CBCS SCHEME

USN | | | | | | | | | | | **21MAT11**

First Semester B.E./B.Tech. Degree Examination, Feb./Mar. 2022
Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the curves $r = a\log\theta$ and $r = \dfrac{a}{\log\theta}$. **(07 Marks)**

c. Find the radius of curvature for the cardioid, $r = a(1+\cos\theta)$. **(07 Marks)**

OR

2 a. With usual notation prove that $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$. **(06 Marks)**

b. Show that $r = 4\sec^2\theta/2$ and $r = 9\operatorname{cosec}^2\theta/2$ the pair of curves cut orthogonally. **(07 Marks)**

c. Find the pedal equation of the curve $r^n = a^n\cos n\theta$. **(07 Marks)**

Module-2

3 a. Expand $\sqrt{1+\sin 2x}$ by Maclaurin's series up to the term containing x^4. **(06 Marks)**

b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{1/x}$. **(06 Marks)**

b. If $z = e^{ax+by} f(ax-by)$ prove that $b\dfrac{\partial z}{\partial x} + a\dfrac{\partial z}{\partial y} = 2abz$. **(07 Marks)**

c. Find the extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

Module-3

5 a. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2x$. **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is a parameter. **(07 Marks)**

c. Solve $x(y')^2 - (2x+3y)y' + 6y = 0$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Solve $(x^2 + y^2 + x)dx + xydy = 0$. (06 Marks)

b. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. (07 Marks)

c. Find the general solutions of $xp^2 + xp - yp + 1 - y = 0$. (07 Marks)

Module-4

7 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. (06 Marks)

b. Solve $(D^3 + D^2 - 4D - 4)y = 3e^{-x} - 4x - 6$ (07 Marks)

c. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ using the method of variation of parameters. (07 Marks)

OR

8 a. Solve $(D^2 + 4)y = x^2$. (06 Marks)

b. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1)$. (07 Marks)

c. Solve $(x^2D^2 + xD + 9)y = 3x^2$. (07 Marks)

Module-5

9 a. Find the rank of the matrix.

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

(06 Marks)

b. Solve by Gauss elimination method

$2x + y + 4z = 12$
$4x + 11y - z = 33$
$8x - 3y + 2z = 20$. (07 Marks)

c. Solve the system of equation by Gauss-Seidel method

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$. (07 Marks)

OR

10 a. Find the values of λ and μ such that the system of equations:

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = \mu$, may have

i) unique solution ii) infinite solution iii) no solution. (06 Marks)

b. Solve by the method of Gauss-Jordan method:

$2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$. (07 Marks)

c. Find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ by using the power method by taking initial vector as $[1, 1, 1]^T$.

(07 Marks)

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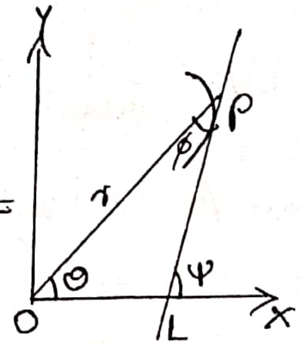
Department: Mathematics
Semester / Branch: Common to all
Subject with Sub. Code: Calculus and Differential Equations(21MAT11)

AY: 2021-2022
VTU QP
Name of Faculty: Prof.Akshata Patil

Q.No.	Solution and Scheme	Marks
1 a.	-: Module 1 :-	
	Let $P(r,\theta)$ be any point on a curve $r=f(\theta)$. Let $OP=r$, $\angle XOP=\theta$. Let PL be the tangent to the curve at P. which makes an angle ψ with +ve x-axis. 	
	Also $\angle OPL=\phi$.	2M
	From Figure, $\psi=\theta+\phi$	
	$\tan\psi=\tan(\theta+\phi)=\dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$ ----(1)	
	Let (x,y) be the Cartesian Co-ordinates of P. $\therefore x=r\cos\theta$, $y=r\sin\theta$	
	Also slope of the tangent is $\tan\psi=\dfrac{dy}{dx}$	2 M
	$\tan\psi=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{r\cos\theta+\frac{dr}{d\theta}\sin\theta}{-r\sin\theta+\frac{dr}{d\theta}\cos\theta}$	
	divide both numerator and denominator by $\frac{dr}{d\theta}\cos\theta$	
	$\tan\psi=\dfrac{r\,d\theta/dr+\tan\theta}{-r\frac{d\theta}{dr}\tan\theta+1}$ ---- (2)	2M
	Comparing equation (1) & (2)	
	$\boxed{\tan\phi=r\dfrac{d\theta}{dr}}$	6M.

