Model Question Paper-I with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

First Semester B.E Degree Examination

Mathematics-I for Electrical and Electronics Engineering Stream (22MATE11)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE.**

		Module -1	Marks
Q.01	a	With usual notations prove that $\tan\varphi = r\frac{d\theta}{dr}$	06
	b	Find the angle between the curves $r = a(1+\cos\theta)$ *and* $r = b(1-\cos\theta)$	07
	c	Show that the radius of curvature at any point of the cycloid $x = a(\theta+\sin\theta),\ y = a(1-\cos\theta)$ *is* $4a\cos\left(\frac{\theta}{2}\right)$	07
		OR	
Q.02	a	If p be the perpendicular from the pole on the tangent, then show that $$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$	06
	b	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$	07
	c	Find the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $\left(\frac{3}{2},\frac{3}{2}\right)$	07
		Module-2	
Q. 03	a	Expand e^{sinx} by Maclaurin's series up to the term containing x^4	06
	b	If $u = f(2x-3y, 3y-4z, 4z-2x)$, show that $6u_x + 4u_y + 3u_z = 0$	07
	c	Show that the function $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at the point (1, 1)	07
		OR	
Q.04	a	Evaluate (i) $\lim_{x\to 0}\left(\frac{tanx}{x}\right)^{\frac{1}{x^2}}$ (ii) $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$	06
	b	If $u = tan^{-1}(y/x)$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$	07
	c	If $x+y+z = u,\ y+z = uv\ and\ z = uvw,\ find\ \frac{\partial(x,y,z)}{\partial(u,v,w)}$	07
		Module-3	
Q. 05	a	Solve $x\frac{dy}{dx} + y = x^3y^6$	06
	b	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\alpha} = 1$, where α is a parameter	07
	c	Solve $xyp^2 - (x^2+y^2)p + xy = 0$	07
		OR	
Q. 06	a	Solve $(x^2+y^2+x)dx + xydy = 0$	06

	b	Show that a differential equation for the current i in an electrical circuit containing an inductance L and resistance R in series and acted on by an electromotive force $Esin\omega t$, satisfies the equation $\frac{di}{dt} + Ri = Esin\omega t$. Find the value of the current at any time t, if initially there is no current in the circuit.	07
	c	Find the general solution of the equation $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form, taking the substitution $X = x^2, Y = y^2$	07
		Module-4	
Q. 07	a	Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)dxdydz$	06
	b	Change the order of integration and evaluate $\int_0^1\int_{\sqrt{y}}^{y} dxdy$	07
	c	Prove that $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{sin\theta}} = \pi$	07
		OR	
Q. 08	a	Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}}(x^2 + y^2)dxdy$ by changing to polar coordinates.	06
	b	Derive the relation between beta and gamma function	07
	c	Using double integration find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$	07
		Module-5	
Q. 09	a	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	06
	b	Solve the system of equations by Jordan method $x + y + z = 10$ $2x - y + 3z = 19$ $x + 2y + 3z = 22$	07
	c	Using Rayleigh's power method find the dominant eigenvalue and the corresponding eigenvector of $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking $[1 \quad 0 \quad 0]^T$ as initial eigenvector [carry out 6 iterations]	07
		OR	
Q. 10	a	Find the rank of the matrix $\begin{bmatrix} 11 & 12 & 13 & 14 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	06
	b	For what values λ and μ the system of equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution and (iii) infinite number of solutions	07

	c	Solve the system of equations $$2x - 3y + 20z = 25$$ $$20x + y - 2z = 17$$ $$3x + 20y - z = -18$$ Using the Gauss-Seidel method, taking (0, 0, 0) as an initial approximate. (Carry out 4 iterations).	07

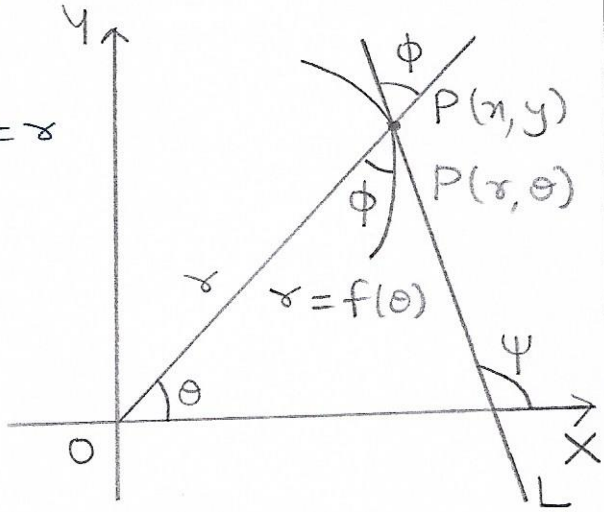
Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.2	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 03
Q.4	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 02
Q.5	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 01
Q.6	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 01
Q.7	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 01
Q.8	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 01
Q.9	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 01
	(c)	L3	CO 05	PO 01
Q.10	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 01
	(c)	L3	CO 05	PO 01

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (Knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

AY: 2022-23

Department: Mathematics **Model Question Paper - I**
Branch: EE/EC **Semester: I**
Subject with Sub. Code: Mathematics-I for Electrical and Electronics Engineering Stream (22MATE11)
Name of Faculty: Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
	Let $P(r,\theta)$ be any point on the curve $r=f(\theta)$. $\therefore X\hat{O}P=\theta$ and $OP=r$ 	
	Let PL be the tangent to the curve at P subtending an angle ψ with the positive direction of the initial line (x-axis) and ϕ be the angle between the radius vector OP and the tangent PL. That is $O\hat{P}L=\phi$	1M
	From the figure we have, $$\psi=\theta+\phi$$ $$\Rightarrow \tan\psi=\tan(\phi+\theta)$$	
	or $$\tan\psi=\frac{\tan\phi+\tan\theta}{1-\tan\phi\tan\theta} \longrightarrow (1)$$	1M
	Let (x,y) be the cartesian coordinates of P so that we have, $$x=r\cos\theta,\quad y=r\sin\theta$$	

Q.No.	Solution and Scheme	Marks
	Also, $\tan\psi = \dfrac{dy}{dx}$ = Slope of the tangent PL	
	i.e. $\tan\psi = \dfrac{dy/d\theta}{dx/d\theta}$ since x and y are functions of θ.	1 M
	$\tan\psi = \dfrac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$ where $r' = dr/d\theta$	
	Dividing both the numerator and denominator by $r'\cos\theta$ we have, $\tan\psi = \dfrac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$	1 M
	$= \dfrac{\frac{r}{r'} + \tan\theta}{1 - \frac{r}{r'}\cdot\tan\theta} \longrightarrow (2)$	1 M
	Comparing equations (1) & (2), $\tan\phi = \dfrac{r}{r'} = \dfrac{r}{dr/d\theta}$	
	$\boxed{\tan\phi = r\cdot\dfrac{d\theta}{dr}}$	1 M
		6 M

