Model Question Paper-I with effect from 2022

USN | | | | | | | | | | |

Third Semester B.E Degree Examination
Transform Calculus, Fourier Series and Numerical Techniques
(21MAT31)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module -1	**Marks**
Q.01	a	Find the Laplace transform of $te^{-t}sin2t + \frac{cos2t-cos3t}{t}$	**06**
	b	Find the Laplace transform of the triangular wave of period 2a given by $$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$$	**07**
	b	Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$	**07**
		OR	
Q.02	a	Find the inverse Laplace transform of (i) $\frac{(s^2-1)^2}{s^5}$ (ii) $\frac{s}{s^2+6s+13}$	**06**
	b	Express the following function in terms of unit step function and hence find its Laplace transform $$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 2t, & 1 < t < 2 \\ 3t, & 2 < t < 3 \end{cases}$$	**07**
	c	Solve by using Laplace transform techniques $y'' - 3y' + 2y = e^{3t}, y(0) = 1, y'(0) = -1$	**07**
		Module-2	
Q. 03	a	Obtain the Fourier series for $f(x) = \frac{\pi - x}{2}$ $in\ 0 \le x \le 2\pi$	**06**
	b	Find half-range Fourier cosine series for the function $f(x) = (x-1)^2, in\ 0 < x < 1$, and hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$	**07**
	c	Find Fourier series expansion of y up to first harmonic if it is given by x: 0, 1, 2, 3, 4, 5 $f(x)$: 9, 18, 24, 28, 26, 20	**07**
		OR	
Q.04	a	Obtain the Fourier series for $f(x) = \|x\|, -\pi \le x \le \pi$	**6**
	b	Obtain half-range sine series for $f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$	**07**

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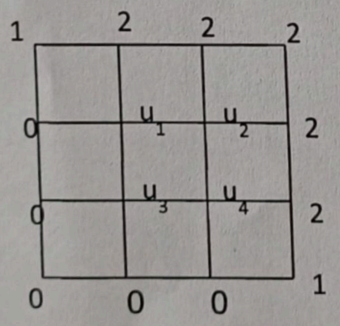
	c	Expand y as a Fourier series up to first harmonic if the values of y given by	07

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$
y	1.98	1.30	1.05	1.30	-0.88	-0.25

Module-3

Q. 05	a	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \lvert x\rvert \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{\sin x - x\cos x}{x^3} \cos\left(\frac{x}{2}\right) dx$	06
	b	Find the Z-transforms of $\cosh n\theta$ and $\sinh n\theta$	07
	c	Using z –transformation, solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$, $u_0 = 0, u_1 = 0$	07
		OR	
Q. 06	a	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$	06
	b	Fin the inverse cosine transform of $F_c(\alpha) = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$ And hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$	07
	c	Fin the inverse z-transform of $\frac{z^2-20z}{(Z-2)(Z-3)(z-4)}$	07

Module-4

Q. 07	a	Solve $u_{xx} + u_{yy} = 0$ for the square mesh with boundary values as given below. Iterate till the mesh values are correct to two decimal places 	10
	b	Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking h = 1 up to t = 1.25. Ihe boundary condition are $u(0,t) = u(5,t) = 0, u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$	10
		OR	

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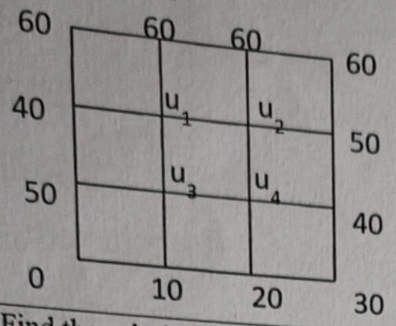
Q. 08	a	Given the values of u(x, y) on the boundary of the square as in the following figure. Evaluate the function u(x, y) satisfying the Laplace equation $u_{xx} + u_{yy} = 0$ at the pivotal points of the figure 	10
	b	Find the solution of the parabolic equation $u_{xx} = 2u_t$ when $u(4,t) = 0, and\ u(x,0) = x(4-x)$, taking $h = 1$. Find the values up to t = 5.	10
		Module-5	
Q. 09	a	Using Runge –Kutta method of order four, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 . Given, $y(0) = 1,\ y'(0) = 1$	06
	b	Find the external of the functional $\int_{x_0}^{x_1}(1 + x^2y')y'dx$	07
	c	Show that the geodesies on a plane are straight lines	07
		OR	
Q. 10	a	Given $y'' = 1 + y', y(0) = 1, y'(0) = 1$, compute y(0.4) for the following data using Milne's predictor – corrector method. $y(0.1) = 1.1103,\quad y(0.2).2427,\quad y(0.3) = 1.344$ $y'(0.1) = 1.2103,\ y'(0.2) = 1.4427,\ y'(0.3) = 1.699$	06
	b	Derive the Euler's equation	07
	c	Find the curves on which the functional $\int_0^1[(y')^2 + 12xy]\,dx$ with $y(0) = 0\ and\ y(1) = 1$	07

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO01
	(b)	L2	CO 01	PO 02
	(c)	L3	CO 01	PO 01
Q.2	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 02
	(c)	L3	CO 01	PO 03

