KLS Vishwanathrao Deshpande Institute of Technology, Haliyal

CBCS SCHEME

USN | | | | | | | | | | | **18MAT31**

Third Semester B.E. Degree Examination, July/August 2022
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Laplace transform,
(i) $e^{-2t}(2\cos 5t - \sin 5t)$ (ii) $\cosh^2 3t$ **(06 Marks)**

b. Find the Laplace transform of the full wave rectifier $f(t) = E\sin\omega t \quad 0 < t < \frac{\pi}{\omega}$ having a period $\frac{\pi}{\omega}$. **(07 Marks)**

c. Find the inverse Laplace transform $\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$. **(07 Marks)**

OR

2 a. Find the Laplace transform, $\frac{\cos at - \cos bt}{t}$. **(06 Marks)**

b. Solve by using Laplace transform method $y'''(t) + 2y''(t) - y'(t) - 2y(t) = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$ **(07 Marks)**

c. Express the function f(t) in terms of unit step function and hence find its inverse LT,

$$f(t) = \begin{cases} \cos t & 0 < t \le \pi \\ 1 & \pi < t \le 2\pi \\ \sin t & t > 2\pi \end{cases}$$ **(07 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$, in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$. **(06 Marks)**

b. Show that the sine half range series for the function, $f(x) = Lx - x^2$, in $0 < x < L$ is $\frac{8L^2}{\pi^3}\sum_0^\infty \frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{L}\right)\pi x$. **(07 Marks)**

c. Obtain the Fourier series of y up to the first harmonics for the following values :

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	−1.5	0	2.6	3.4

(07 Marks)

OR

4 a. Expand the function $f(x) = x\sin x$, as a Fourier series in the interval $-\pi \le x \le \pi$. Deduce that $\frac{1}{1,3} - \frac{1}{3,5} + \frac{1}{5,7} \ldots\ldots = \frac{\pi - 2}{4}$ **(06 Marks)**

b. Obtain the half range cosine series of $f(x) = x\sin x$, $0 \le x \le \pi$. **(07 Marks)**

c. Obtain the constant term and the first three coefficients in the Fourier cosine series for y using the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

5 a. Find the complex Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$.

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. If $\overline{f(z)} = \frac{2z^2 + 3z + 12}{(z-1)^4}$ find the value of u_0, u_1, u_2, u_3 **(07 Marks)**

c. Solve by using z-transforms, $u_{n+2} + 5u_{n+1} + 6u_n = 2^n$: $u_1 = 0, u_0 = 0$ **(07 Marks)**

OR

6 a. Find the Fourier sine transform of e^{-ax}, $a > 0$. **(06 Marks)**

b. Find the Fourier sine and cosine transform of $2e^{-3x} + 3e^{-2x}$. **(07 Marks)**

c. Solve by using Z-transforms,
$y_{n+2} + 2y_{n+1} + y_n = n$, with $y(0) = 0 = y$ **(07 Marks)**

Module-4

7 a. Use Taylor's series method to find y(4.1) given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$ and $y(4) = 4$. **(06 Marks)**

b. Use Fourth order Runge-Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, $y(0.4) = 1$ at $x = 0.5$. Correct to four decimal places. **(07 Marks)**

c. The following table gives the solution of $5xy^1 + y^2 - 2 = 0$, find the value of y at x = 4.5 using Milne's Predictor and Corrector formulae, use the corrector formulae twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

(07 Marks)

OR

8 a. Using modified Euler's method find y at x = 0.2 given $\frac{dy}{dx} = 3x + \frac{y}{2}$, with $y(0) = 1$ taking $h = 0.1$. **(06 Marks)**

b. Using Runge-Kutta method of fourth order find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, y(0) = 1 taking h = 0.2 **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $(y^2 + 1)dy - x^2 dx = 0$, at x = 1, given y(0) = 1, y(0.25) = 1.0026, y(0.5) = 1.0206, y(0.75) = 1.0679. Apply the corrector formulae twice. **(07 Marks)**

Module-5

9 a. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$. Evaluate $y(0.1)$ using Runge-Kutta method of order 4. **(06 Marks)**

b. A necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be extremum that $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. Show that the extremal of the functional $\int_0^1 y^2\{3x(y'^2 - 1) + yy'^3\}dx$, subject to the conditions $y(0) = 0, y(1) = 2$, is the circle $x^2 + y^2 - 5x = 0$. **(07 Marks)**

