CBCS SCHEME

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18EE54

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022
Signals and Systems

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Explain the signals and systems with the help of suitable examples. **(05 Marks)**

b. Obtain the even and odd part of the given signal x(t) shown in Fig.Q1(b).

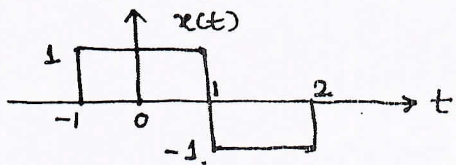

Fig.Q1(b) **(05 Marks)**

c. For the following system, determine whether the system is (i) Linear (ii) Time invariant (iii) Memoryless (iv) Causal (v) Stable.
(A) $y(n) = n\,x(n)$ (B) $y(t) = x(t/2)$ **(10 Marks)**

OR

2 a. Whether the signal shown in Fig.Q2(a) is energy or power signal? Determine energy or power.

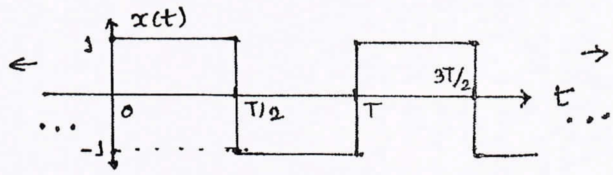

Fig.Q2(a) **(05 Marks)**

b. Check whether the following signals are periodic or not. If periodic, find the fundamental period
(i) $x(n) = \cos 2\pi n$ (ii) $x(t) = \cos 2t + \sin 3t$ **(05 Marks)**

c. For the continuous time signal x(t) shown in Fig.Q2(c). Sketch the following:
(i) $x(2t)$ (ii) $x(t+2)$ (iii) $x(-2t+1)$
(iv) $2x(t-3)$ (v) $x(t+2) + x(t-2)$

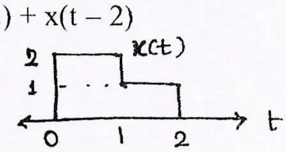

Fig.Q2(c) **(10 Marks)**

Module-2

3 a. Consider a LTI system with unit impulse response, $h(t) = e^{-t} \cdot u(t)$. If the input applied to this system is $x(t) = e^{-3t}[u(t) - u(t-2)]$, find the output y(t) of the system. **(10 Marks)**

b. Find the total response of the system given by $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2.x(t)$

with $y(0) = -1$; $\left.\frac{dy(t)}{dt}\right|_{t=0} = 1$ and $x(t) = \cos t \cdot u(t)$ **(10 Marks)**

OR

4 a. Find the natural response of the system described by difference equation,

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

with $y(-1) = 0$ and $y(-2) = 1$ **(08 Marks)**

b. Draw the direct form I and direct form II of the given system function

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t)$$ **(06 Marks)**

c. Check whether the LTI system which has impulse response given by,

i) $h(t) = \cos(\pi t) \cdot u(t)$ ii) $h(n) = \sin(\frac{1}{2}\pi n)$

is memoryless . causal or stable. **(06 Marks)**

Module-3

5 a. State and prove the following continuous time fourier transform :

(i) Convolution property (ii) Time shift property **(10 Marks)**

b. Find the fourier transform of the following :

(i) $x(t) = e^{-a|t|}$; $a > 0$ (ii) $x(t) = \delta(t)$

Draw the spectrum. **(10 Marks)**

OR

6 a. Using partial expansion, determine the inverse fourier transform of

$$X(jw) = \frac{5jw + 12}{(jw)^2 + 5jw + 6}$$ **(05 Marks)**

b. Find the frequency response and the impulse response of the system having the output y(t) for the input x(t) as given below:

$x(t) = e^{-t}u(t)$ and $y(t) = e^{-3t}u(t) + e^{-2t}u(t)$ **(07 Marks)**

c. Find the frequency response and the impulse response of the system described by the differential equation.

$$\frac{d^2y(t)}{dt^2} + 5\cdot\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$ **(08 Marks)**

Module-4

7 a. Using the appropriate, find the DTFT of the following signal

(i) $x(n) = \left(\frac{1}{2}\right)^n .u(n-2)$ (ii) $x(n) = \sin\left(\frac{\pi}{4}n\right)\left(\frac{1}{4}\right)^n .u(n-1)$ **(10 Marks)**

b. Find the inverse DTFT of

(i) $X(e^{j\Omega}) = \dfrac{6}{e^{-j2\Omega} - 5e^{-j\Omega} + 6}$

(ii) $X(e^{j\Omega}) = 1 + 2\cos\Omega + 3\cos 2\Omega$ **(10 Marks)**

OR

8 a. State and prove the following properties of DTFT :

(i) Linearity property (ii) Frequency shift (iii) Parseval's theorem. **(10 Marks)**

b. Obtain the frequency response and the impulse response of the system having the output y(n) for the input x(n) as given below,

$x(n) = (1/2)^n . u(n)$, $y(n) = \frac{1}{4}(\frac{1}{2})^n . u(n) + (\frac{1}{4})^n . u(n)$ **(10 Marks)**

Module-5

9 a. State and prove the following property of z-transform:
(i) Initial Value theorem (ii) Differentiation in the z-domain. **(08 Marks)**

b. For the signal $x(n) = 7(1/3)^n u(n) - 6(½)^n . u(n).$, find the z-transform and ROC. **(06 Marks)**

c. List the ROC (Region Of Convergence) of z-transform. **(06 Marks)**

OR

10 a. Using partial fraction expansion method, obtain the time domain signal corresponding to the z-transform given below.