Q.No.	Solution and Scheme	Marks
1 b.	Taking log on both sides	
	$\log r = \log a + \log(\log\theta)$; $\log r = \log a - \log(\log\theta)$	→ 1 M
	Differentiate w.r.t 'θ'	
	$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{\log\theta \cdot \theta}$; $\frac{1}{r}\frac{dr}{d\theta} = -\frac{1}{\log\theta \cdot \theta}$	
	$\cot\phi_1 = \frac{1}{\theta\log\theta}$; $\cot\phi_2 = -\frac{1}{\theta\log\theta}$	
	$\tan\phi_1 = \theta\log\theta$; $\tan\phi_2 = -\theta\log\theta$	→ 2 M
	Now consider,	
	$\tan(\phi_1-\phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}$	
	$\tan(\phi_1-\phi_2) = \frac{2\theta\log\theta}{1-(\theta\log\theta)^2}$ ------ (1)	→ 2 M
	we have to find θ by solving the given pair of equation $r = a\log\theta$ and $r = a/\log\theta$	
	Equating the RHS we have,	
	$a\log\theta = a/\log\theta$	
	$(\log\theta)^2 = 1$ or $\log\theta = 1 \Rightarrow \theta = e$	→ 1 M
	Substituting $\theta = e$ in eq 1	
	we get,	
	$\tan(\phi_1-\phi_2) = \frac{2e}{1-e^2}$ $\because \log e = 1$	
	Thus the angle of intersection is	
	$\phi_1 - \phi_2 = \tan^{-1}\left(2e/1-e^2\right) = 2\tan^{-1}e$.	→ 1 M
		7 M.

Q.No.	Solution and Scheme	Marks
1c.	Let, $r = a(1+\cos\theta)$ $\log r = \log a + \log(1+\cos\theta)$ $\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1+\cos\theta} = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$	→ 1 M
	$\frac{1}{r}\frac{dr}{d\theta} = -\tan\theta/2 \Rightarrow \frac{dr}{d\theta} = r_1 = -r\tan(\theta/2)$ $r_2 = -\left[r.\sec^2(\theta/2)\cdot\frac{1}{2} + \tan(\theta/2) r_1\right]$ $r_2 = -\frac{r}{2}\sec^2(\theta/2) + r\tan^2(\theta/2)$	→ 2 M
	we have, $\rho = \frac{[r^2+r_1^2]^{3/2}}{r^2+2r_1^2-rr_2} = \frac{[r^2+r^2\tan^2(\theta/2)]^{3/2}}{r^2+2r^2\tan^2(\theta/2) - r\left[-\frac{r}{2}\sec^2\theta/2 + r\tan^2\theta/2\right]}$ $\rho = \frac{[r^2\sec^2(\theta/2)]^{3/2}}{r^2+r^2\tan^2(\theta/2)+\frac{r^2}{2}\sec^2(\theta/2)}$	→ 2 M
	$\rho = \frac{r^3\sec^3(\theta/2)}{r^2[1+\tan^2(\theta/2)]+\frac{r^2}{2}\sec^2(\theta/2)}$ $\rho = \frac{r^3\sec^3(\theta/2)}{r^2\sec^2(\theta/2)+\frac{r^2}{2}\sec^2(\theta/2)} = r^3\sec(\theta/2)\times\frac{2}{3r^2}$ $\boxed{\rho = \frac{2}{3}r\sec(\theta/2)}$	→ 2 M
		7 M.
2a.	Consider a Cartesian curve $y=f(x)$ $y_1' = \frac{dy}{dx} = \tan\psi$ $y_2'' = \frac{d^2y}{dx^2} = \sec^2\psi.\frac{d\psi}{dx}$	→ 2 M
	$= (1+\tan^2\psi)\frac{d\psi}{ds}.\frac{ds}{dx}$ $y'' = [1+(y')^2]\frac{d\psi}{ds}.\frac{ds}{dx}$	

Q.No.	Solution and Scheme	Marks
	Since, W.K.T $\frac{d\psi}{ds} = \frac{1}{\rho}$; $\frac{ds}{dx} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/2} = \left\{1+(y')^2\right\}^{1/2}$	→ ② M
	$y'' = \frac{\{1+(y')^2\}^{3/2}}{\rho}$	
	OR $\rho = \frac{\{1+(y')^2\}^{3/2}}{y''}$	→ ② M
		6 M.
2 b.	Taking log on both sides.	
	$\log r = \log 4 + 2\log\sec(\theta/2)$; $\log r = \log 9 + 2\log\operatorname{cosec}(\theta/2)$	→ ① M.
	Differentiate w.r.t θ.	
	$\frac{1}{r}\frac{dr}{d\theta} = \frac{2}{\sec(\theta/2)} \cdot \sec(\theta/2)\tan(\theta/2)\,\frac{1}{2}$	
	And, $\frac{1}{r}\frac{dr}{d\theta} = -\frac{2\operatorname{cosec}(\theta/2)\cdot\cot\theta/2\cdot 1/2}{\operatorname{cosec}(\theta/2)}$	→ ② M.
	Therefore, $\frac{1}{r}\frac{dr}{d\theta} = \tan(\theta/2)$: $\frac{1}{r}\frac{dr}{d\theta} = \cot(-\theta/2)$	
	$\cot\phi_1 = \cot(\pi/2 - \theta/2)$: $\cot\phi_2 = \cot(-\theta/2)$	
	$\phi_1 = \pi/2 - \theta/2$: $\phi_2 = -\theta/2$	→ ② M.
	$\therefore \lvert\phi_1 - \phi_2\rvert = \lvert \pi/2 - \theta/2 + \theta/2\rvert = \pi/2$.	→ ② M.
		7 M.