OR

10 a. Apply Milne's method to compute $y(0.8)$. Given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values. **(06 Marks)**

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

b. Find the extremal of the functional $\int_a^b (x^2y'^2 + 2y^2 + 2xy)dx$. **(07 Marks)**

c. Prove that Geodesics on a plane are straight line. **(07 Marks)**

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18MATDIP31

Third Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express $\frac{(3+i)(1-3i)}{(2+i)}$ in the form x + iy. **(06 Marks)**

b. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$. Find the value of 'ρ' such that $\vec{a}-\rho\vec{b}$ is perpendicular to $\vec{c}$. **(07 Marks)**

c. Find the angle between the vector $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(07 Marks)**

OR

2 a. Find the modulus and amplitude of the complex number $1+\cos\alpha+i\sin\alpha$. **(06 Marks)**

b. Prove that $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n=\cos n\theta+i\sin n\theta$. **(07 Marks)**

c. Find the sine of the angle between $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

Module-2

3 a. Find the n^{th} derivative of cosx cos2x. **(06 Marks)**

b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ upto the term containing x^4. **(07 Marks)**

c. If $u=f(y-z, z-x, x-y)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(07 Marks)**

OR

4 a. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(06 Marks)**

b. If $z=xy^2+x^2y$ where $x=at^2$ and $y=2at$. Find $\frac{dz}{dt}$. **(07 Marks)**

c. If $x=e^u\sec v$, $y=e^u\tan v$. Find $J\left(\frac{x,y}{u,v}\right)$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve
$\vec{r}=\cos 2t\hat{i}+\sin 2t\hat{j}+t\hat{k}$ where t is the time variable. Determine the components of velocity and acceleration vectors at $t=\pi/8$ in the direction of $\sqrt{2}\hat{i}+\sqrt{2}\hat{j}+\hat{k}$. **(06 Marks)**

b. Find div $\vec{f}$ for $\vec{f}=\nabla(x^3+y^3+z^3-3xyz)$. **(07 Marks)**

c. Show that $\vec{f}=(2xy+z^2)\hat{i}+(x^2+2yz)\hat{j}+(y^2+2xz)\hat{k}$ is irrotional and find φ such that $\vec{f}=\nabla\varphi$. **(07 Marks)**

OR

6 a. Find the unit normal to the surface $x^3y^3z^2 = 4$ at the point P(-1, -1, 2). (06 Marks)

b. If $\vec{f} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\varphi = 2z - x^3y$, find $\vec{f} \bullet (\nabla\varphi)$ and $\vec{f} \times (\nabla\varphi)$ at (1, -1, 1). (07 Marks)

c. Show that $\vec{f} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. (07 Marks)

Module-4

7 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\, dx\ (n > 0)$. (06 Marks)

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$. (07 Marks)

c. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$. (07 Marks)

OR

8 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x\, dx\ (n > 0)$. (06 Marks)

b. Evaluate $\iint_R xy\, dx\, dy$ where R is the first quadrant of the circle $x^2 + y^2 = a^2$, $x \geq 0$, $y \geq 0$. (07 Marks)

c. Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$. (07 Marks)

Module-5

9 a. Solve $x^2\dfrac{dy}{dx} - 2xy - x + 1 = 0$. (06 Marks)

b. Solve $(3x^2y^2 + x^2)\, dx + (2x^3y + y^2)\, dy = 0$. (07 Marks)

c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. (07 Marks)

OR

10 a. Solve $\left[y\left(1 + \dfrac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]dy = 0$. (06 Marks)

b. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. (07 Marks)

c. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2x$. (07 Marks)

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18MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(06 Marks)

b. Solve the system of equations: $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$ by Gauss elimination method. **(07 Marks)**

c. Find all the eigen values and corresponding eigen vectors of $\begin{pmatrix} -5 & 9 \\ -6 & 10 \end{pmatrix}$ **(07 Marks)**

OR

2 a. Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$

(06 Marks)

b. Using Gauss elimination method solve the system of equations
$x + 2y + 3z = 6$; $2x + 4y + z = 7$; $3x + 2y + 9z = 14$. **(07 Marks)**

c. Find the eigen values of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{pmatrix}$ **(07 Marks)**