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \qquad \therefore |z| > \frac{1}{2}$$ **(06 Marks)**

b. Determine the impulse response h(n) and the system function H(z) of the system, if the input

$$x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$$

$$y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$$ **(06 Marks)**

c. A causal LTI system is described by difference equation

$$y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1)$$

find the system function H(z). Also determine the impulse response of the system. **(08 Marks)**

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Signals & Systems

18EE54

Feb / mar 2022

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HOD

MODULE-1

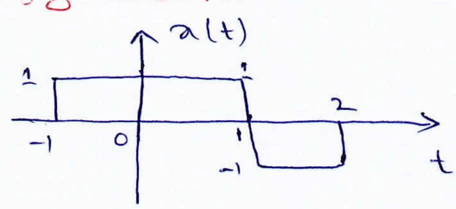
Q.1

a) Explain the signals and systems with help of suitable Example

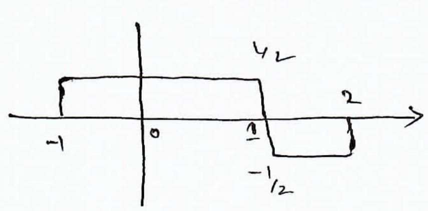
Any thing that carries information can be called as signals It is a single valued function of one or more independent variable which contain some information Any physical quantity which varies w.r.t time space and other independent variable Electric current and voltage are signal Example it may be function of time, temp, position, Pressure, Distance depending on that it is one dimensional and two dimensional signal

A system is defined as an entity that acts on an input signal and transform it into an output signal. It may be also defined as set of Elements which are connected together and produces an output in response to input signals. It is a cause and effect relation between two or more signals various types of Electrical systems are mechanical, Electrical, biological, Electro mechanical systems.

b) obtain the Even and odd part of the given signal x(t) shown in the fig below.



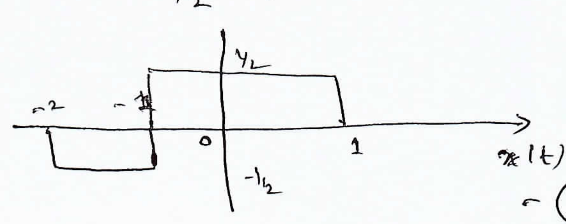
$1/2\, x(t)$



- ①

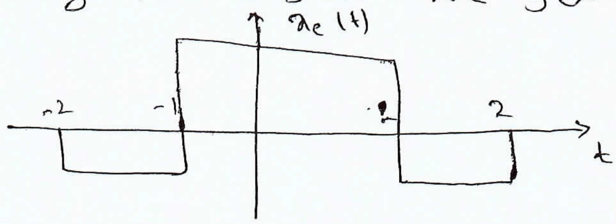
and

$1/2\, x(-t)$

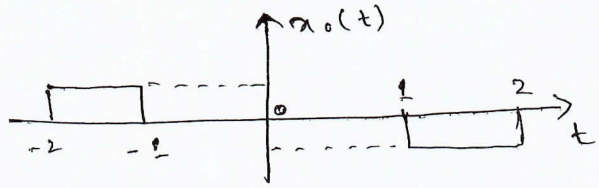


- ②

adding two signal we get ① + ②



subtracting the above two signals i.e ① − ②.



c) For the following system determine wheter the system is
(i) linear (ii) Time invarient (iii) causal (iv) memoryless (v) stable

(A) $y(n) = n\,x(n)$ $\quad y(t) = x(t/2)$

(A) $y(n) = n\,x(n)$

(i) linearity $T\{a x_1(n) + b x_2(n)\} = n\{a x_1(n) + b x_2(n)\}$

$= a n x_1(n) + b n x_2(n)$

$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$

(ii) Time invariance: $T x(n-n_0) = n\,x(n-n_0)$

$y(n-n_0) = (n-n_0)\,x(n-n_0)$

$y(n-n_0) \neq T\,x(n-n_0)$

(iii) Hence Varient

since output depends only on the present value of input it is memoryless

(iv) causality: The output does not depend on the future values of the input so it is causal

(v) stability: Let $|x(n)| \leq B_x$

$y(n) = n(x(n))$

$= |n|\,.|x(n)| = |n|\,B_x$

'n' does not bounded system is unstable

(B) $y(t) = x(t/2)$

linearity $T\{a x_1(t) + b x_2(t)\} = a\,x_1(t/2) + b\,x_2(t/2)$

$= a\,T\,x_1(t) + b\,T\,x_2(t)$

System is linear

B) $y(t) = x(t/2)$

a) linearity $T\{a x_1(t) + b x_2(t)\} = a x_1(t/2) + b x_2(t/2)$

$= a T(x_1(t)) + b T x_2(t)$

System is linear

b) Time invariable

$T x(t-t_0) = x\left(\frac{t}{2} - t_0\right)$

$y(t-t_0) = x\left(\frac{t-t_0}{2}\right)$

$y(t-t_0) \neq T\{x(t-t_0)\}$

System is not time invarient

c) memory less: The output depends on past values of input

$y(1) = x(0.5)$ so the system has memory.

iv) Causal: The output depends on future values of the input

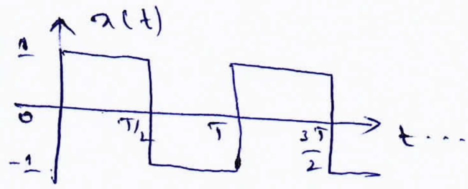
for Ex $y(-1) = x(-0.5)$ system is non causal

(v) stability $|x(t)| \leq B_x$

then $|y(t)| = |x(t/2)| \leq B_x$ system is stable

Q.2

a) whether the signal shown in is Energy or power signal. Determine Energy or power



It is a periodic signal Hence it is a Power signal it's avg Power is given by

$x(t) = 1$ for $0 < t < T/2$

$= -1$ for $T/2 < t < T$

avg Power is given by

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{T}\left(\int_0^{T/2}(1)^2 dt + \int_{T/2}^{T}(-1)^2 dt\right) = \frac{1}{T}\left(t\Big|_0^{T/2} + t\Big|_{T/2}^{T}\right)$$

$$= \frac{1}{T}\left[\left(\frac{T}{2} - 0\right) + \left(T - \frac{T}{2}\right)\right] = 1.$$

b) Check whether the following discrete time signals are Periodic or not

$x(n) = \cos 2\pi n$ $\qquad$ $x(t) = \cos 2t + \sin 3t$

given $x(n) = \cos 2\pi n$

$x(n) = \cos(2\pi f n)$

$2\pi f = 2\pi$ $\qquad$ Here f is a ratio of two integers

$f = 1 = \frac{1}{1} = \frac{k}{N}$ with $k = 1$ and $N = 1$ Hence it is rational

it is a periodic with fundamental period $= N = 1$.