Q.No.	Solution and Scheme	Marks
2c.	$r^n = a^n \cos n\theta$	
	Taking log on both sides.	
	$n \log r = n \log a + \log(\cos n\theta)$	→ ① M.
	Differentiate w.r.t θ.	
	$\frac{n}{r} \frac{dr}{d\theta} = \frac{-n \sin n\theta}{\cos n\theta}$	
	$\frac{1}{r} \frac{dr}{d\theta} = -\tan n\theta$	
	$\cot \phi = \cot\left[\frac{\pi}{2} + n\theta\right] \Rightarrow \phi = \frac{\pi}{2} + n\theta$	→ ② M.
	consider,	
	$p = r \sin \phi$	→ ① M
	$p = r \sin\left(\frac{\pi}{2} + n\theta\right)$ ie $p = r \cos n\theta$	
	Now we have,	
	$r^n = a^n \cos n\theta$ ---- (1)	
	$p = r \cos n\theta$ ---- (2)	
	eqn (1) as consequence of (2)	→ ② M
	$r^n = a^n (p/r)$	
	Thus,	→ ① M
	$r^{n+1} = p a^n$ is the required Pedal Equation.	
		7 M

Q.No.	Solution and Scheme	Marks
	Module - 3	
3a]	We have,	
	$y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0)$	→ 1 M
	$+ \frac{x^4}{4!} y_4(0)$	
	Let,	
	$y = \sqrt{1+\sin 2x} = \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x}$	→ 1 M.
	$y = \sqrt{(\cos x + \sin x)^2} = \cos x + \sin x$	
	$y = \cos x + \sin x$; $y(0) = 1$	
	$y_1 = -\sin x + \cos x$; $y_1(0) = 1$	3 M
	$y_2 = -\cos x - \sin x$; $y_2(0) = -1$	
	$y_2 = -y$; $y_3(0) = -1$	
	$y_3 = -y_1$; $y_4(0) = 1$	
	$y_4 = -y_2$	
	Thus by substituting these values in the expansion of $y(x)$	
	$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - - - - -$	→ 1 M
		6 M.
3b.	Here we need to convert the given function u into a composite function	
	Let, $u = f(p, q, r)$ where $p = x/y$, $q = y/z$, $r = z/x$	→ 1 M
	$\{u \to (p,q,r) \to (x,y,z)\} \Rightarrow u \to x, y, z$	
	$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x}$	→ 2 M.
	$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{1}{y} + \frac{\partial u}{\partial q} \cdot 0 + \frac{\partial u}{\partial r} \cdot \left(\frac{-z}{x^2}\right)$	

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Q.No.	Solution and Scheme	Marks
	Here,	
	$x\frac{\partial u}{\partial x} = \frac{x}{y}\frac{\partial u}{\partial p} - \frac{z}{x}\frac{\partial u}{\partial r}$ ------ (1)	→ 1 M
	Similarly, by symmetry we can write.	
	$y\frac{\partial u}{\partial y} = \frac{y}{z}\frac{\partial u}{\partial q} - \frac{x}{y}\frac{\partial u}{\partial p}$ ------ (2)	
	$z\frac{\partial u}{\partial z} = \frac{z}{x}\frac{\partial u}{\partial r} - \frac{y}{z}\frac{\partial u}{\partial q}$ ------ (3)	→ 2 M
	Thus by adding (1), (2) and (3) we get,	
	$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0}$	→ 1 M
		7 M.
3c.	Let,	
	$u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$	
	$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$	→ 3 M
	It will be easier if the elements of the determinant are evaluated at $(1,-1,0)$	
	$\therefore \frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0) = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$	→ 2 M
	on expanding,	
	$1(0-4)+6(0+4)+0 = 20$	→ 2 M
	Thus,	
	$[J]_{(1,-1,0)} = 20$	→ 1 M
		7 M.

Q.No.	Solution and Scheme	Marks
4a.	Let, $K = \lim_{x\to 0}\left[\frac{a^x+b^x+c^x}{3}\right]^{1/x}$ ------ 1^∞ form	→ 1 M
	Log on both sides $\log K = \lim_{x\to 0} \log\left[\frac{a^x+b^x+c^x}{3}\right]^{1/x}$	
	$= \lim_{x\to 0} \frac{\log\left[\frac{a^x+b^x+c^x}{3}\right]}{x}$ ----- $0/0$ form	→ 2 M
	By L-Hospital rule. $\log K = \lim_{x\to 0}\left[\frac{\frac{3}{a^x+b^x+c^x}\cdot\frac{1}{3}\{a^x\log a + b^x \log b + c^x \log c\}}{1}\right]$	
	$= \frac{1}{a^0+b^0+c^0}\left[a^0\log a + b^0 \log b + c^0 \log c\right]$	
	$= \frac{1}{3}\left[\log a + \log b + \log c\right]$	→ 2 M
	$\log K = \frac{1}{3}\log(abc)$	
	$\log K = \log (abc)^{1/3}$	
	$\boxed{K = (abc)^{1/3}}$	→ 1 M
		6 M.
4b.	we have, $z = e^{ax+by} f(ax-by)$.	
	Let, $r = ax+by$, $s = ax-by$ so that,	→ 1 M
	$z = e^{r} f(s)$	
	Hence, $\{ z \to (r,s) \to (x,y) \Rightarrow z \Rightarrow x, y$.	→ 1 M

