CBCS SCHEME

USN | | | | | | | | | | 18EC52

Fifth Semester B.E. Degree Examination, Jan./Feb. 2023
Digital Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. State and prove circular time shift property. **(06 Marks)**

b. Find the 4-point DFT of the sequence $x(n) = \text{Cos}\left(\frac{\pi}{4}n\right) + \text{Sin}\left(\frac{\pi}{4}n\right)$ use linearity property. **(08 Marks)**

c. Consider 4-point sequences $x(n) = \text{Cos}\left(\frac{\pi n}{2}\right); 0 \le n \le 3$

$h(n) = 2^n;\ 0 \le n \le 3$

Compute circular convolution. Using concentric circle method. **(06 Marks)**

OR

2 a. State and prove Parseval's theorem. **(06 Marks)**

b. Find 6- point DFT of the sequence

$x(n) = n;\quad 0 \le n \le 5$

$= 0;\quad \text{otherwise}$ **(08 Marks)**

c. Find the IDFT of the DFT $X(K) = \{6, -2+j2, -2, -2-j2\}$. **(06 Marks)**

Module-2

3 a. Consider a FIR with filter whose impulse response $h(n) = \{3, 2, 1, 1\}$ if the input is $x(n) = \{1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$, find the output using over lap add method assuming the length of block as 7. **(10 Marks)**

b. Develop Radix- 2 DIT-FFT algorithm and draw complete signal flow graph for N = 8. **(10 Marks)**

OR

4 a. Find the output y(n) of a filter whose impulse response in $h(n) = \{1, 1, 1\}$ and the input signal to the filter is $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$. Using overlap save method. **(10 Marks)**

b. First five point of the Eight point DFT of a real valued sequence is given by

$x(0) = 0,\quad x(3) = 2-2j$

$x(1) = 2+2j\quad x(4) = 0$

$x(2) = -j4$

Determine the remaining points. Hence find the original sequence x(n) using Decimation in frequency FFT algorithm. **(10 Marks)**

Module-3

5 a. List the different types of windowing techniques used in the design of FIR filters. Write the analytical equations, draw the magnitude response and show the largest side lobe value below the dc magnitude. **(08 Marks)**

b. The frequency response of an FIR filter is given by

$H(\omega) = e^{-j3\omega}(1 + 1.8\cos 3\omega + 1.2\cos 2\omega + 0.5\cos\omega)$

Determine the coefficient of the impulse response h(n) of the FIR filter. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

LO

c. Determine the coefficient K_m of the lattice filter corresponding to FIR filter described by the system function $H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$. And also draw the Lattice structure. **(06 Marks)**

OR

6 a. Determine the filter coefficient $h_d(n)$ for the desired frequency response of a Lowpass filter is given by

$$H_d(w) = \begin{cases} e^{-j2w} & ; \quad -\frac{\pi}{4} \le w \le \frac{\pi}{4} \\ 0 & ; \quad \frac{\pi}{4} \le w \le \pi \end{cases}$$

Find h(n) and also frequency response H(w) using Hamming window. **(10 Marks)**

b. Obtain the cascade form realization of system function :

$H(z) = 1 + 5z^{-1} + 2z^{-2} + 2z^{-3}$ **(05 Marks)**

c. Realize the following function in Direct form.

$H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + z^{-2}\right)$ **(05 Marks)**

Module-4

7 a. Discuss the general procedure for IIR filter design using Bilinear transformation. **(06 Marks)**

b. An analog filter is given by $H_a(s) = \frac{s+0.1}{(s+0.1)^2 + 16}$. Obtain digital IIR filter using bilinear transformation method. Digital filter is to have resonant frequency $\omega_r = \frac{\pi}{2}$ radians. **(08 Marks)**

c. Compare FIR and IIR filter. **(06 Marks)**

OR

8 a. Design a Butterworth digital low pass filter with the following specifications.

i) 3dB attenuation at the passband frequency of 1.5KHz

ii) 10dB stopband attenuation at the frequency of 3KHz

iii) Sampling frequency of 8000Hz. **(10 Marks)**

b. A system is represented by a transfer function H(z) is given by $H(z) = 1 + \frac{4z}{z - \frac{1}{2}} - \frac{2}{z - \frac{1}{4}}$

i) Does this H(z) represent a FIR or IIR filter? Why?

ii) Draw direct form – I and Direct form – II realization by showing all differences equations? **(10 Marks)**

Module-5

9 a. Explain IEEE floating point formats using :

i) Single precision format ii) Double precision format. **(08 Marks)**

b. Discuss briefly multiplier and Accumulator unit in Digital signal processor hardware units. **(04 Marks)**

c. Draw the block diagram to TMS320C3X floating point digital signal processor. **(08 Marks)**

OR

10 a. With block diagram explain Digital signal processor based on Harvard architecture. **(06 Marks)**

b. Convert the Q-15 signed number to decimal numbers.

i) 1.110101110000010 ii) 0.100011110110010 **(04 Marks)**

c. Explain the basic architecture of TMS320CS54X used in fixed point Digital signal processor. **(10 Marks)**

05

Digital Signal Processing - 18EC52
Jan./Feb. - 2023 Scheme & Solution

"Module - 1"

1.a) State and Prove circular time shift property.
[Total - 6M]

→ Circular time shift Property :-

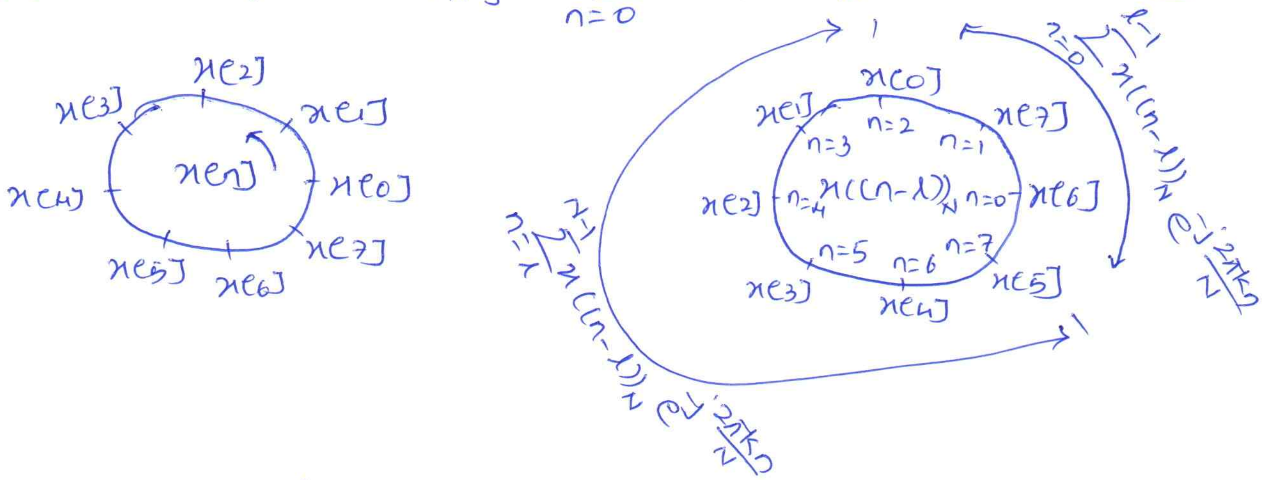
This property states that,

if, $x[n] \xleftrightarrow[N]{DFT} X(k)$

then, $x((n-l))_N \xleftrightarrow[N]{DFT} X(k)\, e^{-j\frac{2\pi kl}{N}}$ — 2M

i.e., shifting the sequence circularly by 'l' samples is equivalent to multiplying its DFT by a complex exponential sequence $e^{-j\frac{2\pi kl}{N}}$

Proof:- $\mathrm{DFT}\{x((n-l))_N\} = \sum_{n=0}^{N-1} x((n-l))_N\, e^{-j\frac{2\pi kn}{N}}$ — ①



On the basis of circular shift we can split the summation in eqⁿ ①. Shifted sequence for $l=2$, the DFT of circularly shifted sequence $x((n-l))_N$ can be split into two parts.

$$\mathrm{DFT}\{x((n-l))_N\} = \sum_{n=l}^{N-1} x((n-l))_N\, e^{-j\frac{2\pi kn}{N}} + \sum_{n=0}^{l} x((n-l))_N\, e^{-j\frac{2\pi kn}{N}}$$

Here, $x((n-l))_N$ can be written as $x(N-l+n)$, hence second summation can be written as

$$\sum_{n=0}^{l-1} x((n-l))_N\, e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{l-1} x(N-l+n)\, e^{-j\frac{2\pi kn}{N}}$$

let, $m = N-l+n$, then when, $n=0$, $m=N-l$ &
$n=l-1$ $m=N-1$

$$\therefore \sum_{n=0}^{l-1} x((n-l))_N e^{-j\frac{2\pi k n}{N}} = \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l-N)}{N}}$$

$$= \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}} \quad \because e^{j\frac{2\pi k N}{N}} = 1.$$ — 1 M

Now, consider the first summation i.e.,

$$\sum_{n=l}^{N-1} x((n-l))_N e^{-j\frac{2\pi k n}{N}}, \quad \text{put, } n-l=m$$

$\therefore$ when $n=l$, $m=0$ &

$n=N-1$, $m=N-1-l$

$$\sum_{n=l}^{N-1} x((n-l))_N e^{-j\frac{2\pi k n}{N}} = \sum_{m=0}^{N-1-l} x[m] e^{-j\frac{2\pi k(m+l)}{N}}$$ — 1 M

$$\therefore DFT\{x((n-l))_N\} = \sum_{m=0}^{N-1-l} x[m] e^{-j\frac{2\pi k(m+l)}{N}} + \sum_{m=N-l}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}}$$ — 1 M

$$= \sum_{m=0}^{N-1} x[m] e^{-j\frac{2\pi k(m+l)}{N}}$$

$$= \sum_{m=0}^{N-1} x[m] e^{-j\frac{2\pi k m}{N}} \cdot e^{-j\frac{2\pi k l}{N}}$$

$$\boxed{\therefore DFT\{x((n-l))_N\} = X(k) e^{-j\frac{2\pi k l}{N}}}, \text{ and}$$ — 1 M

$$\boxed{DFT\{x((n+l))_N\} = X(k) e^{j\frac{2\pi k l}{N}}}, \text{ for } l \text{ negative.}$$

1.b) Find the 4-point DFT of the sequence, $x[n] = \cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{4}n\right)$ use linearity property. [Total - 8M]

→ $x[n] = \cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{4}n\right), \quad 0 \le n \le 3$

From linearity property we know that,

$$x[n] = a_1 x_1[n] + a_2 x_2[n] \xleftrightarrow[N]{DFT} a_1 X_1(k) + a_2 X_2(k) = X(k)$$

$\therefore$ $x_1[n] = \cos\left(\frac{\pi}{4}n\right) = [1, 0.707, 0, -0.707]$, and

$x_2[n] = \sin\left(\frac{\pi}{4}n\right) = [0, 0.707, 1, 0.707]$ — 2M

4-point DFTs of $x_1[n]$ & $x_2[n]$ can be obtained by,

$X_1(k) = [W_4]\, x_1[n]$ $\qquad$ $X_2(k) = [W_4]\, x_2[n]$

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.707 \\ 0 \\ -0.707 \end{bmatrix} \quad \& \quad \begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 0.707 \\ 1 \\ 0.707 \end{bmatrix}$$

$X_1(k) = \{1, 1-1.414j, 1, 1+1.414j\}$ & $X_2(k) = \{2.414, -1, -0.414, -1\}$

— 2M $\qquad$ — 2M

$\therefore X(k) = X_1(k) + X_2(k) = \{3.414, -1.414j, 0.586, 1.414j\}$ — ①

— 2M

And, (Verification)

$x[n] = \cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{4}n) = [1, 1.414, 1, 0]$, $X(k) = [W_4]\, x[n]$.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1.414 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore X(k) = \{3.414, -1.414j, 0.586, 1.414j\}$ — ②

DFTs in ① & ② are same $\therefore$ 4-point DFT of $x[n]$ obtained through linearity property is verified.