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Q.No.	Solution and Scheme	Marks
1 a)	$L[te^{-t}\sin 2t] + L\left[\frac{\cos 2t - \cos 3t}{t}\right] = L[f(t)]$	
	$L[t\sin 2t] = (-1)\frac{d}{ds}\left[\frac{2}{s^2+4}\right] = \frac{4s}{(s^2+4)^2}$	
	$\therefore L[e^{-t}\cdot t\sin 2t] = \frac{4(s+1)}{[(s+1)^2+4]^2}$	(2M)
	$L\left[\frac{\cos 2t - \cos 3t}{t}\right] = \int_s^\infty \left[\frac{s}{s^2+4} - \frac{s}{s^2+9}\right] ds$	
	$= \frac{1}{2}\left[\log(s^2+4) - \log(s^2+9)\right]_s^\infty$	
	$= \log\left(\frac{s^2+9}{s^2+4}\right)^{1/2}$	(3M)
	$\therefore L[f(t)] = \frac{4(s+1)}{[(s+1)^2+4]^2} + \log\left(\frac{s^2+9}{s^2+4}\right)^{1/2}$	(1M)
b)	$L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\, dt$	(1M)
	$= \frac{1}{1-e^{-2as}}\int_0^{2a} e^{-st} f(t)\, dt$	
	$= \frac{1}{1-e^{-2as}}\left[\int_0^a te^{-st}\, dt + \int_a^{2a}(2a-t)e^{-st}\, dt\right]$	(1M)
	$= \frac{1}{1-e^{-2as}}\left[\left\{t\left(\frac{e^{-st}}{-s}\right) - (1)\left(\frac{e^{-st}}{(-s)^2}\right)\right\}_0^a + \left\{(2a-t)\left(\frac{e^{-st}}{-s}\right) - (-1)\left(\frac{e^{-st}}{(-s)^2}\right)\right\}_a^{2a}\right]$	(3M)
	$= \frac{1}{1-e^{-2as}}\left[\frac{1}{s^2} + \frac{e^{-2as}}{s^2}\right]$	
	$L[f(t)] = \frac{1}{s^2(1-e^{-2as})}\left[1+e^{-2as}\right]$	(2M)

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Q.No.	Solution and Scheme	Marks
1 c)	Let $\bar{f}(s)=\frac{s}{s^2+a^2}$ $\quad \bar{g}(s)=\frac{1}{s^2+a^2}$	
	$\Rightarrow f(t)=\cos at \quad g(t)=\frac{1}{a}\sin at$	(1m)
	$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]=\int_{u=0}^{t} f(u)\,g(t-u)\,du$	(1m)
	$=\int_{u=0}^{t} \cos au \cdot \frac{1}{a}\sin(at-au)\,du$	
	$=\frac{1}{2a}\int_{u=0}^{t}[\sin(at-au+au)+\sin(at-au-au)]\,du$	(2m)
	$=\frac{1}{2a}\int_{u=0}^{t}[\sin at+\sin(at-2au)]\,du$	
	$=\frac{1}{2a}\left[t\sin at-\frac{1}{2a}\{(\cos at-\cos au)\}_{u=0}^{t}\right]$	(2m)
	$=\frac{t\sin 2at}{2a}$	
	$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]=\frac{t\sin at}{2a}$	(1m)
2 a)	i) $\bar{f}(s)=\frac{(s^2-1)^2}{s^5}=\frac{s^4+1-2s^2}{s^5}$	
	$\bar{f}(s)=\frac{1}{s}+\frac{1}{s^5}-\frac{2}{s^3}$	
	$L^{-1}[\bar{f}(s)]=L^{-1}\left[\frac{1}{s}\right]+L^{-1}\left[\frac{1}{s^5}\right]-2L^{-1}\left[\frac{1}{s^3}\right]$	
	$f(t)=1+\frac{t^4}{4!}-2\cdot\frac{t^2}{2!}$	
	$f(t)=1+\frac{t^4}{24}-t^2$	(2m)
	ii) $\bar{f}(s)=\frac{s}{s^2+6s+13}$	