$x(t) = \cos 2t + \sin 3t$ $\qquad$ $x_1(t) = \cos 2t$

$x(t) = x_1(t) + x_2(t)$ $\qquad$ with $\cos \omega_1 t$

$\omega_1 = 2$ $\quad$ $2\pi f_1 = 2$ $\quad$ $f_1 = \frac{2}{\pi}$

Period of $x_1(t)$ is $T_1 = \frac{1}{f_1} = \pi$

comparing $x_2(t) = \sin\sqrt{3}\, t$ with $\sin \omega_2 t$

$\omega_2 = \sqrt{3}$ $\quad$ $2\pi f_2 = 3$ $\quad$ $f_2 = \frac{3}{2\pi}$

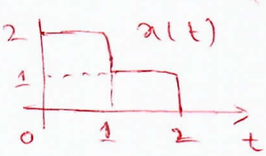
$T_2 = \frac{1}{f_2} = \frac{2\pi}{3}$

Ratio of two periods $\frac{T_1}{T_2} = \frac{\pi}{2\pi/\sqrt{3}} = \frac{3}{2}$

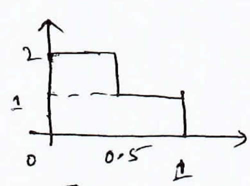
The ratio of two integers given signal $x(t)$ is periodic

c) For the continuous time signal $x(t)$ shown sketch the following

(i) $x(2t)$ (ii) $x(t+2)$ (iii) $x(-2t+1)$ (iv) $2x(t-3)$ (v) $x(t+2) + x(t-2)$

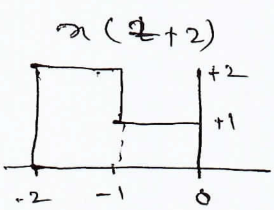


$x(2t) =$



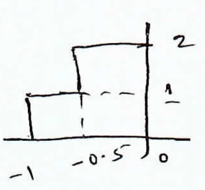
(i)

$x(t+2)$

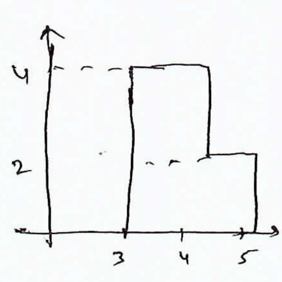


(ii)

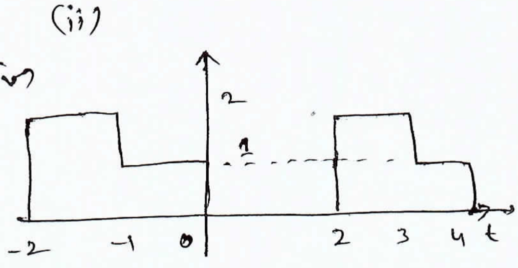
(iii) $x(-2t+1)$



(iv)



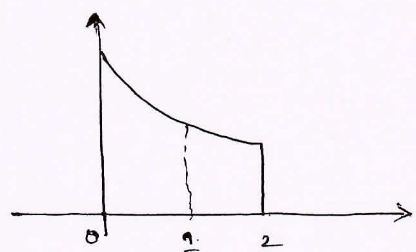
(v)

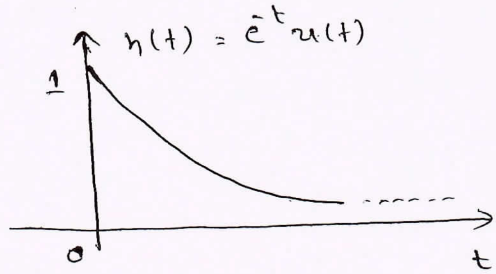


MODULE-2

Q.3

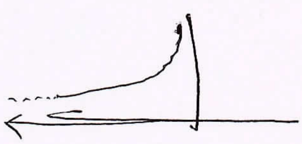
a) consider a LTI system with unit impulse response $h(t) = e^{-t}u(t)$ if the input applied to this system is $x(t) = e^{-3t}[u(t) - u(t-2)]$ find the output $y(t)$ of the system.





w.k.T $\quad y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$h(t-\tau)$



when $t<0$

$x(\tau)h(t-\tau) = 0 \qquad y(t) = 0$

when $t \geq 0$ and $t < 2$

$$\int_0^t e^{-3\tau} e^{-(t-\tau)} d\tau$$

$$= \int_0^t e^{-t-2\tau} d\tau$$

$$= e^{-t} \int_0^t e^{+2\tau} d\tau$$

$$= e^{-t} \left. \frac{e^{-2\tau}}{-2} \right]_0^t$$

$$y(t) = \tfrac{1}{2}\left(1 - e^{-2t}\right) e^{-t}$$

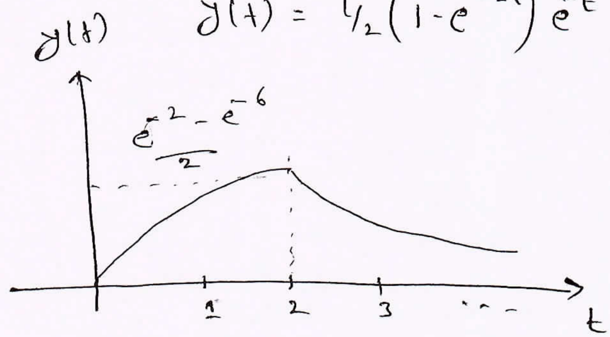
when $t > 2$

$$y(t) = \int_0^2 e^{-3\tau} e^{-(t-\tau)} d\tau$$

$$= e^{-t} \int_0^2 e^{-2\tau} d\tau$$

$$y(t) = \tfrac{1}{2}\left(1 - e^{-4}\right) e^{-t}$$

$$y(t) = \begin{cases} 0 & t < 0 \\ \tfrac{1}{2}(1 - e^{-2t})e^{-t} & 0 < t < 2 \\ \tfrac{1}{2}(1 - e^{-4})e^{-t} & t > 2 \end{cases}$$



b) Find the total response of the system given by $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2x(t)$ with $y(0) = -1$ $\left.\frac{dy(t)}{dt}\right|_{t=0} = 1$

and $x(t) = \cos t \cdot u(t)$

$$y^{(t)}(t) = y^{(n)}(t) + y^{(p)}(t)$$

$$r^2 + 3r + 2 = 0$$

$$r_1 = -2 \qquad r_2 = -1$$

Characteristic Equation is given by

$$y^{(n)}(t) = C_1 e^{-2t} + C_2 e^{-t}$$

since $x(t) = \cos t\, u(t)$ Particular solution is of the form given by $y^p(t) = k_1 \cos t + k_2 \sin t$