1.c) Consider 4-point sequences, $x[n] = \cos(\frac{\pi}{2}n);\ 0 \le n \le 3$, $h[n] = 2^n;\ 0 \le n \le 3$, compute circular convolution using concentric circle method. [Total - 6M]

$\Rightarrow$ $x[n] = \cos(\frac{\pi}{2}n) = [1, 0, -1, 0]$, & — 1M

$h[n] = 2^n = [1, 2, 4, 8]$ — 1M

$$y[m] = x[n] \text{ ④ } h[n] = \sum_{n=0}^{N-1} x[n]\, h((m-n))_N, \quad 0 \le m \le N-1$$

$$= \sum_{n=0}^{3} x[n]\, h((m-n))_4, \quad m = 0, 1, 2, 3$$

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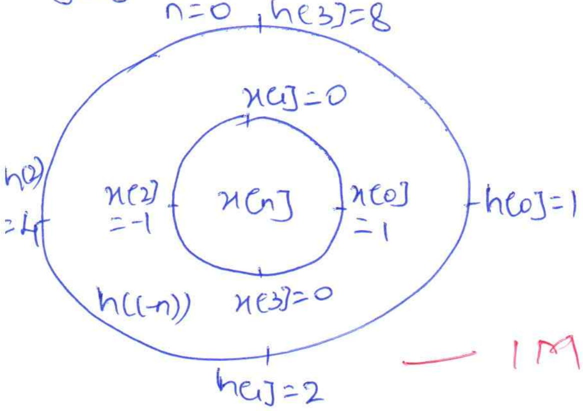
$m=0$

$$y[0] = \sum_{n=0}^{3} x[n]\, h((-n))_4$$



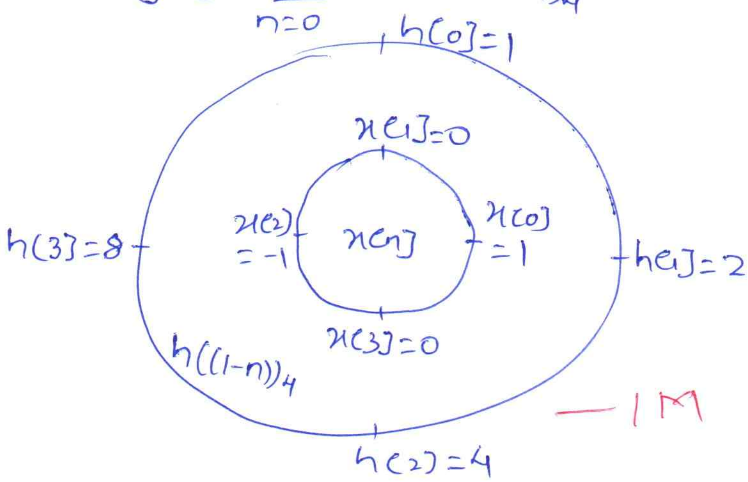
— 1M

$y[0] = 1\times1 + 0\times8 + 4\times-1 + 0\times2$

$y[0] = -3$

$m=1$

$$y[1] = \sum_{n=0}^{3} x[n]\, h((1-n))_4$$



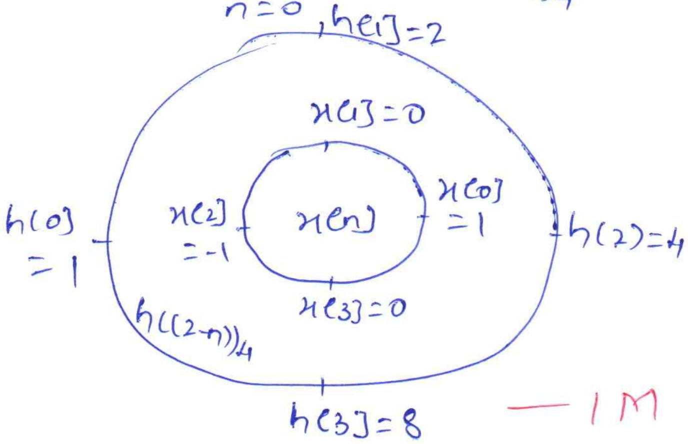
— 1M

$y[1] = 1\times2 + 0\times1 + 8\times-1 + 0\times4$

$y[1] = -6$

$m=2$

$$y[2] = \sum_{n=0}^{3} x[n]\, h((2-n))_4$$



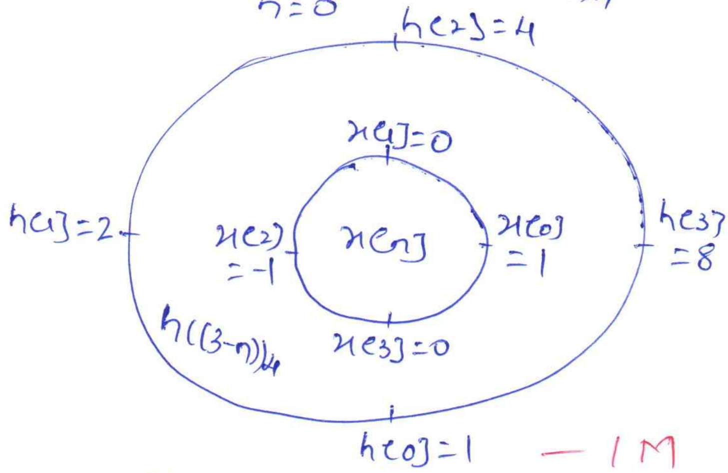
— 1M

$y[2] = 1\times4 + 0\times2 + 1\times-1 + 0\times8$

$y(2) = 3$

$m=3$

$$y[3] = \sum_{n=0}^{3} x[n]\, h((3-n))_4$$



— 1M

$y[3] = 1\times8 + 0\times4 + 2\times-1 + 0\times1$

$y[3] = 6$

$\therefore\ y[n] = x[n] \; ④ \; h[n] = [-3,\ -6,\ 3,\ 6]$

"OR"

2.a) State and prove Parseval's theorem. [Total - 6M]

→ Consider the complex valued sequences $x[n]$ & $y[n]$

Then if, $x[n] \xleftrightarrow[N]{DFT} X(k)$, and

$y[n] \xleftrightarrow[N]{DFT} Y(k)$, then

$$\sum_{n=0}^{N-1} x[n]\, y^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, Y^*(k) \quad \text{——— (1)}$$

when, $y[n] = x[n]$ then, — 2M

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2 \quad \text{——— (2)}$$

Eqⁿ (2) gives the energy of finite duration sequence interms of its frequency components.

Proof:- We know that, circular correlation is given as,

$$\tilde{r}_{xy}(l) = \sum_{n=0}^{N-1} x[n]\, y^*((n-l))_N$$

for $l=0$, $$\tilde{r}_{xy}(0) = \sum_{n=0}^{N-1} x[n]\, y^*((n))_N \quad \text{——— (3)}$$ — 1M

$$DFT\{\tilde{r}_{xy}(l)\} = X(k)\,Y^*(k)$$

$$\tilde{r}_{xy}(l) = IDFT\{X(k)\,Y^*(k)\}$$

$$\tilde{r}_{xy}(l) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\,Y^*(k)\, e^{j\frac{2\pi k l}{N}}$$ — 1M

for $l=0$, $$\tilde{r}_{xy}(0) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\,Y^*(k) \quad \text{— (4)}$$ — 1M

from, eqⁿ (3) & eqⁿ (4)

$$\therefore \boxed{\sum_{n=0}^{N-1} x[n]\, y^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, Y^*(k)}$$ — 1M

2.b) Find 6-point DFT of the sequence,

$$x[n] = n \;;\; 0 \le n \le 5$$
$$= 0 \;;\; \text{otherwise.}$$

[Total - 8M]

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$$x[n] = \begin{cases} n, & 0 \le n \le 5 \\ 0, & \text{otherwise.} \end{cases}$$

$\therefore x[n] = [0, 1, 2, 3, 4, 5], \quad N = 6$ — 2M

we know that, $W_N = e^{-j\frac{2\pi}{N}}$

for $N = 6$, $W_6 = e^{-j\frac{\pi}{3}}$ $\therefore W_6^0 = 1$, $W_6^1 = 0.5 - 0.86j$, $W_6^2 = -0.5 - 0.86j$

$W_6^3 = -1$, $W_6^4 = -0.5 + 0.86j$, $W_6^5 = 0.5 + 0.86j$

$$W_6 = \begin{bmatrix} W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 & W_6^1 & W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 & W_6^2 & W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 & W_6^3 & W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 & W_6^4 & W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 & W_6^5 & W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix}_{6\times 6}$$

$W_6^0 = W_6^6 = W_6^{12} = W_6^{18} \ldots$

$W_6^1 = W_6^7 = W_6^{13} = W_6^{19} \ldots$

$W_6^2 = W_6^8 = W_6^{14} = W_6^{20} \ldots$

$W_6^3 = W_6^9 = W_6^{15} = W_6^{21} \ldots$

$W_6^4 = W_6^{10} = W_6^{16} = W_6^{22} \ldots$

$W_6^5 = W_6^{11} = W_6^{17} = W_6^{23} \ldots$

$$\therefore W_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5-0.86j & -0.5-0.86j & -1 & -0.5+0.86j & 0.5+0.86j \\ 1 & -0.5-0.86j & -0.5+0.86j & 1 & -0.5-0.86j & -0.5+0.86j \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5+0.86j & -0.5-0.86j & 1 & -0.5+0.86j & -0.5-0.86j \\ 1 & 0.5+0.86j & -0.5+0.86j & -1 & -0.5-0.86j & 0.5-0.86j \end{bmatrix}_{6\times 6}$$ — 2M

$\therefore X_6 = [W_6]\, x_6$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5-0.86j & -0.5-0.86j & -1 & -0.5+0.86j & 0.5+0.86j \\ 1 & -0.5-0.86j & -0.5+0.86j & 1 & -0.5-0.86j & -0.5+0.86j \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5+0.86j & -0.5-0.86j & 1 & -0.5+0.86j & -0.5-0.86j \\ 1 & 0.5+0.86j & -0.5+0.86j & -1 & -0.5-0.86j & 0.5-0.86j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

— 2M

$$\therefore \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 15 \\ -3+5.19j \\ -3+1.73j \\ -3 \\ -3-1.73j \\ -3-5.19j \end{bmatrix}$$ — 2M

$\therefore X(k) = \{15, -3+5.19j, -3+1.73j, -3, -3-1.73j, -3-5.19j\}$.