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Q.No.	Solution and Scheme	Marks
	$\bar{f}(s) = \dfrac{s}{(s+3)^2+4} = \dfrac{(s+3)-3}{(s+3)^2+4}$	(1M)
	ie $\bar{f}(s) = \dfrac{s+3}{(s+3)^2+4} - \dfrac{3}{(s+3)^2+4}$	
	$\therefore \mathcal{L}^{-1}[\bar{f}(s)] = e^{-3t}\mathcal{L}^{-1}\left[\dfrac{s}{s^2+4}\right] - 3e^{-3t}\mathcal{L}^{-1}\left[\dfrac{1}{s^2+4}\right]$	(2M)
	$f(t) = e^{-3t}\cos 2t - \dfrac{3}{2}\sin 2t$	(1M)
b)	$f(t) = f_1(t) + [f_2(t) - f_1(t)]u(t-a) + [f_3(t) - f_2(t)]u(t-b)$	(1M)
	$f(t) = 1 + (2t-1)u(t-2) + (3t-2t)u(t-3)$	(1M)
	$\mathcal{L}[f(t)] = \mathcal{L}(1) + \mathcal{L}[(2t-1)u(t-2)] + \mathcal{L}[t\,u(t-3)]$	(1M)
	Let $f(t-2) = 2t-1$ $f(t) = 2(t+2)-1 = 2t+3.$	(2M)
	$g(t-3) = t$ $g(t) = t+3$	(1M)
	$\mathcal{L}[f(t)] = \dfrac{1}{s} + e^{-2s}\left[\dfrac{2}{s^2} + \dfrac{3}{s}\right] + e^{-3s}\left[\dfrac{1}{s^2} + \dfrac{3}{s}\right]$	(1M)
c)	Applying L.T. to the given d.e. we get	
	$\mathcal{L}[y''] - 3\mathcal{L}[y'] + 2\mathcal{L}[y] = \mathcal{L}[e^{3t}]$	(1M)
	$[s^2\bar{y}(s) - sy(0) - y'(0)] - 3[s\bar{y}(s) - y(0)] + 2\bar{y}(s) = \dfrac{1}{s-3}$	
	$[s^2 - 3s + 2]\bar{y}(s) - s + 1 + 3 = \dfrac{1}{s-3}$	(2M)
	$(s-1)(s-2)\bar{y}(s) = s - 4 + \dfrac{1}{s-3}$	
	$\bar{y}(s) = \dfrac{s-4}{(s-1)(s-2)} + \dfrac{1}{(s-1)(s-2)(s-3)}$	(1M)
	$\mathcal{L}^{-1}[\bar{y}(s)] = \mathcal{L}^{-1}\left[\dfrac{s-4}{(s-1)(s-2)}\right] + \mathcal{L}^{-1}\left[\dfrac{1}{(s-1)(s-2)(s-3)}\right]$	
	$= \mathcal{L}^{-1}\left[\dfrac{A}{s-1} + \dfrac{B}{s-2}\right] + \mathcal{L}^{-1}\left[\dfrac{A}{s-1} + \dfrac{B}{s-2} + \dfrac{C}{s-3}\right]$	(2M)

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Q.No.	Solution and Scheme	Marks
	$y(t) = L^{-1}\left[\frac{3}{s-1} - \frac{2}{s-2}\right] + L^{-1}\left[\frac{1/2}{s-1} - \frac{1}{s-2} + \frac{1/2}{s-3}\right]$	
	$y(t) = 3e^{t} - 2e^{2t} + \frac{1}{2}e^{t} - e^{2t} + \frac{1}{2}e^{3t}$	(1M)
3 a)	$f(x) = \frac{\pi - x}{2}$	
	$f(2\pi - x) = \frac{\pi - (2\pi - x)}{2} = \frac{-\pi + x}{2} = -\left(\frac{\pi - x}{2}\right)$	
	$f(2\pi - x) = -f(x)$	
	$\therefore f(x)$ is an odd function $\Rightarrow a_0 = 0, a_n = 0$	(1M)
	$\therefore$ The required F.S. Expansion is	
	$f(x) = \sum_{n=0}^{\infty} b_n \sin nx \longrightarrow (1)$	(1M)
	Where $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$.	
	$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)\sin nx\,dx$	
	$= \frac{1}{2\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$	
	$= \frac{-1}{2\pi}\left[\frac{(\pi - x)}{n}\cos nx\right]_0^{2\pi} = \frac{1}{n}$	(3M)
	$\Rightarrow b_n = \frac{1}{n}$	
	$\therefore f(x) = \sum_{n=0}^{\infty} \frac{\sin nx}{n}$ is the required Fourier Series expansion.	(01M)
b)	The required half range F.C.S. is.	
	$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos n\pi x \longrightarrow (1)$	(1M)
	$a_0 = \frac{2}{1}\int_0^1 f(x)\,dx = \frac{2}{1}\int_0^1 (x-1)^2\,dx = \frac{2}{3}$	(1M)

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Q.No.	Solution and Scheme	Marks

$$a_n = 2\int_0^1 (x-1)^2 \cos n\pi x \, dx.$$

$$= 2\left[(x-1)^2 \frac{\sin n\pi x}{n\pi} - 2(x-1)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + 2\left(\frac{-\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$a_n = \frac{4}{n^2\pi^2}$$ (3M)

$$\therefore f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\cos n\pi x$$

put $x=0$ & $x=1$ we get

$$1 = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{n^2}, \quad 0 = \frac{1}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi$$

Adding we get.

$$\Rightarrow \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ (2M)

c) The required F.S. Expansion is

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$y = \frac{a_0}{2} + a_1\cos\left(\frac{\pi x}{3}\right) + b_1\sin\left(\frac{\pi x}{3}\right)$$

Let $\theta = \frac{\pi x}{3}$

$$y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta \longrightarrow (1)$$ (1M)

x	$\theta = \pi x/3$	y	$y\cos\theta$	$y\sin\theta$
0	0	9	9	0
1	60	18	9	15.588
2	120	24	−12	20.784
3	180	28	−28	0
4	240	26	−13	−22.516
5	300	20	10	−17.32