Q.No.	Solution and Scheme	Marks
1 b.	$r = a(1+\cos\theta)$	
	$\log r = \log a + \log(1+\cos\theta)$	
	Differentiating with respect to 'θ',	1 M
	$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{-\sin\theta}{1+\cos\theta}$	
	$\cot\phi_1 = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$	1 M
	$\cot\phi_1 = -\tan(\theta/2) = \cot(\pi/2+\theta/2)$	1 M
	$\Rightarrow \phi_1 = \pi/2 + \theta/2$	1 M
	$r = b(1-\cos\theta)$	
	$\log r = \log b + \log(1-\cos\theta)$	
	$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1-\cos\theta}$	
	$\cot\phi_2 = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$	1 M
	i.e. $\cot\phi_2 = \cot(\theta/2) \Rightarrow \phi_2 = \theta/2$	1 M
	$\therefore$ angle of intersection $= \lvert\phi_1-\phi_2\rvert$ $= \left\lvert \frac{\pi}{2}+\frac{\theta}{2}-\frac{\theta}{2}\right\rvert = \pi/2$	
	$\therefore \lvert\phi_1-\phi_2\rvert = \pi/2$	1 M
		7 M

Q.No.	Solution and Scheme	Marks

1 c.

$x = a(\theta + \sin\theta)$

Differentiating wrt 'θ',

$$\frac{dx}{d\theta} = a(1+\cos\theta)$$

$$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

1 M

$$= \frac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$$

1 M

$$\therefore y_1 = \tan(\theta/2) \longrightarrow (1)$$

$y = a(1-\cos\theta)$

Differentiating wrt 'θ',

$$\frac{dy}{d\theta} = a\sin\theta$$

1 M

$$\text{Also, } y_2 = \sec^2(\theta/2)\cdot\frac{1}{2}\cdot\frac{d\theta}{dx}$$

(Differentiating eqn (1) wrt 'x')

$$= \sec^2(\theta/2)\cdot\frac{1}{2}\cdot\frac{1}{a(1+\cos\theta)}$$

$$= \frac{\sec^2(\theta/2)}{4a\cos^2(\theta/2)}$$

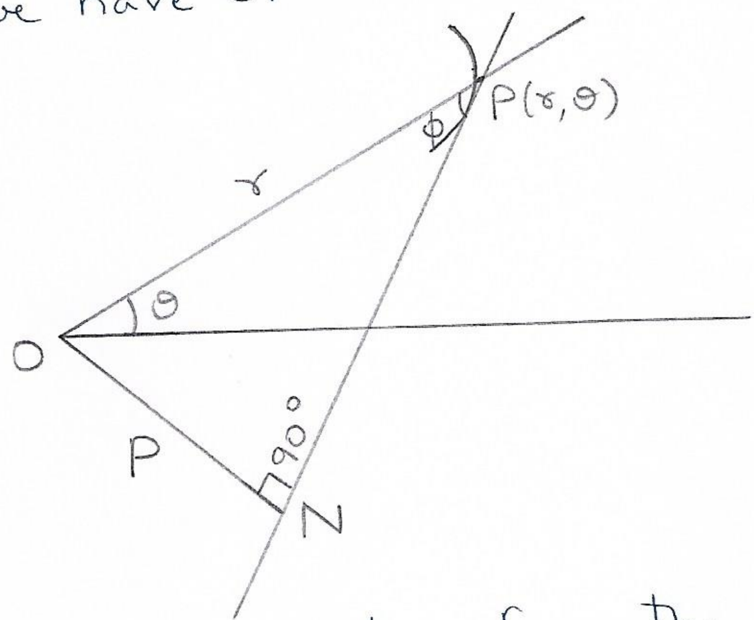
1 M

$$\therefore y_2 = \frac{1}{4a}\sec^4(\theta/2)$$

1 M

We have, $$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{[1+\tan^2(\theta/2)]^{3/2}\cdot 4a}{\sec^4(\theta/2)}$$

Q.No.	Solution and Scheme	Marks
	$\rho = \dfrac{\left[\sec^2(\theta/2)\right]^{3/2} \cdot 4a}{\sec^4(\theta/2)}$	
	$= \dfrac{4a\sec^3(\theta/2)}{\sec^4(\theta/2)} = \dfrac{4a}{\sec(\theta/2)}$	1M
	$\boxed{\therefore \rho = 4a\cos(\theta/2)}$	1M
		7M
Q 02 a	Let 'O' be the pole and OL be the initial line. Let, $P(r,\theta)$ be any point on the curve and hence we have $OP = r$ and $L\hat{O}P = \theta$.	1M
		
	Draw $ON = p$, perpendicular from the pole onto the tangent at P and let ϕ be the angle made by the radius vector with the tangent.	1M
	From the figure $O\hat{N}P = 90°$ and $L\hat{O}P = \theta$ From the right angled triangle ONP, $\sin\phi = \dfrac{ON}{OP} = \dfrac{p}{r}$	

Q.No.	Solution and Scheme	Marks
	or $p = r \sin\phi \longrightarrow (1)$	
	and $\cot\phi = \frac{1}{r}\frac{dr}{d\theta} \longrightarrow (2)$	1M
	squaring equation (1) and taking the reciprocal we get,	
	$\frac{1}{p^2} = \frac{1}{r^2} \cdot \frac{1}{\sin^2\phi}$ or $\frac{1}{p^2} = \frac{1}{r^2}\operatorname{cosec}^2\phi$	1M
	ie. $\frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$	1M
	Now, using (2) we get,	
	$\frac{1}{p^2} = \frac{1}{r^2}\left[1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right]$	
	$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2}$	1M
		6M
2b.	$r^m = a^m(\cos m\theta + \sin m\theta)$	
	$m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$	
	Differentiating wrt 'θ',	
	$\frac{m}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta + \sin\theta}(-m\sin m\theta + m\cos m\theta)$	1M
	$\cot\phi = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$	
	$= \frac{\cos m\theta(1-\tan m\theta)}{\cos m\theta(1+\tan m\theta)}$	1M
	$\cot\phi = \cot(\pi/4 + m\theta)$	1M

Q.No.	Solution and Scheme	Marks
	$\Rightarrow \phi = \pi/4 + m\theta$	
	Consider, $p = r \sin\phi = r\sin(\pi/4 + m\theta)$	
	$p = r\left[\sin(\pi/4)\cos m\theta + \cos(\pi/4)\sin m\theta\right]$	1M
	$p = \frac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta)$ $\quad \sin \pi/4 = \frac{1}{\sqrt{2}}$, $\cos \pi/4 = \frac{1}{\sqrt{2}}$	
	Now, we have	
	$r^m = a^m(\cos m\theta + \sin m\theta) \longrightarrow (1)$	
	$p = \frac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta) \longrightarrow (2)$	1M
	Using (2) in (1), we get	1M
	$r^m = a^m \cdot \frac{p\sqrt{2}}{r}$	
	$\boxed{r^{m+1} = \sqrt{2}\, a^m p}$, is the required pedal equation.	1M
		7M
c.	$x^3 + y^3 = 3xy$	
	Differentiating wrt 'x',	
	$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 3\left(x\frac{dy}{dx} + y\right)$	
	$3(y^2 - x)\frac{dy}{dx} = 3(y - x^2)$	
	$y_1 = \frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$	2M