2.c) Find the IDFT of the DFT, $X(k) = \{6, -2+2j, -2, -2-2j\}$

[Total - 6M]

→ $X(k) = \{6, -2+2j, -2, -2-2j\}$

$x_N = \frac{1}{N}[W_N^*]X_N$, $N = 4$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$\therefore x_4 = \frac{1}{4}[W_4^*]X_4$

and

$$W_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$ — 1M

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$ — 2M

$$= \frac{1}{4} \begin{bmatrix} 6 - 2 + \cancel{2j} - 2 - 2 - \cancel{2j} \\ 6 - \cancel{2j} - \cancel{2} + \cancel{2} + \cancel{2j} - 2 \\ 6 + 2 - \cancel{2j} - \cancel{2} + \cancel{2} + \cancel{2j} \\ 6 + \cancel{2j} + 2 + 2 - \cancel{2j} + 2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0 \\ 4 \\ 8 \\ 12 \end{bmatrix}$$ — 2M

$$\boxed{\therefore x[n] = [0, 1, 2, 3]}$$ — 1M

"Module - 2"

3.a) Consider a FIR filter whose impulse response, $h[n]=[3, 2, 1, 1]$ if the input is $x[n]=[1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1]$, find the output using overlap add method assuming the length of block as 7. [Total - 10M]

→ $N=7$, and $M=4$ $\therefore$ $N=M+L-1$

$\therefore L=4$ — 1M

Taking $L=4$ samples from the input and appending $M-1=3$ zeros we form the input block & appending $L-1$ zeros to impulse response we get,

$h[n]=[3, 2, 1, 1, 0, 0, 0]$, $x_1[n]=[1, 2, 3, 3, 0, 0, 0]$

$x_2[n]=[2, 1, -1, -2, 0, 0, 0]$, $x_3[n]=[-3, 5, 6, -1, 0, 0, 0]$

$x_4[n]=[2, 0, 2, 1, 0, 0, 0]$ — 4M

Calculating 7-point circular convolution by matrix method.

$y_1[n]=h[n] \circledast_7 x_1[n]$

$$y_1[n] = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [3, 8, 14, 18, 11, 6, 3]$$

— 2M

$y_2[n]=h[n] \circledast_7 x_2[n]$

$$y_2[n] = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [6, 7, 1, -5, -4, -3, -2]$$

$y_3[n] = h[n] \circledast_7 x_3[n]$

$$y_3[n] = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \\ 6 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [-9, 9, 25, 11, 9, 5, -1]$$

— 1M

$y_4[n] = h[n] \circledast_7 x_4[n]$

$$y_4[n] = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [6, 4, 8, 9, 4, 3, 1]$$

Overlapping and adding last M-1 samples with first M-1 samples of output segments we obtain the output.

$y_1[n] = [3, 8, 14, 18, 11, 6, 3]$

$y_2[n] = [6, 7, 1, -5, -4, -3, -2]$ — 2M

$y_3[n] = [-9, 9, 25, 11, 9, 5, -1]$

$y_4[n] = [6, 4, 8, 9, 4, 3, 1]$

$y[n] = [3, 8, 14, 18, 17, 13, 4, -5, -13, 6, 23, 11, 15, 9, 7, 9, 4, 3, 1]$

3.b) Develop Radix-2 DIT-FFT algorithm and draw complete signal flow graph for N=8. [Total - 10M]

→ Radix-2 DIT-FFT :-

N-point DFT X(k) can be obtained from $F_1(k)$ & $F_2(k)$ two $\frac{N}{2}$-point DFTs.

Consider, for $N=8$

$x[n] = [x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]]$

$f_1[n] = x[2n] = [x[0], x[2], x[4], x[6]]$ — 2M

$f_2[n] = x[2n+1] = [x[1], x[3], x[5], x[7]]$ $\quad 0 \le n \le \frac{N}{2}-1$

$$\therefore X(k) = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \sum_{n=0}^{N/2-1} x[2n] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] W_N^{k(2n+1)}$$

$$\therefore X(k) = \sum_{n=0}^{N/2-1} f_1[n] W_{N/2}^{kn} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} f_2[n] W_{N/2}^{kn}$$

— 2M

$$\boxed{\begin{aligned} X(k) &= F_1(k) + W_N^k F_2(k) \\ X(k+\tfrac{N}{2}) &= F_1(k) - W_N^k F_2(k) \end{aligned}}$$

$0 \le k \le \frac{N}{2}-1$

and due to periodicity of DFT by $\frac{N}{2}$ samples we can write

Similarly,

$f_1[n]$ & $f_2[n]$ will be split into even numbered & odd numbered samples as,

$V_{11}[n] = f_1[2n]$ and $V_{21}[n] = f_2[2n]$

$V_{12}[n] = f_1[2n+1]$ $\quad V_{22}[n] = f_2[2n+1]$ $\quad 0 \le n \le \frac{N}{4}-1$ — 2M

$\therefore$ their DFTs can be written as,

$F_1(k) = V_{11}(k) + W_{N/2}^k V_{12}(k)$ and $F_2(k) = V_{21}(k) + W_{N/2}^k V_{22}(k)$

$F_1(k+\frac{N}{4}) = V_{11}(k) - W_{N/2}^k V_{12}(k)$ $\quad F_2(k+\frac{N}{4}) = V_{21}(k) - W_{N/2}^k V_{22}(k)$ $\quad 0 \le k \le \frac{N}{4}-1$

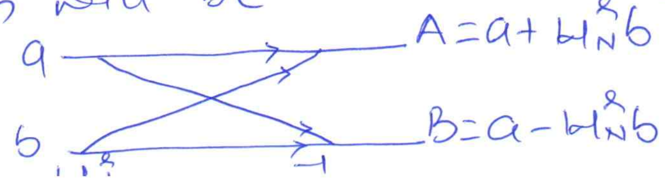
and lastly,

$V_{11}[n] = f_1[2n] = [x[0], x[4]]$

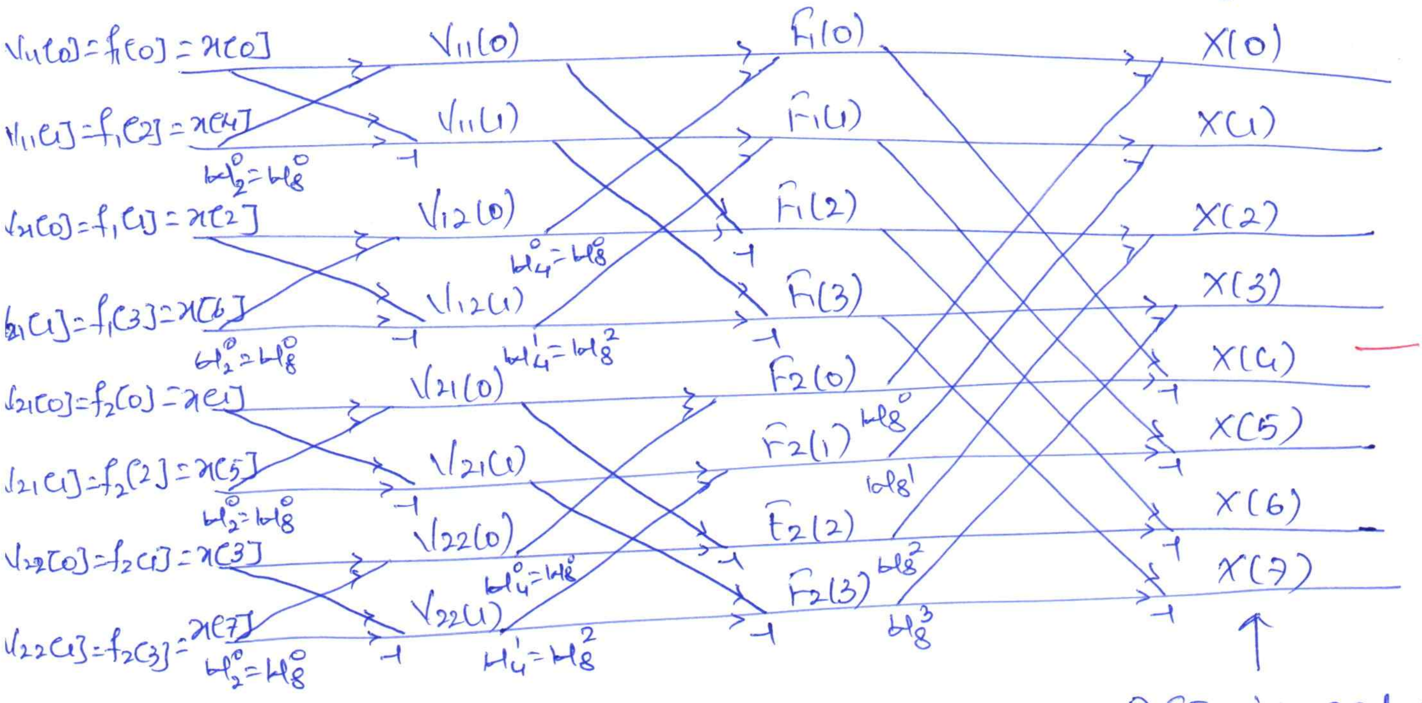
$$\therefore V_{11}(k) = \sum_{n=0}^{1} V_{11}[n] W_2^{kn}, \quad 0 \le k \le 1$$

$k=0, \quad V_{11}(0) = V_{11}[0] + V_{11}[1]$, $\quad k=1, \quad V_{11}(1) = V_{11}[0] + W_2^1 V_{11}[1]$
$= V_{11}[0] - W_2^0 V_{11}[1]$

$\therefore$ Generalization will be



— 1M



— 3M

Shuffled array in bit reversed order

DFT in natural sequence.

"OR"

4. a) Find the output $y[n]$ of a filter whose impulse response is, $h[n] = [1, 1, 1]$ and the input signal to the filter is $x[n] = [3, -1, 0, 1, 3, 2, 0, 1, 2, 1]$. Using overlap save method [Total – 10M]

→ $M = 3$, $N = 2^m = 2^3 = 8$ $\therefore L = 6$ — 1M

$h[n] = [1, 1, 1, 0, 0, 0, 0, 0]$ — 1M

for overlap save method input blocks first M-1 zeros will be included for first segment & L samples will be taken from input & for remaining segments first M-1 samples will be taken from the last M-1 samples of previous segments

$\therefore x_1[n] = [0, 0, 3, -1, 0, 1, 3, 2]$

$x_2[n] = [3, 2, 0, 1, 2, 1, 0, 0]$ — 2M

Calculating 8-point circular convolution by matrix method

$y_1[n] = h[n] \circledast_8 x_1[n]$

$$y_1[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} = [5, 2, 3, 2, 2, 0, 4, 6]$$

— 2M

$y_2[n] = h[n] \circledast_8 x_2[n]$

$$y_2[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = [3, 5, 5, 3, 3, 4, 3, 1]$$

— 2M

To obtain the output $y[n]$ first $M-1$ samples from each output segment will be discarded. — 1M

$\therefore y[n] = [3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1]$ — 1M

4.b) First five points of the Eight point DFT of a real valued sequence is given by,
$X(0)=0$, $X(1)=2+2j$, $X(2)=-j4$, $X(3)=2-2j$, $X(4)=0$.
Determine the remaining points. Hence find the original sequence $x[n]$ using Decimation in frequency FFT algorithm
[Total - 10M]

→ $X(0)=0$, $X(1)=2+2j$, $X(2)=-j4$, $X(3)=2-2j$, $X(4)=0$

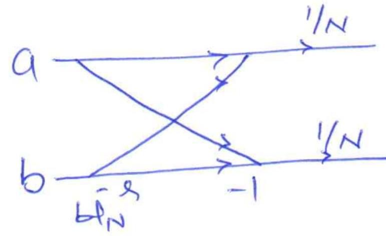
for real valued sequence by symmetry property we know that,
$X(N-k) = X^*(k)$ or $X(k) = X^*(N-k)$

