\frac{1}{2}z^{-1}+1 & \cancel{z^{-1}}+2 \\ & \underset{-}{\cancel{z^{-1}}} \underset{+}{-}2 \\ \hline & \cancel{4} \\ & \underset{(-)}{+\cancel{4}} \underset{+}{-}8z \\ \hline & 8\cancel{z} \\ & \underset{(-)}{\cancel{8z}} \underset{(+)}{-}16z^2 \\ \hline & 16z^2 \end{array}$$ — 4M

$\therefore X(z) = -2-8z-16z^2-32z^3 - \cdots\cdots$ — 2M

taking inverse z-transform.

$$\boxed{x[n] = -2\delta[n] - 8\delta[n+1] - 16\delta[n+2] - 32\delta[n+3] - \cdots}\cdots$$

—2M