(3M)

$\Sigma y = 125$, $\Sigma y\cos\theta = -25$ $\Sigma y\sin\theta = -3.464$

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Q.No.	Solution and Scheme	Marks
	$a_0 = \frac{2}{N}\Sigma y = \frac{1}{3}(125) = 41.67$	
	$\frac{a_0}{2} = 20.83$	
	$a_1 = \frac{2}{N}\Sigma y\cos x = \frac{2}{6}(-25) = -8.33$	(1M)
	$b_1 = \frac{2}{N}\Sigma y\sin\theta = \frac{1}{3}(-3.464) \simeq -1.155$	(1M)
	$\therefore y = 20.83 + (-8.33)\cos\theta + (-1.155)\sin\theta$	(1M)
4 a)	The required Fourier series expansion is	
	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \longrightarrow (1)$	(1M)
	$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ $f(x)$ is Even $\Rightarrow b_n = 0$	
	$= \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} \Rightarrow \boxed{a_0 = \pi}$	(1M)
	$a_n = \frac{2}{\pi}\int_0^{\pi} x \cos nx\,dx$ (1) (2)	
	$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$	
	$= \frac{2}{\pi n^2}\left[\cos nx\right]_0^{\pi}$	(3M)
	$a_n = \frac{-2}{\pi n^2}\left[1-(-1)^n\right]$	
	$\therefore f(x) = \frac{\pi}{2} + \sum_{n=0}^{\infty} \frac{-2}{\pi n^2}\left[1-(-1)^n\right]\cos nx$	
	$\lvert x \rvert = \frac{\pi}{2} + \left(\frac{-2}{\pi}\right)\sum_{n=0}^{\infty}\left[1-(-1)^n\right]\cos nx$	(1M)

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Q.No.	Solution and Scheme	Marks
4.b)	The required half range sine series is	
	$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \longrightarrow (1)$	(1M)
	Where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$	
	$b_n = \frac{2}{\pi}\left[\int_0^{\pi/2} x\sin nx\,dx + \int_{\pi/2}^{\pi}(\pi-x)\sin nx\,dx\right]$	(2M)
	$= \frac{2}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi/2} + \left[(\pi-x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_{\pi/2}^{\pi}$	
	$= \frac{2}{\pi}\left[\frac{-\pi}{2n}\cos\left(\frac{n\pi}{2}\right) + \frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right) + \frac{\pi}{2n}\cos\left(\frac{n\pi}{2}\right) + \frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\right]$	(2M)
	$b_n = \frac{4}{\pi n^2}\sin\left(\frac{n\pi}{2}\right)$	
	$\therefore$ The required half range sine series is	
	$f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin nx$	(2M)
c)	$N = 6 \quad \theta = \frac{\pi x}{l} \quad \theta = \left(\frac{2\pi}{T}\right)t$	

x	y	$y\cos\theta$	$y\sin\theta$
0	1.98	1.98	0
60	1.30	0.65	1.131
120	1.05	-0.525	0.9135
180	1.30	-1.3	0
240	-0.88	0.44	0.7656
300	-0.25	-0.125	0.2765

(3M)

$\Sigma y = 4.5$

$\Sigma y\cos\theta = 1.12$

$\Sigma y\sin\theta = 3.0266$ (1M)

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Q.No.	Solution and Scheme	Marks
	$\frac{a_0}{2} = 0.75$	
	$a_1 = \frac{2}{N}\sum y\cos\theta = \frac{2}{6}(1.12) = 0.3733$	(1M)
	$b_1 = \frac{2}{N}\sum y\sin\theta = \frac{2}{6}(3.0266) = 1.0088$	(1M)
	$\therefore y = 0.75 + \sum_{n=1}^{\infty}(0.3733)\cos\theta + (1.0058)\sin\theta.$	(1M)
5 a)	$F\{f(x)\} = \int_{-1}^{1}(1-x^2)\, e^{isx}\,dx$	(1M)
	$= \left[(1-x^2)\frac{e^{isx}}{is} - (-2x)\frac{e^{isx}}{(is)^2} + (-2)\frac{e^{isx}}{(is)^3}\right]_{-1}^{1}$	
	$= \frac{-2}{s^2}[e^{is}+e^{-is}] - \frac{2i}{s^3}[e^{is}-e^{-is}]$	
	$F(s) = \frac{-4}{s^3}[s\cos s - \sin s]$	(2M)
	By the inversion formula	
	$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}F(s)\,e^{-isx}\,ds$	
	$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-4}{s^3}[s\cos s - \sin s]\,e^{-isx}\,ds = \begin{cases} 1-x^2 & \|x\| \le 1 \\ 0 & \|x\| > 1 \end{cases}$	(2M)
	put $x = 1/2$	
	$\frac{-1}{2\pi}\int_{-\infty}^{\infty}\frac{4}{s^3}[s\cos s - \sin s]\,e^{-is/2}\,ds = 3/4$	
	$2\int_{0}^{\infty}\frac{s\cos s - \sin s}{s^3}[\cos(s/2) - i\sin(s/2)]\,ds = \frac{-6\pi}{16}$	
	$\int_{0}^{\infty}\frac{s\cos s - \sin s}{s^3}[\cos(s/2) - i\sin(s/2)]\,ds = \frac{-3\pi}{16}$	(1M)
	Equating real part on both sides & changing s to x	(1M)
	$\int_{0}^{\infty}\frac{x\cos x - \sin x}{x^3}\cos(x/2)\,dx = \frac{-3\pi}{16}$	