— 2M

$\therefore X(5) = X^*(3) = 2+2j$, $X(6) = X^*(2) = 4j$ & $X(7) = X^*(1) = 2-2j$

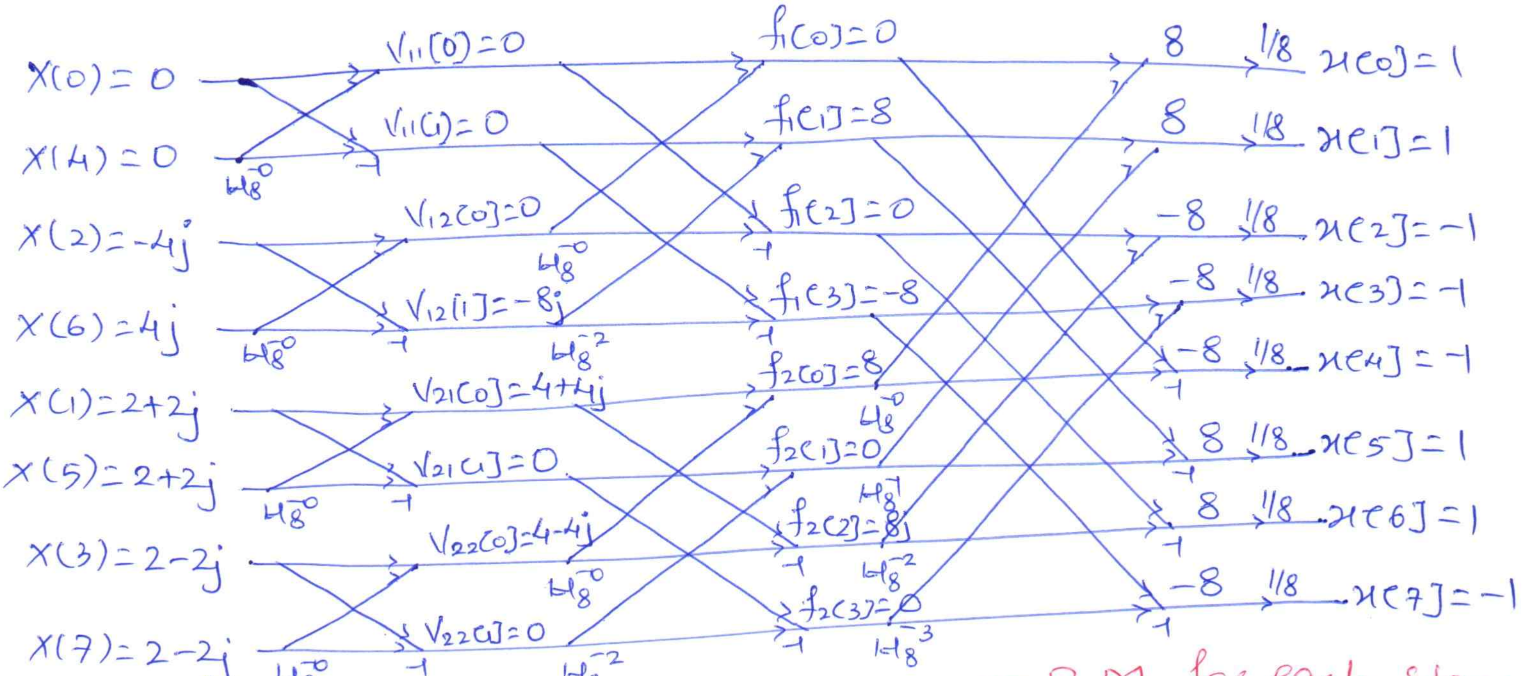
$\therefore X(k) = \{0,\ 2+2j,\ -j4,\ 2-2j,\ 0,\ 2+2j,\ j4,\ 2-2j\}$

Inverse DIF-FFT.



$A = \frac{1}{N}\{a + W_N^{-r} b\}$ — 1M

$B = \frac{1}{N}\{a - W_N^{-r} b\}$.



— 2M for each stage

$\therefore x[n] = [1,\ 1,\ -1,\ -1,\ -1,\ 1,\ 1,\ -1]$. — 1M

"Module - 3"

5.a) List the different types of windowing techniques used in the design of FIR filters. Write the analytical equations, draw the magnitude response and show the largest side lobe value below the dc magnitude. [Total - 8M]

→ Window Functions :-

1) Rectangular Window

2) Bartlett Window (Triangular Window)

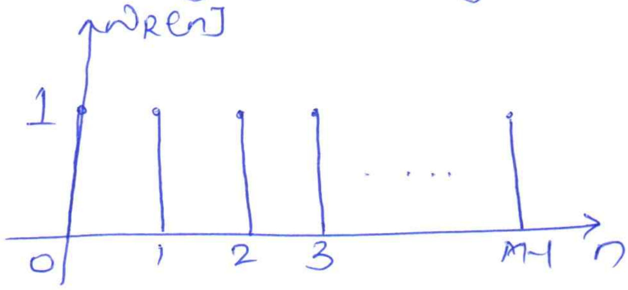
3) Hamming Window

4) Hanning Window

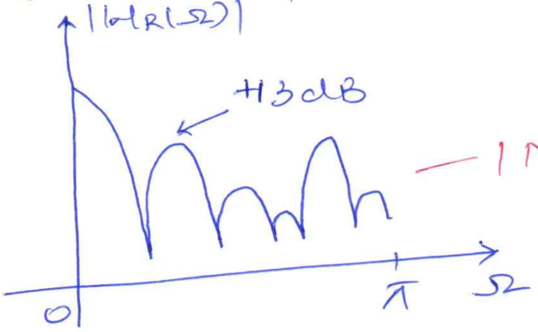
1) Rectangular Window:-

The rectangular window of length 'M' is given as,

$$w_R[n] = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$ — 1M



Magnitude Response



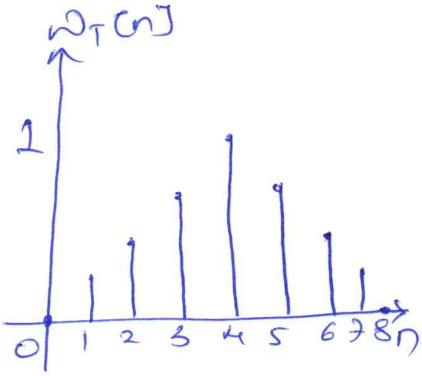
— 1M

Largest side lobe value below the dc magnitude for rectangular window is +13 dB.

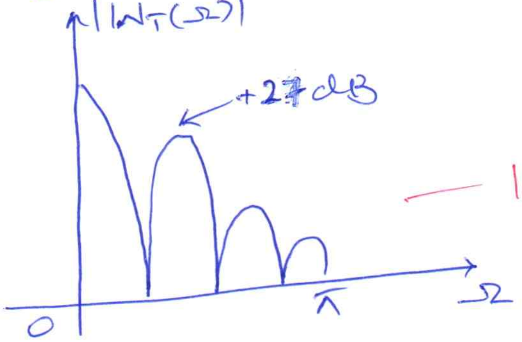
2) Bartlett Window:-

Bartlett window is defined as,

$$w_T[n] = \begin{cases} 1 - \dfrac{2\left|n - \frac{M-1}{2}\right|}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$ — 1M



Magnitude Response



— 1M

Largest side lobe value below the dc magnitude for Bartlett window is +27 dB.

3) Hamming Window:-

Hamming window is defined as,

$$w_H[n] = \begin{cases} 0.54 - 0.46\cos\dfrac{2\pi n}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$ — 1M

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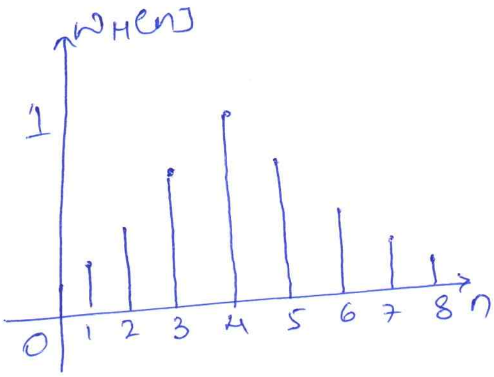
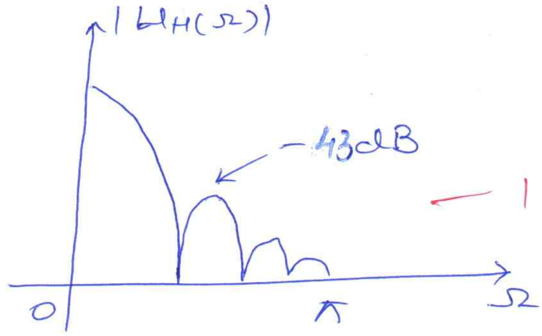




— 1M

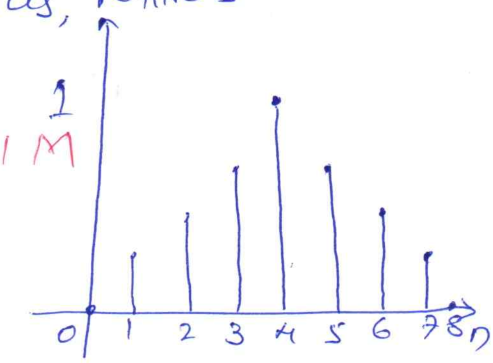
Largest side lobe value below the dc magnitude of Hamming window is -43 dB.

4> Hanning Window:-

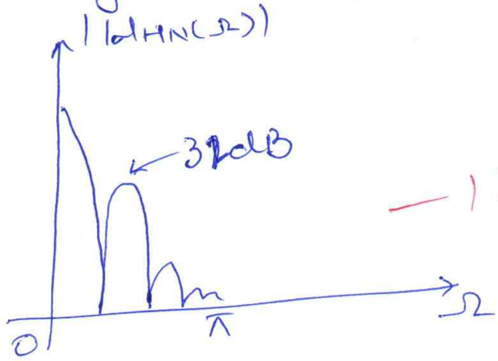
Hanning window is defined as,

$$w_{HN}[n] = \begin{cases} \frac{1}{2}\left(1-\cos\frac{2\pi n}{M-1}\right), & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

— 1M



Magnitude response



— 1M

Largest side lobe value below the dc magnitude for Hanning window is -32dB.

5.b> The frequency response of an FIR filter is given by,

$$H(\omega) = e^{-j3\omega}(1 + 1.8\cos 3\omega + 1.2\cos 2\omega + 0.5\cos\omega)$$

Determine the coefficient of the impulse response $h[n]$ of the FIR filter. [Total - 6M]

$\rightarrow$ $H(\omega) = e^{-j3\omega}(1 + 1.8\cos 3\omega + 1.2\cos 2\omega + 0.5\cos\omega)$

From frequency response, $\alpha = \frac{N-1}{2} = 3$ $\therefore$ $N = 7$ — 1M

— 1M

for N equals odd value, the frequency response of the center symmetric FIR filter is given by,

$$H(w) = e^{-jw\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h[n]\cos\left[w\left(n-\left(\frac{N-1}{2}\right)\right)\right]\right)$$

$$H(w) = e^{-j3w}\left(h(3) + \sum_{n=0}^{2} 2h[n]\cos[w(n-3)]\right)$$

$$\therefore H(w) = e^{-j3w}[h(3) + 2h(0)\cos 3w + 2h(1)\cos 2w + 2h(2)\cos w]$$ — 1 M

Comparing this with the given frequency response,

$h(3)=1$, $h(0)=0.9$, $h(1)=0.6$ & $h(2)=0.25$. — 1 M

Since FIR filter has linear phase,

$h[n] = h[N-1-n]$ — 1 M

$\therefore$ $h[n] = h[6-n]$, $0 \le n \le 6$

$h[0] = h[6] = 0.9$

$h[1] = h[5] = 0.6$ — 1 M

$h[2] = h[4] = 0.25$

$h[3] = 1$.