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Q.No.	Solution and Scheme	Marks
	we have Chain rule,	
	$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial z}{\partial s} \cdot \frac{\partial s}{\partial x}$	
	$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial y} + \frac{\partial z}{\partial s} \cdot \frac{\partial s}{\partial y}$	$\to$ 1 M
	ie	
	$\frac{\partial z}{\partial x} = e^{r} f(s) a + e^{r} f'(s) a$	
	$\partial z / \partial y = e^{r} f(s) b + e^{r} f'(s)(-b)$	$\to$ 2 M
	$\frac{\partial z}{\partial x} = a e^{r}\left[f(s) + f'(s)\right]$	
	$\frac{\partial z}{\partial y} = b e^{r}\left[f(s) - f'(s)\right]$	
	Now consider,	
	$b \frac{\partial z}{\partial x} + a\, \partial z / \partial y$	
	$= b \cdot a e^{r}\left[f(s) + f'(s)\right] + a \cdot b e^{r}\left[f(s) - f'(s)\right]$	$\to$ 1 M
	$= ab e^{r} f(s) + ab e^{r} f'(s) + ab e^{r} f(s) - ab e^{r} f'(s)$	
	$= 2ab e^{r} f(s) = 2abz$	$\to$ 1 M
	Thus,	
	$b \cdot \partial z / \partial x + a \cdot \partial z / \partial y = 2abz.$	
		7 M.
4c.	$f(x,y) = x^3 + y^3 - 3x - 12y + 20$	
	$f_x = 3x^2 - 3, \quad f_y = 3y^2 - 12.$	
	We shall find points (x,y) such that	
	$f_x = 0$ and $f_y = 0$	$\to$ 1 M
	ie $3x^2 - 3 = 0$ and $3y^2 - 12 = 0$ or $x^2 - 1 = 0$	
	and $y^2 - 4 = 0$	
	ie $x = \pm 1, \quad y = \pm 2$	$\to$ 1 M
	$\therefore (1,2)\ (1,-2)\ (-1,2)\ (-1,-2)$ are the Stationary pts	

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Q.No.	Solution and Scheme	Marks

Let,

$A = f_{xx}$, $B = f_{xy}$, $C = f_{yy}$

	(1,2)	(1,−2)	(−1,2)	(−1,−2)
$A = 6x$	$6>0$	6	-6	$-6<0$
$B = 0$	0	0	0	0
$C = 6y$	12	-12	12	-12
$AC-B^2$	$72>0$	$-72<0$	$-72<0$	$72>0$
Conclusion	Min pt	Saddle pt	Saddle pt	Max. pt

→ (3) M

Maximum value of $f(x,y)$ is,

$f(-1,-2) = -1-8+3+24+20 = 38$ → (1) M

Minimum value of $f(x,y)$ is $f(1,2)$

$\therefore f(1,2) = 1+8-3-24+20 = 2.$ → (1) M

Thus,

Maximum value is 38 and Minimum value is 2. 7 M

Module 3:

5a. This is Bernoullis equation. Dividing the given equation by y^2 we have,

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{yx} = x \text{ ------ } (1)$$ → (1) M

Put, $\frac{1}{y} = t$ $\therefore$ $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

Hence (1) becomes,

$$-\frac{dt}{dx} + \frac{t}{x} = x \quad \text{or} \quad \frac{dt}{dx} - \frac{t}{x} = -x$$ → (1) M

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Q.No.	Solution and Scheme	Marks
	This equation is a linear equation of the form,	
	$\frac{dt}{dx} + Pt = Q$, where $P = -1/x$ and $Q = -x$	$\rightarrow$ 1 M
	$\therefore e^{\int P dx} = e^{-\int 1/x \, dx} = e^{-\log x} = 1/x$	$\rightarrow$ 1 M
	The solution is, $t e^{\int P dx} = \int Q e^{\int P dx} dx + C.$	$\rightarrow$ 2 M
	$t \cdot 1/x = \int -x \cdot 1/x \, dx + C.$	
	Thus, $\frac{1}{xy} = -x + C$ is the required solution.	6 M.
5b.	We have,	
	$\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ ------ (1)	
	Diff w.r.t 'x' we have,	
	$\frac{2x}{a^2} + \frac{2yy_1}{b^2+\lambda} = 0$ where $y_1 = \frac{dy}{dx}$.	$\rightarrow$ 1 M
	$\frac{x}{a^2} = \frac{-yy_1}{b^2+\lambda}$ ------ (2)	
	Also from (1), $\frac{x^2}{a^2} - 1 = \frac{-y^2}{b^2+\lambda}$	
	or $\frac{x^2-a^2}{a^2} = \frac{-y^2}{b^2+\lambda}$ ----- (3)	$\rightarrow$ 2 M
	Now dividing (2) by (3)	
	$\frac{x}{x^2-a^2} = \frac{yy_1}{y^2}$ or $\frac{x}{x^2-a^2} = \frac{y_1}{y}$	