Module-2

3 a. Use an appropriate Interpolation formula to compute f(6).

x	1	2	3	4	5
y	1	-1	1	-1	1

(07 Marks)

b. Evaluate $\int_0^6 3x^2 dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking $n = 6$. **(07 Marks)**

c. Find a real root of the equation $x^3 - 2x - 5 = 0$ by Newton Raphson method. **(06 Marks)**

OR

4 a. Find solution using Newton's Interpolation formula, at $x = -1$.

x	0	1	2	3
f(x)	1	0	1	10

(07 Marks)

b. Find the real root of the equation $\cos x = 3x - 1$ using Regula Falsi method. (07 Marks)

c. Evaluate $\int_{4}^{5.2} \log_e x$ taking $n = 6$ by Weddle's rule. (06 Marks)

Module-3

5 a. Solve : $(D^3 - 2D^2 + 4D - 8)y = 0$ (06 Marks)

b. Solve : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$ (07 Marks)

c. Solve : $\frac{d^2y}{dx^2} + 4y = \cos 4x$ (07 Marks)

OR

6 a. Solve : $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$ (06 Marks)

b. Solve : $(D^2 - 6D + 9)y = 7e^{-2x} - \log 2$ (07 Marks)

c. Solve : $\frac{d^2y}{dx^2} - 16y = \sin 16x$ (07 Marks)

Module-4

7 a. Form the partial differential equation by eliminating the arbitrary constants from $z = (x - a)^2 + (y - b)^2$ (06 Marks)

b. Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ (07 Marks)

c. Solve : $\frac{\partial^2 z}{\partial y^2} - z = 0$; given that $z = \cos x$ and $\frac{\partial z}{\partial y} = \sin x$, when $y = 0$. (07 Marks)

OR

8 a. Form the partial differential equation by eliminating the arbitrary function 'f' from $f(x^2 + y^2, z - xy) = 0$ (06 Marks)

b. Solve the equation $\frac{\partial^2 z}{\partial y^2} = \sin xy$ (07 Marks)

c. Form the partial differential equation by eliminating the arbitrary constants $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (07 Marks)

Module-5

9 a. Define : (i) Mathematical definition of probability
(ii) Mutually exclusive events
(iii) Independent events (06 Marks)

b. If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.
Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(\overline{A}/\overline{B})$ (iv) $P(\overline{B}/\overline{A})$ (07 Marks)

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3%, 2% are defective. If a bolt drawn at random was found defective, what is the probability that it was manufactured by A? (07 Marks)

OR

10 a. State and prove Baye's theorem. **(06 Marks)**

b. A card is drawn at random from a pack of cards. (i) What is the probability that it is a heart? (ii) If it is known that the card drawn is red, what is the probability that it is a heart? **(07 Marks)**

c. An Urn 'A' contains 2 white and 4 black balls. Another Urn 'B' contains 5 white and 7 black balls. A ball is transferred from the Urn A to the Urn B. Then a ball is drawn from the Urn B. Find the probability that it is white. **(07 Marks)**

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18MAT41

Fourth Semester B.E. Degree Examination, July/August 2022
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive Cauchy-Riemann equation in Polar form. **(06 Marks)**

b. Find the analytic function f(z) whose real part is
$x \sin x \cosh y - y \cos x \sinh y$ **(07 Marks)**

c. If f(z) is analytic show that
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$$ **(07 Marks)**

OR

2 a. Find the analytic function f(z) given that the sum of its real and imaginary part is
$x^3 - y^3 + 3xy(x - y)$ **(06 Marks)**

b. Find the analytic function $f(z) = u + iv$ if
$v = r^2\cos 2\theta - r\cos\theta + 2$ **(07 Marks)**

c. If f(z) is analytic function then show that
$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$ **(07 Marks)**

Module-2

3 a. State and prove Cauchy's Integral formula. **(06 Marks)**

b. Evaluate $\int_0^{2+i} \bar{z}^2 dz$ along (i) the line $y = \frac{x}{2}$ (ii) The real axis to 2 and then vertically to 2 + i. **(07 Marks)**

c. Find the bilinear transformation which maps the points 1, i, –1 onto the points i, 0, –i respectively. **(07 Marks)**