Q.No.	Solution and Scheme	Marks
	At $(3/2, 3/2)$, $y_1 = \dfrac{(3/2) - (3/2)^2}{(3/2)^2 - (3/2)}$	1M
	$y_1 = -1$	
	Next, differentiating eqn ①,	
	$y_2 = \dfrac{(y^2-x)(y_1-2x) - (y-x^2)(2yy_1-1)}{(y^2-x)^2}$	
	$= \dfrac{y^2y_1 - 2xy^2 - xy_1 + 2x^2 - 2y^2y_1 + y + 2x^2yy_1 - x^2}{(y^2-x)^2}$	
	$y_2 = \dfrac{x^2 + y - y^2y_1 - 2xy^2 + 2x^2yy_1 - xy_1}{(y^2-x)^2}$	2M
	$y_2 = -\dfrac{32}{3}$ at $(3/2, 3/2)$	
	$\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2} = \dfrac{[1+(-1)^2]^{3/2}}{-32/3}$	1M
	$\rho = 2\sqrt{2} \times \dfrac{3}{-32} = -\dfrac{3\sqrt{2}}{16}$	
	$\boxed{\lvert\rho\rvert = \dfrac{3}{8\sqrt{2}}}$	1M
		7M

Q.No.	Solution and Scheme	Marks
Q. 3	a. $y(x) = y(0) + x\,y_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \frac{x^4}{4!}y_4(0) + \text{-----} \longrightarrow (\mathrm{I})$	
	$y(x) = e^{\sin x} \Rightarrow y(0) = e^0 = 1$	1 M
	$y_1 = \cos x\, e^{\sin x}$ $y_1 = y\cos x \;;\; y_1(0) = 1$	
	$y_2 = y(-\sin x) + y_1\cos x$ $y_2 = y_1\cos x - y\sin x$ $\therefore y_2(0) = 1$	1 M
	$y_3 = y_1(-\sin x) + y_2\cos x - y\cos x - y_1\sin x$ $y_3 = -y_1\sin x - y_1\sin x - y\cos x + y_2\cos x$ $y_3 = -2y_1\sin x - y\cos x + y_2\cos x$ $y_3(0) = 0 - 1 + 1 = 0$	1 M
	$y_4(0) = -2y_1\cos x - 2y_2\sin x - y_1\cos x + y\sin x + y_3\cos x - y_2\sin x$ $y_4(0) = -3$	1 M
	Substituting these values in expansion (I), $y(x) = 1 + x(1) + \frac{x^2}{2}\times(1) + \frac{x^3}{6}\times 0 - \frac{x^4}{24}\times(-3)$	1 M
	$y(x) = 1 + x + \frac{x^2}{2} + \frac{x^4}{8}$	1 M
		6 M

Q.No.	Solution and Scheme	Marks
3b.	$u = f(2x-3y,\ 3y-4z,\ 4z-2x)$	
	Let, $u = f(p, q, r)$ where $p = 2x-3y,\ q = 3y-4z,\ r = 4z-2x$	1M
	Using Chain rule, $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x}$	1M
	$= 2\frac{\partial u}{\partial p} - 2\frac{\partial u}{\partial r}$	1M
	$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial \cancel{x}y} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial y} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x}$ $= -3\frac{\partial u}{\partial p} + 3\frac{\partial u}{\partial q}$	1M
	$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial z} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial z} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial z}$ $= -4 \cdot \frac{\partial u}{\partial q} + 4\frac{\partial u}{\partial r}$	1M
	Now, consider, $6u_x + 4u_y + 3u_z = 6\left(2\frac{\partial u}{\partial p} - 2\frac{\partial u}{\partial r}\right) + 4\left(-3\frac{\partial u}{\partial p} + 3\frac{\partial u}{\partial q}\right) + 3\left(-4\frac{\partial u}{\partial q} + 4\frac{\partial u}{\partial r}\right)$ $= 12\frac{\partial u}{\partial p} - 12\frac{\partial u}{\partial r} - 12\frac{\partial u}{\partial p} + 12\frac{\partial u}{\partial q} - 12\frac{\partial u}{\partial q} + 12\frac{\partial u}{\partial r}$	1M
	$\therefore 6u_x + 4u_y + 3u_z = 0$	1M
		7M

Q.No.	Solution and Scheme	Marks
3c.	$f(x,y) = x^3+y^3-3xy+1$	
	$f_x = 3x^2-3y \; ; \; f_y = 3y^2-3x$	
	Let, $A = f_{xx} = 6x \; ; \; B = f_{xy} = -3$	2M
	$C = f_{yy} = 6y$	
	At $(1,1)$: $f_x = 0$ & $f_y = 0$	1M
	Also, $A = 6$, $B = -3$, $C = 6$	1M
	$\therefore AC - B^2 = 27 > 0$	
	Now, at $(1,1)$, $f_x = 0$ & $f_y = 0$	
	$AC - B^2 > 0$ & $A = 6 > 0$.	1M
	Hence, $f(x,y)$ at $(1,1)$ satisfy the necessary and sufficient conditions for minimum.	1M
	Thus, $f(x,y)$ is minimum at $(1,1)$.	
		7M
Q 04. a.	(i) $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ ---→ 1^∞ form	
	$\log k = \lim_{x\to 0} \frac{\log(\tan x/x)}{x^2}$ ⟶ $(0/0)$ form	1M
	$= \lim_{x\to 0} \frac{\left[\frac{x}{\tan x}\left(\frac{x\sec^2 x - \tan x}{x^2}\right)\right]}{2x}$	1M
	$= \lim_{x\to 0} \frac{x}{\tan x} \cdot \lim_{x\to 0} \left(\frac{x\sec^2 x - \tan x}{2x^3}\right)$	1M

Q.No.	Solution and Scheme	Marks
	$= 1 \cdot \lim\limits_{x \to 0} \dfrac{\sec^2 x \tan x}{3x^3}$	
	$= \lim\limits_{x \to 0} \dfrac{\tan x}{x} \cdot \lim\limits_{x \to 0} \dfrac{\sec^2 x}{3}$	
	$= \dfrac{\sec^2 0}{3} = \dfrac{1}{3}$	1M
	$\boxed{k = e^{1/3}}$	
	(ii) $k = \lim\limits_{x \to 0} (\cos x)^{1/x^2} \dashrightarrow 1^{\infty}$ form	
	$\log k = \lim\limits_{x \to 0} \dfrac{1}{x^2} \log(\cos x) \longrightarrow \infty \times 0$	
	$= \lim\limits_{x \to 0} \dfrac{\log \cos x}{x^2} \to \dfrac{0}{0}$ form	1M
	By L'Hospital's rule	
	$= \lim\limits_{x \to 0} \dfrac{\left(\dfrac{\sin x}{\cos x}\right)}{2x} = \dfrac{1}{2} \lim\limits_{x \to 0} \dfrac{\tan x}{x}$	1M
	$\log_e k = \dfrac{1}{2} \cdot 1$	1M
	$\Rightarrow k = e^{1/2} = \sqrt{e}$	7M
b.	$u = \tan^{-1}(y/x) \longrightarrow (1)$	
	Differentiating (1) partially with respect to x and y,	
	$\dfrac{\partial u}{\partial x} = \dfrac{1}{1+(y/x)^2} \cdot \left(\dfrac{-y}{x^2}\right) = -\dfrac{y}{x^2+y^2}$	1M