5.c) Determine the coefficient K_m of the lattice filter corresponding to FIR filter described by the system function $H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$. And also draw the Lattice structure.

[Total - 6M]

$\rightarrow$ $H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$.

$K_m = a_m(m)$

$a_2(2) = \frac{1}{3}$ and $a_2(1) = 2$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2}$$

$1 \le i \le m-1$

for, $m=2$,

$K_2 = a_2(2) = \frac{1}{3}$ — 2M

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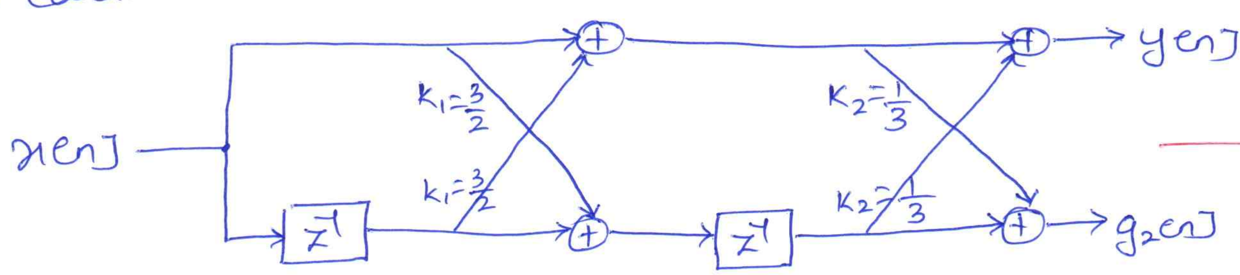
for, $m=1$, $K_1 = a_1(1)$

$$\therefore a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} = \frac{a_2(1)[1-a_2(2)]}{1-a_2^2(2)}$$

$$a_1(1) = \frac{a_2(1)}{1+a_2(2)} = \frac{2}{1+\frac{1}{3}} = \frac{6}{4}$$

$$\therefore K_1 = a_1(1) = \frac{3}{2} \quad \text{— 2M}$$

$\therefore$ Lattice structure



— 2M

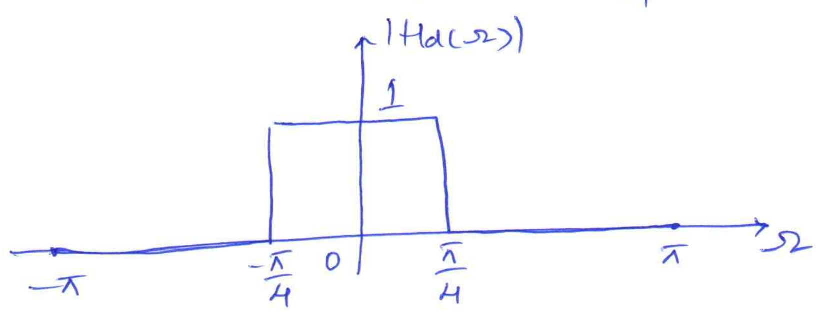
"OR"

6.a) Determine the filter coefficients $h_d[n]$ for the desired frequency response of a lowpass filter is given by,

$$H_d(\omega) = \begin{cases} e^{-j2\omega} & ; -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & ; \frac{\pi}{4} \le \omega \le \pi \end{cases}$$

Find $h[n]$ and also frequency response $H(\omega)$ using Hamming window. [Total - 10M]

$$H_d(\omega) = \begin{cases} e^{-j2\omega} & , -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & , \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$



$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n}\, d\omega$$

— 1M

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$$h_d[n] = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2\omega} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)} d\omega$$

$$\therefore h_d[n] = \frac{1}{2\pi}\left[\frac{e^{j\omega(n-2)}}{j(n-2)}\right]_{-\frac{\pi}{4}}^{\pi/4} \quad \text{— 1M}$$

$$\therefore h_d[n] = \begin{cases} \dfrac{\sin\frac{\pi(n-2)}{4}}{\pi(n-2)} & \text{for } n \neq 2 \\ \dfrac{1}{4} & \text{for } n = 2 \end{cases} \quad \text{— 1M}$$

$$\therefore h_d[n] = [0.1591, 0.2250, 0.25, 0.2250, 0.1591] \quad \text{— 1M}$$

Hamming window,

$$w_H[n] = \begin{cases} 0.54 - 0.46\cos\frac{2\pi n}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases} \quad \text{— 1M}$$

$$\therefore w_H[n] = 0.54 - 0.46\cos\frac{2\pi n}{4}, \quad 0 \le n \le 4$$

$$w_H[n] = [0.08, 0.54, 1, 0.54, 0.08] \quad \text{— 1M}$$

$$h[n] = h_d[n]w[n]$$

$$\therefore \boxed{h[n] = [0.012, 0.121, 0.25, 0.121, 0.012]} \quad \text{— 1M}$$

Frequency response,

$$H(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)}\left\{h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h[n]\cos\omega\left(n - \frac{M-1}{2}\right)\right\} \quad \text{— 1M}$$

M=5,

$$H(\omega) = e^{-j2\omega}\left(h(2) + 2\sum_{n=0}^{1} h[n]\cos\omega(n-2)\right)$$

$$H(\omega) = e^{-j2\omega}[0.25 + 2\times 0.012\cos 2\omega + 2\times 0.121\cos\omega]$$

$$\therefore \boxed{H(\omega) = e^{-j2\omega}[0.25 + 0.024\cos 2\omega + 0.242\cos\omega]} \quad \text{— 2M}$$

6.b> Obtain the cascade form realization of system function: 11

$$H(z) = 1 + 5z^{-1} + 2z^{-2} + 2z^{-3}$$ [Total - 5M]

→ $H(z) = 1 + 5z^{-1} + 2z^{-2} + 2z^{-3}$

Given system function is of order 3. So, it has to be divided into 2 FIR systems one of 2nd order & another of 1st order.

$$\therefore H(z) = \frac{z^3 + 5z^2 + 2z + 2}{z^3}$$

$$H(z) = \frac{(z+4.66)}{z} \frac{(z+0.16-0.63j)(z+0.16+0.63j)}{z^2}$$ — 1M

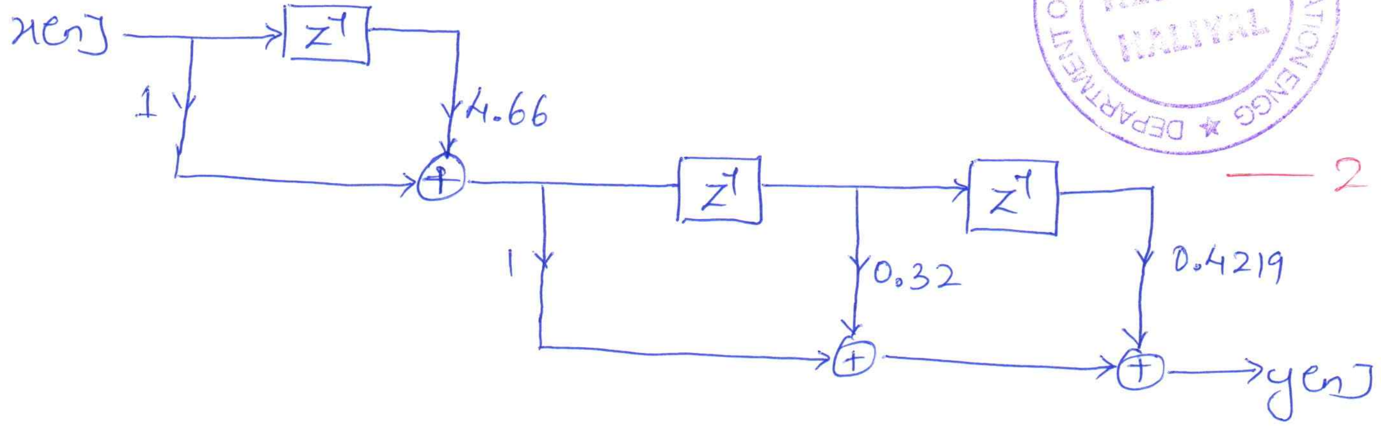
Complex conjugate roots will be combined to have real coefficient values.

$$\therefore H(z) = \frac{(z+4.66)}{z} \frac{(z^2+0.32z+0.4219)}{z^2}$$

$$H(z) = (1+4.66z^{-1})(1+0.32z^{-1}+0.4219z^{-2})$$ — 2M

$$H(z) = H_1(z)\,H_2(z)$$

Cascade form realization:-



— 2M

6.c> Realize the following function in Direct form.

$$H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + z^{-2}\right)$$

[Total - 5M]

→ $H(z) = \left(1 + \frac{1}{2} z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4} z^{-1} + z^{-2}\right)$

$\frac{Y(z)}{X(z)} = H(z) = 1 + 0.75 z^{-1} + 2.125 z^{-2} + 0.75 z^{-3} + z^{-4}$

— 2M

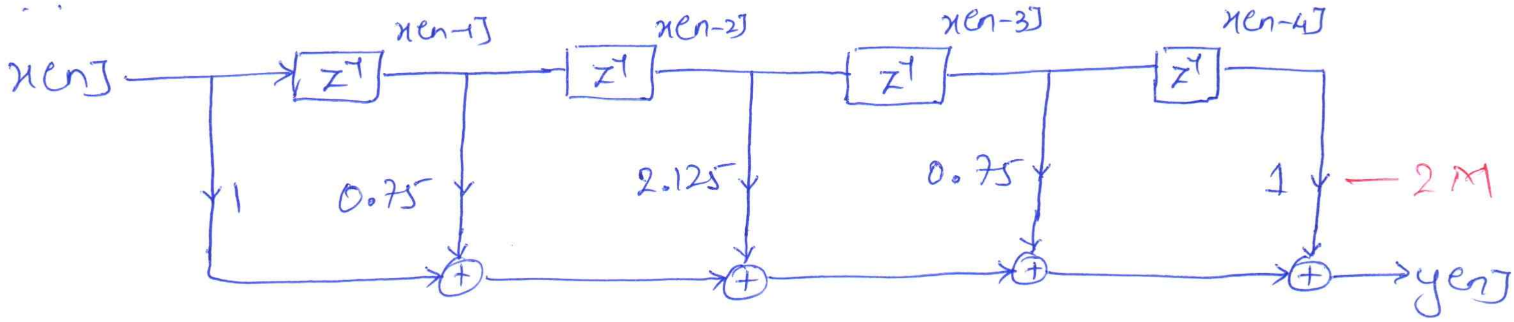
$\therefore Y(z) = X(z) + 0.75 z^{-1} X(z) + 2.125 z^{-2} X(z) + 0.75 z^{-3} X(z) + z^{-4} X(z)$.

taking inverse Z-transform.