OR

4 a. Discuss the transformation $w = e^z$, with respect to straight lines parallel to x and y axis. **(06 Marks)**

b. Using Cauchy's integral formula evaluate
$$\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz \text{ , where } c : |z| = 3$$ **(07 Marks)**

c. Find the bilinear transformation which maps the points 0, 1, ∞ on to the points –5, –1, 3 respectively. **(07 Marks)**

Module-3

5 a. A random variable X has the following probability function for various values of X.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find i) k ii) $P(X < 6)$ iii) $P(3 < X \le 6)$ **(06 Marks)**

b. Out of 800 families with 5 children each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls, assuming equal probabilities for boys and girls. **(07 Marks)**

c. The length in time (minutes) that a certain lady speaks on a telephone is a random variable with probability density function

$$f(x)=\begin{cases}Ae^{-x/5} & \text{for } x>0\\ 0 & \text{elsewhere}\end{cases}$$

Find the value of the constant A. What is the probability that she will speak over the phone for (i) More than 10 minutes (ii) Less than 5 minutes (iii) Between 5 and 10 minutes. **(07 Marks)**

OR

6 a. Find the constant C such that the function $f(x)=\begin{cases}Cx^2, & 0<x<3\\ 0 & \text{otherwise}\end{cases}$ is a probability density function. Also compute $P(1 < x < 2)$, $P(x \le 1)$ and $P(x > 1)$ **(06 Marks)**

b. 2% fuses manufactured by a firm are found to be defective. Find the probability that the box containing 200 fuses contains
(i) No defective fuses (ii) 3 or more defective fuses (iii) At least one defective fuse. **(07 Marks)**

c. If x is a normal variate with mean 30 and standard deviation 5 find the probabilities that (i) $26 \le x \le 40$ (ii) $x \ge 45$ (iii) $|x-30| > 5$
Given that $\phi(1)=0.3413$, $\phi(0.8)=0.2881$, $\phi(2)=0.4772$, $\phi(3)=0.4987$ **(07 Marks)**

Module-4

7 a. The following table gives the ages (in years) of 10 married couples. Calculate Karl Pearson's coefficient of correlation between their ages:

Age of husband (x)	23	27	28	29	30	31	33	35	36	39
Age of wife (y)	18	22	23	24	25	26	28	29	30	32

(06 Marks)

b. In a partially destroyed laboratory record of correlation data only the following results are available:
Variance of x is 9 and regression lines are $8x-10y+66=0$, $40x-18y=214$. Find
(i) Mean value of x and y
(ii) Standard deviation of y
(iii) Coefficient of correlation between x and y. **(07 Marks)**

c. Fit a parabola of the form $y = ax^2 + bx + c$ for the data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

OR

8 a. Obtain the lines of regression and hence find the coefficient of correlation of the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(06 Marks)

b. Show that if θ is the angle between the lines of regression

$$\tan\theta=\frac{\sigma_x\sigma_y}{\sigma_x^{\,2}+\sigma_y^{\,2}}\left(\frac{1-r^2}{r}\right)$$

(07 Marks)

c. Fit a straight line $y = a + bx$ to the data

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

Module-5

9 a. The joint probability distribution of the random variables X and Y is given below.

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Find (i) E[X] and E[Y] (ii) E[XY] (iii) cov(X, Y) iv) ρ(X, Y).
Also, show that X and Y are not independent. **(06 Marks)**

b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory confirmed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5% ($z_{0.05} = 1.96$, $z_{0.01} = 2.58$). **(07 Marks)**

c. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f. is 2.201). **(07 Marks)**

OR

10 a. Define the terms :
(i) Null hypothesis (ii) Type-I and Type – II errors (iii) Significance level **(06 Marks)**

b. In an experiment of pea breeding the following frequencies of seeds were obtained:

Round Yellow	Wrinkled Yellow	Round Green	Wrinkled Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1
Is the experiment in agreement with theory ($\chi^2_{0.5}$ for 3 d.f is 7.815) **(07 Marks)**

c. The joint probability distribution of two discrete random variable X and Y is given by $f(x, y) = k(2x + y)$ where x and y are integers such that $0 \le x \le 2$, $0 \le y \le 3$. Find k and the marginal probability distribution of X and Y. Show that the random variables X and Y are dependent. Also, find $P(X \ge 1, Y \le 2)$. **(07 Marks)**

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