Q.No.	Solution and Scheme	Marks
	$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right)$	
	$= (-1)y \frac{\partial}{\partial x}(x^2+y^2)^{-1}$	
	$= y(x^2+y^2)^{-2} \times \frac{\partial}{\partial x}(x^2+y^2)$	
	$= \frac{y}{(x^2+y^2)^2} \times 2x = \frac{2xy}{(x^2+y^2)^2}$	2M
	$\therefore \frac{\partial^2 u}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2} \longrightarrow (1)$	1M
	$\frac{\partial u}{\partial y} = \frac{1}{1+(y/x)^2} \times \frac{\partial}{\partial y}(y/x)$	
	$= \frac{x^2}{x^2+y^2} \times \frac{1}{x} = \frac{x}{x^2+y^2}$	1M
	$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = x\frac{\partial}{\partial y}\left[(x^2+y^2)^{-1}\right]$	
	$= x(-1)(x^2+y^2)^{-2} \times 2y$	
	$\frac{\partial^2 u}{\partial y^2} = \frac{-2xy}{(x^2+y^2)^2} \longrightarrow (2)$	1M
	Adding equations (1) & (2),	
	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	1M
		7M

Q.No.	Solution and Scheme	Marks
4c.	$x+y+z=u$, $y+z=uv$, $z=uvw$	
	Here we need to express x, y, z in terms of u, v, w.	
	$x = u-(y+z) = u-uv$ $y = uv - z = uv - uvw$ $z = uvw$	2M
	$\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$	1M
	$= \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$	1M
	$= (1-v)\left[(u-uw)uv + uv(uw)\right] + u\left[(v-vw)uv + uv(vw)\right]$ $= (1-v)\left[u^2v - u^2vw + u^2vw\right] + u\left[uv^2 - uv^2w + uv^2w\right]$	2M
	$= (1-v)\left[u^2v\right] + u\left[uv^2\right]$ $= u^2v - u^2v^2 + u^2v^2$	
	$\boxed{\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v}$	1M
		7M

Q.No.	Solution and Scheme	Marks
	Module - 3	
Q.05 a	$x \frac{dy}{dx} + y = x^3 y^6$	
	Dividing by xy^6	
	$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{xy^5} = x^2$	
	put, $\frac{1}{y^5} = t$	1 M
	$-5\frac{1}{y^6}\frac{dy}{dx} = dt/dx$	
	$-\frac{1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2$	
	$\frac{dt}{dx} - \frac{5}{x}t = -5x^2$, which is linear in 't',	1 M
	$P = -\frac{5}{x}$, $Q = -5x^2$	
	Now, $e^{\int P dx} = e^{-\int 5/x \, dx} = e^{-5\log x} = \frac{1}{x^5}$	1 M
	The general solution is,	
	$t e^{\int P dx} = \int \left(Q e^{\int P dx}\right) dx + c$	1 M
	$\frac{t}{x^5} = \int \left(-5x^2 \times \frac{1}{x^5}\right) dx + c$	1 M
	$= -5 \int \frac{1}{x^3} dx + c$	
	$\boxed{\frac{1}{x^5 y^5} = \frac{5}{2} \times \frac{1}{x^2} + c}$	1 M
		6 M

Q.No.	Solution and Scheme	Marks
5 b.	$\frac{x^2}{a^2} + \frac{y^2}{b^2+\alpha} = 1 \longrightarrow (1)$	
	Differentiating wrt 'x',	
	$\frac{2x}{a^2} + \frac{2yy_1}{b^2+\alpha} = 0$, where $y_1 = dy/dx$	
	$\frac{x}{a^2} = \frac{-yy_1}{b^2+\alpha} \longrightarrow (2)$	1M
	Also, from eqn (1) $\frac{x^2}{a^2} - 1 = -\frac{y^2}{b^2+\alpha}$	
	or $\frac{x^2-a^2}{a^2} = -\frac{y^2}{b^2+\alpha} \longrightarrow (3)$	1M
	Now, dividing (2) by (3) we get	
	$\frac{x}{x^2-a^2} = \frac{yy_1}{y^2}$ or $\frac{x}{x^2-a^2} = \frac{y_1}{y}$	1M
	Replace $y_1 = \frac{dy}{dx}$ by $-dx/dy$	
	$\therefore \frac{x}{x^2-a^2} = \frac{1}{y}\left(-\frac{dx}{dy}\right)$	1M
	$y\,dy = -\frac{(x^2-a^2)}{x}\,dx$, by separating variables	1M
	$\Rightarrow \int y\,dy = -\int x\,dx + a^2\int \frac{dx}{x} + c$	1M
	$\frac{y^2}{2} = -\frac{x^2}{2} + a^2 \log x + c$	1M
	Thus, $x^2+y^2-2a^2\log x - b = 0$, where $b = 2c$, is the required orthogonal trajectory.	7M

Q.No.	Solution and Scheme	Marks
5c.	$xyp^2 - (x^2+y^2)p + xy = 0$	
	$p = \dfrac{(x^2+y^2) \pm \sqrt{(x^2+y^2)^2 - 4x^2y^2}}{2xy}$	1M
	[using $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ formula]	
	$= \dfrac{(x^2+y^2) \pm \sqrt{x^4+y^4+2x^2y^2-4x^2y^2}}{2xy}$	1M
	$= \dfrac{(x^2+y^2) \pm \sqrt{(x^2-y^2)^2}}{2xy}$	
	$= \dfrac{(x^2+y^2) \pm (x^2-y^2)}{2xy}$	1M
	$p = \dfrac{x}{y}$ or $p = y/x$	1M
	$\dfrac{dy}{dx} = \dfrac{x}{y}$ or $\dfrac{dy}{dx} = y/x$	
	$y\,dy = x\,dx$; $\dfrac{dy}{y} = \dfrac{dx}{x}$	
	Integrating ; $\log y - \log x = \log c$	
	$\dfrac{y^2}{2} = \dfrac{x^2}{2} + c_1$; $\log(y/x) = \log c$	2M
	or $y^2 - x^2 - c = 0$; $y/x = c$	
	or $y = cx$	
	Thus, the general solution is,	
	$(y^2-x^2-c)(y-cx) = 0$	1M
		7M