Substituting above Equation in main Equation

$$\frac{d^2}{dt^2}\left[k_1 \cos t + k_2 \sin t\right] + 3\frac{d}{dt}\left(k_1 \cos t + k_2 \sin t\right) + 2\left[k_1 \cos t + k_2 \sin t\right] = 2\cos t$$

$$(k_1 + 3k_2)\cos t + (k_2 - 3k_1)\sin t = 2\cos t$$

comparing RHS & LHS we get

$$k_1 + 3k_2 = 2 \qquad k_2 - 3k_1 = 0$$

on solving we get $k_1 = 1/5 \quad k_2 = 3/5$

$$y(t) = C_1 e^{-2t} + C_2 e^{-t} + 1/5 \cos t + 3/5 \sin t$$

Initial condition $y(0) = -1 \qquad \left.\frac{dy(t)}{dt}\right|_{t=0} = 1$

$$-1 = C_1 + C_2 + 1/5 \qquad 1 = -2C_1 - C_2 + 3/5$$

on solving above Equation we get

$$C_1 = 4/5 \qquad C_2 = -2$$

The Response is

$$y(t) = 4/5\, e^{-2t} - 2e^{-t} + 1/5 \cos t + 3/5 \sin t$$

$$y(t) = \frac{1}{5}\left[\cos t + 3\sin t + 4e^{-2t} - 10e^{-t}\right]$$

Q. 4 a) Find the natural response of the system described by the difference Equation

$$y(n) - \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) = x(n) + x(n-1) \quad \text{with } y(-1) = 0$$

and $y(-2) = \cancel{0}\,1$

Homogeneous Equation is

$$y(n) - \tfrac{1}{4} y(n-1) - \tfrac{1}{8} y(n-2) = 0$$

characteristic Equation is

$$1 - \tfrac{1}{4} r^{-1} - \tfrac{1}{8} r^{-2} = 0 \quad \text{or} \quad r^2 - \tfrac{1}{4} r - \tfrac{1}{8} = 0$$

$r_1 = \tfrac{1}{2}$ and $r_2 = -\tfrac{1}{4}$ The roots are real and non-repeated The natural response is of the form

$$y^{(n)}(n) = C_1 (\tfrac{1}{2})^n + C_2(-\tfrac{1}{4})^n$$

$$y(n) = \tfrac{1}{4} y(n-1) + \tfrac{1}{8} y(n-2)$$

Using initial conditions in the above Equations we get

$$y(0) = \tfrac{1}{4} y(-1) + \tfrac{1}{8} y(-2) = \tfrac{1}{4} y(0) + \tfrac{1}{8}(1) = \tfrac{1}{8} = C_1 + C_2$$

$$y(1) = \tfrac{1}{4} y(0) + \tfrac{1}{8} y(-1) = \tfrac{1}{4}(\tfrac{1}{8}) + \tfrac{1}{8}(0) = \tfrac{1}{32} = \tfrac{1}{2} C_1 - \tfrac{1}{4} C_2$$

on solving we get $C_1 = \tfrac{1}{12}$ & $C_2 = \tfrac{1}{24}$

The natural response is given by

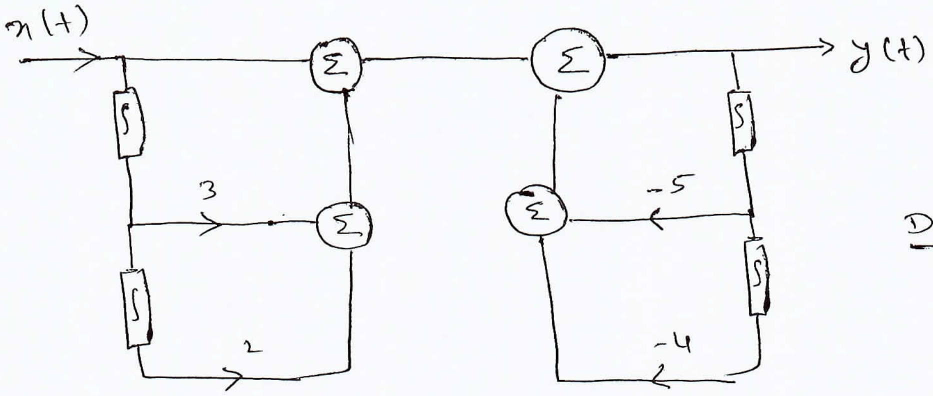
$$y^{(n)}(n) = \tfrac{1}{12}(\tfrac{1}{2})^n + \tfrac{1}{24}(-\tfrac{1}{4})^n \quad ; n > 0$$