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Q.No.	Solution and Scheme	Marks
5 b)	$\cosh n\theta = \frac{1}{2}[e^{n\theta} + e^{-n\theta}] = \frac{1}{2}[(e^{\theta})^n + (e^{-\theta})^n]$	(1M)
	$Z[\cosh n\theta] = \frac{1}{2} Z[(e^{\theta})^n + (e^{-\theta})^n]$	(1M)
	$= \frac{1}{2}\left[\frac{z}{z-e^{\theta}} + \frac{z}{z-e^{-\theta}}\right]$	
	$= \frac{z}{2}\left[\frac{z-e^{-\theta}+z-e^{\theta}}{(z-e^{\theta})(z-e^{-\theta})}\right]$	(1M)
	$= \frac{z}{2}\left[\frac{2z-2\cosh\theta}{z^2-2z\cosh\theta+1}\right]$	(1M)
	Thus $Z[\cosh n\theta] = \frac{z[z-\cosh\theta]}{z^2-2z\cosh\theta+1}$	(1M)
	$Z[\sinh n\theta] = \frac{1}{2} Z[(e^{\theta})^n - (e^{-\theta})^n]$	
	$= \frac{z}{2}\left[\frac{z-e^{-\theta}-(z-e^{-\theta})}{z^2-2z\cosh\theta+1}\right]$	(2M)
	Thus $Z[\sinh n\theta] = \frac{z\sinh\theta}{z^2-2z\cosh\theta+1}$	
c)	Applying Z-transform on both sides	
	$Z[u_{n+2}] + 6Z[u_{n+1}] + 9Z[u_n] = Z[2^n]$	(1M)
	$z^2[U(z)-u_0-u_1z^{-1}] + 6z[U(z)-u_0] + 9U(z) = \frac{z}{z-2}$	
	Using the condition $u_0=0$, $u_1=0$	
	$(z^2+6z+9)U(z) = \frac{z}{z-2}$.	(2M)
	$U(z) = \frac{z}{(z-2)(z+3)^2}$.	(1M)
	$\frac{U(z)}{z} = \frac{z}{(z-2)(z+3)^2} = A\cdot\frac{z}{(z-2)} + B\frac{z}{(z+3)} + C\frac{z}{(z+3)^2}$	
	$A = 1/25$, $B = -1/25$, $C = -1/5$	(2M)
	Taking inverse Z-transform	
	$Z^{-1}\left[\frac{z}{(z-2)(z+3)^2}\right] = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n + \frac{1}{15}(-3)^n \cdot n$	(1M)

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Q.No.	Solution and Scheme	Marks

6 a) $F_s(s) = \int_0^\infty f(x)\sin sx\,dx$ (1M)

$= \int_0^1 x \sin sx\,dx + \int_1^2 (2-x)\sin sx\,dx$ (1M)

$= \left[x\left(\frac{-\cos sx}{s}\right) - (1)\left(\frac{-\sin sx}{s^2}\right)\right]_0^1 + \left[(2-x)\left(\frac{-\cos sx}{s}\right) - (-1)\left(\frac{-\sin sx}{s^2}\right)\right]_1^2$

$= \left[-\cancel{\frac{\cos s}{s}} + \frac{\sin s}{s^2} + 0 - 0\right] + \left[0 - \frac{\sin 2s}{s^2} + \cancel{\frac{\cos s}{s}} + \frac{\sin s}{s^2}\right]$ (3M)

$= \frac{\sin s}{s^2} - \frac{\sin 2s}{s^2} + \frac{\sin s}{s^2}$ (1M)

$= \frac{1}{s^2}\left[2\sin s - \sin 2s\right]$

b) We have to find $f(\theta)$ and we shall consider inverse F.C.T. with

~~$F(s) =$~~ $F(\alpha) = \begin{cases} 1-\alpha & 0 \le \alpha \le 1 \\ 0 & \alpha > 1 \end{cases}$

$f(\theta) = \frac{2}{\pi}\int_0^\infty F(\alpha)\cos\alpha\theta\,d\alpha$ (1M)

$= \frac{2}{\pi}\int_0^\infty (1-\alpha)\cos\alpha\theta\,d\alpha$

$= \frac{2}{\pi}\left[(1-\alpha)\left(\frac{\sin\alpha\theta}{\theta}\right) - (-1)\left(\frac{-\cos\alpha\theta}{\theta^2}\right)\right]_0^1$ (2M)

$= \frac{2}{\pi\theta^2}\left[1 - \cos\theta\right]$

$f(\theta) = \frac{2}{\pi\theta^2} 2\sin^2\theta/2 = \frac{4\sin^2(\theta/2)}{\pi\theta^2}$ (2M)