Q.No.	Solution and Scheme	Marks
6a.	$(x^2+y^2+x)\,dx + xy\,dy = 0$	
	Let, $M = x^2+y^2+x$, $N = xy$	
	$\frac{\partial M}{\partial y} = 2y$ $\quad \frac{\partial N}{\partial x} = y$	
	$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y$ ----near to N	1M
	$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{y}{xy} = \frac{1}{x} = f(x)$	1M
	Hence, Integrating factor $= e^{\int f(x)\,dx}$	
	$= e^{\int 1/x\,dx} = e^{\log x} = x$	1M
	Multiplying the given equation by x,	
	$M = x^3 + xy^2 + x^2$, $N = x^2 y$	1M
	$\frac{\partial M}{\partial y} = 2xy$, $\frac{\partial N}{\partial x} = 2xy$	
	The solution is $\int M\,dx + \int N\,dy = c$ (terms in N not containing x)	1M
	$\int (x^3+xy^2+x^2)\,dx + \int 0\,dy = c$	
	$\boxed{\frac{x^4}{4} + \frac{x^2}{2}y^2 + \frac{x^3}{3} = c,}$ is the required solution	1M
		6M

Q.No.	Solution and Scheme	Marks
b.	$L\frac{di}{dt} + Ri = E\sin\omega t$	
	Applying the solution for the linear differential equation $\frac{dy}{dx} + Py = Q$,	
	$i e^{Rt/L} = \int \frac{E}{L} \sin\omega t \, e^{Rt/L} dt + c$	
	We use the following result to find the integral in RHS.	
	$\int e^{at} \sin bt \, dt = \frac{e^{at}}{\sqrt{a^2+b^2}} \sin[bt - \tan^{-1}(b/a)]$	1 M
	Here, $a = R/L$ and $b = \omega$	
	$\therefore i e^{Rt/L} = \frac{E}{L} \frac{e^{Rt/L}}{\sqrt{(R/L)^2+\omega^2}} [\sin\omega t - \tan^{-1}(\omega L/R)] + c$	2 M
	Denoting $\tan^{-1}(\omega L/R) = \phi$ or $\tan\phi = \omega L/R$	
	$i = \frac{E}{L} \frac{L}{\sqrt{R^2+\omega^2 L^2}} \sin(\omega t - \phi) + c e^{-Rt/L}$ $\to (1)$	1 M
	Using the initial conditions, $i = 0$ when $t = 0$ in eqn (1) we have	
	$0 = \frac{E\sin(-\phi)}{\sqrt{R^2+\omega^2 L^2}} + c$	1 M

Q.No.	Solution and Scheme	Marks
	$c = \dfrac{E\sin\phi}{\sqrt{R^2+\omega^2L^2}}$, since $\sin(-\phi) = -\sin\phi$	
	Substituting this value of c in eqn (1) to get the value of current at any time t. $i = \dfrac{E\sin(\omega t-\phi)}{\sqrt{R^2+\omega^2L^2}} + \dfrac{E\sin\phi}{\sqrt{R^2+\omega^2L^2}} e^{-Rt/L}$	1M
	Thus, $i = \dfrac{E}{\sqrt{R^2+\omega^2L^2}}\left\{\sin(\omega t-\phi)+e^{-Rt/L}\sin\phi\right\}$	1M
	where $\phi = \tan^{-1}(\omega L/R)$	7M
6c.	$(px-y)(py+x) = a^2p$,	
	$X = x^2 \Rightarrow \dfrac{dX}{dx} = 2x$	1M
	$Y = y^2 \Rightarrow \dfrac{dY}{dy} = 2y$	
	Now, $p = \dfrac{dy}{dx} = \dfrac{dy}{dY}\cdot\dfrac{dY}{dX}\cdot\dfrac{dX}{dx}$ and let	
	$P = \dfrac{dY}{dX}$ (capital P)	
	$p = \dfrac{1}{2y}\cdot P\cdot 2x$ or $p = \dfrac{x}{y}P$	1M

Q.No.	Solution and Scheme	Marks
	$p = \frac{\sqrt{X}}{\sqrt{Y}} P$	
	Consider, $(px - y)(py + x) = a^2 p$	
	$\left[\frac{\sqrt{X}}{\sqrt{Y}} P\sqrt{X} - \sqrt{Y}\right]\left[\frac{\sqrt{X}}{\sqrt{Y}} P\sqrt{Y} + \sqrt{X}\right] = a^2 \frac{\sqrt{X}}{\sqrt{Y}} P$	2 M
	$\frac{PX - Y}{\sqrt{Y}} (P+1)\sqrt{X} = a^2 \frac{\sqrt{X}}{\sqrt{Y}} P$	
	$(PX - Y)(P+1) = a^2 P$	
	or $PX - Y = \frac{a^2 P}{P+1}$	
	$Y = PX - \frac{a^2 P}{P+1}$	1 M
	This is in the Clairaut's form and hence the general solution is	
	$Y = cX - \frac{a^2 c}{c+1}$	1 M
	$\boxed{y^2 = cx^2 - \frac{a^2 c}{c+1}}$, is the required general solution of the given equation.	1 M
		7 M

Q.No.	Solution and Scheme	Marks

Module - 4

Q 07

a. Let, $I = \displaystyle\int_{x=-c}^{c}\int_{y=-b}^{b}\int_{z=-a}^{a} (x^2+y^2+z^2)\, dz\, dy\, dx$

Integrating wrt 'z',

$$I = \int_{x=-c}^{c}\int_{y=-b}^{b} \left(x^2 z + y^2 z + \frac{z^3}{3}\right)_{z=-a}^{a} dy\, dx$$ 1 M

$$= \int_{x=-c}^{c}\int_{y=-b}^{b} \left[x^2(a+a) + y^2(a+a) + \left(\frac{a^3}{3}+\frac{a^3}{3}\right)\right] dy\, dx$$

$$= \int_{x=-c}^{c} \left[2ax^2 y + 2a\left(y^3/3\right) + \frac{2a^3}{3}(y)\right]_{y=-b}^{b} dx$$ 1 M

(integrating wrt 'y')

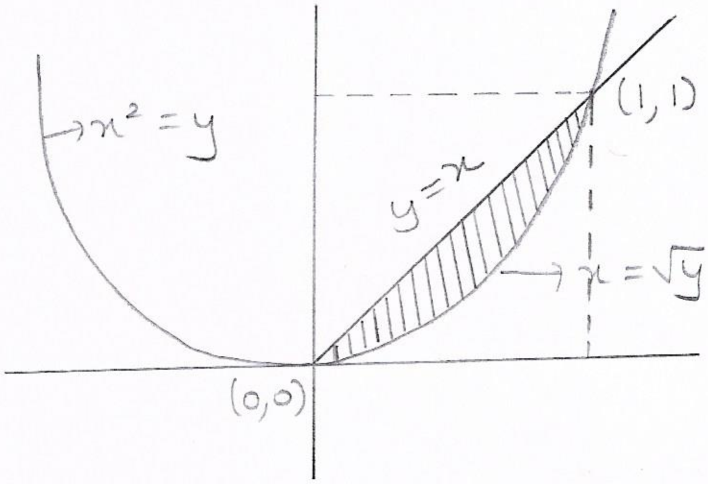
$$= \int_{x=-c}^{c} \left[2ax^2(b+b) + \frac{2a}{3}(b^3+b^3) + \frac{2a^3}{3}(b+b)\right] dx$$

$$= \int_{x=-c}^{c} \left[4ax^2 b + \frac{4ab^3}{3} + \frac{4a^3 b}{3}\right] dx$$ 1 M