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Q.No.	Solution and Scheme	Marks
	Replacing, $y_1 = \frac{dy}{dx}$ by $-\frac{dx}{dy}$, $\frac{x}{x^2-a^2} = \frac{1}{y}\left(-\frac{dx}{dy}\right)$	→1M
	$y\,dy = -\frac{(x^2-a^2)}{x}\,dx$ by separating the variables. $\int y\,dy = -\int x\,dx + a^2\int \frac{dx}{x} + c.$	→2M
	$\frac{y^2}{2} = \frac{-x^2}{2} + a^2\log x + c.$ Thus, $x^2+y^2-2a^2\log x - b = 0$	→1M
	where, $b = 2c$ is the required Orthogonal Trajectory.	7M
5c	The given equation with the usual notation $xp^2 - (2x+3y)p + 6y = 0$ $\therefore p = \frac{(2x+3y) \pm \sqrt{(2x+3y)^2 - 24xy}}{2x}.$	→2M
	$p = \frac{(2x+3y) \pm (2x-3y)}{2x} = 2$ or $3y/x$	
	we have, $\frac{dy}{dx} = 2 \Rightarrow y = 2x + c$ or $(y-2x-c) = 0$	
	Also, $\frac{dy}{dx} = \frac{3y}{x}$ or $\frac{dy}{y} = 3\frac{dx}{x} \Rightarrow$ $\int \frac{dy}{y} = 3\int \frac{dx}{x} + k.$	→2M

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Q.No.	Solution and Scheme	Marks
	i.e. $\log y = 3\log x + K$ or $\log y = \log x^3 + \log c$,	$\rightarrow$ 1 M
	where,	
	$K = \log c$	
	$\log y = \log(cx^3) \Rightarrow y = cx^3$ or $y - cx^3 = 0$	$\rightarrow$ 2 M
	Thus the general solution is,	
	$\boxed{(y-2x-c)(y-cx^3) = 0.}$	
		7 M
6a.	Let,	
	$M = x^2+y^2+x$ and $N = xy$	
	$\frac{\partial M}{\partial y} = 2y$ and $\frac{\partial N}{\partial x} = y$	
	$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y$ ---- near to N.	$\rightarrow$ 1 M
	Now, $\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{y}{xy} = \frac{1}{x} = f(x)$	$\rightarrow$ 1 M
	Hence, $I.F = e^{\int f(x)dx} = e^{\int 1/x\, dx} = e^{\log x} = x$	
	Multiplying the given equation by x,	
	we now have,	$\rightarrow$ 2 M
	$M = x^3+xy^2+x^2$ and $N = x^2y$	
	$\frac{\partial M}{\partial y} = 2xy$ and $\frac{\partial N}{\partial x} = 2xy$	
	The solution is,	
	$\int M\,dx + \int N(y)\,dy = c$	$\rightarrow$ 2 M
	$\int (x^3+xy^2+x^2)\,dx + \int 0\,dy = c$	
	Thus, $\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} = c$ is the required solution.	$\rightarrow$ 1 M 6 M

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Q.No.	Solution and Scheme	Marks
6b.	According to the Newton's Law of Cooling the expression for the temperature function at any time t is given by,	→ (1) M
	$T = t_2 + (t_1 - t_2)\, e^{-kt}$	
	we have, by data	
	$t_1 = 100$, $t_2 = 30$ and $T = 70$	→ (1) M
	when $t = 15$	
	$\therefore T = 30 + 70\, e^{-kt}$.	
	By applying the initial condition.	
	$70 = 30 + 70\, e^{-15k}$ or $e^{-15k} = \frac{40}{70} = \frac{4}{7}$	→ (2) M
	or $e^{15k} = 7/4 = 1.75$	
	$e^{15k} = 1.75 \Rightarrow 15k = \log_e(1.75)$	
	or $k = \frac{1}{15}\log_e(1.75) \simeq 0.0373$.	→ (1) M
	Hence we have,	
	$T = 30 + 70\, e^{-0.0373t}$.	
	we have to find t when $T = 40$	
	$\therefore 40 = 30 + 70\, e^{-0.0373t}$ or $e^{-0.0373t} = 1/7$	
	Equivalently,	→ (2) M
	$e^{0.0373t} = 7 \Rightarrow 0.0373t = \log_e 7$	
	$\therefore t = \frac{\log_e 7}{0.0373} = 52.17 \simeq 52.2$	
	Thus we conclude that it will take 52.2 minutes for the metal ball to reach a temperature of 40°C	7 M

Q.No.	Solution and Scheme	Marks

6c. $xp^2 + px - py + 1 - y = 0$

$xp^2 + px + 1 = y(p+1)$ → (2) M

$y = \dfrac{xp(p+1)+1}{p+1}$, or $y = px + \dfrac{1}{p+1}$ — (1) → (1) M

Equation (1) is in the Clairaut's form $y = px + f(p)$. whose general solution is $y = cx + f(c)$ → (2) M

Thus the General solution is → (2) M

$$\boxed{y = cx + \frac{1}{c+1}}$$

7 M

:- Module 4 :-

7a. $[4D^4 - 8D^3 - 7D^2 + 11D + 6]y = 0$

AE is $4m^4 - 8m^3 - 7m^2 + 11m + 6 = 0$ → (2) M

If, $m = -1$: $4 + 8 - 7 - 11 + 6 = 18 - 18 = 0$

$\therefore m = -1$ is a root by inspection.