$y[n] = x[n] + 0.75 x[n-1] + 2.125 x[n-2] + 0.75 x[n-3] + x[n-4]$

— 1M

$\therefore$

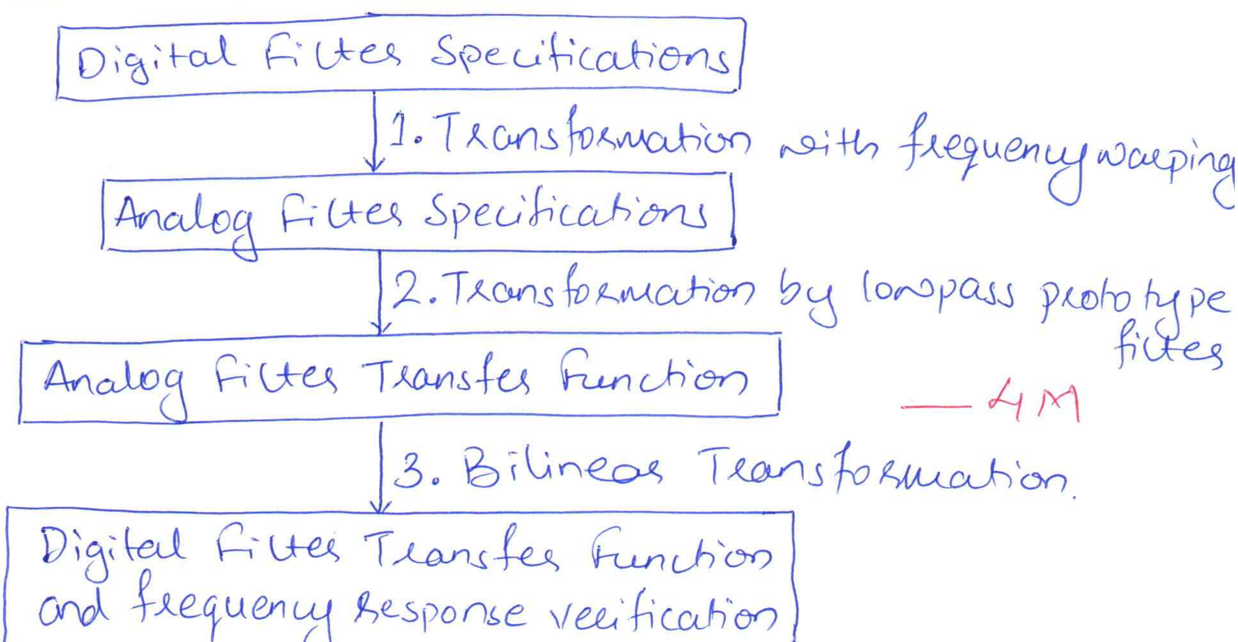


— 2M

"Module – 4"

7.a) Discuss the general procedure for IIR filter design using Bilinear transformation. [Total – 6M]

→ Bilinear Transformation:-



— 4M

Fig 7.a
General Procedure for IIR filter design using Bilinear Transformation.

The design procedure includes the following steps:-

1) Transforming digital filter specifications into analog filter specifications.

2) Performing analog filter design, and — 2M

3) Applying bilinear transformation and verifying its frequency response.

7.b) An analog filter is given by $Ha(s) = \frac{s+0.1}{(s+0.1)^2+16}$.
Obtain digital IIR filter using bilinear transformation method. Digital filter is to have resonant frequency $\omega_r = \frac{\pi}{2}$ radians. [Total - 8M]

→ $$Ha(s) = \frac{s+0.1}{(s+0.1)^2+16}$$

Poles,

$$(s+0.1)^2+16 = s^2+0.2s+16.01 = (s+0.1-4j)(s+0.1+4j)$$

$$\therefore s = -0.1+4j \quad \& \quad s = -0.1-4j$$

$$\therefore \sigma = -0.1 \quad \& \quad \Omega = \pm 4 \qquad \text{— 2M}$$

Bilinear Transformation should map $\Omega = 4$ into $\omega_r = \frac{\pi}{2}$

$$\therefore \Omega = \frac{2}{T}\tan\frac{\omega}{2} \quad \text{or} \quad T = \frac{2}{\Omega}\tan\frac{\omega}{2}$$

for $\Omega = 4$ & $\omega_r = \frac{\pi}{2}$

$$T = \frac{2}{4}\tan\frac{\pi}{4} = \frac{1}{2} \qquad \text{— 1M}$$

for, $T = \frac{1}{2}$ the resonant frequency $\Omega = 4$ of analog filter will map into $\omega_r = \frac{\pi}{2}$ of digital filter in bilinear transformation

$\therefore$ Bilinear Transformation, $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ — 1M

At, $T = \frac{1}{2}$, $S = 4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ — 1M

$$\therefore H(z) = H_a(s)\Big|_{s=4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1}{\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1\right]^2+16}$$ — 2M

$$\therefore H(z) = \frac{0.128 + 0.006z^{-1} - 0.122z^{-2}}{1 + 0.0006z^{-1} + 0.975z^{-2}}$$

or

$$\boxed{H(z) = \frac{0.128z^2 + 0.006z - 0.122}{z^2 + 0.0006z + 0.975}}$$ — 1M

Poles of $H(z)$, $z = -0.0003 \pm j0.9874$ in polar form can be written as,

$$z = 0.9874\, e^{\pm j1.5711001} \qquad (z = re^{j\omega})$$

here, $r = 0.9874$ & $\omega = \pm 1.5711001 \simeq \pm\frac{\pi}{2}$

Conjugate poles are located at $\omega = \pm\frac{\pi}{2}$. Hence, $H(z)$ will Resonant at $\omega = \frac{\pi}{2}$.

7. c) Compare FIR and IIR filters. [Total - 6M]

IIR Filters	FIR filters
1) Impulse Response is of infinite duration.	1) Impulse Response is of finite duration.
2) IIR filters use feedback, hence recursive filters	2) FIR filters do not use feedback, hence non-recursive.
3) Non-linear phase	3) Linear Phase
4) Filters are designed to be stable	4) Inherently stable systems.

— 4M

IIR filters	FIR filters
5) IIR filters are complex to design & implement	5) FIR filters are simple to design & implement. — 2M
6) Require lower order systems to get same magnitude response.	6) Require higher order system to get same magnitude response.

"OR"

8.a) Design a Butterworth digital low pass filter with the following specifications.

i) 3 dB attenuation at the passband frequency of 1.5 KHz

ii) 10 dB stopband attenuation at the frequency of 3 KHz

iii) Sampling frequency of 8000 Hz. [Total - 10M]

→ Digital frequencies in radians per second:

$$\omega_{dp} = 2\pi f = 2\pi(1500) = 3000\pi \text{ rad/sec}$$

$$\omega_{ds} = 2\pi f = 2\pi(3000) = 6000\pi \text{ rad/sec}$$ — 2M

$$T = \frac{1}{f_s} = \frac{1}{8000} \text{ sec}$$ — 1M

i) Applying the warping equations,

$$\omega_{ap} = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right) = 16000 \times \tan\left(\frac{3000\pi/8000}{2}\right) = 1.069 \times 10^4 \text{ rad/sec}$$

$$\omega_{as} = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right) = 16000 \times \tan\left(\frac{6000\pi/8000}{2}\right) = 3.862 \times 10^4 \text{ rad/sec}$$

Lowpass prototype specifications,

$$V_s = \frac{\omega_{as}}{\omega_{ap}} = \frac{3.862 \times 10^4}{1.069 \times 10^4} = 3.613 \text{ rad/sec}$$

$$A_s = 10 \text{ dB}$$

Filter order is computed by, $\varepsilon^2 = 10^{0.1 \times A_p} - 1 = 10^{0.1 \times 3} - 1$

$$\therefore \varepsilon^2 = 1$$

$$n = \frac{\log_{10}\left(\frac{10^{0.1\times A_s}-1}{\varepsilon^2}\right)}{\left[2.\log_{10}(V_s)\right]} = \frac{\log_{10}\left(10^{0.1\times 10}-1\right)}{2\log_{10}(3.613)} = 0.8553 \simeq 1 \quad \text{— 2M}$$

ii) We choose $n=1$ for the lowpass prototype.

$$H_p(s) = \frac{1}{s+1}$$

Applying prototype transformation,

$$H(s) = H_p(s)\Big|_{s=\frac{s}{\Omega_{ap}}} = \frac{1}{\frac{s}{\Omega_{ap}}+1} = \frac{\Omega_{ap}}{s+\Omega_{ap}}$$

$$\therefore H(s) = \frac{1.0691\times 10^4}{s+1.069\times 10^4} \quad \text{— 2M}$$

iii) Using bilinear transformation.

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{1.069\times 10^4}{s+1.069\times 10^4}\Bigg|_{s=\frac{16000(z-1)}{z+1}}$$

$$H(z) = \frac{1.069\times 10^4}{\frac{16000(z-1)}{z+1}+1.069\times 10^4}$$; dividing by 16000

— 2M

$$H(z) = \frac{0.6682}{\frac{z-1}{z+1}+0.6682} = \frac{0.6682z+0.6682}{1.6682z-0.3318}$$

dividing numerator & denominator by $1.6682z$.

$$\boxed{H(z) = \frac{0.4006+0.4006z^{-1}}{1-0.1989z^{-1}}} \quad \text{— 1M}$$

8.b) A system is represented by a transfer function $H(z)$ is given by $H(z) = 1 + \frac{4z}{z-\frac{1}{2}} - \frac{2}{z-\frac{1}{4}}$.

i) Does this $H(z)$ represent a FIR or IIR filter? Why?

ii) Draw direct form-I & direct form-II realization by showing all differences equations? [Total - 10M]

$$H(z) = 1 + \frac{4z}{z-\frac{1}{2}} - \frac{2}{z-\frac{1}{4}}$$

$$\therefore H(z) = \frac{5z^2 - 3.75z + 1.125}{z^2 - 0.75z + 0.125}$$

i) IIR filter, because of denominator polynomial which constitute for feedback terms (recursive systems). — 1M

ii) $$H(z) = \frac{Y(z)}{X(z)} = \frac{5z^2 - 3.75z + 1.125}{z^2 - 0.75z + 0.125} \text{ or } \frac{5 - 3.75z^{-1} + 1.125z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$Y(z)[z^2 - 0.75z + 0.125] = X(z)[5z^2 - 3.75z + 1.125]$$

OR

$$Y(z)[1 - 0.75z^{-1} + 0.125z^{-2}] = X(z)[5 - 3.75z^{-1} + 1.125z^{-2}].$$

$$\therefore Y(z) = 5X(z) - 3.75z^{-1}X(z) + 1.125z^{-2}X(z) + 0.75z^{-1}Y(z) + 0.125z^{-2}Y(z)$$

— 2M

taking inverse z-transform.