Integrating wrt 'x',

Q.No.	Solution and Scheme	Marks
	$I = \left[4a\left(\frac{x^3}{3}\right)b + \frac{4ab^3}{3}(x) + \frac{4a^3b}{3}(x) \right]_{x=-c}^{c}$	1M
	$I = \frac{4ab}{3}(c^3+c^3) + \frac{4ab^3}{3}(c+c) + \frac{4a^3b}{3}(c+c)$	1M
	$I = \frac{4ab}{3} \times 2c^3 + \frac{4ab^3}{3}(2c) + \frac{4a^3b}{3}(2c)$	
	$I = \frac{8ac^3b}{3} + \frac{8ab^3c}{3} + \frac{8a^3bc}{3}$	
	$I = \frac{8abc}{3}(a^2+b^2+c^2)$	1M 6M
b.	Let $I = \int_0^1 \int_{\sqrt{y}}^{y} dx\,dy$	
	Here, y varies from 0 to 1, and for each y, x varies from $x=\sqrt{y}$ to $x=y$. Thus, the lower value of x lies on the curve $x^2=y$ (which is a parabola) and the upper value of x lies on the curve $x=y$ (which is a straight line passing through the origin, subtending an angle of $45°$).	
	The region R of integration is as shown in the figure.	

Q.No.	Solution and Scheme	Marks



Since, $x = \sqrt{y}$ and $x = y$

$\sqrt{y} = y \Rightarrow y = y^2$ (on squaring)

$y^2 - y = 0 \Rightarrow y(y-1) = 0 \Rightarrow y = 1, 0$

Hence, the points of intersection of the curves are $(0,0)$ & $(1,1)$.

On changing the order of integration,

$$I = \int_{x=0}^{1} \int_{y=x^2}^{x} dy\, dx$$

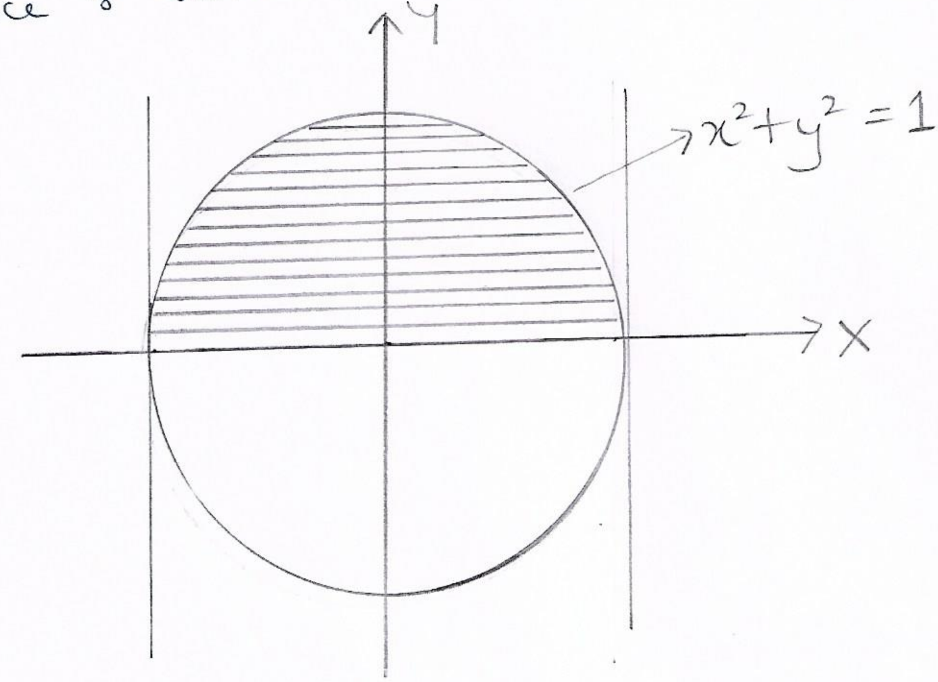
$$= \int_{x=0}^{1} [y]_{y=x^2}^{x} dx$$

$$= \int_{x=0}^{1} (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{x=0}^{1}$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

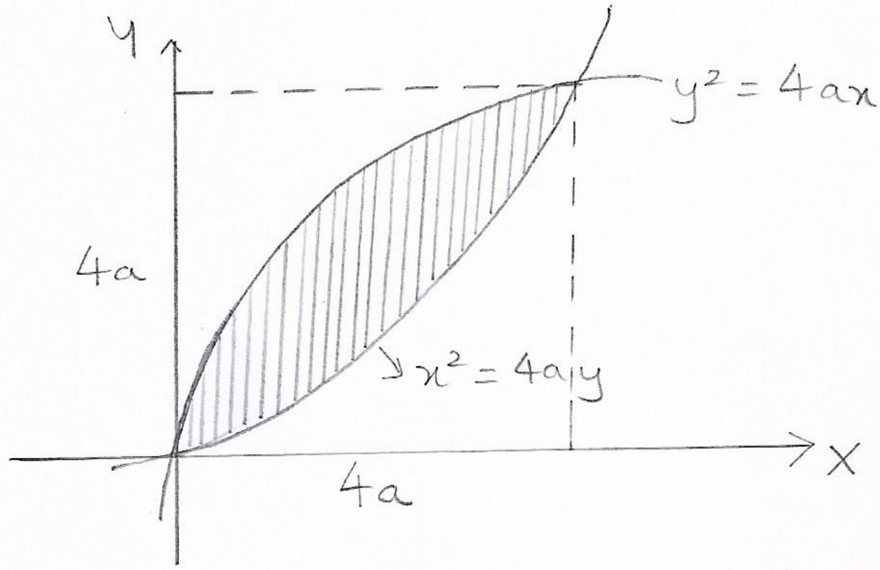
$$\boxed{\therefore I = \frac{1}{6}}$$

Q.No.	Solution and Scheme	Marks
7c.	Let, $I_1 = \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \int_0^{\pi/2} (\sin\theta)^{1/2}\, d\theta$	
	$= \int_0^{\pi/2} \sin^{1/2}\theta \cos^0\theta\, d\theta$	
	Using, $\int_0^{\pi/2} \sin^p\theta \cos^q\theta\, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) \rightarrow (I)$	2M
	$I_1 = \frac{1}{2}\beta\left(3/4, 1/2\right)$	1M
	Let, $I_2 = \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \int_0^{\pi/2} \sin^{-1/2}\theta\, d\theta$	
	$= \int_0^{\pi/2} \sin^{-1/2}\theta \cos^0\theta\, d\theta$	
	Using result (I), we get	1M
	$I_2 = \frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$	
	$\therefore I_1 \times I_2 = \frac{1}{2}\beta\left(3/4, 1/2\right) \cdot \frac{1}{2}\beta\left(1/4, 1/2\right)$	1M
	$= \frac{1}{4} \frac{\Gamma(1/4)\,\Gamma(1/2)}{\Gamma(3/4)} \cdot \frac{\Gamma(3/4) \cdot \Gamma(1/2)}{\Gamma(5/4)}$	1M
	$= \frac{1}{4} \Gamma(1/4) \sqrt{\pi} \cdot \frac{\sqrt{\pi}}{1/4\, \Gamma(1/4)} = \pi$	
	$\boxed{\therefore I_1 \times I_2 = \pi}$	1M
		7M