b) Draw the direct form I and direct form II of the given system function

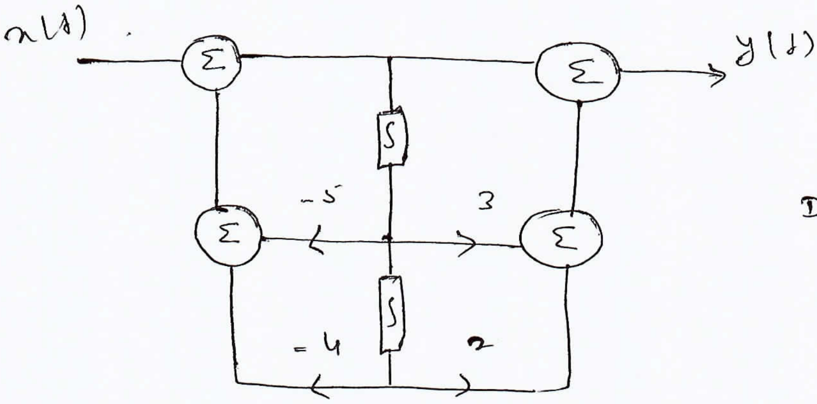
$$\frac{d^2 y(t)}{dt^2} + 5\frac{d y(t)}{dt} + 4 y(t) = \frac{d^2 x(t)}{dt^2} + 3\frac{d x(t)}{dt} + 2 x(t)$$

$$y(t) + 5y'(t) + 4y''(t) = x(t) + 3x'(t) + 2x''(t)$$

$$y(t) = x(t) + 3x'(t) + 2x''(t) - 5y'(t) - 4y''(t)$$



Direct form-I



Direct form-II

Q) Check whether LTI system which has impulse response given by (i) $h(t) = \cos(\pi t)\, u(t)$ (ii) $h(n) = \sin(\frac{1}{2}\pi n)$ is memory less, causal or stable.

(i) $h(t) = \cos(\pi t)\, u(t)$

$$\int_0^\infty |h(t)|\, dt = |1|$$

Hence the system is not memory less

it is causal because output depends on past and present input only

it is a unstable system as it runs up to ∞

(ii) $h(n) = \sin\left(\frac{1}{2}\pi n\right)$

$$= \sum_{k=-\infty}^{\infty} h(n) \quad \text{for } n < 0 \; = \text{it is a not memory less}$$

it is a non causal system & unstable system

MODULE-3

Q.5 a) State and prove the following continuous time F.T.

(i) convolution property (ii) Time shifting property

(i) convolution in time domain is equivalent to multiplication in frequency domain

If $x(t) \xleftrightarrow{F.T} X(j\omega)$

and $y(t) \xleftrightarrow{F.T} Y(j\omega)$ then

$$z(t) = x(t) * y(t) \xleftrightarrow{F.T} Z(j\omega) = X(j\omega)\, Y(j\omega)$$

Proof

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$Z(j\omega) = \int_{-\infty}^{\infty} z(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} [x(t) * y(t)]\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau\right] e^{-j\omega t}\, dt$$ Changing the order of the integration

$$Z(j\omega) = \int_{-\infty}^{\infty} x(\tau)\left(\int_{-\infty}^{\infty} y(t-\tau)\, e^{-j\omega t}\, dt\right) d\tau$$

Put $t-\tau = a$ then $dt = da$

$$Z(j\omega) = \int_{-\infty}^{\infty} x(\tau) \int_{-\infty}^{\infty} y(a)\, e^{-j\omega(a+\tau)}\, da\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau \int_{-\infty}^{\infty} y(a)\, e^{-j\omega a}\, da$$

$$= Z(j\omega) = X(j\omega)\, Y(j\omega)$$

Time shifting property

If $x(t) \xleftrightarrow{F.T} X(j\omega)$

then $y(t) = x(t-t_0) \xleftrightarrow{F.T} Y(j\omega) = e^{-j\omega t_0} X(j\omega)$

Proof

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t-t_0)\, e^{-j\omega t}\, dt$$ Put $t - t_0 = a$ then $dt = da$

$$Y(j\omega) = \int_{-\infty}^{\infty} x(a)\, e^{-j\omega(a+t_0)}\, da$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} x(a)\, e^{-j\omega a}\, da \qquad Y(j\omega) = e^{-j\omega t_0} X(j\omega)$$

Hence the Proof

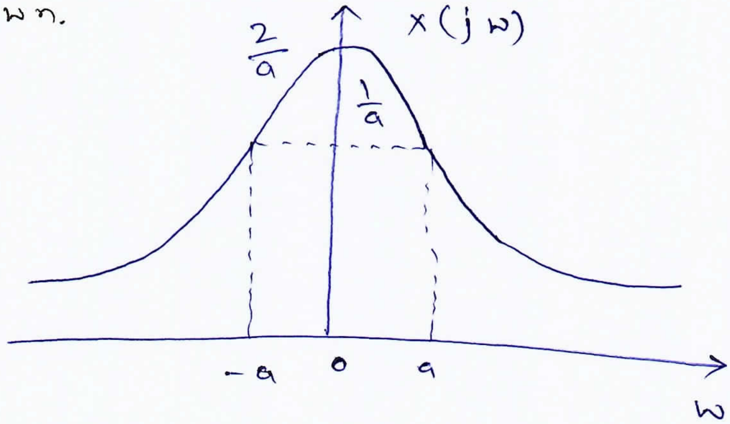
5) Find the F.T of the following
(i) $x(t) = e^{-a|t|}$; $a>0$ (ii) $x(t) = \delta(t)$ draw the spectrum

$x(t) = e^{-a|t|}$, $a>0$ we have $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-a(a+j\omega)t}\, dt = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}$$ Since the given $x(t)$ is even symmetric $X(j\omega)$ is purely real the spectrum is shown.



we have

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

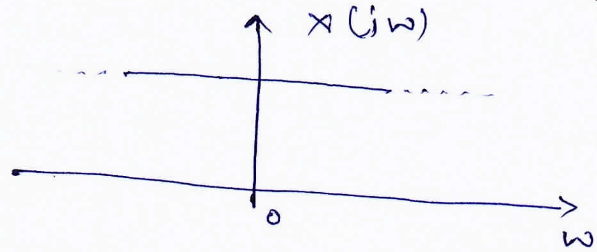
But we know that

$\delta(t) = 0 \qquad t \neq 0$

$$X(j\omega) = \delta(0)\, e^{j\omega(0)}$$

$$X(j\omega) = 1$$

The spectrum is shown in fig



Q.6

a) Using Partial Expansion to determine the IFT of

$$X(j\omega) = \frac{5j\omega + 12}{(j\omega)^2 + 5j\omega + 6}$$

$$X(j\omega) = \frac{5j\omega + 12}{(j\omega)^2 + 5j\omega + 6} = \frac{5j\omega + 12}{(j\omega + 3)(j\omega + 2)}$$