$\therefore \int_0^\theta f(\theta)\cos s\theta\,d\theta = \int_0^\infty \frac{4\sin^2\theta/2}{\pi\theta^2}\cos s\theta\,d\theta = F(\alpha)$

$\int_0^\infty \frac{\sin^2(\theta/2)}{(\theta/2)^2}\cos s\theta = \pi F(\alpha)$ put $\theta/2 = t$, put $\alpha = 0$

$\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$ (2M)

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Q.No.	Solution and Scheme	Marks
6 c)	$U(z) = \dfrac{z^2-20z}{(z-2)(z-3)(z-4)}$	
	$\dfrac{U(z)}{z} = \dfrac{z-20}{(z-2)(z-3)(z-4)} = \dfrac{A}{z-2} + \dfrac{B}{z-3} + \dfrac{C}{z-4}$	(1M)
	$z-20 = A(z-3)(z-4) + B(z-2)(z-4) + C(z-2)(z-3)$ put $z=2 \Rightarrow A=-9.$ put $z=3 \Rightarrow B=17$ put $z=4 \Rightarrow C=-10.$	(3M)
	$\dfrac{U(z)}{z} = \dfrac{-9}{z-2} + \dfrac{17}{z-3} + \dfrac{-10}{z-4}$	
	$U(z) = \dfrac{-9z}{z-2} + \dfrac{17z}{z-3} - \dfrac{10z}{z-4}$	(2M)
	Taking inverse z-transform. $u_n = -9(2)^n + 17(3)^n - 10(4)^n$	(1M)
7. a)	To get the initial values assume that $u_4 = 0$	
	Ist Iteration $u_1^{(0)} = \frac{1}{4}[1+2+u_4+0] = 0.75$ diagonal formula. $u_2^{(0)} = \frac{1}{4}[u_1^{(0)}+2+2+0] = 1.1875$ std. formula. $u_3^{(0)} = \frac{1}{4}[0+u_1^{(0)}+u_4+0] = 0.1875$ std. formula. $u_4^{(0)} = \frac{1}{4}[u_3^{(0)}+u_2^{(0)}+2+0] = 0.84375$ std formula.	(2M)
	2nd iteration $u_1^{(1)} = 0.84375$ $u_2^{(1)} = 1.4218$ $u_3^{(1)} = 0.4218$ $u_4^{(1)} = 0.9609$	(2M)
	3rd Iteration $u_1^{(2)} = 0.9609$ $u_2^{(2)} = 1.4804$, $u_3^{(2)} = 0.4804$ $u_4^{(2)} = 0.9902$	(2M)
	$u_1^{(3)} = 0.9902$ $u_2^{(3)} = 1.4951$, $u_3^{(3)} = 0.4951$, $u_4^{(3)} = 0.9975$ $u_1^{(4)} = 0.9975$ $u_2^{(4)} = 1.4987$ $u_3^{(4)} = 0.4987$, $u_4^{(4)} = 0.99935$ $u_1^{(5)} = 0.99935$, $u_2^{(5)} = 1.4996$, $u_3^{(5)} = 0.4996$, $u_4^{(5)} = 0.9998$ $u_1^{(6)} = 0.9998$, $u_2^{(6)} = 1.4999$ $u_3^{(6)} = 0.4999$, $u_4^{(6)} = 0.9999$ $\therefore u_1 = 0.9998$, $u_2 = 1.4999$, $u_3 = 0.4999$, $u_4 = 0.9999$	4M

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Q.No.	Solution and Scheme	Marks
b)	The difference equation for the given Eq^n is	

$$u_{i,j+1} = 2(1-16\alpha^2)u_{i,j} + 16\alpha^2(u_{i-1,j} + u_{i+1,j}) - u_{i,j-1} \quad (1)$$ (2m)

where $\alpha = k/h$. Taking $h=1$ and choosing k so that the coefficient of $u_{i,j}$ vanishes.

We have $16\alpha^2 = 1$ i.e. $k = h/4 = 1/4$

Eq^n (1) reduces to.

$$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1} \longrightarrow (2)$$ (2m)

Since $u(0,t) = u(5,t) = 0$

$\therefore$ ~~ut~~ $u_{0,j} = 0$ and $u_{5,j} = 0$ for ~~vo~~ all values of j (2m)

$u(x,0) = x^2(5-x)$

$\therefore$ ~~ut u₀. u(x,0)~~

$\therefore u_{i,0} = i^2(5-i) = 4, 12, 18, 16$ for $i = 1,2,3,4$ at $t=0$ (2M)

$\frac{u_{i,j+1} - u_{i,j-1}}{2k} = 0$ when $j=0$ giving $u_{i,1} = u_{i,-1}$

Putting $j=0$ in (2) $u_{i,1} = \frac{1}{2}[u_{i-1,0} + u_{i+1,0}]$

Taking $i = 1, 2, 3, 4$ successively

$u_{1,1} = 6$, $u_{2,1} = 11$, $u_{3,1} = 14$, $u_{4,1} = 9$

Similarly putting $j = ~~1~~, 2, 3, 4$ successively we get,

j \ i	0	1	2	3	4	5
0	0	4	12	18	16	0
1	0	6	11	14	9	0
2	0	7	8	2	-2	0
3	0	2	-2	-8	-7	0
4	0	-9	-14	-11	-6	0
5	0	-16	-18	-12	-4	0