Q.No.	Solution and Scheme	Marks
Q.08 a.	Let, $I = \int_{y=0}^{1}\int_{x=0}^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$	
	Here, $x=\sqrt{1-y^2}$ or $x^2+y^2=1$ is a circle with centre origin and radius 1.	
	Since, y varies from 0 to 1, the region of integration is the first quadrant of the circle.	
	In polars, we have $x=r\cos\theta$, $y=r\sin\theta$	2M
	$\therefore x^2+y^2=r^2$	
	i.e. $r^2=1^2 \Rightarrow r=1$	
	Also, $x=0$, $y=0$ will give $r=0$ and hence r varies from 0 to 1.	
		
	In, the first quadrant θ varies from 0 to $\pi/2$.	

Q.No.	Solution and Scheme	Marks
	We know that, $dx\,dy = r\,dr\,d\theta$	
	$$I = \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r.r\,dr\,d\theta$$	1M
	$$= \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r^2\,dr\,d\theta$$	
	$$= \int_{r=0}^{1} r^2[\theta]_{\theta=0}^{\pi/2}\,dr$$ [Integrating wrt θ]	1M
	$$= \int_{r=0}^{1} r^2\frac{\pi}{2}\,dr$$	1M
	Integrating wrt 'r',	
	$$I = \frac{\pi}{2}\left[\frac{r^3}{3}\right]_{r=0}^{1} = \frac{\pi}{2}\times\frac{1}{3}[1-0]$$	
	$$I = \frac{\pi}{6}$$	1M
		6M
8b.	By the definition of Beta & Gamma Functions	
	$$\beta(m,n) = 2\int_{\theta=0}^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\,d\theta \rightarrow (1)$$	1M
	$$\Gamma n = 2\int_{0}^{\infty} e^{-x^2}x^{2n-1}\,dx \longrightarrow (2)$$	1M

Q.No.	Solution and Scheme	Marks
	$\Gamma m = 2\int_0^\infty e^{-y^2}\, y^{2m-1}\, dy \longrightarrow (3)$	
	$\Gamma m+n = 2\int_0^\infty e^{-r^2}\, r^{2(m+n)-1}\, dr \longrightarrow (4)$	
	Now, $\Gamma m . \Gamma n = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, x^{2n-1}\, y^{2m-1}\, dx dy \to (5)$	1 M
	Evaluate RHS by changing into polars. Putting $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$ Also, $dx dy = r\, dr d\theta$. r varies from 0 to ∞, θ varies from 0 to $\pi/2$.	
	$\Gamma m . \Gamma n = 4\int_{r=0}^\infty\int_{\theta=0}^{\pi/2} e^{-r^2}(r\cos\theta)^{2n-1}(r\sin\theta)^{2m-1}\, r\, dr d\theta$	1 M
	$= 4\int_{r=0}^\infty\int_{\theta=0}^{\pi/2} e^{-r^2}\, r^{2m+2n-1}\sin^{2m-1}\theta\cos^{2n-1}\theta\, dr d\theta$	
	$= \left[2\int_{r=0}^\infty e^{-r^2}\, r^{2(m+n)-1}\, dr\right] \times \left[2\int_{\theta=0}^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta\right]$	1 M
	$\therefore \Gamma m . \Gamma n = \Gamma m+n . \beta(m,n)$ $\{$ by using (1) & (4),	1 M

Q.No.	Solution and Scheme	Marks
	Thus, $\boxed{\beta(m,n) = \dfrac{\Gamma m \cdot \Gamma n}{\Gamma m+n}}$	1M
		7M
8c.		
	The region between the given parabolas is shown in the above figure. In this region, x varies from 0 to 4a and for each x, y varies from a point on the parabola $x^2 = 4ay$ to a point on the parabola $y^2 = 4ax$; that is from $y = \dfrac{x^2}{4a}$ to $y = 2\sqrt{ax}$.	1M
	The required area is $A = \displaystyle\int_{x=0}^{4a} \int_{y=x^2/4a}^{2\sqrt{ax}} dy\, dx$	1M
	Integrating wrt 'y', $= \displaystyle\int_{0}^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$	1M

Q.No.	Solution and Scheme	Marks
	$= 2\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^{4a} - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{4a}$	1M
	$= \frac{4}{3}\sqrt{a}\left[(4a)^{3/2}\right] - \frac{1}{12a}\left[(4a)^3\right]$	1M
	$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2$	1M
	$\boxed{A = \frac{16}{3}a^2 \text{ sq. units}}$	1M
		7M
	<u>Module - 5</u>	
9a.	$A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	
	$R_1 \longleftrightarrow R_2$ $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 1 & -1 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	1M
	$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - 3R_1$ $A \sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -3 & -9 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -9 & -3 \end{bmatrix}$	2M

Q.No.	Solution and Scheme	Marks
	$A \sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -3 & -9 & -3 \\ 0 & -3 & -9 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \leftrightarrow R_4$	1M
	$R_3 \to R_3 - R_2$ $A \sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & -3 & -9 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	1M 1M
	$\therefore \rho(A) = 2$	6M
b	$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 19 \\ 22 \end{bmatrix}$	
	$[A:B] = \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 10 \\ 2 & -1 & 3 & 19 \\ 1 & 2 & 3 & 22 \end{array}\right]$	1M
	$[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 10 \\ 0 & -3 & 1 & -1 \\ 0 & 1 & 2 & 12 \end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$	2M
	$[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 10 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 7 & 35 \end{array}\right]$ $R_3 \to 3R_3 + R_2$	1M
	$[A:B] \sim \left[\begin{array}{ccc\|c} 3 & 0 & 4 & 29 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 7 & 35 \end{array}\right]$ $R_1 \to 3R_1 + R_2$	1M