By Partial fraction

$$X(j\omega) = \frac{3}{j\omega + 3} + \frac{2}{j\omega + 2}$$

Taking the inverse fourier transform

$$\left(e^{-at} u(t) \xleftrightarrow{F.T} \frac{1}{a + j\omega} \right)$$

$$x(t) = 3e^{-3t} u(t) + 2e^{-2t} u(t)$$

$$x(t) = \left(3e^{-3t} + 2e^{-2t}\right) u(t)$$

b) Find the frequency response and the impulse response of the system having the output $y(t)$ for the input $x(t)$ as given below.

$$x(t) = e^{-t} u(t) \quad \text{and} \quad y(t) = e^{-2t} u(t) + e^{-3t} u(t)$$

Taking F.T of the given input and output signal we get

$$X(j\omega) = \frac{1}{1 + j\omega} \qquad Y(j\omega) = \frac{1}{2 + j\omega} + \frac{1}{3 + j\omega}$$

The frequency response $H(jw) = \dfrac{Y(jw)}{X(jw)}$

$$= \frac{1+jw}{2+jw} + \frac{1+jw}{3+jw}$$

$$H(jw) = 2 - \frac{1}{2+jw} - \frac{2}{3+jw}$$

Taking IFT we get the impulse response

$$h(t) = 2\delta(t) - e^{-2t}u(t) - 2e^{-3t}u(t)$$

$$h(t) = 2\delta(t) - e^{-2t}\left[1+2e^{-t}\right]u(t)$$

c) Find the frequency response and the impulse response of the system described by the differential Equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$

Taking F.T on both sides we get

$$(jw)^2 Y(jw) + 5(jw)Y(jw) + 6Y(jw) = -jw\,X(jw)$$

The frequency response is

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{-jw}{(jw)^2 + 5jw + 6} = \frac{-jw}{(2+jw)(3+jw)}$$

By Partial fraction Expansion we get

$$H(jw) = \frac{2}{2+jw} - \frac{3}{3+jw}$$

Taking inverse F.T we get the impulse response

$$h(t) = 2e^{-2t}u(t) - 3e^{-3t}u(t)$$

$$h(t) = \left(2e^{-2t} - 3e^{-3t}\right)u(t)$$

Q.7 a)

MODULE-4

Using the appropriate Find the DTFT of the following signal

(i) $x(n) = \left(\frac{1}{2}\right)^n u(n-2)$ (ii) $x(n) = \sin\left(\frac{\pi}{4}n\right)\left(\frac{1}{4}\right)^n u(n-1)$

Given $x(n) = \left(\frac{1}{2}\right)^n u(n-2)$

This can be re-written as

$$x(n) = \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} u(n-2)$$

W.K.T

$$\left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{D.T.F.T} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

using the time shifting property

$$\left(\frac{1}{2}\right)^{n-2} u(n-2) \xleftrightarrow{D.T.F.T} e^{-j2\Omega} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

using the linearity property we get

$$\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} u(n-2) \xleftrightarrow{D.T.F.T} \left(\frac{1}{2}\right)^2 \frac{e^{-j2\Omega}}{1-\frac{1}{2}e^{-j\Omega}}$$

$$\therefore x(n) = \left(\frac{1}{2}\right)^n u(n-2) \xleftrightarrow{D.T.F.T} \frac{1}{4} e^{-j2\Omega} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

$$X(e^{j\Omega}) = \frac{1}{4} e^{-j2\Omega} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

(ii) Given $x(n) = \sin\left(\frac{\pi}{4}n\right)\left(\frac{1}{4}\right)^n u(n-1)$

$$\left(\frac{e^{j\pi/4 n} - e^{-j\pi/4 n}}{2j}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)^{n-1} u(n-1)$$

$$\therefore x(n) = \frac{1}{8j}\left[e^{j\frac{\pi}{4}n}\left(\frac{1}{4}\right)^{n-1} u(n-1) - e^{-j\frac{\pi}{4}n}\left(\frac{1}{4}\right)^{n-1} u(n-1)\right]$$

W.K.T $\left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{D.T.F.T} \frac{1}{1-\frac{1}{4}e^{-j\Omega}}$

Using time shift property we get

$$\left(\frac{1}{4}\right)^{n-1} u(n-1) \xleftrightarrow{D.T.F.T} e^{-j\Omega}\frac{1}{1-\frac{1}{4}e^{-j\Omega}} = \frac{1}{e^{j\Omega}-\frac{1}{4}}$$

using the frequency shift property we get

$$e^{j\frac{\pi}{4}n}\left(\frac{1}{4}\right)^{n-1} u(n-1) \xleftrightarrow{D.T.F.T} \frac{1}{e^{j(\Omega-\pi/4)}-\frac{1}{4}}$$

using the linearity property we get

$$\frac{1}{8j}e^{j\frac{\pi}{4}n}\left(\frac{1}{4}\right)^{n-1} u(n-1) \xleftrightarrow{D.T.F.T} \frac{1}{8j}\frac{1}{e^{j(\Omega-\pi/4)}-\frac{1}{4}}$$

$||^{ly}$

$$\frac{1}{8j}e^{-j\frac{\pi}{4}n}\left(\frac{1}{4}\right)^{n-1} u(n-1) \xleftrightarrow{D.T.F.T} \frac{1}{8j}\frac{1}{e^{j(\Omega+\pi/4)}-\frac{1}{4}}$$

$$x(e^{j\Omega}) = \frac{1}{8j}\left[\frac{1}{e^{j(\Omega-\frac{\pi}{4})}-\frac{1}{4}} - \frac{1}{e^{j(\Omega+\pi/4)}-\frac{1}{4}}\right]$$

b) Find the inverse DTFT of

(i) $X(e^{j\Omega}) = \dfrac{6}{e^{-j2\Omega}-5e^{-j\Omega}+6}$ (ii) $X(e^{j\Omega}) = 1+2\cos\Omega+3\cos 2\Omega$