-1	4	-8	-7	11	6
	0	-4	12	-5	-6
	4	-12	5	6	0

→ (2) M

Now,

$4m^3 - 12m^2 + 5m + 6 = 0$

If $m = 2$: $32 - 48 + 10 + 6 = 48 - 48 = 0$

$\therefore m = 2$ is also a root. → (1) M

2	4	-12	5	6
	0	8	-8	-6
	4	-4	-3	0

Q.No.	Solution and Scheme	Marks
	Now, $4m^2-4m-3=0$ $4m^2-6m+2m-3=0$ $2m(2m-3)+1(2m-3)=0$ or $m=-\frac{1}{2}, \frac{3}{2}$ Hence the roots of the AE are $-1, 2, -\frac{1}{2}, \frac{3}{2}$. Thus, $y=c_1e^{-x}+c_2e^{2x}+c_3e^{-x/2}+c_4e^{3x/2}$ is the general sol^n.	→(1)M 6M.
7b.	AE is, $m^3+m^2-4m-4=0$ $m^2(m+1)-4(m+1)=0$ or $(m+1)(m^2-4)=0$ $m=-1, \pm 2$ $\therefore y_c=c_1e^{-x}+c_2e^{2x}+c_3e^{-2x}$ $y_p=\dfrac{3e^{-x}}{D^3+D^2-4D-4}-\dfrac{(4x+6)}{D^3+D^2-4D-4}=P_1-P_2$ $P_1=\dfrac{3e^{-x}}{D^3+D^2-4D-4}=\dfrac{3e^{-x}}{-1+1+4-4}$ $[D_r=0]$ $P_1=x.\dfrac{3e^{-x}}{3D^2+2D-4}=x.\dfrac{3e^{-x}}{3-2-4}=-xe^{-x}$ $P_2=\dfrac{4x+6}{-4-4D+D^2+D^3}$ PI is found by division $-4-4D+D^2+D^3$) $4x+6$ $4x+4$ ——— 2 2 ——— 0 Complete solution : $y=y_c+P_1-P_2$ $y=c_1e^{-x}+c_2e^{2x}+c_3e^{-2x}-xe^{-x}+x+(\frac{1}{2})$.	→(1)M →(1)M →(3)M →(2)M 7M

Q.No.	Solution and Scheme	Marks
7c.	we have, $[D^2+1]y = \sec x \tan x$. AE is, $m^2+1=0 \Rightarrow m=\pm i$ $\therefore y_c = C_1\cos x + C_2 \sin x$ Let, $y = A(x)\cos x + B(x)\sin x$. be the complete solution of the given DE where $A(x), B(x)$ are to be found. we have, $y_1 = \cos x$; $y_2 = \sin x$. $y_1' = -\sin x$; $y_2' = \cos x$. $W = y_1y_2' - y_2y_1' = 1$ Also $\phi(x) = \sec x \tan x$ $A' = \frac{-y_2\phi(x)}{W}$; $B' = \frac{y_1\phi(x)}{W}$. $A' = -\sin x \sec x \tan x$; $B' = \cos x \sec x \tan x$ $A' = -\tan^2 x = 1-\sec^2 x$; $B' = \tan x$ $A = \int (1-\sec^2 x)dx + k_1$; $B = \int \tan x\, dx + k_2$ $A = x - \tan x + k_1$; $B = \log(\sec x) + k_2$ Substituting these in $y = A\cos x + B\sin x$ we have, $y = \{x - \tan x + k_1\}\cos x + \{\log(\sec x) + k_2\}\sin x$ $y = k_1\cos x + k_2\sin x + x\cos x - \sin x + \sin x \log(\sec x)$ The term, $-\sin x$ can be neglected since of the term $k_2 \sin x$ is present in the solution. Thus, $y = k_1\cos x + k_2 \sin x + x\cos x + \sin x \log(\sec x)$	$\rightarrow$ 1 M $\rightarrow$ 2 M $\rightarrow$ 2 M $\rightarrow$ 2 M 7 M