$$y[n] = 5x[n] - 3.75x[n-1] + 1.125x[n-2] + 0.75y[n-1] + 0.125y[n-2]$$

$\therefore b_0 = 5,\ b_1 = -3.75,\ b_2 = 1.125$

$a_1 = -0.75,\ a_2 = 0.125$ — 1M

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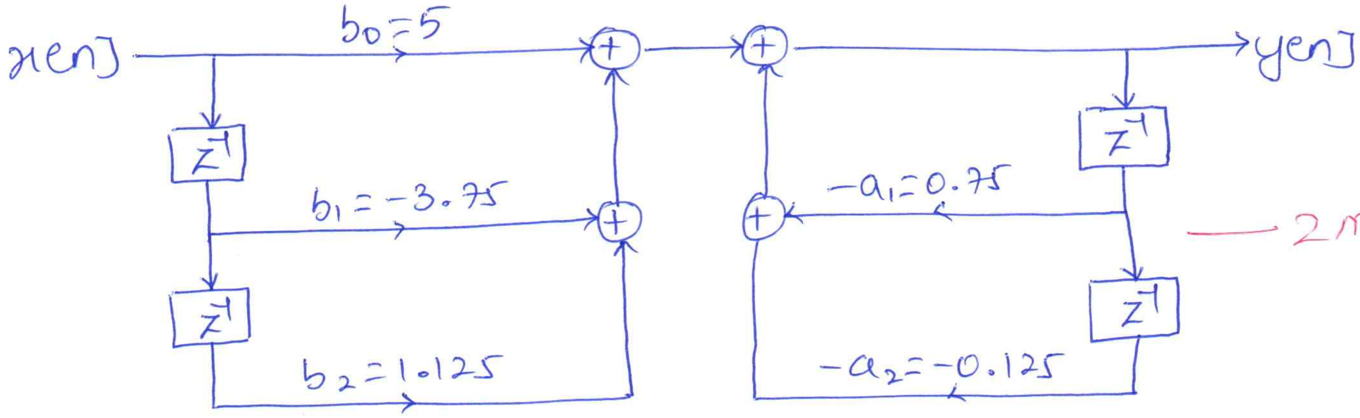
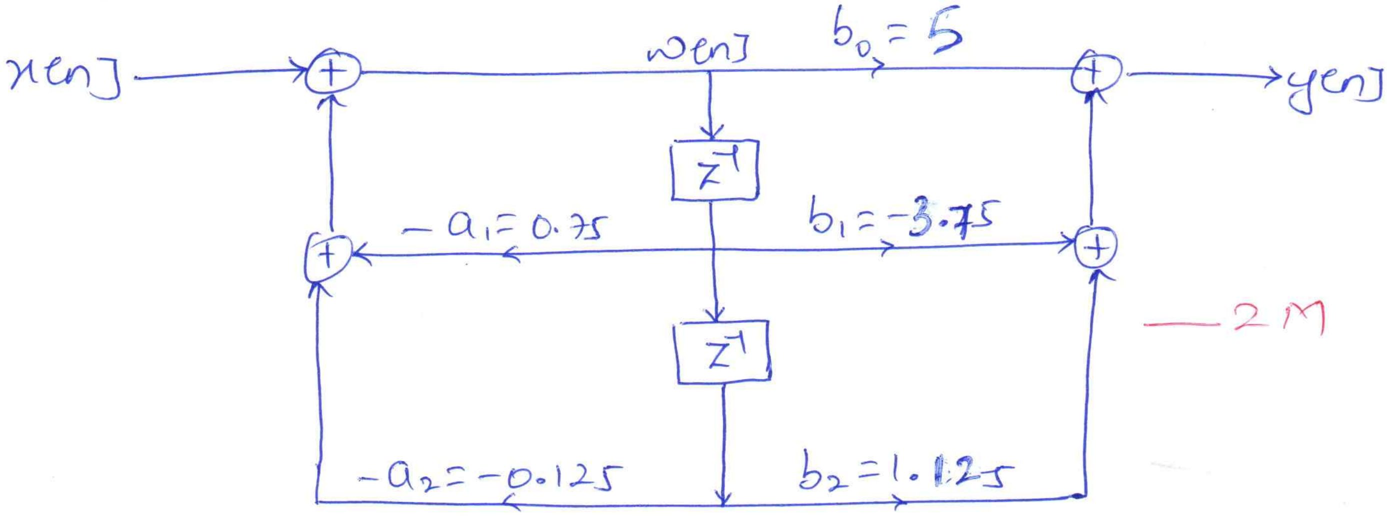
Direct Form - I



— 2M

Direct Form - II



— 2M

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\therefore \frac{W(z)}{X(z)} = \frac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$\therefore W(z) = X(z) - 0.75z^{-1}W(z) + 0.125z^{-2}W(z)$$

taking inverse Z-transform.

$$w[n] = x[n] - 0.75w[n-1] + 0.125w[n-2]$$ and

$$\frac{Y(z)}{W(z)} = 5 - 3.75z^{-1} + 1.125z^{-2}$$ — 2M

$$Y(z) = 5W(z) - 3.75z^{-1}W(z) + 1.125z^{-2}W(z)$$

taking inverse Z-transform.

$$y[n] = 5w[n] - 3.75w[n-1] + 1.125w[n-2].$$

"Module-5"

9.a) Explain IEEE floating point formats using:
i) Single precision format ii) Double precision format.
(Total - 8M)

→ i) IEEE single precision format:-

31	30 … 23	22 … 0
S	Exponent	Fraction

— 1M

$$X = (-1)^S \times (1.F) \times 2^{E-127}$$ — 1M

IEEE single precision floating-point standard representation requires 23 fraction bits F, 8 exponent bits E, and 1 sign bit S, with a total of 32 bits each word. F is mantissa in 2's complement positive binary fraction. The mantissa is within the normalized range limits between +1 & +2. The sign bit is employed to indicate the sign of the numbers. The Exponent E is in excess 127 form. The value of 127 is the offset from the 8-bit exponent range from 0 to 255, so that E-127 will range from -127 to 128. $X = (-1)^S \times (1.F) \times 2^{E-127}$ can be applied to convert IEEE 754 single precision to the decimal number. — 2M

ii) IEEE double precision format:-

31	30 … 20	19 … 0	31 … 0
S	Exponent	Fraction	Fraction
Odd Register			Even Register

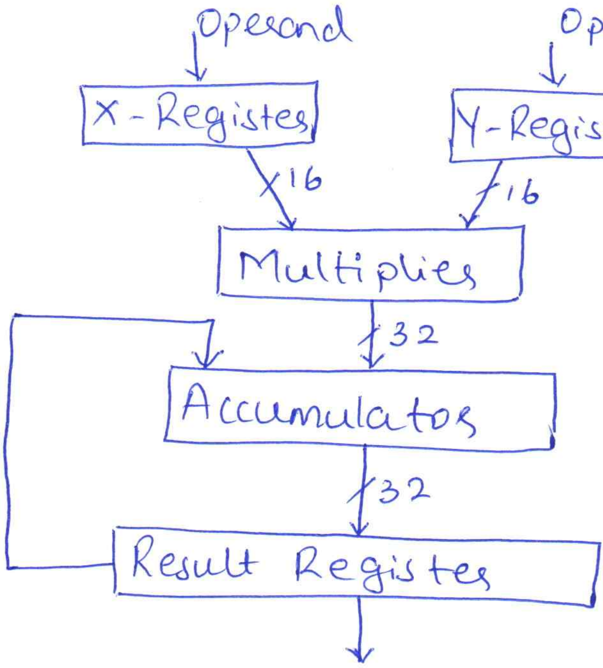
— 1M

$$X = (-1)^S \times (1.F) \times 2^{E-1023}$$ — 1M

IEEE double precision floating-point standard requires 64-bit word. The first bit is the sign bit S, the next 11-bits are the exponent bits E, and the final 52-bits are the fraction bits F. Double precision format increases the dynamic range of number representation, since there are 11 exponent bits, the double precision format also reduces the interval size in the mantissa normalized range of +1 to +2, since there are 52 mantissa bits as compared with the single precision case of 23 bits. — 2M

9.b) Discuss briefly multiplier and Accumulator unit in Digital Signal Processor hardware units. [Total - 4M]

→ Multiplier and Accumulator:-



— 2M

Multiplier & accumulator is a special hardware unit used to enhance the speed of digital filtering, instruction is generally referred to as MAC operation. Multiplier has a pair of input registers, each holding the 16-bit input to the multiplier. The result of the multiplication is accumulated in a 32-bit accumulator unit. The result register holds the double precision data from the accumulator. — 2M

9.c) Draw the block diagram of TMS320C3x floating point digital signal processor. [Total - 8M]

→ TMS320C3X:-

Floating point digital signal processor perform operations using floating-point arithmetic. The advantages of using floating point processor include getting rid of finite word length effects such as overflows, round-off errors, truncation errors, and coefficients quantization errors. Hence, shifting of input samples to perform scaling can be avoided.

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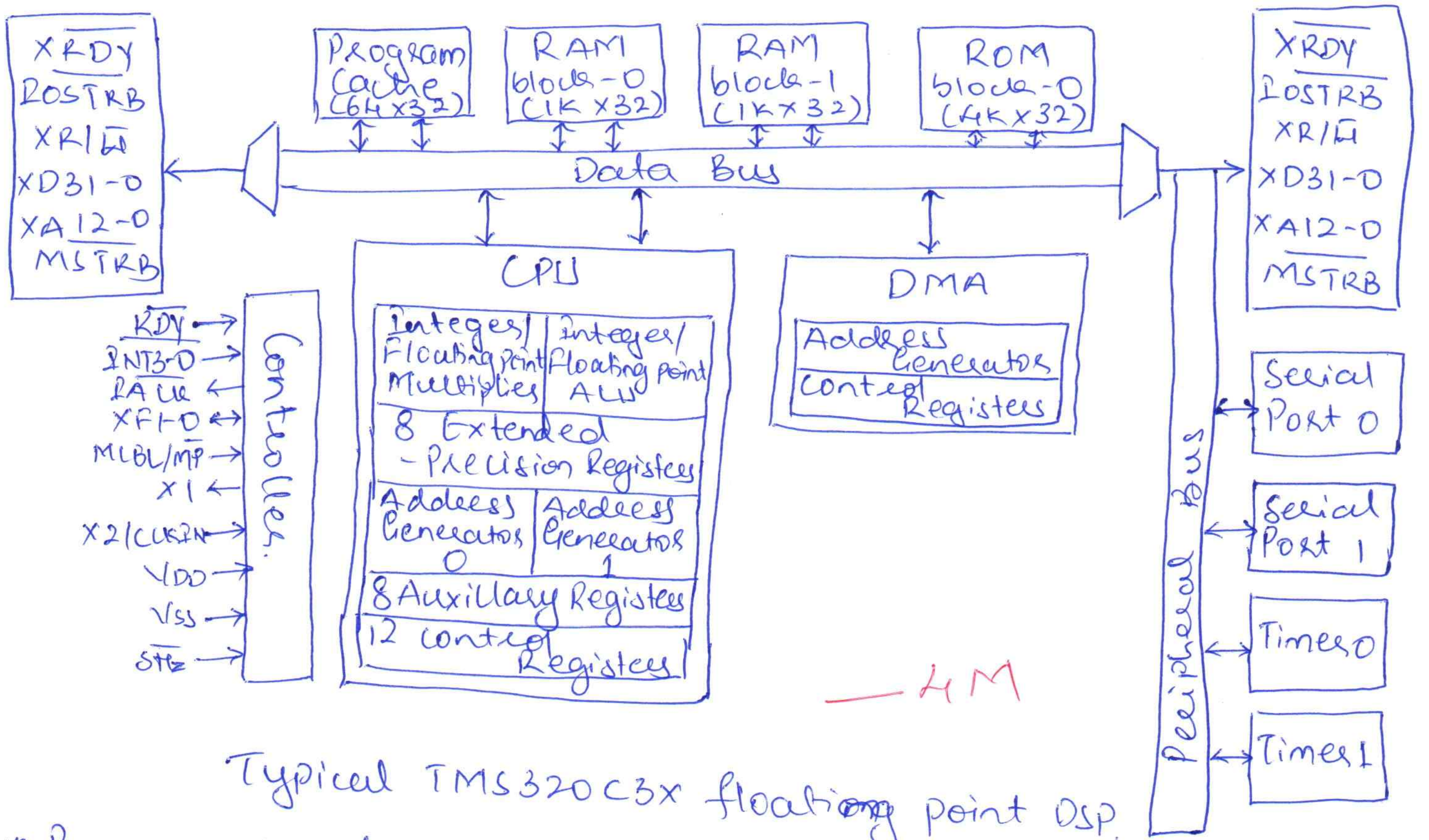
3



— 4M

Typical TMS320C3x floating point DSP.