Given

$$x(e^{j\Omega}) = \frac{6}{e^{-j2\Omega}-5e^{-j\Omega}+6}$$

By Partial fraction Expansion we get

$$= \frac{6}{(e^{-j\Omega}-2)(e^{-j\Omega}-3)}$$

$$x(e^{j\Omega}) = \frac{-6}{(e^{-j\Omega}-2)} + \frac{6}{(e^{-j\Omega}-3)}$$

$$x(e^{+j\Omega}) = \frac{3}{(1-\frac{1}{2}e^{-j\Omega})} - \frac{2}{(1-\frac{1}{3}e^{-j\Omega})}$$

Taking inverse DTFT we get

$$x(n) = \left[3\left(\tfrac{1}{2}\right)^n u(n) - 2\left(\tfrac{1}{3}\right)^n u(n)\right]$$

$$x(n) = \left[3\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{3}\right)^n\right] u(n)$$

(ii) $X(e^{j\Omega}) = 1 + 2\cos\Omega + 3\cos 2\Omega$

$$= 1 + 2\left[\frac{e^{j\Omega} + e^{-j\Omega}}{2}\right] + 3\left[\frac{e^{j2\Omega} + e^{-j2\Omega}}{2}\right]$$

$$= 1 + e^{j\Omega} + e^{-j\Omega} + \frac{3}{2}e^{j2\Omega} + \frac{3}{2}e^{-j2\Omega}$$

Taking inverse D.T.F.T we get

$$x(n) = \delta(n) + \delta(n+1) + \delta(n-1) + \frac{3}{2}\delta(n+2) + \frac{3}{2}\delta(n-2)$$

$$x(n) = \left\{\frac{3}{2}, 1, \underset{\uparrow}{1}, 1, \frac{3}{2}\right\}$$

Q.8
a) State and Prove the following properties of D.T.F.T
(i) linearity Property (ii) Frequency shift (iii) Parseval's theorem.

Linearity Property :- If $x(n) \xleftrightarrow{D.T.F.T} X(e^{j\Omega})$

and $y(n) \xleftrightarrow{D.T.F.T} Y(e^{j\Omega})$

then $z(n) = a x(n) + b y(n) \xleftrightarrow{D.T.F.T} Z(e^{j\Omega}) = a X(e^{j\Omega}) + b Y(e^{j\Omega})$

we have
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\Omega n}$$

$$Z(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} z(n) e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} [a x(n) + b y(n)] e^{-j\Omega n}$$

$$= a\sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} + b\sum_{n=-\infty}^{\infty} y(n) e^{-j\Omega n}$$

$$Z(e^{j\Omega}) = a X(e^{j\Omega}) + b Y(e^{j\Omega})$$

Frequency shift If $x(n) \xleftrightarrow{D.T.F.T} X(e^{j\Omega})$

then $y(n) = e^{j\beta n} x(n) \xleftrightarrow{D.T.F.T} Y(e^{j\Omega}) = X(e^{j(\Omega-\beta)})$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{j\beta n}\, x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\Omega-\beta)n}$$

$$Y(e^{j\Omega}) = X\left(e^{j(\Omega-\beta)}\right)$$

Parseval's theorem

If $x(n) \xleftrightarrow{D.T.F.T} X(e^{j\Omega})$

then $$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2\, d\Omega$$

is known as Energy Density spectrum of the signal $x(n)$

W.K.T

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{2\pi}\int_{2\pi} X^*(e^{j\Omega})\, e^{-j\Omega n}\, \partial\Omega\right]$$

Changing the order of the summation and integration we get

$$E = \frac{1}{2\pi}\int_{2\pi} X^*(e^{j\Omega}) \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}\, \partial\Omega$$

$$= \frac{1}{2\pi}\int_{2\pi} X^*(e^{j\Omega})\, X(e^{j\Omega})\, \partial\Omega = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2\, \partial\Omega$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2\, \partial\Omega$$

b) Obtain the frequency response and the impulse response of the system having the output $y(n)$ for the input $x(n)$ as given below

$x(n) = \left(\frac{1}{2}\right)^n u(n)$, $y(n) = \frac{1}{4}\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{4}\right)^n u(n)$

$$x(n) = \left(\tfrac{1}{2}\right)^n u(n) \qquad y(n) = \tfrac{1}{4}\left(\tfrac{1}{2}\right)^n u(n) + \left(\tfrac{1}{4}\right)^n u(n)$$

Taking the D.T.F.T we get

$$X(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \qquad Y(e^{j\Omega}) = \frac{1/4}{1-1/2\, e^{-j\Omega}} + \frac{1}{1-1/4\, e^{-j\Omega}}$$

The frequency response is $H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})}$

$$= \frac{1}{4} + \frac{1-1/2\, e^{-j\Omega}}{1-1/4\, e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{4} + \frac{1}{1-1/4\, e^{-j\Omega}} - \frac{1/2\, e^{-j\Omega}}{1-1/4\, e^{-j\Omega}}$$

Taking inverse D.T.F.T we get

the impulse response $h(n) =$

$$\frac{1}{4}\delta(n) + \left(\frac{1}{4}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{4}\right)^{n-1} u(n-1)$$

MODULE-5

Q.9

a) State and prove the following property of Z-Transform.