(2M)

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Q.No.	Solution and Scheme	Marks
8 a)	To get the initial values assume that $u_4 = 0$.	
	1st iteration	
	$u_1^{(0)} = \frac{1}{4}[60+60+50+u_4] = 42.5$ diagonal formula	
	$u_2^{(0)} = \frac{1}{4}[u_1+60+50+0] = 38.125$ std. formula,	
	$u_3^{(0)} = \frac{1}{4}[50+u_1+u_4+10] = 25.625$ std. formula,	
	$u_4^{(0)} = \frac{1}{4}[u_3+u_2+40+20] = 30.9375$ std. formula,	(2M)
	2nd Iteration	
	$u_1^{(1)} = \frac{1}{4}[40+60+u_2+u_3] = 40.9375$	
	$u_2^{(1)} = \frac{1}{4}[40.9375+60+50+30.9375] = 45.46875$	
	$u_3^{(1)} = \frac{1}{4}[60+40.9375+30.9375] = 32.96875$	(2M)
	$u_4^{(1)} = \frac{1}{4}[45.46875+32.96875+60] = 34.609375$.	
	3rd Iteration	
	$u_1^{(2)} = 44.60$, $u_2^{(2)} = 47.30$, $u_3^{(2)} = 34.80$, $u_4^{(2)} = 35.52$	
	4th Iteration	
	$u_1^{(3)} = 45.52$, $u_2^{(3)} = 47.76$, $u_3^{(3)} = 35.26$, $u_4^{(3)} = 35.75$.	(2M)
	5th Iteration	
	$u_1^{(4)} = 45.75$, $u_2^{(4)} = 47.87$, $u_3^{(4)} = 35.37$, $u_4^{(4)} = 35.81$	
	6th Iteration	
	$u_1^{(5)} = 45.81$, $u_2^{(5)} = 47.90$, $u_3^{(5)} = 35.40$, $u_4^{(5)} = 35.82$	(2M)
	7th Iteration	
	$u_1^{(6)} = 45.82$, $u_2^{(6)} = 47.91$, $u_3^{(6)} = 35.41$, $u_4^{(6)} = 35.83$	
	8th Iteration	
	$u_1^{(7)} = 45.83$, $u_2^{(7)} = 47.91$, $u_3^{(7)} = 35.41$, $u_4^{(7)} = 35.83$	
	The 7th and 8th iterations are identical	(3M)
	$\therefore u_1 = 45.83$, $u_2 = 47.91$, $u_3 = 35.41$, $u_4 = 35.83$	

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Q.No.	Solution and Scheme	Marks
b)	$h=1 \quad c^2=\frac{1}{2}$	(1)
	$k^2 = \frac{h^2}{2c^2} = 1$ if $\alpha = 1/2$	
	Values of t are 0, 1, 2, 3, 4	(3M)
	The values of x lies in $0 \le x \le 4$	

t \ x	0	1	2	3	4
0	0	3	4	3	0
1	0	2	3	2	0
2	0	1.5	2	1.5	0
3	0	1	1.5	1	0
4	0	0.75	1	0.75	0
5	0	0.5	0.75	0.5	0

(6M)

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Q.No.	Solution and Scheme	Marks
9 a)	put $\frac{dy}{dx} = z = f(x,y,z)$ $\frac{dz}{dx} = xz^2 - y^2$ with $y=1,\ z=1$ at $x=0$ $k_1 = hf(x_0, y_0, z_0) = 0.2f(0,1,1) = 0.2\times 1 \Rightarrow \boxed{0.2 = k_1}$ $l_1 = hg(x_0, y_0, z_0) = 0.2f(0,1,1) = 0.2\times(0-1^2) = -0.2$ $\boxed{l_1 = -0.2}$	(1M)
	$k_2 = hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}) = 0.2f(0.1, 1.1, 0.9)$ $\boxed{k_2 = 0.18}$ $l_2 = hg(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}) = 0.2g(0.1, 1.1, 0.9)$	(1M)
	$\boxed{l_2 = -0.2258}$ $k_3 = hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}) = 0.2f(0.1, 1.09, 0.88)$ $\boxed{k_3 = 0.176}$	(1M)
	$l_3 = hg(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}) = 0.2g(0.1, 1.09, 0.88)$ $l_3 = -1.1106\times 0.2 \Rightarrow \boxed{l_3 = -0.2221}$	(1M)
	$k_4 = hf(x_0+h, y_0+k_3, z_0+l_3) = 0.2f(0.2, 1.176, 0.7779)$ $\boxed{k_4 = 0.15558}$ $l_4 = hg(x_0+h, y_0+k_3, z_0+l_3) = 0.2g(0.2, 1.176, 0.7779)$ $\boxed{l_4 = -1.2619}$ $k = \frac{1}{6}(k_1+2k_2+2k_3+k_4) = 0.1779$ $y(0.2) = y_0 + k = 1 + 0.1779$ $y(0.2) = 1.1779$	(2M)
b)	$f(x, y, y') = (1+x^2y')y' = y' + x^2y'^2$ By Euler's Eqn $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$	(1M)