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Q.No.	Solution and Scheme	Marks
8a	we have, AE is $m^2+4=0$ or $m^2=-4 \Rightarrow m=\pm 2i$	$\rightarrow$ 1 M
	$\therefore y_c = c_1\cos 2x + c_2 \sin 2x$	$\rightarrow$ 1 M
	$y_p = \dfrac{x^2}{4+D^2}$ PI is found by division	$\rightarrow$ 1 M
	$4+D^2 \overline{\big) \; x^2 \;}$ quotient $x^2/4$; $x^2 - x^2 = 0$	$\rightarrow$ 2 M
	$\therefore y_p = x^2/4$. $\therefore$ complete solution is $y = y_c + y_p \Rightarrow c_1\cos 2x + c_2\sin 2x + x^2/4$	$\rightarrow$ 1 M
		6 M.
8b.	we have, $[D^2-4]y = \cosh(2x-1)$ AE is, $m^2-4=0$ or $(m-2)(m+2)=0 \Rightarrow m=2,-2$	$\rightarrow$ 2 M
	$\therefore y_c = c_1 e^{2x} + c_2 e^{-2x}$ $y_p = \dfrac{\cosh(2x-1)}{D^2-4} = \dfrac{1}{2}\left[\dfrac{e^{2x-1}}{D^2-4} + \dfrac{e^{-(2x-1)}}{D^2-4}\right]$ $= P_1 + P_2$	$\rightarrow$ 1 M
	$P_1 = \dfrac{1}{2}\cdot\dfrac{e^{2x-1}}{D^2-4} = \dfrac{1}{2}\cdot\dfrac{e^{2x-1}}{2^2-4}$ $[D_r=0]$ $= \dfrac{1}{2}\cdot x\cdot\dfrac{e^{2x-1}}{2D} = \dfrac{1}{2}\cdot x\cdot\dfrac{e^{2x-1}}{4} = \dfrac{x}{8}e^{2x-1}$	$\rightarrow$ 2 M
	$P_2 = \dfrac{1}{2}\cdot\dfrac{e^{-(2x-1)}}{D^2-4} = \dfrac{1}{2}\cdot\dfrac{e^{-(2x-1)}}{(-2)^2-4}$ $[D_r=0]$ $= \dfrac{1}{2}\cdot x\cdot\dfrac{e^{-(2x-1)}}{2D} = \dfrac{1}{2}\cdot x\cdot\dfrac{e^{-(2x-1)}}{-4} = \dfrac{-x}{8}\cdot e^{-(2x-1)}$	$\rightarrow$ 2 M
		7 M.

$y = y_c + y_p \Rightarrow y = c_1 e^{2x} + c_2 e^{-2x} + \dfrac{x}{8}e^{2x-1} - \dfrac{x}{8}e^{-(2x-1)}$

$y = c_1 e^{2x} + c_2 e^{-2x} + x/4 \sinh(2x-1)$ //

Q.No.	Solution and Scheme	Marks
8c.	$\left[x^2D^2+xD+9\right]y=3x^2$	
	we have,	
	$xy'=Dy \quad , \quad x^2y''=D(D-1)y \qquad$ where $D=\frac{d}{dt}$	
	The given equation becomes	2 M
	$\left[D(D-1)+D+9\right]y=3x^2$	
	put $t=\log x$ or $e^t=x$.	
	$\left[D(D-1)+D+9\right]y=3e^{2t}$.	1 M
	$\left[D^2+9\right]y=3e^{2t}$	1 M
	AE is	
	$m^2+9=0 \Rightarrow m=\pm 3i$	1 M
	$\therefore y_c=C_1\cos 3t+C_2\sin 3t$	
	$y_p=\dfrac{3e^{2t}}{D^2+9} \qquad [D\to 2]$	2 M
	$y_p=\dfrac{3e^{2t}}{2^2+9}=\dfrac{3e^{2t}}{4+9}=\dfrac{3e^{2t}}{13}$.	
	Complete solution : $y=y_c+y_p$	
	$y=C_1\cos(3\log x)+C_2\sin(3\log x)+\dfrac{3x^2}{13}$ //	7 M

Module 5 :-

Q.No.	Solution and Scheme	Marks
9a.	$A=\begin{bmatrix}0&1&-3&-1\\1&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$	
	$R_1 \leftrightarrow R_2$	
	$A\sim\begin{bmatrix}1&0&1&1\\0&1&-3&-1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$	2 M
	$R_3\to -3R_1+R_3 \ , \ R_4\to -R_1+R_4$.	

Q.No.	Solution and Scheme	Marks

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

→ 2 M

$R_3 \to -R_2 + R_3$, $R_4 \to -R_2 + R_4$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix A in the row echelon form has two non zero rows.

→ 2 M

Thus,

$$\boxed{\rho[A] = 2}.$$

6 M.

9b. The Augmented matrix of the system is,

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$$

$R_1 \to -2R_1 + R_2$, $R_3 \to -4R_1 + R_3$

→ 2 M

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & -7 & -14 & -28 \end{array}\right]$$

$\frac{1}{9} \cdot R_2$, $-\frac{1}{7} R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 2 & 4 \end{array}\right]$$

→ 2 M

$R_3 \to -R_2 + R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

→ 1 M

Q.No.	Solution and Scheme	Marks
	Hence we have,	
	$2x+y+6z=12$	
	$y-z=1$	
	$3z=3 \quad \therefore z=1$	$\Rightarrow$ 2 M
	By Back substitution; $y=2$ and $x=3$	
	Thus,	
	$\boxed{x=3,\ y=2,\ z=1}$ is the required solution.	1 M.
9c.	The equation are diagonally dominant and hence we first write them in the following form.	
	$x=\frac{1}{20}[17-y+2z]$	$\Rightarrow$ 2 M
	$y=\frac{1}{20}[-18-3x+z]$	
	$z=\frac{1}{20}[25-2x+3y]$	
	we start with the trial solution	
	$x=0,\ y=0,\ z=0$	
	First iteration:	
	$x^{(1)}=17/20=0.85$	$\Rightarrow$ 2 M
	$y^{(1)}=\frac{1}{20}[-18-3(0.85)]=-1.0275$	
	$z^{(1)}=\frac{1}{20}[25-2(0.85)+3(-1.0275)]=1.0109$	
	Second iteration:	
	$x^{(2)}=\frac{1}{20}[17-(-1.0275)+2(1.0109)]=1.0025$	
	$y^{(2)}=\frac{1}{20}[-18-3(1.0025)+1.0109]=-0.9998$	$\Rightarrow$ 2 M
	$z^{(2)}=\frac{1}{20}[25-2(1.0025)+3(-0.9998)]=0.9998$	