(i) Initial value theorem (ii) Differentiation in the Z-domain

Initial value theorem

If $x(n) = 0$ for $n < 0$ (i.e $x(n)$ is causal)

then $\lim_{n\to 0} x(n) = x(0) = \lim_{z\to\infty} X(z)$

$$Z[x(n)] = X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= x(0) + x(1) z^{-1} + x(2) z^{-2} + \cdots\cdots$$

Take limit $z \to \infty$ on both sides we get

$$\lim_{z \to \infty} X(z) = x(0) + 0 + 0 + \cdots\cdots$$

$$\lim_{n \to 0} x(n) = x(0) = \lim_{z \to \infty} X(z)$$

Differentiation in the Z-Domain

If $x(n) \xleftrightarrow{Z.T} X(z)$ with $RoC = R$

then $n\,x(n) \xleftrightarrow{Z.T} -z \dfrac{\partial X(z)}{\partial z}$ with $RoC = R$

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Differentiating both the sides w.r.t 'z' we get

$$\frac{\partial X(z)}{\partial z} = \sum_{n=-\infty}^{\infty} x(n) (-n) z^{-n-1}$$

$$= -z^{-1} \sum_{n=-\infty}^{\infty} (n\,x(n)) z^{-n}$$

$$= -z^{-1} Z(x(n)\,n)$$

$$Z[n\,x(n)] = -z \frac{\partial}{\partial z} X(z)$$

5) For the signal $x(n) = 7\left(\frac{1}{3}\right)^n u(n) - 6\left(\frac{1}{2}\right)^n u(n)$ Find the Z-transform and RoC.

We have $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(7\left(\frac{1}{3}\right)^n - 6\left(\frac{1}{2}\right)^n\right) u(n)\, z^{-n}$$

$$= 7 \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n \cancel{u(n)}\, z^{-n} - 6 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= 7 \sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n - 6 \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{7}{1 - \frac{1}{3} z^{-1}} - \frac{6}{1 - \frac{1}{2} z^{-1}}$$

$$X(z) = \frac{21}{3z - 1} - \frac{12}{2z - 1}$$ both sum should

Converge

$$\left|\frac{1}{3} z^{-1}\right| < 1 \quad \text{and} \quad \left|\frac{1}{2} z^{-1}\right| < 1$$

$$|z| > \frac{1}{3} \quad \text{and} \quad |z| > \frac{1}{2}$$

ROC $\quad |z| > 1/2$

() List the ROC (Region of Convergence) of Z-Transform.

The basic properties

* The ROC of $X(z)$ consists of a ring in the z-plane centred about the origin

* The ROC does not contain any poles

* If $x(n)$ is of finite duration then ROC is entire z-plane except possibly $z = 0$ and $|z| = \infty$

* If $x(n)$ is a right sided sequence and if the circle $|z| = r_0$ is in the ROC then all the finite values of 'z' for which $|z| > r_0$ will also be in the ROC

* If $x(n)$ is left sided sequence and if the circle $|z| = r_0$ is in the ROC then all the values of 'z' for which 'z' for which $0<|z|<r_0$ will also be in the ROC

* If $x(n)$ is a two sided & if the circle $|z| = r_0$ is in the ROC then the ROC will consists of a ring in the z-plane that contain include the circle $|z| = r_0$

* If the z-transform $X(z)$ of $x(n)$ is rational then it's ROC is bounded by poles or extends to infinity

Q.10

a) Using a partial fraction method obtain the time domain signal corresponding to the z-transform given below

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \quad ; \quad |z| > \frac{1}{2}$$

Factorising the denominator

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)} \quad ; \quad |z| > \frac{1}{2}$$

$$= \frac{K_1}{1 + \frac{1}{2}z^{-1}} + \frac{K_2}{1 + \frac{1}{4}z^{-1}}$$

$$K_1 = \left(1 + \frac{1}{2}z^{-1}\right) X(z) \Big|_{z=-\frac{1}{2}} = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{1}{4}z^{-1}} \Bigg|_{z=-\frac{1}{2}} = 4$$

$$K_2 = \left(1 + \frac{1}{4}z^{-1}\right) X(z) \Big|_{z=-\frac{1}{4}} = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{1}{2}z^{-1}} \Bigg|_{z=-\frac{1}{4}} = -3$$

$$\therefore \quad X(z) = \frac{4}{\left(1+\frac{1}{2}z^{-1}\right)} - \frac{3}{\left(1+\frac{1}{4}z^{-1}\right)}$$

Since ROC $|z| > \frac{1}{2}$: $x(n)$ must be right side sequence

Taking inverse z-transform

$$x(n) = \left[4\left(-\frac{1}{2}\right)^n - 3\left(\frac{-1}{4}\right)^n\right] u(n)$$

b) Determine the impulse response $h(n)$ and the system function $H(z)$ of the system if the input

$$x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$$

$$y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$$

Taking z-transform of Eqn

$$X(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} \qquad Y(z) = 1 - \frac{3}{4}z^{-1}$$

Transfer function of the system is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{3}{4}z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \frac{1-\frac{3}{4}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

By Partial fraction Expansion we get

$$H(z) = \frac{-2/3}{1-1/4\, z^{-1}} + \frac{5/3}{1+\frac{1}{2}z^{-1}}$$

It is given that the system is causal

i.e $h(n) = 0$ for $n<0$

Taking inverse z-transform of the Equation we get

$$h(n) = -\frac{2}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{5}{3}\left(-\frac{1}{2}\right)^n u(n)$$

$$= \frac{1}{3}\left[-2\left(\frac{1}{4}\right)^n + 5\left(-\frac{1}{2}\right)^n\right] u(n)$$

c) A causal LTI system is described by difference equation

$y(n) - \frac{1}{4} y(n-1) - \frac{3}{8} y(n-2) = -x(n) + 2x(n-1)$ find the system function $H(z)$ also determine the impulse response of the system.

Taking the z-transform on the both side we get

$$Y(z)\left(1 - \tfrac{1}{4} z^{-1} - \frac{3}{8} z^{-2}\right) = X(z)\left[-1 + 2z^{-1}\right]$$

Transfer function $H(z) = \frac{Y(z)}{X(z)} = \frac{-1 + 2z^{-1}}{1 - \frac{1}{4} z^{-1} - \frac{3}{8} z^{-2}}$

$$H(z) = \frac{-1 + 2z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 - \frac{3}{4} z^{-1}\right)}$$

By partial fraction we get

$$H(z) = \frac{-2}{\left(1 + \tfrac{1}{2} z^{-1}\right)} + \frac{1}{\left(1 - \tfrac{3}{4} z^{-1}\right)}$$

Taking the inverse z-transform we get

impulse response $h(n) = \left[-2\left(-\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n\right] u(n)$