* Processor has large memory space & equipped with dual-access on-chip memory. Program cache is used to enhance the execution of commonly used codes.
* There exist memory & data buses for DMA, concurrent I/O, CPU operations & peripheral access.
* CPU contains floating point / integer multiplier an ALU, capable of operating both integer & floating point arithmetic and a 32-bit barrel shifter.
* Multiplier operates single-cycle multiplications on 24-bit integers and on 32-bit floating-point values.
* ARAU's support addressing modes specific to DSP such as circular buffering & bit reversal.

— 4M

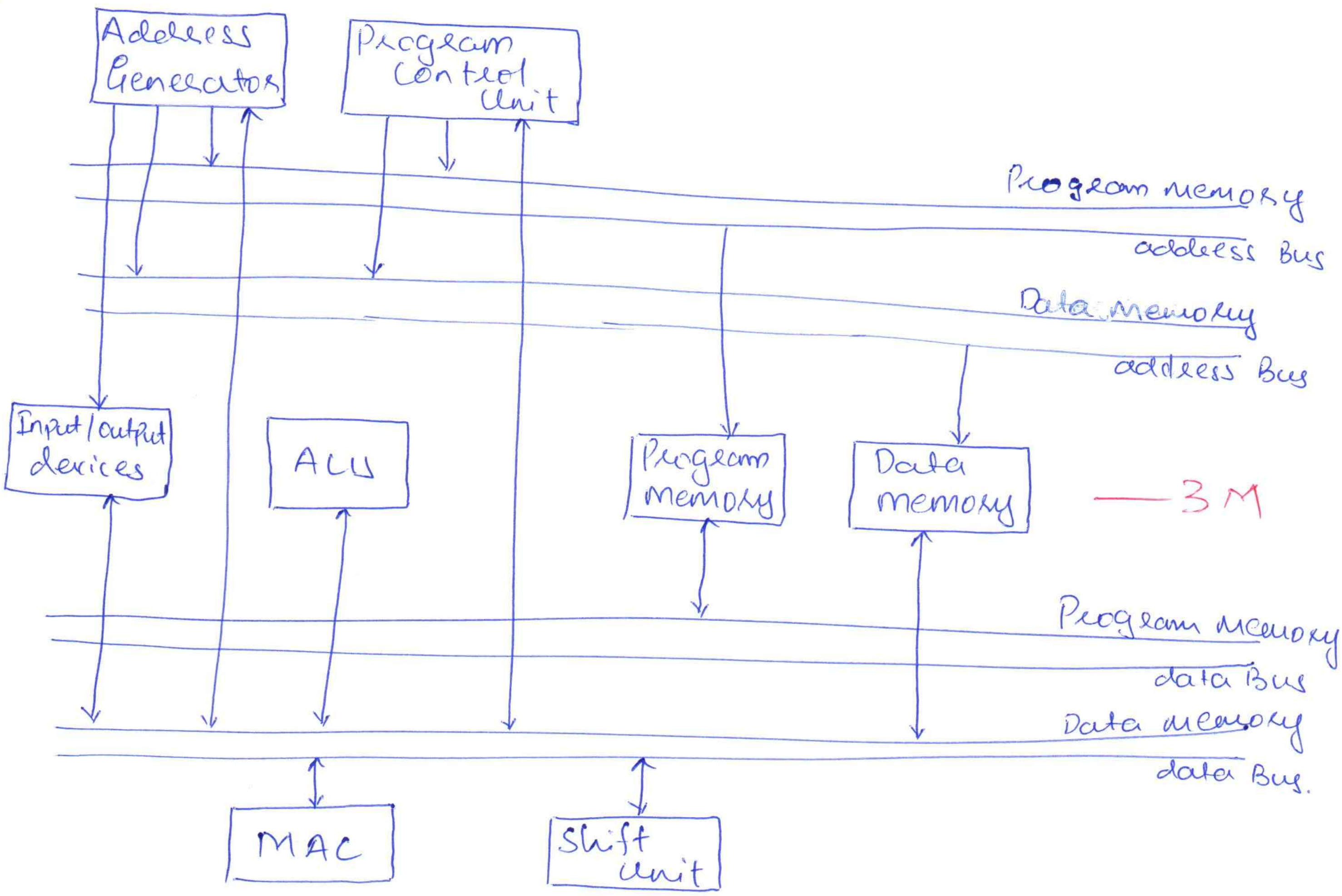
* Three floating point formats are supported, a short 16-bit having 4 Exponent bits, 1 sign bit & 11 mantissa bits. a 32-bit single precision format has 8 Exponent bits, 1 sign bit & 23 fraction bits. A 40-bit extended precision format contains 8 Exponent bits, 1 sign bit & 31 fraction bits.
* TMS320C3x offers high-speed performance with 60-nanosecond single cycle instruction execution time, which is equivalent to 16.7 MIPS.

"OR"

35

10. a) With block diagram explain Digital signal processor based on Harvard architecture. [Total - 6 M]

→



— 3 M

Digital Signal Processor based on Harvard architecture.

* DSP designed on Harvard architecture has two seperate memory spaces, one dedicated to the program code, while the other is employed for data.

* Two corresponding address buses & data buses are used. Harvard Processor can fetch the program instruction and data in parallel at the same time, the former via the program memory bus and the latter via the data memory bus.

— 3 M

* Multiplier and Accumulator (MAC) & Shift unit are the two additional units for digital filtering & scaling operations for fixed-point implementation.

10.b) Convert the Q-15 signed numbers to decimal numbers.
i) 1.110101110000010   ii) 0.100011110110010.
[Total - 4M]

i) 1.110101110000010.
Since the number is negative, applying the 2's complement yields,

$$0.001010001111110$$

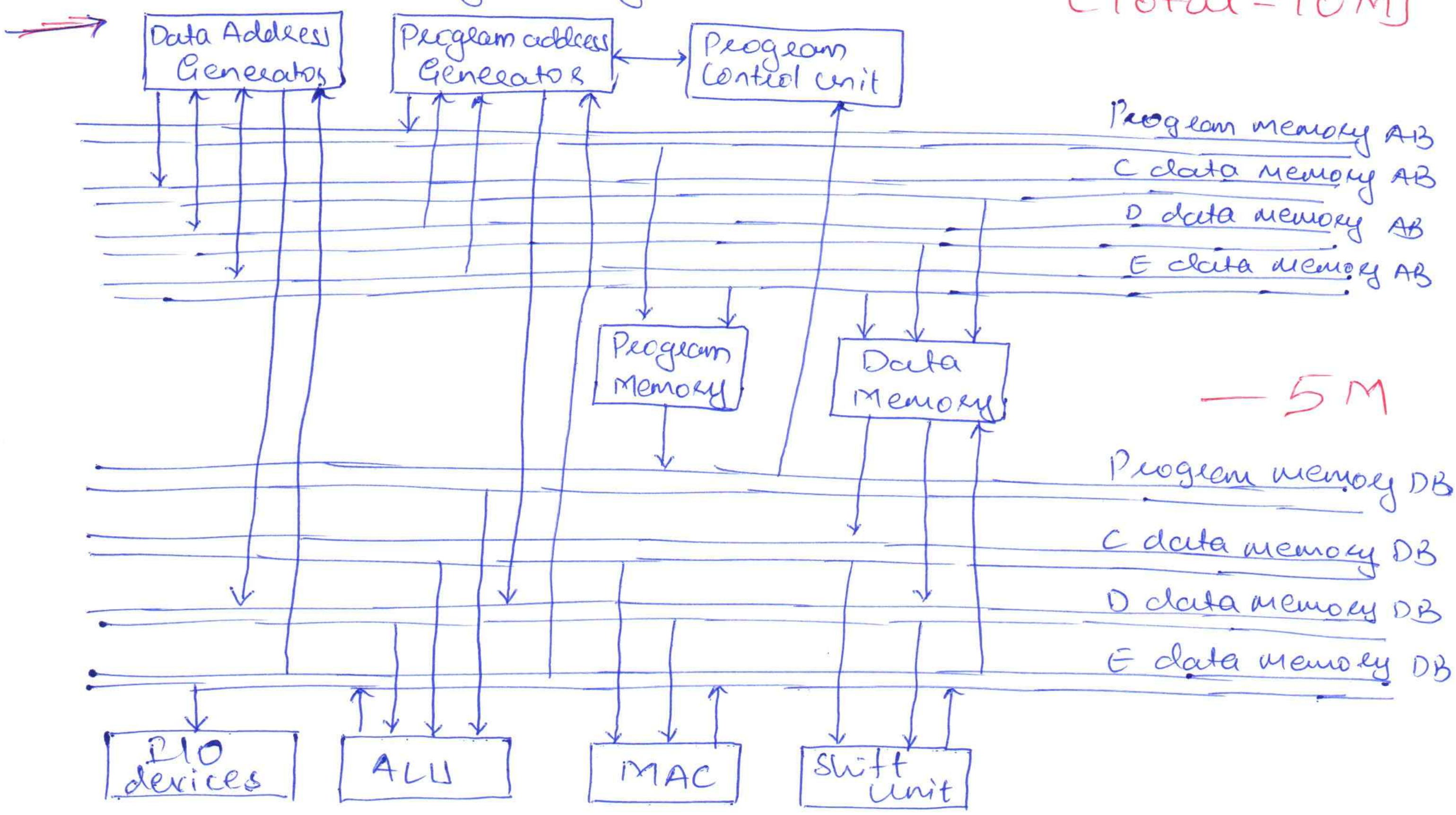
Then the decimal number is,

$$-(2^{-3} + 2^{-5} + 2^{-9} + 2^{-10} + 2^{-11} + 2^{-12} + 2^{-13} + 2^{-14}) = -0.160095. \quad -2M$$

ii) 0.100011110110010
The decimal number is,

$$2^{-1} + 2^{-5} + 2^{-6} + 2^{-7} + 2^{-8} + 2^{-10} + 2^{-11} + 2^{-14} = 0.560120 \quad -2M$$

10.c) Explain the basic architecture of TMS320C54x used in fixed point Digital Signal processor.
[Total - 10M]



— 5M

Basic architecture of TMS320C54x family.

* Four data and address buses are accommodated to work with the data memories & program memories. The program memory address & data bus are responsible for fetching the program instruction.

* C & D data memory address & data buses deal with fetching data from the data memory, while the E data memory address and data bus are dedicated to moving data into data memory, in addition, the E memory data bus can access the I/O devices.

* ALU can fetch data from the C, D, & program memory data buses and access the E memory data Bus. — 5 M

* It has two independent 40-bit accumulators, capable of operating 40-bit addition. The multiplier can fetch data from C & D memory data buses and write data via the E memory data bus, is capable of operating 17-bit × 17-bit multiplications.

* Program control unit fetches instruction via the program memory data bus. Inorder to speed up memory access, there are two address generators, one responsible for program addresses & one for data addresses.

* Several instructions operate at the same time for a given single instruction cycle processing performance offers 40 MIPS.

"Scheme & Solution Prepared by,"

06/03/2023
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