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Q.No.	Solution and Scheme	Marks
	$0 - \frac{d}{dx}\left(1+2x^2y'\right) = 0.$	(1M)
	$\frac{d}{dx}\left[1+2x^2y'\right] = 0$ Integrating.	
	$1+2x^2y' = C$	(2M)
	$2x^2y' = C-1$	
	$y' = \frac{C-1}{2x^2}$ Integrating.	(2M)
	$y = \frac{C-1}{2}\left(\frac{-1}{x}\right) + C_2.$	
	$y = \frac{-C_1}{x} + C_2$	(1M)
c)	Let $A(x,y)$ & $B(x,y)$ be any two points on the surface. then	
	$S = \int_{x_1}^{x_2} \frac{ds}{dx}\, dx.$	
	$= \int_{x_1}^{x_2} \sqrt{1+(y')^2}\, dx$	(1M)
	$\therefore f(x,y,y') = \sqrt{1+y'^2}.$	
	By Euler's Eqn $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$	(1M)
	$0 - \frac{d}{dx}\left[\frac{1}{2\sqrt{1+y'^2}} \times 2y'\right] = 0.$ Integrating	(2M)
	$\frac{y'}{\sqrt{1+y'^2}} = c$ squaring	
	$(y')^2 = c^2(1+y'^2)$	(1M)
	$(y')^2 = \frac{c^2}{1-c^2} \Rightarrow y' = \left(\frac{c}{\sqrt{1-c^2}}\right) = A$	
	$\frac{dy}{dx} = A$ Integrating	
	$\boxed{y = Ax+B}$ which is a st. line.	(1M)

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Q.No.	Solution and Scheme	Marks
10 a)	Putting $\frac{dy}{dx} = z = f(x,y,z)$	
	$z' = \frac{dz}{dx} = 1 + z = g(x,y,z)$	
	$z'(0) = 1 + z(0) = 1 + 1 = 2$	
	$z'(0.1) = 1 + 1.2103 = 2.2103$	
	$z'(0.2) = 1 + 1.4427 = 2.4427$	(1m)
	$z'(0.3) = 1 + 1.699 = 2.699.$	
	$y_4^{(P)} = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3]$	
	$z_4^{(P)} = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3']$	
	$\boxed{y_4^{(P)} = 1.5834}$ and $\boxed{z_4^{(P)} = 1.9834}$	(3m)
	$y_4^{(C)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4)$	
	$z_4^{(C)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4')$	
	where $z_4' = 1 + z_4^{(P)} = 1 + 1.9834 = 2.9834$	
	$y_4^{(C)} = 1.5834$ and $z_4^{(C)} = 1.9834$	(2m)
	$\therefore \boxed{y(0.4) = 1.5834}$	
b)	By the variational problem for functional $\Rightarrow \delta I = 0$	
	$\Rightarrow \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial y'}\delta y' \right) dx = 0$	(1m)
	W.K.T $\delta y' = \frac{d}{dx}(\delta y)$	
	$\therefore \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial y'} \cdot \frac{d}{dx}(\delta y) \right) dx = 0$	(1m)
	$\int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\delta y \right) dx + \int_{x_1}^{x_2} \underbrace{\frac{\partial f}{\partial y'}}_{1} \cdot \underbrace{\frac{d}{dx}(\delta y)}_{2} dx = 0$	
	$\int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\delta y \right) dx + \left(\frac{\partial f}{\partial y'}\delta y \right)_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) \delta y \, dx$	(2m)

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Q.No.	Solution and Scheme	Marks
	Since $\delta y = 0$ is the boundry condition	
	$\therefore \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\delta y\right)dx - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right)\delta y\, dx = 0$	
	$\int_{x_1}^{x_2}\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_1}\right)\right]\delta y\, dx = 0$	(1m)
	Since δy is the condition imposed only on the boundry pt $\therefore \delta y \neq 0$ for $x_1 < x < x_2$	
	$\therefore \boxed{\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_1}\right) = 0}$	(1m)
c)	$f(x, y, y') = (y')^2 + 12xy.$	
	By Euler's Eqn	
	$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_1}\right) = 0$	(1m)
	$12x - \frac{d}{dx}(2y') = 0$	
	$12x - 2y'' = 0$	
	$6x - y'' = 0$	(2m)
	$y'' = 6x$	
	$\frac{d^2y}{dx^2} = 6x$ Integrating	
	$\frac{dy}{dx} = 6 \cdot \frac{x^2}{2} = 3x^2 + C_1$ Integrating.	
	$\boxed{y = x^3 + C_1 x + C_2}$	(2m)
	$y' = 3x^2 + C_1$ using given condn.	
	$C_2 = 0$, $C_1 = 0$	(1m)
	$\Rightarrow \boxed{y = x^3}$	
	— END —	

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