Q.No.	Solution and Scheme	Marks
	Third iteration:	
	$x^{(3)} = \frac{1}{20}[17-(-0.9998)+2(0.9998)] = 0.99997 \approx 1$	
	$y^{(3)} = \frac{1}{20}[-18-3(0.99997)+0.9998] = -1.000005 \approx -1$	→(2)M
	$z^{(3)} = \frac{1}{20}[25-2(0.99997)+3(-1.000005)] = 1.0000022 \approx 1$	
	Thus, $\boxed{x=1,\ y=-1,\ z=1}$ is the required solution.	7M
10a.	$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$ is the augmented matrix.	
	$R_2 \to -R_1+R_2,\ R_3 \to -R_1+R_3$	
	$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$	→(1)M
	$R_3 \to -R_2+R_3$	→(1)M
	$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$	
	a] Unique solution: We must have $\rho[A] = \rho[A:B] = 3$, $\rho[A]$ will be 3 if $(\lambda-3) \neq 0$ since the other two entries in the last row of A are zero. or $\lambda \neq 3$. If $(\lambda-3) \neq 0$. irrespective of the value of μ, $\rho[A:B]$ will also be 3.	→(2)M
	b] Infinite Solutions: Here we have $n=3$ and we need. $\rho[A] = \rho[A:B] = r < 3$. we must have $r=2$ Since first row and second row are non-zero.	→(1)M

Q.No.	Solution and Scheme	Marks
	$\therefore \rho[A] = \rho[A:B] = 2$ only when the last row of $[A:B]$ is completely zero. This is possible if $\lambda - 3 = 0$, $\mu - 10 = 0$ Hence, the system will have infinite solution if $\lambda = 3$ and $\mu = 10$	
	c] <u>No solution</u>:- We must have $\rho[A] \neq \rho[A:B]$ By case (a) $\rho[A] = 3$ if $\lambda \neq 3$ and hence	$\rightarrow$ 1 M
	if $\lambda = 3$ we obtain $\rho[A] = 2$ If we impose $(\mu - 10) \neq 0$ then $\rho[A:B]$ will be 3. Hence, the system has no solution if $\lambda = 3$ and $\mu \neq 10$.	1 M
10b]	$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right]$	
	$R_2 \rightarrow -2R_1 + R_2$, $R_3 \rightarrow -2R_1 + R_3$ $\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right]$	$\rightarrow$ 2 M
	$R_1 \rightarrow R_2 + R_1$, $R_3 \rightarrow 3R_2 + R_3$ $\sim \left[\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$	$\rightarrow$ 2 M
	$R_3 \rightarrow -1/4 . R_3$ $\sim \left[\begin{array}{ccc:c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 1 & 5 \end{array}\right]$	$\rightarrow$ 1 M

Q.No.	Solution and Scheme	Marks
	$R_1 \to 2R_3 + R_1$, $R_2 \to 3R_3 + R_2$	
	$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & -1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$	$\to$ 2 M
	Hence, we have $x = 1, -y = -3, z = 5$.	
	Thus, $\boxed{x = 1, y = 3, z = 5}$ is the required solution	7M
10C	$X^{(0)} = [1, 1, 1]^T$.	
	we have,	
	$AX^{(0)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \lambda^{(1)} X^{(1)}$	$\to$ 1 M
	$AX^{(1)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \lambda^{(2)} X^{(2)}$	$\to$ 1 M
	$AX^{(2)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} = 4\begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix} = \lambda^{(3)} X^{(3)}$	$\to$ 1 M
	$AX^{(3)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.75 \\ -1 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -3.5 \\ 2.5 \end{bmatrix} = 3.5\begin{bmatrix} 0.71 \\ -1 \\ 0.71 \end{bmatrix} = \lambda^{4} X^{(4)}$	$\to$ 1 M
	$AX^{(4)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.71 \\ -1 \\ 0.71 \end{bmatrix} = \begin{bmatrix} 2.42 \\ -3.42 \\ 2.42 \end{bmatrix} = 3.42\begin{bmatrix} 0.708 \\ -1 \\ 0.708 \end{bmatrix} = \lambda^{5} X^{(5)}$	$\to$ 1 M
	$AX^{(5)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.708 \\ -1 \\ 0.708 \end{bmatrix} = \begin{bmatrix} 2.416 \\ -3.416 \\ 2.416 \end{bmatrix} = 3.416\begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix}$	$\to$ 1 M
	$AX^{(6)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0.7073 \\ -1 \\ 0.7073 \end{bmatrix} = \begin{bmatrix} 2.4146 \\ -3.4146 \\ 2.4146 \end{bmatrix} = 3.4146\begin{bmatrix} 0.7071 \\ -1 \\ 0.7071 \end{bmatrix}$	$\to$ 1 M
		7M

Largest eigen value is 3.4146 and eigen vectors $[0.7071, -1, 0.7071]'$

Solution and Scheme	Marks

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