Q.No.	Solution and Scheme	Marks
	$[A:B] \sim \left[\begin{array}{ccc|c} 21 & 0 & 0 & 63 \\ 0 & -21 & 0 & -42 \\ 0 & 0 & 7 & 35 \end{array}\right]$ $\begin{array}{l} R_1 \to 7R_1 - 4R_3 \\ R_2 \to 7R_2 - R_3 \end{array}$	
	$21x = 63 \Rightarrow x = 3$ $-21y = -42 \Rightarrow y = 2$ $7z = 35 \Rightarrow z = 5$	2 M
	$\therefore x = 3,\ y = 2,\ z = 5$	7M
9c.	Let, $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$, $X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	
	$AX^{(0)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.5 \\ -0.5 \end{bmatrix}$ $= \lambda^{(1)} X^{(1)}$	1M
	$AX^{(1)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -4 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix}$ $= \lambda^{(2)} X^{(2)}$	1M
	$AX^{(2)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 5.2 \\ -5.2 \end{bmatrix}$ $= 5.6 \begin{bmatrix} 1 \\ 0.928 \\ -0.928 \end{bmatrix} = \lambda^{(3)} X^{(3)}$	1M

Q.No.	Solution and Scheme	Marks
	$AX^{(3)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.928 \\ -0.928 \end{bmatrix} = \begin{bmatrix} 5.857 \\ 5.714 \\ -5.714 \end{bmatrix}$	
	$= 5.857 \begin{bmatrix} 1 \\ 0.97 \\ -0.97 \end{bmatrix} = \lambda^{(4)} X^{(4)}$	1M
	$AX^{(4)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.97 \\ -0.97 \end{bmatrix} = \begin{bmatrix} 5.95 \\ 5.90 \\ -5.90 \end{bmatrix}$	1M
	$= 5.95 \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} = \lambda^{(5)} X^{(5)}$	
	$AX^{(5)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} = \begin{bmatrix} 5.98 \\ 5.93 \\ -5.93 \end{bmatrix}$	1M
	$= 5.98 \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} = \lambda^{(6)} X^{(6)}$	
	Largest Eigenvalue is $\lambda = 5.98$	
	Largest Eigenvector is $X = [1 \ 0.99 \ -0.99]'$	1M
		7M

Q.No.	Solution and Scheme	Marks
10a.	$A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	
	$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ R_4 \to R_4 - R_1$	
	$A \sim \begin{bmatrix} 11 & 12 & 13 & 14 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$ R	2M
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 11 & 12 & 13 & 14 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$ $R_1 \leftrightarrow R_2$	1M
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 - 11R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 3R_1 \end{matrix}$	2M
	Rank $\underline{\underline{\rho(A)}} = 2$	1M
		6M
10b	$[A:B] = \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$	
	is the augmented matrix.	

Q.No.	Solution and Scheme	Marks
	$R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$	
	$[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$	1M
	$R_3 \to R_3 - R_2$	
	$[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$	1M
	(i) <u>No Solution</u> : We must have $\rho[A] \neq \rho[A:B]$ $\rho[A] = 3$ if $\lambda \neq 3$ and hence if $\lambda = 3$ we obtain $\rho[A] = 2$. If, we impose $(\mu - 10) \neq 0$, then $\rho[A:B]$ will be 3. Hence, the system has no solution if $\lambda = 3$ and $\mu \neq 10$.	1M
	(ii) <u>a unique solution</u>: We must have $\rho[A] = \rho[A:B] = 3$, $\rho[A]$ will be 3 if $(\lambda - 3) \neq 0$ since the other two entries in the last row of A are zero. If, $(\lambda - 3) \neq 0$ or $\lambda \neq 3$ irrespective of the value of μ, $\rho[A:B]$ will also be 3. Hence, the system will have unique solution if $\lambda \neq 3$.	2M

Q.No.	Solution and Scheme	Marks
	(iii) infinite solutions: $n=3$ and we need $\rho[A]=\rho[A:B]=r<3$. We must have $r=2$, since first row and second row are non-zero.	
	$\therefore \rho[A]=\rho[A:B]=2$ only when the last row of $[A:B]$ is completely zero. This is possible if $\lambda-3=0$, $\mu-10=0$	2M
	Hence, the system will have infinite solution if $\lambda=3$ and $\mu=10$.	7M
10c.	Given, $$2x-3y+20z=25$$ $$20x+y-2z=17$$ $$3x+20y-z=-18$$ Rearrange the equations, so that we obtain diagonally dominant system $$20x+y-2z=17$$ $$3x+20y-z=18$$ $$2x-3y+20z=25$$	
	$$x=\frac{1}{20}[17-y+2z]$$ $$y=\frac{1}{20}[18-3x+z]$$ $$z=\frac{1}{20}[25-2x+3y]$$	1M
	Considering, trial solution $x=0, y=0, z=0$	

Q.No.	Solution and Scheme	Marks
	First iteration: $x^{(1)} = \frac{17}{20} = 0.85$	
	$y^{(1)} = \frac{1}{20}[-18-3(0.85)] = -1.0275$	
	$z^{(1)} = \frac{1}{20}[25-2(0.85)+3(-1.0275)]$	
	$= 1.0109$	1 M
	$\therefore x = 0.85,\ y = -1.0275,\ z = 1.0109$	
	Second iteration:	
	$x^{(2)} = \frac{1}{20}[17-(-1.0275)+2(1.0109)]$	
	$= 1.0025$	
	$y^{(2)} = \frac{1}{20}[-18-3(1.0025)+1.0109]$	
	$\simeq -0.9998$	1 M
	$z^{(2)} = \frac{1}{20}[25-2(1.0025)+3(-0.9998)]$	
	$\simeq 0.9998$	
	$\therefore x = 1.0025,\ y = -0.9998,\ z = 0.9998$	
	Third iteration:	
	$x^{(3)} = \frac{1}{20}[17-(-0.9998)+2(0.9998)]$	
	$= 0.99997 \approx 1$	
	$y^{(3)} = \frac{1}{20}[-18-3(0.99997)+0.9998]$	
	≈ -1.0000055	

Q.No.	Solution and Scheme	Marks
	$z^{(3)} = \frac{1}{20}[25 - 2(0.99997) + 3(-1.0000055)]$	
	≈ 1.0000022	
	$\therefore x = 0.99997,\ y = -1.0000055$	1M
	$z = 1.0000022$	
	<u>Fourth iteration:</u>	
	$x^{(4)} = \frac{1}{20}[17 - (-1.0000055) + 2(1.0000022)]$	
	$\simeq 1.000000495 \simeq 1$	
	$y^{(4)} = \frac{1}{20}[-18 - 3(1.000000495) + 1.0000022]$	2M
	$\simeq -0.999999 \simeq -1$	
	$z^{(4)} = \frac{1}{20}[25 - 2(1.000000495) + 3(-0.999999)]$	
	$= 1.000000101 \approx 1$	1M
	$\therefore x \simeq 1,\ y \simeq -1,\ z = 1$	7M
	Faculty: Dr. Meenal Kaliwal (Mkf) 20/01/2023 Asst. Prof. & Head Dept. of Mathematics KLS, VDIT, Haliyal	